INTERACTIVE CONTROL ENGINEERING
Principles and Applications

Volume 17 Control of Energy Conversion and Storage

CHAPTER 10

# A Vanadium Flow Battery Simulator for Inventory-Constrained Dispatch on a Duck Curve

Pages 248–274

Edited by John D. Hedengren

**How to cite this chapter.** "A Vanadium Flow Battery Simulator for Inventory-Constrained Dispatch on a Duck Curve," in *Control of Energy Conversion and Storage*, John D. Hedengren, Ed., vol. 17 of *Interactive Control Engineering: Principles and Applications*. Advanced Process Solutions, LLC, 2026, ch. 10, pp. 248–274.

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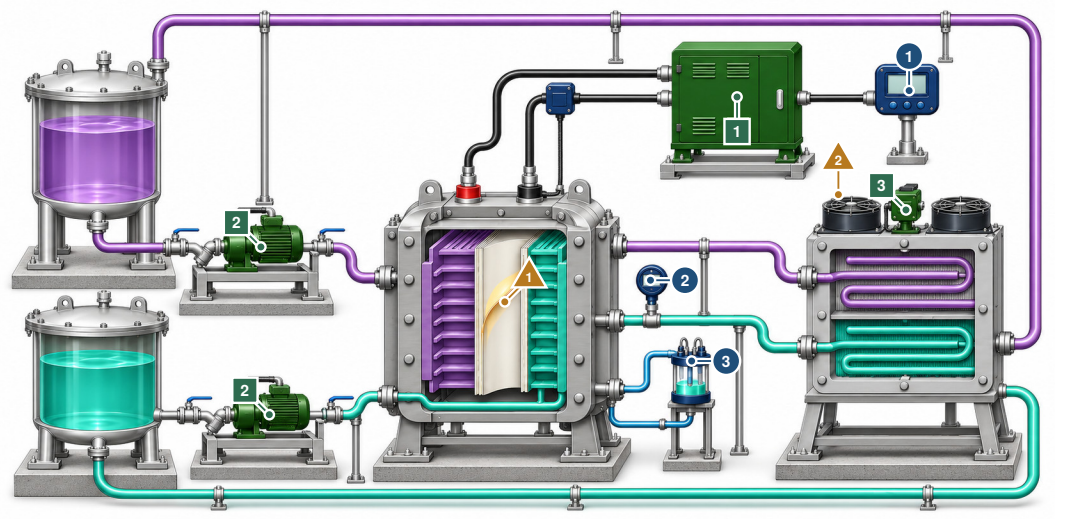
**The app for this chapter**
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CHAPTER 10

# A Vanadium Flow Battery Simulator for Inventory-Constrained Dispatch on a Duck Curve



**The controller moves** 1 the power converter, which sets the current 2 the two electrolyte pumps, which set the flow 3 the cooler on the return lines

**The controller reads** 1 battery power on the line to the grid 2 electrolyte temperature at the stack outlet 3 the charge-state estimate

**What disturbs it** 1 vanadium crossing the membrane 2 hot weather at the cooler

*Notice that the energy sits in the two tanks of liquid while the stack and the converter only set how fast it can flow in or out. Illustration, not to scale.*

IN PLAIN TERMS **A Battery That Keeps Its Charge in Tanks**

**The main idea.** A flow battery stores its energy in two big tanks of liquid. Pumps carry the liquid through stacks of cells that turn it into electricity, so more tank means more hours and more cells mean more power.

**The challenge.** The tanks hold only so much, and the same power is worth far more at some hours than at others. Deciding when to empty them is the hard part. A forecast helps only on days when the price jumps at an hour the grid's demand does not show.

**Why it matters.** Spending the stored charge at the wrong hour can cost thousands of dollars in a single day.

THREE WAYS TO READ THIS CHAPTER

| | | |
|---|---|---|
| **Explore** | 12 min | The first page, the two history boxes of Section 10.2, the fact box of Section 10.3, the try-it box of Section 10.4, the try-it and fact boxes of Section 10.7, and the key points |
| **Study** | 50 min | The whole chapter with its pause points and the twelve exercises |
| **Practice** | 15 min | The first page, Section 10.7, Section 10.8, Section 10.9 and the open challenges |

**The control question** A flow battery buys its megawatts as stacks and its megawatt-hours as electrolyte, and a converter answers a power command in seconds. When should a finite inventory be spent on a duck-curve feeder, and when is a forecast worth more than feedback on the net load?

**Learning objectives**

- Compute the energy, the working band and the hours at rated power of a tank of vanadium electrolyte, and what an imbalance takes off them.
- Explain why the transport margin, set by the pump, limits rated power before the tanks are empty.
- Compare fixed, feedforward and margin-trimmed electrolyte flow by pumping energy and worst margin.
- Explain when forecast-based dispatch beats a net-load feedback controller, and why a finite-horizon plan needs a terminal condition.

**Prerequisites** Sections 1.4.4 (storage as an inventory), 1.4.6 (horizon and terminal value), 1.6.2 (velocity-form PID) and 1.6.4 (scenarios and data).

**The app** https://apmonitor.com/apps/vanadium-flow/

**See also** Volume 9, Chapter 6 and Volume 9, Chapter 11 for a stored inventory scheduled against a price over a day, with a predictive or integer plan. Chapter 13 and Chapter 11 in this volume for a quadratic dispatch program and a water-value plan.

## 10.1 Introduction

A battery in a control course is usually a bucket. Charge goes in at one efficiency, comes out at another, and the state of charge is the integral of the difference. That model is adequate for a lithium-ion cell on a bench and it is misleading for a flow battery on a feeder, because a flow battery does not have one size. Its energy is a volume of dissolved vanadium in two tanks and its power is an area of membrane in the stacks, and the two are bought on separate purchase orders [1]. The consequence is a plant that can be full and unable to deliver its rating, or empty with every stack healthy, and neither condition is visible in a lumped-efficiency model.

A common misconception is that storage control is a power problem. It is not. A grid-scale converter answers a megawatt command in seconds and there is very little to tune, so a student who spends an afternoon on the power loop has learned almost nothing about storage. Everything difficult sits above that loop. The tanks hold a finite quantity of charge, the price of spending it moves by more than a factor of ten across a day, and the evening ramp arrives whether or not the midday surplus was used well. The decision of when to spend the inventory is the control problem, and it is the one that a transfer function and a step test cannot pose.

The class of problem is inventory-constrained economic dispatch under a moving price, and it is general. A pumped hydro reservoir, a compressed air cavern, a hydrogen buffer, a chilled water tank, a refinery intermediate tank and a warehouse of finished product all share the shape. Each has a state that integrates the manipulated variable, a capacity constraint at both ends, a time-varying value on the contents and a horizon shorter than the physics that fills it. The formal statement of that shape is economic model predictive control, in which the optimizer minimizes a cost rather than a tracking error [2]. Once a student has seen why a finite-horizon optimizer empties the tank at the edge of its horizon, the same reasoning transfers to every plant in that list.

The chapter rests on one controlled comparison on one plant, between feedback dispatch and forecast-based dispatch, and it reports the case where the forecast is worth nothing as carefully as the case where it is worth almost seventeen thousand dollars in a day. The plant is a twelve-state electrochemical and thermal model with a supervisory layer that runs at interactive speed in a browser. The supervisory layer is a quadratic program of twenty-four variables solved by a fixed count of projected-gradient steps rather than a nonlinear program. Eleven scenarios and a manual mode beneath them let the reader reproduce each result.

This chapter gives the model, the control structure and the measured behavior of the vanadium flow battery simulator in enough detail to rebuild it, and uses it to show why the inventory rather

than the converter sets the value of grid storage. Section 10.2 traces the domain from the NASA redox program to grid-scale vanadium plants. Section 10.3 gives the electrochemical, hydraulic and thermal model and the synthetic feeder. Section 10.4 measures the open-loop character in manual mode. Section 10.5 describes the four regulatory loops and the battery management limits, and Section 10.6 the dispatch layer above them. Section 10.7 reports the scenarios: the idle day, the flow policy, feedback against forecast dispatch, the terminal charge state, degradation and the graded challenge. Section 10.8 describes the hardware of a real plant, Section 10.9 states the limitations and Section 10.10 the conclusions. In Chapter 1 the plant is an example of storage as an inventory (Section 1.4.4) and of dispatch against a forecast with a terminal value (Section 1.4.6).

## 10.2 From the NASA Redox Program to a Feeder

The redox flow battery began as an iron-chromium storage program at NASA, in which shunt currents through the shared electrolyte, pumping power and cell performance had to be traded against one another [3].

> ⧖ HISTORY **Pumping Power in the NASA Redox Program (1982)**
>
> A NASA laboratory in Cleveland developed a redox storage system from the mid-1970s, with iron and chromium solutions pumped from tanks through stacks of up to forty cells [3]. Its 1982 report called the sizing of each cell's flow ports the single most important design feature. Narrow ports suppress the shunt currents that leak through the shared electrolyte, wide ports cut the pumping power, and the cell's performance sets a minimum flow, so no port size wins on all three [3]. A throttling valve had been used to vary the flow, which saved no pumping energy. The report suggested a variable-frequency motor and pump instead, and estimated that a variable flow schedule could cut the parasitic losses by about 65 % against a fixed flow sized for the worst case [3].

The iron-chromium chemistry was abandoned because the two half-cells contained different metals, so any transport across the membrane contaminated both sides permanently. The all-vanadium cell solved that by using one element in four oxidation states, so crossover causes an imbalance that can be corrected rather than a contamination that cannot [4]. The first full characterization of a working all-vanadium cell followed two years later and reported the coulombic and voltaic efficiencies that the technology still quotes [5]. Commercial development through the 1990s and 2000s was slow and largely Japanese and Australian, and the review literature of the period reads as a catalog of stack engineering rather than of system operation [6].

Control of these plants developed late and in a particular order. Stack-level electrochemical models capable of predicting the transient voltage appeared in 2008 [7], thermal models of a complete battery including self-discharge followed in 2012 [8], and quantitative membrane transport models were published in the same year [9]. Flow rate was treated as a design constant until pressure-drop and efficiency optimization studies showed that a fixed flow is wrong at almost every operating point [10], and gain-scheduled flow control was proposed once it was clear that the gain from flow to mass-transfer margin falls as the flow rises [11]. The shunt current that the NASA report identified was quantified for a modern multi-stack plant in 2013 [12].

The application context arrived from outside the battery literature, with the duck chart of a solar-heavy grid [13].

