Cover Letter

Traveling to mars and beyond has transformed from a dream into a possibility with the entrance of private companies into space transportation. Optimization of navigation is essential to traveling to mars however, and this paper will explore this concept.

Optimizing our travel time to mars, decreases the cost of travel and the time spent in space. Both are quintessential to becoming a multiplanetary species. One method of decreasing travel time is to include electric propulsion aboard space ships which have the ability to increase their velocity throughout the journey.

We have developed a controller that can take advantage of an engine, using constant adjustments to keep the ship on course towards its target destination.

Highlights

A controller has been designed with the following capabilities:

- Transporting a ship between A and B.
- Adjusting to unknown gravitational forces from objects such as asteroids to stay on track.
- Correcting course to continually adjust to known gravitational forces from known objects.
- Estimating its speed based on past external gravitational forces and thrust.

Rocket Process Control

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Abstract

In this project we take a process control approach to maintaining rocket trajectory for interplanetary travel. To deal with the forces that cannot be predicted through a simple physical model, we have attempted to develop a PID controller using FOPDT parameters to account for unknown forces acting on the rocket ship during its travel. Using a sensitivity analysis and simulation, we determined the characteristics of the rocket's travel and simulated the noise and unknown forces acting on the rocket on top of the gravitational forces from the planet and sun.

Introduction

To guide a rocketship from an initial position in space to another position in space requires significant understanding of the many forces acting on the object from celestial bodies and other phenomena. Process control proves useful in these scenarios where the forces acting on a system aren't entirely understood or practical to calculate. With the many forces of asteroids, dark matter, planetary and stellar forces, and unpredictable forces, we will be developing a model that uses process control to supplement a classical physics based model and help guide the rocket on the most direct course between two points in space. Taking the shortest path between 2 points is essential in this application as fuel is a precious commodity in space travel and avoiding objects requires a precise ability to maneuver the ship along a projected course.

Literature Review

Electric powered rocket engines have the advantage of being energetically autonomous or nearly so with the help of solar panels ("Micropropulsion"). These types of engines use a voltage gradient to accelerate heavy cations out of the rocket nozzle where they rejoin their electron to maintain neutral charge in the engine (Marcuccio).

In order to navigate through the solar system we will examine the following studies which are all based on fundamental principles of physics and in general a flavor of polar coordinate systems.

The type of coordinate plane must be considered to navigate electric powered spacecrafts through our solar system. The author uses polar coordinates to describe movement. His method of space navigation consists of breaking the complete trajectory into segments and optimizing each one individually. He demonstrates this using collocation and non-linear programming (Marec 24).

Vladimir, in Orbital Mechanics, describes keplerian study of ellipticity and its capability to describe position of an object around a gravitational object as long as the parameters of the ellipse are known. There are many parameters involved in this approach and not all of them readily available (Chobotov 35)

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Theory

Our simulation used Newton's first and second laws of motion to predict the position of the rocket at time t as a function of the forces acting on the rocket. Because more than two objects are considered in our simulations it is necessary to use numerical approximations in place of the more exact closed form equations. We used the cartesian coordinate system, labeling the two dimensions x and y and assuming that all motion occurs in the z plane because the solar system is relatively planar. The calculations for calculating the x and y position were carried out independently with sets of equations for each. Since we expect that our probe will never approach the speed of light, we have assumed that special relativity does not apply and can therefore simplify our equations by leaving special relativity equations out.

During the simulation we first calculated the force acting on the rocket, then the responding velocity, and finally the new position at time t. The second law is represented by equation one, shown below, using forces acting on the rocket to calculate the resulting acceleration at time t.

$$a = \frac{\Sigma F_{on\ rocket}}{m}$$

Equation 1

And then the first law and its derivatives were used to calculate the velocity and position at time t.

$$x = vt + x_0 \qquad \textit{Equation 2}$$

$$v = at + v_0 \qquad \textit{Equation 3}$$

The above process was repeated for the x and y acceleration, velocity and position after calculating the resulting sum of x and y forces on the rocket.

Methods

In Figure 1, the x and y forces for the engine in the x and y direction are plotted over time. The total force is always kept constant.

