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CHAPTER 7

Cycling Exertion Simulator to Teach Control of a Depleting Actuator

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CHAPTER 7

Cycling Exertion Simulator to Teach Control of a Depleting Actuator



The controller moves 1 how hard the rider tries, through the legs

The controller reads 1 crank power, from strain gauges in the crank 2 heart rate, from a chest strap

What disturbs it 1 the grade of the road 2 a headwind

Notice that the controller can only set how hard the rider tries, while the watts that effort buys come from the legs and fall when a hard effort is held. Illustration, not to scale.

IN PLAIN TERMS **Pacing a Rider Whose Strength Runs Down**

The main idea. A controller on a bicycle cannot set the rider's power, only how hard the rider tries. Above the level a rider can keep up, the same effort buys less power as a small store of energy runs low.

The challenge. Heart rate and blood lactate, a substance that rises in the blood with hard effort, warn of too much effort only minutes later. One effort setting has to serve a power target and these slow limits at once.

Why it matters. A battery that weakens as it is used poses the same problem.

THREE WAYS TO READ THIS CHAPTER

Explore	11 min	The first page, the fact box and the history box of Section 7.2, the two try-it boxes of Section 7.4, the fact box of Section 7.7 and the key points
Study	60 min	The whole chapter with its pause points and the twelve exercises
Practice	20 min	The first page, Section 7.7, Section 7.8, Section 7.9 and the open challenge of Section 7.10

The control question A rider sets exertion, and the watts that exertion buys fall as a finite work capacity drains. How should a pacing loop and two slow physiological limits share that one actuator, and what is the terrain or the end of the ride worth to a controller that plans ahead?

Learning objectives

- Compute the gain from exertion to power below and above the critical-power exertion, and the fade time constant of a held exertion.
- Distinguish saturation against a drained store from integral windup on a run of the power loop.
- Explain how a low selector with margin-scaled external reset shares one actuator, and why the margin is scaled and never negative.
- Compute how much speed a watt buys on the flat and on a climb, and judge what terrain preview is worth on a timed ride.
- Compare the regulator and the predictive controller on the challenge hour and say where the predictive lead is earned.

Prerequisites Sections 1.4.5 (late samples), 1.4.6 (stores and overrides), 1.6.2 (velocity form) and 1.6.4 (scenarios and data). Section 1.6.3 and Chapter 6, which shares the athlete model, are useful but not required.

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See also Volume 4, Chapter 4 and Volume 4, Chapter 3 for one actuator shared among a governing loop and its limits through a selector with external reset. Volume 4, Chapter 14 for a store that drains as it is used and a defense that ends when it runs out.

7.1 Introduction

An actuator is usually treated as a fixed map from the controller output to the process input. A valve at half travel passes about the same flow tomorrow as it does today, and a tuning written against it stays written. Some actuators do not keep their gain. A battery sags as it discharges, a heat exchanger loses duty as it fouls, a catalyst bed loses activity and an accumulator delivers less flow as it empties. In each case what a given actuator position buys depends on how hard the actuator has been driven in the recent past, and a controller that assumes a fixed gain saturates against a limit that is not a mechanical stop and winds its integral up while it waits [1].

A person on a bicycle is the most familiar actuator of that kind. The quantity a rider decides is effort, which has its own psychophysical scales of perceived exertion [2]. What the effort buys in watts is not fixed. Above critical power the rider draws on a finite work capacity, and the greatest power available at any instant falls in proportion to what remains of that capacity [3, 4]. Cyclists asked to hold a constant rating of perceived exertion let their power output decline steadily, and fastest in the heat [5, 6]. Power is therefore something a rider produces rather than something a rider sets, and effort is the natural manipulated variable.

Three control problems follow from that structure, and none of them is particular to cycling. The first is saturation against a store instead of a stop, where the actuator reaches its limit because the capacity behind it has run down and the integral has to be kept from winding up while it is there. The second is override selection, where one actuator serves a pacing loop and several limits whose measurements trail the effort by minutes, and the losing loops have to stay ready to take over without winding up and without deadlocking the selector [1]. The third is economic operation over a fixed horizon, where the objective grows with the quantity the limits forbid and a stored capacity is worth nothing once the ride ends [7]. A peaking battery system, a generator on a fuel allowance and a batch unit with a fixed cycle time present the same three.

Road-cycling power models predict the power a speed requires on a given road from aerodynamic drag, rolling resistance, gravity and drivetrain loss, and they reproduce measured road power closely [8, 9]. Optimal-pacing studies use such models to schedule power over a course, either against a fixed power budget [10] or against individualized dynamics of fatigue and recovery [11]. Feedback

control of heart rate on a cycle ergometer adjusts the work load to hold a heart-rate reference, and nonlinear and robust designs have been demonstrated in closed-loop experiments [12, 13]. However, both lines of work take power as the decision variable, so the loss of authority as the work capacity drains appears as a constraint on an optimizer or does not appear at all, and neither exercises the selector logic that one actuator shared by a pacing target and several slow limits requires.

This chapter works with a cycling simulator whose manipulated variable is exertion rather than power, so that pacing, override selection and anti-windup all have to work through an actuator whose gain depends on how it has been used. Three measured observations about that plant run through it. A held exertion above the sustainable level settles onto critical power with a closed-form time constant. An external reset whose margin is not scaled to each override's distance from its limit gives up more than a third of the hour's distance, and one whose margin is allowed to go negative ratchets the actuator down. Preview of the terrain is worth almost nothing on this plant, because on a timed ride the grade enters the objective and not the constraints. The road mechanics, the critical-power structure and the control laws are standard.

Section 7.2 traces ergometry, critical power and pacing. Section 7.3 carries the nineteen-state model of the rider, the bicycle and the road, with every equation and constant needed to reimplement it, and Section 7.3.11 says where each assumption breaks down. Section 7.4 measures the gain from exertion to power, the lags of the limits behind the effort and what the terrain changes. Section 7.5 sets out the instruments, the three proportional-integral loops on a low selector and the receding-horizon alternative, and Section 7.6 defines the scorecard. Section 7.7 reports the twenty cases, from open-loop mechanics through saturation and slow overrides to an hour on a rolling circuit and a time trial, and Section 7.7.5 measures what each selector rule is worth. Section 7.8 describes the real power meters, heart-rate straps and head units, and Sections 7.9 to 7.11 give the limitations, the discussion and the conclusion.

7.2 Background: Ergometry, Critical Power and Pacing

The bicycle ergometer and the critical-power construct came out of one laboratory tradition, whose early steps are told in the history box at the end of this section. In its two-parameter form the tolerable duration of a constant power above critical power is the work capacity W' divided by the excess, and critical power is the highest rate of work that can be sustained without drawing on W' [3].

Two refinements made the construct usable inside a simulation. The three-parameter model added the maximal instantaneous power and implied that the power available at any instant is proportional to the work capacity remaining [4], which is the structure of the exertion map in Section 7.3. The three-minute all-out test turned that structure into a measurement. An all-out effort long enough to exhaust W' finishes on a stable power equal to critical power, and the work done above that power matches W' [14]. For intermittent riding the W' balance models track depletion above critical power and reconstitution below it, in an integral and a differential form that rest on different assumptions about recovery [15]. Critical power and the blood-lactate threshold markers correlate closely in recreational cyclists, but with limits of agreement wide enough that the two cannot be used interchangeably [16], and the simulator carries them as separate parameters for that reason.

> **✦ DID YOU KNOW? A Store That Refills Slowly**
>
> In one laboratory study, the work capacity above critical power came back to 37, 65 and 86 % of its full value after 2, 6 and 15 min of recovery at a nominal 20 W. Its half time of 234 s was slower than that of oxygen uptake, 74 s, and faster than that of blood lactate, 1366 s [17].

The mechanics of the moving bicycle were written down separately. Towing experiments gave the resistance of a cyclist as a rolling term plus an aerodynamic term quadratic in air speed, and an equation of motion for the rider followed from it [8]. A validated model of road-cycling power later reproduced power measured on the road with $R^2 = 0.97$ and a standard error of 2.7 W [9]. Pacing

studies built on that model predict that varying a 289 W mean power by 10 % with gradient and wind saves 126 s on a hilly course against constant power [10], and optimal-control formulations treat the rider's maximal power as a time-varying constraint driven by fatigue and recovery [11]. Feedback control reached the ergometer before it reached the road. Heart rate has been regulated through the work load by a nonlinear generalization of PI control [12] and by robust linear designs with root-mean-square tracking errors of 2.5 to 3.1 bpm over 73 closed-loop experiments [13].

The experiments that fix effort instead of power are the ones closest to this simulator. At a fixed rating of perceived exertion, cyclists let power decline linearly in cool, normal and hot trials, fastest in the heat [5], and over one hour at a constant rating in the moderate domain, power output, oxygen uptake and ventilation all fell [6]. Those studies measure what a held effort buys and do not close a loop around it, and the controllers in the pacing and heart-rate literature do not take effort as their output. The application of this chapter does both on one plant, with the exertion map carrying the three-parameter structure and the controllers acting on the exertion alone.

> ⧖ HISTORY **From One Muscle Group to the Bicycle (1913 to 1981)**
>
> The bicycle entered the physiology laboratory as an instrument for imposing a known rate of work, and an ergometer built for respiration measurements during pedaling was described in 1913 [18]. That the power a person can hold falls with the time it must be held had long been established. A simple mathematical frame for it came in 1965 with the concept of critical power, defined for a single synergic muscle group. The most work that can be done grows in a straight line with the time it lasts, with a fixed energy reserve as the intercept and critical power as the slope [17, 19]. In 1981 the concept was applied to cycling, where critical power correlated highly with the anaerobic threshold [17, 20]. Measuring it stayed costly. A valid value took two to seven laboratory visits until a single all-out test of three minutes arrived [17].

7.3 Rider, Bicycle and Road Model

The plant is one rider on one bicycle on a straight road, either for a fixed 3600 s or over a fixed distance, starting from a standstill. Exertion $E \in [0, 1]$ is the only manipulated variable. The road is either set by hand as a percent grade $g_{\%} = 100 \tan\theta$ or supplied as a function of distance by one of two fixed routes, and a headwind w is a second disturbance. The quantities that appear in the equations are labeled on the drawing in Figure 7.1. The model works in SI units, except that heart rate stays in beats per minute and lactate and glucose in mmol/L, the units in which their limits are written. The recreational preset is the design case throughout.

The model integrates nineteen states, and Table 7.1 classifies the ones a controller or a student acts on. Five belong to the road and the drivetrain, being speed, distance, altitude, cumulative climbing and the applied exertion. Eleven are physiological, being oxygen uptake, heart rate, lactate, the work capacity, fatigue, core temperature, respiratory frequency, muscle and liver glycogen, circulating glucose and carbohydrate in the gut. One is an interstitial glucose estimate that is displayed and not controlled, and two accumulate the energy absorbed by the brakes and the work done at the cranks. Every parameter is adjustable and displayed, and nothing is withheld.

7.3.1 Exertion and the Power It Buys

The applied exertion follows the command with a first-order neuromuscular lag, as shown in Equation (7.1).

$$\tau_a \frac{\mathrm{d}E}{\mathrm{d}t} = u - E \tag{7.1}$$

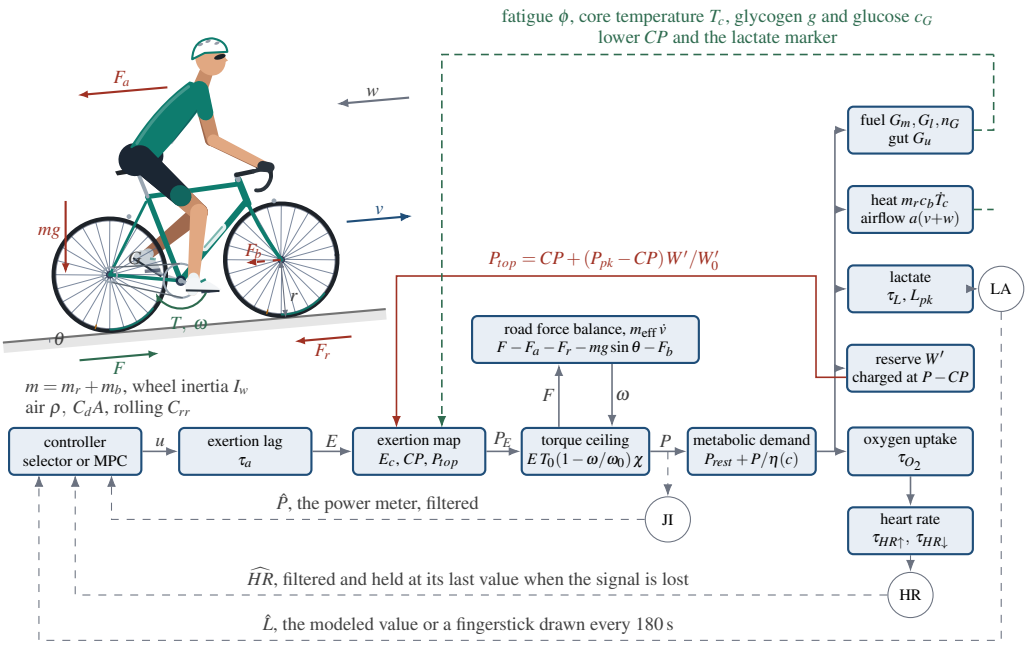


Figure 7.1 One rider on one bicycle on a road of grade angle θ. The controller sets the exertion command u, the rider applies exertion E after a neuromuscular lag τ_a, and the exertion map turns it into the power P_E the rider asks for, from critical power CP and a ceiling P_{top} that falls with the remaining work capacity W'. The torque ceiling bounds the crank torque T at crank speed ω, and the drive force F through gear G, wheel radius r and drivetrain efficiency η_d works against aerodynamic drag F_a in the apparent wind $v+w$, rolling resistance F_r, gravity mg and the brakes F_b. Crank power P sets the metabolic demand, which drives oxygen uptake, heart rate, lactate, the reserve, heat and fuel in parallel, and the slow states return to critical power and the lactate marker as one bundle. The controller reads a filtered power $\hat{P}$, a filtered heart rate $\widehat{HR}$ and a lactate value $\hat{L}$, and commands exertion alone. The rider and bicycle are the application's own drawing.

where $u \in [0,1]$ is the command and $\tau_a = 1.0\,\text{s}$. Critical power at the cranks falls with accumulated fatigue and with the fuel state, as shown in Equation (7.2).

$$\begin{aligned} CP &= CP_0\,(1-\phi)\,\Psi_G\,\Psi_c, \qquad g = \frac{G_m + G_l}{G_{m,0} + G_{l,0}} \\ \Psi_G &= 0.80 + 0.20\,\text{clamp}\left(\tfrac{g}{0.15}, 0, 1\right) \\ \Psi_c &= 0.85 + 0.15\,\text{clamp}(c_G - 3,\, 0,\, 1) \end{aligned} \tag{7.2}$$

where CP_0 is the value for a fresh rider of the preset, ϕ is the fatigue accumulated so far, g is the share of the starting glycogen still in the stores and c_G is the blood glucose concentration. Both fuel terms stay at one until the stores drop under 15 % and glucose under 4 mmol/L, which leaves critical power almost constant through a fueled hour at moderate effort.

