INTERACTIVE CONTROL ENGINEERING
Principles and Applications

Volume 19 Control of Biological Systems and Human Performance

CHAPTER 11

Feeding an Anaerobic Digester Toward Its Fold on a Daily Titration

Pages 280–303

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How to cite this chapter. "Feeding an Anaerobic Digester Toward Its Fold on a Daily Titration," in *Control of Biological Systems and Human Performance*, John D. Hedengren, Ed., vol. 19 of *Interactive Control Engineering: Principles and Applications*. Advanced Process Solutions, LLC, 2026, ch. 11, pp. 280–303.

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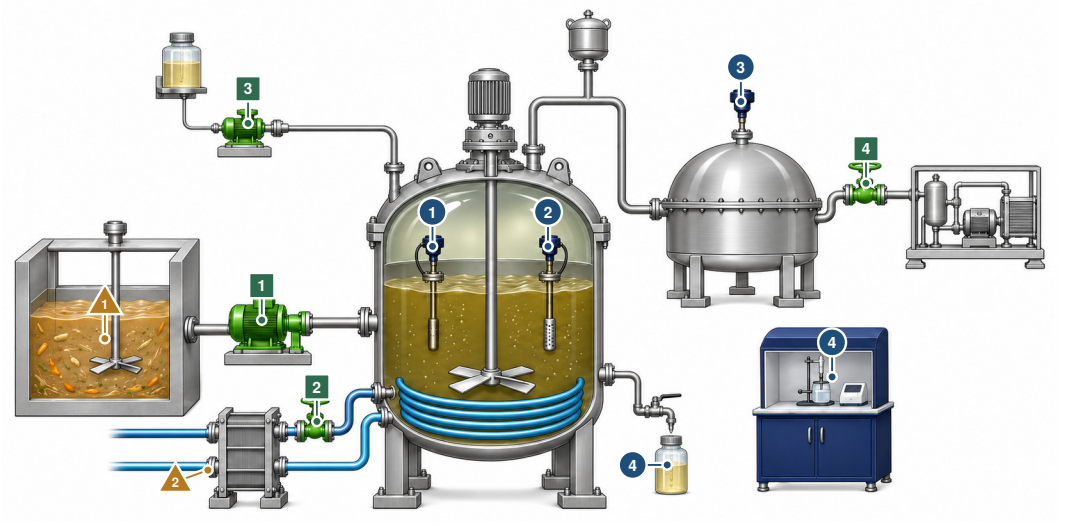
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CHAPTER 11

Feeding an Anaerobic Digester Toward Its Fold on a Daily Titration



The controller moves 1 feed pump 2 heating valve 3 bicarbonate dosing 4 gas draw to the upgrader

The controller reads 1 tank temperature 2 pH probe 3 gas holder level 4 daily acids test of a sample

What disturbs it 1 change in the feed blend 2 heater fault

Notice that the controller reads the tank's temperature and pH at once, but its acids only through a sample tested once a day, hours after it was drawn. Illustration, not to scale.

IN PLAIN TERMS **Feeding a Digester Without Souring It**

The main idea. A digester is a warm, closed tank where microbes turn manure and food waste into a gas that burns. Fast microbes make acids within hours, and slow ones eat the acids over days to give off the gas.

The challenge. More feed makes more gas for a day or two, then the acids pile up until the slow microbes stop and the tank turns sour. The test that shows the acids comes from a laboratory once a day, hours after the sample was taken.

Why it matters. A sour tank takes weeks to bring back, so the feed must be held back before the test warns.

THREE WAYS TO READ THIS CHAPTER

Explore	11 min	the first page, the fact on farm digesters (Section 11.1), the two history segments (Section 11.2), the try-it segments and the fact of Section 11.6, and the key points
Study	45 min	the whole chapter with its three pause points and the twelve exercises of Section 11.11
Practice	15 min	the first page, the measured cases (Section 11.6), Technology in Practice (Section 11.8), Limitations (Section 11.9) and the open challenge

The control question More feed raises a digester's biogas for a day before the acids consume its buffer and the tank sours for weeks. How close to that fold can a feed loop run when the only timely warning of the acids is a titration drawn once a day and reported six hours later?

Learning objectives

- Locate the fold of the Haldane growth law and explain why the retention, not the feed's strength, sets it.
- Explain why the biogas rises past the fold before the titration warns, and why bicarbonate buys time without removing acid.
- Predict how far a heater fault moves the fold from the heat balance and the temperature factors.
- Relate the gain of a loading loop to the age of the titration it acts on, and recognize a loop that rings on a sampled measurement.
- Weigh an economic supervisor's revenue against the margin it spends near the fold.

Prerequisites Sections 1.4.3 (Haldane growth and the fold), 1.4.5 (sparse and late samples), 1.4.6 (one-sided actuators and stores), 1.6.2 (velocity form) and 1.6.4 (scenarios). Section 1.6.3 is needed only for the predictive feed controller.

The app https://apmonitor.com/apps/anaerobic-digester/

See also Volume 7, Chapter 2 for the ignition and extinction folds of an exothermic reactor and Volume 7, Chapter 4 for a loop held at the peak of its plant characteristic. Volume 5, Chapter 3 for a tuning limited by a slow analyzer and Volume 5, Chapter 4 for a cook graded by late analyzer and laboratory results.

11.1 Introduction

A 4000 m^3 digester fed 160 m^3/d of dairy manure with 12 % food waste makes 4674 Nm^3/d of biogas at 60.0 % methane in the simulator of this chapter. On the second day of its overload case the feed is raised to 400 m^3/d. The biogas climbs to a peak of 9504 Nm^3/d on day 2.63, twice where it started, and an operator paid by the cubic meter would read that as success. The titration that shows the acids past the limit is drawn at 08:00 on day 3 and reported on day 3.58, almost a day after the gas has turned down, and the digester sours on day 4.51. Bringing a soured tank back takes 45.3 days at half feed with the pH held by bicarbonate, and without the bicarbonate it does not come back at all.

The behavior has a simple cause that a block diagram hides. Acidogens turn the feed into volatile fatty acids within hours, and methanogens turn the acids into methane over days while being slowed by those same acids. Above a critical feed no methanogen population can grow as fast as the liquid carries it out, and the steady states on either side of that point form a fold. The misconception the simulator corrects is that more gas means a healthy digester. Near the fold more gas is the methanogens burning a surplus they cannot keep up with, plus the carbon dioxide that the accumulating acids drive out of the bicarbonate.

The class of problem is wider than biogas. Growth on an inhibitory substrate can make a continuous culture unstable, a result published in 1968 with wastes containing phenols, ammonia and volatile acids named as the cases where it matters [1]. The controlled variable comes from a laboratory on a slow cycle, as in any plant run on a delayed assay, such as the kraft digester of Volume 5, Chapter 4, and the economics push the operator toward the edge, the setting of extremum seeking and best-operating-point studies for digesters [2, 3]. A reader who has run a nitrifying basin behind organic removal (Volume 9, Chapter 10), a fermentation with an inhibitory product or a reactor near a washout limit has met this shape.

✦ DID YOU KNOW? Four Hundred Farm Digesters

As of June 2024 the United States had 400 anaerobic digestion systems on livestock manure, 343 of them taking dairy manure, and 105 of them complete-mix tanks like the one simulated here. Vehicle fuel was by then the most common use of their biogas [4].

However, the pieces sit in different literatures. The operating diagram of the two-step model is mathematical analysis [5, 6], the titration ratio is operating guidance [7], feed control is reviewed for energy production [8], and none of them lets a student watch a gas peak, a day-late titration and a fold move with temperature on one tank. This chapter works with one digester simulator in which the fold, the daily titration, bicarbonate dosing, a heater fault and the revenue from renewable natural gas act on one plant, and it measures each one's effect on that plant. The model is published, its methanogen constants are declared teaching values, and feed control for maximum biogas is established. The simulator has a 22-state model whose chemical oxygen demand and gas ledgers close to round-off, a souring map computed from the same growth law the engine integrates, and a strict split between what the controllers may read and the populations and true acids they may not. Its application grades ten cases, from a feed raised by hand to a sixty-day season.

Section 11.2 follows digester control from failing digesters to the titration loop. Section 11.3 gives the model: the two populations, the pH and the gas, the heat balance, the events and the measurements. Section 11.4 locates the fold, shows that the retention sets it and that temperature moves it. Section 11.5 describes the three control layers and the predictive feed controller. Section 11.6 measures the ten cases, the season and the predictive controller against the sampled loop. Section 11.7 sorts the evidence for the model into code verification, agreement with published results and the missing educational evaluation, Section 11.8 names the devices and the controller platform of a real plant, and Section 11.9 states the limitations. The exercises in Section 11.11 use the cases of Table 11.3.

11.2 Failing Digesters, Inhibition and the Titration

Digesters earned a reputation for instability long before they earned revenue, and the first dynamic models were built to explain it.

⧖ HISTORY A Model That Could Fail (1969 to 1971)

Andrews opened his 1969 model of the process by naming its major problem: a poor record of stability, as the reports of failing digesters showed. A model that predicts failure, he argued, needs an inhibition function that relates the volatile acids to the growth rate of the methane bacteria. He took the un-ionized acid as both the growth-limiting substrate and the inhibitor, which settled the conflict between inhibition by the acids and inhibition by pH, since the un-ionized share depends on both. His simulations reproduced what operators saw in the field at start-up, at failure and in recovery [9]. Andrews and Graef then added the interactions within and between the liquid, the gas and the biological phases of the digester [10].

Operators read their own indicators from a laboratory titration.

⧖ HISTORY The Operator's Ratio (1976 to 1987)

A federal operations manual for anaerobic sludge digestion, published in 1976, ranked the indicators of an upset in order of importance: the ratio of volatile acids to alkalinity first, then the production rates of methane and carbon dioxide, then pH. It found that the ratio can vary from less than 0.1 to about 0.35 without significant changes in digestion and called a rising ratio the first warning, while the pH, held up by the alkalinity, may not move until the digester is completely upset [11]. A survey of seventy observations in 1987 related failure to acetic acid above 800 mg/L or a propionic-to-acetic ratio above 1.4 [12], rules for a person reading a laboratory result.