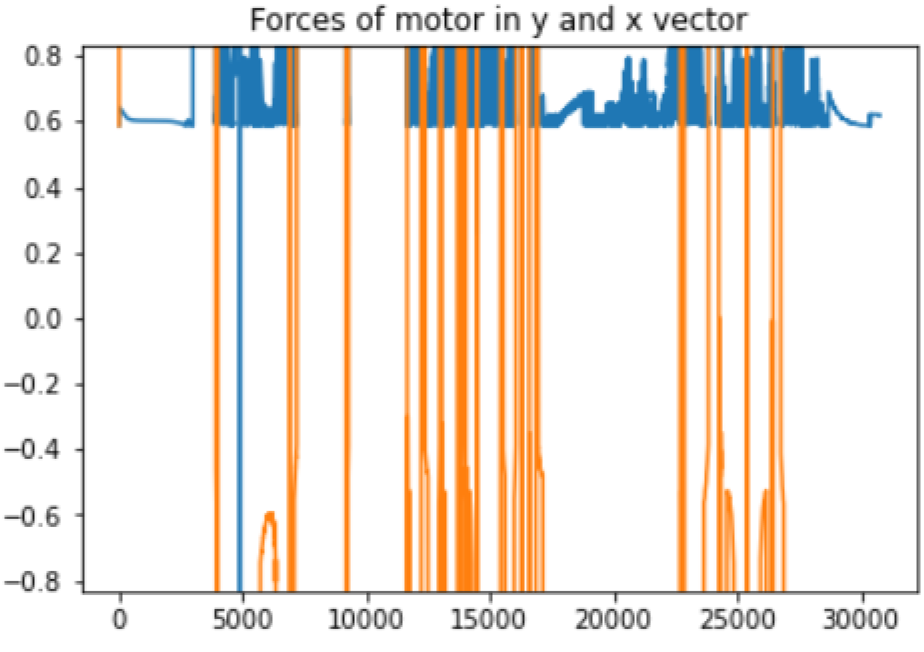

Figure 1: Forces of the motor in the x (blue) and y (orange) direction

In Figure 2, the position of the rocket is plotted starting from its initial position to the target.

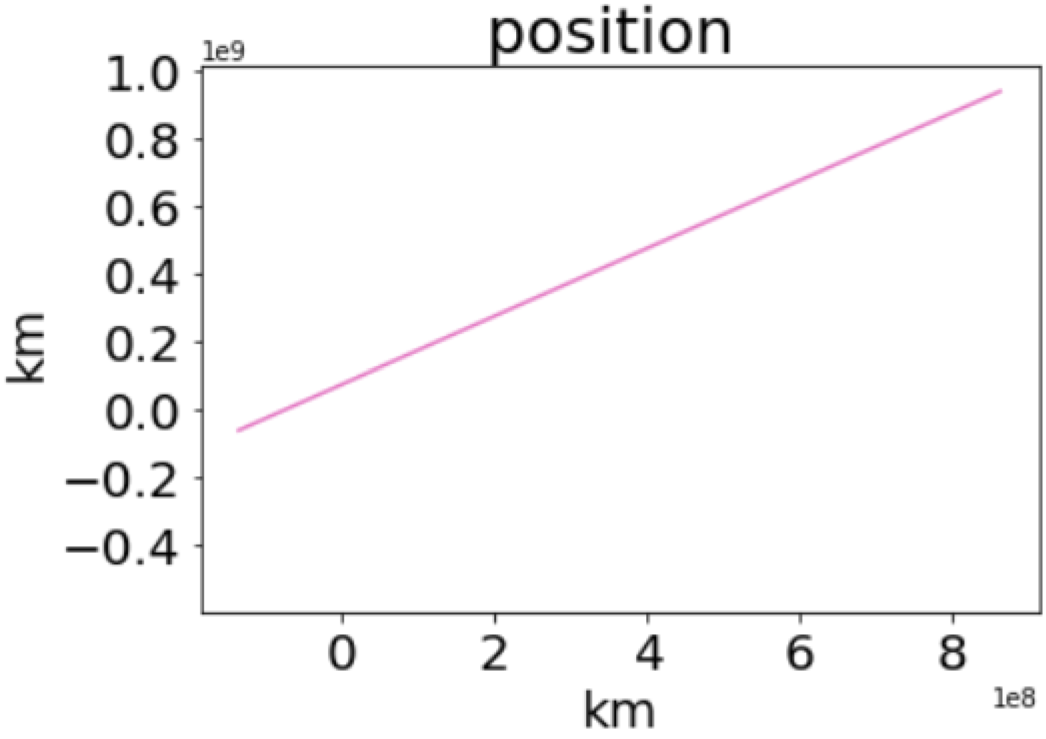

Figure 2: Depicts the course of the rocket from it's initial and final points. The rocket follows a perfectly linear path as no noise or PID controllers have been introduced to bring offset and response into the system.

