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April 23, 2019

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Subject: Optimization of Home Energy System

Dr. Hedengren,

It is with great delight that we present our final report on the Optimization of our Home Energy System. As stewards of the earth we are responsible for using energy responsibly. Also, to avoid the calamities of the finals days, we must reduce our carbon footprint. Additionally, reducing cost is a high priority for any member of society.

Our project takes into account these concerns for the average residential home. The energy system of the home includes solar panels, Tesla Powerwall batteries, and a Tesla car. Electricity from the grid is also used. Energy cost is minimized using the various sources of electricity mentioned above based on the weather and the peak electricity costs. The daily electricity demand is varied based on the average electricity usage for a residential home over a 24 hour period. Revenue is generated by selling stored energy back to the grid when the price of electricity is high. We recommend a system with 1 Powerwall battery and 70 solar modules. Optimization of this system results in a cost savings of $17.18. Future work should include weather forecasting and a more involved cost analysis including the initial investment on the equipment.

Sincerely,

Emilee Hunter
Nicole Burchfield
Hunter Rawson

Please see the enclosed file.

Highlights

- The cost of electricity for a single family home was minimized by varying the source of energy used to provide electricity
- A first principles model of the system is simulated with Python
- Models developed include the following power sources used in the home: Tesla Model S car battery, Tesla Powerwall battery, and solar panels corrected for orientation with respect to the sun
- The optimizer shows when certain power sources should be used throughout the day and reports a daily income of $17.18 for a 1 Powerwall, 70 solar module system
- Future work will include weather forecasting and more involved cost analysis including initial investment

Optimization of Home Energy System

Nicole Burchfield, Emilee Hunter, Hunter Rawson

April 23, 2019

Abstract

The cost of electricity for a single family home was minimized by varying the source of energy used to provide electricity throughout an entire day. The sources available for this home include Powerwall batteries, solar panels, grid power, and a Tesla Model S car battery. The Tesla car battery can be discharged to provide power, but the car is also required to be fully charged by the morning for use throughout the day. The daily weather determines the electricity available from the solar panels. Daily electricity demand varies based on the family's power needs at home. We first present a literature review, in which we have obtained information regarding weather, Tesla batteries, and electricity pricing and usage. Next, a model overview includes the constants, parameters, equations, and variables employed in the solar and battery models. Each of these models is further described, and model simulations show the inputs and outputs from each model, including parameters used. The solar model includes corrections for the orientation of the panels with respect to the sun's rays. To optimize the entire system, the models were connected and run using Python GEKKO. We explored the effects of using 1 or 2 Powerwall batteries, as well as various numbers of solar modules. The optimal system includes one Powerwall battery, 70 solar modules, a Tesla car battery, and grid power. By selling power back to the grid, this system results in a daily cost savings of $17.18.

Introduction

The cost of electricity for a single family home (Figure 1) was minimized by varying the source of energy used to provide electricity throughout a 24 hour day. Electricity is available from solar panels, Tesla Powerwall batteries, the grid, and from discharging a Tesla Model S car battery. The Tesla car battery also requires electricity to be charged for use during the day. The cost of electricity from the grid varies based on the on-peak and off-peak hours, and excess electricity is sold back to the grid. This assumes the grid will purchase all excess energy. The daily weather determines the electricity available from the solar panels. Daily electricity demand varies based on the family's power needs at home.

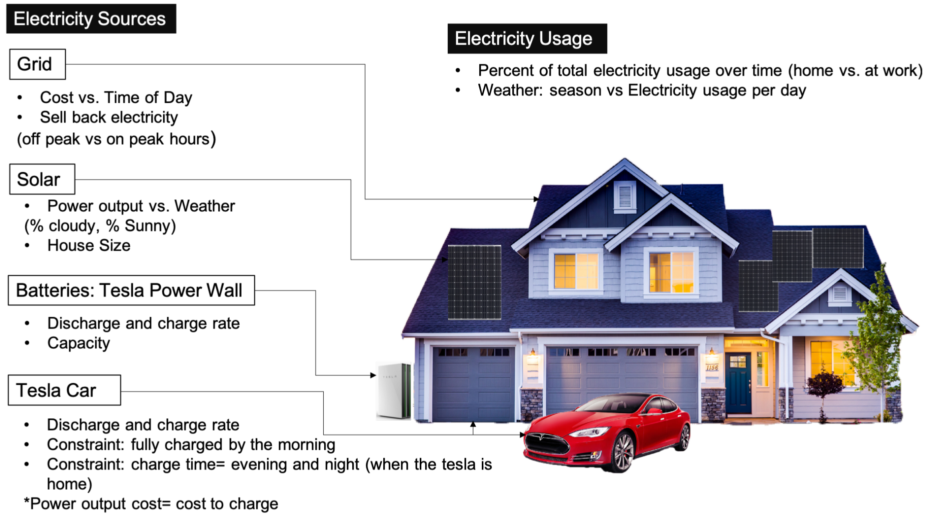


Figure 1: System Diagram

Literature Review

Solar and Weather

Håvard Vika derived a model of solar array performance in a 2014 paper[1]. This solar array model requires manufacturer data for the solar cells, the solar array operating temperature, and intensity of the light on the solar array. Navin Sharma et al. related solar intensity to the weather (temperature, dew point, wind speed, precipitation, humidity, and cloud cover) using various models in their 2011 paper[2]. Solar radiation data and temperature data are also available by location for up to two weeks prior to the current date[7].

