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CHAPTER 8

Hunting, Reverse Flow and an Unmeasured Customer on a Solar Distribution Feeder

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Edited by John D. Hedengren

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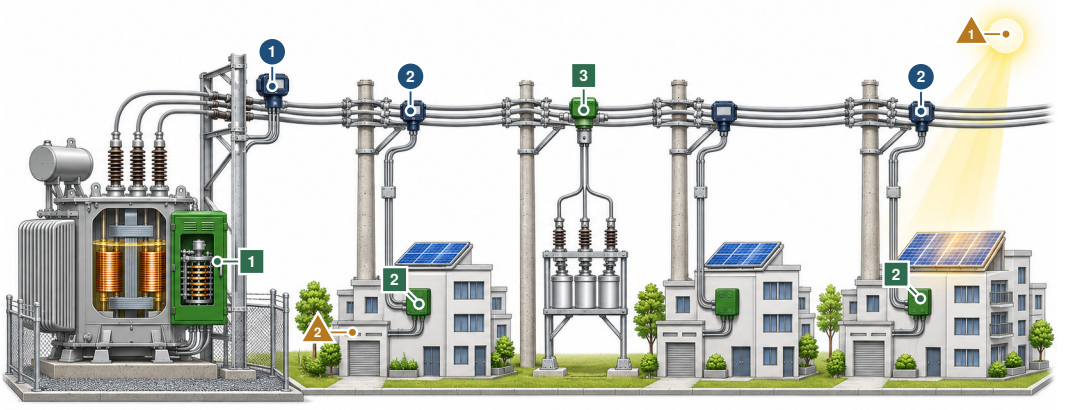
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CHAPTER 8

Hunting, Reverse Flow and an Unmeasured Customer on a Solar Distribution Feeder



The controller moves 1 tap changer on the substation transformer 2 solar inverters on every block 3 capacitor bank switch

The controller reads 1 voltage and power at the substation 2 three meters along the line

What disturbs it 1 changing sunlight 2 household demand

Notice that the tap changer sits at the substation end while the sunlit roofs at the far end push the voltage up, and that only three meters see the line between them. Illustration, not to scale.

IN PLAIN TERMS **Holding Every Home's Voltage on a Sunny Street**

The main idea. A power line loses voltage along its length, so a transformer at its start steps the voltage up or down for every home at once. Rooftop solar pushes the far end up instead, and at noon power can flow backward.

The challenge. The transformer waits before each step and watches only its own end, so it misses a cloud over the far end. The panels' electronics answer in seconds, but if they react too hard, they and the transformer undo each other's work.

Why it matters. Every home needs a voltage inside a narrow range, all day and in any weather.

THREE WAYS TO READ THIS CHAPTER

Explore	11 min	The first page, the history box of Section 8.2, the try-it and fact boxes of Section 8.6, and the key points
Study	50 min	The whole chapter with its pause points and the twelve exercises
Practice	20 min	The first page, Section 8.6, Section 8.7, Section 8.8 and its open challenge

The control question A substation tap changer moves the whole feeder in steps after a timer, while rooftop solar reverses the flow at noon, clouds move the far end in seconds and inverters answer at every node. Which device should set the level and which should trim the spread, and what does a central controller that reads only the utility's measurements add?

Learning objectives

- Compute the line-drop compensator's correction under reverse flow and state when it raises the bus instead of lowering it.
- Relate the bandwidth and the time delay of a tap changer to its operations count and to the lowest customer voltage.
- Explain how a steep Volt-VAR slope and a reactance-weighted compensator make the tap changer hunt, and name the settings that remove it.
- Measure a conservation voltage reduction factor and say why it depends on the composition of the load.
- Compare a linear program and a mixed-integer predictive controller on the same estimate in minutes out of band and tap operations.

Prerequisites Sections 1.3 (loop action), 1.4.3 (Volt-VAR droop and the tap changer as the slow layer), 1.6.1 and 1.6.4 (engine, scenarios and scorecard). Section 1.6.3 is needed only for the predictive controller.

The app https://apmonitor.com/apps/feeder-volt-var/

See also Volume 9, Chapter 11 for integer decisions by branch and bound and Volume 9, Chapter 5 for releases planned ahead of demand. Volume 5, Chapter 3 for a loop behind a late measurement and Volume 5, Chapter 11 for a limiting state that no instrument reads.

8.1 Introduction

At noon on a clear summer day in the simulator of this chapter, the twelve blocks of houses on a 9.6 km feeder make more power than they use, and 1.08 MW flows back through the substation transformer. The houses' air conditioners still draw reactive power, so 0.64 Mvar keeps flowing forward. The tap changer's line-drop compensator, set on the evening's load with $R_{set} = 0.01$ pu and $X_{set} = 0.15$ pu, reads that reactive power as load and holds the bus 0.0083 pu above its setpoint, more than one tap step of 0.006 25 pu. The far end of the feeder passes the 1.05 pu limit at 10:44 and spends 203 min above it before 15:00, while every instrument at the substation reads a voltage inside its band.

The scene contradicts three readings of the problem that a block diagram invites. The first is that the substation sees the feeder. When a bank of cloud reaches the far end, the far end falls 0.031 pu and the bus 0.001 pu, so the regulated voltage never stays outside its band for the 45 s a step needs. The second is that devices that each regulate correctly alone will regulate correctly together. With the inverters' Volt-VAR slope raised from the default 7.33 to 22 on a cloudy day, the tap changer makes 18 operations between 09:00 and 17:00 instead of 2, because its compensator reads the inverters' reactive swing as load coming and going. The third is that a compensator designed for forward flow fails in a known direction when the flow reverses. Its correction is the in-phase part of a phasor drop, and whether it raises or lowers the bus depends on the ratio of its own reactance to resistance against the ratio of exported to imported power.

The class of problem is general. A slow, discrete, integrating actuator with a deadband and a timer shares a plant with fast proportional devices that it cannot see, and each reads the other's action as a disturbance. It appears between substation tap changers and switched capacitors [1], and distributed generation added voltage rise, reverse flow and interaction with regulators and capacitors [2, 3]. Photovoltaic generation forces tap changers and line regulators to operate almost continuously, and uncoordinated reactive power control can drive a regulator to its limit [4]. In process terms the tap changer is a large valve with a slow positioner, the inverters are small fast trim valves, and the compensator is a feedforward computed from a flow meter that the trim valves also move.

These interactions are usually studied on detailed feeder models [4, 5], and the interconnection standard states the curves an inverter must offer rather than what they do to a regulator [6]. This chapter works on a smaller plant that turns the bandwidth, the timer, the compensator dials, the Volt-VAR slope and the meter delay into settings whose effects read in tap operations, minutes out of band and kilowatt-hours. It describes the feeder simulator in enough detail to rebuild it and measures what each local device and the central program contribute when solar power reverses the flow and clouds move it. The power flow, the curves, the compensator and conservation voltage reduction follow the cited literature and standards. The hunting of a reactance-weighted compensator against steep Volt-VAR curves belongs to a class of regulator runaway already described [4], and it is reported here as something measured on this teaching plant, within a stated window. The model has 29 states, and its energy ledger closes to 1.6×10^{-14} over a day. The tap changer and the capacitor are explicit discrete controllers, the estimator and the linear program read only what a utility measures, and a day runs in 24 s in a browser. Ten cases, graded in the application, range from a tap moved by hand to a cloudy day at 125 % solar. An optional mixed-integer model predictive controller on the same estimate and weights is measured against the program and against stronger forms of it.