To control our ship from its initial launch point to its destination, we've implemented a controller that adjusts the angle of the ship's thruster with a constant force. The goal of the controller is to keep the ship on a straight path between the launch point and destination planet with as little deviation as possible. We'll be optimizing the deviation from the path, and in turn optimize flight time.

The gravitational pull of each planet in the solar system acting on the ship will be broken into component x and y vectors. The thruster will initially aim against these vectors to provide an opposite force to keep the ship on course. However, due to antimatter, asteroids, and measurement error the ship is unlikely to maintain a perfect course in spite of this effort. Deviation from the course after these measures will be considered the "error" (regardless of the source) and implemented in a first-order plus dead-time model (FOPDT). For the purpose of the simulation, these external forces will be simulated with a position dependent noise for the ship to deal with. Parameters K_p, θ_p, and τ_p, will be optimized to reduce oscillations and keep the ship on course in spite of the noise forces.

We could improve the simulations by incorporating the motion of the planets as the ship moves, as well as specifying a target velocity or position boundaries, however for the sake of our initial simulation we'll be assuming the ship moves fast enough that planetary motion has a negligible effect on the forces the ship experiences, and that the velocity and oscillatory position of the ship does not have restrictive bounds.

To achieve this, we'll discretize the path into time steps, and at each step calculate the predicted gravitational force on the rocket using vector algebra. A vector for the thruster will then be calculated to keep the rocket on the desired path, and the offset/error of the rocket on it's path will be used in a FOPDT model to add a correction to the next timestep's thruster angle.

Sensitivity Analysis

The adjustable parameters in this model are the angle of the rocket, and thrust is also being examined to determine if adjusting the parameter of thrust can affect flight time as well.

The controlled variable in this model is flight time, however this may not be plausible to analyze if the rocket never reaches its destination. To analyze flight time, we've instead analyzed the percentage of the trip completed in 1 iteration given the chosen parameters. This will give a normalized result of the effect of angle and thrust on flight time without requiring the rocket to actually reach its intended target. The rocket is approaching a target 10 billion kilometers north and 10 billion kilometers east of the launch point. We are assuming the rocket, launch point, and destination exist and travel on the same z-plane. The results are given below in Table 1.

Table 1: Results from the sensitivity analysis. Columns are the force of the thruster in Newtons, and rows are the angle of thrusters in degrees. The table results are the percentage of the trip made in the first iteration, lasting 1 second.

	60	**70**	**80**	**90**	**100**
25	0.00591	0.00591	0.00715	0.00591	0.00849
35	0.0062	0.0062	0.00749	0.0062	0.00891
45	0.0063	0.0063	0.00761	0.0063	0.00905
55	0.0062	0.0062	0.0071	0.0062	0.00891
65	0.00591	0.00591	0.00715	0.00591	0.00849

Simulation Results

We have estimated the distance remaining at each time point using the sum of the x and y forces acting on the rocket and its mass. Newton's laws of motion were used to convert the x and y forces applied over one time step to the resulting change in x and y displacement. In an actual flight, there would be software designed to view the relationship between a near object, such as a planet, relative to the stars in the background and infer the probes location for confirmation but this would only recycle data that we already have in our case and not result in a confirmation. The graph below shows the distance, in millions meters, of the rocket vs time, in minutes. The graph was produced without noise being added to the simulator.