Batteries

The Tesla Powerwall[3] is a promising battery that can be used to store electricity in the home. Each Powerwall has a capacity limit of 13.5 kWh, a maximum peak power of 7 kW, and a maximum continuous power of 5 kW. The round-trip efficiency of a Powerwall battery is about 91.5%.

Electricity Pricing and Usage

Data on Electricity usage is shown below for one average residential home [8]. This data was adjusted from overall electricity usage data and scaled down to match the average (30 kWh/day) used in a residential home. The data is dependent on location and the time of the year, which introduces some uncertainty. The typical peak hour information we used correlates well with the electricity demand data[11].

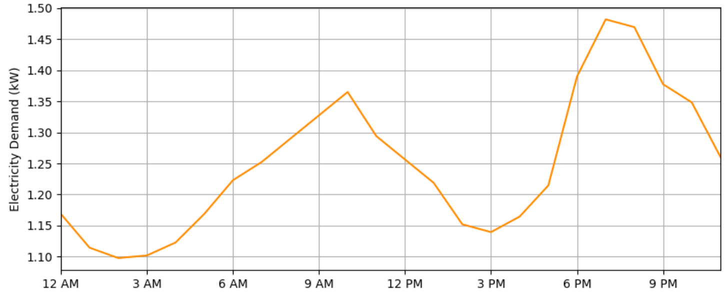


Figure 2: Electricity usage over a 24 hr time period with an average usage of 30kWh

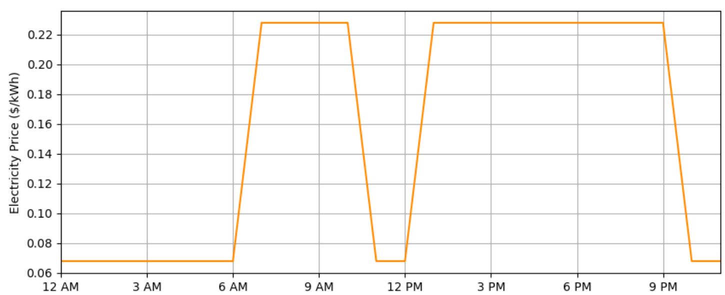


Figure 3: Grid electricity prices over a 24 hour time period

Model Overview

The following tables outline the constants, parameters, intermediates, and variables used in the model.

Table 1: Constants

Constant	Symbol	Value
Roof Area	A_r	1500 ft^2
Roof Pitch	θ	26.6°
Roof Direction	ϕ	0.0° from North
Powerwall Battery Maximum Continuous Discharge Rate	D_1	5.0 kW
Powerwall Battery Maximum Continuous Charge Rate	C_1	15.0 kW
Powerwall Battery Capacity	Q_1	13.5 kWh
Powerwall Battery Round-Trip Efficiency	$\eta_{B,1}$	91.5%
Model S Battery Maximum Continuous Discharge Rate	D_1	30.0 kW
Model S Battery Maximum Continuous Charge Rate	C_1	60.0 kW
Model S Battery Capacity	Q_2	87 kW
Model S Battery Round-Trip Efficiency	$\eta_{B,2}$	91.5%
Constants in the charge rate	a,b,c,d	1, 1+1e-14, -9.11e-3, 1.593965
Constants in the discharge rate	a,b,c,d	-1e29, 9.12e3, 1.593965, 1
Solar Module Open Circuit Voltage	V_{OC}	69.7 V
Solar Module Maximum Power Point Voltage	V_{MPP}	58.0 V
Solar Cell Maximum Power Point Current	I_{MPP}	5.70 A
Solar Cell Short Circuit Current	I_{SC}	6.07 A
Solar Cells Per Module	N_{cells}	96
Solar Module Area	A_{Mod}	18.02 ft^2
Inverter Efficiency	η_{inv}	90%

Table 2: Parameters

Parameter	Symbol	Typical Values
Solar Irradiance	G_{sol}	0-1200 W/m^2
Solar Array Temperature	T_{sol}	0-65$°C$
Electricity Price	E_p	Off peak: \$6.79, On peak: \$22.78
Electricity Usage	E_u	Average 30 kWh per day

Table 3: Intermediates

Variable	Equation	Typical Values
Solar Array Power	$P_{sol} = a(b^{T_{sol}})(G_{sol}^c)$	0 - 20 kW
Solar Array Voltage	$V_{pv} = d(e^{T_{sol}})(G_{sol}^f)$	0 - 500 V
Grid Electricity	$P_{grid} = E_u + R_{C1} + R_{C2} - R_{D1} - R_{D2} - P_{sol}$	-30 - 30 kW
Cost Rate	$\dot{cost} = E_p * P_{grid}$	-3 - 3 \$/hr

Table 4: Variables

Variable	Symbol	Type	Bounds
Solar Modules in Parallel	N_{mp}	fixed	1 minimum
Solar Modules in Series	N_{ms}	fixed	5-7
Electricity Cost	E_{cost}	controlled	none
SOC	F	state	0 - 100%
Battery charge rate	R_{Ci}	manipulated	0 - D_i
Battery discharge rate	R_{Di}	manipulated	0 - C_i

Equations

Solar Equations

An accurate solar model is outlined in Vika's paper. It is an implicit equation of the form shown in Equation 1.

$$I_{sol} = f(G_{sol}, T_{sol}, V_{pv}) \tag{1}$$

It is assumed that the solar array always operates at the maximum power point voltage. Due to the complexity of Vika's model, simple equations were fitted to simulation results for use in the GEKKO model (see Table 3 and Parameter Estimation section).