Table 11.1 Variables by Role at the Design Point (160 m^3/d, 35 °C)

Variable	Role	Design	Range
r_{lab}	CV, daily titration	0.095	limit 0.4
T	CV, thermometer	35.0 °C	setpoint 30–40
pH	CV, probe (Auto in cases 3, 5)	7.39	setpoint 6.0–7.8
L	CV, holder level	50 %	setpoint 20–90
Q	MV, feed	160 m^3/d	0–400
Q_h	MV, heater duty	215 kW	0–600
u_{alk}	MV, bicarbonate	0	0–80 kmol/d
F_{up}	MV, upgrader feed	—	0–8640 Nm^3/d
f_{fw}	DV, food-waste share	12 %	slug 50, change 25
Q_{max}	DV, heater fault	600 kW	0 for 6 d
I	DV, inhibitor shock	0	0.35, decays
X_1, X_2	states, populations	0.889, 2.120	kg VSS/m^3, truth
S_1	state, substrate	0.374	kg COD/m^3, truth
S_2	state, acids	733 mg/L	12.20 mmol/L, truth
Z, C	states, alkalinity, carbon	154.4, 153.9	mmol/L
T	state, temperature	35.0 °C	—
H_m, H_c	states, holder gas	1000 Nm^3 total	methane, CO_2
q_{gas}	output, biogas	4674 Nm^3/d	60.0 % CH_4

Model-based control needed a model small enough to identify. Bernard and coauthors reduced the process to acidogenesis and methanogenesis with an alkalinity balance, identified it step by step from designed dynamical experiments, validated it on 70 days of a 1 m^3 fermenter, and made the alkalinity central to monitoring and control [13]. The International Water Association's task group published its structured model, ADM1, a year later, with disintegration, hydrolysis, acidogenesis, acetogenesis and two methanogenic steps, ion equilibria and gas transfer in 26 dynamic states [14]. Fifteen years of instrumentation, control and automation on high-rate digesters were then reviewed with the sensors, the model complexity and the strategies each one needs [15]. The two-step model's bifurcations were analyzed [5] and its operating diagram, the map of feed and dilution on which washout and bistability occur, was drawn in full [6]. Its constants identified on a full-scale 6000 m^3 reactor at a water resource recovery facility differ widely from the original values [16].

Feed control followed the models. Volatile fatty acids and total alkalinity have been regulated together [7], robust control has been applied to improve digester stability [17], biogas production has been intensified by extremum seeking with a variable gain [2], the operating conditions that maximize biogas in simple models have been characterized [3], and feed control for renewable energy has been reviewed [8]. A recent model predictive controller moves the feed against temperature disturbances without controlling the temperature [18]. Biogas upgrading and utilization have their own review literature [19], and in this simulator the declared fuel credits on the upgraded gas are what make the last cubic meter of feed valuable.

11.3 The Digester as Modeled

The simulated plant is one generic mesophilic digester with synthetic teaching values, drawn in Figure 11.1 in the arrangement of the application's Plant view. A pit blends dairy manure with food waste, and a pump feeds the blend continuously to a 4000 m^3 stirred tank held at 35 °C by a heater that burns the tank's own gas. The biogas collects in a 2000 Nm^3 membrane holder. A membrane upgrader draws from the holder and sells the methane as renewable natural gas (RNG), and the holder relieves to a flare when it is nearly full. A dosing pump can add sodium bicarbonate. Table 11.1 classifies the variables at the design point, a feed of 160 m^3/d for a hydraulic retention of 25 days and an organic loading rate (OLR) of 2.18 kg COD/m^3/d.

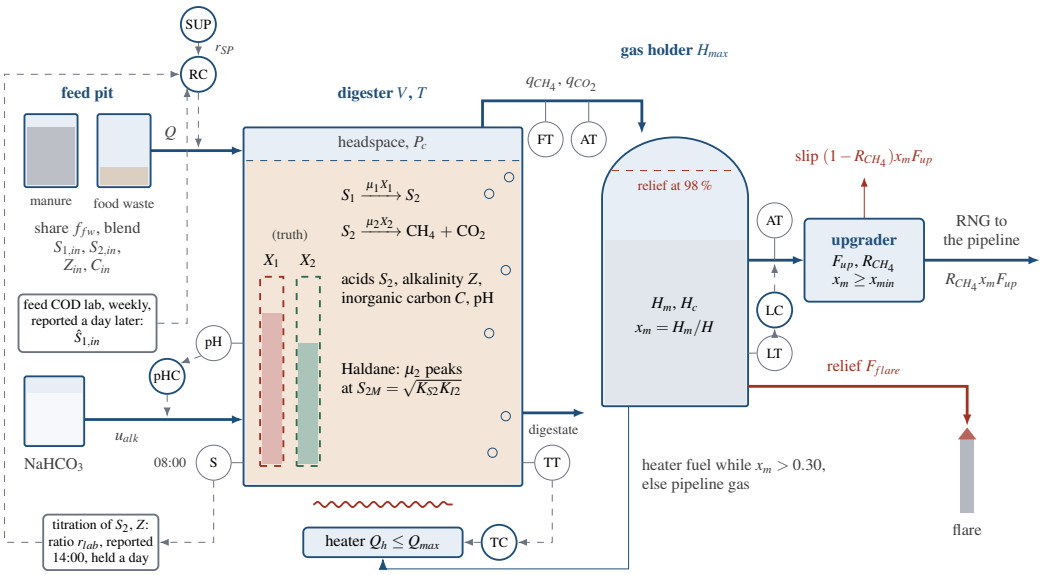


Figure 11.1 The digester as modeled. The feed pit blends manure and food waste at share f_{fw} and pumps Q into the tank of volume V, where acidogens X_1 and methanogens X_2 (teaching truth, dashed) turn the substrate S_1 into acids S_2 and the acids into methane and carbon dioxide, with alkalinity Z, inorganic carbon C and the headspace pressure P_c setting the pH. Bicarbonate enters at u_{alk}, the heater supplies Q_h, and the gas fills the holder (H_m, H_c) that feeds the upgrader at F_{up} and relieves to the flare above 98 % of H_{max}. A sample drawn at 08:00 is titrated and reported at 14:00, and the ratio controller (RC) acts on it under the supervisor (SUP). The temperature (TC), pH (pHC) and holder level (LC) loops act hourly. Not to scale.

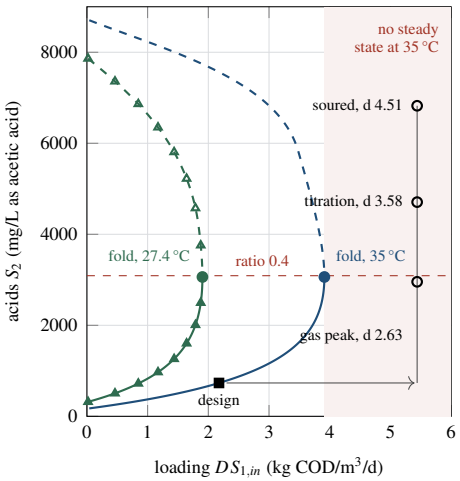


Figure 11.2 The souring map. Steady acids against loading from the application's map functions at the design alkalinity and dissolved CO_2, at 35 °C (blue, no markers) and at the heater fault's 27.4 °C (green, triangle markers). Solid branches are stable and dashed ones bound the basin the tank sours into. The thin dashed horizontal line is the ratio limit of 0.4 at the design alkalinity. The circles are case 2's true state at the gas peak, the first titration over 0.4 and the souring.

11.3.1 Two Populations

The liquid is the two-step model of Bernard and coauthors [13] written in units in which the chemical oxygen demand (COD) closes, with ADM1's first-order decay and pH inhibition [14] and a temperature factor per population. The populations grow on their substrates, decay, and leave with the liquid, as shown in Equation (11.1).

$$\begin{aligned}
\frac{\mathrm{d}X_1}{\mathrm{d}t} &= (\mu_1 - \alpha D - k_d)X_1 \\
\frac{\mathrm{d}X_2}{\mathrm{d}t} &= (\mu_2 - \alpha D - k_d)X_2 \\
\mu_1 &= \mu_{1,max}\,\theta_1^{T-35}\,I_{pH,1}\,\frac{S_1}{K_{S1}+S_1} \\
\mu_2 &= \mu_{2,max}\,\theta_2^{T-35}(1-I)\,I_{pH,2} \\
&\quad \times \frac{S_2}{K_{S2}+S_2+S_2^2/K_{I2}}
\end{aligned} \tag{11.1}$$

where $D = Q/V$ is the dilution rate in d^{-1}, $\alpha = 1$ says that the stirred tank keeps no biomass back (the fixed bed of the original work had 0.5), k_d is the decay rate, θ_1 and θ_2 are temperature factors per degree from 35 °C, and I is the activity an inhibitor shock removes. The acids S_2 are counted as acetic-acid equivalents in mmol/L. The Haldane term makes μ_2 peak at $S_{2M} = \sqrt{K_{S2}K_{I2}}$ and fall beyond it, the form of Equation (1.5). The pH factor is ADM1's lower inhibition, as shown in Equation (11.2).

$$I_{pH} = \begin{cases} 1 & \mathrm{pH} \geq \mathrm{pH}_{UL} \\ \exp\Big[-3\Big(\dfrac{\mathrm{pH}-\mathrm{pH}_{UL}}{\mathrm{pH}_{UL}-\mathrm{pH}_{LL}}\Big)^2\Big] & \mathrm{pH} < \mathrm{pH}_{UL} \end{cases} \tag{11.2}$$

with limits 7 and 6 for the methanogens (ADM1's acetate degraders) and 5.5 and 4 for the acidogens.