Section 8.2 traces how feeder voltage regulation reached its present form. Section 8.3 gives the model, Section 8.4 four mechanisms that explain the cases, and Section 8.5 the controllers, with the predictive one in Section 8.5.5. Section 8.6 measures the nine cases and the challenge, Section 8.6.10 sets the predictive controller beside the program and its stronger forms, and Section 8.6.11 states what was verified and what is not claimed, Section 8.7 describes the hardware that does this work on a real feeder, and Section 8.8 lists the limitations.

8.2 Voltage Regulation on Radial Feeders

A radial feeder loses voltage along its length in proportion to the current it carries, so the voltage a distant customer receives falls as the load rises. The first remedy was to regulate the sending end on a measurement of the far end. A voltage-regulating relay combined with a line-drop compensator, described in 1942, combined the voltage at the instrument with a measured line drop so that the relay acted on the voltage at a remote point of the feeder, and it named tap-changing transformers and capacitor switching as the equipment it would control [7]. The step-voltage regulator with that compensator is now textbook material for distribution engineers [8]. Supervisory schemes later used the measurements available at substations to improve both the regulator and the capacitor controllers there [1].

Running the voltage low on purpose has its own history. A 1978 experimental investigation related the energy and demand a utility saved by voltage reduction to the composition of its load [9], and a national evaluation in 2010 simulated conservation voltage reduction on 24 prototypical feeders with appliance-level load models and found energy reductions of about 0.5 to 4 %, nearly all of it in the end-use loads [10]. Analysis of the feeder itself became systematic when the DistFlow branch equations were introduced for capacitor sizing on radial systems [11], and power-flow approximations built on them guided feeder reconfiguration for loss reduction [12]. A weighted least-squares state estimator for distribution systems was then proposed to improve forecast load data with real-time measurements [13].

Distributed generation changed the assumptions every one of these devices was set on. Interconnected resources raise the voltage, interact with capacitor, regulator and tap changer operations, and break the premise that the substation is the sole source of power [2]. Voltage control by on-load tap changers in medium-voltage feeders was then examined in the presence of distributed generation [3]. How inverters came to be allowed to hold the voltage is told in the history box below. In 2011 an industry collaboration defined Volt-VAR as one of the smart inverter functions for high penetration [14]. Local Volt-VAR control by photovoltaic inverters was shown to mitigate voltage rise on detailed feeders [5], an optimal coordination of reactive power was proposed to limit the tap operations and the regulator runaway that photovoltaics cause [4], and the optimality and stability of local reactive

Table 8.1 Variables by Role, With Values at 12:00 in Case 3

Variable	Role	Value	Range
V_j	CV of the challenge, node voltages (truth)	1.040–1.057	0.95–1.05
V_{reg}	CV of the tap changer, computed	1.0295	$V_{set} \pm BW/2$
V_{bus}	measured, bus voltage	1.0394	—
P, Q	measured, into the transformer	−1.093, 0.649	MW, Mvar
n	MV, tap position (integer)	+7	−16 to +16
V_{set}	setpoint, or MV of the program	1.025	0.95–1.05
u_j	MVs of the program, reactive offsets	0	± capability
b_{cap}	MV, capacitor in or out	out	0 or 0.06 pu
$p_{L,j}$, κ_j	DVs, load and sunlight share	—	shape, seeded
V_{grid}	DV, transmission voltage	1.00	0.97–1.03
V_3, V_7, V_{10}	measured, meters, 15 min old	1.045–1.053	—
q_j, w_j	states, inverter Q and Volt-Watt limit	0, 1	12 each
E	states, five energy ledgers	—	pu s
s_L, s_{PV}	estimator, load and solar scales	0.989, 0.994	see text

control under limited capability were analyzed [15]. The design of the curves within the ranges that IEEE 1547-2018 allows has itself become an optimization problem [16].

> ⧖ HISTORY **Inverters Join Voltage Regulation (2010 to 2018)**
>
> For years, United States interconnection standards largely kept inverters from supporting voltage regulation and required a distributed system to disconnect at set levels of grid disturbance instead [17]. A study first posted in 2010 noted that cloud-driven swings of photovoltaic output could outpace slowly responding utility equipment such as tap changers and switched capacitors, and that fast inverters able to inject or absorb reactive power, although not permitted under the standards of the day, could hold the voltage [18]. In May 2014 an amendment, IEEE 1547a, allowed voltage regulation support by distributed resources [17]. IEEE 1547-2018 then set reactive power categories for every resource it covers [6]: category A requires constant power factor, constant reactive power and Volt-VAR capability, and category B adds watt-VAR and Volt-Watt [19].

8.3 The Feeder as Modeled

The simulated plant is one generic 12.47 kV suburban feeder with synthetic teaching values, drawn in Figure 8.1(a) in the arrangement of the application's Feeder view. A substation transformer with an on-load tap changer feeds a bus, node 0, from which twelve overhead segments of 0.8 km reach twelve nodes. Each node stands for a block of 80 houses, and the share of houses with rooftop solar rises from 10 % at node 1, the older blocks, to 90 % at node 12, so that half the houses carry solar. A 600 kvar capacitor bank sits at node 6 and smart meters at nodes 3, 7 and 10. Every power is in per unit of $S_b = 10\,\text{MVA}$ and every voltage in per unit of 12.47 kV, so the base impedance is 15.55 Ω and the whole feeder is $0.2642 + j0.2485$ pu, an R/X of 1.06. Time is in seconds from the case's start, which a hidden parameter places at its time of day. Table 8.1 classifies the variables.