Battery Equations

Simplified battery model equations are shown in Equations 2 and 3. The subscript 1 represents the Powerwall battery and subscript 2 represents the Model S battery. The terms Q, F, Rc, and Rd, are the capacity of the batteries, state of charge (SOC), charge rate, and discharge rate, respectively.

$$Q_1 \frac{dF_1}{dt} = R_{C1} - R_{D1} \tag{2}$$

$$Q_2 \frac{dF_2}{dt} = R_{C2} - R_{D2} \tag{3}$$

Equations 4 and 5 describe the maximum charging and discharging rates as a function of the SOC. The form Equation 4 above was chosen to depict the charging rate approaching 0kW as the SOC approaches 100%, while not exceeding the maximum charging rate. Similarly, the equation for the discharge rate (Equation 5) was chosen to approach 0kW as the SOC approaches 0%, while not exceeding the maximum continuous discharging rate over the range of SOC values. These two equations were used solely for the Python simulation, since SOC bounds can be set in the GEKKO model.

$$\hat{R}_{C,max} = \frac{1}{a + (\frac{SOC-b}{c})^d} \tag{4}$$

$$\hat{R}_{D,max} = \frac{d}{1 + (\frac{b}{SOC-a})^c} \tag{5}$$

Electricity Equations

The cost of electricity is shown below (Equation 6) and is dependent on the amount of electricity needed from the grid. This is a function of time because of the price changes for on-peak and off-peak hours. The amount of grid electricity used is a function of the electricity demand of the home, the electricity generated by the solar panels, and the charge and discharge rates of the Powerwall battery and the Model S battery (see Table 3).

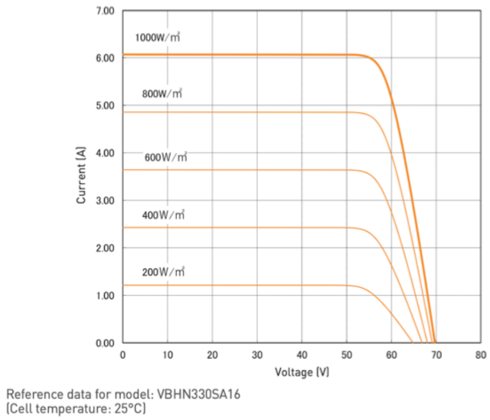
$$E_{cost} = E_p(t) * P_{grid} \tag{6}$$

Simulation

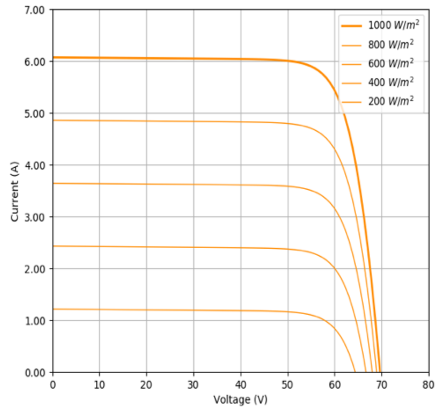
Solar Model Simulation

Specifications for the Panasonic VBHN330SA16 photovoltaic module were obtained from the Panasonic website[10]. Figure 4 shows a comparison of the manufacturer provided current-voltage curves (4a) and simulated current-voltage curves (4b) for varying levels of solar irradiance at $25°C$. The general shapes and positions of the curves are quite similar.

Weather data was obtained including temperature ($°C$) and solar irradiance (W/m^2) from March 31, 2019 in Provo, UT[7]. This data (shown in Figure 5 below) was the input for the solar cell simulation. Figure 6 shows simulated solar module performance based on the data shown in Figure 5. Solar array power increases with irradiance and decreases with temperature.