The substrate, the acids, the alkalinity and the inorganic carbon follow Equation (11.3).

$$\begin{aligned}
\frac{\mathrm{d}S_1}{\mathrm{d}t} &= D(S_{1,in} - S_1) - k_1\mu_1 X_1 + 1.42\,k_d(X_1 + X_2) \\
\frac{\mathrm{d}S_2}{\mathrm{d}t} &= D(S_{2,in} - S_2) + k_2\mu_1 X_1 - k_3\mu_2 X_2 \\
\frac{\mathrm{d}Z}{\mathrm{d}t} &= D(Z_{in} - Z) + \frac{u_{alk}}{V} \\
\frac{\mathrm{d}C}{\mathrm{d}t} &= D(C_{in} - C) - q_C + k_4\mu_1 X_1 + k_5\mu_2 X_2 + \frac{u_{alk}}{V}
\end{aligned} \tag{11.3}$$

where S_1 is the biodegradable COD in $\mathrm{kg/m^3}$, Z and C are in mmol/L, the dead biomass returns to the substrate at 1.42 kg COD per kg of volatile suspended solids, and bicarbonate adds one equivalent of alkalinity and one mole of carbon per mole, so it raises the pH and removes no acid. The methane flow is $q_M = k_6\mu_2 X_2$. Bernard and coauthors' identified yields close the COD of both steps with no single acid, since their $k_2 = 116.5$ would need 350 g COD per mole of acid while acetic acid carries 64. The application keeps their $k_1 = 42.14$ and $k_3 = 268$ and derives the other two so that COD closes exactly, as shown in Equation (11.4), which gives $k_2 = 636.25$ and $k_6 = 245.81$ mol/kg.

$$k_2 = \frac{k_1 - 1.42}{0.064}, \qquad k_6 = \frac{0.064\,k_3 - 1.42}{0.064} \tag{11.4}$$

11.3.2 pH, Headspace and Gas

The pH is the root of the charge balance on the liquid, as shown in Equation (11.5).

$$\begin{aligned} Z = C\,\frac{K_1 h + 2K_1K_2}{h^2 + K_1 h + K_1K_2} \\ + S_2\,\frac{K_a}{K_a + h} + \frac{K_w}{h} - h \end{aligned} \tag{11.5}$$

where $h = 10^{-\mathrm{pH}}$ in the same units, $\mathrm{p}K_1 = 6.31$ is ADM1's 6.35 at 25 °C moved to 35 °C, $\mathrm{p}K_a = 4.76$, $K_2 = 5 \times 10^{-11}$ and $K_w = 2.1 \times 10^{-14}$. The root is found by 48 bisection steps between pH 2 and 13, a fixed count so that the derivative stays a pure function of the state. Between pH 5.5 and 8 the balance reduces to the two-step model's closed form, and at the design point the two agree within 0.0063 (7.3884 against 7.3947). The headspace is the two-step model's quasi-steady one, with the carbon dioxide transfer and its partial pressure given by Equation (11.6).

$$\begin{aligned} q_C &= k_L a\,(\mathrm{CO_2} - K_H P_c) \\ P_c &= \frac{\phi - \sqrt{\phi^2 - 4K_H P_T\,\mathrm{CO_2}}}{2K_H} \\ \phi &= \mathrm{CO_2} + K_H P_T + \frac{q_M}{k_L a} \end{aligned} \tag{11.6}$$

where $\mathrm{CO_2}$ is the dissolved carbon dioxide from the speciation of C, $k_L a = 20\,\mathrm{d}^{-1}$, $K_H = 27\,\mathrm{mmol/(L\,atm)}$ and $P_T = 1\,\mathrm{atm}$. The gas flows are $q_{CH_4} = q_M V v_m$ and $q_{CO_2} = q_C V v_m$ with $v_m = 0.022\,414\,\mathrm{Nm^3/mol}$.

The holder keeps methane and carbon dioxide apart, so that its methane share $x_m = H_m/(H_m + H_c)$ is a state, and every draw takes gas at that share, as shown in Equation (11.7).

$$\begin{aligned} \frac{\mathrm{d}H_m}{\mathrm{d}t} &= q_{CH_4} - x_m(aF_{up} + F_{flare}) - F_{fuel} \\ \frac{\mathrm{d}H_c}{\mathrm{d}t} &= q_{CO_2} - (1 - x_m)(aF_{up} + F_{flare}) \\ &\quad - F_{fuel}\,\frac{1 - x_m}{x_m} \\ F_{flare} &= 200\,\max(0,\ H - 0.98H_{max}) \\ a &= \min\left(1, \frac{H}{0.02H_{max}}\right) \end{aligned} \tag{11.7}$$

where $H = H_m + H_c$, the relief gain is $200\,\mathrm{d}^{-1}$, and a stops an empty holder from being drawn. The heater needs $Q_h \cdot 86.4/(\eta_h L_{CH_4})$ $\mathrm{Nm^3}$ of methane per day at an efficiency $\eta_h = 0.85$ and a lower heating value of $35.88\,\mathrm{MJ/Nm^3}$, and the holder supplies the fraction F_{fuel} of it scaled by a and by a burner factor that falls from 1 at 40 % methane to 0 at 30 %, the rest being bought as pipeline gas. The upgrader sells $R_{CH_4} x_m a F_{up}$ with $R_{CH_4} = 0.97$, and the other 3 % of its methane is the declared slip. Revenue accrues at 0.03774 MMBtu per $\mathrm{Nm^3}$ of RNG (a higher heating value of $39.82\,\mathrm{MJ/Nm^3}$) times \$48 per MMBtu, the sum of a \$3 gas price, a \$25 federal fuel-standard credit and a \$20 state low-carbon fuel credit, all declared teaching prices.

11.3.3 Heat, Feed and Events

The tank's temperature follows Equation (11.8).

$$\rho c_p V\,\frac{\mathrm{d}T}{\mathrm{d}t} = 86.4\,Q_h - \rho c_p Q(T - T_{feed}) - 86.4\,UA(T - T_{amb}) \tag{11.8}$$

with $\rho c_p = 4.18\,\mathrm{MJ/(m^3\,K)}$, Q_h and UA in kW and kW/K, a 15 °C feed and 5 °C outdoors. At $250\,\mathrm{m^3/d}$ with the heater tripped the tank cools with a time constant of $\rho c_p V/(86.4\,G) = 13.73\,\mathrm{d}$, where $G = 14.09\,\mathrm{kW/K}$ is the feed's and the shell's conductance, toward 13.58 °C. The feed is a blend of manure ($40\,\mathrm{kg\,COD/m^3}$, 8 mmol/L acids, 170 meq/L alkalinity, 75 mmol/L carbon) and food waste (160, 80, 40 and 10 in the same units) at the food-waste share f_{fw}, which gives $S_{1,in} = 54.4\,\mathrm{kg\,COD/m^3}$ at the design share of 12 %. Five events change the plant between steps, each leaving a mark on the trends: a slug that makes half of one day's feed food waste (1.84 times the COD, 32 % less alkalinity), a feedstock change to 25 % food waste, a heater fault that sets Q_{max} to zero for six days, an inhibitor shock that removes 35 % of the methanogens' activity and decays as $\exp(-t/25\,\mathrm{d})$, and a drift of the pH probe at $0.06\,\mathrm{d^{-1}}$.

11.3.4 Measurements

The controllers read only declared measurements. Online, the biogas meter (2 %), the methane analyzer (0.3 points), the pH probe (0.02), the thermometer (0.05 °C), the holder level (0.5 points) and the upgrader's inlet methane analyzer (0.3 points) carry first-order filtered Gaussian noise with a correlation time of 0.05 d (0.02 d for the level), seeded so that a case replays exactly, and the heater reports a trip contact. The titration samples the true acids and alkalinity at 08:00 every $T_{lab} = 1\,\mathrm{d}$ and reports them $\tau_{lab} = 6\,\mathrm{h}$ later with 3 % and 1 % relative noise, as the ratio

$$r_{lab} = \frac{60.05\,S_2}{50.04\,Z}, \tag{11.9}$$

of acids in mg/L as acetic acid to alkalinity in mg/L as calcium carbonate, held until the next report. A weekly laboratory reports the feed COD a day after its Monday sample with 4 % noise. The populations, the true acids and the true pH are logged under a `truth_` prefix, and one of the application's tests searches the controller source to prove that it never reads them.

11.3.5 Parameters, Initialization and Numerics

Table 11.2 lists the parameters. The acidogen kinetics and k_1, k_3 and k_4 are the values identified by Bernard and coauthors, read through the table that Attar and Haugen reproduce [16], because the original was not available to read. The methanogen constants are declared for teaching and place the fold at 1.8 times the design feed. With the original methanogens ($\mu_{2,max} = 0.74\,\mathrm{d^{-1}}$, $K_{S2} = 9.28$, $K_{I2} = 256\,\mathrm{mmol/L}$) in a stirred tank the fold would sit at a retention of about 1.9 days, and the full-scale fit to a municipal reactor has $K_{I2} = 2396\,\mathrm{mmol/L}$ [16], so no single published set fits a farm digester on manure.

Every case starts lined out. The liquid begins at the analytic steady state of Equation (11.1) and (11.3) at the case's feed and 35 °C, the two Monod and lower Haldane roots giving S_1 and S_2, a linear solve giving the populations and 80 bisection steps on the carbon balance giving C. The holder starts at its level setpoint with the gas's own composition. Case 3 starts instead from the design state after eight days at twice the design feed with no dosing. The 22 states (seven for the liquid and the temperature, two for the holder and thirteen ledgers) advance by fourth-order Runge-Kutta steps of $\Delta t = 600\,\mathrm{s}$ in an engine whose time base is the day, the controllers execute every hour, and a row is logged every hour (the engine of Section 1.6.1). Every event and every laboratory sample and report happens once, after a step.

11.3.6 Assumptions

The two-step model lumps hydrolysis, acidogenesis and acetogenesis into one Monod step and every acid into acetic-acid equivalents, so a fibrous manure is digested faster here than in a real tank, where hydrolysis often limits. The tank is perfectly mixed and keeps no solids back. The methanogens are inhibited only by the total acids and by pH, so free ammonia, sulfide and the sodium that bicarbonate adds are absent, and the inhibitor shock is a declared factor rather than a speciation. The headspace is quasi-steady and the holder's pressure is not modeled. The carbon dioxide yield k_5 is a teaching

Table 11.2 Model Parameters. A: Adjustable in the Parameters Panel, H: Hidden, D: Derived. Units of Concentration in mmol/L Unless Given