8.3.1 Branch Flows and Voltages

The flow entering segment k is the sum of the injections it feeds, and the voltage falls along each segment by its resistance times the active flow plus its reactance times the reactive flow, as shown in Equation (8.1).

$$\begin{aligned} P_k &= \sum_{j \geq k} p_j, \qquad Q_k = \sum_{j \geq k} q_j \\ V_0 &= V_{grid}(1 + 0.00625n) - r_t P_h - x_t Q_h \\ V_k &= V_{k-1} - (r_k P_k + x_k Q_k), \quad k = 1 \ldots 12 \end{aligned} \tag{8.1}$$

where p_j and q_j are the net active and reactive loads at node j, P_h and Q_h are the flows at the feeder's head including its losses, and the nominal voltage that divides the drop is 1 pu. This is the DistFlow

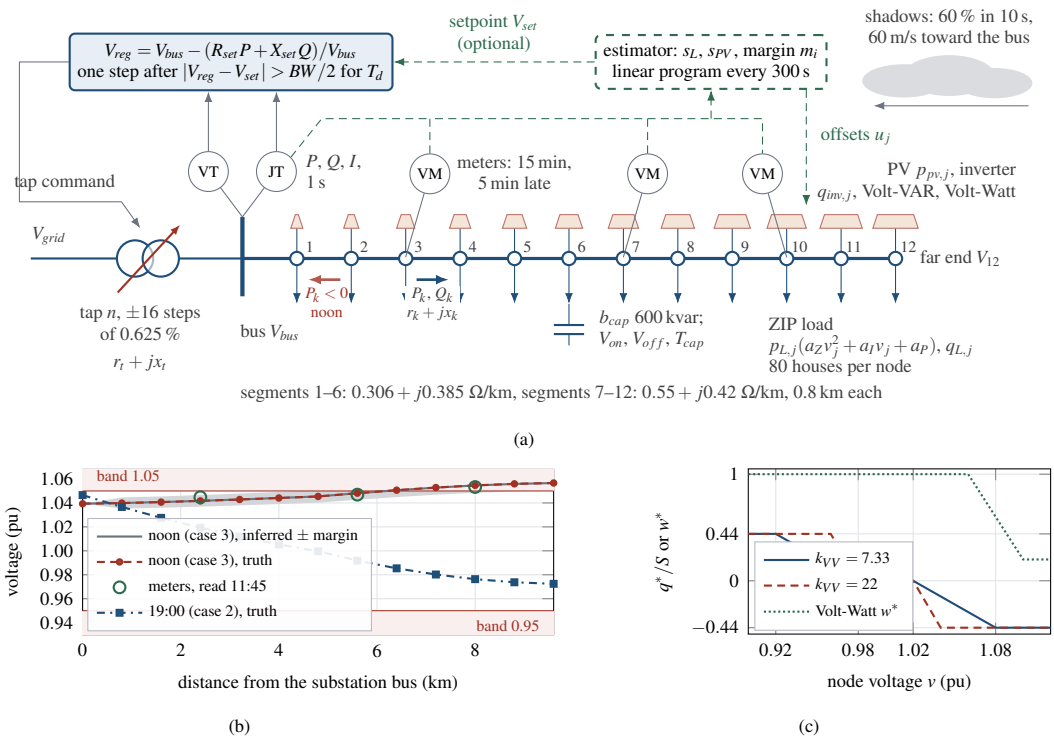


Figure 8.1 The feeder as modeled. (a) One-line diagram, not to scale: the tap n sets the source behind the transformer impedance $r_t + jx_t$, the voltage transformer (VT) and power transducer (JT) give the tap changer's control V_{bus}, P, Q and I, and the control steps the tap once V_{reg} has stayed outside $V_{set} \pm BW/2$ for T_d. Segment k carries P_k and Q_k through $r_k + jx_k$, each node j has a ZIP load and rooftop solar whose panels grow toward the far end, the capacitor b_{cap} sits at node 6 and the smart meters (VM) at nodes 3, 7 and 10. Dashed green: the optional estimator and linear program. (b) Voltage along the feeder at 12:00 in case 3 (reverse flow: the inferred profile, solid, inside its gray margin, the truth dashed with dots and the meters' last readings as circles) and at 19:00 in case 2 (the evening peak, no solar, dash-dotted with squares), against the 0.95 pu to 1.05 pu teaching band. (c) The Volt-VAR curve at the category B default slope (solid) and at the steepest the standard allows (dashed), and the Volt-Watt limit (dotted).

branch model [11] with the loss terms dropped from the voltage equation, the form usually called LinDistFlow. The losses still enter the power balance, so the substation supplies them, as shown in Equation (8.2).

$$P_h = P_1 + \sum_k r_k \frac{P_k^2 + Q_k^2}{V_{k-1}^2}, \quad Q_h = Q_1 + \sum_k x_k \frac{P_k^2 + Q_k^2}{V_{k-1}^2}$$
$$P = P_h + r_t \frac{P_h^2 + Q_h^2}{V_0^2}, \quad Q = Q_h + x_t \frac{P_h^2 + Q_h^2}{V_0^2} \tag{8.2}$$

where P and Q enter the transformer from the grid. Segments 1 to 6 are $0.306 + j0.385\,\Omega$/km and segments 7 to 12 are $0.55 + j0.42\,\Omega$/km, a heavier trunk and a lighter outer conductor, and the transformer is $r_t = 0.006$ pu, $x_t = 0.08$ pu. Against the full nonlinear DistFlow, solved by a backward and forward sweep that an independent complex phasor flow reproduces to 10^{-14}, the linearized far end reads high by 0.11 % of nominal at clear noon with 75 % solar, by 0.46 % with 125 % and by 0.16 % at the evening peak, because the linear form leaves out the drop that the losses cause.

8.3.2 Loads, Sun and Clouds

Each node's injection depends on its own voltage through a ZIP load and on the capacitor, as shown in Equation (8.3).

$$\begin{aligned} p_j &= p_{L,j}\, z(v_j) - p_{pv,j} \\ q_j &= q_{L,j}\, z(v_j) - q_j^{inv} - b_{cap,j} v_j^2 \\ z(v) &= a_Z v^2 + a_I v + a_P \\ p_{L,j} &= \tfrac{1}{12} P_{pk}\, \lambda(t)\,\big(1+\omega_j(t)\big), \quad q_{L,j} = p_{L,j}\tan\varphi \end{aligned} \tag{8.3}$$

where $(a_Z, a_I, a_P) = (0.3, 0.2, 0.5)$ are the constant-impedance, constant-current and constant-power shares, $P_{pk} = 4.416\,\text{MW}$ is the evening peak of 960 houses at 4.6 kW each, $\lambda(t)$ is a declared hot-day shape of 24 hourly values with its peak of 1.00 at 19:00 and 0.40 at midday, cosine-smoothed between hours, ω_j is a slow random wander with a 3 % standard deviation and a 15 min correlation time, and $\tan\varphi = 0.329$ for a power factor of 0.95. Because the injections depend on the voltages, the flow is iterated three fixed times from a flat start. Three passes agree with thirty to 3.9×10^{-6} pu at the three check points, and the fixed count keeps the work per call constant, so the derivative function stays pure and runs replay bit for bit.

The solar a node's inverters can deliver is its rating times a clear-sky curve times the share of sunlight that reaches it, as shown in Equation (8.4).

$$p_{av,j} = S_j\, c_s(t)\, \kappa_j(t), \qquad c_s = 0.92 \sin^{1.3}\!\left(\pi \frac{t_d - t_r}{t_s - t_r}\right) \tag{8.4}$$

where the ratings S_j add to $\pi_{pv} P_{pk}$ with a penetration $\pi_{pv} = 75\,\%$ (3.31 MW, about 6.9 kW on each solar house) and are shared in proportion to $0.1 + 0.8(j-1)/11$, t_d is the time of day, sunrise t_r is 06:00, sunset t_s is 20:00 and solar noon 13:00. The sunlight share κ_j is 1 under a clear sky. A cloud field of strength c draws, from the case's seed, shadows between 08:00 and 18:00 that last 1 to 10 min and remove 55 to 80 % of the sunlight at $c = 1$ (up to 90 % above it). Each edge takes 10 s to cross a roof, so a 60 % shadow cuts a roof's output by 60 % in 10 s, and the shadows sweep from the far end toward the bus at 60 m/s, reaching node j $(12 - j)\,0.8\,\text{km}/(60\,\text{m/s})$ after node 12.