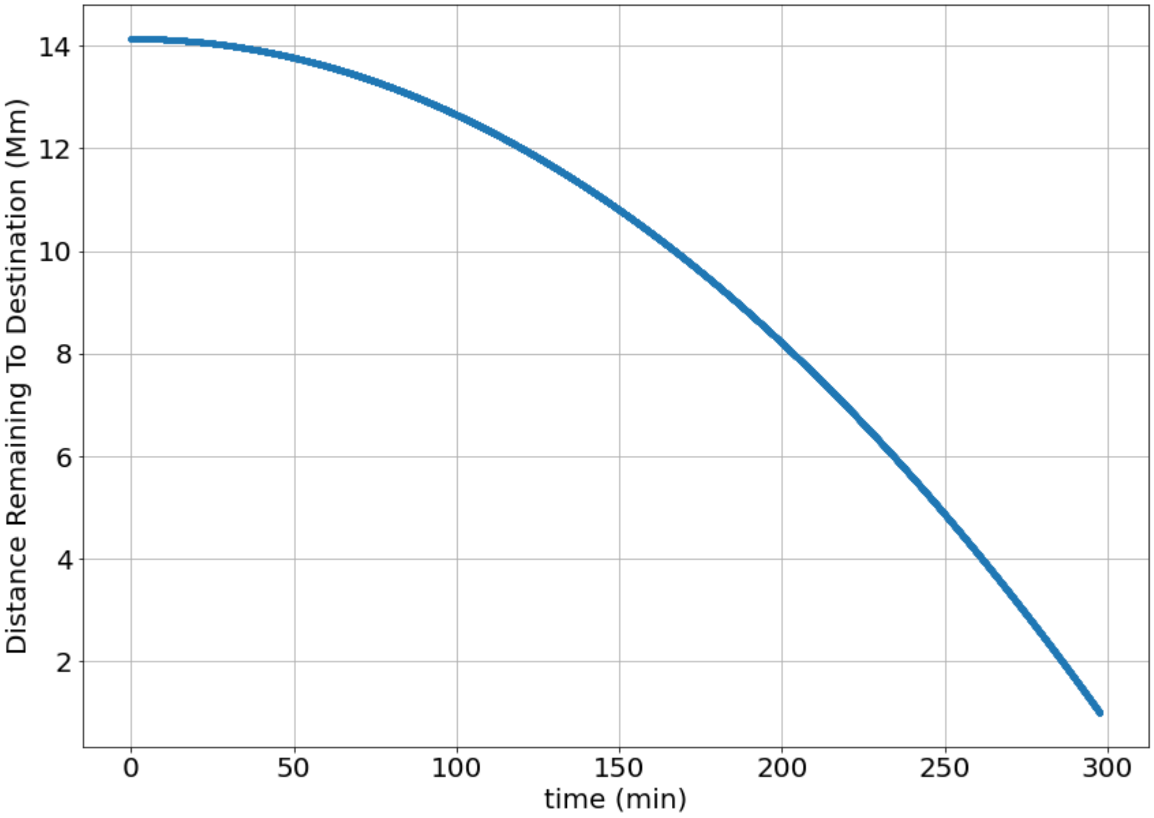

Figure 3: Distance to the target destination vs time without noise.

Random error was added to the calculated displacement at every time step in order to simulate five percent error in the distance measuring device that would be carried above the space probe. The first of the two graphs below show that the target is still reached with noise added and the second shows the noise added to the x and y vectors.