> ⧖ HISTORY **The Duck Chart (2013)**
>
> In 2013 California's grid operator published a chart of the net load, the demand minus wind and solar output, on a spring day as solar capacity grew [13]. The curve sagged at midday, where solar output peaks, and its name came from its resemblance to the profile of a duck. The chart raised the concern that the conventional power system could not meet the ramp rate and range needed to use all the solar energy, so output would be curtailed. Negative prices had already appeared in that market, driven in part by wind. In October 2013

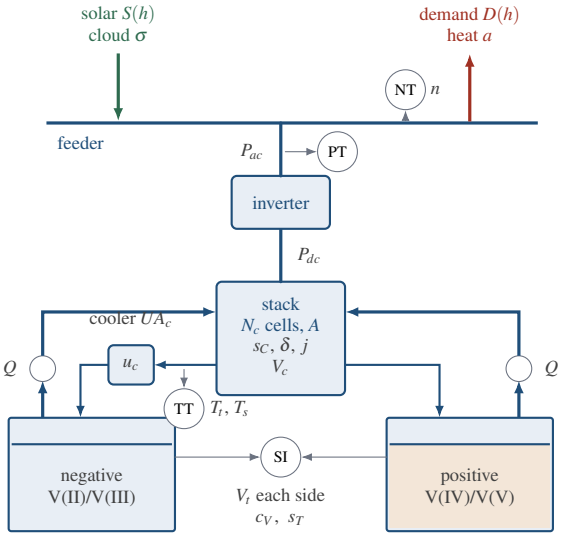



**Figure 10.1** Plant arrangement and the symbols used in the equations. Two tanks of vanadium electrolyte hold the energy, the stacks hold the power, and one pump per circuit sets the rate at which reactant reaches the membrane. The cooler acts on the combined return header. PT is the power transducer, TT the electrolyte thermometer, SI the charge-state estimator and NT the feeder net-load measurement.

> the state's utility regulator set targets for 1325 MW of energy storage by 2020 [13].

Storage is the answer being built at scale, and reviews of the storage portfolio place flow batteries in the multi-hour bulk role rather than in the fast frequency role [14]. Predictive dispatch of a flow battery for peak shaving is now an active subject in its own right [15]. The plant of this chapter sits at that junction, where a well-understood electrochemical device is asked to answer a scheduling question.

## 10.3 Process Model

The simulated plant is a 100 MW converter connected to 176 000 cells of 4000 $cm^2$ active area, arranged as 1100 stacks of 160 cells. Each side of the plant holds 10 000 $m^3$ of 1600 $mol/m^3$ vanadium electrolyte in a tank, plus 150 $m^3$ of hold-up in the stacks and manifolds. At the formal cell potential of 1.40 V that inventory is 600.4 MWh between empty and full and 480.3 MWh inside the 10 % to 90 % working band. Two pumps circulate electrolyte, one cooler removes the loss heat, and one inverter connects the direct-current stacks to the feeder. Figure 10.1 shows the arrangement and every symbol used in the equations that follow.

The sign convention is positive for discharge throughout. A positive current density, a positive direct-current power and a positive alternating-current power all mean that the battery is supplying the feeder. The clock is in hours, power is in megawatts and energy in megawatt-hours, current density is in amperes per square centimeter, and charge state is a fraction rather than a percentage.

### 10.3.1 Variables

Table 10.1 classifies every quantity the app exposes. Four controlled variables are paired with four manipulated variables, and the pairing is one-to-one because each loop acts on a different time scale. The disturbances split into three that the dispatch layer is told about in its forecast, namely the feeder demand scale, the afternoon air conditioning load and the scarcity price adder, and eight that it is not.

**Table 10.1** Variable Classification

| Symbol | Meaning | Unit | Range |
|---|---|---|---|
| *Controlled variables* | | | |
| $n$ | feeder net load | MW | −100 to 700 |
| $P_{ac}$ | battery power at the bus | MW | −100 to 100 |
| $m$ | transport margin $j_\ell/\lvert j\rvert$ | — | 1.2 to 8 |
| $T_s$ | stack outlet temperature | °C | 18 to 40 |
| *Manipulated variables* | | | |
| $P_b$ | battery power request | MW | −100 to 100 |
| $j$ | stack current density | % rated | −100 to 115 |
| $Q$ | electrolyte flow per circuit | m³/s | 2 to 30 |
| $u_c$ | cooler duty | % | 0 to 100 |
| $r$ | rebalance cell flow | % | 0 to 100 |
| *Disturbances* | | | |
| $\sigma_\infty$ | sky clarity | × clear | 0.10 to 1.05 |
| $\ell$ | feeder demand scale | × | 0.80 to 1.30 |
| $a_\infty$ | afternoon air conditioning | MW | 0 to 200 |
| $T_a$ | ambient temperature | °C | 0 to 45 |
| $w_p$ | hydraulic resistance | × | 0.8 to 3.0 |
| $w_c$ | membrane and contact resistance | × | 0.8 to 2.5 |
| $\chi$ | crossover rate | × | 0 to 5 |
| $\Delta\pi$ | scarcity price adder | \$/MWh | 0 to 700 |
| $\varepsilon$ | forecast error scale | × | 0 to 1.5 |
| $A_{reg}$ | regulation signal, when in service | MW | 0 to 30 |
| $\sigma_P$ | power transducer noise | MW | 0 to 6 |
| *States* | | | |
| $s_T$ | tank charge state | — | 0 to 1 |
| $s_C$ | charge state inside the stacks | — | 0 to 1 |
| $\delta$ | half-cell imbalance | — | −0.45 to 0.45 |
| $T_t$ | tank electrolyte temperature | °C | −5 to 80 |
| $j_a$ | applied current density | A/cm² | −0.120 to 0.138 |
| $Q_a$ | applied electrolyte flow | m³/s | 0.5 to 32 |
| $u_c^a$ | applied cooler duty | — | 0 to 1 |
| $R_m$ | membrane resistance growth | — | 0 to 3 |
| $\phi$ | capacity fade | — | 0 to 0.5 |
| $\bar{n}$ | filtered daily mean net load | MW | — |
| $\sigma$ | filtered sky clarity | — | 0 to 1.05 |
| $a$ | filtered air conditioning load | MW | 0 to 400 |

### 10.3.2 Cell Voltage

The open-circuit potential of one cell follows the Nernst expression in the two half-cell charge states, which differ from each other by the imbalance $\delta$,

$$E = E^0 + \frac{RT}{F}\left[\ln\frac{x}{1-x} + \ln\frac{y}{1-y}\right] \tag{10.1}$$

with $x = s_C + \delta/2$ and $y = s_C - \delta/2$, with $T$ in kelvin and both arguments clipped to $[10^{-4}, 1-10^{-4}]$. With no imbalance this reduces to the usual $2(RT/F)\ln[s_C/(1-s_C)]$ and returns $E^0$ exactly at half charge. The two half-cell charge states are carried separately because a plant with crossover has capacity set by whichever half runs out first, and that is invisible in a single charge-state variable.

Three loss terms are subtracted on discharge and added on charge. Activation loss uses the

symmetric Butler-Volmer inverse,

$$\eta_{act} = \frac{2RT}{F}\,\operatorname{asinh}\!\left(\frac{|j|}{2j_0}\right), \quad j_0 = j_{00}\,\mathrm{e}^{E_{a0}\left(\frac{1}{303.15}-\frac{1}{T}\right)} \tag{10.2}$$

ohmic loss uses an area-specific resistance that improves with temperature and degrades with membrane age and the fouling disturbance,

$$\eta_{ohm} = R_\Omega |j|, \quad R_\Omega = \mathrm{ASR}_0\, w_c\,(1+R_m)\,\mathrm{e}^{E_{aR}\left(\frac{1}{T}-\frac{1}{303.15}\right)} \tag{10.3}$$

and concentration loss follows the limiting-current logarithm,

$$\eta_{conc} = -\frac{RT}{F}\ln(1-u), \qquad u = \min\!\left(\frac{|j|}{j_\ell}, 0.985\right). \tag{10.4}$$

The clip at 0.985 keeps the logarithm defined and caps the term at about 0.11 V at 30 °C, so the concentration loss alone never stops the current. That is the job of the supply limit in Equation (10.10) below. The limiting current density is where the pump enters the polarization,

$$j_\ell = j_{\ell 0}\,\alpha_C \left(\frac{Q}{Q_0}\right)^{n_Q}, \quad \alpha_C = \begin{cases} \min(x,\,y) & j \geq 0 \\ \min(1-x,\,1-y) & j < 0 \end{cases} \tag{10.5}$$

where $\alpha_C$ is the availability of whichever species is being consumed, so the same current is harder to sustain near an empty tank on discharge and near a full tank on charge. The cell terminal voltage and the stack powers follow,

$$\eta_{tot} = \eta_{act} + \eta_{ohm} + \eta_{conc} \tag{10.6}$$

$$V = \max(0.15,\; E - \operatorname{sgn}(j)\,\eta_{tot}) \tag{10.7}$$

$$P_{dc} = 10^{-6} N_c A\, V\, j \tag{10.8}$$

$$P_{loss} = 10^{-6} N_c A\, \eta_{tot}\, |j|. \tag{10.9}$$

The transport margin that the flow loop controls is $m = j_\ell / \max(|j|, 0.08\, j_R)$, capped at 12 so that an idle plant reports a finite number.

The current density $j$ in Equations (10.2) to (10.9) and in the inventory balance below is the current the stacks actually pass. It is the applied current $j_a$ limited by Faraday's law to the reactant that the flow brings from the tanks, at one electron per vanadium ion,

$$j = \mathrm{clip}(j_a, -j_F, j_F), \quad j_F = \frac{Q\, c_V F}{N_c A}\left[\alpha_T - \frac{|\delta|}{2}\right]^+ \tag{10.10}$$

with $\alpha_T = s_T$ on discharge and $1 - s_T$ on charge. At design flow and half charge $j_F$ is 1.64 A/cm$^2$, almost fourteen times the rated current density, so the stoichiometric flow that just supplies rated current at half charge is 1.09 m$^3$/s per circuit and the limit is idle in normal operation. It binds at low flow, where 3 m$^3$/s cannot supply rated current below 18 % charge, and near an exhausted tank, where it tapers charge and discharge to zero in manual as well as automatic mode while the pumps and auxiliaries go on drawing power. The same limited current sets the power, the losses and the charge consumed, so no current delivers energy that the tanks could not supply.

### 10.3.3 Inventory and Transport

The charge state inside the stacks and the charge state in the tanks are separate states because they move on separate time scales. Electrolyte passes through the 150 m$^3$ of cell and manifold hold-up in

10 s at design flow and through the 10 000 m$^3$ tank in 11.1 min, so

$$\frac{\mathrm{d}s_C}{\mathrm{d}t} = 3600\left[\frac{Q}{V_c}(s_T - s_C) - \frac{N_c A\, j}{F c_V V_c}\right] \tag{10.11}$$

$$\frac{\mathrm{d}s_T}{\mathrm{d}t} = 3600\,\frac{Q}{V_t}(s_C - s_T). \tag{10.12}$$

The factor 3600 converts the second-based flow and Faraday terms to the hour clock. Vanadium is conserved by construction, because $V_t\,\dot{s}_T + V_c\,\dot{s}_C$ equals the reaction term alone and vanishes at zero current. The gap between $s_T$ and $s_C$ is the entire transport story, and it is what makes the charge state measured at the stack outlet a biased estimate of the charge state in the tanks whenever current is flowing.

Crossover moves vanadium across the membrane at a rate that grows with temperature and with current, and the rebalance cell is the only thing that reverses it,

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \chi k_x\left(0.35 + 0.65\frac{|j|}{j_R}\right)(s_T - 0.5)\,\mathrm{e}^{E_{aX}\left(\frac{1}{303.15} - \frac{1}{T}\right)} - k_{rb}\frac{r}{100}\,\delta. \tag{10.13}$$

The driving term changes sign at half charge, which reproduces the observed behavior that a cell parked full or parked empty drifts in opposite directions [9].

