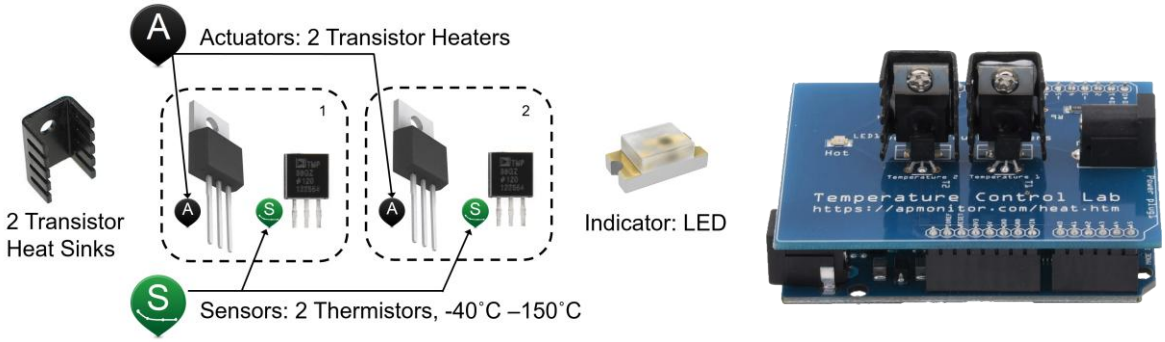
Worksheet: Dynamic Model Step Tests

Learning Objectives

- Formulate energy balance equations for two interacting heaters.
- Analyze and compare model results to experimental behavior.
- Reflect on system dynamics and modeling assumptions.



Estimated Time Allocation

Section	Task	Time
1	Model Understanding & Equations	10 min
2	Simulation & Measurement	45 min
3	Analysis & Reporting	5 min
Total		60 min

1. Model Understanding & Equations (10 min)

a. Initial Setup:
Note down the given parameters (all in SI units) and what each represents:

- Initial temperature (T_0): 296.15 K (23 °C)
- Ambient temperature ($T\infty$): same (296.15 K)
- Heater 1 output (Q_1): up to 1 W; factor α_1 = 0.01 W per % heater
- Heater 2 output (Q_2): up to 0.75 W; factor α_2 = 0.0075 W per % heater
- Heat capacity (C_p): 500 J/kg·K
- Mass (m): 0.004 kg
- Surface area outside heaters (A): 1.0×10^{-3} m^2
- Surface area between heaters (A_s): 2.0×10^{-4} m^2
- Heat transfer coefficient (U): 10 $W/m^2 \cdot K$

- Emissivity (ε): 0.9
- Stefan-Boltzmann constant (σ): 5.67×10^{-8} $W/m^2\cdot K^4$

b. Heat Transfer Terms:

Convective: $Q\_C12 = U * A\_s * (T_2 - T_1)$
Radiative: $Q\_R12 = \varepsilon * \sigma * A\_s * (T_2^4 - T_1^4)$

c. Energy Balance Equations:

For T_1: $m\ c_p\ dT_1/dt = U\ A\ (T\infty - T_1) + \varepsilon\ \sigma\ A\ (T\infty^4 - T_1^4) + Q\_C12 + Q\_R12 + \alpha_1\ Q_1$

For T_2: $m\ c_p\ dT_2/dt = U\ A\ (T\infty - T_2) + \varepsilon\ \sigma\ A\ (T\infty^4 - T_2^4) - Q\_C12 - Q\_R12 + \alpha_2\ Q_2$

2. Simulation & Analysis (45 min)

a. Simulate TCLab with separate step tests for Heater 1 (0-100%) and Heater 2 (0-100%): https://tclab.apopt.com

b. Observe Behavior:
- Did temperatures reach steady state?
- What is the Gain, Time Constant, and Dead Time for each response (4 total)?

	Gain (K/%)	Time Constant (sec)	Dead Time (sec)
Q1 effect on T1			
Q1 effect on T2			
Q2 effect on T1			
Q2 effect on T2			

c. Compare simulation with TCLab data: https://apmonitor.com/tclab/index.html
- Run same test with TCLab device.
- What is the Gain, Time Constant, and Dead Time for each response (4 total)?

	Gain (K/%)	Time Constant (sec)	Dead Time (sec)
Q1 effect on T1			
Q1 effect on T2			
Q2 effect on T1			
Q2 effect on T2			

d. Dynamics Discussion:
- Does system resemble first-order, second-order, or more complex behavior?

3. Reflection & Reporting (5 min)

Write a concise summary (2–3 sentences) addressing:
- Did the model capture main dynamics?

- Possible sources of discrepancies.

- Potential improvements (e.g., adjusting α values, including conduction or time delays).

Quick Checklist

[ ] Energy balance equations correctly written
[ ] Gain, Time Constant, and Dead Time calculated for 4 Simulated Responses
[ ] Gain, Time Constant, and Dead Time calculated for 4 Measured Responses
[ ] Screenshots of T_1 and T_2 generated and interpreted
[ ] Brief reflection on modeling accuracy and improvements

Additional Resources

See https://apmonitor.com/pdc/index.php/Main/ArduinoModeling2 for additional resources.