Symbol	Meaning	Value	Unit	
$\mu_{1,max}$, K_{S1}	acidogen growth, half-saturation[a]	1.2, 7.1	d^{-1}, kg/m^3	A
$\mu_{2,max}$	methanogen growth[t]	0.164	d^{-1}	A
K_{S2}, K_{I2}	Haldane constants[t]	20, 130	—	A
k_d	decay, both populations[b]	0.02	d^{-1}	A
α	biomass leaving with liquid	1	—	H
k_1, k_3, k_4	yields[a]	42.14, 268, 50.6	per kg VSS	H
k_2, k_6	COD-consistent yields	636.25, 245.81	mol/kg VSS	D
k_5	CO_2 per methanogen[t]	170	mol/kg VSS	H
θ_1, θ_2	temperature factors[t]	1.04, 1.07	per °C	A
pH_{UL}, pH_{LL}	methanogen pH limits[b]	7.0, 6.0	—	A
$pH_{UL,1}$, $pH_{LL,1}$	acidogen pH limits[b]	5.5, 4.0	—	H
pK_1, pK_a	carbonic, acetic acid	6.31, 4.76	—	A, H
K_H, k_La	CO_2 solubility, transfer	27, 20	–, d^{-1}	A
P_T	headspace pressure	1	atm	H
V, UA	volume, shell loss	4000, 2.0	m^3, kW/K	A
T_{amb}, T_{feed}	outdoor, feed temperature	5, 15	°C	A
Q_{max}, η_h	heater capacity, efficiency	600, 0.85	kW, —	A, H
$S_{1,man}$, Z_{man}	manure COD, alkalinity	40, 170	kg/m^3, meq/L	A
$S_{2,man}$, C_{man}	manure acids, carbon	8, 75	—	H
$S_{1,fw}$, Z_{fw}	food-waste COD, alkalinity	160, 40	kg/m^3, meq/L	A
$S_{2,fw}$, C_{fw}	food-waste acids, carbon	80, 10	—	H
f_{fw}, f_{new}, f_{slug}	food-waste shares	12, 25, 50	%	A
I_{NH_3}, t_{fault}	inhibitor shock, fault length	35, 6	%, d	A
H_{max}	holder volume	2000	Nm^3	A
$F_{up,max}$, $F_{up,min}$	upgrader capacity, turndown	360, 30	Nm^3/h, %	A
R_{CH_4}, x_{min}	methane recovery, inlet floor	97, 45	%	A
prices	gas, federal, state credits	3, 25, 20	$/MMBtu	A
c_{alk}	bicarbonate price	0.40	$/kg	A
T_{lab}, τ_{lab}	titration period, delay	1, 0.25	d	A
T_{COD}	feed COD period	7	d	A

[a]Bernard et al. through Attar and Haugen. [b]ADM1. [t]Teaching value.

value that puts the design biogas at 60.0 % methane, and the inorganic carbon is not balanced against the COD. The ratio limit of 0.4, the supervisor's back-off at 0.30, the upgrader's 45 % methane floor, its 97 % recovery and the 98 % relief are teaching values. A plant takes its ratio limit from its own titration history (a healthy range of 0.1 to 0.35 is the published guidance [7]), and its upgrader limits from the membrane maker's data and the pipeline's gas-quality tariff.

11.4 The Fold

11.4.1 Where the Methanogens Lose

A steady state with methanogens present needs $\mu_2(S_2) = \alpha D + k_d$, the condition Section 1.4.3 sets out for Equation (1.5). Below the peak of the Haldane curve there are two roots, a lower one that is stable and an upper one that bounds the basin of the sour state, and above the peak there are none. With the teaching constants $S_{2M} = \sqrt{20 \cdot 130} = 51.0\,\text{mmol/L}$ (3062 mg/L) and $\mu_2(S_{2M}) = 0.0919\,d^{-1}$, so the critical dilution is $D_c = 0.0719\,d^{-1}$, a feed of 287.6 m^3/d, a retention of 13.9 days and 1.80 times the design. At S_{2M} with the design alkalinity the pH is 7.25, above ADM1's limit of 7, so the pH term does not move the fold at the design point. It sharpens the collapse once the acids have consumed the bicarbonate. The application's map function computes the fold and both branches from the same growth law the engine integrates, at the present temperature, inhibitor, alkalinity and dissolved carbon dioxide, as shown in Figure 11.2. An exothermic reactor has steady states of the same shape, with an ignition and an extinction fold (Volume 7, Chapter 2).

11.4.2 A Fold Set by the Retention

At a steady state the acids depend on D alone, because $\mu_2(S_2) = \alpha D + k_d$ contains no feed concentration. At 250 m^3/d the steady acids are 25.184 mmol/L whether the feed is 12, 25 or 50 % food waste. A richer feed needs more methanogens, which grow into it at the same retention, not less volume. The steady ratio still moves a little with the feed's alkalinity, from 0.196 to 0.220 at 250 m^3/d after the change to 25 % food waste. The consequence for control is that a feedforward dividing the loading by a measured feed COD answers the wrong model. At a ratio of 0.25 the steady feed falls only 3.3 %, from 271.0 to 261.9 m^3/d, after the feedstock change, while the feedforward cuts it 22.3 %. This mechanism is behind the revenue the feedforward costs in case 6 and in the challenge.

🕮 **Pause.** Try Exercise 11.2 before reading on.

11.4.3 A Ratio That Stays Quiet

The steady ratio along the stable branch is 0.084, 0.104, 0.163, 0.215, 0.259, 0.303, 0.328 and 0.396 at 50, 60, 80, 90, 95, 98, 99 and 100 % of the critical feed. The ratio barely moves until the last few percent, so in this model 0.3 already marks the edge and 0.4 is the fold itself. The 0.4 limit is therefore a teaching value placed at this model's fold, between the published healthy range of 0.1 to 0.35 [7] and the value near 0.5 that operator training material reads as a sour digester.

11.4.4 Temperature Moves the Fold

The methanogens' factor $\theta_2 = 1.07$ against $\theta_1 = 1.04$ makes a cooling tank produce acids almost as fast while consuming them more slowly. A fall of 7 °C costs the methanogens 38 % of their rate. In case 7 the tank reaches 27.41 °C on day 8.42, and the fold at that temperature and the state's alkalinity and dissolved carbon dioxide is 139.9 m^3/d, about half its warm value, while S_{2M} is unchanged. The same feed is safe warm and beyond the fold cold, as the green branch with triangle markers in Figure 11.2 shows.

11.5 The Control Layers as Implemented

Three layers act on the measurements of Section 11.3. Every loop uses the velocity-form PID routine of Section 1.6.2 with derivative action off, so its clamp and its back-calculation anti-windup, whose tracking constant is τ_I, are those of Equations (1.14) and (1.15). In the convention of Section 1.3, the temperature, pH and loading loops are reverse acting, because more heat, more bicarbonate and a higher loading raise the temperature, the pH and the ratio. The level trim of the upgrader is direct acting, because a holder above its setpoint calls for a larger draw. Each layer-one loop can be switched between Manual and Auto on its own, and the loading loop follows the header's switch.

11.5.1 Temperature, pH and the Holder

Every hour the temperature loop moves the heater duty with $K_c = 300\,\mathrm{kW/°C}$ and $\tau_I = 1\,\mathrm{d}$ between 0 and the capacity the trip contact says is available, so the integral does not wind up during a fault. The pH loop doses bicarbonate between 0 and 80 kmol/d with $K_c = 100\,\mathrm{kmol/d}$ per pH unit and $\tau_I = 0.5\,\mathrm{d}$, and it ships in Manual at zero dose except in cases 3 and 5. The upgrader's feed follows the metered biogas less the heater's fuel, trimmed on the holder level, as shown in Equation (11.10).

$$F_{up} = q_{gas,m} - \frac{86.4\,Q_h}{\eta_h L_{CH_4} \max(0.3,\, x_{m,m})} + K_L (L_m - L_{SP}) H_{max} \tag{11.10}$$

where the subscript m marks a measurement, $K_L = 2\,\mathrm{d}^{-1}$ and $L_{SP} = 50\,\%$. The feed is held between its turndown of 30 % and its capacity of 8640 Nm^3/d, runs at the turndown only while the holder is above its setpoint, and stops below a level of 10 % (restarting above 40) or below the 45 % methane floor (restarting five points above it).

11.5.2 The Loading Loop on the Titration

The loading loop acts once per report, on a measurement of the kind Section 1.4.5 describes. At the k-th report it runs the PI with a step equal to the time since the previous report and holds the requested loading until the next one, as shown in Equation (11.11).

$$\begin{aligned} e_k &= r_{SP,k} - r_{lab,k}, \qquad \Delta t_k = t_k - t_{k-1} \\ OLR_k &= \mathrm{PI}(e_k;\ K_c,\ \tau_I,\ \Delta t_k), \qquad 0.2 \le OLR_k \le 6 \\ Q &= \min\left(400,\ \frac{OLR_k V}{\hat{S}_{1,in}}\right) \end{aligned} \tag{11.11}$$

where OLR is in kg COD/m^3/d, $\hat{S}_{1,in}$ is the nameplate 54.4 kg COD/m^3 of the design blend or, with the feedforward on, the weekly laboratory's last report, and a ratio below its setpoint raises the loading. The defaults are $K_c = 8$ kg COD/m^3/d per unit of ratio, $\tau_I = 8$ d and $r_{SP} = 0.20$, about twice the design ratio. When the pump limit cuts the requested loading, the integral is moved by the shortfall times $\Delta t_k / \tau_I$, a second back-calculation against what the pump delivered. In Manual the loop tracks the operator's feed.

11.5.3 The Economic Supervisor

With the loading loop in Auto, the supervisor acts on the same report before the PI does. If the report exceeds the back-off ratio $r_{off} = 0.30$ it drops the setpoint to its floor of 0.20, so the PI's proportional action cuts the loading on that report. Otherwise, if the report lies within 0.03 of the setpoint, it raises the setpoint by 0.01 up to a ceiling of 0.28. A variant that also cut the loading by a fifth at once was measured when the application was built, and it lost revenue while breaking the limit, and it was not kept.

11.5.4 Plant, Map and Controller Models

The plant model of Section 11.3 is integrated. The map of Figure 11.2 is a second model, the steady branches at the present slow quantities, used only for display. The sampled controllers carry no internal biological state. The predictive feed controller (MPC) instead integrates an independent six-state nominal AM2 observer. It reads measured temperature and methane flow, reported acid and alkalinity, the selected feed COD and applied feed and dose. Feed COD is the declared design value unless COD feedforward is On, when the available laboratory report is used. Neither actual populations, hidden inhibition, current kinetic parameters nor future events are inputs.

Every six hours the observer balance is centrally linearized and discretized with its affine term. The framework's quadratic program of Section 1.6.3 predicts sixty steps, fifteen days, in twelve move blocks. Ratio weight 100, feed-move weight 0.001 and unit methane reward compete. Feed bounds 0–400 m^3/d and planned block moves of 10 m^3/d are hard. The smaller of 0.38 and the supervisor back-off ratio is a soft ceiling, with quadratic and linear prices 10^5 and 10^3. The methane reward omits holder and upgrader constraints. This follows the predictive optimization framework reviewed in [20]. Bounded output-bias correction follows the offset-correction principle of [21]: ratio reports are compared with predictions at their sample times, with gain 0.4 and bound 0.25, while hourly methane correction uses gain 0.05 and bound 5 10^3 Nm^3/d. It does not identify biomass or inhibitor kinetics. The other loops and supervisor remain active. The sampled PI tracks the applied feed. A rejected plan holds feed on the handover scan, marks the historian and returns to PI until a valid plan resumes. The first plan is computed while the engine primes.