The power that an exertion asks for is written in two domains joined at the exertion E_c at which a

Table 7.1 Roles of the Variables for the Recreational Preset, With Values Read at 30 min of the Level Hour Under the Selector and a 215 W Power Target.

Symbol	Role	Nominal	Range
E	MV	0.702	0–1
P	CV, measured	206 W	0 to P_{top}
HR	CV, measured limit	156.8 bpm	SP 168 bpm
L	CV, estimated	3.74 mmol/L	SP 3.7, limit 4.0
s	objective, timed	16.95 km	—
$g_{\%}$	DV	0 %	−10 to +12
w	DV	0 m/s	−8 to +15
v	state, shown	34.28 km/h	≥ 0
W'	state, shown	16.0 kJ	0–16 kJ
T_c	state, shown	37.91 °C	34–41.5 °C
$G_m + G_l$	state, shown	350 g	0–425 g
$\dot{V}_{O_2}$	state, shown	37.7	3.5–48
ϕ	state, shown	0.000	0–0.40
c	output	88.8 rpm	0–150 rpm
G	discrete, automatic	3.06	1.21–4.55

$\dot{V}_{O_2}$ in mL/(kg min).

fresh rider produces exactly critical power, as shown in Equation (7.3).

$$P_E = \begin{cases} CP\,\dfrac{E}{E_c}, & E \leq E_c \\ CP + (P_{top} - CP)\,\dfrac{E - E_c}{1 - E_c}, & E > E_c \end{cases} \qquad P_{top} = CP + \max\left(0,\ P_{pk}\left(1 - \tfrac{\phi}{2}\right) - CP\right)\frac{W'}{W'_0} \tag{7.3}$$

where P_{pk} is the sprint power of a fresh rider at the optimal cadence, W' is the work capacity above critical power that remains and W'_0 is its full value. Below E_c the rider is inside the sustainable domain and exertion buys a proportional share of critical power. Above it the extra exertion buys a share of the margin between critical power and the ceiling P_{top}, and that ceiling falls linearly with the W' left, which is the three-parameter form in which the power available at any instant is proportional to the remaining capacity [4]. At an empty reserve every exertion above E_c buys critical power and no more. The actuator has then lost its whole upper range without moving, and $E_c = 0.75$ for every preset.

7.3.2 Torque, Cadence and Gearing

Power is torque times crank speed, and a bicycle is started from rest by torque alone. The torque the rider will apply is bounded by exertion times a torque-cadence line through the sprint peak, as shown

in Equation (7.4).

$$\begin{aligned} \omega &= \frac{v}{rG}, \qquad c = \frac{60\,\omega}{2\pi} \\ T_{\max} &= E\,T_0\left(1 - \frac{\omega}{\omega_0}\right)\chi \\ \chi &= \operatorname{clamp}\left(\frac{c_{\max} - c}{15\,\text{rpm}}, 0, 1\right) \\ T &= \min\left(\frac{P_E}{\omega},\ T_{\max}\right) \\ F &= \min\left(\frac{\eta_d\,T}{G\,r},\ F_{\max}\right) \end{aligned} \tag{7.4}$$

where G is the gear in wheel turns per crank turn, r is the wheel radius, η_d is the drivetrain efficiency and $F_{\max}$ is the largest drive force the tire and the rider's balance allow. The constants $\omega_0 = 2\omega_{opt}$ and $T_0 = 2P_{pk}(1-\phi/2)/\omega_{opt}$ put the apex of $T_{\max}\omega$ at the optimal cadence $c_{opt} = 120\,\text{rpm}$, where it equals the sprint power when $E = 1$. The torque-velocity and power-velocity relations of sprint cyclists peak in power at an optimal pedaling rate [21], and the model takes the torque-cadence relation as a straight line through that peak, giving $T_0 = 151.2\,\text{N m}$ and ω_0 equivalent to 240 rpm for the recreational preset. When the traction cap binds, the torque is recomputed from $F_{\max}$, and the delivered crank power is $P = T\,\omega$ in every case. At a standstill $\omega = 0$, the torque is $E\,T_0$ and the power is zero, so the bicycle starts on force and a gentle exertion gives a gentle start. The taper χ replaces a hard cutoff at $c_{\max} = 150\,\text{rpm}$, the highest cadence the rider will pedal, because a cutoff made a rider in too small a gear alternate between pedaling and coasting on successive integration steps. Pedaling stops when $E \le 0.004$ or $\chi = 0$, and the freewheel then leaves the cranks still while the wheels turn.

The drivetrain is a ladder of eleven ratios spaced geometrically from $G_{lo} = 1.21$, which is a 34-tooth chainring on a 28-tooth cog, to $G_{hi} = 4.55$, which is 50 on 11, with $G_i = G_{lo}(G_{hi}/G_{lo})^{i/10}$. Automatic shifting moves one gear per 1 s scan whenever cadence leaves the band $c_{pref} \pm 10\,\text{rpm}$ around the preferred cadence $c_{pref} = 88\,\text{rpm}$, and it selects the lowest gear below 0.3 m/s. The lowest, the middle ($\sqrt{G_{lo}G_{hi}}$) and the highest gear can also be fixed as teaching settings.

7.3.3 Road Force Balance

Road speed follows from the force balance along the road, as shown in Equation (7.5).

$$\begin{aligned} m_{\text{eff}}\frac{\mathrm{d}v}{\mathrm{d}t} &= F - F_a - F_r - m\,g\sin\theta - F_b \\ F_a &= \tfrac{1}{2}\,\rho\,C_d A\,(v+w)\,|v+w| \\ F_r &= C_{rr}\,m\,g\cos\theta \\ F_b &= k_b \max(0,\ v - v_{cap}), \qquad m_{\text{eff}} = m + I_w/r^2 \end{aligned} \tag{7.5}$$

where $m = m_r + m_b$ is the mass of the rider and the bicycle, I_w is the rotational inertia of both wheels, $\theta = \arctan(g_{\%}/100)$, w is the headwind and v_{cap} is the descending speed the rider brakes to hold. The resistance terms are the standard road-cycling model [8, 9]. A bicycle at rest with nothing pushing it forward stays at rest, because the rider has a foot down, so $\dot{v} = 0$ whenever $v = 0$ and the net force is not positive. Distance, altitude, cumulative climbing, braking energy and crank work are the integrals of v, $v\sin\theta$, $\max(0, v\sin\theta)$, $F_b v$ and P. For the recreational preset $m_{\text{eff}} = 84.75\,\text{kg}$, and 50 % exertion on the level gives 146.67 W and a steady 30.10 km/h, at which the wheel power equals drag plus rolling resistance to within 0.1 %, an equality the application's test suite checks.

7.3.4 Metabolic Demand and Gross Efficiency

Metabolic demand is the resting turnover plus crank power divided by a gross efficiency that depends on cadence, as shown in Equation (7.6).

$$\begin{aligned} P_{met} &= P_{rest} + \frac{P}{\eta(c)}, \qquad P_{rest} = \frac{3.5\,\varepsilon_{O_2}}{60}\,m_r \\ \eta(c) &= \max\left(0.10,\ \eta_0\left[1-\kappa\left(\frac{c-c_\eta}{60}\right)^2\right]\right) \end{aligned} \tag{7.6}$$

where $\varepsilon_{O_2} = 20.9\,\mathrm{J/mL}$ is the energy equivalent of oxygen, $\eta_0 = 0.235$ is the peak gross efficiency, $c_\eta = 60\,\mathrm{rpm}$ is the most economical cadence and $\kappa = 0.08$ is the fraction of efficiency lost 60 rpm away from it. The parabola is a declared teaching curve and not a fit. Energy expenditure in cycling is close to linear in work rate, with cadence a secondary factor, and a review of the evidence finds no firm energetically optimal cadence [22]. Equation (7.6) is the only route by which gearing reaches the physiology. At the preferred 88 rpm the efficiency is 0.2309, against 0.2350 at 60 rpm and 0.2162 at 120 rpm. Grade does not appear in it, so a climb at constant exertion changes the speed of the bicycle and leaves the demand on the body where it was.

7.3.5 Oxygen Uptake, Heart Rate and the Work Capacity

The physiology is the lumped athlete model that the treadmill of Chapter 6 also runs, charged here through a bicycle adapter. Oxygen uptake, expressed as the metabolic power it delivers, trails the demand by a single time constant and cannot exceed the aerobic maximum, as shown in Equation (7.7).

$$\begin{aligned} P_{aer} &= \frac{m_r\,\dot{V}_{O_2,\max}\,\varepsilon_{O_2}}{60} \\ f(x) &= \max\left(0,\ \frac{x-P_{rest}}{P_{aer}-P_{rest}}\right) \\ \tau_{O_2}\,\frac{\mathrm{d}\dot{V}}{\mathrm{d}t} &= \min(P_{met},\ P_{aer}) - \dot{V} \end{aligned} \tag{7.7}$$

where $\dot{V}$ is the delivered oxygen uptake in watts and f is the fraction of the oxygen-uptake reserve that a metabolic power represents. For the recreational preset $P_{rest} = 91.4\,\mathrm{W}$ and $P_{aer} = 1254.0\,\mathrm{W}$. The work capacity is charged in crank watts against critical power in crank watts, as shown in Equation (7.8).

$$\frac{\mathrm{d}W'}{\mathrm{d}t} = \begin{cases} -(P-CP), & P > CP \\ (CP-P)\,k_r\left(1-\dfrac{W'}{W'_0}\right), & P \le CP \end{cases} \tag{7.8}$$

where $k_r = 0.5$ is a recovery efficiency. Below critical power this is the differential form of the W' balance, with reconstitution proportional to the deficit [15], slowed by k_r. Heart rate is driven by the delivered oxygen uptake rather than by the demand, with separate onset and recovery lags and drift terms in core temperature and fatigue, as shown in Equations (7.9) and (7.10).

$$\begin{aligned} HR_{eq} &= HR_{rest} + (HR_{\max} - HR_{rest})\,f(\dot{V}) \\ &\quad + k_T \max(0, T_c - 37.8) + k_F\,\frac{\phi}{\phi_{\max}} \end{aligned} \tag{7.9}$$

$$\tau_{HR}\,\frac{\mathrm{d}HR}{\mathrm{d}t} = \mathrm{clamp}(HR_{eq},\ 35,\ 1.02\,HR_{\max}) - HR \tag{7.10}$$

where τ_{HR} is $\tau_{HR\uparrow}$ while HR_{eq} exceeds HR and $\tau_{HR\downarrow}$ otherwise. The two drift terms make heart rate climb at a constant power once the rider warms or tires.

7.3.6 Blood Lactate

Lactate is modeled in a single mixed compartment. Its steady value rises as a power of the metabolic intensity above a knee and is calibrated to cross the constraint $L_4 = 4\,\mathrm{mmol/L}$ at the rider's lactate-marker power, as shown in Equation (7.11).

$$\begin{aligned} f_T &= \frac{P_L/\eta(c_{pref})}{P_{aer} - P_{rest}} \\ f_{T,e} &= \max\bigl(0.40,\ f_T\,(1-\phi)\,\Psi_G\Psi_c\bigr) \\ x &= \max\left(0,\ \frac{f(P_{met}) - f_0}{f_{T,e} - f_0}\right) \\ L_{ss} &= L_{rest} + (L_4 - L_{rest})\,x^q\,\Lambda \\ L^* &= \begin{cases} L_{ss}, & L_{ss} \le 8 \\ 8 + \Delta_L\left(1 - \mathrm{e}^{-(L_{ss}-8)/\Delta_L}\right), & L_{ss} > 8 \end{cases} \\ \tau_L \frac{\mathrm{d}L}{\mathrm{d}t} &= L^* - L \end{aligned} \tag{7.11}$$

where P_L is the crank power at which steady lactate crosses L_4 at the preferred cadence, $f_0 = 0.35$ is the knee, $q = 3$ is the curvature, $\Lambda = 0.6 + 0.4\,\mathrm{clamp}(g/0.2, 0, 1)$ carries a depleted store, the denominator $f_{T,e} - f_0$ is floored at 0.05, and $\Delta_L = L_{pk} - 8$ with $L_{pk} = 20\,\mathrm{mmol/L}$ sets a soft ceiling. The ceiling is the one change the bicycle needs in the lactate model. A sprint reaches several times the metabolic power at maximal oxygen uptake, where a power law calibrated on submaximal work asks for hundreds of millimoles per liter, so L^* leaves the curve alone up to 8 mmol/L and bends it with a continuous slope toward L_{pk} above that. The marker $P_L = 210\,\mathrm{W}$ sits 10 W below critical power for the recreational preset, and the application's test suite checks that every preset settles on exactly 4 mmol/L at its own marker power.