(a) Manufacturer Results



(b) Simulation Results

Figure 4: Solar Cell Validation

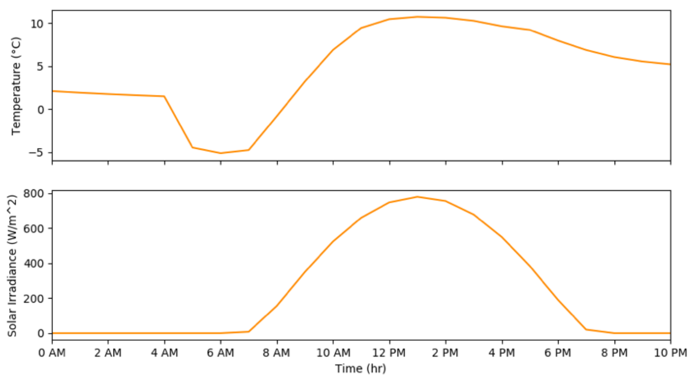


Figure 5: Solar Cell Simulation Input Data

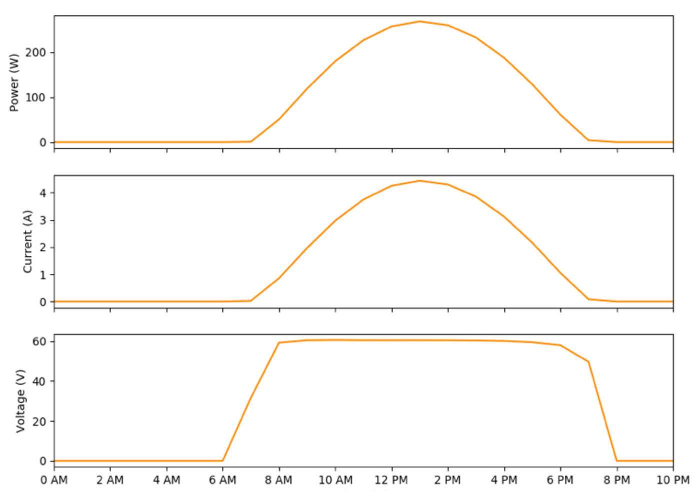


Figure 6: Solar Cell Simulation Output Data

Battery Model

The profiles for the normalized charge and discharge rates as a function of the fractional SOC are shown below (Figure 7).

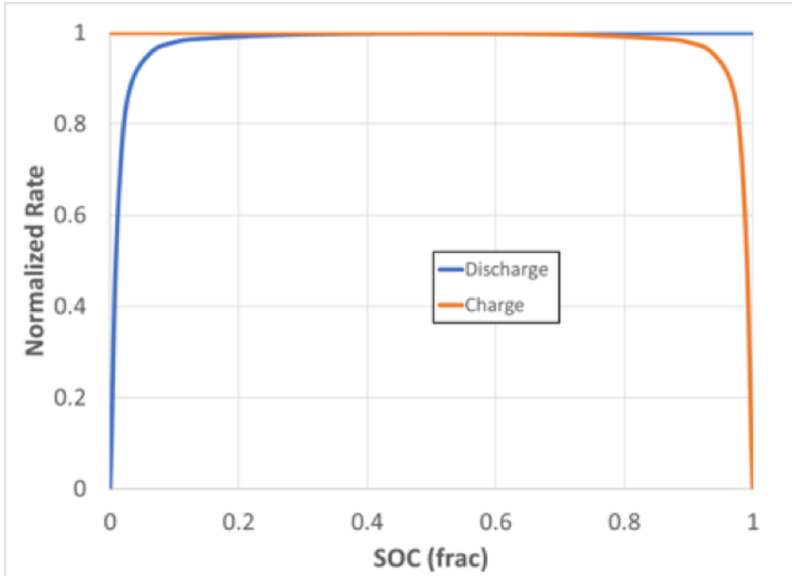


Figure 7: Normalized charge and discharge rates used for simulating the Powerwall and model S batteries

The model was tested by varying the inputs to the model as shown in Figure 8. The expected trends are observed for the model output (e.g. rates of increase, decrease, and saturation values) in Fig. 8b.

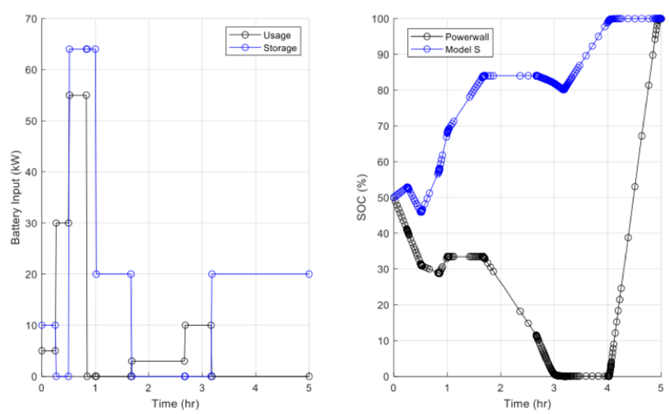


Figure 8: (a) Example of varying the inputs to battery model and (b) transient SOC model output for the Tesla Powerwall and Model S batteries

Parameter Estimation

Data

Input data for the system includes weather (Figure 9) as well as electricity demand (Figure 2) and electricity prices (Figure 3). Electricity demand and prices will determine the percentages of each source of electricity (battery wall, car battery, solar panels, or grid power) used over time in the GEKKO model.

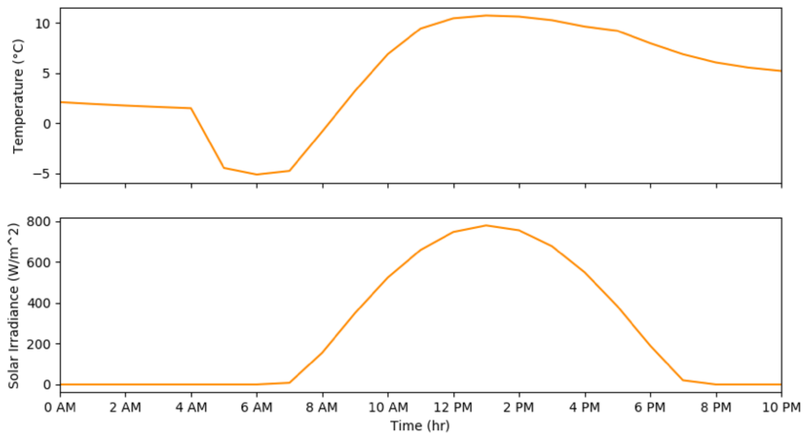


Figure 9: Weather Data from March 31, 2019

Solar Model

The solar model implemented in GEKKO (the digital twin) uses Equations 7 and 8 below.

$$P = a(b^{T_{sol}})(G_{sol}^{c}) \tag{7}$$

$$V = d(e^{T_{sol}})(G_{sol}^{f}) \tag{8}$$

The parameters and variables used are found in Table 5.