A hand dilution Jacobian agrees within 1.4×10^{-13} and the ratio Jacobian within 6.6×10^{-14}. Over one six-hour interval, feed perturbations up to ± 10 m^3/d give ratio prediction errors below 6.3×10^{-6} against the nonlinear nominal observer. This is a local check, not a bound on a fifteen-day forecast.

Table 11.3 The Cases as Shipped

#	Setting and scripted events	Teaching point
1	Manual, 160 m^3/d, learner raises to 200 at day 1	fast gas, slow acids
2	Manual, feed to 400 m^3/d at day 1.35	gas peak before the warning
3	Soured start, 80 m^3/d, pH held at 7.0	recovery takes weeks
4	Auto, r_{SP} 0.20, K_c 8, τ_I 8 d	gain a daily sample allows
5	Feed to 300 m^3/d at day 1.35, pH held at 7.2	a buffer, not a cure
6	Auto at 0.22, slug day 3.35, change day 13.9	co-digestion, feedforward
7	Manual 250 m^3/d, heater fault day 2.4	the fold moves with T
8	Auto with supervisor, inhibitor day 20.4	revenue against margin
9	Auto MPC, target 0.25, inhibitor day 20.4	nominal forecast versus PI
10	Challenge: 250 m^3/d held (gain 0), change day 15.5, fault day 38.5	the whole season

Table 11.4 Measured Results at the Default Seed (Days From the Start)

#	Setting	Quantity	Value
1	200 at day 1	biogas day 0, 2, 15	4674, 5667, 5779 Nm^3/d
		acids start, peak (day 4.5)	732, 1106 mg/L
2	400 at day 1.35	upgrader trip, gas peak	day 2.01, 2.63
		titration over 0.4, soured	day 3.58, 4.51
3	pH held at 7	recovered, bicarbonate	day 45.3, 1227 kmol
	seeded before Run	recovered, bicarbonate	day 25.4, 866 kmol
	no bicarbonate	pH at day 30	4.42, still sour
4	$K_c = 8$	feed day 20, 40; worst	233, 248; 0.207
	$K_c = 30$	days at 400, under 100	7, 6 of 40
5	pH held at 7.2	soured, bicarbonate	day 18.85, 1002 kmol
	no bicarbonate	soured	day 10.22
6	feedforward off, on	titration peak after change	0.289, 0.275
		revenue to day 25	$159,425, $153,734
7	held at 250	minimum T, soured	27.4 °C, day 10.60
	cut to 160 at day 7.58	reports over 0.4	4, not soured
8	supervisor on, off	revenue to day 40	$190,704, $187,176
		worst titration	0.326, 0.305

11.6 Nine Cases and a Season, Measured

The cases appear in Table 11.3. Every value below was measured without a display on the engine the browser runs, at the default seed 20261026, and any change named in the text is applied at its stated day after Run. Section 1.6.4 describes how a case, its completion rule and its scorecard are set up. Table 11.4 gathers the headline values.

> ✦ **DID YOU KNOW? Knowing the Feed Lost Money**
>
> From the simulator: in case 6 a feedforward on the weekly laboratory's feed COD lowers the titration peak after the feedstock change only from 0.289 to 0.275, and the revenue to day 25 from $159,425 to $153,734 (Table 11.4). The steady acids do not depend on the feed's strength.

11.6.1 A Long Forecast With Unmeasured Biology

Case 9 compares the same 0.25 target and instruments for forty days (Figure 11.3). MPC ratio IAE is 3.130 versus 1.403 ratio-days for PI, feed travel is 640 versus 462 m^3/d, and six reports exceed 0.4 versus one. Revenue is $206,058 versus $213,896, with 4.04% flared versus zero. The hidden inhibitor at day 20.4 makes the nominal biological forecast optimistic. Scalar output correction cannot reconstruct lost methanogen activity, and methane production reward does not imply sold gas revenue. Halving the integration step retains both revenue and IAE orderings (MPC 3.086 versus PI 1.403 ratio-days). A hindsight PI grid of targets 0.20, 0.24 and 0.26 with gains 4 and 8 includes two settings

Table 11.5 Matched Feed Control Comparisons, PI / MPC

Case	Ratio IAE	High reports	Revenue ($1000)	Plans / µs
New, 40 d	1.403 / 3.130	1 / 6	213.90 / 206.06	160 / 539
Ratio, 40 d	1.188 / 1.145	0 / 0	215.05 / 233.63	160 / 390
Co-feed, 40 d	1.157 / 2.690	1 / 5	274.27 / 279.36	160 / 353
Supervisor, 40 d	1.593 / 2.963	0 / 5	190.70 / 204.33	160 / 395
Season, 60 d*	2.000 / 2.412	0 / 3	428.20 / 435.39	240 / 359

*Original suggested settings. Plans and mean Node solve microseconds are for MPC, with zero rejected plans in these runs.

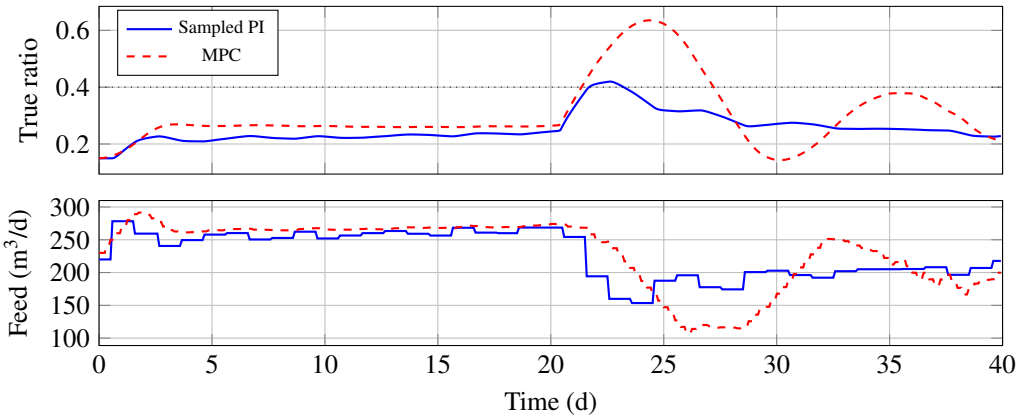


Figure 11.3 Case 9 with matched target and measurements, sampled PI as the solid line and MPC as the dashed line. Hidden inhibition begins on day 20.4. The nominal MPC accumulates more acids and flares more gas than sampled PI, despite its longer forecast. The dotted line is the 0.4 ratio indicator, not a hard state constraint.

with no report over 0.4. Fixed feeds 185, 220 and 250 m^3/d earn less than either matched controller, and the latter two sour the tank.

The ordinary ratio-loop case instead earns $233,627 with MPC versus $215,050 with PI, and ratio IAE is 1.145 versus 1.188. Co-digestion earns $279,363 versus $274,268, but has five high titrations versus one. In the suggested sixty-day challenge MPC earns $435,389 versus $428,198, but three high reports make it fail while PI passes. Thus the MPC does not replace the suggested settings of the challenge. The manual heater-fault case gives identical outputs when only the architecture option is changed. Active Auto operation is checked separately on all ten cases.

Forty browser plans per viewport average 1.13–2.46 ms, excluding observer propagation. At 0.05 d/s playback this costs 0.023–0.049% of one core, and 0.45–0.98% at the top 1 d/s. The six-hour decision is held between plans. Table 11.5 uses identical seeded instruments and no preview.

11.6.2 The Gas Answers in a Day, the Acids in a Week

Case 1 raises the feed from 160 to 200 m^3/d on day 1. The biogas goes from 4674 to 5667 Nm^3/d by day 2, and by day 7 it is within 0.11 % of its day-15 value of 5779 Nm^3/d, 24 % above the start. The acids rise from 732 to a peak of 1106 mg/L on day 4.5 and still read 1044 mg/L on day 15. The methanogens already present burn more acid at once because their rate rises with the acids, but only a larger population brings the acids back down, and at a 20-day retention it needs more acids to grow as fast as it is washed out.

11.6.3 The Overload

Case 2 raises the feed to 400 m^3/d at day 1.35, past the fold, as shown in Figure 11.4. The methane rises 41 % to its peak on day 2.61, while the biogas doubles to 9504 Nm^3/d on day 2.63, because the

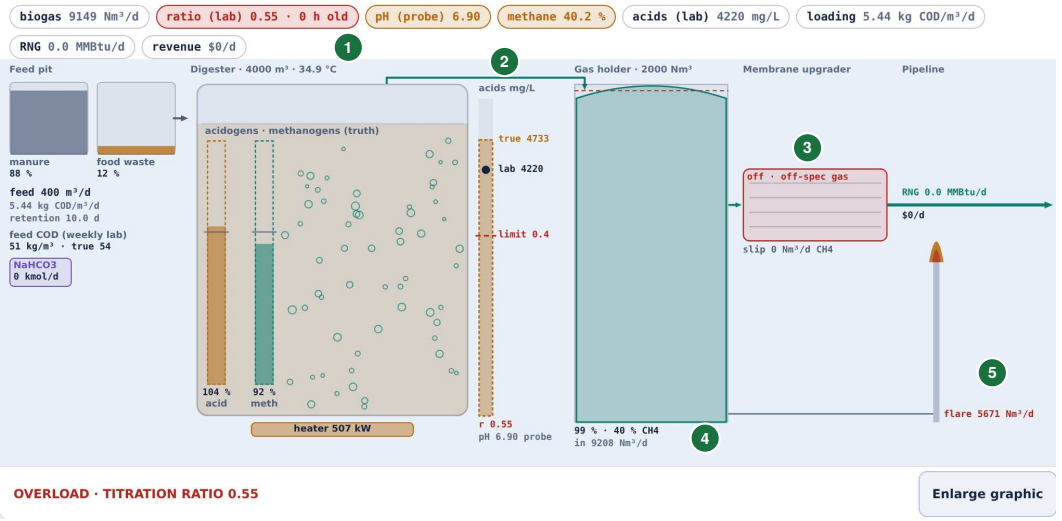


Figure 11.4 The overload of case 2 on the Plant view at $t = 3.60$ d, just after the first titration over the limit, cropped to the plant view. The feed has run at 400 m^3/d since day 1.35. Callout 1 is the report of the 08:00 sample, a ratio of 0.55, in the badge row, where the meter reads 9149 Nm^3/d of biogas at 40.2 % methane and the probe still reads pH 6.90. Callout 2 is the acids column, with the laboratory's 4220 mg/L against the true 4733 mg/L (a true ratio of 0.61). Callout 3 is the upgrader, off on off-spec gas since day 2.01. Callout 4 is the holder, 99.4 % full at 40.5 % methane, and callout 5 the flare, which takes 5671 Nm^3/d of relief.

acids displace bicarbonate and the carbon dioxide they release leaves with the gas, which reaches 2.97 times its design flow on day 3.08. That carbon dioxide gives the first warning of all. On day 2.01 the holder's gas falls below the upgrader's 45 % methane floor, the upgrader trips and the RNG sales stop, after the revenue rate had peaked 43 % above design on day 1.88. The true ratio passes 0.4 on day 2.71. The titration that shows it is drawn at 08:00 on day 3 and reported on day 3.58, 0.95 days after the gas peak, and the digester sours on day 4.51 (methane below half its three-day high with the true ratio above 0.4), when 42.5 % of the biogas made so far has been flared and the run has earned $8,268. On the map the case's state sits at a loading of 5.44 kg COD/m^3/d, beyond the fold where no steady state exists, and its acids climb from 2956 mg/L at the gas peak to 6824 mg/L at the souring.

