Mathcad Lecture #2 In-class Worksheet
Working With Units in Mathcad

At the end of this lecture, you will be able to:
- Use dimensional analysis to aid engineering calculations
- Assign units to variables
- Perform mathematical operations using units
- Correctly convert temperature units

Introduction

- Mathematics considers relationships with numbers.
- Engineers usually associate numbers with dimensions and units.
- Dimensions and units are not the same
  - All **heights** have the same *dimensions.*
  - The height can be expression in different units: ft, in, m, mm, km, etc.
- Units are important in engineering.
    - 1+1 does not always equal 2 (e.g. 1 ft + 1 in = 13 in)
    - Equations must be dimensionally consistent (e.g. 1 ft + 1 hr is meaningless)

1. Dimensional Analysis

Explanation

Dimensions are generally categorized into two types. Basic dimensions are things you can measure and include mass (M), length (L), and time (T). Derived units are obtained by multiplying or dividing basic units and include force, power, and velocity. Units, though related to dimensions, are not identical concepts. A unit is a standard amount of a dimension. A foot is a standard measure of length as is a meter.

While a thorough treatment on dimensional analysis is beyond the scope of this course, several key concepts relating to dimensional consistency must be understood when working with units and can actually aid you in performing calculations.

Consider the following in regard to dimensions and units.

1. The dimensions on the Left Hand Side (LHS) of the equation must equal the dimensions on the Right Hand Side (RHS) of the equation (i.e. a mass cannot equal a length). This can be written as LHS [=] RHS.
2. Two quantities must have the same units to add or subtract.
3. Units can help get you to the answer

Class Exercises to do BY HAND

This is a key concept that we will visit over and over, especially when we talk about solving systems of equations.

1. Is the following equation dimensionally consistent: mv=tF

m = mass, v = velocity, t = time, and F = Force

$$M \cdot \frac{L}{T} = T \cdot M \cdot \frac{L}{T^2}$$

yes, dimensionally consistent

2. If the dimensions of power are ML^2T^{-3}, how do you calculate your power exerted when you bench press 70 kg? (What phyical quantities do I need to multiply/divide?)

- multiply the mass by g to get the force
- multiply force by distance moved
- divide by time it takes to do the rep

3. How can I calculate the distance traveled given a initial position (d_0), a velocity (v) and an acceleration (a)?

$$d = d_0 + v \cdot t + 0.5 \cdot a \cdot t^2$$

Note: the 0.5 come from solving a differential equation, but you can get the time dependence from analyzing the units.

4. What mass of salt is needed to make 150 mL of 2.0 M NaCl solution if the molecular weight of salt is 58.4 g mol^{-1}.

$$2.0M \cdot \frac{molNaCl}{L \cdot M} \cdot \frac{58.4gmNaCl}{molNaCl} \cdot \frac{L}{1000mL} \cdot 150mL = 17.52gmNaCl$$

2. Using Units in Mathcad

Mathcad can do the units for you!

Demonstration

Variable can be assigned units

$h := 10km$

$h = 1 \times 10^4 m$

Key Points:

- When Mathcad displays the variable, by default it does so in SI-like units (volumes are displayed in liters).
- You can change the default under Tools/Worksheet Options

You can do unit conversions by typing the desired units in the little box on the RHS of displayed results when they are selected.

$h = 6.214 \cdot mi$ $h = 2.187 \times 10^4 \cdot cubit$ $h = 49.71 \cdot furlong$

Caution: Because Mathcad must be dimensionally consistent, inputting incorrect units will give undesired results.

$$h = 2.778 \frac{m}{s} \cdot hr$$

You can define you own units.