7.3.7 Fatigue, Core Temperature and Breathing

Fatigue is what lowers critical power in (7.2) when lactate stays above the constraint or the core stays hot, and it recovers whenever the rider works below critical power, as shown in Equation (7.12).

$$\begin{aligned} \Gamma &= k_L\,\frac{\max(0, L - L_4)}{\max(0.5,\ L_4)} + k_{TF}\max(0, T_c - 38.5) \\ \frac{\mathrm{d}\phi}{\mathrm{d}t} &= \Gamma\left(1 - \frac{\phi}{\phi_{\max}}\right) - k_\phi\,\phi\,\mathrm{clamp}\left(1 - \frac{P}{CP}, 0, 1\right) \end{aligned} \tag{7.12}$$

Heat is balanced in a single body node whose losses are referenced to the resting state, and every crank joule leaves the body as work, as shown in Equation (7.13).

$$\begin{aligned} m_r c_b \frac{\mathrm{d}T_c}{\mathrm{d}t} &= \max(0,\ P_{met} - P) - Q_{loss} \\ Q_{loss} &= P_{rest} + G_d\,(T_c - 37.0) \\ &\quad + \min\bigl(G_e(T_c - 37.05),\ E_{\max}\bigr) \end{aligned} \tag{7.13}$$

The conductances scale with the rider and the air, and the airflow over the rider is the apparent wind, as shown in Equation (7.14).

$$\begin{aligned} a &= \mathrm{clamp}\left(a_f + \sqrt{\max(0,\ v+w)/v_{ref}},\ 0,\ 1.6\right) \\ \Phi &= \frac{m_r \dot{V}_{O_2,\max}}{75 \times 45} \\ G_d &= (g_0 + g_a a)\,\frac{m_r}{75}\,\mathrm{clamp}\left(1 + \tfrac{20-T_{air}}{25}, 0.3, 1.8\right) \\ G_e &= k_s \Phi\,(1 - 0.55\varphi)(0.45 + 0.55a) \\ E_{\max} &= E_0 \Phi\,(1 - 0.6\varphi)(0.45 + 0.55a) \end{aligned} \tag{7.14}$$

where a_f is any extra airflow from a fan or a breeze, $v_{ref} = 8\,\mathrm{m/s}$ is the air speed that gives full convective cooling, φ is the relative humidity, and Φ scales the rider's aerobic capacity by that of a 75 kg rider at 45 mL/(kg min). In this equation a is the airflow and not the loop action of Section 1.3, and $E_{\max}$ and E_0 are the evaporative ceiling and its constant, not the exertion. The apparent wind is the single change the bicycle makes to the thermal model, and it is why a slow climb with the wind behind is the hottest riding in the application. The ride is stopped when T_c reaches 40.5 °C. Writing the loss about 37.0 °C makes a resting rider in still air at 20 °C exactly steady. Respiratory frequency follows $\tau_R \dot{f}_R = \min\big(70,\ f_{R,0} + k_R f(\dot{V}) + 2\max(0, L-2) + 3\max(0, T_c - 38)\big) - f_R$ in breaths per minute and is displayed rather than controlled.

7.3.8 Carbohydrate, Glucose and Feeding

Both fuels burn at once. The carbohydrate share rises with intensity, fat oxidation has a rate ceiling, and anything supplied above the aerobic maximum is charged to carbohydrate, so the work capacity is never free energy, as shown in Equation (7.15).

$$\begin{aligned} \sigma &= \mathrm{clamp}\big(0.20 + 0.78\, f(P_{met}),\ 0.15,\ 0.98\big) \\ P_F &= \min\left(P_{F,\max}\left[0.7 + 0.3\left(1 - \tfrac{G_m}{G_{m,0}}\right)\right],\ (1-\sigma)P_{del}\right) \\ \dot{n}_C &= \frac{P_{del} - P_F + P_{an}}{e_G}, \qquad P_{del} = \min(P_{met}, P_{aer}) \end{aligned} \tag{7.15}$$

where $P_{an} = P_{met} - P_{del}$ and $e_G = 3153\,\mathrm{J/mmol}$ is the energy in a millimole of glucose equivalent. The carbohydrate flux is drawn from muscle glycogen and from circulating glucose, the liver and gluconeogenesis defend the circulating pool, and carbohydrate eaten during the ride empties from the gut into the blood, as shown in Equation (7.16).

$$\begin{aligned} \beta &= \mathrm{clamp}\left(0.18 + 0.35\left(1 - \tfrac{G_m}{G_{m,0}}\right), 0, 0.6\right) \\ \dot{n}_w &= \max\big(0,\ \beta\dot{n}_C + 0.02\,(c_{G,0}V_G - n_G)\big) \\ \dot{n}_g &= \min(k_g,\ \dot{n}_w), \qquad \ell = \mathrm{clamp}\left(\tfrac{G_l}{0.15\,G_{l,0}}, 0, 1\right) \\ \dot{n}_H &= \mathrm{clamp}(\dot{n}_w - \dot{n}_g,\ 0,\ \dot{n}_{H,\max}\,\ell) \\ \frac{\mathrm{d}G_m}{\mathrm{d}t} &= -(1-\beta)\,\dot{n}_C M_G, \qquad \frac{\mathrm{d}G_l}{\mathrm{d}t} = -\dot{n}_H M_G \\ \frac{\mathrm{d}G_u}{\mathrm{d}t} &= -\frac{G_u}{\tau_u}, \qquad \dot{n}_r = k_u \max\left(0,\ \frac{n_G}{V_G} - c_{G,0}\right) \\ \frac{\mathrm{d}n_G}{\mathrm{d}t} &= \dot{n}_H + \dot{n}_g - \beta\dot{n}_C + \frac{G_u}{\tau_u M_G} - \dot{n}_r \end{aligned} \tag{7.16}$$

where n_G is circulating glucose in millimoles, $c_G = n_G/V_G$, $V_G = 0.25\,\mathrm{L/kg}$ times body mass is its distribution volume, $M_G = 0.180\,16\,\mathrm{g/mmol}$, $\tau_u = 600\,\mathrm{s}$ is the gastric emptying constant and $k_u = 0.12\,\mathrm{L/s}$ is an illustrative uptake above the resting concentration. The uptake term is not an insulin model. It keeps a meal from accumulating in a small circulating compartment, it is zero whenever glucose is at or below its starting value, and the glucose it removes is oxidized in place of fat, which is not a tracked store. A feeding event adds 40 g to G_u. Carbohydrate eaten during prolonged cycling holds plasma glucose up and postpones fatigue without slowing the use of muscle glycogen [23], which is the behavior (7.16) reproduces. An interstitial glucose estimate follows c_G with a 480 s lag for display. After each step the states are bounded to $W' \in [0, W'_0]$, $\phi \in [0, \phi_{\max}]$, $L \geq 0.2$, $T_c \in [34, 41.5]$, $f_R \in [5, 75]$, nonnegative stores and $c_G \geq 0.45\, c_{G,0}$.

7.3.9 Finishing Allowances

Both scored limits open over the end of the ride, as shown in Equation (7.17), because work capacity carried over the finish line was never used.

$$\begin{aligned} \zeta_t &= \mathrm{clamp}\left(\tfrac{t-(T-t_w)}{t_w}, 0, 1\right) \\ \zeta_s &= \mathrm{clamp}\left(\tfrac{s-(S-s_w)}{s_w}, 0, 1\right) \\ L_{lim} &= L_4 + (L_f - L_4)\,\zeta^2 \\ HR_{lim} &= HR_{hi} + (HR_{\max} - HR_{hi})\,\zeta^2 \end{aligned} \tag{7.17}$$

where ζ is ζ_t on a timed ride of length T and ζ_s on a route of length S. With $T = 3600\,\mathrm{s}$, $t_w = 300\,\mathrm{s}$, $s_w = 1000\,\mathrm{m}$ and $L_f = 6\,\mathrm{mmol/L}$, the lactate limit holds at 4.0 mmol/L until the last five minutes or the last kilometer, and the heart-rate limit $HR_{hi} = 0.92\,HR_{\max}$ is 170.2 bpm for the recreational preset before it opens toward the maximum. A timed ride ends at T, a route ends at $s = S$, and either ends early if core temperature reaches the stop.

7.3.10 Routes, Parameters, Presets and Numerics

Two routes are fixed by distance, so every comparison meets each hill at the same place and nothing about the terrain depends on a random stream. The rolling circuit is a 13.2 km lap of fifteen segments with grades from −5 to +8 %. Its two climbs rise at 3, 6 and 2 % over 2.1 km and at 4, 8 and 3 % over 1.5 km, and the lap climbs and descends 163 m with its high point 106.8 m above the start. The time-trial climb is 10 km in eight pitches of 3 to 7 %, gaining 518 m at a mean of 5.2 %. Each change of grade is blended over 100 m with a smoothstep, because a step in grade is a step in the gravity term.

Table 7.2 lists the constants of the recreational preset. Three presets write the rider's mass, aerobic capacity, heart-rate range, critical power, lactate marker, work capacity, sprint power, efficiency, glycogen stores and physiological lags in one action, and every value stays editable afterward. Table 7.3 gives the three and the quantities derived from them. Each preset's critical power sits between 78 and 86 % of the crank power at which oxygen uptake reaches its maximum, and each lactate marker sits 5 to 15 W below critical power, and the test suite fails if either leaves its band.

The framework of Section 1.6.1 integrates the model by the classical fourth-order Runge-Kutta method with a fixed step of 0.25 s, and halving that step changes the first ten minutes of scenario 8, which include two surges, by less than a meter, a convergence check the test suite repeats. The controllers execute once per simulated second, the predictive controller replans once per five seconds, and the historian keeps one sample per second. Measurement noise is drawn from a seeded stream, and the routes use no random numbers at all, so reloading a case reproduces its run to the last digit. Reset, a change of road and the four playback speeds, from real time to sixty times faster, behave as that section describes.

Table 7.2 Model Parameters of the Recreational Preset. All Are Adjustable and Displayed in the Application.

Symbol	Meaning	Value	Unit
m_r	rider mass	75	kg
$\dot{V}_{O_2,\max}$	maximal oxygen uptake	48	mL/(kg min)
HR_{rest}, $HR_{\max}$	resting, maximum heart rate	60, 185	bpm
CP_0	critical power at the cranks	220	W
P_L	lactate-marker power	210	W
W'_0	work capacity above CP	16000	J
P_{pk}	sprint power	950	W
E_c	exertion at critical power	0.75	—
τ_a	exertion lag	1.0	s
c_{opt}, $c_{\max}$	sprint cadence, highest cadence	120, 150	rpm
η_0	peak gross efficiency	0.235	—
c_η, κ	most economical cadence, curvature	60, 0.08	rpm, —
c_{pref}	preferred cadence	88	rpm
m_b	bicycle mass	8.5	kg
I_w	wheel inertia, both wheels	0.14	kg m^2
r	wheel radius	0.335	m
η_d	drivetrain efficiency	0.975	—
G_{lo}, G_{hi}	lowest and highest gear	1.21, 4.55	—
C_dA	drag area	0.32	m^2
C_{rr}	rolling-resistance coefficient	0.0045	—
$F_{\max}$	traction limit	300	N
v_{cap}, k_b	braking speed, brake gain	16, 300	m/s, N s/m
ρ	air density	1.20	kg/m^3
T_{air}, φ	air temperature, humidity	20, 0.45	°C, —
a_f, v_{ref}	extra airflow, full-cooling air speed	0, 8	—, m/s
τ_{O_2}	oxygen-uptake lag	35	s
$\tau_{HR\uparrow}$, $\tau_{HR\downarrow}$	heart-rate onset, recovery lags	40, 60	s
k_T	heart-rate drift with heat	7	bpm/°C
k_F	heart-rate drift at full fatigue	10	bpm
τ_L, L_{rest}	lactate clearance, resting lactate	230, 1.0	s, mmol/L
f_0, q	lactate knee, curvature	0.35, 3.0	—
L_{pk}	lactate soft ceiling	20	mmol/L
τ_R, $f_{R,0}$, k_R	respiration lag, rest, gain	20, 12, 38	s, 1/min
k_r	W' recovery efficiency	0.5	—
k_L	fatigue from lactate	1.7×10^{-4}	1/s
k_{TF}	fatigue from heat	4.6×10^{-5}	1/(s °C)
k_ϕ, $\phi_{\max}$	fatigue recovery, largest loss	2.0×10^{-4}, 0.40	1/s, —
c_b	specific heat of the body	3500	J/(kg K)
k_s, E_0	evaporative gain, ceiling	900, 1820	W/K, W
g_0, g_a	dry loss, still air and airflow	16, 22	W/K
T_{stop}	core temperature that stops the ride	40.5	°C
$G_{m,0}$, $G_{l,0}$	muscle and liver glycogen	350, 75	g
$c_{G,0}$, V_G	starting glucose, its volume	5.0, 0.25	mmol/L, L/kg
k_g	gluconeogenesis	0.02	mmol/s
$\dot{n}_{H,\max}$	largest hepatic output	0.18	mmol/s
$P_{F,\max}$	fat oxidation ceiling	420	W
τ_u, k_u	gastric emptying, glucose uptake	600, 0.12	s, L/s
L_4, L_f	lactate limit, at the finish	4.0, 6.0	mmol/L
HR_{hi}	heart-rate limit	$0.92\,HR_{\max}$	bpm
t_w, s_w, T	finishing window, distance, ride	300, 1000, 3600	s, m, s

7.3.11 Assumptions

Every assumption below is made for teaching purposes. The application does not prescribe training, estimate anybody's fitness or give medical guidance, and its numbers belong to three invented riders. Each assumption is listed with the situation in which it breaks down.

Exertion is a normalized effort and not a measured signal. Perceived effort has a psychophysical scale and a literature of its own [2, 5], and the two-domain map of (7.3) is a modeling choice built to reproduce the three-minute all-out test. At a fixed rating of perceived exertion real cyclists reduce

Table 7.3 The Three Rider Presets and the Actuator Gains They Imply, Fresh. The Gains Are the Slopes of (7.3) Below and Above E_c.

