Mid-Term Exam Solution Key

ChEn 693R, Section 2: Dynamic Optimization

May 15-May 18, 2015

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Open Book, Notes, Homework, Internet

Time Limit: 6 hours

Time(s) started: ____________________________________

Time(s) stopped: ___________________________________

Cumulative time:___________________________________

Name___Solution Key________________________________

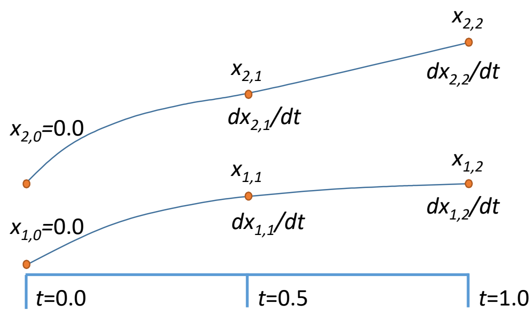
1. (30 pts) Orthogonal Collocation on Finite Elements

Objective: Solve a set of differential equations with orthogonal collocation on finite elements.

Solve the following 2 coupled differential equations from time starting at 0 until a final time of 1. Solve the system of equations with orthogonal collocation on finite elements with 3 nodes (time points) at $t = [0 \quad 0.5 \quad 1.0]$ for discretization points.

$$5\frac{dx_1}{dt} = -x_1 + x_2 + 2\,u$$

$$3\frac{dx_2}{dt} = -x_2 + x_1$$



The initial conditions for x_1 and x_2 are 0 and the value of the input, u is 1.0. The following is available for approximation of the derivative values at $x_{i,1}$ and $x_{i,2}$, where i is either variable 1 or 2 (see link for additional details on orthogonal collocation):

$$t_2\begin{bmatrix}0.75 & -0.25\\ 1.00 & 0.00\end{bmatrix}\begin{bmatrix}\frac{dx_{i,1}}{dt}\\ \frac{dx_{i,2}}{dt}\end{bmatrix} = \begin{bmatrix}x_{i,1}\\ x_{i,2}\end{bmatrix} - \begin{bmatrix}x_{i,0}\\ x_{i,0}\end{bmatrix}$$

Report the solution of x_1 and x_2 and derivative values ($\dot{x}_1$ and $\dot{x}_2$) at $t = 0.5$ and $t = 1.0$ (8 values total). Show work such as the system of 8 variables $\left(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}, \frac{dx_{1,1}}{dt}, \frac{dx_{1,2}}{dt}, \frac{dx_{2,1}}{dt}, \frac{dx_{2,2}}{dt}\right)$ and 8 equations for full credit.

Hint: Because these are linear differential equations, the problem can be set up as a system of linear equations that are solved with a single matrix inversion. You can also check your end points with the following (just use this to check the solution):

APM Model File (model.apm)

```
Parameters
  u = 1
Variables
  x[1:2] = 0
Equations
  5 * $x[1] = -x[1] + x[2] + 2*u
  3 * $x[2] = -x[2] + x[1]
```

APM Data File (model.csv)

```
Time
0
1
```

MATLAB Script

```
clear all; close all; clc
addpath('apm')
y = apm_solve('model',7);
disp(y.x.x1); disp(y.x.x2);
```

```
States with Orthogonal Collocation
x11 = 0.18958
x12 = 0.36556 (Matrix) vs 0.36556 (APM)
x21 = 0.017372
x22 = 0.057402 (Matrix) vs 0.057402 (APM)

Derivatives with Orthogonal Collocation
d(x11)/dt = 0.36556
d(x12)/dt = 0.33837
d(x21)/dt = 0.057402
d(x22)/dt = 0.10272
```

2. (70 pts) Simulation, Estimation, and Control

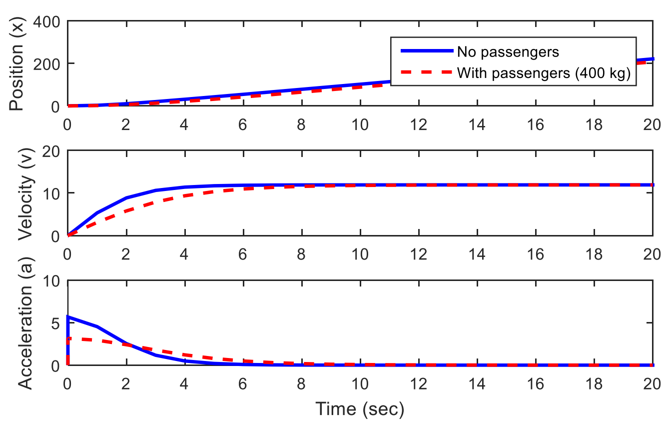
Objective: You have just purchased a boat for water-skiing and want a cruise control to maintain a desired velocity. Your objective is to design an estimator to determine unknown parameters and implement a model predictive controller to maintain a desired velocity.