### 10.3.4 Balance of Plant

Pumping power for both circuits is cubic in flow on a fixed hydraulic system, which is why the last few cubic meters per second are the expensive ones,

$$P_{pump} = \frac{2k_Q\, w_p\, Q^3}{\eta_p\, 10^6}. \tag{10.14}$$

Inverter loss carries a no-load term, a conduction term linear in the direct-current power and a resistive term quadratic in it,

$$P_{inv} = P_{nl} + k_1|P_{dc}| + \frac{k_2 P_{dc}^2}{P_R}. \tag{10.15}$$

The power delivered at the point of connection subtracts everything the plant consumes, so on charge the feeder pays for the losses as well,

$$P_{ac} = P_{dc} - P_{inv} - P_{pump} - P_{aux,0} - P_{reb}\,\frac{r}{100}. \tag{10.16}$$

At rated current density, half charge and 15 m$^3$/s of flow the plant produces 98.71 MW of direct-current power at 1.1685 V per cell against an open-circuit 1.3924 V, loses 18.91 MW in the stacks and 1.45 MW in the pumps, and delivers 94.90 MW. The same 100 MW requires 106.4 % of rated current on discharge and only 72.2 % on charge, because charging runs at a higher cell voltage. The converter rating and the current rating are therefore different constraints, and which one binds depends on the direction.

Heat is generated by the stack loss and the pumps and removed through the tank walls and the cooler,

$$C_T\frac{\mathrm{d}T_t}{\mathrm{d}t} = 3600\Big[10^6(P_{loss} + P_{pump}) - UA_{amb}(T_t - T_a) - UA_c\,u_c^a\,(T_t - T_a)^+\Big] \tag{10.17}$$

where $(\cdot)^{+}$ denotes the positive part, so the cooler cannot heat the inventory. With $2.9\times10^{10}$ J/°C of thermally active electrolyte the open-loop time constant is 25.1 h and the closed-loop constant at full cooler duty is 3.63 h. The sensor reads the stack outlet rather than the tank, and the stacks run a fraction of a degree above the tanks because the flow through them is large,

$$T_s = T_t + \frac{10^{6} P_{loss}}{2 Q \rho_e c_{p,e}}. \tag{10.18}$$

Two health proxies integrate use. Membrane resistance grows and capacity fades in proportion to current, and the fade accelerates above 35 °C,

$$\frac{\mathrm{d}R_m}{\mathrm{d}t} = k_{Rm}\frac{|j|}{j_R}, \qquad \frac{\mathrm{d}\phi}{\mathrm{d}t} = k_{fade}\frac{|j|}{j_R}\left[1+0.08\,(T_t-35)^{+}\right]. \tag{10.19}$$

### 10.3.5 The Feeder, the Price and the Target

The feeder is a synthetic day built from a base load and three gaussian shoulders, each evaluated at the hour and at the hour plus and minus a day so that the evening peak is continuous across midnight,

$$w(h;c,s) = \sum_{k\in\{-1,0,1\}} \mathrm{e}^{-\left(\frac{h+24k-c}{s}\right)^{2}} \tag{10.20}$$

$$D(h) = \ell\Big[L_b + L_m w(h;8,2.6) + L_e w(h;19.5,2.4) + L_c w(h;14,3.5) + a\,w(h;17,3)\Big] \tag{10.21}$$

$$S(h) = \sigma S_{max}\sin^{1.3}\!\left(\pi\frac{h-h_{up}}{h_{dn}-h_{up}}\right), \qquad h_{up}<h<h_{dn} \tag{10.22}$$

and the raw net load is $n_{raw} = D - S$. With the default parameters the raw net load reaches 585.2 MW at 19:27, falls to −35.2 MW at 12:28 when the feeder exports, and averages 282.3 MW over the day. The steepest evening climb is 2.75 MW/min measured over a ten-minute window and 2.73 MW/min measured over an hour, so the ramp is not an artifact of the measurement window.

> ✦ DID YOU KNOW? **Curtailment at a Modest Solar Share**
>
> A 2015 national laboratory study of California found that, without new flexibility, solar supplying as little as 20 % of annual energy could lead to marginal curtailment rates above 30 %. Letting distributed solar and storage provide grid services could greatly reduce it [13].

Energy price is built the same way and deliberately does not peak at the same hour as the scarcity adder,

$$\pi(h) = p_b + p_{pk}w(h;19,1.7) - p_{bl}w(h;13,2.6) + \Delta\pi\,w(h;h_{sp},w_{sp}) + 6\cos\frac{2\pi(h-6)}{24}. \tag{10.23}$$

The default day runs from \$−13.6 per megawatt-hour at 13:07 to \$158 per megawatt-hour at 19:01. The scarcity event is centered at 15:00 by default, away from the evening peak, because on an ordinary duck-curve day the highest net load and the highest price are the same hour and a controller that watches only the net load is already pointing the right way.

The trajectory the feeder asks the plant to follow is peak shaving and valley filling and nothing more ambitious,

$$\begin{aligned} \hat{n} &= \mathrm{clip}(n_{raw},\ \bar{n}-W,\ \bar{n}+W) \\ n^{*} &= \mathrm{clip}(\hat{n},\ n_{raw}-0.85P_R,\ n_{raw}+0.85P_R) \end{aligned} \tag{10.24}$$

where $\bar{n}$ is a one-day first-order average of the raw net load, carried as a state, and $W$ is a deadband inside which the feeder is left alone. The outer clip to within 85 % of the converter rating matters. A target the storage could not meet with full tanks is a statement about the size of the purchase order rather than a setpoint, and a controller scored against one is being measured on somebody else's decision. Flattening to the daily mean is the tempting alternative and it is wrong on this shape, because the overnight net load sits above the daily mean and a target of the mean asks the battery to discharge all night and arrive at the evening peak empty.

### 10.3.6 Actuators, Instruments and Filtered Disturbances

Three actuators are first-order lags on their commands, with time constants of 4 s for the inverter current, 20 s for pump speed and 60 s for the cooler. Three measurements are first-order filters, with time constants of 8 s for the power transducer, 3 min for the thermometer and 72 s for the charge-state estimator. Two disturbances are lagged rather than stepped, because a cloud front takes about a quarter hour to cross a feeder and an afternoon air conditioning load takes most of an hour to build,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{\sigma_\infty - \sigma}{\tau_{sky}}, \qquad \frac{\mathrm{d}a}{\mathrm{d}t} = \frac{a_\infty - a}{\tau_{ac}}. \tag{10.25}$$

A step in either would give the net load an unbounded derivative, which makes a ramp metric meaningless and does not happen on a real feeder. The net-load ramp reported to the operator is measured against a ten-minute lag rather than differentiated,

$$\dot{n} = \frac{n - \mathrm{lag}(n, \tau_r)}{60\,\tau_r}, \qquad \tau_r = 0.1667\,\mathrm{h}. \tag{10.26}$$

### 10.3.7 Parameters

Table 10.2 gives every parameter with its default value. The column marked V indicates whether the value is visible in the app's parameter panel, where A means adjustable and shown, H means hidden from the beginner view so that the reader must infer it from behavior, and F means fixed physical constant.

### 10.3.8 Initialization and Numerics

The initial state places the tanks at the scenario charge state, sets the imbalance and both health proxies to zero, and computes the temperature at which the loss heat produced by the initial commands is exactly removed,

$$T_t(0) = T_a + \frac{10^6\,[P_{loss}(0) + P_{pump}(0)]}{UA_{amb} + UA_c u_c(0)}. \tag{10.27}$$

The cell charge state is initialized to its algebraic steady value rather than to the tank value, because the hold-up empties in 10 s and starting it equal produces a transient that is a numerical artifact.

Integration is classical fourth-order Runge-Kutta on a fixed step of 0.001 h, which is 3.6 s and about one time constant of the fastest actuator. The regulatory controllers execute every 0.002 h, the dispatch layer every 0.25 h, and the historian logs one sample every 0.017 h. The declared and realized rate is 58.8 samples per simulated hour, with a logging interval of 61.2 s. The engine keeps these clocks apart from the screen, as Section 1.6.1 describes. A post-step clip holds every state inside its physical range, including the charge states (0 to 1), the imbalance and the cooler fraction.

### 10.3.9 Assumptions

1. **The electrolyte is perfectly mixed in each tank and in the stack hold-up.** A real tank stratifies, and the charge-state estimate at the return header then lags the bulk by more than the 11.1 min residence time used here. The assumption breaks down at very low flow, which is where the model is least trustworthy.

**Table 10.2** Model Parameters and Default Values

| Symbol | Meaning | Value | Unit | V |
|---|---|---|---|---|
| $N_c$ | cells in the plant | 176 000 | — | A |
| $A$ | active area per cell | 4000 | $\mathrm{cm}^2$ | A |
| $j_R$ | rated current density | 0.12 | $\mathrm{A/cm}^2$ | A |
| $E^0$ | formal cell potential | 1.40 | V | A |
| $\mathrm{ASR}_0$ | area resistance at 30 °C | 1.00 | $\Omega\,\mathrm{cm}^2$ | A |
| $E_{aR}$ | resistance temperature coefficient | 1800 | K | A |
| $j_{00}$ | exchange current at 30 °C | 0.020 | $\mathrm{A/cm}^2$ | A |
| $E_{a0}$ | kinetic temperature coefficient | 2500 | K | A |
| $j_{\ell 0}$ | limiting current, design flow | 0.90 | $\mathrm{A/cm}^2$ | A |
| $n_Q$ | mass-transfer exponent on flow | 0.5 | — | H |
| $V_t$ | tank volume, each side | 10 000 | $\mathrm{m}^3$ | A |
| $V_c$ | stack and manifold hold-up | 150 | $\mathrm{m}^3$ | A |
| $c_V$ | vanadium concentration | 1600 | $\mathrm{mol/m}^3$ | A |
| $\rho_e$ | electrolyte density | 1350 | $\mathrm{kg/m}^3$ | H |
| $c_{p,e}$ | electrolyte heat capacity | 3000 | J/(kg K) | H |
| $Q_0$ | design flow, each side | 15 | $\mathrm{m}^3/\mathrm{s}$ | A |
| $k_Q$ | hydraulic resistance, one circuit | 155 | $\mathrm{Pa\,s}^2/\mathrm{m}^6$ | A |
| $\eta_p$ | pump and drive efficiency | 0.72 | — | A |
| $C_T$ | thermal inventory | $2.9 \times 10^{10}$ | J/°C | A |
| $UA_{amb}$ | tank and pipework heat loss | $3.2 \times 10^5$ | W/°C | A |
| $UA_c$ | cooler conductance at full duty | $1.9 \times 10^6$ | W/°C | A |
| $P_R$ | converter rating | 100 | MW | A |
| $\eta_{plan}$ | one-way efficiency in the plan | 0.84 | — | A |
| $P_{nl}$ | inverter no-load loss | 0.25 | MW | A |
| $k_1$ | inverter conduction coefficient | 0.0075 | — | A |
| $k_2$ | inverter resistive coefficient | 0.011 | — | A |
| $P_{aux,0}$ | balance-of-plant auxiliary load | 0.30 | MW | A |
| $P_{reb}$ | rebalance cell load at full flow | 0.15 | MW | A |
| $k_x$ | crossover imbalance rate | 0.020 | 1/h | A |
| $E_{aX}$ | crossover temperature coefficient | 3500 | K | H |
| $k_{rb}$ | rebalance cell effectiveness | 0.55 | 1/h | A |
| $k_{fade}$ | capacity fade at rated current | $2.0 \times 10^{-4}$ | 1/h | A |
| $k_{Rm}$ | membrane resistance growth | $3.0 \times 10^{-4}$ | 1/h | A |
| $L_b$ | overnight feeder demand | 340 | MW | A |
| $L_m$ | morning shoulder | 90 | MW | A |
| $L_e$ | evening peak above base | 240 | MW | A |
| $L_c$ | afternoon cooling load | 60 | MW | A |
| $S_{max}$ | photovoltaic capacity | 430 | MW | A |
| $h_{up}, h_{dn}$ | sunrise, sunset | 6.3, 19.0 | h | A |
| $p_b$ | overnight energy price | 34 | \$/MWh | A |
| $p_{pk}$ | evening price adder | 130 | \$/MWh | A |
| $p_{bl}$ | midday price depression | 46 | \$/MWh | A |
| $\pi_{ref}$ | price scale in the objective | 45 | \$/MWh | H |
| $c_{curt}$ | cost charged for curtailed solar | 12 | \$/MWh | A |
| $c_{deg}$ | degradation cost on throughput | 4.0 | \$/MWh | A |
| $s_{min}, s_{max}$ | working charge-state band | 0.10, 0.90 | — | A |
| $T_{min}, T_{max}$ | precipitation limits | 15, 40 | °C | A |
| $T_{trip}$ | stack high-temperature trip | 45 | °C | A |
| $W$ | shaving deadband | 110 | MW | A |
| $\tau_{tgt}$ | averaging time behind $\bar{n}$ | 24 | h | A |
| $\tau_j, \tau_Q, \tau_c$ | actuator lags | 4, 20, 60 | s | A |
| $\tau_{sky}, \tau_{ac}$ | disturbance lags | 0.25, 0.6 | h | A |
| $F$ | Faraday constant | 96 485 | C/mol | F |
| $R$ | gas constant | 8.314 | J/(mol K) | F |