Pause. Try Exercise 11.1 before reading on.

> **TRY IT IN THE APP** **2 · The Overload: Feast, Then the Crash**
>
> Load the case and press run. Watch the biogas trend after the feed jumps to 400 m^3/d on day 1.35: the gas doubles to a peak on day 2.63, and the titration that shows the acids over the limit is reported only on day 3.58. The tank sours on day 4.51.

11.6.4 Recovery

Case 3 starts from the overload continued for eight days. The pH is 5.33, the ratio 1.50, and the methanogens are still 79 % of their design population but make almost no methane at that pH. At half feed with the pH loop holding 7.0, the methanogens fall to 52 % of design on day 12.6 before regrowing, and the digester is recovered (true pH above 7, ratio below 0.3, methane above 45 % of design) on day 45.3 after 1227 kmol of bicarbonate, $41,218 at the teaching price. Without bicarbonate the pH stands at 4.42 on day 30 and the tank stays in its sour steady state. Replacing

a tenth of the contents with healthy digestate before Run shortens the recovery to 25.4 days and 866 kmol. Dilution alone at half feed needs 50 days per e-fold, which is why the acids must be consumed rather than washed out.

11.6.5 The Gain a Daily Sample Allows

Case 4 closes the loading loop at $r_{SP} = 0.20$. The feed rises from 160 to 233 m^3/d by day 20 and 248 m^3/d by day 40, with no titration above 0.207 and every report within 0.03 of the setpoint after day 19.6. At a gain of 30 the first report sends the feed to 390 m^3/d and the next to 69 m^3/d, and at the day boundaries over 40 days it reads the pump's 400 m^3/d seven times and less than 100 m^3/d six times. Each step is decided on acids that have not yet answered the step before. A column tuned against a slow analyzer meets the same limit (Volume 5, Chapter 3). At the shipped gain a report 0.05 above the setpoint moves the loading by 0.4 kg COD/m^3/d, about 29 m^3/d of the design blend.

11.6.6 Bicarbonate, a Buffer and Not a Cure

Case 5 raises the feed to 300 m^3/d, a 13.3-day retention, just past the fold, with the pH loop holding 7.2. The titration passes 0.4 on day 5.58 and the digester sours on day 18.85 after 1002 kmol (84 t, $33,666) of bicarbonate, against day 10.22 without it. Holding the pH bought 8.6 days and removed no acid. Past the fold the Haldane term inhibits the methanogens whatever the pH, and the time bought is time to cut the feed.

Pause. Try Exercise 11.3 before reading on.

> **TRY IT IN THE APP** **5 · Bicarbonate on pH: A Buffer, Not a Cure**
>
> Run the case as shipped, with the pH loop holding 7.2: the tank sours on day 18.85 after 1002 kmol of bicarbonate. Reload it, set the pH loop to Manual at zero dose and run again: the tank sours on day 10.22. The time the bicarbonate buys is time to cut the feed.

11.6.7 Co-Digestion and the Feedforward

Case 6 runs the loop at 0.22 from 240 m^3/d. The food-waste slug on day 3.35 raises the biogas from 7133 to 11 844 Nm^3/d, more than the upgrader's 8640 Nm^3/d, so the holder relieves to the flare from day 3.97 to 4.83 and 3.9 % of the biogas to day 25 is flared, while one titration reads 0.454 and the loop rides it out. After the feedstock change on day 13.9 the weekly laboratory reports a COD of 73.7 on day 15.33 against a true 70.0. With the feedforward on, the titrations after the change peak at 0.275 instead of 0.289 and the revenue to day 25 falls from $159,425 to $153,734. The feedforward cuts the volume of a feed whose steady acids it does not change, which is the mechanism of Section 11.4.

11.6.8 The Heater Fault and an Infeasible Variant

Case 7 feeds 250 m^3/d warm and trips the heater on day 2.4 for six days. The tank falls from 35.0 to 27.4 °C by day 8.4, the fold moves to 140 m^3/d, the titration crosses 0.4 on day 7.58 and the digester sours on day 10.60 if nothing is done. Cut to 160 m^3/d at that report, it survives with four titrations over the limit, the last reported on day 10.58. Held at 250 m^3/d, it sours at every bicarbonate dose from 0 to 80 kmol/d in steps of 10, on days 10.60, 12.78, 18.30, 22.22, 25.26, 27.95, 30.49, 33.00 and 35.56. Only less feed answers a fold that has moved below the feed, and the application says so. A fault that cooled the tank by 8 °C in two days would need a mean loss of 774 kW, 2.6 times the 302 kW this tank loses at 35 °C, so the fault cools it by 7.6 °C over six days instead.

11.6.9 The Economic Supervisor

Case 8 adds the supervisor to case 4's loop and removes 35 % of the methanogens' activity on day 20.4. The feed reaches 240 m^3/d before the shock (233 with the setpoint fixed at 0.20), the shock cuts it to 143 (152), and the setpoint reaches 0.25 on day 31.6. The revenue to day 40 is $190,704 against

Table 11.6 The Sixty-Day Challenge at the Default Seed

Policy	over 0.4	worst ratio	flared (%)	revenue ($)	verdict
shipped, gain 0 at 250	41	sour	9.11	166,751	fails
constant 185	0	0.368	0.00	341,870	fails
constant 190	1	0.411	0.00	350,866	fails
loop, fixed 0.20	0	0.303	0.00	406,506	fails
loop, fixed 0.24	0	0.360	0.00	426,881	passes
loop, supervisor (suggested)	0	0.359	0.00	428,198	passes
suggested + feedforward	0	0.331	0.00	416,550	passes
loop, fixed 0.26	0	0.389	0.00	434,400	passes
loop, fixed 0.28	1	0.417	0.02	440,312	fails

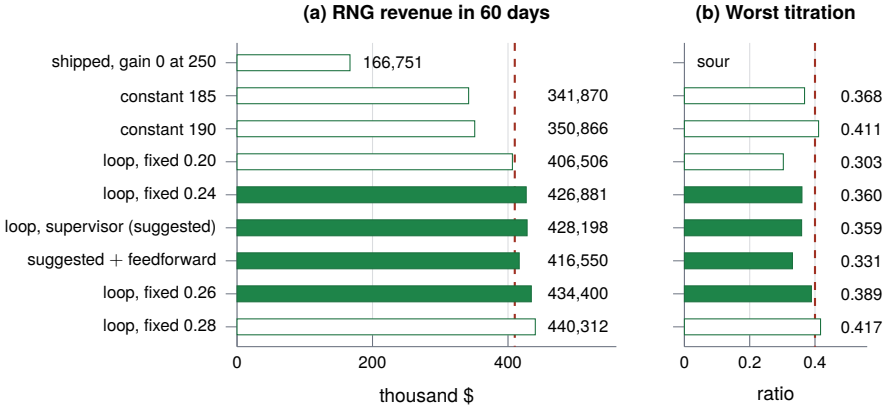


Figure 11.5 The nine policies of Table 11.6 drawn as bars: the RNG revenue of the sixty-day challenge against its $410,000 floor, and the worst titration against the 0.4 limit (dashed lines). Solid bars pass and white bars fail. Notice that revenue rises with the setpoint while the worst report climbs toward the limit, so the policy that earns most, the fixed 0.28, fails on one report, and that the constant feeds fail on revenue even where they keep the ratio low.

$187,176, 1.9 % more, with a worst titration of 0.326 against 0.305. Near a fold the last few percent of gas cost most of the margin.

11.6.10 The Season

The challenge runs 60 days at 250 m^3/d, about 87 % of the warm fold, with the feedstock change on day 15.5 and the heater fault on day 38.5. It passes with no titration report above 0.4, at most 2 % of the biogas flared and at least $410,000 of RNG revenue, on the case's own parameters. Table 11.6 lists nine policies, each with the same measurements, and Figure 11.5 draws their revenue and worst titration against the two limits. As shipped, the loop in Auto with no gain holds the feed, the methanogens cannot grow into the richer feed fast enough (their spare growth at 87 % of the fold is 0.0094 d^{-1}), the titration passes 0.4 on day 19.58 and the tank sours on day 26.74. The suggested loop with the supervisor passes. A fixed setpoint of 0.26 earns more than the supervisor with a worst report of 0.389, and the supervisor's share is finding the edge from 0.20 without hindsight. The feedforward, added to the suggested settings, costs $11,648. The first report of the richer feed arrives on day 22.33, when the loop has already cut the feed from 267 to 221 m^3/d on the titrations, and the feedforward cuts it again to 173 m^3/d. With a noise-free COD it costs $18,164, so the laboratory's noise is not the mechanism. Across seeds 1 to 5 the shipped run sours on days 26.65 to 26.78 and earns $166,101 to $167,274, and the suggested run passes every time with $426,332 to $429,879 and worst reports of 0.351 to 0.369.

11.6.11 Margin, Revenue and the Age of the Titration

Two further measurements bear on the supervisory question. The first is the distance to the fold that each policy keeps. Computed every hour from day 2 on from the true state with the map’s fold at the present temperature, inhibitor, alkalinity and dissolved carbon dioxide, the mean margin over the season falls from 54.7 to 41.8, 40.0, 36.8 and 32.7 m³/d for the fixed setpoints 0.20 and 0.24, the supervisor, and 0.26 and 0.28, while the revenue rises from $406,506 to $440,312. Every one of these policies runs past the instantaneous fold during the heater fault, by 9.3 to 34.9 m³/d at its worst hour, and none of them sours. A short excursion past the fold is not a souring, because the acids need days to climb through the basin, which is why the titration’s reaction time matters more than its threshold.