	Novice	Recreational	Trained
m_r (kg), $\dot{V}_{O_2,\max}$	82, 35	75, 48	68, 68
HR_{rest}, $HR_{\max}$ (bpm)	66, 182	60, 185	48, 188
CP_0, P_L (W)	150, 140	220, 210	315, 300
W_0' (kJ), P_{pk} (W)	12, 700	16, 950	22, 1250
η_0	0.215	0.235	0.245
$G_{m,0}$, $G_{l,0}$ (g)	300, 70	350, 75	430, 90
$\tau_{HR\uparrow}$, $\tau_{HR\downarrow}$, τ_L (s)	45, 70, 250	40, 60, 230	30, 50, 200
CP_0 over crank power at $\dot{V}_{O_2,\max}$	0.789	0.820	0.856
$\partial P/\partial E$ below E_c (W)	200	293	420
$\partial P/\partial E$ above E_c (W)	2200	2920	3740
ratio of the two	11.0	9.95	8.90
τ_W at $E = 1$ (s)	21.8	21.9	23.5

their power in anticipation, at a rate set by the rate of heat storage [5], and the map carries that only through the fatigue and heat terms.

The power ceiling falls linearly with W'. That is the three-parameter form [4], and it implies that a held exertion never quite exhausts W'. Riders at exhaustion do not always use the whole capacity [4], and measured reconstitution follows time constants that the single recovery efficiency k_r only approximates [15].

The torque-cadence relation is a straight line through the sprint peak. Grinding a low cadence is fatiguing in ways the model does not carry, standing changes the available torque, and the model has neither effect.

Gross efficiency is a parabola with its apex at 60 rpm. The evidence does not fix an energetically optimal cadence [22], and the curve is a declared teaching value.

The road is straight and the rider is alone. There is no cornering, no drafting, no crosswind, no bicycle handling and no change of position, so the drag area is constant.

The physiology is lumped. Lactate has a single compartment, heat has a single body node without any dehydration, heart rate responds only to intensity, temperature and fatigue, and the glucose sink above the resting concentration is illustrative rather than an insulin model.

Critical power and the lactate marker are separate but fixed. Both fall with fatigue and fuel in the same proportion, which keeps them close throughout a ride, while the literature finds them related but not interchangeable [16].

Rides repeat exactly. A scenario run twice gives identical numbers, which simplifies grading and describes no real rider, whose second hour is never the same as the first.

7.4 Dynamic Analysis

7.4.1 An Actuator Whose Gain Drains

The distinctive feature of this plant is the gain from the actuator to power. Below E_c it is $\partial P_E/\partial E = CP/E_c$, which is 293 W per unit exertion for the recreational preset and falls only slowly with fatigue and fuel. Above E_c it is $(P_{top} - CP)/(1 - E_c)$, which is 2920 W per unit for a fresh rider, ten times larger, and it falls in proportion to the W' left. Table 7.3 gives the same pair for the three presets, with a ratio of 8.90 to 11.0 between the two slopes.

The consequence of holding an exertion above E_c follows in closed form. With critical power constant and the torque ceiling inactive, (7.3) and (7.8) give $\dot{W}' = -(P_{pk} - CP)(E - E_c)W'/[(1 -$

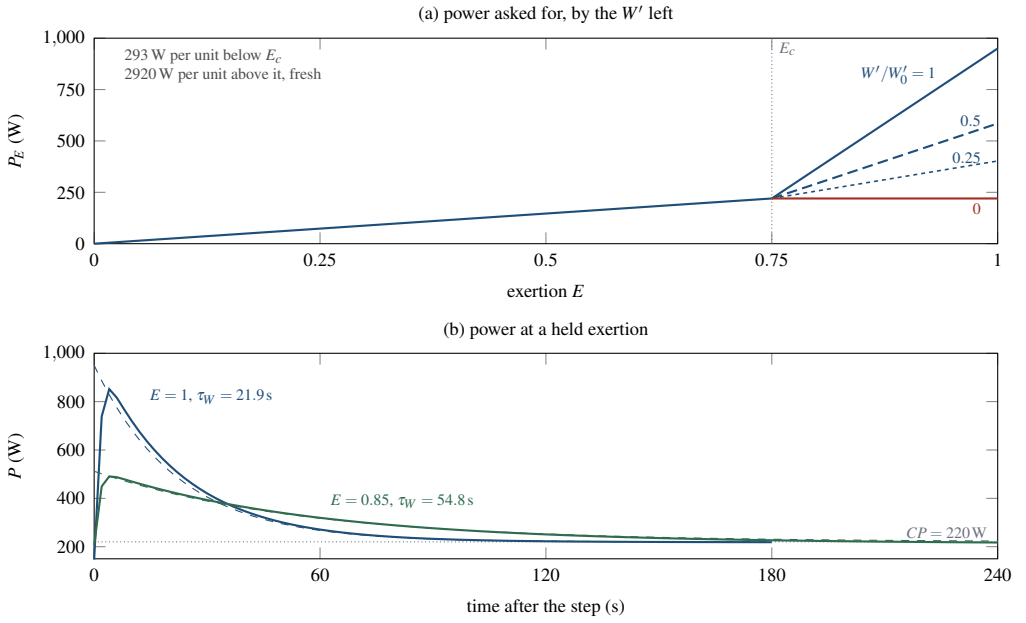


Figure 7.2 The actuator of the recreational preset. (a) Power asked for as a function of exertion for four fractions of the work capacity left, from Equation (7.3) with $CP = 220$ W. (b) Power after a step to a held exertion from full W', in scenario 3 at $E = 1$ and scenario 2 at $E = 0.85$. Solid traces are the application's trajectories and dashed traces are the closed form of Equation (7.18).

$E_c)W'_0]$, and the power settles exponentially onto critical power, as shown in Equation (7.18).

$$P(t) = CP + (P_{pk} - CP)\,\frac{E - E_c}{1 - E_c}\,\mathrm{e}^{-t/\tau_W}$$
$$\tau_W = \frac{W'_0\,(1 - E_c)}{(P_{pk} - CP)(E - E_c)} \tag{7.18}$$

For the recreational preset τ_W is 21.9 s at $E = 1$ and 54.8 s at $E = 0.85$, the time constant of Equation (1.11) in Section 1.4.6. A held exertion therefore never exhausts W' in finite time. It asymptotically spends it, and the power it buys decays onto critical power while the exertion itself does not change. That is also why the three-minute all-out test measures critical power [14]. At $E = 1$ the model's power is within 0.2 W of critical power after 180 s, with W' spent to within 5 J. Figure 7.2 compares (7.18) with the application's own trajectories. They fall short of the closed form in the first seconds, where the exertion lag τ_a delays the rise and, at full effort, the torque ceiling holds the power below 882 W at 88 rpm, and they then follow it apart from the few watts of critical power that fatigue takes during the effort.

> **TRY IT IN THE APP** **3 · Three Minutes All-Out**
>
> Load scenario 3 and press Run. The scripted step to 100 % exertion holds until nine minutes. Watch Crank power against the dashed critical power line while the exertion trace stays flat. The power is 839 W five seconds into the effort, 271 W after a minute and 219 W after three, settled onto critical power with the work capacity spent.

A power loop tuned in the sustainable domain therefore sees its loop gain rise by a factor of ten the moment its output crosses E_c and then fall as W' drains. With the gains of Section 7.5 the

Table 7.4 Apparent Time Constants After a Step in Exertion From Zero to 70 % at $t = 30$ s on the Level, Recreational Preset, Taken as the Time to 63 % of the Change Reached by 90 min, at the 0.25 s Integration Step.

Output	Initial	Final	t_{63} (s)
Crank power (W)	0	205.3	1.0
Cadence (rpm)	0	88.7	2.5
Road speed (km/h)	0	34.24	10.5
Oxygen uptake (mL/(kg min))	3.50	37.57	35.75
Respiratory frequency (1/min)	12.0	44.4	67.75
Heart rate (bpm)	60.0	156.4	81.75
Blood lactate (mmol/L)	1.00	3.67	229.5
Core temperature (°C)	37.00	37.91	328.25

dimensionless loop gain $K_c\,\partial P/\partial E$ is 0.667 below E_c for every preset and 6.64 just above it for a fresh recreational rider. Holding 245 W above the knee in scenario 15, with the meter noise off, the power standard deviation between five and ten minutes is 1.05 W, so the higher gain does not destabilize the loop. What it does do is hand the loop an actuator that runs out. In the case as loaded, with the meter noise on, the exertion needed to hold the target climbs from 76.0 % at five minutes to 81.4 % at ten, the command reaches the stop at 688 s, and the power then falls to within 4 W of critical power with the actuator saturated.

Pause. Try Exercise 7.1 before reading on.

7.4.2 Lags Between the Effort and the Limits

The time each output needs to cover 63 % of its eventual change after a step from rest to 70 % exertion on the level, a step inside the sustainable domain, is listed in Table 7.4 from one run of the application.

The ordering in Table 7.4 is the second half of the control problem. The watts follow the effort within a second and the bicycle within eleven, while heart rate is 82 s behind and blood lactate 230 s behind. A surge above E_c therefore spends W' at the rate of (7.8) for minutes before either physiological limit reports it, and an override on lactate is structurally too slow to protect W'. It can hold the average effort at a level it can measure, and it cannot stop a fast pacing loop from overspending in the meantime, which scenario 16 measures.

> **TRY IT IN THE APP** **16 · Pacing by Lactate Alone**
>
> Load scenario 16, which sets the power target out of reach, and press Run. Exertion reaches 92 % in the first minute, and the lactate loop takes over at 68 s, yet lactate keeps rising to 6.24 mmol/L. Press Reset, set Power target to 215 W and press Run again. The lactate loop now takes over at 445 s, and the hour stays compliant.

7.4.3 Terrain Enters the Objective and Not the Constraints

At constant exertion the grade reaches the force balance (7.5) and nothing else, so a climb changes the speed and leaves the watts and the body where they were. On a timed ride what matters for pacing across terrain is the speed one more watt buys, $\partial v/\partial P = 1/(\partial P/\partial v)$, where $\partial P/\partial v$ is the derivative of the resistance power $(F_a + F_r + mg\sin\theta)\,v/\eta_d$. For the recreational preset it is 53.4 W per m/s on the level at 33 km/h and 59.9 W per m/s on a 6 % climb at 11.3 km/h, so a watt buys 0.0187 m/s on the flat and 0.0167 m/s on the climb. Aerodynamic drag dominates the first and gravity the second, and the two sensitivities differ by 11 %. That is the contrast with a treadmill, where the same grade at the same belt speed raises the metabolic cost of every meter and moves the constraint itself.

The small difference has a measurable consequence. The challenge hour on the rolling circuit was ridden open loop with the exertion schedule $u = u_0 + k\,g_{\%}$ for slopes k between -0.0025 and

0.02 per percent of grade, each paired with the largest base exertion u_0 that keeps the hour compliant, found by bisection. The best constant exertion, $k = 0$ with $u_0 = 71.34\,\%$, covers 28 999 m. The best grade proportional schedule, $k = 0.00125$ per percent, covers 29 024 m, a gain of 25 m or 0.09 %. At $k = 0.01$ the hour is 102 m shorter and at $k = 0.02$ it is 692 m shorter, because the binding limit is a four-minute average of the effort that a climb lasting several minutes fills. The prediction that variable power saves time [10] rests on a fixed-distance objective and a power budget, and on a timed ride under a lactate limit neither condition holds.

7.5 Controller Design

Exertion can be set in three ways. In manual the operator's value goes straight to the rider, in the regulatory mode three proportional-integral loops compete for it through a low selector, and in the predictive mode a receding-horizon search sets it instead. The instruments are common to all three. In either automatic mode the commanded exertion may change by no more than 4 % per second, so switching between modes, or between the two automatic architectures, never produces a step in effort.

7.5.1 Instruments

The power meter reads crank power with colored noise of 4 W standard deviation and a 2 s correlation time, through a 3 s first-order filter. The heart-rate monitor reads heart rate with colored noise of 0.8 bpm and a 3 s correlation time, through a 6 s filter, as shown in Equation (7.19), and holds its last value while the signal is lost.

$$\begin{aligned} \widehat{HR}_k &= \alpha\,\widehat{HR}_{k-1} + (1-\alpha)\,(HR_k + \sigma_H n_k) \\ \alpha &= \mathrm{e}^{-\Delta t_c/\tau_f} \end{aligned} \tag{7.19}$$

where n_k is a first-order filtered Gaussian sequence rescaled to unit stationary variance and $\Delta t_c = 1\,\mathrm{s}$. Lactate is read one of two ways. The default is the modeled value, labeled on every display as an estimate because no continuous blood-lactate sensor exists. The alternative is a fingerstick, a blood sample drawn every 180 s and read 45 s later, held between readings, so that a reading is up to 225 s old when the next one arrives.

7.5.2 Velocity Form PID Algorithm

Each loop runs the shared incremental, or velocity, form algorithm of Equation (7.20) on its own error, with the output bounded to the exertion range.

$$\begin{aligned} e_k &= \mathrm{SP}_k - \mathrm{PV}_k, \qquad p_k = K_c\,e_k \\ s_k &= s_{k-1} + (p_k - p_{k-1}) \\ c_k &= u_{b,k} + s_k + I_{k-1} \\ u_k &= \mathrm{clamp}(c_k,\ 0,\ 1) \\ I_k &= I_{k-1} + \left[\frac{K_c}{\tau_I}\,e_k + \frac{u_k - c_k}{T_t}\right]\Delta t_c \end{aligned} \tag{7.20}$$

These are Equations (1.12) to (1.15) of Section 1.6.2 in the loop's own symbols, with $s_k = S_k$, $c_k = v_k$ the unclipped demand and no derivative term, and anti-windup is back-calculation on I_k alone with the tracking time $T_t = \tau_I$, so the running output is never truncated. All three loops are reverse acting, $a = +1$ in the convention of Section 1.3, because more exertion raises power, heart rate and lactate. The bounds of 0 and 1 are the only limit the back-calculation sees, because the slew limit is applied after the selector and is handled by the conditional integration described below. Derivative action is available on the filtered measurement and is zero on all three loops, because Table 7.4 shows nothing fast enough in the limits to anticipate. In manual, I_k is solved so that c_k equals the operator's exertion, and transfer into automatic starts from there.