$Mm := 10^6 m$ $h = 0.01 \cdot Mm$

$knotts := 69in$ $h = 5.706 \times 10^3 \cdot knotts$

$knotts\_min := 2min$ $test\_time := 50knotts\_min$ $test\_time = 1.667 \cdot hr$

CAUTIONS / HINTS / TIPS / AXIOMS FOR LIFE / DON'T FORGET

1. Never define a variable (like a mass) using **m**. m is meter in Mathcad.
2. Never define a variable (like the gas constant) as **R**. R is Rankine (a unit of temperature discussed below). I usually use R_g:=8.314 J/mol/K for the gas constant.
3. g is **NOT** grams, gm is grams.
4. Use caution when defining **g** as a variable. It is already defined as 9.8 m/s^2

$m := 10gm$ $a := 9.8 \frac{m}{s^2}$ $F := m \cdot a$ $F = 9.8 \times 10^{-4} \frac{kg^2}{s^2}$

Notice that you don't get Newtons!

Practice

1. Convert 13 kilograms to pounds and slugs.

$13kg = 28.66 \cdot lb$ $13kg = 0.891 \cdot slug$

2. Convert 200,000 Pascals into kilopascals, millimeters of mercury, atmospheres, bars, and pounds per square foot.

$P := 200000Pa$ $P = 200 \cdot kPa$ $P = 1.5 \times 10^{3} \cdot torr$ $P = 1.974 \cdot atm$

$P = 2 \cdot bar$ $P = 4.177 \times 10^{3} \cdot \frac{lbf}{ft^{2}}$ $P = 29.008 \cdot psi$

Demo: Letting Mathcad Do the Unit Conversions

In ChE 374, you will learn that the following equation, the Mechanical Energy Equation, is used to describe steady-state flow of an incompressible fluid:

$$w_s = \frac{P_2 - P_1}{\rho} + \frac{1}{2} \cdot \left(v_2^{\ 2} - v_1^{\ 2}\right) + g \cdot \left(z_2 - z_1\right)$$

Consider a section of pipe where the liquid is pumped up a hill. Calculate w_s (the pump work required) for the following values of the other parameters: P_1 = 4 psi, P_2 = 0 atm, v_1 = 1 ft/s, v_2 = 3 ft/s, ρ = 62.4 lb_m/ft^3, z_1 = 0 ft, z_2 = 100 ft. What do the unit mean (try multiplying the top and bottom by kg).

$P_1 := 4psi$ $P_2 := 0psi$ $v_1 := 1\frac{ft}{s}$ $v_2 := 3\frac{ft}{s}$ $\rho := 62.4\frac{lb}{ft^3}$ $z_1 := 0ft$ $z_2 := 100ft$

$w_s := \frac{P_2 - P_1}{\rho} + \frac{1}{2} \cdot \left(v_2^{\ 2} - v_1^{\ 2}\right) + g \cdot \left(z_2 - z_1\right)$ $w_s = 271.687\frac{m^2}{s^2}$ $w_s = 271.687 \cdot \frac{J}{kg}$

3. Temperature Units in Mathcad

Discussion

Temperature units are often problematic in Mathcad. There are two main reasons. The first is that converting between certain temperature units is not a simple multiplication but requires a more complex algebraic equation. For example, converting Celsius to Fahrenheit requires the following operation: f=9/5c+32. The second reason is that we talk about two different types of temperature in engineering. The first is the type you measure with a thermometer and the second is combined with other units and really signifies a change in temperature (e.g. °F/ft or J mol^{-1} K^{-1}. You handle the two differently.

- "Natively", Mathcad only knows about two temperature units: Kelvin (K) and Rankine (R).
- A change of 1 degree Kelvin is the same as 1 degree Celsius; the Kelvin scale simply moves the freezing point of water to be 273.15 so that 0 K is the lowest possible temperature (absolute 0).
- A change of 1 degree Rankine is the same as 1 degree Fahrenheit; the Rankine scale simply moves the freezing point of water to 459.67+32 = 491.67 so that 0 R is the lowest possible temperature (absolute 0).
- Kelvin and Rankine are absolute temperature scales (there are no negative temperatures).

Dealing with temperatures you actually measure.