8.3.3 Inverters

Each node's inverters are one aggregate that follows its reactive power target through a first-order lag, and a second lag carries its Volt-Watt limit, as shown in Equation (8.5).

$$\begin{aligned} \tau \frac{\mathrm{d}q_j^{inv}}{\mathrm{d}t} &= \operatorname{clip}\!\big(S_j \phi(v_j) + u_j,\ \pm \bar{q}_j\big) - q_j^{inv} \\ \phi(v) &= -\operatorname{sgn}(v-1)\min\big(0.44,\ k_{VV}\,(|v-1| - \delta)^+\big) \\ \tau_w \frac{\mathrm{d}w_j}{\mathrm{d}t} &= w^*(v_j) - w_j, \qquad p_{pv,j} = \min(p_{av,j},\ w_j S_j) \end{aligned} \tag{8.5}$$

where $\tau = T_{ol}/\ln 10$ makes the open-loop response time $T_{ol} = 5\,\text{s}$ the time to 90 %, $\delta = 0.02\,\text{pu}$ is the deadband, $k_{VV} = 0.44/0.06 = 7.33$ puts the 44 % limit at 0.92 pu and 1.08 pu, u_j is the central program's offset (zero without it) and $\bar{q}_j = 0.44 S_j \min(1, p_{av,j}/0.05 S_j)$ takes the capability away at night. These are the category B defaults of IEEE 1547-2018, read through a paper that states the standard's table and its ranges, which allow the saturation as close as 0.02 pu beyond the deadband, a slope of 22 [16], as plotted in Figure 8.1(c). The Volt-Watt limit w^* falls linearly from 1 at 1.06 pu to 0.2 at 1.10 pu with $\tau_w = 10\,\text{s}/\ln 10$, a teaching value after the category B form that no source available here confirmed. In the convention of Section 1.3 the curve acts on the reactive power it injects as a reverse-acting proportional controller with a deadband: a node voltage above the deadband makes the inverter absorb.

Table 8.2 Model and Controller Parameters. A: Adjustable, F: Fixed, H: Hidden

Symbol	Meaning	Value	Unit	
S_b, V_b	base power, voltage	10, 12.47	MVA, kV	F
$r_k + jx_k$	segments 1–6, 7–12 (0.8 km)	see text	Ω/km	F
x_t, r_t	transformer	0.08, 0.006	pu	A, F
p_{pk}, pf	peak per house, power factor	4.6, 0.95	kW, —	A
a_Z, a_I	ZIP shares (a_P the rest)	0.3, 0.2	—	A
π_{pv}	solar rating / evening peak	75	%	A
c, v_c	cloud field, shadow speed	0, 60	—, m/s	A
n	tap, step	±16, 0.625	–, %	F
V_{set}, BW, T_d	setpoint, band, delay	1.025, 1.67, 45	pu, %, s	A
R_{set}, X_{set}	compensator (Forward)	0.01, 0.15	pu	A
δ, k_{VV}	deadband, slope (2.75–22)	0.02, 7.33	pu, pu/pu	A
T_{ol}	inverter response time (3–90)	5	s	A
w^*	Volt-Watt 1.06 to 1.10 pu	1 to 0.2	—	A
V_{on}, V_{off}, T_{cap}	capacitor rule (when on)	0.98, 1.035, 60	pu, s	A
—	meter period, delay	900, 300	s	F
m	program's added margin	0.005	pu	A
w_{tap}, w_q, w_s	program weights	3, 1, 300	—	F
T_s, N	MPC period, horizon	300, 24	s, steps	F
L_b	MPC blocks	1,1,1,1,2,2,4,6,6	steps	F
—	MPC node, band-row caps	100, 72	—	F
—	MPC preview	On	—	A
t_0, n_0	start time, starting tap	case	s, —	H

8.3.4 Tap Changer, Capacitor and Ledgers

The tap changer and the capacitor are discrete, and their steps are applied once per step after integration, with a historian mark. One tap step is 0.625 % of the source voltage, ±16 steps give ±10 %, and the capacitor adds $b_{cap} = 0.06$ pu of susceptance at node 6 with a 5 min discharge lockout after it opens. Five ledgers integrate the substation's energy, the losses, the loads' energy, the available solar and the delivered solar, so the balance $E_{sub} = E_{load} - E_{pv} + E_{loss}$ closes by construction. The 29 states (twelve reactive powers, twelve Volt-Watt limits and five ledgers) advance by fourth-order Runge-Kutta steps of 1 s with the power flow inside each stage and the weather and loads held at the step's midpoint, the local controllers run every second, the central program every 300 s, and a row of 93 columns is logged every 5 s, on the engine of Section 1.6.1.

8.3.5 Measurements and Initialization

Controllers see only a measurement packet. The substation publishes V_{bus} with 0.05 % noise and P and Q with 0.5 %, each a function of the seed and the whole second, so that halving the step hands the controllers the same readings. The capacitor has its own voltage sensor. The three meters record their node voltages every 15 min with 0.1 % noise, report 5 min later through the framework's analyzer, and hold the last reading through an outage, marked stale. A late sample of this kind limits the composition loop of Volume 5, Chapter 3 as well. Every other node voltage, the clouds, the loads and the inverters' outputs are teaching truth, logged under a `truth_` prefix, and a source test rejects any read of them by the controller, estimator or program. A run starts at the tap that puts the regulated voltage nearest its setpoint with the inverters at unity power factor, unless the case pins one, and with the inverters relaxed onto their curves. Table 8.2 lists the parameters.

8.3.6 Assumptions

The feeder is a balanced single-phase equivalent, so unbalance, single-phase laterals, three-phase line regulators and the service transformers are out of scope, and a node voltage stands for its houses' service voltage. LinDistFlow leaves out the losses' effect on the voltages, an error of 0.11 to 0.46 % of nominal at the check points, which makes it a teaching approximation rather than a planning tool. The loads follow one declared day, the ZIP shares are fixed, and thermostatic recovery after a voltage

change is absent. The inverters are one aggregate per node with no ride-through or trip settings, and the tap changer has no mechanism time or intertap delay. The regulator, estimator and program read P and Q entering the transformer, which include its own reactive losses, about 0.003 pu of correction at the evening peak with $X_{set} = 0.15$ pu. The band 0.95 pu to 1.05 pu is the ANSI C84.1 Range A service band used as a teaching value, the Volt-VAR defaults are IEEE 1547-2018 category B as teaching values, and nothing is a utility's or a manufacturer's setting or data.