Table 7.5 Loop Settings. Every Loop Is PI With $\tau_D = 0$, a 1 s Scan and Output Bounds of 0 and 1.

	Power	Heart rate	Lactate
K_c	$0.5/CP_0$ per W	0.004 per bpm	0.03 per mmol/L
τ_I (s)	12	150	420
bias u_b	$E_0(P_{\mathrm{SP}})$	$E_0(P_L)$	$E_0(P_L)$
default SP	200 W	168 bpm	3.7 mmol/L
margin range R_j	—	15 bpm	1.0 mmol/L

The three loops are the power loop, with $e = P_{\mathrm{SP}} - \hat{P}$ in watts, the heart-rate loop, with $e = HR_{\mathrm{SP}} - \widehat{HR}$, and the lactate loop, with $e = L_{\mathrm{SP}} - \hat{L}$. Their settings are given in Table 7.5. The power-loop gain is carried per unit of fresh critical power, $K_c = 0.5/CP_0$ per watt, so that its loop gain below E_c is $0.5/E_c = 0.667$ for every preset. The heart-rate and lactate loops have loop gains of 0.546 and 0.682 for the recreational preset and 0.49 to 0.74 across the three. The heart-rate setpoint is the limit itself, 168 bpm by default, with an optional ramp from the resting value for a ride paced on heart rate.

Every loop trims around a feedforward rather than integrating its way up from a standstill. The bias is the exertion a fresh rider needs for a power, obtained by inverting (7.3) with full stores, as shown in Equation (7.21).

$$E_0(P) = \begin{cases} E_c\, P/CP_0, & P \le CP_0 \\ E_c + (1 - E_c)\,\dfrac{P - CP_0}{P_{pk} - CP_0}, & P > CP_0 \end{cases} \tag{7.21}$$

The power loop is biased at $E_0(P_{\mathrm{SP}})$ and the two physiological loops at the nominal marker exertion $E_{nom} = E_0(P_L) = 0.716$. Both come from the configured profile, which is what a coach has on paper, and neither knows about the fatigue, the fuel or the heat that move the real rider during a ride. Closing that gap is the integral's job.

7.5.3 Low Selector With Margin-Scaled External Reset

The applied exertion is the smallest of the loop requests, and the loops that lose are reset from the exertion actually applied, as shown in Equation (7.22).

$$\begin{aligned} u^* &= \min_{j \in \mathscr{L}} u_j \\ u_k &= \mathrm{clamp}(u^*,\ u_{k-1} - r_u \Delta t_c,\ u_{k-1} + r_u \Delta t_c) \\ m_j &= \max\left(p_{j,k},\ \frac{\mathrm{SP}_j - \mathrm{PV}_j}{R_j}\right), \qquad m_P = p_{P,k} \\ I_j &\leftarrow u_k - u_{b,j} - s_{j,k} + \max(0,\ m_j) \end{aligned} \tag{7.22}$$

where $\mathscr{L}$ is the set of live loops, $r_u = 0.04/\mathrm{s}$ is the slew rate, R_j is the margin range of an override, and the reset of the last line is applied to every loop j that was not selected. The reset places each losing loop exactly its margin m_j above the applied exertion. For the power loop the margin is its proportional term, so it competes on its present error. For an override it is the larger of its proportional term and its distance from its limit scaled to exertion, so a lactate loop 1 mmol/L below its target sits a whole unit of exertion above the applied value and cannot be selected by accident, while one at its target competes on its proportional term alone and comes down onto the applied value as the limit approaches. The same structure shares the ground speed of a combine among a feed-rate loop and two overrides in Volume 4, Chapter 4, and the fuel of a jet engine among a governor and its limiters in Volume 4, Chapter 3.

Three details of (7.22) decide whether the selector works, and Section 7.7.5 measures each. The margin is scaled to the distance from the limit because a lactate error is at most a few millimoles

per liter whatever the gain, so an unscaled override that is nowhere near binding looks like the most restrictive loop. The margin is never negative, because a loser reset below the applied exertion wins the next comparison, and two overrides past their limits then take turns resetting below each other. On entry into the selector, at a reset or from manual or predictive operation, every loop has been tracking the applied exertion and the smallest increment would win, so the leader is chosen as the live loop with the smallest margin, it starts exactly from the applied exertion, and the others start at their margins above it. The same entry rule governs the return from a signal loss. The fallback is treated as a separate law, so leaving it once the signal is back is an entry, while during the loss the cap is one more candidate that the loops meet with their own requests.

Two further rules complete the selector. When the slew limit holds the applied exertion short of the selected loop's request, that loop's integral keeps the value it had before the scan, which is conditional integration against the rate limit rather than the output bound. When the heart-rate signal is lost the heart-rate loop leaves $\mathscr{L}$ and a conservative cap of $0.9\,E_{nom}$ joins it, because a controller holding a limit on a measurement it no longer has must not chase the frozen value.

7.5.4 Receding-Horizon Alternative

Every 5 s the predictive controller solves a new plan for the next 300 s, a horizon cut to the time remaining on a timed ride but never below 60 s. The plan is an exertion held piecewise constant on six blocks that begin at 0, 5, 15, 30, 50 and 75 % of the horizon. Each plan starts from the controller's own estimate, never from the plant's state: the declared preset stepped each second under the measured crank power, overwritten by the measured heart rate, lactate, speed and distance, a fingerstick through its innovation, with the unmeasured headwind inferred from power and speed. An edited parameter is thus a rider the controller was not told about, and only the ride's limits, duration and route length come from the run. The predictor steps this copy in 5 s increments. The exertion lag is dropped, the gear is the one an ideal automatic shifter would choose, and speed is integrated in five substeps with the crank power of the step held constant. From a standstill there is no cadence and so no crank power, and a force computed from power is zero, so the predictor then evaluates the drive force of (7.4) at every substep. Without that rule every plan predicted a bicycle that never moved, and a rider started under the predictive controller on a level road stayed at rest for the whole hour. The physiology takes one step of the same equations per predictor step, with the four first-order lags advanced by their exact exponential update so that no step length makes them ring, and the slow stores advanced explicitly.

This predictor is the application's own nonlinear search through a copy of the rider and the road, not the linear option of Section 1.6.3. Its objective is economic. On a timed ride the objective is Equation (7.23).

$$
\begin{aligned}
J &= \Delta s_N + s_{tail}(\mathbf{z}_N) + R(\mathbf{z}_N) \\
&\quad - 20\lambda \sum_k |u_k - u_{k-1}| - 12\,\Omega \\
R &= k_R \,\frac{W'_N}{\max(20,\ \partial P/\partial v)}\,\mathrm{clamp}\Big(\tfrac{T - t_N}{t_w}, 0, 1\Big) \\
\Omega &= 5 \sum_k \Big[40\,(L_k - L_{lim} + 0.25)_+ \\
&\qquad + 1.2\,(HR_k - HR_{lim} + 2)_+ + 400\,(T_{c,k} - 40.2)_+ \Big]
\end{aligned}
\tag{7.23}
$$

Here Δs_N is the distance the predictor covers within the horizon, $\mathbf{z}_N$ is the state it predicts at the horizon's end, $\lambda = 0.35$ is the move penalty, $k_R = 0.5$, and Ω accumulates constraint violation over the 5 s prediction steps against margins of 0.25 mmol/L, 2 bpm and 0.3 °C inside the true limits. The tail term s_{tail} estimates the distance that remains available once the horizon ends, under a base policy that holds the nominal marker exertion, rolled out to the end of the ride through a reduced model. The road is integrated in 10 s semi-implicit steps, each charged for the height it actually gains from a

Table 7.6 The Application's Own Performance Measures, Accumulated at the Logging Rate and Frozen When the Ride Ends.

Measure	Definition	Purpose
Distance or time	$s(T)$, or t at $s = S$	the objective
Average power	$\int P\,\mathrm{d}t / t$	the number a rider reads
Compliant	lactate excess below 1 and heart-rate excess below 10	pass or fail over both limits
Lactate excess	$\int \max(0, L - L_{lim})\,\mathrm{d}t$	area above a moving limit
Above HR limit	time with $HR > HR_{lim}$	duration of the excursion
W' empty	time with $W' < 0.05\,W'_0$	the actuator's lost range
Torque limited	time with $T_{\max}$ or $F_{\max}$ binding	the gear, not the rider
Peak core	$\max_t T_c$	the limit with no actuator
Glucose minimum	$\min_t c_G$	the circulating store
Glycogen left	$g(T)$	what the ride cost

5 m altitude table, and the fatigue, glycogen, glucose and core temperature states are advanced every 150 s, so the tail integrates through the knee in the fuel factor instead of sampling one side of it. The reserve term R prices the W' the base policy never spends at the speed it would buy, $W'_N/(\partial P/\partial v)$ meters, with $\partial P/\partial v$ close to 53 W per m/s on the level at 33 km/h and 60 W per m/s on a 6 % climb at 11 km/h, and ramps that value to zero over the finishing window, where it is worth nothing. On a route the objective is the negative of the predicted finishing time, written as meters at 5 m/s so that the penalties keep their scale, with the reserve ramped over the last kilometer and set to zero once the line is inside the horizon.

Plans are ranked by their constraint violation before their objective, so a plan with less violation wins however much farther the other one goes. The search starts from six seeds, being the previous plan shifted one block, the applied exertion held, the nominal marker exertion held, the smaller override request held, and two plans that hold the applied exertion for two blocks and ease by 3 and 8 % afterward. It then refines one block at a time over steps of 0.12, 0.04 and 0.012 in exertion, capped at 320 plan evaluations. When no feasible plan exists the applied exertion is limited to the nominal marker exertion, which is survivable by construction. By default the controller knows only the grade underfoot and holds it over the horizon. A teaching setting hands it the whole route profile, and every comparison that uses it says so. On the circuit hour a replan costs 57.7 plan evaluations on average and 114 at most, and over two repeated runs the mean solve time is 5.1 to 5.2 ms with a maximum of 20 ms in Node.js 26 on an Arm Cortex-X925 core shared with other work, against a replan period of 5 s.

7.6 Performance Measures

Besides the error integrals, peak errors, travel and saturation time that the framework of Section 1.6.4 keeps for each loop, the scorecard carries the ten measures of Table 7.6. They are competing costs and are never combined into a weighted score.

A ride counts as compliant when the areas above both limits stay under small thresholds. Riding on a limit on purpose then does not fail for one sample over it, and a large excursion still fails when it is brief. The integral absolute error of each loop accrues from the standing start and includes the time a loop spends unselected, so it is quoted here only between runs of identical length. Travel sums the scan-to-scan changes of the commanded exertion, $\sum_k |u_k - u_{k-1}|$, in percentage points, as the exported ride file records it. The loop scorecard's own travel figure is taken from the logged exertion after the neuromuscular lag and reads lower, 549 points against 713 for the 190 W hour of Table 7.8. Travel separates a controller that pays for its distance with continual changes of effort from one that does not, and with the power meter's noise it also separates a loop that reads a noisy measurement from one that reads a smooth one.

Table 7.7 Scenarios and Completion Requirements

#	Scenario	Behavior isolated	Required to complete
1	A standing start *Experiment*	force before power	Ride through the step to 50 % at 30 s, then hold 50 % in manual for 2 min.
2	Exertion steps *Experiment*	a step that fades	Ride all five scripted steps, holding 85 % until W' is below 10 % of W'_0.
3	Three minutes all-out *Experiment*	ceiling decays onto *CP*	Hold 100 % to the end of the effort at 9 min, then ride 6 min at 30 % in manual.
4	A steady climb *Experiment*	grade in the road only	Hold 60 % in manual for 9 min of the 6 % climb, then ride on to level road.
5	Chasing the flat speed *Demonstration*	the cost of chasing speed	Keep the scripted 95 % on the 6 % climb until W' is below 5 %.
6	Descending *Experiment*	freewheel and braking	Coast on −8 % until the brakes engage and heart rate is below 80 bpm.
7	A headwind *Experiment*	drag in the air speed	Ride 2 min at 65 % into the wind, then 80 % or more until W' is below half.
8	Repeated surges *Experiment*	slow refill of W'	Ride all four surges and recoveries, then 1 min more at 55 % in manual.
9	Gears and cadence *Experiment*	efficiency, spinning out	Ride 60 % in the highest gear, the lowest past 11 min, then 1 min on automatic.
10	Too big a gear *Demonstration*	a torque limit of the gear	Stall in the highest gear before 4 min, then restart above 8 km/h after the shift.
11	Hot climb, tailwind *Experiment*	cooling by apparent wind	Finish the hour with core temperature above 39.0 °C, then Reset, set a 3 to 4 m/s headwind and finish below 38.5 °C.
12	Under-fueled, 90 min *Experiment*	a store that lowers *CP*	Hold 66 % until blood glucose is below 2.5 mmol/L and on to 90 min.
13	The same ride, fed *Experiment*	delayed glucose	See glucose above 5.5 mmol/L after the 30 min feed, then finish 90 min at 66 %.
14	Three riders *Experiment*	effort is relative	Ride 30 min at 70 % as each preset in turn, pressing Reset between them.
15	Target above *CP* *Demonstration*	saturation on a store	Under the PID selector, saturate the request between 5 and 12 min, then drop below 90 % within 1 min of the 200 W target.
16	Lactate pacing *Demonstration*	a slow override	At 600 W, let lactate peak above 5.5 mmol/L, then Reset, set 210 to 215 W within 1 min and stay compliant to 30 min.
17	Heart-rate dropout *Experiment*	fallback and re-entry	Reach the fallback cap during the dropout, then ride 1 min under PID after it.
18	Fingerstick lactate *Experiment*	a sampled, late reading	Finish the hour on the fingerstick, then Reset, switch to the estimate within 1 min and finish again.
19	10 km time trial *Experiment*	a fixed distance	Summit at a steady 72 % with W' above 90 %, then Reset, start at 88 to 92 %, return to 72 % and summit with W' below 30 %.
C	Hour on the circuit *Challenge*	the economic problem	Over 28.8 km in the hour and compliant, on the case as given.[a]

[a] With the shipped 190 W power target the hour is 27.93 km, compliant but 0.87 km short of the target. With the suggested 215 W target it is 28.90 km, compliant and 0.10 km past it, whether the target is chosen before Run or carried through Reset.