Table 5: Solar Digital Twin Parameters

Parameter or Variable	Value or Definition
a	6.5126737778450083E-01
b	9.9680804877945284E-01
c	1.0391858697506762E+00
d	1.1834672231963125E+02
e	9.9667039815506886E-01
f	4.0600537403361850E-02
P	Power (W)
V	Voltage (V)
T	Temperature (K)
G	Irradiance (W/m^2)

The results of the digital twin for the solar panels are shown as surfaces in Figure 10. The points in this figure represent the simulation results of Vika's model. The relative error between these two models is quite small.

The solar panel orientation was corrected in order to produce a more accurate representation of the irradiance. The function used for this correction takes the direct tracking irradiance, solar azimuth angle, solar zenith angle, and roof orientation as inputs, then outputs the corrected solar irradiance for each side of the roof. This model assumes that the home has a simple roof like the one shown in Figure 11 below[13]. We assumed that the home was oriented such that one side of the

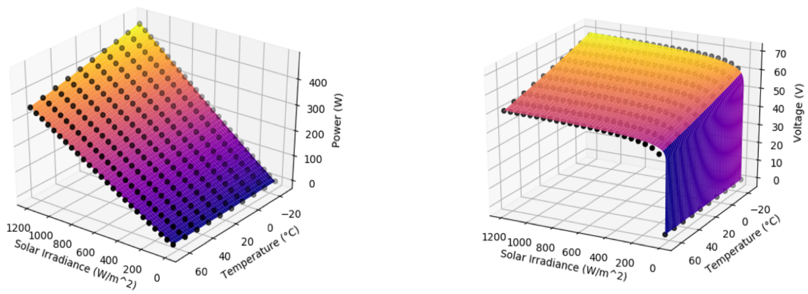


Figure 10: Solar Panel Digital Twin Results

roof faces north and the other side faces south.

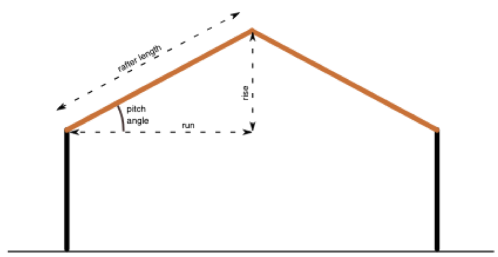


Figure 11: Simple Roof Diagram

A python package called Pysolar[14] calculates the solar azimuth and zenith angles, as well as the clear sky direct tracking irradiance. Figure 12 shows the results of the orientation correction using weather data from March 31, 2019 in Provo, UT.

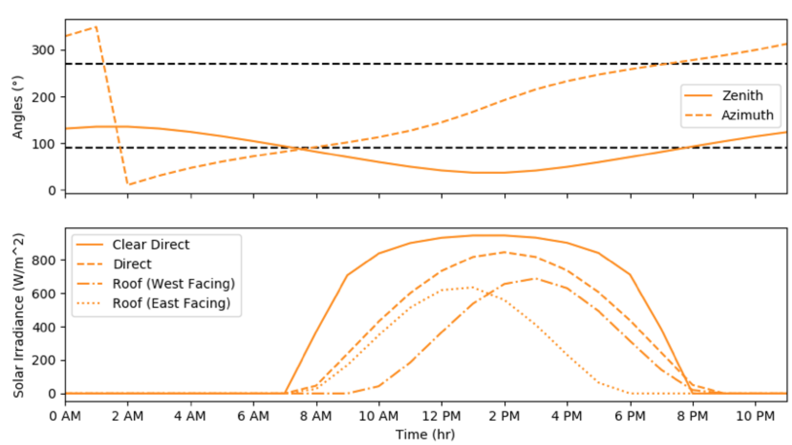


Figure 12: Orientation Correction Results

Battery Model

The interaction between the Tesla Powerwall and Tesla car battery are shown in Figure 13 below.

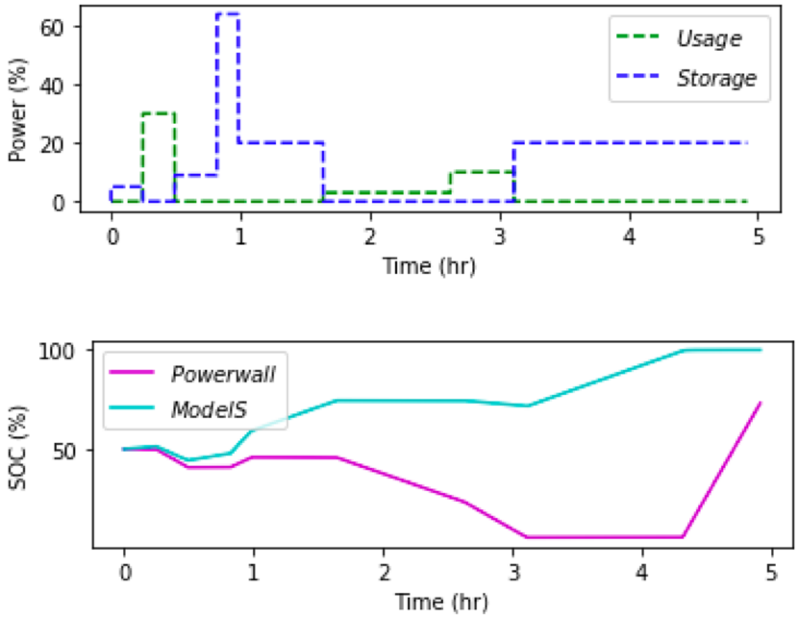


Figure 13: Tesla Car and Wall Batteries with priority on charging the car battery