8.4 Four Mechanisms Behind the Cases

8.4.1 A Tap Moves Everyone

A tap step scales the source, and to first order every node moves with the bus, because the drop along each segment depends on the flows, which one step barely changes. In case 1 at 11:30 one step down takes 0.0062 pu off the bus and 0.0060 pu off the far end. The profile's shape, a rise of 0.017 pu from the bus to node 12 at noon in case 3 and a fall of 0.074 pu at the evening peak in case 2, belongs to the flows, as shown in Figure 8.1(b). One setting of the tap therefore has to keep the far end under 1.05 pu at noon and over 0.95 pu at 19:00, and only a device at the nodes can change the spread.

8.4.2 The Compensator's Sign

The regulator controls the compensated voltage of Equation (8.6) rather than the bus.

$$V_{reg} = V_{bus} - c, \qquad c = \frac{R_{set}P + X_{set}Q}{V_{bus}} \tag{8.6}$$

The correction c is the in-phase part of the phasor drop $(R_{set} + jX_{set})I$ that a control computes from its current transformer. With $P < 0$ and $Q > 0$ it is positive, and holds the bus up, exactly when $X_{set}/R_{set} > -P/Q$. At 12:00 in case 3, $P = -1.081$ MW and $Q = 0.645$ Mvar put the threshold at 1.68, so the shipped X/R of 15 raises the bus, by 0.008 27 pu in the application's form against 0.008 14 pu from the phasor magnitude $|V - (R_{set} + jX_{set})I|$, while a compensator matched to the line, $R = X = 0.06$ pu, lowers it by 0.0025 pu. A design note for the simulator expected the opposite failure, a compensator that lowers the bus when it should raise it. On one feeder with solar spread along its length the far end is the high point, so that error needs generation concentrated beyond the loads, and the application teaches the sign rule instead. On forward flow at a steady power factor the correction reads only $R_{set} + X_{set}\tan\varphi$, so evening data cannot tell the shipped setting from a line-matched one. Case 2 makes 7 operations with $R = 0.01$, $X = 0.15$ and 5 with $R = X = 0.045$, whose largest corrections are 0.031 pu and 0.028 pu. The shipped dials equal 11.7, 15.6 or 23.3 V of reactance on a 120 V base for a 300, 400 or 600 A current transformer, inside a regulator's range, but the textbook procedure sets both dials in proportion to the impedance to the regulated point, which gives $R \approx X$ on this line. The shipped setting is a teaching choice.

🕮 **Pause.** Try Exercise 8.1 before reading on.

8.4.3 Steep Curves Against the Compensator

The inverters alone are stable at any slope. Linearizing Equation (8.5) with the tap fixed, the voltages respond to the reactive powers through the shared path reactance X_{ij} of the LinDistFlow model, $\partial v/\partial q = X$, and the Jacobian of the lags is $-(I + KX)/\tau$ with K the diagonal of the slopes times the ratings. KX is similar to $K^{1/2}XK^{1/2}$, which is positive semidefinite, so every eigenvalue lies at or left of $-1/\tau$. The stability condition on the norm of the sampled update that the Volt-VAR literature derives [16] applies to an inverter that jumps to its curve once per sample, which the application does not model.

The hunting comes from the coupling to the tap changer. The inverters' reactive power returns through the substation, where the compensator reads X_{set} times its change as a change of load. A common change Δv of the node voltages inside the curves' slope moves the correction by about $X_{set}k_{VV}\sum_j S_j\,\Delta v$, a factor of 0.36 at the default slope and 1.09 at 22 for $X_{set} = 0.15$ pu at 75 % solar.

On case 5's cloudy day the range of the correction over each 10 min has a median of 0.0031 pu and a 90th percentile of 0.0050 pu at slope 7.33, against 0.0082 pu and 0.0142 pu at 22, close to the 0.0167 pu band. At the steep slope the inverters swing their whole range with the cloud edges, with a median 10 min range of 0.664 Mvar against 0.302 Mvar and a lowest total of −1.41 Mvar against −0.50 Mvar, and all 8 steps down come with a shadow over the far end and all 10 steps up in sun, the wrong way each time.

8.4.4 Conservation Voltage Reduction

A ZIP load's power changes with its voltage at the rate $\partial \ln p / \partial \ln v = 2a_Z + a_I$, 0.8 for the declared shares, so lowering the voltage by 1 % lowers a constant-impedance and constant-current load by about 0.8 %. A constant-power load draws more current instead, and the losses rise. The feeder's measured factor is the energy saved over the load-weighted mean voltage reduction.

8.5 The Controllers as Implemented

No PID loop runs in this application, so the velocity form of Section 1.6.2 does not appear. Every controller is written explicitly in its own module and executes on the measurement packet every second, except the estimator, the program and the optional predictive controller, which execute every 300 s.

8.5.1 Tap Changer

At each second k the tap changer computes $V_{reg,k}$ from Equation (8.6) and the error $e_k = V_{reg,k} - V_{set}$, and runs the timer law of Equation (8.7).

$$
\begin{aligned}
d_k &= \begin{cases} -\operatorname{sgn}(e_k) & |e_k| > BW/2 \\ 0 & \text{otherwise} \end{cases} \\
\theta_k &= \begin{cases} \theta_{k-1} + \Delta t & d_k = d_{k-1} \neq 0 \\ \Delta t & d_k \neq d_{k-1},\ d_k \neq 0 \\ 0 & d_k = 0 \end{cases} \\
n_k &= \operatorname{clip}(n_{k-1} + d_k,\ \pm 16),\ \theta_k \leftarrow 0 \ \text{ if } \theta_k \geq T_d
\end{aligned}
\tag{8.7}
$$

where θ is the timer, which counts while V_{reg} stays outside the band on one side, restarts on a side change and clears inside the band, and the tap otherwise holds. In the convention of Section 1.3 the tap changer is reverse acting: a regulated voltage below the band steps the tap up. The compensator is Off, Forward (always on) or Reverse-aware, which drops the correction while the measured P is negative, the cogeneration-style mode of regulator controls. In Manual the tap is the operator's station, rounded, and the timer is held at zero. The application reports the tap the regulator asked for as its manual-follow value.

8.5.2 Capacitor

The voltage rule switches the bank in when its own sensor reads below V_{on} for T_{cap} and out above V_{off} for T_{cap}. A schedule keeps it in from 13:00 to 21:00. A closing command waits out the 5 min lockout, and an out-of-service event keeps it open.