2. **One lumped thermal capacitance.** The tanks, the pipework and the stack inventory are one temperature with a fraction of a degree of stack rise from Equation (10.18). This is reasonable at the flows modeled and it is wrong during a pump trip, when the stacks would heat independently in seconds.

3. **Shunt currents are folded into the area-specific resistance rather than modeled.** A multi-stack plant loses current through shared manifolds in a way that depends on stack count and manifold geometry [12], and no such term appears here. The consequence is that the modeled efficiency does not degrade with plant size.

4. **All cells are identical.** The model carries one voltage for 176 000 cells, so cell-to-cell distribution, the reason a real battery management system exists, is absent.

5. **Crossover is a single scalar.** The four vanadium species diffuse at different rates through a real membrane [9], and the single imbalance state cannot reproduce the resulting composition drift. It reproduces the effect that matters for dispatch, which is a loss of usable capacity that no controller reports.

6. **The feeder, the solar output and the price are smooth synthetic profiles.** They are teaching data shaped like a spring day in California and they are never labeled as measured. Real net-load data has structure at every time scale that no smooth profile reproduces.

7. **The dispatch layer plans with one constant round-trip efficiency and constant megawatt-hours per charge point.** That is a linearization of a logarithm, and it is corrected every fifteen minutes by re-planning from the measured state rather than by making the plan more accurate.

8. **Capacity fade and membrane growth are first-order in current and do not feed back into the charge-state capacity.** They are indicators rather than a life model, and a plant run for a simulated month would report a fade number that no accelerated-aging study supports.

## 10.4 Open-Loop Character

Manual mode hands the operator the stack current, the electrolyte flow, the cooler duty and the battery power request, and the first useful experiment is to decide which of those four moves megawatts, which moves megawatt-hours and which does neither. Current moves power at once and charge state slowly, because the tanks hold 600 MWh and the stacks hold about 9 MWh. Flow moves almost nothing until it gets low, because the concentration term in Equation (10.4) is flat until $|j|$ approaches $j_\ell$. In the first scenario, cutting the flow from 10 m³/s to 3 m³/s at 60 % of rated current and 44 % charge lowers the transport margin from 4.2 to 1.7 within a quarter hour and the power only from 59.6 MW to 58.2 MW. The margin is still above its starvation line of 1.15, which takes a lower flow or an emptier tank to reach.

Three time scales separate cleanly. The converter answers a command in 4 s and the pump follows in 20 s, so under the power loop of the fourth scenario a 140 MW swing from 60 MW of discharge to 80 MW of charge is 63 % complete in 14 s and 90 % complete in 65 s. The tank charge state moves at 0.34 % per minute when discharging at 100 MW and 0.23 % per minute when charging at that power, so the working band lasts 3.9 h at rated discharge and 5.7 h at rated charge. The electrolyte temperature has a 25.1 h open-loop time constant, which is longer than the day the plant is being dispatched over. A student who sets the temperature setpoint at two in the afternoon has already lost, because the cooler cannot change anything before six.

The inventory constraint is the lesson manual mode exists to deliver, and it is delivered by omission. Nothing in manual mode holds the plant inside its working band. In the third scenario, holding −70 % of rated current from thirty percent charge at 10:00 brings the tank to the working ceiling of 90 % at 14:24, with the 100 MW converter rating having trimmed the command to −64.5 % on the way.

Charging continues at full current to 96.7 % and then tapers as the supply limit of Equation (10.10) runs out of uncharged reactant. It stops by 16:00 with the tank at 99.0 % and the positive half-cell full, which is where the positive electrolyte is nearly all V(V) and vanadium pentoxide precipitates on a warm day. From then on the plant draws 1.29 MW for its pumps and auxiliaries and stores nothing. Current is a command and charge state is a consequence, and only the operator is watching the consequence.

> **TRY IT IN THE APP 3 · Charge, Discharge, and the Inventory Nobody Mentioned**
>
> Load scenario 3 and press Run. Watch the tank charge state. The plant charges at −70 % of rated current from 30 % at 10:00 and reaches the 90 % working ceiling at 14:24, and nothing in manual mode stops it there. It stops by 16:00 at 99.0 % and then draws 1.29 MW for its pumps and auxiliaries.

Coming back down, the interesting number is not where the plant stops but where it stops being a 100 MW battery. Reversed to 100 % of rated current at the ceiling alarm, with the flow held at 12 $m^3/s$, the plant makes 106 MW at first and falls below 100 MW at 63 % charge as the open-circuit voltage falls. The limiting current in Equation (10.5) is proportional to the availability of the species being consumed in the cell, so the transport margin reaches its starvation line of 1.15 at 22 % tank charge (17:53) with 88 MW still flowing, and the concentration overpotential takes a growing share from there. The floor alarm sounds at 10.4 % with 78 MW still flowing, the supply limit cuts the current only below 4.7 %, and the plant stops exporting at 19:39 with the tank at 0.2 %. The tanks still held about 130 MWh when the cell began to starve. Usable energy and usable energy at rated power are different numbers, and the gap between them is set by the pump.

**Pause.** Try Exercise 10.2 before reading on.

The pump trade is worth measuring directly. At rated current, half charge and 30 °C, stack loss falls from 20.90 MW at 4 $m^3/s$ to 18.70 MW at 25 $m^3/s$ while pumping rises from 0.03 MW to 6.73 MW. The sum has a minimum at 8.92 $m^3/s$, which the app reports from a golden-section search of twenty-four iterations and twenty-six evaluations. That optimum is well below the design flow of 15 $m^3/s$, although still about eight times the stoichiometric flow, and it moves with charge state, rising to 16.09 $m^3/s$ at twenty percent charge and falling to 7.20 $m^3/s$ at eighty percent. At twenty percent charge the sum is lower again below 2.74 $m^3/s$, but only because the supply limit then cuts the current, and the golden-section search settles on the local minimum at 16.09 $m^3/s$, where the full current flows. A little concentration overpotential is cheaper than a lot of pumping, and how much cheaper depends on the inventory.

## 10.5 Regulatory Control

### 10.5.1 The Discrete Algorithm

Four loops share the velocity-form PID of Section 1.6.2, Equations (1.10) to (1.13), with the derivative on the measurement and back-calculation on the integral alone. The chapter writes the action flag as $\gamma$, the framework's $a$, with $+1$ for a reverse-acting loop and $-1$ for a direct-acting one in the ISA sense. The settings specific to this app are few. The tracking constant $T_t$ is $\tau_I$ on every loop, no loop sets a slew rate, and all four loops run with $\tau_D = 0$, so derivative action is off everywhere in this app. The regulatory sample period is $\Delta t_c = 0.002\,\mathrm{h}$, which is 7.2 s. In manual mode the integral is set so that the controller output equals the operator's actuator value, so transfer is bumpless in both directions with no separate logic. The supply limit of Equation (10.10) acts outside the power loop's output bounds, so the power integral would wind against it, but no automatic case reaches it.

### 10.5.2 Four Loops on Three Time Scales

Table 10.3 gives the pairings and the default tunings. Two of the four are direct acting. More cooling makes the electrolyte colder, and more discharge lowers the feeder net load, so both process gains are negative.

**Table 10.3** Regulatory Loops as Implemented

| Loop | CV | MV | $K_c$ | $\tau_I$ (h) | Action |
|---|---|---|---|---|---|
| net load | $n$ | $P_b$ | 0.9 | 0.08 | direct |
| power | $P_{ac}$ | $j$ | 0.6 | 0.006 | reverse |
| flow | $m$ | $Q$ | 3.0 | 0.03 | reverse |
| temperature | $T_s$ | $u_c$ | 18 | 1.5 | direct |

The power loop is the trivial one and it is included because a student needs to see how trivial it is. One megawatt of alternating-current power costs about one percent of rated current on discharge (0.98 MW per percent at 60 MW) and less on charge (1.37 MW per percent at −80 MW), through a four-second converter. In the fourth scenario the loop reaches 63 % of a 60 MW step in 18 s and 90 % in 83 s, and there is no interaction to manage. Few plants answer a load change that fast, the grid-forming inverter of Chapter 7 being one.

The flow loop is a feedforward path with an optional feedback trim. The feedforward takes the flow the cell needs from the current command, which is known exactly,

$$Q_{ff} = \mathrm{clip}\left( k_{ff} \frac{|j_{cmd}|}{100} Q_0 + Q_{min},\ 2,\ 30 \right) \tag{10.28}$$

and the trim, when it is selected, adds a margin-controlled correction bounded so that the sum stays inside the pump range. That structure is the standard division of labor. Feedforward handles the current, which is measurable and instantaneous, and feedback handles the charge state, which is not. The gain from flow to transport margin falls as the flow rises and as the tanks empty, so a trim tuned at high flow is too aggressive at low flow and one tuned at low flow is sluggish at high flow. Gain scheduling on that loop is the answer the flow battery literature gives [11], and the app reproduces the problem it solves.