These results show the effect of storing vs. using power on the state of charge for the Powerwall battery and the Tesla Model S battery. In this simulation, the car battery is prioritized and charged first. As the figure shows, when the Model S SOC reaches 100% the Powerwall SOC starts to increase.

Sensitivity Discussion

The solar validation plots previously discussed show irradiance sensitivity (Figure 4). The solar cells have a temperature coefficient (max power) of -0.3%/°C[15].

Figure 14 shows the individual responses of the Tesla car battery and Powerwall battery to changes in battery usage and storage. The Powerwall is much more sensitive to changes in usage and storage than the car battery due to charging and discharging rate parameters.

Control and Optimization

Objectives and Constraints

The objective of the model is to minimize the total cost of electricity at the end of the day. This objective can be see in Equation 9. W_1 is the weight added to magnify the importance of the main objective. The cost is integrated over the 24 hour time period.

$$Objective_1 = \min(W_1 \int_0^{t=24hrs} \dot{cost} dt) \tag{9}$$

There is a constraint on the Tesla car battery to be fully charged at 6 am. This is accomplished by fixing the SOC_2 (Tesla car battery) to 1.0 (100%) by 6am (Equation 10). This ensures that the

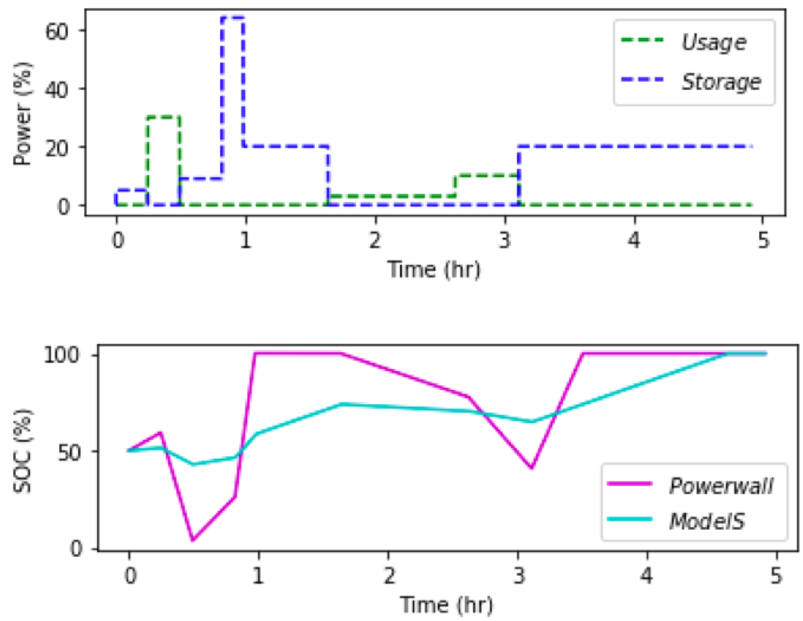


Figure 14: Tesla Car and Wall Batteries - Independent responses

owner can use the car to drive to work in the morning.

$$SOC_2(6am) = 1.0 \tag{10}$$

Additionally, to simulate the Tesla car being used to commute to work, an objective is placed on the SOC_2 to decrease to 0.4 by 4pm. In other words 60% of the battery was used in driving to and from work. This is applied in our model as a sum squared difference between the desired final SOC_2 at 4pm and the actual SOC_2 as shown in Equation 11. The parameter W_2 is the weight added to show the importance of this second objective.

$$Objective_2 = \min((SOC_2(4pm) - 0.4)^2 \times W_2) \tag{11}$$

Results and Discussion

The optimized system results are shown below. Figures 15 and 16 show the battery rate and SOC for the Powerwall and Tesla Model S car batteries for 1 or 2 Powerwall batteries, respectively. As described above, the Model S car battery is required to reach 100% charge by 6am in order to be ready for use. The car does not affect the system during the day when it is not home.

Figure 17 shows the effect of decreasing the number of installed solar panels from 70 to 56, which would save on initial equipment costs.

Figures 18 through 20 show the price of electricity during the day, the power used by each of the power sources, and the total (integrated) cost rate throughout the day for 1 or 2 Powerwall batteries. The daily revenue for each of these cases is shown on each graph. The “battery” label on Figures 18 through 20 includes both the Powerwall and Model S batteries.