8.5.3 Estimator

Every 300 s the estimator fits a load scale s_L and a solar scale s_{PV} on the declared shapes to the substation's 1 min averages now and to the meters' last readings, each compared with the model at the time the meter recorded it, as shown in Equation (8.8).

$$\min_{s_L,s_{PV}} \left(\frac{\hat{P}-\bar{P}}{\sigma_P}\right)^2 + \left(\frac{\hat{Q}-\bar{Q}}{\sigma_Q}\right)^2 + \sum_{m\in\{3,7,10\}} \left(\frac{\hat{V}_m(t_m)-V_m}{\sigma_m}\right)^2 \tag{8.8}$$

where $\sigma_P = \sigma_Q = 0.005\,\text{pu}$, $\sigma_m = 0.002 + 0.1|\bar{P}_{now} - \bar{P}(t_m)|$ discounts a meter read before a change at the substation, and four Gauss-Newton iterations with a finite-difference Jacobian solve it with s_L held in 0.3 to 2.5 and s_{PV} in 0 to 1.3. The model is the utility's: the declared impedances, shapes, ratings, curves and ZIP shares, with neither the clouds nor the loads' wander nor the inverters' outputs. The profile $\hat{V}$ is then drawn from the fit, anchored at the measured bus, and each node carries the margin of Equation (8.9).

$$m_i = 2\max(0.002,\ \rho) + 0.1|\hat{V}_i - V_{bus}| + \left|\frac{\partial \hat{V}_w}{\partial s_{PV}}\right| \frac{\Delta \bar{P}_{10}}{\sum_j S_j c_s} + 0.01\,\chi_{st} \tag{8.9}$$

where ρ is the root mean square of the meter residuals, $\Delta\bar{P}_{10}$ is the range of the substation's 1 min power over the last 10 min, the clouds the fit cannot place, and $\hat{V}_w$ is the inferred worst node, the one nearest or furthest past its band edge, and χ_{st} is 1 while the meters are stale and 0 otherwise.

8.5.4 Central Program

With the central controller on, every 300 s a linear program on the framework's bounded two-phase simplex chooses a bus move $d^+ - d^-$ and an offset $u_j^+ - u_j^-$ per node, as shown in Equation (8.10).

$$\begin{aligned} \min\ & \frac{w_{tap}}{0.00625}(d^+ + d^-) + \sum_j (w_q \mp \lambda_j) u_j^{\pm} \\ & + w_s \sum_i (s_i^+ + s_i^-) \\ \text{s.t. } & V_i = \hat{V}_i^0 + d^+ - d^- + \textstyle\sum_j X_{ij}(u_j^+ - u_j^-) \\ & 0.95 + m_i' - s_i^- \le V_i \le 1.05 - m_i' + s_i^+ \\ & 0 \le d^{\pm} \le 0.06, \quad 0 \le u_j^{\pm} \le \bar{q}_j \end{aligned} \tag{8.10}$$

where $\hat{V}_i^0 = \hat{V}_i - \sum_j X_{ij}\bar{u}_j$ is the inferred profile without the offsets $\bar{u}_j$ in force, which the new ones replace, $m_i' = m_i + m + \Delta n/2$, $\lambda_j = 2\sum_{k\le j} r_k \hat{Q}_k$ is the loss gradient, so offsets that inject where reactive power flows forward pay less, Δn is one tap step, the capabilities $\bar{q}_j$ are taken at the lower of the clear-sky output now and in 5 min and are zero while the curves are Off, when the inverters run at unity power factor and apply no offset, and conservation voltage reduction adds a cost of 600 per pu on the bus voltage. The bus move is rounded to whole steps and sent as a target tap, which the tap changer reaches one step per T_d, together with the setpoint that target implies. When the band slack the program needs exceeds 0.02 pu summed over the nodes, a sign that the margins fill the band, it holds its last target and sends no offsets. Without the central controller, conservation voltage reduction sets the setpoint every 300 s to put the inferred lowest node at 0.95 pu plus its margin, the added margin and half the band.

8.5.5 Mixed-Integer Predictive Controller

As a third setting of the central controller, a model predictive controller (MPC) takes the program's call. It decides on the same 300 s clock from the same estimate and drives the same tap target, setpoint and offsets, but it plans the tap as an integer over the next 2 h instead of rounding one bus move afterwards, and it sees the declared load shape and clear-sky curve coming. It is written for this application rather than taken from the framework's option of Section 1.6.3, because its main

decision is an integer tap. Receding-horizon control under constraints has a settled theory [20], a plant that mixes logical decisions with continuous dynamics turns the predictive problem into a mixed-integer program [21], and centralized predictive schemes that correct a multi-step plan with real-time measurements have been proposed for the voltages of distribution networks that host many generators [22].

The controller's model is the estimator's fitted feeder, so nothing is identified. At a decision at time t with tap n_0, the source behind the transformer is estimated as $\hat{v}_{src} = V_{bus} + r_t P + x_t Q$ and corrected twice so that the model's bus equals the measured bus with the offsets in force, and $\hat{v}_{grid} = \hat{v}_{src}/(1+0.00625 n_0)$. The base profile is evaluated at the 24 five-minute steps of the horizon with no offsets and linearized in the tap and the offsets, as shown in Equation (8.11).

$$\begin{aligned} B_{i,k} &= \mathscr{V}_i\big(s_L, s_{PV}, t+300k, \hat{v}_{src}, 0\big), \quad k = 0 \ldots 23 \\ V_{i,k} &\approx B_{i,k} + S_i(n_k - n_0) + \textstyle\sum_j G_{ij} u_{j,k} \end{aligned} \tag{8.11}$$

where $\mathscr{V}_i(s_L, s_{PV}, \tau, v, u)$ is the estimator's LinDistFlow profile at node i for the declared shapes at time of day τ, source voltage v and offsets u, with the inverters on their declared curves through four fixed-point passes, S_i is the change at node i for one tap step and $G_{ij} = \partial V_i/\partial u_j$, both taken once per decision at $\tau = t$ by finite differences (an offset of 0.002 pu) through the same profile. With the preview Off the shapes are held at their values now, $B_{i,k} = B_{i,0}$. The preview is only the declared day, so its error is every cloud and the loads' wander, which only the next fit sees. With no load and no sun the computed S_i equals $0.00625 \hat{v}_{grid}$ to a relative 2.3×10^{-14}. At noon in case 3 with the curves Off, $G_{ij} - X_{ij}$ lies between 0.074 and 0.080 against $x_t = 0.08$ pu, because this source sits behind the transformer while the program's X holds the bus. A tap step moves the far end 0.0039 pu with the curves On against 0.0061 pu with them Off, since the curves answer inside the profile. The model leaves out the clouds, the wander, the curves' saturation in G, the band acting between decisions, the capacitor's rule inside the horizon and the change of S and G over 2 h. It is offset-free in the output-bias sense [23], re-anchored on the measured bus at every decision while the estimator refits its two scales at the same call, and no Kalman filter is used.