The second is the age of the titration. With the case’s parameters changed (so the attempt is no longer graded), sampling every half day with a 3 h delay moves case 2’s first warning from day 3.58 to 2.96, and every quarter day with 1.5 h to 2.90, while a daily sample reported at once warns on day 3.33. On the challenge the suggested settings then earn $432,438 and $435,857 instead of $428,198, with worst reports of 0.356 and 0.337. The literature treats the same trade with extremum seeking for biogas [2], best operating conditions for the two-step model [3] and predictive feed control against temperature disturbances [18]. The two measurements here are properties of this teaching plant.

11.7 Three Kinds of Evidence

The first kind is code verification, which asks whether the engine solves the equations written above. The COD balance closes at the design steady state with a relative residual of 4.2×10^{-15} and over the 60 days of the suggested challenge, through both events, to 3.9×10^{-14} of the feed. The holder balance closes to 2.1×10^{-15} of the production over the same run and to 4.0×10^{-15} over case 6 to day 25, where no flaring step occurs below 98 % full, and the slip is 3.000 000 % of the upgrader’s methane. The pH root agrees with the closed form within 0.0063 at the design point and within 0.008 at three checked points. Halving the step to 300 s moves case 2’s souring from day 4.5069 to 4.5035, and the suggested challenge’s revenue from $428,198 to $428,254 with every verdict and the ranking unchanged. Sixty days of the challenge take 170 ms headless and 60.1 s of browser time at the fastest setting, 1 d/s. The application runs 28 checks of its own and 18 generic ones, and every case passes the same run loaded first, from the menu and after Reset.

The second kind, agreement with published results, is weaker. With the parameter set of Sari and Benyahia’s published case (from Benyahia and coauthors [5]), the engine’s equations reproduce their critical dilution of 0.818557467 d^{-1} and their $S_{2M} = 36.332\,\mathrm{mmol/L}$ to round-off [6], and the integrated steady state at $D = 0.7\,d^{-1}$ equals their closed form. That is code verification against a published steady state, not validation. The acidogen constants are identified values from a fixed bed on another substrate, the methanogen constants are declared, and no digester’s data were fitted, so the model has not been validated against any operating digester. The third kind, educational evaluation, is absent. No study of learning has been run, so no gain in learning is claimed.

11.8 Technology in Practice

The simulated digester carries four hourly loops, a daily titration, a weekly feed analysis and an economic supervisor. Figure 11.6 draws them with the devices a farm plant would install.

What Measures It

The loading loop’s measurement is not an instrument on the tank but a laboratory result: the ratio of volatile acids to alkalinity from a sample drawn at 08:00. The chapter models it as the true acids and alkalinity of the sample hour, with 3 % and 1 % relative noise, reported 6 h later and held for a day. The online instruments serve the faster loops: the biogas meter, the two methane analyzers, the pH probe, the thermometer and the holder level, each with filtered noise.

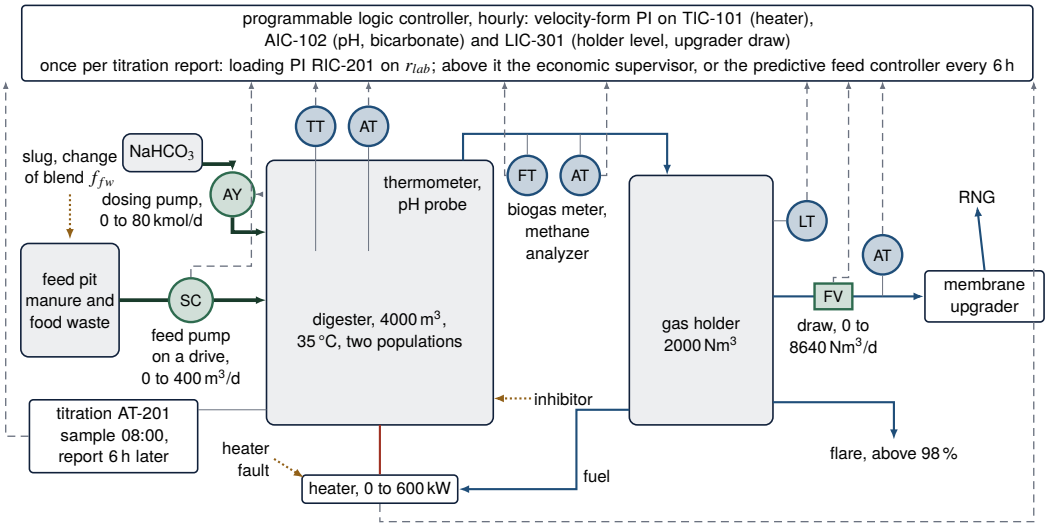


Figure 11.6 The chapter's loops with the devices a farm plant would install. Thick green lines carry the feed, blue lines the gas, dashed gray lines the signals and dotted amber arrows the disturbances. Notice that every instrument on the tank and on the gas is continuous except the one the loading loop acts on: the titration reports once a day, six hours after its sample.

> **TECHNOLOGY** **Sensor: Volatile Acids and Alkalinity by Titration**
>
> The ratio comes from two titrations of one sample. In the 1976 method a settled sample is titrated with sulfuric acid to pH 4.0, which gives the total alkalinity, then boiled lightly to drive off carbon dioxide and titrated with sodium hydroxide from pH 4.0 to 7.0, which gives the volatile acid alkalinity. The method was judged acceptable for digester control but not accurate enough for research [11]. Titration assays remain common for their low cost, while offline sampling introduces delays and limits the time resolution [22]. A sample taken just after feeding, or from a line not run clear, is not representative [11], and on a trend it shows as one report out of step with its neighbors. The chapter's 6 h report delay and its relative noise stand for the laboratory.

What Moves It

The feed pump sets the loading, a dosing pump adds bicarbonate, the heater duty holds the temperature and the upgrader's draw holds the holder level. In the model the feed is a flow between 0 and 400 m^3/d that arrives at once, so the delay of the loading loop lies wholly in the titration and in the biology.

> **TECHNOLOGY** **Final Element: Feed Pump on a Variable-Speed Drive**
>
> A positive-displacement pump discharges a fixed volume of fluid with every revolution of its shaft, so at a constant motor speed it delivers a fixed flow. A variable frequency drive sets the speed of an alternating-current motor by varying the frequency of its supply, and such drives are important final control devices in industry [23]. A blocked discharge stalls a positive-displacement pump and overloads its motor unless a relief path exists [23]. The 1976 manual found that the hydraulic overload of a digester is caused most often by leaving the wrong valve open or a pump on by accident [11]. On a trend that fault shows as a feed that does not match the request. The simulated pump delivers the requested feed at once, between 0 and 400 m^3/d.

Where the Controller Runs

On a farm plant these loops would run on a programmable logic controller or in a small distributed control system. A programmable logic controller holds instructions for input and output, logic, timing,

counting and three-mode PID control, and in a distributed control system it serves as a local controller within a supervisory scheme [24]. Supervisory and lower loops run continuously with cycle times from milliseconds to minutes [24]. The chapter's temperature, pH and level loops execute once an hour, and the loading loop once per titration report, so the platform's scan is never the limit. In a function block program the loading loop is a PI block whose input is a laboratory value entered once a day, and the supervisor that moves its setpoint, like the predictive feed controller that replans every six hours, belongs one layer above it.

What Is Modernizing It

Three lines of work aim at the day the titration costs. The first is online acid sensing. Microbial electrochemical sensors have measured volatile fatty acids with an accuracy comparable to gas chromatography, but electrochemical sensors in digestate are still limited by their longevity, reproducibility and biofouling, and need cleaning and recalibration [22]. The second is the soft sensor. Machine learning models have predicted the ratio of volatile acids to alkalinity in real time from the operating data of a pilot bio-electrochemical digester, the best of them a neural network with an explained variance of 0.77 [25]. The third is model-based feed control. A predictive controller has moved the feed rate to hold production against weather-driven temperature changes without controlling the temperature itself [18], and a supervisory policy has been learned offline on a calibrated digital twin of a full-scale digester [26]. Each would shorten or bridge the delay that sets the gain of the chapter's loading loop in case 4.

> ⚠ **IN PRACTICE** **Read the Ratio as a Trend**
>
> Operator training for wastewater plants reads the ratio in bands: 0.05 to 0.1 optimum, 0.3 to 0.4 stress, 0.5 to 0.7 deep trouble, 0.8 and above failure. It calls the acid-alkalinity relationship the first warning and the pH one of the last, and it notes that gas with more than 45 % carbon dioxide will not burn. A sour digester is cured by adding alkalinity, most often lime, or sodium bicarbonate at a higher cost [27]. The 1976 manual cautions the operator against an absolute number: the rate of change is much more significant, and trends of the indicators best predict an upset [11].

11.9 Limitations

All values describe a consistent generic digester, not a measured one. The methanogen kinetics are chosen to place the fold at 1.8 times the design, and the published constants span an order of magnitude. Hydrolysis is fast, the acids are one pool, and ammonia, sulfide and sodium are absent, so a real tank on manure has loading limits that this model does not have and a bicarbonate dose that this model treats as harmless. The steady state is set by the retention alone, which is what makes the feedforward lose, and a model with hydrolysis limitation or an ammonia balance could reward it. The ratio limit, the upgrader's limits and the prices are teaching values. The titration is the only acids measurement, so the online analyzers and direct online acid measurements that shorten its delay in practice are absent. The nominal MPC observer is an unvalidated forecast, and its soft ratio ceiling does not ensure stability. A simulated pass is not a guarantee of a real digester's stability.

OPEN CHALLENGE **Feeding Closer to the Edge on a Real Plant**

Full-scale digesters are often operated conservatively because their process feedback is delayed, their stability constraints are nonlinear and their state is hard to observe. A supervisory policy learned offline on a calibrated digital twin of one plant showed a potential gain of 2.7 % in biogas on a year of its operating data, with a probability of 0.524 that the gain is real [26]. A review of digester sensing asks for more reliable sensors, open datasets and interpretable models [22]. Progress would look like a feed supervisor proven on a live plant, with a gain that holds outside its own model.