7.7 Results

Table 7.7 lists the twenty cases in the order the application presents them, from open-loop mechanics to closed-loop pacing, with the behavior each one isolates and what completes it. Each case sets the mode, the controller, the rider, the tuning, the road and the timed events in one step and opens with a short briefing on its purpose, and credit follows the completion rules of Section 1.6.4. The challenge is judged on the case as given, so a changed model parameter or disturbance voids the attempt.

7.7.1 Open-Loop Behavior at a Held Exertion

Scenario 1 separates the time scales before any controller is involved. At 50 % exertion from a standstill the standing torque is half of T_0, the rider reaches 17.9 km/h ten seconds after setting

off and 26.8 km/h at thirty seconds, and settles at 30.1 km/h on 146.7 W. Scenario 2 then steps the exertion to 30, 55 and 70 %, which settle at 88, 161 and 205 W, and finally to 85 %, which does not settle. The fourth step buys 491 W five seconds after it lands, 320 W a minute later, 251 W after two minutes and 217 W after four, by which time W' is down to 0.2 kJ and fatigue has cut critical power to 214 W. Equation (7.18) predicts 318 W at one minute and 253 W at two for a constant critical power. Scenario 3 is the all-out test itself, at 839, 537, 271 and 219 W at 5, 20, 60 and 180 s, and afterward at 30 % exertion W' recovers to 12.3 kJ in six minutes and 14.9 kJ in eleven. Scenario 8 repeats a one-minute surge to 95 % four times with two minutes at 55 % between them. The surges peak at 736, 369, 332 and 326 W, because W' recovers only to 4.6, 3.5 and 3.4 kJ between them. The same request buys less each time, because reconstitution below critical power is slower than depletion above it.

The road scenarios separate what the grade does from what the effort does. At a constant 60 %, scenario 4 holds 176.0 W while speed falls from 32.3 to 11.3 km/h on a 6 % climb and cadence falls from 95 to 74 rpm in the lowest gear, and heart rate does not step, drifting 1.3 bpm down because 74 rpm is nearer the most economical cadence. Scenario 5 tries to hold the flat-road speed instead by going to 95 %. It buys 729 W five seconds in, 303 W at 55 s and 216 W after five minutes, and the speed is 13.8 km/h at 200 s against 11.3 km/h for the rider who stayed at 60 %, so 58 % more exertion buys 23 % more watts once W' is gone. Scenario 7 turns a 6 m/s headwind into the rider at 190.7 W, and the speed falls from 33.3 to 21.5 km/h while the aerodynamic force rises from 16.4 to 27.5 N. Scenario 6 coasts down 8 %, where the bicycle reaches the 57.6 km/h braking speed 14.5 s after the rider stops pedaling and the brakes absorb 196 W, 64.8 kJ over the descent, while heart rate falls from 142 to 61 bpm.

The gearing scenarios put an actuator limit outside the rider. Scenario 9 holds 60 % in the highest gear, where cadence falls from 95 to 56 rpm, torque rises from 17.6 to 29.9 N m for the same 176 W, efficiency rises from 22.8 to 23.5 % and heart rate settles 2 bpm lower. In the lowest gear at 32 km/h the legs would have to turn at 211 rpm, the taper holds the rider at 148 rpm, and the power falls to 73 W. Scenario 10 rolls onto 8 % at 15 km/h in the highest gear at 70 %. Within five seconds the rider is torque limited at 97 N m and delivers 184 W of the 204 W asked for, the power falls with the cadence to 84 and 21 W at 30 and 60 s, and the bicycle stops 77 s after the start. After a shift to the lowest gear the rider restarts and settles at 10.2 km/h on 205 W. No exertion setting fixes a gear that is wrong for the grade.

The slow stores then move the actuator itself. Scenario 11 climbs 6 % for an hour at 72 % in 35 °C air at 70 % humidity with a 3.5 m/s tailwind that nearly matches the rider's speed. Evaporation reaches its ceiling after 35 min, core temperature reaches 39.2 °C, thermal fatigue takes critical power from 220 to 206 W, and at the same exertion the power falls from 211 W at ten minutes to 198 W at sixty. Riding the same climb into a 3.5 m/s headwind instead, the rider finishes at 38.2 °C with the full 211 W, slower on the road and far cooler. Scenario 12 rides 90 min at 66 % from a third of the usual glycogen. The liver is down to 1.4 g of its 24 g by thirty minutes, glucose reaches its 2.25 mmol/L floor at forty, critical power falls to 173 W at sixty minutes and 160 W at ninety, and the power at the same exertion falls from 194 to 141 W. Scenario 13 adds 40 g of carbohydrate at thirty and fifty minutes. Glucose rises from 4.1 to 6.1 mmol/L in five minutes and the rider produces 176 W at sixty minutes against 153 W unfed, but by ninety minutes muscle glycogen is down to 7 g and the power is 135 W, slightly below the unfed rider's 141 W. Feeding moved the collapse and did not remove it. Scenario 14 holds 70 % for the three presets, which produce 140, 205 and 294 W at thirty minutes, a factor of 2.1, while heart rate sits at 83, 85 and 86 % of each rider's maximum. Per watt the three hearts differ by a factor of 1.6, while at the same exertion they use 74, 77 and 81 % of their heart-rate reserves, which is what makes exertion a better-conditioned actuator than power across riders.

7.7.2 Saturation and Slow Overrides

Scenario 15, captured in Figure 7.4, asks the power loop for 245 W, 25 W above critical power, with both overrides set out of reach. The loop holds a mean of 243 W between five and ten minutes, and

Table 7.8 Measured Scorecard for One Hour on the Rolling Circuit, Recreational Preset. Hindsight Is the Best Constant Exertion Found by Bisection Over Whole Hours, Which No Rider Could Know in Advance.

	PID 190 W	PID 215 W	PID 600 W	Hind-sight	MPC grade	MPC route
Distance (km)	27.93	28.90	28.80	29.00	29.05	28.97
Compliant	yes	yes	no	yes	yes	yes
Lactate excess	0	0	2292	0.96	0	0
Peak lactate	2.74	3.91	6.45	4.02	5.69	5.75
Peak HR (bpm)	150	159	171	160	173	173
W' left (kJ)	16.0	16.0	6.1	16.0	11.4	12.2
Average power (W)	188	206	205	209	210	208
Travel (points)	713	154	190	0	838	1611
Laps at (s)	—	1643	—	—	1630	1641
	—	3268	—	—	3251	3272

Lactate in mmol/L and mmol s/L. Travel is the total variation of the exertion command in percentage points. Laps are the times at which 13.2 and 26.4 km are passed, to the nearest second.

the exertion it needs rises because every minute the same exertion buys fewer watts. At 688 s the exertion command reaches its stop and the power falls to 212 W, within 4 W of a critical power that fatigue has lowered to 208 W. The integral is not allowed to wind up against the stop, so when the target drops to 200 W at twelve minutes the exertion is at 89.4 % six seconds later and 80.7 % after a minute. The loop has not failed. The actuator has.

Scenario 16 shows how slow an override is. With the power target out of reach the power loop leads from the start and exertion reaches 92 % in the first minute. The lactate loop takes over at 68 s, by which time W' is down to 7.5 kJ, and lactate, four minutes behind the effort, keeps rising until 149 s, to 6.24 mmol/L, while W' falls below 5 % at 135 s. The level hour covers 34.15 km and is not compliant, with 2646 mmol s/L above the limit and W' below 5 % for 2636 s. With a 215 W target the power loop leads until lactate reaches its 3.7 mmol/L target, the lactate loop takes over at 445 s, and the hour is 34.09 km and compliant, 57 m shorter and inside every limit. The target gives this one run whether it is chosen before Run or carried through Reset. Scenario 18 runs that 215 W hour with a fingerstick lactate. The sampled reading lets lactate reach 4.03 mmol/L, 3.5 mmol s/L over the limit and non-compliant, for 34.10 km, against 34.09 km, a peak of 3.88 mmol/L and compliance on the continuous estimate. The stale reading bought 8 m and a violation.

🕮 **Pause.** Try Exercise 7.2 before reading on.

Scenario 17 loses the heart-rate signal for five minutes at ten minutes, with the limit set to 158 bpm so that it binds, in turn with the lactate loop, from three to six minutes. At ten minutes the lactate loop holds the effort, with heart rate at 155.9 bpm. While the signal is missing the fallback cap holds exertion at 64.4 % and 189 W for the whole five minutes, below every live loop's request, and the heart rate the controller cannot see drifts from 156 to 148 bpm under the frozen 156 bpm. When the signal returns the selector is re-entered from the applied 64.4 %, and the power loop ramps it back toward its target at the slew rate, to 66.2 % after four seconds, until the lactate loop takes over again at sixteen minutes.

7.7.3 The Level Hour and the Rolling Circuit

Table 7.8 reports the scorecard for the challenge, one hour on the rolling circuit, under six ways of driving the same rider, and Figure 7.3 draws three of its rows. The predictive controller holds lactate within 0.07 mmol/L of the 3.75 mmol/L it plans against through the middle of the hour and covers 29.05 km, 151 m more than the suggested regulator with every limit respected. The regulator's hour is spent 87 % under the lactate loop and 13 % under the power loop. Both clear the challenge's 28.8 km, and the loaded 190 W target does not.

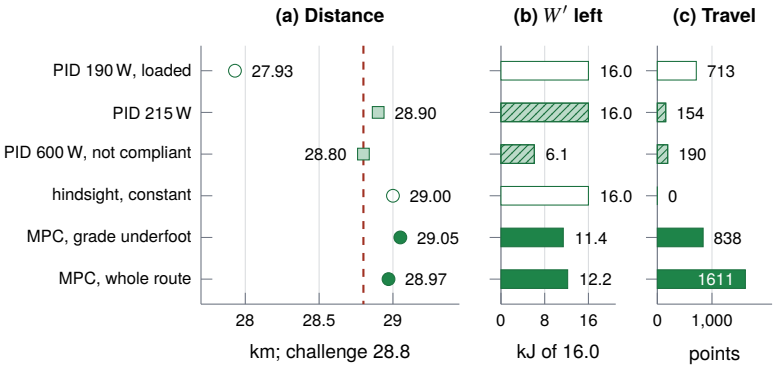


Figure 7.3 The challenge hour of Table 7.8 for its six columns: distance as dots against the challenge's 28.8 km (dashed), and the W' left at the end and the exertion travel as bars. Open marks and white bars are the loaded 190 W target and the hindsight constant exertion, squares and hatched bars the two other PID targets, and the solid ones the predictive controller. Notice that the two predictive columns go furthest while leaving work capacity unspent, and that handing the predictive controller the whole route nearly doubles its travel and shortens its hour.

Table 7.9 Where the Predictive Controller's Advantage Over the Regulator Is Accumulated on the Level Hour. Identical Rider and Limits, and Both Compliant.

Elapsed	Regulator (m)	Predictive (m)	Gap (m)
1 min	320	503	183
3 min	1454	1675	221
10 min	5502	5691	189
30 min	16951	17123	173
55 min	31232	31410	179
60 min	34088	34366	279

The columns of Table 7.8 separate four effects. The loaded 190 W target is compliant and 0.97 km short, and its travel of 713 points is noise, because the power loop reads a meter with 4 W of noise and is selected for the whole hour. The 600 W target reproduces scenario 16 on the hills, and it is 101 m shorter than the compliant 215 W hour despite breaking the limit for most of it, because lactate above the limit accumulates fatigue, and by the end of the hour critical power is down to 201 W against an unchanged 220 W for the compliant regulator. The hindsight column covers 28.999 km at a constant 71.34 %, which is 102 m more than the regulator and 48 m less than the predictive controller, so most of what separates the automatic modes is how quickly each reaches the right exertion and what each does at the ends of the hour. Handing the predictive controller the whole route makes it worse, 28.97 km against 29.05 km, and nearly doubles its exertion travel. A plan that eases before a climb to bank lactate headroom finds the headroom gone four minutes later, which is what Section 7.4 predicts for a limit that is a slow filter on effort, and the route-aware plan is 2 s ahead of the regulator at the first lap and 4 s behind at the second.

🕮 **Pause.** Try Exercise 7.3 before reading on.

On the circuit the gap between two riders at a fixed time depends on where each is on the lap, so Table 7.9 decomposes the predictive controller's advantage on the level hour instead, against the compliant 215 W regulator.

About two thirds of the 279 m exists after one minute. The predictive controller asks for 100 % exertion from the start, reaches it at the slew limit after 24 s, makes 737 W and 43.7 km/h at thirty seconds and spends 7.1 kJ of W' while lactate is still far below its limit, and from ten minutes to fifty

Table 7.10 The 10 km Climb, 518 m of Ascent, From a Standstill. Recreational Preset.

Strategy	Time	Compliant	Peak L (mmol/L)	W' left (kJ)
Steady 72 %	40:22	no	4.15	16.0
90 % for 5 min, then 72	40:20	no	6.17	6.9
85 % for 5 min, then 72	40:39	no	6.80	6.9
PID at 215 W	40:58	yes	3.83	16.0
MPC, grade underfoot	40:14	yes	5.65	10.5
MPC, whole route	40:08.8	yes	5.69	9.6

its exertion is within 1.1 points of the regulator's. The last five minutes add 100 m as the limits open. Removing the finishing allowance from the same controller gives 34.27 km on the level and 28.97 km on the circuit, so the finish is worth 94 and 82 m. Even so, the predictive controller crosses the line with 11.2 kJ of W' on the level and 11.4 kJ on the circuit, because lactate reaches the opening limit before W' runs out. The binding store at the finish is lactate headroom, not work capacity.