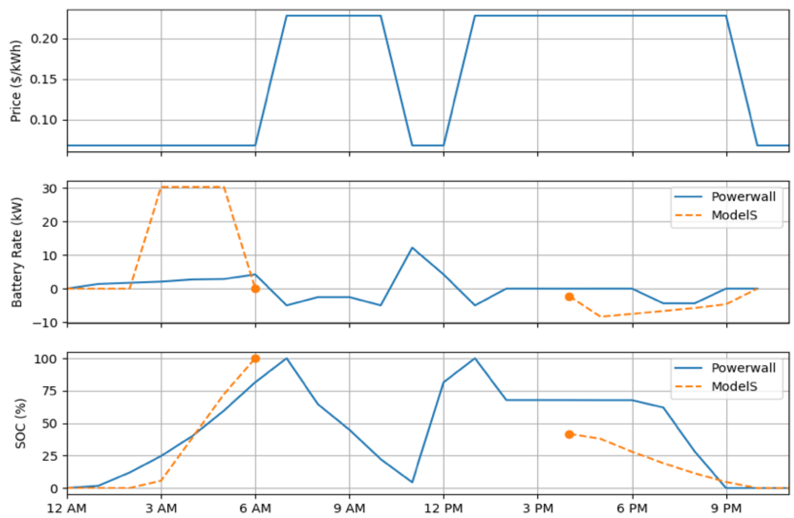


Figure 15: 1 Powerwall Battery; 70 Solar Modules

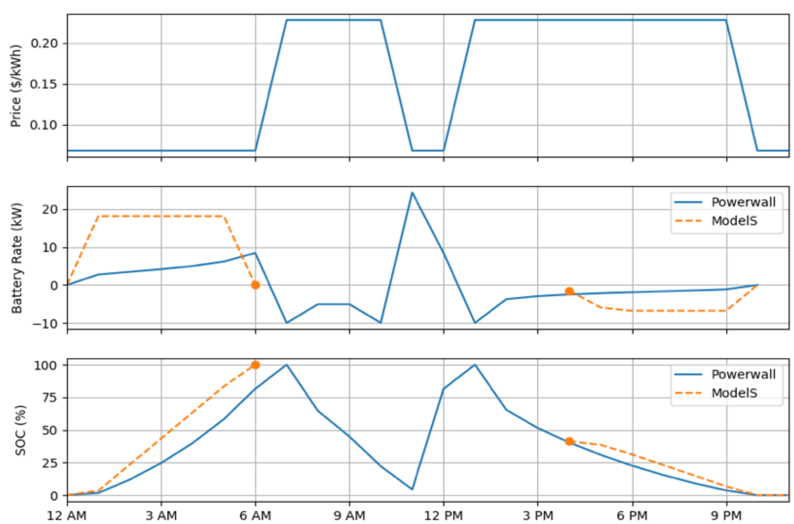


Figure 16: 2 Powerwall Batteries; 70 Solar Modules

Figure 20 shows the impact on the final cost over the 24 hour period with 56 solar panels instead of 70.

The optimized results show that a total of $17.18 is made each day with a 1 Powerwall battery system and $21.47 with a 2 Powerwall battery system (both with 70 solar modules). However, due to the high cost of extra Powerwall batteries, it would take a little over 5 years for it to be profitable to add one extra battery.

We recommend using 1 Powerwall battery due to the high cost of these batteries and the limited difference in final cost due to adding a second battery.

We also investigated the effects of varying the number of solar modules while using 1 Powerwall.

