ME 575: Two-bar Truss

Name______________________

Consider the design of a simple tubular symmetric truss shown in Fig. 1.1 below (problem originally from Fox[1]). A *design* of the truss is specified by a unique set of values for the analysis variables: height (H), diameter, (d), thickness (t), separation distance (B), modulus of elasticity (E), and material density (ρ). Suppose we are interested in designing a truss that has a minimum weight, will not yield, will not buckle, and does not deflect "excessively," and so we decide our model should calculate weight, stress, buckling stress and deflection as analysis functions.

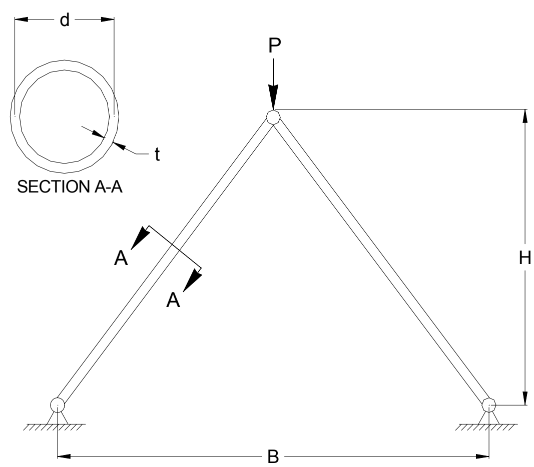

Fig. 1.1 - Layout for the Two-bar truss model.

In this case we can develop a model of the truss using explicit mathematical equations. These equations are:

$$Weight = \rho \cdot 2 \cdot \pi \cdot d \cdot t \sqrt{\left(\frac{B}{2}\right)^2 + H^2}$$

$$Stress = \frac{P \cdot \sqrt{\left(\frac{B}{2}\right)^2 + H^2}}{2 \cdot t \cdot \pi \cdot d \cdot H}$$

$$Buckling\ Stress = \frac{\pi^2 E(d^2 + t^2)}{8[\left(\frac{B}{2}\right)^2 + H^2]}$$

---

[1] R.L. Fox, *Optimization Methods in Engineering Design*, Addison Wesley, 1971

$$Deflection = \frac{P \cdot \left[\left(\frac{B}{2} \right)^2 + H^2 \right]^{(3/2)}}{2 \cdot t \cdot \pi \cdot d \cdot H^2 \cdot E}$$

The analysis variables and analysis functions for the truss are also summarized in Table 1.1. We note that the analysis variables represent all of the quantities on the right hand side of the equations give above. When all of these are given specific values, we can *evaluate* the model, which refers to calculating the functions.

Table 1.1 - Analysis variables and analysis functions for the Two-bar truss.

Analysis Variables	Analysis Functions
B, H, t, d, P, E, ρ	Weight, Stress, Buckling Stress, Deflection

An example design for the truss is given as,

Analysis Variables	Value
Height, H (in)	30.
Diameter, d (in)	3.
Thickness, t (in)	0.15
Separation distance, B (inches)	60.
Modulus of elasticity (1000 lbs/in^2)	30,000
Density, ρ (lbs/in^3)	0.3
Load (1000 lbs)	66
Analysis Functions	**Value**
Weight (lbs)	35.98
Stress (ksi)	33.01
Buckling stress (ksi)	185.5
Deflection (in)	0.066

We can obtain a new design for the truss by changing one or all of the analysis variable values. For example, if we change thickness from 0.15 in to 0.10 in., we find that weight has decreased, but stress and deflection have increased, as given below,

Analysis Variables	Value
Height, H (in)	30.
Diameter, d (in)	3.
Thickness, t (in)	0.1

Separation distance, B (inches)	60.
Modulus of elasticity (1000 lbs/in^2)	30,000
Density, ρ (lbs/in^3)	0.3
Load (1000 lbs)	66
Analysis Functions	**Value**
Weight (lbs)	23.99
Stress (psi)	49.52
Buckling stress (psi)	185.3
Deflection (in)	0.099

SPECIFYING AN OPTIMIZATION PROBLEM

Variables, Objectives, Constraints

Optimization problems are often specified using a particular form. That form is shown in Fig. 1.2. First the design variables are listed. Then the objectives are given, and finally the constraints are given. The abbreviation “s.t.” stands for “subject to.”

Find height and diameter to**:**

Minimize Weight

s. t.

Stress ≤ 100
(Stress-Buckling Stress) ≤ 0
Deflection ≤ 0.25

Fig. 1.2 Example specification of optimization problem for the Two-bar truss

Note that to define the buckling constraint, we have combined two analysis functions together. Thus we have *mapped* two analysis functions to become one design function.

We can specify several optimization problems using the same analysis model. For example, we can define a different optimization problem for the Two-bar truss to be:

Find thickness and diameter to:

Minimize Stress

s.t.