The horizon is held in nine blocks of L_b = 1, 1, 1, 1, 2, 2, 4, 6 and 6 steps. In block b the tap moves by m_b at the block's start, an integer in the first four blocks (the first 20 min) with $|m_0| \le 1$, and a continuous ramp of at most one step per 5 min, $|m_b| \le L_b$, afterwards. The offsets $u_{j,g} = u^+_{j,g} - u^-_{j,g}$ are held in three groups $g(b)$: the first block, the next three and the rest. The program is Equation (8.12).

$$\begin{aligned} \min \sum_b \Big[& w_{tap} a_b + L_b \sum_j (w_q \mp \lambda_j) u^{\pm}_{j,g(b)} \\ & + L_b w_s \sum_i (s^+_{i,b} + s^-_{i,b}) \Big] \\ \text{s.t. } & a_b \ge |m_b|, \quad N_b = \textstyle\sum_{c \le b} m_c, \quad |n_0 + N_b| \le 16 \\ & \max_{k \in b} B_{i,k} + S_i N_b + \textstyle\sum_j G_{ij} u_{j,g(b)} - s^+_{i,b} \le 1.05 - m''_i \\ & \min_{k \in b} B_{i,k} + S_i N_b + \textstyle\sum_j G_{ij} u_{j,g(b)} + s^-_{i,b} \ge 0.95 + m''_i \\ & 0 \le u^{\pm}_{j,g} \le \bar{q}_{j,g}, \quad s^{\pm}_{i,b} \ge 0 \end{aligned} \tag{8.12}$$

where a_b counts the block's tap operations, $m''_i = m_i + m$ is the estimator's margin plus the added margin without the program's half step, since the tap is already whole, $\bar{q}_{j,g}$ is the capability of Equation (8.10) at the lowest clear-sky output of the group's steps, zero while the curves are Off, and the weights are the program's, $w_{tap} = 3$ per operation, $w_q = 1$ and $w_s = 300$ per pu and step, with conservation voltage reduction adding $600 \sum_b L_b S_0 N_b$. A block's decision is constant, so a node stays inside the band at every step of the block exactly when the block's highest and lowest base voltages do, and the band rows judge all 24 steps. One slack per node, side and block prices a violation per 5 min, as the program prices its own.

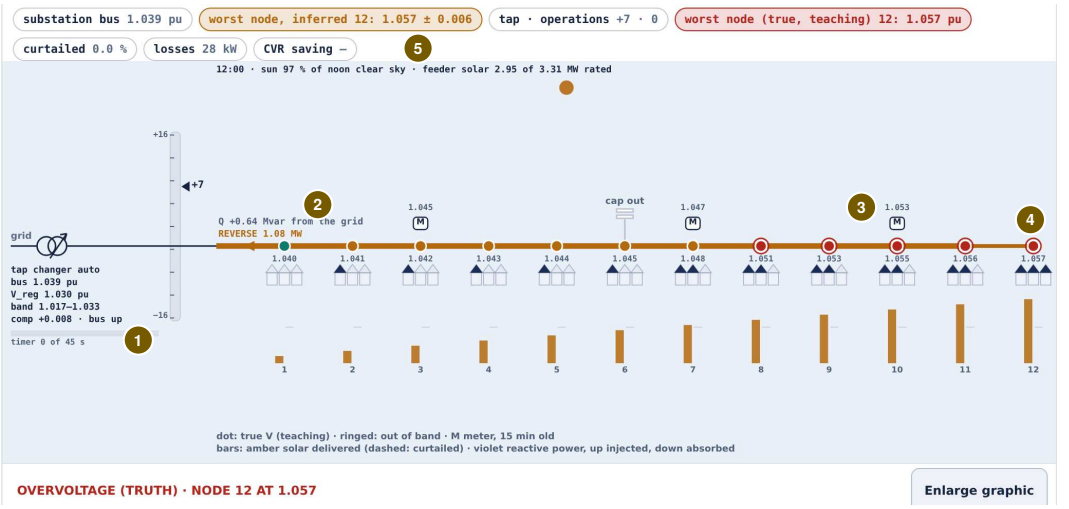


Figure 8.2 Reverse flow at solar noon on the Feeder view, case 3 at 12:00 as shipped, cropped to the plant view. The substation exports 1.08 MW while 0.64 Mvar still flows in (2), and the compensator ($R_{set} = 0.01$, $X_{set} = 0.15$ pu) holds the bus up by 0.008 pu, so the tap stays at +7 with $V_{reg} = 1.030$ pu inside its band of 1.017 to 1.033 pu and the bus at 1.039 pu (1). Nodes 8 to 12 are ringed over 1.05 pu, the far end at 1.057 pu (4), which the estimator infers as 1.057 ± 0.006 pu (5) from meter readings 15 min old (3, one of the three meters). Solar delivers 2.95 MW of the 3.31 MW rated, the losses are 28 kW, and the tap has made no operation since 09:00.

Depth-first branch and bound on the framework's bounded simplex solves the program from the incumbent with no move and no offset. The pumps of Volume 9, Chapter 11 are scheduled as integers by the same kind of search. Row generation starts from the three nodes nearest each limit per block and side and adds every row the plan crosses, for six rounds at most, and a final check verifies all 216 band rows. The first program has 144 variables, 81 rows and 4 integers. With all nine moves integer it cost 1.1 s a decision on average, which set the integer window at 20 min. The controller sends the tap target $n_0 + m_0$, the bus voltage the plan implies as the setpoint and the first offsets, and the band and timer still act between decisions.

The MPC first runs the program's hold test on the same estimate and, when the margins fill the band, keeps the last target and sends no offsets, as the program does. A plan is accepted only if it is optimal with a zero gap, finite, inside every box, integral, consistent and within every band row, including those left out. A plan that needs more than 100 branch-and-bound nodes or 72 band rows, or fails a check, hands the period to the program, marked in the historian, until the next valid plan. Replayed on the measurement packets of a run over a different hidden plant, the program and the MPC made 0 different calls in 46,800. In Chrome, under another simulation's load, a solve took 1.7 to 4.8 ms on average over six cases and at most 208 ms, at most 0.0015 % of a core at 1× and 5.2 % at 3600×, where a day plays in 24.1 s with 2 of 1,369 frames over 50 ms. A decision falls once per 300 simulated seconds, so no frame guard is used.

8.5.6 Three Models

The plant is Section 8.3, with the true ZIP shares, clouds and wander. The estimator's model is the utility's declared feeder without them. The program's model is linear, the bus move and the shared reactance X_{ij} with the bus held, and it omits the inverters' own curves, whose response the next call corrects. The MPC's model is the estimator's declared feeder again, evaluated over 2 h and linearized once per decision with the curves answering inside it. None of the local devices carries a model.