> ✦ DID YOU KNOW? **The Highest Peaks With No Excess**
>
> From the simulator: on the rolling circuit the predictive controller's lactate peaks at 5.69 mmol/L and its heart rate at 173 bpm, against 3.91 and 159 bpm for the regulator at 215 W, yet neither hour records lactate excess (Table 7.8). Both limits open over the last five minutes of the hour (Equation (7.17)).

7.7.4 The Time Trial

Scenario 19 changes the objective to time over a fixed distance, the 10 km climb, and Table 7.10 reports six ways of riding it.

A hard start saves 2.8 s at 90 % and loses 16.5 s at 85 %, and both starts break the lactate limit for the rest of the climb. The longer excursion of the 85 % start leaves more fatigue, with critical power at the summit at 209.9 W against 212.4 W after the 90 % start and 219.1 W after the steady ride. The steady 72 % also breaks it narrowly, at 71.4 mmol s/L, so the fastest compliant strategy by hand lies slightly below it. The predictive controller is 44.25 s faster than the regulator and compliant, and here the route profile helps by 5.0 s. On a fixed-distance objective the slow pitches carry more of the time, so a watt moved from the gentler pitches to the steeper ones is worth something, which is the mechanism behind the variable-pacing predictions [10], and the gain is small because the pitches differ by only 3 to 7 %.

7.7.5 What Each Selector Rule Is Worth

Table 7.11 removes the rules of (7.22) one at a time from a copy of the controller and measures the case each rule exists for.

The band row is the design of Chapter 6, in which a losing loop is only prevented from sitting more than a fixed band above the selection, which suits two slow loops. On the bicycle an automatic start breaks it within four scans. At the first scan the applied exertion is zero, so the band pins both overrides near 5 %, and from there they move only by their own slow increments. The power loop leads from rest, and at the fourth scan its request of 7.8 % passes the lactate loop's 5.0 %, so the lactate loop has the smallest request and is selected. From then on the power loop is itself a loser held inside the band, and the exertion rises only as fast as the lactate loop's 420 s integral moves it, to 5.3 % at twenty seconds while lactate sits at rest. The negative-margin row reproduces the ratchet the non-negative rule was written to prevent. The conditional-integration row is recorded because the rule buys nothing on the shipped cases, and without it the circuit hour is 3 m longer and the time trial 2.5 s shorter. Its purpose is to keep the selected loop from accumulating error that the rate limit

Table 7.11 The Selector With One Rule Removed at a Time. Each Row Is Measured on the Case the Rule Exists For, Against the Shipped Controller.

Rule removed	Measured effect
Margin scaled to the limit (losers held within a 0.05 band instead)	circuit hour 18.50 km against 28.90 km. The lactate loop is selected 99.9 % of the time, and exertion is 6.1 % at one minute and 39.6 % at thirty.
Margin never negative	power target 250 W and heart-rate limit 160 bpm on the circuit. Exertion falls from 76.2 to 46.2 % between 160 and 200 s with heart rate still 4.3 bpm over its limit, and the hour is 28.65 km against 28.91 km.
Leader chosen by margin on entry	return from the dropout of scenario 17. Exertion jumps from 64.4 to 74.9 % in four seconds at the slew limit, against a ramp to 66.2 % in the same time.
Conditional integration	circuit hour 28.900 km against 28.896 km, and the time trial under the power loop 2.5 s faster. The rate limit binds only in the first seconds.

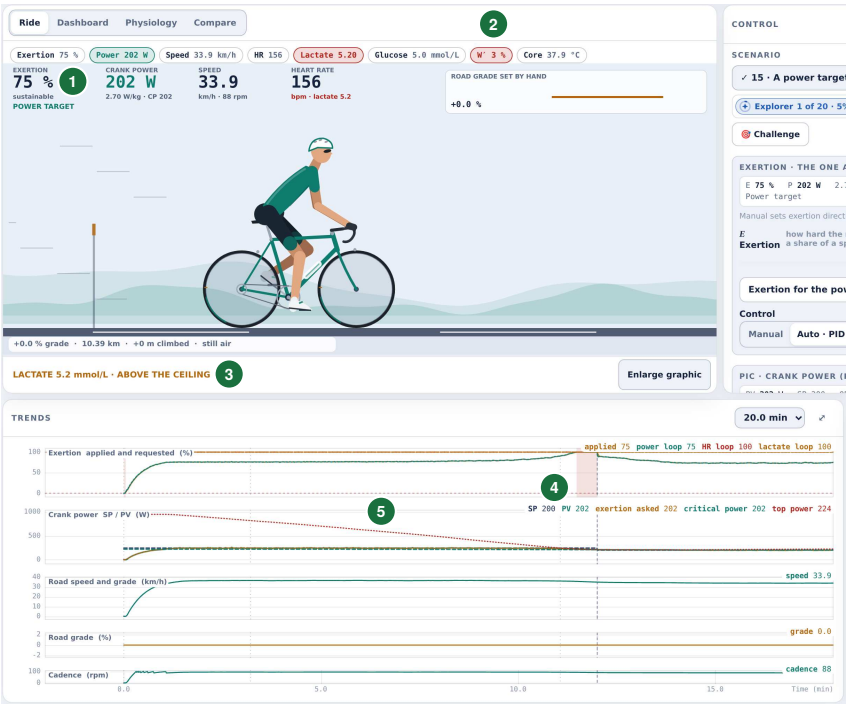


Figure 7.4 Scenario 15 at $t = 18$ min, a 245 W power target above critical power with both overrides out of reach, cropped to the Ride view and the trends. The exertion trace climbs while the power holds its target, reaches its stop at 11.5 min in the shaded saturation band, and the power then falls onto the dashed critical-power line while the dotted ceiling P_{top} decays from 950 W with the work capacity. After the target drops to 200 W at twelve minutes the exertion comes straight off the stop, and at eighteen minutes the rider holds 202 W at 75 % with W' at 3 %, a ceiling of 224 W and a critical power that fatigue has lowered to 202 W. The run records the ride as non-compliant, because with the overrides out of reach lactate peaked at 7.8 mmol/L at eleven minutes and is still 5.2 mmol/L, 2095 mmol s/L above the limit so far. Callouts: (1) the applied exertion with the power loop selected, (2) the W' left, (3) the status line on lactate, (4) the exertion command on its stop in the shaded saturation band, (5) the dotted ceiling decaying onto the dashed critical-power line.

prevented the actuator from answering, and on this plant that happens only in the first seconds of an automatic start.

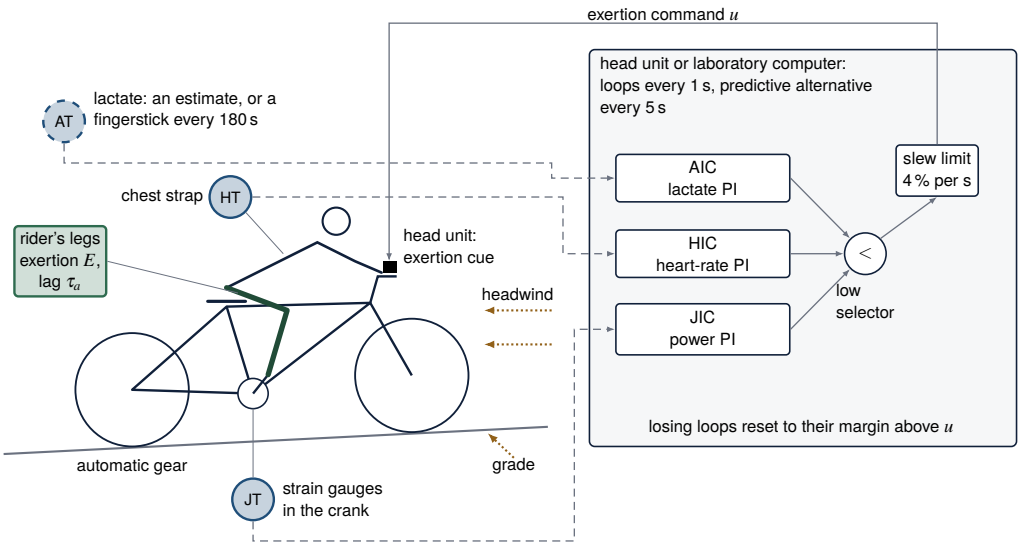


Figure 7.5 The selector loop of Section 7.5 with the equipment a rider carries in place of the schematic's symbols. Strain gauges in the crank (JT) report crank power and a chest strap (HT) heart rate, while the lactate signal (AT, dashed) is an estimate or a sampled blood reading. Three PI loops (JIC, HIC, AIC) meet in a low selector, and the slew-limited exertion command u goes to the head unit as a cue. Notice that the final element is the rider's legs, drawn solid, and that no instrument in this loop reads the work capacity that sets what the legs can deliver.

7.8 Technology in Practice

The power meter, the heart-rate monitor, the lactate reading and the exertion command of Figure 7.1 stand for the equipment a rider carries and a laboratory uses. Figure 7.5 draws the selector loop of Section 7.5 with that equipment in place of the schematic's symbols.

What Measures It

Crank power comes from strain gauges in the drivetrain, read by a head unit on the handlebar [24]. The chapter's power meter, with 4 W of colored noise and a 3 s filter, stands for those gauges and for the averaging of the two impulses of each crank revolution. Heart rate comes from a chest strap, as in Chapter 6, and the held reading of scenario 17 stands for a strap that stops reporting. Lactate has no continuous sensor. The chapter's fingerstick, drawn every 180 s and read 45 s later, stands for a blood sample, and its default lactate signal is an estimate.

> **TECHNOLOGY Sensor: The Strain-Gauge Power Meter**
>
> A cycling power meter measures pedaling force with strain gauges, most often in the crank spindle, a crank arm or a pedal spindle, and a head unit on the handlebar records the power in real time on the road. Each crank revolution delivers two impulses, one per leg, so a reported power is a mean over many of them. Gauges at different positions read differently, because the drivetrain loses some energy between them. In tests cited by a 2022 review, power meters averaged a trueness of -0.9 ± 3.2 %, and some units deviated by more than 5 %. The review recommends calibration and a day-to-day zero offset before the data are used [24].

What Moves It

The final element is the rider. The exertion command reaches the legs through the neuromuscular lag τ_a of Equation (7.1) and a slew limit of 4 % per second, which stand for a rider following a cue on a

screen. An automatic shifter picks the gear between the legs and the road, and scenario 10 shows that no exertion setting fixes a gear that is wrong for the grade.

> **TECHNOLOGY Final Element: The Rider, Cued by a Screen**
>
> Nothing on a road bicycle turns the pedals but the rider, so a pacing controller can only show a target and let the rider follow it. In a laboratory pilot a virtual coach displayed the optimal power to hold, recomputed once every 100 m and held constant over each interval so that it was comfortable to follow. The rider saw only that display, and the trial time fell by 3 % against a self-paced trial [11]. A heart-rate loop on a computer-controlled ergometer has a machine as its final element instead: the ergometer's own microcontroller adjusts the flywheel load to meet the target work rate it is sent [13].

Where the Controller Runs

On the road the platform is the head unit, a cycle computer that receives power and heart rate over a wireless link. Ten such computers recording one ergometer's power once a second agreed at constant loads and differed significantly in short all-out efforts [25]. In the laboratory a personal computer runs the loop over a serial link to the ergometer [13]. The chapter's three loops and their selector scan once a second, the rate at which those cycle computers recorded, and the predictive controller replans every 5 s, the period of the laboratory heart-rate loop. The selector, the slew limit and the fallback cap form one pacing layer above the rider, who closes the innermost loop.

> **IN PRACTICE Choosing the Sample Interval of a Heart-Rate Loop**
>
> A laboratory heart-rate loop on a cycle ergometer shows how a sample interval is chosen in practice. Its designers followed the guideline that a controller should sample 10 to 30 times faster than its closed-loop bandwidth. A bandwidth of 0.01 Hz gave a range of 0.1 to 0.3 Hz, and the loop ran every 5 s on the mean of five heart-rate samples taken once a second. Cadence was a declared test condition: one series held 70 rpm from a visual display, and another let each rider choose [13].

What Is Modernizing It

Three developments bear on this loop. The first is pacing planned on a model of the rider. An optimal pace, computed in advance by dynamic programming from individually identified models of fatigue and recovery, was read from a table every 100 m of the ride [11]. The second is an estimate of the work capacity left, shown to the rider in real time, which a differential balance model allows once its recovery time constant is known for that rider [17]. The third is a reading of the body's state without a blood sample. The pilot's authors estimated the remaining anaerobic energy open loop and expected that wearable sensors such as a sweat lactate sensor might close that loop [11]. A wearable sensor on the forehead has since tracked sweat lactate continuously through an incremental arm-crank test in 27 healthy young adults [26]. Such a measurement would turn the chapter's work capacity into a soft-sensed variable, as Section 7.10 proposes.

7.9 Limitations

No part of the model is a physiological measurement. The constants in (7.3), (7.11), (7.12) and (7.14) were set so that three invented riders behave plausibly, not fitted to data from any person or group, and while each relation follows published work, none of the parameter sets does. The application has not been validated for planning training, judging fitness or advising a rider.

Exertion is a normalized effort, and its map to power is a modeling choice made to reproduce the all-out test. A rider holding a perceived exertion also lets power fall in anticipation of the effort still to come, which the map represents only through fatigue and heat. The road has no corners, no drafting, no crosswind and no change of riding position, and the gearing is an eleven-step ladder behind one drivetrain efficiency.