The temperature loop uses a setpoint rather than a limit because both precipitation boundaries matter. V(II) precipitates below about 15 °C and V(V) above about 40 °C, so the electrolyte has to be held in a band rather than kept cold. With a 1.5 h integral time against a 3.63 h closed-loop plant the loop is deliberately slow, and the practical consequence is that its setpoint is a planning decision made in the morning.

### 10.5.3 Battery Management as Cascade Anti-Windup

Charge and discharge limits are computed before the supervisory layer runs, not after it. Charging tapers to nothing as the fuller half-cell approaches the working ceiling and discharging as the emptier half approaches the floor, over a three-point band so that the limit arrives as a derate rather than as a trip. The halves are the tank values $s_T \pm \delta/2$,

$$\begin{aligned} \beta_{up} &= \mathrm{clip}\left( \frac{s_{max} - s_T - |\delta|/2}{0.03},\ 0,\ 1 \right) \\ \beta_{dn} &= \mathrm{clip}\left( \frac{s_T - |\delta|/2 - s_{min}}{0.03},\ 0,\ 1 \right) \end{aligned} \tag{10.29}$$

and the same range is handed to the net-load controller as its output bounds. That is ordinary cascade anti-windup and it is the difference between a battery that recovers the moment the tanks refill and one that spends an hour unwinding an integrator. Section 1.4.4 treats tapered limits in the same terms, and Chapter 7 tapers its battery's limits over a wider band. The taper applies in automatic mode only, because in manual the operator is allowed to drive the plant past its own band, which is the point of the third scenario. There only the supply limit of Equation (10.10) stops the plant, at exhaustion rather than at the band.

The current limits are recomputed every scan rather than fixed, because the current that produces the rated charging power moves with charge state, temperature and membrane age. The lower limit is found by bisecting Equation (10.16) for the current density that produces $-P_R$, using sixty halvings on a bracket of $\pm 1.35 j_R$. The same limit clips the operator's current in manual mode, which is why the third scenario's $-70\,\%$ command is trimmed as the tank fills and the cell voltage rises.

**Pause.** Try Exercise 10.3 before reading on.

## 10.6 The Dispatch Layer

The supervisory layer answers the only question the regulatory loops cannot, which is when to spend the inventory. It uses twenty-four one-hour blocks, one decision variable per block, and a quadratic objective in the battery power $u_k$,

$$J = \sum_{k=1}^{N} \left[ \frac{w_t (n_k - n_k^{*})^2}{P_R^2} + \frac{w_r (n_k - n_{k-1})^2}{P_R^2} - \frac{w_p \pi_k u_k}{\pi_{ref} P_R} \right] + w_f (s_N - s_f)^2 \tag{10.30}$$

with $n_k = f_k - u_k$, where $f_k$ is the forecast net load in block $k$ and $s_N$ is the charge state at the end of the horizon predicted by the plan. Every term is quadratic or linear in $u$, so the gradient is affine and no line search is needed. The step size comes from a Gershgorin bound on the Hessian computed once per execution,

$$L = \frac{2 w_t}{P_R^2} + \frac{8 w_r}{P_R^2} + 2 w_f N c_T^2, \quad c_T = \frac{\Delta}{\eta_{plan} E_{cap}} \tag{10.31}$$

with a step size $\alpha = 1/L$, where the tracking term is diagonal, the ramp term has a spectral radius of at most four and the terminal term is a dense rank-one block whose largest eigenvalue is $N$ times its diagonal entry. The layer then takes twenty projected-gradient steps with no convergence test. The cost per execution is therefore fixed and known in advance, which is what makes the layer run at interactive speed in a browser. The plant beneath it is already closed loop, and the only thing this layer can see is an integrator with a price on it, so a nonlinear model integrated over a horizon would buy nothing. The layer is the app's own and not the predictive control option of Section 1.6.3, which this app does not offer. Chapter 13 solves a box-constrained quadratic program of twice as many variables on the same fifteen-minute cadence.

The projection is where the physics lives. Each iteration restores feasibility in three passes. The box constraint clips every block to the converter rating, the ramp constraint clips forward from the power currently in force, and a causal forward pass on the charge state reduces any block that would spend energy an earlier block has not yet stored,

$$s_{k+1} = s_k + \frac{1}{E_{cap}} \begin{cases} -u_k \Delta / \eta_{plan} & u_k > 0 \\ -u_k \Delta \, \eta_{plan} & u_k \le 0 \end{cases} \tag{10.32}$$

with $u_k$ reduced to whatever hits $s_{min}$ or $s_{max}$ exactly. One efficiency is used in each direction because the layer below already carries the real polarization and the plan is not permitted to pretend that it knows it. The forward pass is greedy feasibility restoration rather than an exact projection onto the reachable set, it is order $N$, and it is what a production dispatch code does. The planning capacity is Faraday capacity at the formal potential,

$$E_{cap} = \frac{c_V V_t F E^0}{3.6 \times 10^9} = 600.4\,\mathrm{MWh}. \tag{10.33}$$

The plan sets a reference and the net-load controller trims it. A one-hour block cannot follow a peak that is two hours wide, and the plan also steers toward its own terminal charge state, so the

**Table 10.4** Effect of the Trim Authority on the Eighteen-Hour Dispatch Day of the Seventh Scenario

| $P_{trim}$ (MW) | Peak net (MW) | Off target (h) | Margin ($) | Final SOC |
|---|---|---|---|---|
| 0 | 571.6 | 6.926 | 27 920 | 0.182 |
| 10 | 551.0 | 4.706 | 31 545 | 0.167 |
| 50 | 502.8 | 0.607 | 35 067 | 0.101 |
| 100 | 502.5 | 0.002 | 34 675 | 0.101 |

**Table 10.5** Scenarios

| # | Start, SOC | Architecture | Subject |
|---|---|---|---|
| 1 | 00:00, 0.50 | manual | which handle does what |
| 2 | 00:00, 0.50 | manual | the duck curve, battery idle |
| 3 | 10:00, 0.30 | manual | charging past the ceiling |
| 4 | 00:00, 0.50 | power | a fast power loop |
| 5 | 16:00, 0.65 | power | fixed, feedforward and trimmed flow |
| 6 | 13:00, 0.40 | net load | feedback and an unseen price event |
| 7 | 06:00, 0.20 | dispatch | the plan meets the midday belly |
| 8 | 06:00, 0.20 | dispatch | the terminal charge state |
| 9 | 09:00, 0.30 | dispatch | a cloud front the plan did not know |
| 10 | 06:00, 0.30 | dispatch | heat, crossover and lost capacity |
| 11 | 04:00, 0.45 | power | graded challenge, twenty hours |

reference and the target part company. On the dispatch day of the seventh scenario the interpolated plan sits 13 MW above what the target asks for at the 19:27 peak and falls 56 MW short by 20:48 and 80 MW short by 21:00, as it holds charge for the end of its horizon. The base is interpolated across the block rather than held, because consecutive blocks on that day differ by a median of 38 MW and by up to 100 MW, and a block held flat and then stepped would put that discontinuity into the feeder every hour, which is a ramp the plan created rather than one it removed.

The trim authority is bounded to $\pm P_{trim}$, defaulting to 50 MW. Given the whole range the trim simply tracks the target and the plan becomes decoration, and the energy policy is then written by a loop that cannot see past the present instant. Table 10.4 shows what the bound is worth in both directions on the seventh scenario's day. With no trim the plan alone leaves the feeder 6.9 h off target and books $27 920. At the default bound it books $35 067 with 0.61 h off target. A 100 MW bound removes almost all of the off-target time and gives back $392, because the inventory is then spent on regulation rather than on price. Figure 10.4 catches the default trim at its bound late in the evening, when the plan has begun to hold charge back.

The forecast is deliberately imperfect and deliberately biased. A multiplicative error is drawn once per reset from the seeded generator and attached to the hour of the day rather than to the execution, so a forecast that is wrong at three in the afternoon stays wrong every time the day is re-planned. The sky disturbance is not in the forecast at all, which is the subject of the ninth scenario.

## 10.7 Scenarios and Measured Results

Table 10.5 lists the eleven scenarios. Each sets a clock start, an initial charge state, a control architecture and a tuning, and the first three run in manual mode. Table 10.6 gives the scorecard at the end of each scenario window with the shipped settings and no learner action. Pumping is the logged pumping energy and the other columns are the app's scorecard tiles. The ninth scenario is the only row that depends on the seeded generator, through its forecast error. Across five other seeds its peak net load ranges from 504 MW to 516 MW and its margin from $30 042 to $32 704.

**Table 10.6** Scorecard at the End of Each Scenario Window

| # | Window (h) | Peak net (MW) | Worst ramp (MW/min) | Off target (h) | Pumping (MWh) | Margin ($) |
|---|---|---|---|---|---|---|
| 1 | 4 | 349.4 | 0.14 | 0.00 | 1.7 | −160 |
| 2 | 24 | 585.9 | 2.74 | 10.74 | 2.2 | −1468 |
| 3 | 12 | 586.5 | 2.74 | 4.80 | 8.9 | −2290 |
| 4 | 3 | 420.8 | 11.66 | 1.50 | 1.1 | −1269 |
| 5 | 6 | 585.7 | 6.81 | 4.71 | 7.2 | 21470 |
| 6 | 12 | 500.2 | 4.80 | 0.94 | 13.7 | 14035 |
| 7 | 18 | 502.8 | 5.55 | 0.61 | 17.0 | 35067 |
| 8 | 18 | 503.2 | 5.64 | 0.84 | 16.0 | 34740 |
| 9 | 14 | 503.3 | 7.36 | 2.91 | 14.5 | 32806 |
| 10 | 18 | 574.3 | 5.82 | 1.98 | 21.8 | 33852 |
| 11 | 20 | 618.3 | 3.50 | 10.73 | 81.3 | −320 |

**Table 10.7** Six Hours From 65 % Charge at an 85 MW Request, Each Flow Policy Held Throughout

| Policy | Pumping (MWh) | Peak flow ($m^3/s$) | Worst margin | Margin ($) |
|---|---|---|---|---|
| fixed at 18 $m^3/s$ | 15.04 | 18.00 | 0.69 | 20720 |
| feedforward | 6.34 | 20.18 | 0.75 | 21598 |
| feedforward and trim | 13.47 | 30.00 | 1.01 | 20546 |

Peak flow and worst margin are taken at the 3.6 s physics step. The logged trend reads worst margins of 0.71, 0.76 and 1.03. Settings made before Run and carried through Reset give identical results.

### 10.7.1 The Day with Nothing Smoothing It

The second scenario runs twenty-four hours with the battery idle, and it establishes the numbers every later scenario is measured against. The net load reaches 585.9 MW in the evening, the worst ten-minute ramp is 2.74 MW/min, 63.9 MWh of solar is curtailed because the feeder exports at midday, and the feeder spends 10.74 h more than 25 MW away from the shaving target. The margin is −$1 468, which includes $766 of curtailment and the 15.42 MWh the idle plant buys for its pumps, its auxiliaries and its inverter. A plant that does nothing is not neutral on a feeder like this one.