Weight ≤ 25.0
Deflection ≤ 0.25

Fig. 1.3 A second specification of an optimization problem for the Two-bar truss

The specifying of the optimization problem, i.e. the selection of the design variables and functions, is referred to as the *mapping between the analysis space and the design space*. For the problem defined in Fig. 1.2 above, the mapping looks like:

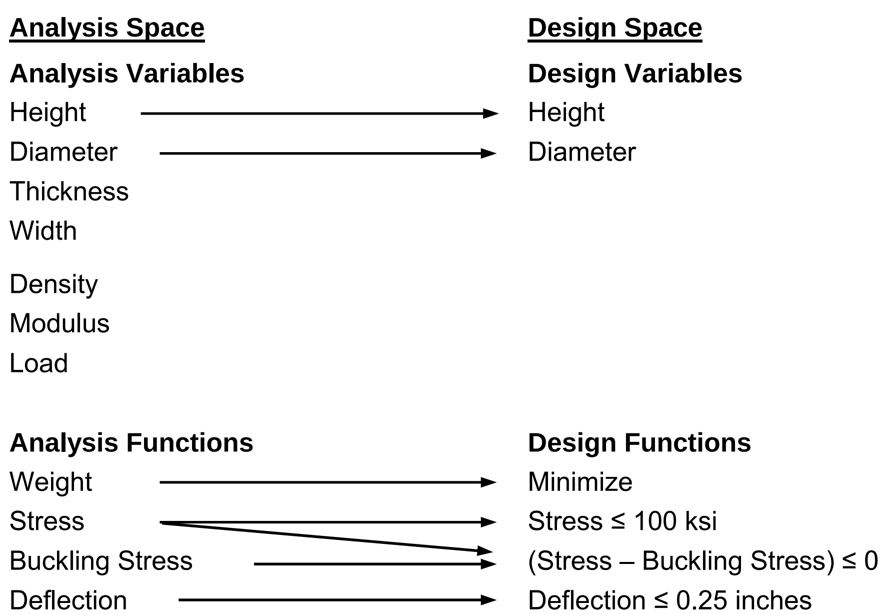

Fig. 1.4- Mapping Analysis Space to Design Space for the Two-bar Truss.

We see from Fig. 1.4 that the design variables are a subset of the analysis variables. This is always the case. In the truss example, it would never make sense to make load a design variable—otherwise the optimization software would just drive the load to zero, in which case the need for a truss disappears! Also, density and modulus would not be design variables unless the material we could use for the truss could change. In that case, the values these two variables could assume would be linked (the modulus for material A could only be matched with the density for material A) and would also be discrete. The solution of discrete variable optimization problems is discussed in Chapters 4 and 5. At this point, we will assume all design variables are continuous.

In like manner, the design functions are a subset of the analysis functions. In this example, all of the analysis functions appear in the design functions. However, sometimes analysis functions are computed which are helpful in understanding the design problem but which do not become objectives or constraints. Note that the analysis function “stress” appears in two design functions. Thus it is possible for one analysis function to appear in two design functions.

In the examples given above, we only have one objective. This is the most common form of an optimization problem. It is possible to have multiple objectives. However, usually we are limited to a maximum of two or three objectives. This is because objectives usually are conflicting (one gets worse as another gets better) and if we specify too many, the algorithms can’t move. The solution of multiple objective problems is discussed in more detail in Chapter 5.

We also note that all of the constraints in the example are “less-than” inequality constraints. The limits to the constraints are appropriately called the allowable values or right hand sides. For an optimum to be valid, all constraints must be satisfied. In this case, stress must be less than or equal to 100; stress minus buckling stress must be less than or equal to 0, and deflection must be less than or equal to 0.25.

Most engineering constraints are inequality constraints. Besides less-than (≤) constraints, we can have greater-than (≥) constraints. Any less-than constraint can be converted to a greater-than constraint or vice versa by multiplying both sides of the equation by -1. It is possible in a problem to have dozens, hundreds or even thousands of inequality constraints. When an inequality constraint is equal to its right hand side value, it is said to be active or binding.

Besides inequality constraints, we can also have equality constraints. Equality constraints specify that the function value should equal the right hand side. Because equality constraints severely restrict the design space (each equality constraint absorbs one degree of freedom), they should be used carefully.

Complete the following activities:

1. Define the following design problem: With height and thickness as design variables, minimize deflection, subject to weight less than 26 pounds, stress less than 90 ksi, and stress minus buckling stress less than zero (i.e. no buckling).

Analysis Variables	**Value**
Diameter, d (in)	2.
Width of separation at base, B (inches)	60.
Modulus of elasticity (1000 lbs/in^2)	30,000
Density, ρ (lbs/in^3)	0.3
Load (1000 lbs)	66
Design Variables	**Initial Guess**
Height, H (in)	30.
Thickness, t (in)	0.15
Analysis Functions	**Value**
Weight (lbs)	
Stress (ksi)	
Buckling stress (ksi)	
Deflection (in)	

Set lower bound of thickness to 0.05, upper bound to be 0.20; lower bound of height to be 10; upper bound to be 50.)

   a. Optimize the design.
   b. Turn in a hardcopy of the starting design and the optimal design.
   c. What is the optimal solution? What constraints are binding?

2. Keeping the same problem description given in Part 1, construct 2d contour and 3d surface plots (similar to plots in the tutorials, but with height and thickness as variables, deflection as the function contoured, and weight included as a constraint with stress and buckling).
   a. Turn in hardcopy of the plots.
   b. Shade the feasible region. Show the optimum point.
   c. Do the optimum variable values on the plot match the optimum values given in Part 1?

3. Keeping the problem from 2 above, change the value of diameter from 2.0 to 3.0 Re-optimize the problem. Redo the 2d plot.
   a. Include hardcopy. Show the feasible region and optimum.
   b. What has changed? Why?
   c. Why does the buckling constraint boundary no longer show up? Verify your explanation.

4. Select width and load as variables for a 3d optimal surface plot and plot the solution of the optimization problem from Part 1 to minimize deflection at each of the width / load combinations.
    a. Turn in hardcopy of the plot.
    b. How is this plot different from the 2d and 3d plots in parts 2 and 3 in terms of the variables we can select? Could we have chosen height and thickness as the graph variables? Why or why not?
    c. How is this plot different from the 2d and 3d plots in parts 2 and 3 in terms of the computational expense? Explain your answer.
    d. Why don't we usually plot constraint boundaries on such a plot?