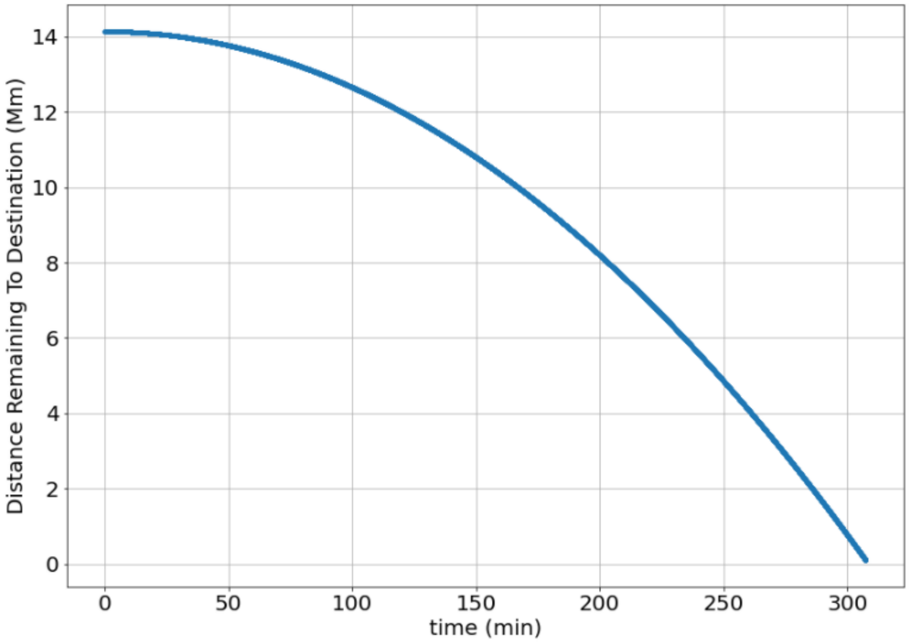

Figure 4: Distance to the target destination vs time with 5% random noise added.

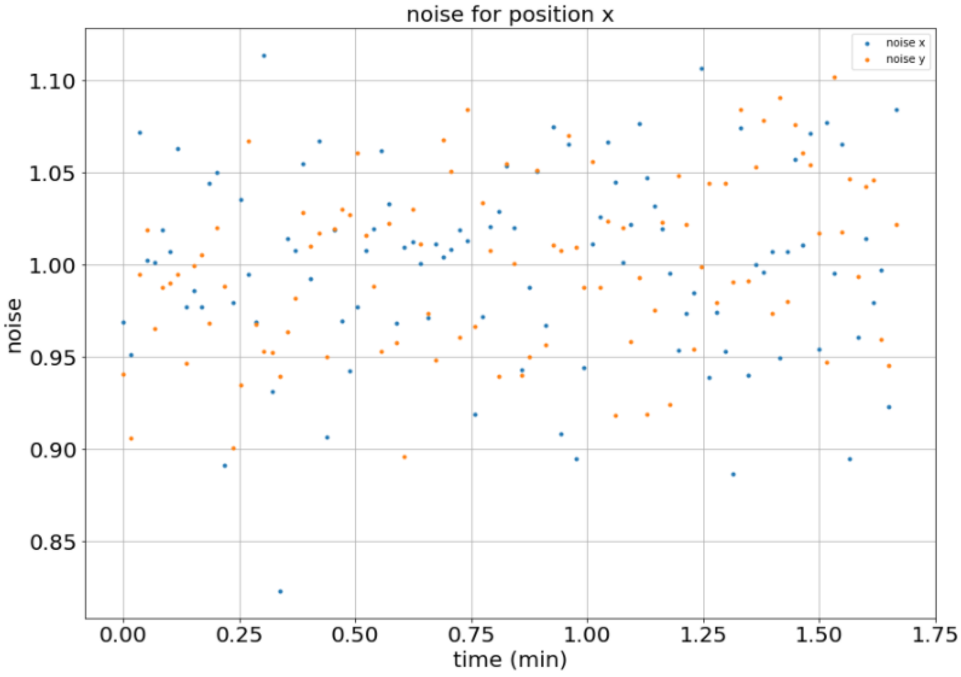

Figure 5: noise in the x and y over time.