> **TRY IT IN THE APP** **2 · The Duck, with Nothing to Smooth It**
>
> Load scenario 2 and press Run for the full twenty-four hours with the battery idle. Watch the feeder net load. It reaches 585.9 MW in the evening, its worst ten-minute ramp is 2.74 MW/min, and 63.9 MWh of solar is curtailed while the feeder exports at midday.

### 10.7.2 Flow Policy Is an Economic Decision

The fifth scenario requests 85 MW of discharge from 65 % charge at 16:00 and switches the flow policy from fixed to feedforward at 1.5 h and to feedforward with trim at 3.0 h. The power cannot be held past about 2.86 h, and at 3.0 h the tank is at 11.3 % and the power is down to 36 MW, so the margin recovery after the second switch is the current collapsing at the floor rather than the trim at work. Table 10.7 therefore holds each policy for the whole six hours, which a learner does by putting the policy back, with the 18 $m^3/s$ flow for the fixed policy, after each scheduled switch. The differences are large and they do not all point the same way.

Feedforward alone cuts pumping energy by 58 % against fixed flow and raises the margin by $878. Only 1.2 MWh of that saving comes while the plant is discharging, because the flow the current calls for at 85 MW is 16.5 $m^3/s$ to 20 $m^3/s$, close to the fixed 18 $m^3/s$. The other 7.5 MWh comes after the tanks reach their floor, when the current falls to zero and the feedforward drops to its 3 $m^3/s$ minimum




**Figure 10.2** Six hours from 65 % charge at an 85 MW request with each flow policy held throughout, from Table 10.7: (a) pumping energy, (b) worst transport margin and (c) margin. Hatched: fixed at 18 $m^3/s$. Solid: feedforward. Light: feedforward with the margin trim. Notice that the trim buys the only worst margin above one at more than twice the feedforward's pumping energy.

**Table 10.8** Eighteen Hours From 06:00, Feedback Against Forecast Dispatch

| Day | Architecture | Peak net (MW) | Worst ramp (MW/min) | Off target (h) | Margin ($) |
|---|---|---|---|---|---|
| clean | net load | 500.2 | 2.61 | 0.000 | 35 083 |
| clean | dispatch | 502.8 | 5.55 | 0.607 | 35 067 |
| scarcity | net load | 500.2 | 2.61 | 0.000 | −2 242 |
| scarcity | dispatch | 506.6 | 5.67 | 2.968 | 14 599 |

while the fixed policy goes on pumping 2.5 MW. Adding the transport-margin trim at its default setpoint of 3.0 raises pumping back to 13.47 MWh and gives back most of the gain. The trim is not failing. It is buying what it was asked to buy, which is transport margin, and it lifts the worst margin of the run from 0.75 to 1.01 by driving the pump to its 30 $m^3/s$ limit as the tanks empty. Pumping power is cubic, so a flow excursion costs far more than its duration suggests. The lesson is that the margin setpoint is a purchased quantity rather than a safety setting, and at 3.0 on this plant at this charge state it is expensive. Figure 10.2 draws the three policies side by side.

The pump-wear disturbance sharpens the same point. Raising the hydraulic resistance by 70 % costs the fixed-flow plant 10.5 MWh of extra pumping, the trimmed plant 9.6 MWh and the feedforward plant 4.8 MWh. Pumping power at a given flow is proportional to the resistance, so each policy pays roughly seventy percent of its own pumping again. The trim does not avoid the loss, because it holds the margin rather than the pumping, and the feedforward pays least because it spends the empty-tank hours at its flow floor.

### 10.7.3 Feedback Dispatch Against Forecast Dispatch

The native sixth scenario runs twelve hours through the scheduled scarcity event. Feedback books $14 035 and dispatch $18 863, a $4 828 difference. The comparison the app is built around runs the same eighteen-hour day from six in the morning at twenty percent charge, once with a net-load controller that has no forecast and once with the dispatch layer. This is the seventh scenario, run as shipped and again with the architecture switched to the net-load controller before Run. Table 10.8 gives both, with and without a $480 per megawatt-hour scarcity event at three in the afternoon, set before Run with the scarcity price control. Each of these settings gives the same result to the dollar whether it is made before Run or carried through Reset. Figure 10.3 draws the same table as bars.

On the clean duck-curve day the forecast is worth nothing. The two architectures book $35 083 and $35 067, a difference of $16 or 0.05 %, and the feedback controller is the better of the two on every regulation metric, holding a worst ramp of 2.61 MW/min against 5.55 MW/min and spending

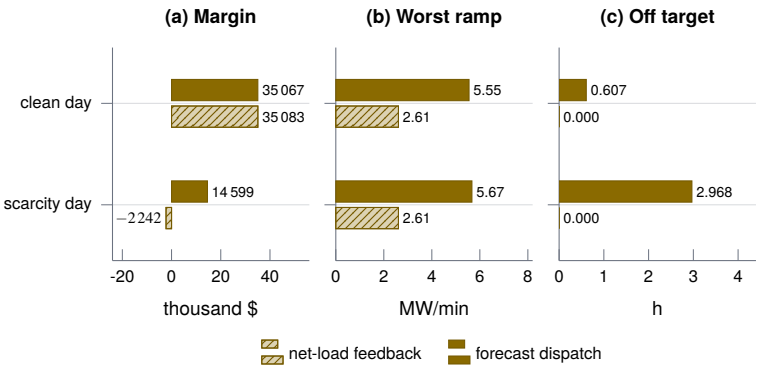



**Figure 10.3** Eighteen hours from 06:00 under net-load feedback and forecast dispatch, from Table 10.8: (a) margin, (b) worst ten-minute ramp and (c) time off target, on the clean day and on the day with a scarcity hour at 15:00. Hatched: net-load feedback. Solid: forecast dispatch. Notice that on the clean day the two margins are equal while feedback regulates better, and that only the scarcity day separates them.

no time off target at all against 0.61 h. That result is half of the comparison and it carries as much weight as the other half. On an ordinary duck-curve day the highest net load and the highest price fall in the same hour, so a controller that watches only the net load is already pointing the right way.

🕮 **Pause.** Try Exercise 10.1 before reading on.

The scarcity day separates them completely. The feedback controller does nothing about a $480 per megawatt-hour event at three in the afternoon, because nothing it measures has changed. It does worse than nothing, in fact, since the midday net load is below its target and the controller is charging through the most expensive hour of the day. The same eighteen hours book −$2 242 under feedback and $14 599 under the dispatch layer, a swing of $16 841 produced by one hour of one day. A controller can act only on what it measures, so a decision that depends on something it does not measure has to be made somewhere else.

The dispatch layer pays for that capability in regulation quality. Its worst ramp is more than twice the feedback controller's on both days and it spends 3.0 h off target on the scarcity day against none for feedback, because the block schedule creates ramps at its own boundaries and the bounded trim cannot remove all of them. A plant that must meet a ramp contract and a price signal at once needs both layers, which is how the app is wired.

> ✦ **DID YOU KNOW? A Forecast Worth Sixteen Dollars**
>
> From the simulator: on the clean eighteen-hour day the net-load controller books $35 083 and the dispatch layer $35 067, and the controller with no forecast holds the worst ramp to 2.61 MW/min against 5.55 MW/min (Table 10.8). One scarcity hour turns the same pair into −$2 242 and $14 599.

### 10.7.4 The Terminal Charge State

An optimizer with a finite horizon and no terminal condition sells everything at the edge of its horizon, because energy left in the tank beyond the horizon is worth zero to the objective. The eighth scenario runs the seventh scenario's day with a terminal target of 55 % at a weight of 12, and the learner repeats it at weights of zero and 60. At midnight the plan's end-of-horizon charge state, the tank and the margin read 11.6 %, 10.1 % and $34 998 at weight zero, 18.4 %, 11.7 % and $34 740 at the shipped weight, where the net-load trim spends most of what the plan holds back, and 38.8 %, 26.2 % and $32 088 at the maximum. Holding a quarter of the tank for tomorrow morning costs $2 910

against the plan that sells everything, and it should. The terminal term is in charge-fraction units, so even the maximum weight does not bring the plan to its 55 % target against tracking and ramp terms summed over twenty-four blocks. The terminal condition is where the horizon's ignorance is priced, and every storage plant has one whether it is stated or not. Section 1.4.6 prices the same edge with a water value taken from the merit order of the forecast prices, and Chapter 11 plans its cavern with a rolling thirty-day water-value plan for the same reason.

### 10.7.5 Degradation That No Controller Reports

The tenth scenario runs a hot day with 34 °C ambient, 120 MW of extra air conditioning, crossover at three and a half times nominal and an imbalance of 9 % already present. The imbalance costs the most and says the least. The two half-cells no longer hold the same charge, so whichever reaches its limit first sets the capacity of the whole plant, and a nine percent imbalance takes 4.5 points of charge state off each end of the working band, cutting it from 480 MWh to 426 MWh. Crossover drives the imbalance up to 15.6 % by 19:00, while the tanks are above half charge, and back to 9.2 % by midnight as they empty. The cooler runs at full duty from 07:49, yet the electrolyte reaches 37.0 °C by 13:56 and 39.7 °C at 20:58, against a 32 °C setpoint and 0.3 °C short of the V(V) precipitation limit. A 26 °C setpoint from the start gives an identical run. The peak net load is 574.3 MW against 502.8 MW on the clean day. The power and flow loops hold their setpoints through all of it and nothing on the screen reports the lost capacity. It is a fault that degrades the plant while the control system reports success.

### 10.7.6 The Graded Challenge

The eleventh scenario opens twenty hours from four in the morning with weak tuning, fixed flow at 20 $m^3/s$, the battery on a hand-set power request, and four timed events including a cloudy midday, a hot afternoon and a pump losing its impeller. A run passes when, over the twenty hours, the net load stays within 50 MW of target over the last third, the peak net load stays below 560 MW, pumping stays below 45 MWh, the electrolyte stays within 15 °C to 40 °C and the margin ends above $25 000. The model parameters and disturbances must stay as given: changing one voids the attempt, as does an extra learner disturbance event when it fires. The default configuration books −$320, reaches 618.3 MW of peak net load, spends 81.3 MWh on pumping and misses the target by up to 86.9 MW in the evening, so it fails every criterion except temperature. The suggested answer is the dispatch layer, feedforward flow with trim at a margin setpoint of 2.6, a 28 °C temperature setpoint chosen in the morning and a 32 % terminal target. It books $34 713, holds the peak to 535.8 MW, spends 25.1 MWh on pumping and keeps the evening error to 47.0 MW, 3.0 MW inside the band and 4.6 MW to 6.9 MW inside with five other seeds of the forecast error. Settings made before Run and carried through Reset give identical results. The structural decisions are worth $35 033 and 56.2 MWh, and no gain change in the default structure comes close.

## 10.8 Technology in Practice

The instruments and actuators of Section 10.3 are, in a plant, instrument transformers with a power transducer, a temperature sensor, a charge-state computation, pumps on variable-speed drives, a cooler and a power converter. Figure 10.5 draws the chapter's loops with that hardware in place of the schematic's symbols.