11.10 Conclusion

An anaerobic digester is a plant in which more feed first looks like success, and this simulator shows why with the published two-step model, a carbonate charge balance, ADM1's pH inhibition, a heated tank, a holder, an upgrader and a flare. The fold sits at 287.6 m^3/d, 1.8 times the design, and is set by the retention, so a richer feed needs more methanogens rather than less volume, and a feedforward on the weekly COD costs $11,648 over the season. At 400 m^3/d the gas peaks on day 2.63, the titration warns on day 3.58 and the tank sours on day 4.51, after the upgrader has already tripped on off-spec gas on day 2.01, and recovery takes 45.3 days with bicarbonate. On a daily titration a loading gain of 8 settles in about three weeks while 30 swings the feed between its limits, and a supervisor that raises the setpoint toward 0.25 earns $428,198 in sixty days with no report over 0.4, against $166,751 for a constant feed that sours. The model is lumped, its methanogens are declared, and its only acids measurement is the titration. A fuller model would add hydrolysis, ammonia and sodium inhibition, an online acids analyzer and biological state identification for the nominal predictive feed controller. A digester is fed on its slowest measurement, and the simulator shows what that costs. The fifteen-day MPC earns more in ordinary ratio control but loses under hidden biological inhibition, so forecast horizon alone does not replace state information.

KEY POINTS

- More feed makes more gas for a day or two, and then the acids the slow methanogens cannot keep up with sour the tank for weeks.
- The fold sits at 287.6 m^3/d, 1.8 times the design, and is set by the retention, so a feedforward on the weekly COD costs $11,648 over the season.
- At 400 m^3/d the gas peaks on day 2.63, the titration warns on day 3.58 and the tank sours on day 4.51.
- On a daily titration a loading gain of 8 settles in about three weeks while 30 swings the feed between its limits.

Where else this applies. Volume 7, Chapter 2 has ignition and extinction folds of the same shape, Volume 5, Chapter 3 tunes a column against a slow analyzer, and Volume 5, Chapter 4 grades a cook on late laboratory results.

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11.11 Exercises

11.1 Concept check · 2 min

In case 2 the feed rises from 160 to 400 m^3/d, beyond the fold. The biogas doubles to 9504 Nm^3/d on day 2.63 while the methane rises by only 41 %. What makes up the rest of the rise?

(a) The acidogens grow within hours and turn the extra feed straight into methane

(b) The heater burns less of the holder's gas, so more of it passes the biogas meter

(c) The higher feed strips dissolved methane out of the liquid into the headspace

(d) The acids drive carbon dioxide out of the bicarbonate, and it leaves with the gas

11.2 Concept check · 1 min

A digester at 250 m^3/d is moved slowly from 12 to 25 % food waste, slowly enough that its methanogens grow into the richer feed. Where do its steady acids settle?

(a) At the same 25.184 mmol/L, because the steady acids depend on the dilution rate alone

(b) Higher in proportion to the feed COD, because a richer feed brings more acids each day

(c) Lower, because the richer feed grows more methanogens than this dilution rate needs

(d) Nowhere, because the loading now exceeds the fold loading of the design blend

11.3 Concept check · 2 min

In case 5 the feed rises to 300 m^3/d, just past the fold, and the pH loop holds 7.2 with bicarbonate. The tank sours on day 18.85 instead of day 10.22. Why does it sour at all?

(a) Bicarbonate removes the acids only slowly, at one mole of acid per mole dosed

(b) Past the fold the Haldane term inhibits the methanogens whatever the pH

(c) The sodium the bicarbonate adds builds up and inhibits the methanogens

(d) The pH probe drifts high, so the true pH sits below the loop's setpoint

11.4 Theory and calculation · 6 min

The methanogens of the chapter have $\mu_{2,max} = 0.164\,\mathrm{d}^{-1}$, $K_{S2} = 20\,\mathrm{mmol/L}$, $K_{I2} = 130\,\mathrm{mmol/L}$ and $k_d = 0.02\,\mathrm{d}^{-1}$. The tank holds 4000 m^3, keeps no biomass back ($\alpha = 1$) and is designed for 160 m^3/d. Take $T = 35\,^{\circ}\mathrm{C}$ and the pH factor as 1.

(a) Find the acids S_{2M} at the peak of the Haldane curve and the peak growth rate. *Checkpoint:* 51.0 mmol/L and 0.0919 d^{-1}.

(b) Find the critical dilution rate, the feed and retention it corresponds to, and its ratio to the design feed. *Checkpoint:* 0.0719 d^{-1}, 287.6 m^3/d, 13.9 days and 1.80.

(c) Find both steady acids at the design feed and say which one the tank sits at. *Checkpoint:* 12.2 and 213 mmol/L; the lower one.

11.5 Theory and calculation · 8 min

The tank of case 7 is fed 250 m^3/d at 15 °C and loses heat through a shell of $UA = 2.0\,\mathrm{kW/K}$ to 5 °C outdoors, with $\rho c_p = 4.18\,\mathrm{MJ/(m^3\,K)}$ and $V = 4000\,\mathrm{m}^3$. The heater trips at 35.0 °C and stays off for six days. Use Equation (11.8).

(a) Find the heater duty that holds the tank at 35 °C at this feed. *Checkpoint:* 302 kW.

(b) With the heater off, find the conductance G, the time constant, the temperature the tank cools toward and its temperature after six days. *Checkpoint:* 14.09 kW/K, 13.73 d, 13.58 °C and 27.4 °C.

(c) With $\theta_2 = 1.07$ and the methanogen constants of the previous exercise, estimate the fold at that temperature, taking the pH factor as 1. *Checkpoint:* About 140 m^3/d.

11.6 Theory and calculation · 6 min

The titration reports acids in mg/L as acetic acid over alkalinity in mg/L as calcium carbonate, Equation (11.9). The design state has $S_2 = 12.20\,\mathrm{mmol/L}$ and $Z = 154.4\,\mathrm{mmol/L}$.

(a) Find the design ratio. *Checkpoint:* 0.095.

(b) At the design alkalinity, find the acids at which the ratio reaches the limit of 0.4, in mg/L and in mmol/L, and compare them with S_{2M}. *Checkpoint:* 3090 mg/L, 51.5 mmol/L, against 51.0 mmol/L.

(c) The loading loop runs with $K_c = 8$ per unit of ratio on the nameplate feed COD of $54.4\,\mathrm{kg\,COD/m^3}$. Find the proportional part of its step, in loading and in feed, when a report comes in 0.05 above the setpoint, and the same at $K_c = 30$. *Checkpoint:* $0.4\,\mathrm{kg\,COD/m^3/d}$ or $29\,\mathrm{m^3/d}$; $1.5\,\mathrm{kg\,COD/m^3/d}$ or $110\,\mathrm{m^3/d}$.

11.7 App challenge · 9 min

Load case 2, The overload, and change no setting. Run at the default speed of 0.25 d/s with the default seed until the run pauses at the souring, about 18 s, and export the CSV file.

(a) Find the largest true biogas, column `truth_biogas_Nm3d`, and the day of that row. *Checkpoint:* $9504\,\mathrm{Nm^3/d}$ on day 2.63, within $20\,\mathrm{Nm^3/d}$.

(b) Find the first titration report above 0.4 in column `ratio_lab`: its day, its value, and how long after the gas peak it arrives. *Checkpoint:* Day 3.58, 0.55, about 0.95 days later.

(c) Find the first row in which `upgrader_on` reads 0, and the day the souring tile names. Explain why the upgrader stopped first. *Checkpoint:* Day 2.04 (the trip at day 2.01); soured day 4.5.

11.8 App challenge · 8 min

Load case 4, The ratio loop on the daily titration, and run 40 days at the default seed at 1 d/s (40 s). Export the CSV file.

(a) Read the feed, column `feed_m3d`, at days 20 and 40, and the largest report in column `ratio_lab`. *Checkpoint:* 233 and $248\,\mathrm{m^3/d}$, within 2; 0.207, within 0.005.

(b) Set Loading gain to 30, press Reset, run 40 days again and export. The first two reports come on days 0.58 and 1.58. Read the feed in the row after each, at days 0.625 and 1.625, since the row at a report still holds the old feed. *Checkpoint:* 390 and $69\,\mathrm{m^3/d}$, within 2.

(c) Explain the swing with the step size of the previous exercise and the timing of the reports. *Checkpoint:* Each step is decided on acids that have not yet answered the one before.

11.9 App challenge · 8 min

Load case 7, The heater fault, with the default seed.

(a) Run without acting until the run pauses at the souring. Read the lowest true temperature, column `truth_T_C`, and its day, the day of the first report above 0.4, and the souring day on the tile. *Checkpoint:* 27.4 °C on day 8.42, report on day 7.58, soured day 10.6.

(b) Press Reset. Run at 1 d/s to day 7, change the speed to 0.05 d/s, and at the report of day 7.58 cut Feed rate to $160\,\mathrm{m^3/d}$. Return to 1 d/s and run to day 25. Read the titrations tile. *Checkpoint:* Not soured, four reports over 0.4 if the cut is made before day 7.8.

(c) The previous exercise put the cold fold near $140\,\mathrm{m^3/d}$. Explain why a feed of $160\,\mathrm{m^3/d}$ saves the tank. *Checkpoint:* The excursion past the fold is short.

11.10 Application area · 3 min

Published guidance keeps a healthy digester between 0.1 and 0.35 on the acids-to-alkalinity ratio. The chapter's steady ratio along the stable branch is 0.259, 0.303, 0.328 and 0.396 at 95, 98, 99 and 100 % of the critical feed.

(a) What share of the critical feed does the supervisor's back-off ratio of 0.30 correspond to in this model? *Checkpoint:* About 98 %.

(b) Why should a real plant take its ratio limit from its own titration history rather than from this model? *Checkpoint:* The mapping from ratio to margin depends on the kinetics and the alkalinity.

11.11 Application area · 3 min

The upgraded gas is sold at \$3 per MMBtu with a \$25 federal and a \$20 state credit, at 0.037 74 MMBtu per Nm^3. Sodium bicarbonate costs \$0.40 per kg and weighs 84.0 kg/kmol.

(a) Find the revenue per Nm^3 of renewable natural gas and the share of it that the credits pay. *Checkpoint:* \$1.81 per Nm^3; 94 %.

(b) The recovery of case 3 takes 1227 kmol of bicarbonate. Find its mass and cost, and compare the cost with the \$3,528 the supervisor of case 8 earns over 40 days. *Checkpoint:* 103 t, about \$41,200, about 12 times as much.

11.12 Review the agent · 5 min

An AI assistant is asked how to raise the digester's revenue and replies as follows. "The titration is a day late but the biogas meter answers within hours. Close the feed loop on the gas instead: raise the feed by 5 % each day the metered biogas has risen since the day before, and cut it by 5 % each day it has fallen. The climb stops by itself at the feed that makes the most gas." Find the flaw in the proposal and say how it could be checked.