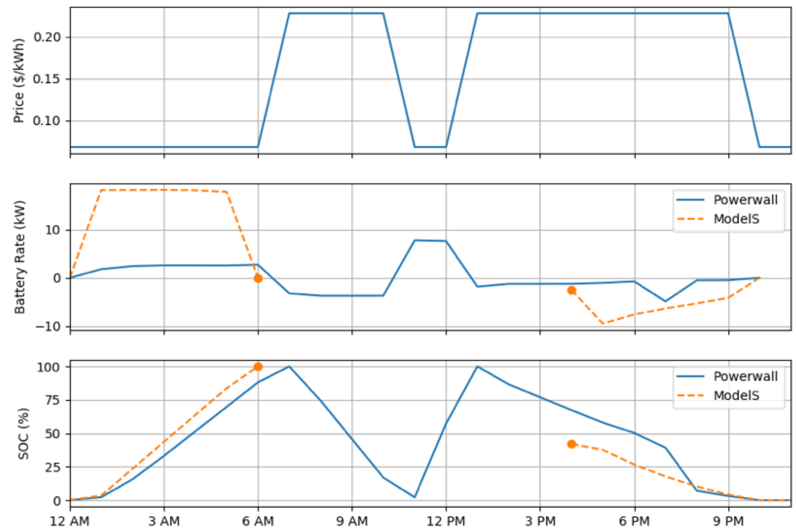


Figure 17: 1 Powerwall Battery; 56 Solar Moduless

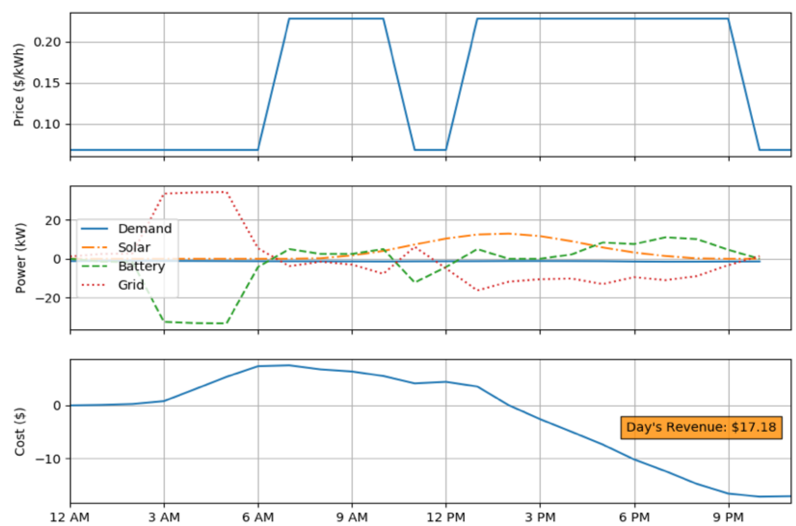


Figure 18: 1 Powerwall Battery; 70 Solar Modules

The results are found in Table 6 below. A total of $13.94 is made each day with a 1 Powerwall battery, 56 solar module system. It would take just over 3.5 years to pay for the 14 extra solar panels used in Figures 18 and 19 with the revenue from the electricity they generate. The decrease in the number of solar panels decreases the possible revenue that can be generated by about $3.24 per day.

From these results, we recommend using 70 solar panels (the maximum possible for our roof) since the savings per day is high ($17.18), and the payoff period is approximately equal to buying any number of panels for this system.

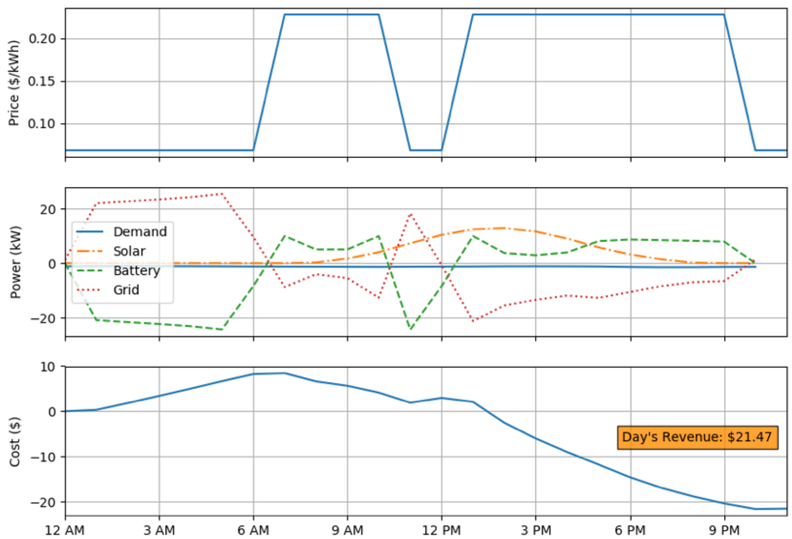


Figure 19: 2 Powerwall Batteries; 70 Solar Modules

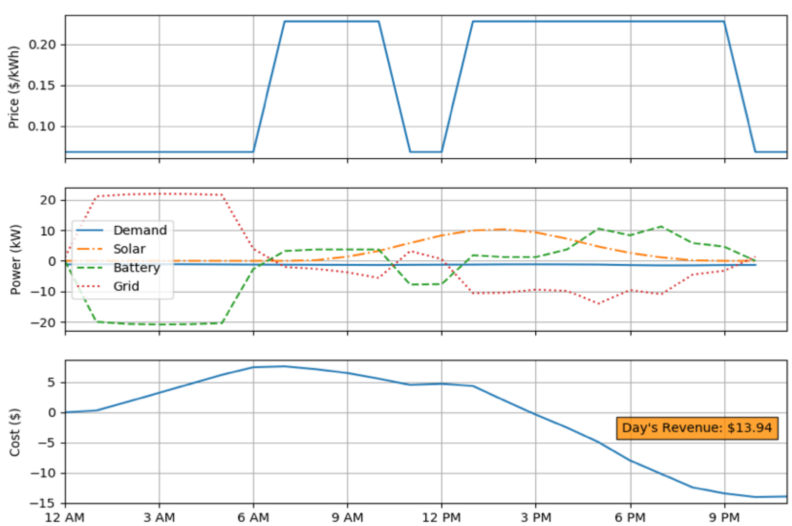


Figure 20: 1 Powerwall Battery; 56 Solar Modules

Table 6: Solar Module Investigation

N Solar Modules	Savings Per Day	Years to break even (from 0 solar modules)
0	$1.86	0
14	$5.00	3.66
28	$7.96	3.77
42	$10.80	3.86
56	$13.94	3.81
70	$17.18	3.76

Conclusions and Future Work

The cost of electricity for a single family home was minimized by varying the source of energy used to provide electricity over a 24 hour day. The optimal system includes 1 Powerwall battery, 70 solar modules, a Tesla car, and grid power. The daily weather determines the electricity availability from the solar panels. Daily electricity demand varies based on the family's power needs at home. By selling power back to the grid, this system results in a cost savings of $17.18.

Future work could improve the accuracy of this project. It would be beneficial to do a more involved cost analysis that includes initial equipment costs and equipment maintenance and lifetimes, and then figure out the return on investment based on those costs and the savings obtained by the optimized system. Restrictions on the amount of power that can be sold back to the grid should also be investigated. The model for the inverter efficiency and voltage requirements can be improved. The simulations could be run for more than one day at a time. A model that predicts solar irradiance based on the weather forecast could be produced. The solar cell temperatures were assumed to be at the same temperature as the outside air. However, this work can be improved in the future by predicting how the solar cell temperature actually changes throughout the day based on how much sunlight the cells receive.

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