#### What Measures It

The net load and the battery's own power are electrical quantities. They are read through instrument transformers and turned into megawatts by a transducer. Utilities collect such readings from dispersed assets with supervisory systems that display them to an operator in near real time [16]. The charge state has no direct sensor in the chapter. Its estimator SI is a filter with a 72 s time constant, and a charge state read at the stack outlet is a biased estimate of the tanks whenever current flows.

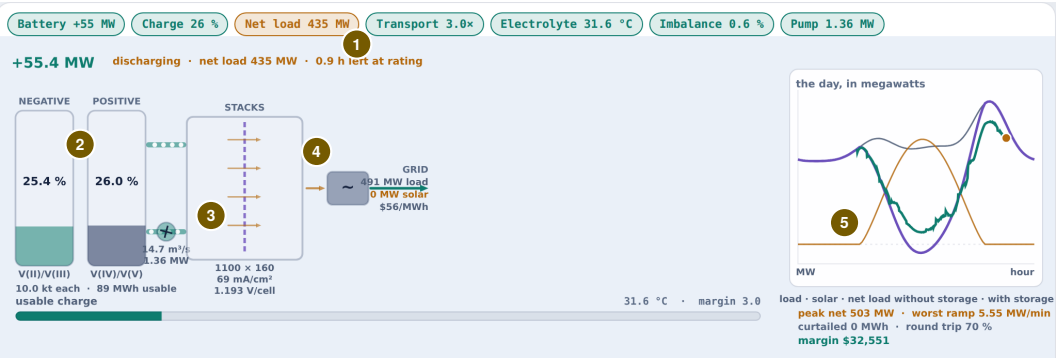



**Figure 10.4** The app at 21:09 on the seventh scenario, 15.15 h into an eighteen-hour dispatch day that began at six in the morning at twenty percent charge. The plan filled the tanks to 86.8 % through the midday belly and has spent them into the evening neck, so the tank holds 25.7 %, 89 MWh above the floor (2). The plan's own request has fallen to 5.4 MW as it holds charge for the end of its horizon, and the net-load trim sits at its 50 MW bound, so the battery delivers 55.4 MW through the converter (4) and the feeder net load reads 435.4 MW (1). The trim holds the transport margin at its setpoint of 3.0 with 14.7 m³/s of flow and 1.36 MW of pumping (3). The scorecard line under the day inset (5) shows a peak net load of 503 MW and a margin of $32 551.

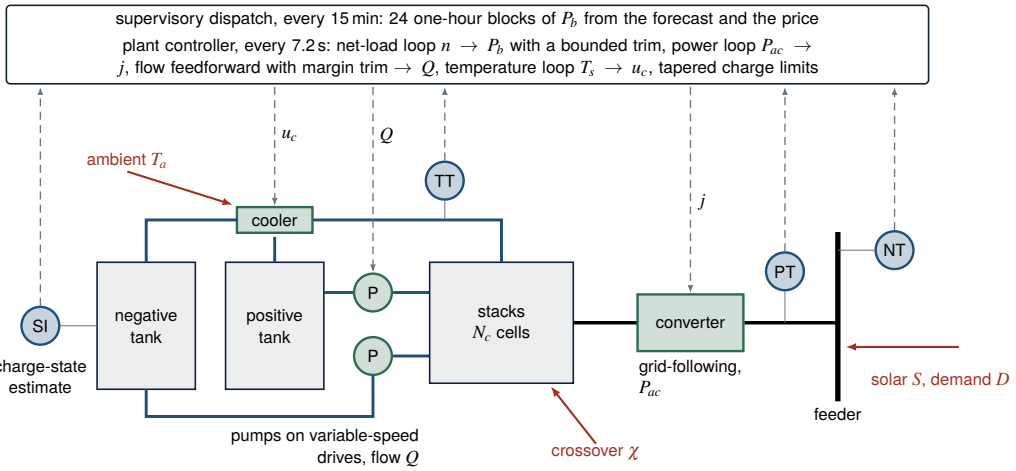



**Figure 10.5** The chapter's loops drawn with their hardware. The power transducer PT and the feeder measurement NT read through instrument transformers, TT reads the stack outlet and SI estimates the charge state. Every 7.2 s the plant controller sets the converter current $j$, the pump flow $Q$ and the cooler duty $u_c$, and every 15 min the dispatch layer above it re-plans the battery power. Crossover acts on the stacks, ambient temperature on the cooler, and solar and demand on the feeder. Notice that no device measures the energy left in the tanks directly.

**TECHNOLOGY Sensor: Instrument Transformers and a Power Transducer**

A potential transformer steps the line voltage down to about 120 V, and a current transformer steps the line current down to about 5 A at full load [17]. A metering-class current transformer rated 0.3B1.8 is accurate to 0.3 % of its ratio while it drives a 1.8 Ω burden, and a larger burden makes the signal sag, so the power reads low [17]. An open secondary on an energized current transformer can develop an extremely high

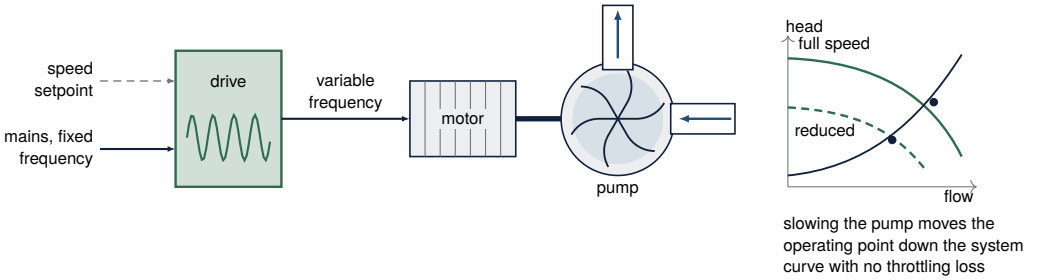



**Figure 10.6** Principle of a variable-speed drive on a centrifugal pump. The drive changes the motor's speed, and the pump curve moves along one system curve. Notice that a lower speed reaches a lower flow without a throttling valve, which is how the feedforward policy of Section 10.5 saves pumping energy.

voltage, which is why its circuits carry shorting switches [17]. The chapter's transducer PT is a first-order filter of 8 s, with a noise disturbance of up to 6 MW.

### What Moves It

Three final elements act on the plant. The converter sets the current. In the common grid-following design it estimates the angle of the grid voltage with a phase-locked loop and regulates the current it injects, so for fixed power commands it behaves as a constant real and reactive power source [18]. That is why the chapter's power loop is easy: the converter follows its command with a 4 s lag. The cooler is the second element, with a 60 s lag, and the pumps are the third and the one with an economic trade. Figure 10.6 shows the principle of their drive.

**TECHNOLOGY Final Element: Electrolyte Pumps on Variable-Speed Drives**

A variable frequency drive sets the speed of an alternating-current motor by varying the frequency of its supply, and its torque by varying the voltage and current [17]. On a flow battery it replaces the throttling valve, which in the NASA tests varied the flow without saving any pumping energy [3]. The same report found the small chemically resistant pumps of kilowatt systems only 5 to 20 % efficient [3]. Pumping power is cubic in flow in Equation (10.14), so a worn pump or fouled piping shows on the trend as more pumping power at the same flow. The chapter's pumps follow their command with a 20 s lag between 2 and 30 $m^3/s$.

### Where the Controller Runs

The four loops of Table 10.3 run in the plant's own controller, and the dispatch layer runs above them in supervisory software that also gathers the feeder data. Supervisory and lower-level loops of this kind run with cycle times from milliseconds to minutes [16]. In the chapter the regulatory loops execute every 7.2 s on a 3.6 s physics step, and the dispatch layer re-plans every 15 min. The tapered limits of Section 10.5.3 belong to the battery management layer between the two.

**IN PRACTICE** **Spill Containment, Permits and the Queue**

A water-based electrolyte cannot burn and has no thermal runaway, so the main operating risk of a flow battery is a leak, handled with containment and neutralizing spill kits as NFPA 855 describes [19]. A battery plant also answers to the local authority having jurisdiction, often the fire department, whose setback and siting rules vary from place to place. Following current codes and standards and engaging that authority early minimizes delays, and an interconnection request typically takes over three years to complete [19].

### What Is Modernizing It

Three developments touch this plant. The first is the pump drive. The NASA report already pointed to a variable-frequency motor and pump as the way to make variable flow efficient [3], and the chapter's feedforward flow schedule is that idea with a modern drive. The second is the converter. Grid-forming inverters act as voltage sources and can black-start a power system. Replacing synchronous machines with them on the bulk grid is a 10 to 30 year prospect that first needs protection, controls, interoperability and standards [18]. The third is the network. Operational systems increasingly resemble information systems as they adopt standard computers, operating systems and protocols for remote access, which leaves them less isolated from the outside world and in greater need of security [16]. A plant dispatched from a forecast depends on that network every fifteen minutes.

## 10.9 Limitations

The feeder, the solar profile and the price curve are synthetic. They are shaped like a spring day on a California distribution system and no claim is made that they reproduce one. Any conclusion about how much a forecast is worth on a real system requires real data, and the numbers here should be read as the shape of an answer rather than as its magnitude.

**OPEN CHALLENGE** **Proving Long Life at Scale**

A 2024 national laboratory assessment put a 100 MW, ten-hour vanadium flow battery at about $446 per kilowatt-hour installed, against about $379 for a lithium iron phosphate system of the same size. The electrolyte allows projected lives of 20 years or more, but few plants in the United States have run long enough to prove it, and some investors wait for operating data from the first deployments [19]. Progress would be successful demonstration plants, which the assessment names as the key to confidence and lower cost [19].

The dispatch layer runs a fixed twenty projected-gradient iterations with no convergence test, so the plan is a bounded-effort approximation to the solution of Equation (10.30) rather than the solution. Receding-horizon re-planning every fifteen minutes makes the approximation adequate, and it also hides it. A student who reads the objective and expects the plan to be optimal will find small inconsistencies between successive plans that the framework does not explain.

The electrochemical model is a lumped single-cell model scaled by a cell count, so everything that makes a real stack differ from the sum of its cells is missing. Shunt currents, cell-to-cell voltage distribution, electrode compression, flow-field maldistribution and gas evolution at the extremes of charge are all absent. The concentration overpotential in Equation (10.4) is a limiting-current form with a flow exponent fitted to nothing in particular, and it should be read as the right qualitative shape rather than as a prediction.

Crossover is one scalar where the physical process is four coupled diffusion fluxes at different rates. The model reproduces the operational consequence, which is capacity that disappears without an alarm, and not the composition drift that a plant operator would actually measure through electrolyte sampling.

Round-trip efficiency is reported only when the run is close to a closed cycle, because an uncorrected ratio of energy out to energy in exceeds unity whenever a run ends emptier than it started. The

application shows a dash instead of a number when the stored-energy change exceeds forty percent of the energy moved, which is honest and occasionally frustrating.

**OPEN CHALLENGE Where the Vanadium Comes From**

The same assessment rated vanadium supply a moderate risk, because the ore and the processed precursor are both geographically concentrated and the price is volatile. China holds 39 % of global vanadium reserves and leads production, largely from the slag of steelmaking, and supply deficits are predicted through 2030 if flow batteries are deployed widely [19]. Progress would be chemistries built from abundant elements such as iron or zinc, which the assessment rates low risk [19].