Estimation and Dynamic Optimization Results

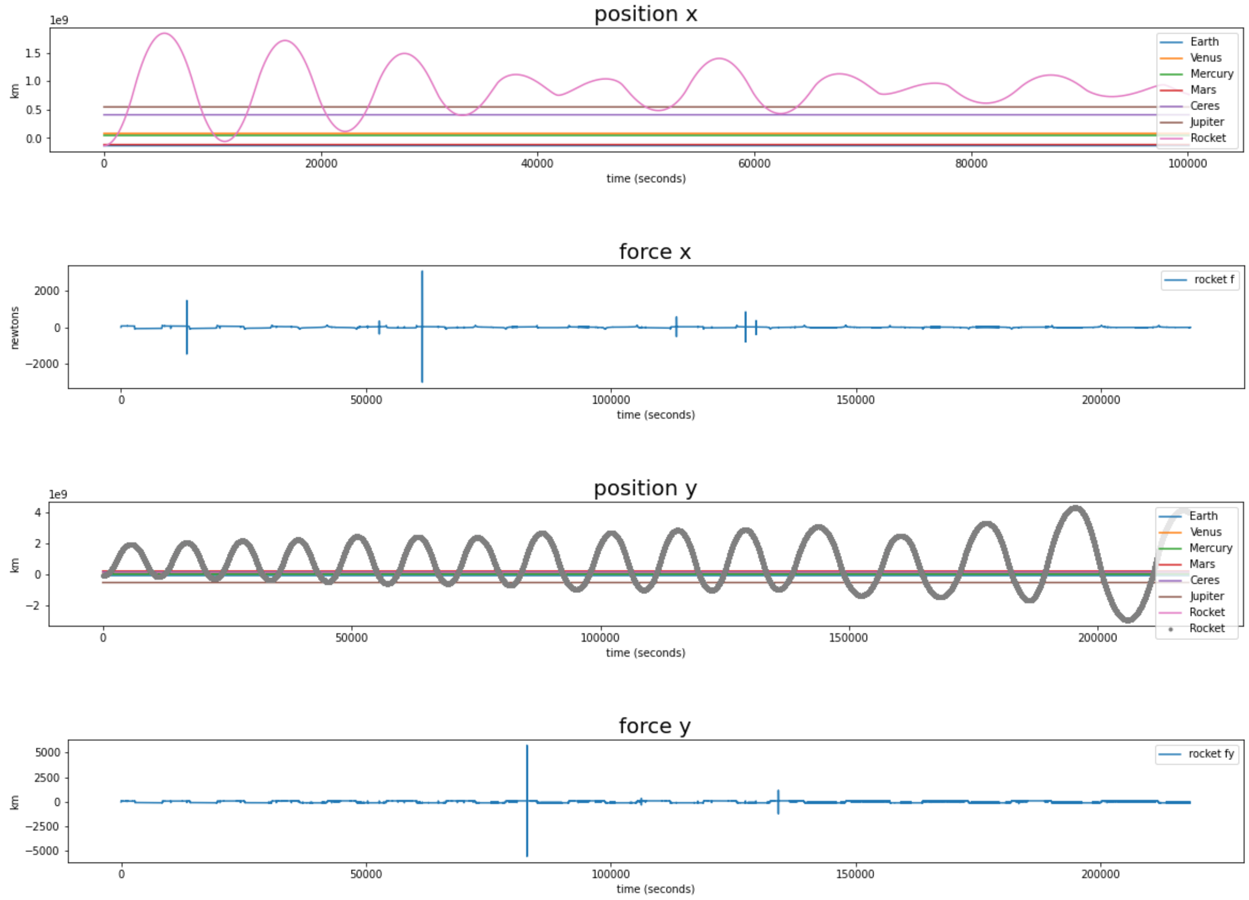

Figure 6: Labeling from top to bottom: the position of the rocket in the x direction, the force on the rocket from the thruster in the x direction, the position of the rocket in the y direction, and the force on the rocket from the thruster in the y direction.

Discussion

Our sensitivity analysis and preliminary simulations revealed that decreasing reactor thrust made the rocket more susceptible to planetary forces, and may have caused the rocket to be pulled into the orbit of nearby planets. On the other hand, increasing the force past a certain point may have made slowing down the rocket at the destination too difficult of a task. To prepare for a PID controller, we implemented a FOPDT model with initial Kp, τ, and θ values of 1, 10, and 0, respectively. Unfortunately, we were unable to determine the source of the oscillation in our x and y position, and as a result were unable to successfully implement a PID controller into our design. Our plan was to optimize these parameters until they described the model well, then implement them into a PID controller to add a thruster value to the value predicted by our physics based model. Our position in x seemed to be approaching the destination value with extreme oscillation, however our position in y appeared to be increasing in amplitude with time. We believe our thruster force may have been too great, causing the ship to overshoot the destination and turn around, however we struggled at lower forces to get the ship to maintain its course without being dragged into planetary orbits around it.

Conclusion/Recommendations

Although our rocket was able to move between points without the introduction of noise or extra-physical forces, when we introduced the unpredicted forces our FOPDT model was unable to converge to a reasonable trajectory. We believe this may be due to an improper value for thruster force, or a missing breaking force for when the rocket approaches its destination. Once we can pinpoint the oscillatory behavior found in our simulations, we recommend tuning the FOPDT parameters and implementing them into a PID controller to optimize the flight time/fuel usage of the rocket.

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