Relevant information on your boat and factors that affect the dynamic relationship between propeller rotation rate (rounds per minute or RPM), position (measured), and velocity (controlled) are given below.

- Mass of the boat and driver: $m = 500\ kg$
- Density of water: $\rho = 1000\ kg/m^3$
- Drag coefficient of the boat in water: $C_d = 0.05\ (unitless)$
- Cross-sectional area for drag correlation: $A_c = 0.8\ m^2$
- Propeller rotation rate: $0 \leq RPM \leq 5000$
- Drag force resisting forward movement: $\frac{1}{2}\rho v^2 C_d A_c$
- Propeller force forward: $F_p = 40\sqrt{RPM}$
- Position: $\frac{dx}{dt} = v$
- Velocity: $\frac{dv}{dt} = a$
- Acceleration: a
- Force balance: $ma = F_p - F_d$

Using a force balance, simulate a change in boat velocity with and without passengers. Assume that the additional passengers have a combined weight of $400\ kg$. In each case, the propeller is stepped from $RPM = 0$ to the maximum, $RPM = 5000$. Report the velocity for both cases at 5 sec after the step change in RPM.

A. Velocity without passengers (at 5 sec): _**11.7**_________________ m/s
B. Velocity with passengers (at 5 sec): __**10.3**________________ m/s



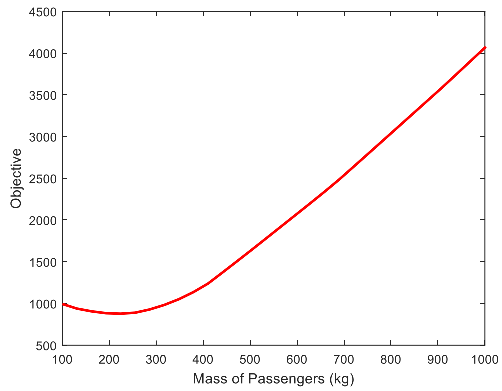
While a better estimate of the weight would improve the controller performance, it is not desirable to ask each boat passenger for weight information. Use dynamic estimation to predict the weight of the passengers given the data on the next page for RPM and GPS supplied position.

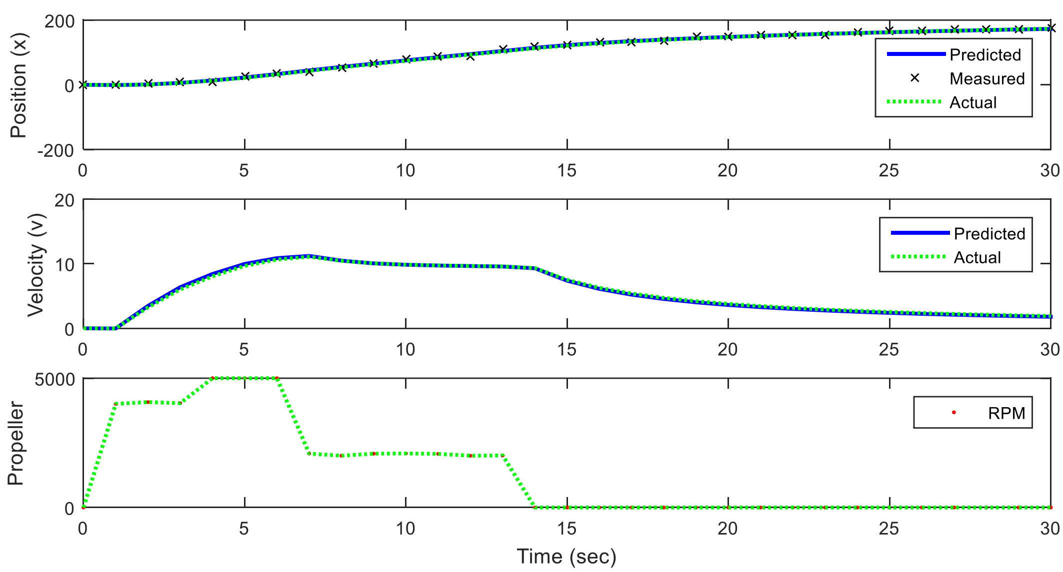
C. Estimated weight of passengers:

~~**With 2 Nodes: 594.2 (l1-norm)** or **633.7 (sq error)** kg~~

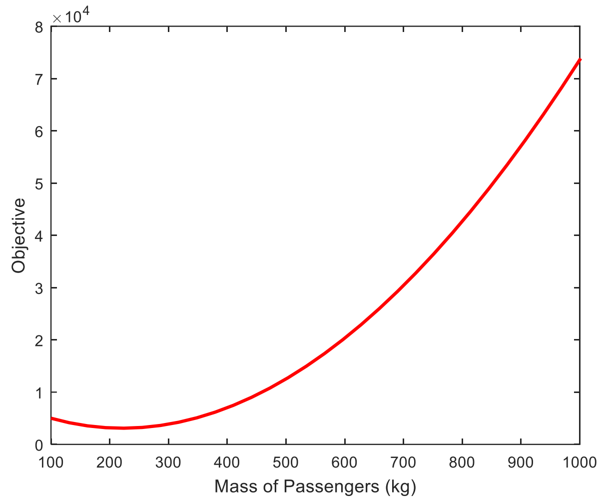
With 3 Nodes: 206.7861 (l1-norm) or **633.7 (sq error)** kg

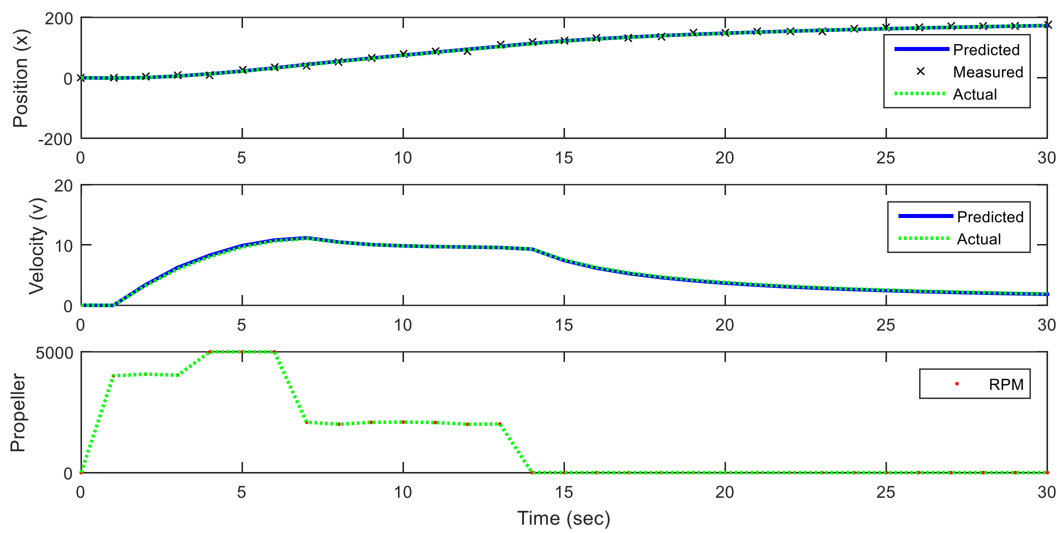
Solution with an l1-norm objective



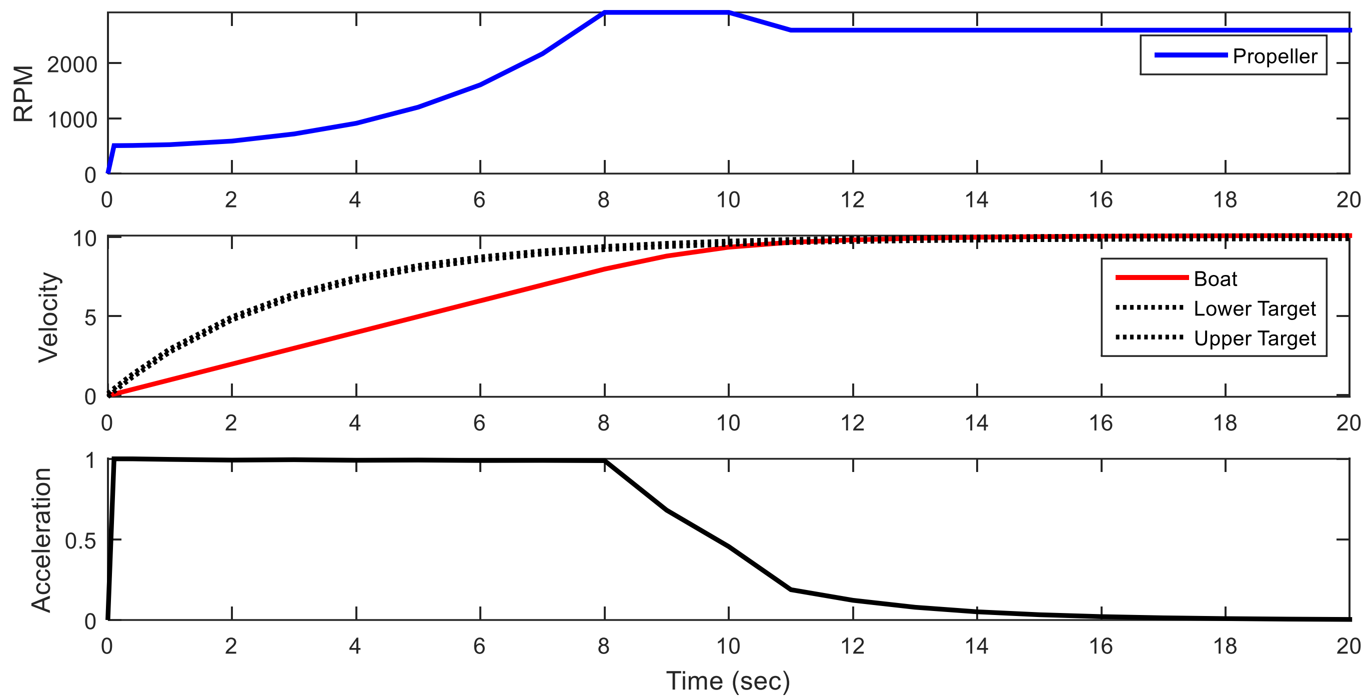


Solution with a squared error objective



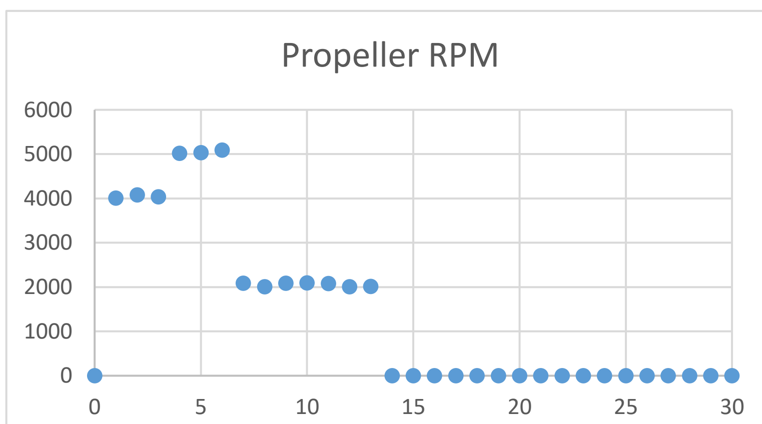


D. Demonstrate that your new boat cruise control can reach $10 \pm 0.1 \frac{m}{s}$ with passengers. Use a passenger weight of $400\ kg$, not the value estimated from part C. Limit the acceleration to no more than $1.0\ m/s^2$. Include a plot of cruise control performance including velocity, acceleration, and RPM.



Data collected with passengers of unknown weight

time (s)	rpm	position (x)
0.00	0.00	0.00
1.00	4007.30	0.00
2.00	4076.12	3.96
3.00	4032.04	9.59
4.00	5018.40	8.84
5.00	5033.08	25.65
6.00	5085.69	33.06
7.00	2084.71	38.92
8.00	2002.84	51.49
9.00	2082.02	64.40
10.00	2091.07	78.46
11.00	2074.44	88.33
12.00	2002.35	89.96
13.00	2013.06	107.71
14.00	0.00	117.54
15.00	0.00	121.15
16.00	0.00	131.14



17.00	0.00	130.36
18.00	0.00	138.19
19.00	0.00	147.58
20.00	0.00	150.32
21.00	0.00	155.60
22.00	0.00	155.86
23.00	0.00	152.69
24.00	0.00	163.63
25.00	0.00	167.09
26.00	0.00	166.98
27.00	0.00	170.07
28.00	0.00	172.09
29.00	0.00	170.63
30.00	0.00	175.21