## 10.10 Conclusion

A vanadium flow battery separates the purchase of power from the purchase of energy, and the control structure has to respect that separation. The power loop on a 100 MW converter reaches 63 % of a 60 MW step in 18 s and 90 % in 83 s, and offers nothing to a control course beyond a demonstration of how easy it is. The flow loop carries a real trade-off, because pumping is cubic and the loss-minimizing flow of 8.92 $m^3/s$ at rated current is well below the 15 $m^3/s$ design flow and moves with charge state. The temperature loop is slower than the day it is dispatched over, so its setpoint is a planning decision. Above all three, the question of when to spend 480 MWh of usable inventory is the one that decides whether the plant is worth what it cost.

The measured comparison between feedback dispatch and forecast dispatch is the result worth carrying away, and it has two halves. On a clean duck-curve day the two are within 0.05 % of each other and the feedback controller regulates better, holding 2.61 MW/min of worst ramp against 5.55 MW/min. On a day with a price event away from the evening peak the same two architectures book −$2 242 and $14 599. A forecast is not worth having because it is a forecast. It is worth having exactly when the quantity that decides the right action is one the feedback path cannot measure, and naming that condition is more useful to a student than any comparison of architectures in the abstract.

**KEY POINTS**

- A flow battery's energy is the liquid in its tanks and its power is the membrane in its stacks.
- The power loop reaches 63 % of a 60 MW step in 18 s, so the hard question is when to spend the inventory.
- The loss-minimizing flow of 8.92 $m^3/s$ at rated current is well below the 15 $m^3/s$ design flow and moves with charge state.
- Feedback and forecast dispatch are within 0.05 % on a clean day, and a price event splits them to −$2 242 and $14 599.
- A forecast is worth having when the quantity that decides the right action is one the feedback path cannot measure.

**Where else this applies.** A stored inventory scheduled against a price over a day also sets the plan in Volume 9, Chapter 6 and Volume 9, Chapter 11, and Chapter 11 prices its terminal state with a water value.

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## 10.11 Exercises

**10.1** ○ Concept check · 2 min

On the clean eighteen-hour day of scenario 7 the net-load feedback controller books \$35 083 and the dispatch layer, which has a forecast, books \$35 067. Why is the forecast worth nothing on that day?

(a) The twenty fixed projected-gradient steps stop well short of the optimum of the dispatch objective

(b) The terminal charge weight makes the plan hold back energy that it would otherwise sell in the evening

(c) The highest net load and the highest price fall in the same hour, so feedback on the net load already points the right way

(d) The shaving target is built from the price curve, so the net-load controller sees the price through its setpoint

**10.2** ○ Concept check · 2 min

In scenario 3 the plant is reversed to full discharge current at the ceiling alarm with the flow held at 12 $m^3/s$. The transport margin reaches its starvation line of 1.15 at about 22 % tank charge, with 88 MW still flowing. What sets the charge state at which this happens?

(a) The battery management taper, which derates discharge over the last three points above the floor

(b) The Faradaic supply limit, which cuts the current once the flow cannot carry enough reactant

(c) The converter rating, which trims the current as the open-circuit voltage falls with the charge

(d) The limiting current, which is proportional to the availability of the consumed species and rises with the flow

**10.3** Concept check · 1 min

The charge and discharge limits of Equation (10.29) are handed to the net-load controller as its output bounds. What does that buy?

(a) The controller's integral does not wind up at the limit, so the battery responds as soon as the tanks have room again

(b) The plant can no longer charge past its working ceiling in manual mode, which protects the positive electrolyte

(c) The dispatch plan no longer needs its causal pass on the charge state, because the bounds keep it feasible

(d) The loop changes its action near the floor, so it charges the tanks back toward the middle of the band

**10.4** Theory and calculation · 6 min

Use the parameters of Table 10.2: $c_V = 1600\,\mathrm{mol/m^3}$, $V_t = 10\,000\,\mathrm{m^3}$ a side, $V_c = 150\,\mathrm{m^3}$ of hold-up, $E^0 = 1.40\,\mathrm{V}$, $Q_0 = 15\,\mathrm{m^3/s}$ and a working band from 0.10 to 0.90.

(a) Find the energy of the tanks between empty and full from Equation (10.33), and the energy inside the working band. *Checkpoint:* 600.4 MWh and 480.3 MWh.

(b) Find the energy held in the stack hold-up and the time the electrolyte takes to pass through the hold-up and through one tank at design flow. *Checkpoint:* About 9.0 MWh, 10 s and 11.1 min.

(c) An imbalance $\delta = 0.09$ is present. Using the half-cell charge states $s_T \pm \delta/2$ of Equation (10.29), find the energy left inside the working band. *Checkpoint:* About 426 MWh.

**10.5** Theory and calculation · 7 min

The plant has $N_c = 176\,000$ cells of $A = 4000\,\mathrm{cm^2}$ with a rated current density $j_R = 0.12\,\mathrm{A/cm^2}$. At half charge and design flow, 100 MW at the connection point takes 106.4 % of rated current on discharge and 72.2 % on charge. Take $F = 96\,485\,\mathrm{C/mol}$ and $c_V = 1600\,\mathrm{mol/m^3}$.

(a) Find the plant current at rated current density. *Checkpoint:* 84.48 MA.

(b) The tank and the hold-up move together once the transient of the stacks has passed, so the vanadium in $V_t + V_c = 10\,150\,\mathrm{m^3}$ changes state together. Find the rate of change of the tank charge state in percent per minute at 100 MW of discharge and of charge. *Checkpoint:* 0.344 %/min and 0.234 %/min, within 0.002.

(c) Find how long the working band of 80 points lasts at each rate, and say why charging takes less current than discharging for the same power. *Checkpoint:* 3.87 h and 5.71 h, within 0.05 h.

**10.6** Theory and calculation · 6 min

Pumping power follows Equation (10.14) with $k_Q = 155\,\mathrm{Pa\,s^2/m^6}$, $\eta_p = 0.72$ and $w_p = 1$.

(a) Find the pumping power for both circuits at the design flow of 15 $\mathrm{m^3/s}$ and at the pump limit of 30 $\mathrm{m^3/s}$. *Checkpoint:* 1.45 MW and 11.6 MW.

(b) From Equation (10.10) with no imbalance, find the flow per circuit that just supplies rated current at half charge. Compare the design flow and the loss-minimizing flow of 8.92 $\mathrm{m^3/s}$ with it. *Checkpoint:* 1.09 $\mathrm{m^3/s}$, about 13.7 and 8.2 times.

(c) Over six hours of scenario 5 the fixed policy at 18 $m^3/s$ pumps 15.04 MWh. Predict the extra pumping when the hydraulic resistance rises by 70 %, and say why the prediction is exact for this policy and only approximate for the other two. *Checkpoint:* 10.5 MWh.

**10.7** App challenge · 7 min

Load scenario 2, *The duck, with nothing to smooth it*, and change no setting. Run the full 24 h at speed 5 with the default seed, then export the CSV file.

(a) Read the Peak net load, Worst ramp, Curtailed and Margin tiles. *Checkpoint:* 586 MW, 2.74 MW/min, 64 MWh and −$1 468.

(b) In the last row of the file, at `time_h` 23.987, read `metric_curtail_MWh` and `metric_in_MWh`. With the curtailment charge $c_{curt} = \$12$ per megawatt-hour, find the share of the margin's loss that the curtailment accounts for, as a percentage of $1 468, within one point. *Checkpoint:* 63.87 MWh and 15.42 MWh; $766, about 52 %.

(c) From the columns `net_raw_MW` and `clock_h`, find the clock times of the lowest and the highest raw net load, within two minutes. *Checkpoint:* 12:28 and 19:27.

**10.8** App challenge · 9 min

Load scenario 7, *The dispatch layer meets the belly*, with the default seed. Each run lasts 18 h at speed 5. Read the Margin, Worst ramp and Off target tiles at the end of each run, and give dollar values to within $2.

(a) Run the scenario as loaded, on the Dispatch architecture. *Checkpoint:* $35 067, 5.55 MW/min, 0.61 h.

(b) Pause, Reset, set Power architecture to Ramp before Run and run again. Find what the forecast is worth on this day. *Checkpoint:* $35 083, 2.61 MW/min, 0.00 h. The forecast is worth −$16.

(c) Reload the scenario, set the Scarcity price event slider to $480 per megawatt-hour before Run, and run on Dispatch. Then Reset, set Power architecture to Ramp and run again. Find the swing between the two margins and explain it. *Checkpoint:* $14 599 and −$2 242, a swing of $16 841.

**10.9** App challenge · 8 min

Load scenario 8, *The terminal charge state, and why it is not optional*, with the default seed. It runs the day of scenario 7 with a terminal target of 55 % at a weight of 12. Switch to the Advanced level. Run 18 h at speed 5, then Pause, Reset, set Dispatch: terminal charge weight to 0 and run again, then Reset, set it to 60 and run a third time. After each run read the Charge and Margin tiles and, from the exported CSV file, `soc_plan_pct` in the last row (`time_h` 17.986).

(a) Record the planned end-of-horizon charge, the tank charge and the margin for the three weights, the charges within 0.2 points and the margins within $2. *Checkpoint:* Weight 12: 18.4, 11.7, $34 740. Weight 0: 11.6, 10.1, $34 998. Weight 60: 38.8, 26.2, $32 088.

(b) Find what holding charge for the next morning costs at the maximum weight, against the plan with no terminal term. *Checkpoint:* $2 910.

(c) Explain why even the maximum weight leaves the plan well short of its 55 % target. *Checkpoint:* The terminal term is in charge-fraction units.

**10.10** Application area · 4 min

Scenario 10 runs a hot day at an ambient of 34 °C. The cooler goes to full duty in the morning, the electrolyte still reaches 39.7 °C against a 32 °C setpoint, and a 26 °C setpoint from the start gives an identical run.

(a) From $C_T = 2.9 \times 10^{10}\,\mathrm{J/°C}$, $UA_{amb} = 3.2 \times 10^{5}\,\mathrm{W/°C}$ and $UA_c = 1.9 \times 10^{6}\,\mathrm{W/°C}$, find the time constant of the electrolyte temperature at full cooler duty. *Checkpoint:* 3.63 h.

(b) Using Equation (10.17), explain why no setpoint below the ambient can change this run, and why the setpoint is held in a band rather than kept as low as possible. *Checkpoint:* The cooler rejects heat to the ambient air.

**10.11** Application area · 3 min

A planner proposes to use the chapter's $16 841 swing between feedback and forecast dispatch to justify buying a price-forecast service for a real 100 MW flow battery. Name two limitations stated in the chapter that stop the number from transferring, and the data that would be needed to replace it.

**10.12** Review the agent · 5 min

An AI assistant reviews scenario 5, *Fixed flow, feedforward flow, and the pump bill*, and writes: "Run the flow on FF + trim at the default transport margin setpoint of 3.0. The trim adds flow only when the margin falls, so it pumps less than feedforward alone and pays for itself, and when a pump loses its impeller the trim finds that it needs less flow and recovers most of the loss." Find the flaw and say how you would check it in the app.