The ***most sure way*** to deal with a temperature that you **measure** is to convert all °C to K and all °F to R.

$$T_R = (T_F + 459.67)\cdot R \quad T_K = (T_c + 273.15)\cdot K$$

For example, if the temperature of a gas is 30 °C, you would define:

$T_{gas} := (30 + 273.15)\cdot K$ $\quad$ $T_{gas} = 303.15\,K$

If the temperature of the water is is 60 °F, you would define:

$T_{water} := (60 + 459.67)\cdot R$ $\quad$ $T_{water} = 288.706\,K$

If you define all temperatures in terms of K and R, Mathcad will always do the calculations correctly because R = 0.555556 K (a simple multiplication conversion).

$T_{gas} = 545.67\cdot R$ $\quad$ $T_{water} = 519.67\cdot R$

Dealing with temperatures found with other units.

Certain physical properties, such as heat capacities or thermal conductivities, describe the ability of a substance with respect to heat inputs. The temperatures found in these quantities actually represent *changes in temperature*. For example, the heat capacity (or specific heat) (Cp) of liquid water at room temperature is 1 cal g^{-1} $°C^{-1}$. Physically, this means that if you add 1 calorie of heat to 1 gram of water, you will change the temperature by 1 °C.

- When you encounter temperatures with other units, simply replace C with K and F with R.

$$Cp_{water} := 1\,\frac{cal}{gm\cdot K}$$

Practice

1. What is the final temperature, T_{final}, in °F of 10 grams of gold after 11 calories of heat, q, are applied to it. The initial temperature, $T_{initial}$, is 70 °F. The specific heat of gold is 0.030 BTU lb_m^{-1} $°F^{-1}$. Note: q = mCp(T_{final}-$T_{initial}$) where m is the mass of the substance.

$T_{initial} := (70 + 459.67)\cdot R$ $\quad$ $Cp := 0.030\,\frac{BTU}{lb\cdot R}$ $\quad$ $mass := 10gm$ $\quad$ $q := 11cal$

$T_f := \frac{q}{mass\cdot Cp} + T_{initial}$ $\quad$ $T_f = 595.67\cdot R$ $\quad$ $T_{final} := 595.67 - 459.67$ $\quad$ $T_{final} = 136$ $\quad$ °F

Key Point

- Notice that we convert the "measured" temperature to Rankine before putting it into the equation.

More Practice

Compact Fluorescent Lamps (CFL's) are gaining popularity in replacing the traditional incandescent light bulbs that have been used since the time of Edison. CFL's usually use less than a third of the energy that a traditional lightbulb does. For example, a 14 W CFL gives the same light output as a 60 W incandescent bulb.

Currently, a CFL costs more than an incandescent light bulb does. For example, a 14 W CFL will cost about $2.50 while an incandescent will cost about $0.50. The question is, how much savings does the bulb give you.

Say you run a light bulb in your bedroom for 4 hours a day on average. The cost of electricity in Provo is 6.98 cents/kWhr. What is the elecricity cost of running a 14 W CFL for a year? What is the cost for a 60 W incandescent bulb?

$kWhr := 1kW \cdot 1hr$ $\quad elec_{cost} := \frac{.0698¤}{kWhr}$ $\quad time := 1yr \cdot 4\frac{hr}{day}$ $\quad cfl := 14W$ $\quad incan := 60W$

$elec\_cost_{cfl} := elec_{cost} \cdot time \cdot cfl$ $\quad elec\_cost_{cfl} = 1.428\ ¤$

$elec\_cost_{incan} := elec_{cost} \cdot time \cdot incan$ $\quad elec\_cost_{incan} = 6.119\ ¤$

A typical CFL lasts 9 years while an incandescent bulb will burn out after about a year. Estimate how much money you save over the life of a CFL bulb.

$n := 9$ $\quad bulb\_cost_{cfl} := 2.50¤$ $\quad bulb\_cost_{incan} := 0.5¤$

$total\_cost_{cfl} := bulb\_cost_{cfl} + elec\_cost_{cfl} \cdot n$ $\quad total\_cost_{cfl} = 15.349\ ¤$

$total\_cost_{incan} := n \cdot (bulb\_cost_{incan} + elec\_cost_{incan})$ $\quad total\_cost_{incan} = 59.567\ ¤$