The predictive controller returns a local improvement found by a bounded coordinate search over six blocks with a rolled-out tail, with no guarantee of optimality, so Table 7.8 compares two implementations rather than an architecture with its bound. On the circuit the plan that is given the route does worse than the plan that sees only the grade underfoot, and the application reports that result as it is. Giving a controller the route is a teaching privilege no real controller has, and any result obtained with it is marked as such on screen and in the exported file. Where each modeling assumption fails is set out in Section 7.3.11.

7.10 Discussion

Nothing in this structure is specific to the bicycle. Any actuator whose authority is a store that drains with use and refills slowly poses the same three problems, as a battery pack under a peak load, a hydraulic accumulator behind a press, a catalyst bed between regenerations and a heat exchanger between cleanings all do. The vault of reserves that defends a currency peg in Volume 4, Chapter 14 is a store that drains with use in the same way. A controller that reads the shortfall as a process disturbance winds up against a limit that is really a state. The correct response is to recognize that the actuator has run out, hold the integral, and let whichever loop can still act take the decision. The bicycle makes the point easily because its store has a name, a unit and a known recovery law.

Set beside the treadmill of Chapter 6, the bicycle also qualifies the value of preview. The treadmill's incline multiplies the energy cost of every meter, so an approaching hill moves the constraint and a controller that sees it early can prepare. On the bicycle at constant exertion the grade changes the speed and not the constraints, a watt buys nearly the same speed on the flat as on a 6 % climb, and the binding lactate limit averages the effort over about four minutes. Preview therefore costs 82 m on the timed circuit and buys 5.0 s on the fixed-distance climb. Knowing which of the two cases a plant belongs to settles whether a disturbance forecast is worth building before any controller is designed around one.

The equations support two methods beyond those implemented. Written with its constraints, the nineteen-state model can be posed as nonlinear model predictive control in APMonitor [27] or the Gekko Optimization Suite [28], replacing the blocked search of (7.23) with a gradient-based solution over a fully discretized horizon in which W', critical power and the lactate limit are explicit path constraints. Estimation is the more interesting extension. A moving horizon estimator on the same model could infer W' and the present critical power from the power meter and the heart-rate monitor, and so turn the authority the actuator has left, which is the most important and least observable quantity in the plant, into a soft-sensed variable instead of a model state. The predictive controller's estimate already integrates W' from the meter, but under the declared critical power, so a rider stronger than declared drains it in the estimate and not on the road.

> **△ OPEN CHALLENGE Knowing How Much Work Capacity Is Left**
>
> In the chapter the work capacity left is a model state, and the predictive controller integrates its own estimate of it from the power meter. A 2020 review notes that no technique measures it directly. Balance models estimate it from power, and a differential form lets a cyclist watch the estimate in real time, provided its recovery time constant was individually and accurately calculated beforehand. That constant varied widely between riders, refilling slowed after repeated efforts, and a model built on untrained riders underestimated the refill of elite athletes [17]. Progress would be recovery parameters identified for each rider and checked against protocols and riders unlike those that produced them, which is where the review calls for more research.

7.11 Conclusion

This chapter has presented a cycling simulator in which the manipulated variable is exertion and the power it buys is a consequence of the rider's state. The model integrates nineteen states covering the road, the drivetrain and a lumped athlete, with the road-cycling force balance, a torque ceiling from

a torque-cadence line, gross efficiency as a declared function of cadence, and an exertion map in which the power ceiling falls linearly with the work capacity left. The gain from exertion to power is 293 W per unit below the sustainable exertion and 2920 W per unit above it for a fresh recreational rider, and a held exertion above that level settles onto critical power with a time constant of 21.9 s at full effort. The limits trail the effort, with heart rate 82 s and lactate 230 s behind a step. A power loop with heart-rate and lactate overrides on a low selector covers 28.90 km of the rolling circuit in the hour and a receding-horizon controller 29.05 km, both compliant, and on the level hour two thirds of the predictive advantage is earned in the first minute. The selector depends on details that a block diagram omits, and removing the margin scaling costs 10.4 km of the hour. The terrain preview costs 82 m on the timed circuit and buys 5.0 s on the fixed-distance climb, because on a bicycle the grade enters the objective and not the constraints. A fuller application would add an indoor-trainer mode with a virtual road, drafting and position changes in the drag area, measured rider data for identification, estimation of the remaining work capacity from the power and heart-rate signals, and a gradient-based predictive controller. A rider on a bicycle is an accessible plant for learning to control an actuator whose authority is a store.

KEY POINTS

- A rider sets how hard to try, and the watts that effort buys fall when an effort above the sustainable level is held, because a finite work capacity drains.
- The gain from exertion to power is 293 W per unit below the sustainable exertion and 2920 W per unit above it for a fresh recreational rider.
- Heart rate trails the effort by 82 s and lactate by 230 s, so an override on lactate is too slow to protect the work capacity.
- On the rolling circuit the selector covers 28.90 km in the hour and the predictive controller 29.05 km, both compliant. Removing the margin scaling costs 10.4 km.

Where else this applies. One actuator shared among limits by a selector returns in Volume 4, Chapter 4 and Volume 4, Chapter 3, and a store that drains as it is used in Volume 4, Chapter 14.

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7.12 Exercises

7.1 ○ Concept check · 2 min

In scenario 2 the rider steps from 70 % to 85 % exertion at 15 min on the level and holds it. What does the crank power do over the next four minutes?

(a) It settles at a new steady value, because every exertion buys a fixed share of critical power

(b) It falls to zero once W' is empty, because the rider can no longer turn the cranks

(c) It falls from a peak toward critical power while the exertion trace stays flat

(d) It rises slowly as oxygen uptake and heart rate catch up with the new exertion

7.2 Concept check · 2 min

In scenario 16, with the power target out of reach, the lactate loop takes the selector at 68 s. Lactate still rises until 149 s, and W' falls below 5 % at 135 s. Why does the override not protect W'?

(a) Its integral wound up while it was losing, so it could not pull the exertion down

(b) The slew limit of 4 % per second holds the exertion near its peak for minutes

(c) Its margin went negative, and the reset ratcheted the exertion upward

(d) Lactate trails the effort by minutes, so W' is spent before the reading shows it

7.3 Concept check · 2 min

On the timed rolling circuit, handing the predictive controller the whole route gives 28.97 km in the hour against 29.05 km with the grade underfoot only. Which explanation does the chapter support?

(a) The grade changes the speed and not the physiology, and the binding lactate limit averages the effort over minutes

(b) The route-aware plan is solved less often than the plan that sees only the grade, so it answers each climb later

(c) The route profile makes the controller's copy of the rider less accurate, so its plans violate the limits

(d) The move penalty suppresses the extra changes of exertion that a route-aware plan needs

7.4 Theory and calculation · 6 min

Use the recreational preset, fresh: $CP = 220\,\mathrm{W}$, $E_c = 0.75$, $P_{pk} = 950\,\mathrm{W}$ and $W'_0 = 16\,000\,\mathrm{J}$.

(a) From Equation (7.3), compute the gain $\partial P_E/\partial E$ below and above E_c at full W', and their ratio. *Checkpoint:* 293 W and 2920 W per unit exertion, a ratio of 9.95.

(b) From Equation (7.18), compute τ_W for a held $E = 0.85$, and the power 60 s and 120 s after the step. *Checkpoint:* $\tau_W = 54.8\,\mathrm{s}$, 318 W and 253 W.

(c) For a held $E = 1$, compute τ_W and the time after which the closed form is within 0.2 W of critical power. Say what this means for the length of the all-out test. *Checkpoint:* $\tau_W = 21.9\,\mathrm{s}$, about 180 s.

7.5 Theory and calculation · 6 min

Take the road model of Equation (7.5) with $\rho = 1.20\,\mathrm{kg/m^3}$, $C_dA = 0.32\,\mathrm{m^2}$, $C_{rr} = 0.0045$, $m = 83.5\,\mathrm{kg}$, $\mathrm{g} = 9.81\,\mathrm{m/s^2}$, $\eta_d = 0.975$ and no wind.

(a) At a steady 30.10 km/h on the level, compute the power absorbed by drag and by rolling resistance, and the crank power that supplies both. *Checkpoint:* 112.2 W, 30.8 W, 146.7 W.

(b) Compute $\partial P/\partial v$ of the crank power $(F_a + F_r + mg\sin\theta)\,v/\eta_d$ on the level at 33 km/h and on a 6 % climb at 11.3 km/h. *Checkpoint:* 53.4 W and 59.9 W per m/s.

(c) Convert both to speed per watt and give the percentage by which the climb's value falls short of the level one. What would the same comparison show on a treadmill? *Checkpoint:* 0.0187 m/s and 0.0167 m/s per watt, about 11 %.

7.6 Theory and calculation · 6 min

Use Table 7.5, Equation (7.21) and Equation (7.22) with the fresh recreational preset ($CP_0 = 220\,\mathrm{W}$, $E_c = 0.75$, $P_L = 210\,\mathrm{W}$).

(a) Compute the power-loop gain K_c per watt and the loop gain $K_c\,\partial P/\partial E$ just below and just above E_c. *Checkpoint:* 0.00227 per W, 0.667 and 6.64.

(b) Compute the nominal marker exertion $E_{nom} = E_0(P_L)$, the fallback cap that replaces a lost heart-rate signal, and the power a fresh rider makes at that cap. *Checkpoint:* $E_{nom} = 0.716$, a cap of 64.4 %, 189 W.

(c) At 30 min of the level hour in Table 7.1, take the measurements as $HR = 156.8$ bpm and $P = 206$ W, with the 168 bpm limit and a 215 W power target. The lactate loop is selected. Compute the margins m_j at which the heart-rate loop and the power loop are reset above the applied exertion. *Checkpoint:* 0.747 for the heart-rate loop, 0.020 for the power loop.

7.7 App challenge · 8 min

Load scenario 3, Three minutes all-out, and keep the default seed and every setting it loads (Manual, 50 % exertion, the all-out effort from 6:00 to 9:00, then 30 %). Run to 20:00 at speed 20 and export the CSV file. The time column is `Time (min)`.

(a) Read column `wp_kJ` in the rows at 9.00, 15.00 and 20.00 min, within 0.1 kJ, and columns `power_W` and `cp_W` at 15.00 min, within 1 W. *Checkpoint:* 0.0, 12.3 and 14.9 kJ; 87 W against a critical power of 218 W.

(b) Below critical power Equation (7.8) is linear in W'. Write its time constant for a constant P and CP, and evaluate it with $k_r = 0.5$ and the readings of (a). *Checkpoint:* $\tau = W'_0/[k_r(CP - P)] \approx 244$ s.

(c) Predict W' six and eleven minutes after the effort from $W' = 0$ at 9:00, compare with (a), and compare the recovery with the three minutes that emptied W'. *Checkpoint:* 12.3 kJ and 14.9 kJ.

7.8 App challenge · 7 min

Load scenario 15, A power target above critical power, and keep the default seed and every setting it loads (Auto · PID, power target 245 W dropping to 200 W at 12:00, overrides out of reach). Run to 13:00 at speed 5 and export the CSV file (time column `Time (min)`).

(a) Read the time at which the exertion command first reaches 100 % and the SATURATED flag appears, within 0.1 min, and columns `power_W` and `cp_W` in the last row before 12.00 min, within 1 W. *Checkpoint:* 11.5 min; 212 W against 208 W.

(b) Read column `exertion_pct` 6 s and 60 s after the target drops (rows at 12.10 and 13.00 min), within 0.5 points. Explain from Equation (7.20) why the exertion leaves the stop at once. *Checkpoint:* 89.4 % and 80.7 %.

7.9 App challenge · 9 min

Load the challenge, One hour on the rolling circuit, and keep the default seed and every setting it loads (Auto · PID, power target 190 W). Run each hour to its end at speed 60 and read the Distance, Compliant and Lactate excess tiles. Make each change below, press Reset, then Run.

(a) Run the case as loaded. *Checkpoint:* 27.93 km, compliant.

(b) Set Power target to 215 W. *Checkpoint:* 28.90 km, compliant.

(c) Set Power target to 220 W. Explain the Compliant tile. *Checkpoint:* 28.94 km, not compliant, 14 mmol s/L.

(d) Set Control to Auto · MPC, with the grade underfoot only. *Checkpoint:* 29.05 km, compliant.

Give each distance within 0.01 km. Which runs pass the challenge, and what does (c) say about raising the power target further?

7.10 Application area · 3 min

Scenario 18 paces the level hour on a fingerstick: a blood sample drawn every 180 s and read 45 s later.

(a) How old can the lactate reading be just before a new one arrives, and how does that compare with the lag of blood lactate behind the effort in Table 7.4? *Checkpoint:* 225 s, about the 230 s lag.

(b) The chapter compares the hour on the fingerstick with the hour on the continuous estimate. State the result, and name one change an engineer could make to keep the sampled hour compliant. *Checkpoint:* 8 m more and non-compliant.

7.11 Application area · 4 min

Scenario 11 climbs 6 % for an hour at 72 % exertion in 35 °C air, first with a 3.5 m/s tailwind, at about 13.2 km/h at the end, then into a 3.5 m/s headwind, at about 12.0 km/h. No fan is fitted ($a_f = 0$).

(a) From Equation (7.14), compute the airflow a in each case and the factor $0.45 + 0.55a$ that scales evaporative cooling. *Checkpoint:* $a = 0.14$ and 0.92; factors 0.53 and 0.96.

(b) The chapter reports core temperature at the end of each hour. Use the factors to explain the difference, and say what a rider on a stationary trainer indoors needs. *Checkpoint:* 39.2 °C against 38.2 °C.

7.12 Review the agent · 5 min

An AI assistant reviews scenario 15 and writes: “The loop gain of the power loop jumps from 0.667 to 6.64 above E_c, so the loop is too aggressive. It drives the exertion into saturation at 11.5 min and winds up there. Halve the power loop gain to 0.25 and the actuator no longer saturates, so the rider holds 245 W.” Find the flaw, and say how you would check it in the app.