Table 8.3 The Cases as Shipped

#	Setting and scripted events	Teaching point
1	Manual from 10:00, tap +6, clear	a tap moves every node
2	12:00–24:00, no solar	band, timer, operations
3	09:00–15:00, clear, Forward 0.01 + *j*0.15	reverse flow, the compensator
4	12:00, cloud bank at 12:15 for 25 min	the cloud the tap misses
5	09:00–17:00, curves on, cloud field 1.5	the slope that hunts
6	16:00–22:00, band 1.25 %, capacitor rule	two timers
7	16:00–22:00, no solar	conservation voltage reduction
8	10:00, curves on, meter outage at 12:00	the unmeasured customer
9	24 h, 150 % solar, field 1.5, suggested local settings, MPC	the MPC against the program
C	Challenge: 24 h, 125 % solar, field 1.5	coordination

Table 8.4 Measured Results. Voltages in Per Unit

#	Setting	Quantity	Value
1	tap +6 held, +5	far-end peak	1.0516, 1.0455
2	band 1.67, 2.5, 1.25 %	operations	7, 5, 8
	band 1.67, 2.5 %	lowest node	0.9724, 0.9698
3	shipped, at 12:00	correction, far end	0.0083, 1.0567
	shipped, Off at 11:00, +center	highest	1.0575, 1.0514, 1.0453
4	curves off, on	far-end fall in 30 s	0.0144, 0.0105
5	slope 7.33, 22	operations to 17:00	2, 18
	slope 22, seeds 1–3	operations	10, 13, 18
6	shipped, tap delay 20 s	tap, capacitor ops	14 + 12, 5 + 2
7	baseline, CVR, CVR + central	energy (kWh)	24 053, 23 863, 23 720
	constant power, CVR	energy change	+16.9 kWh
8	before, after outage	fits bracketing	23/24, 24/24
9	program, MPC	min out (operations)	21.2 (17), 0 (15)

8.6 Nine Cases and a Challenge

The cases appear in Table 8.3 and the headline measurements in Table 8.4. The numbers in this section were taken headlessly from the same engine code the page loads, seeded with 20261026 and started from the case's own settings (Section 1.6.4), and a change a sentence names is made at the time it states. Clock times are rounded to the nearest minute.

8.6.1 The Tap by Hand

Case 1 starts at 10:00 in Manual with the tap at +6. The far end passes 1.05 pu at 11:55 and, held at +6, peaks at 1.0516 pu at 12:20 after 77 min over the band, while at +5 it peaks at 1.0455 pu. Lowering the tap one step at 11:30 moves the bus from 1.0331 pu to 1.0269 pu and the far end from 1.0486 pu to 1.0426 pu.

8.6.2 Band and Timer

Case 2 runs the afternoon and evening without solar. As shipped, with a 1.67 % band (2 V on 120 V) and a 45 s delay, the tap changer makes 7 operations from 12:00 to 24:00 with the lowest node at 0.9724 pu. Widening the band to 2.5 % cuts the count to 5 and lets the lowest node fall to 0.9698 pu, a 1.25 % band makes 8, and a 120 s delay makes 5. A band must exceed one step, or a step taken at one edge would cross the other. At 1.0 %, 1.6 steps, the regulator makes 8 operations without hunting. At 19:00 the tap stands at +10, the bus at 1.0465 pu and the far end at 0.9724 pu, as shown in Figure 8.1(b).

8.6.3 Reverse Flow and the Compensator

Case 3 runs a clear day from 09:00. Power flows back from 09:34, the far end passes 1.05 pu at 10:44, and it peaks at 1.0575 pu at 12:20 with 203 min over the band by 15:00. The state at 12:00 appears in Figure 8.2. Switching the compensator Off at 11:00 brings one step down to +6 by 11:15, after

which the far end peaks at 1.0514 pu because the bus is then inside its band. Pressing the application's Center the taps helper after switching it Off takes the tap to +5, and the far end peaks at 1.0453 pu. A Reverse-aware compensator would also stop the correction, but it switches with every change of sign and is no cure on a cloudy day, as case 5 measures.

> **TRY IT IN THE APP** **3 · Solar Noon: Reverse Flow and the Compensator**
>
> Load scenario 3 and press Run. Watch the far end of the feeder on the Feeder view. Power flows back to the substation from 09:34, and the far end passes 1.05 pu at 10:44. The tap changer's regulated voltage stays inside its band, and by 12:00 no tap has moved while nodes 8 to 12 are ringed over the band.

8.6.4 The Cloud

Case 4 adds a cloud bank at 12:15 that takes 60 % of the sunlight in 10 s, sweeps the feeder in 160 s and stays 25 min. Node 12's rooftops fall from 449 to 179 kW in 10 s. With the inverters at unity power factor, the far end falls from 1.0447 pu to 1.0303 pu in 30 s and to 1.0137 pu by 12:40, while the bus moves only from 1.0268 pu to 1.0258 pu. The regulated voltage stays between 1.0145 pu and 1.0284 pu against a band of 1.0166 to 1.0333 pu, never outside for 45 s, and the tap does not move. With the curves on, the far-end inverters had been absorbing 0.294 Mvar at the high noon voltage and fall to 0.154 Mvar within 30 s, so the far end starts lower, at 1.0369 pu, and falls 0.0105 pu instead of 0.0144 pu in the first 30 s.

Pause. Try Exercise 8.2 before reading on.

8.6.5 Hunting

Case 5 runs a heavily clouded day from 09:00 with the curves on and the evening compensator. At slope 7.33 the tap changer makes 2 operations by 17:00. At 22, set before Run, it makes 18, the score marks HUNTING (two reversals within 30 min) first at 11:54, and the tap steps between +9 and +10 with the clouds, as shown in Figure 8.3. On seeds 1 to 3 the counts are 10, 13 and 18 against 2 each, and halving the time step leaves 18 and 2 unchanged.

The hunting needs three settings at once, and the application states the window. Each of the following changes, made alone at slope 22, removes it: a cloud field of 1.0 instead of 1.5 (3 operations, while 1.2 gives 8), a 2.5 % band (1), a setpoint of 1.0125 pu (1), $R_{set} = 0.02$ pu (3), $X_{set} = 0.12$ pu (2), the compensator Off (0) and slopes of 18 or 16 (2 each), while slope 20 still hunts with 10. Settings in proportion to the line remove it too, $R = X = 0.06$ pu with 4 operations and the far end's uniform-load equivalent $0.124 + j0.132$ pu with 9 and no HUNTING mark, while a reactance-only $X_{set} = 0.15$ pu hunts harder, with 29. The steep slope, a compensator weighted on reactance (X/R 15 on a line whose X/R is 1.06) and a heavy cloud field are teaching choices that put the mechanism on the screen, not a claim that every steep curve hunts. The Reverse-aware mode makes 8 operations at 7.33, with HUNTING at 14:59, and 4 at 22, because it turns its compensation on and off with each shadow that reverses the flow.

> **TRY IT IN THE APP** **5 · The Slope: A Steep Curve That Hunts**
>
> Load scenario 5, set the Volt-VAR slope to 22 before pressing Run, and run to 17:00. The tap changer makes 18 operations instead of the 2 it makes at the default slope of 7.33. The status marks HUNTING first at 11:54, and the tap steps between +9 and +10 with the clouds.

8.6.6 Two Timers

Case 6 runs the evening with a 1.25 % band and the capacitor on a voltage rule of 1.008 pu and 1.016 pu, both with 45 s delays. The capacitor's own step, 1.16 % at node 6 and 0.48 % at the bus at 19:00, is wider than its 0.8 % rule, and with equal delays both timers start on the same change and run out together. The evening takes 14 tap operations and 12 capacitor switchings, against 4 tap

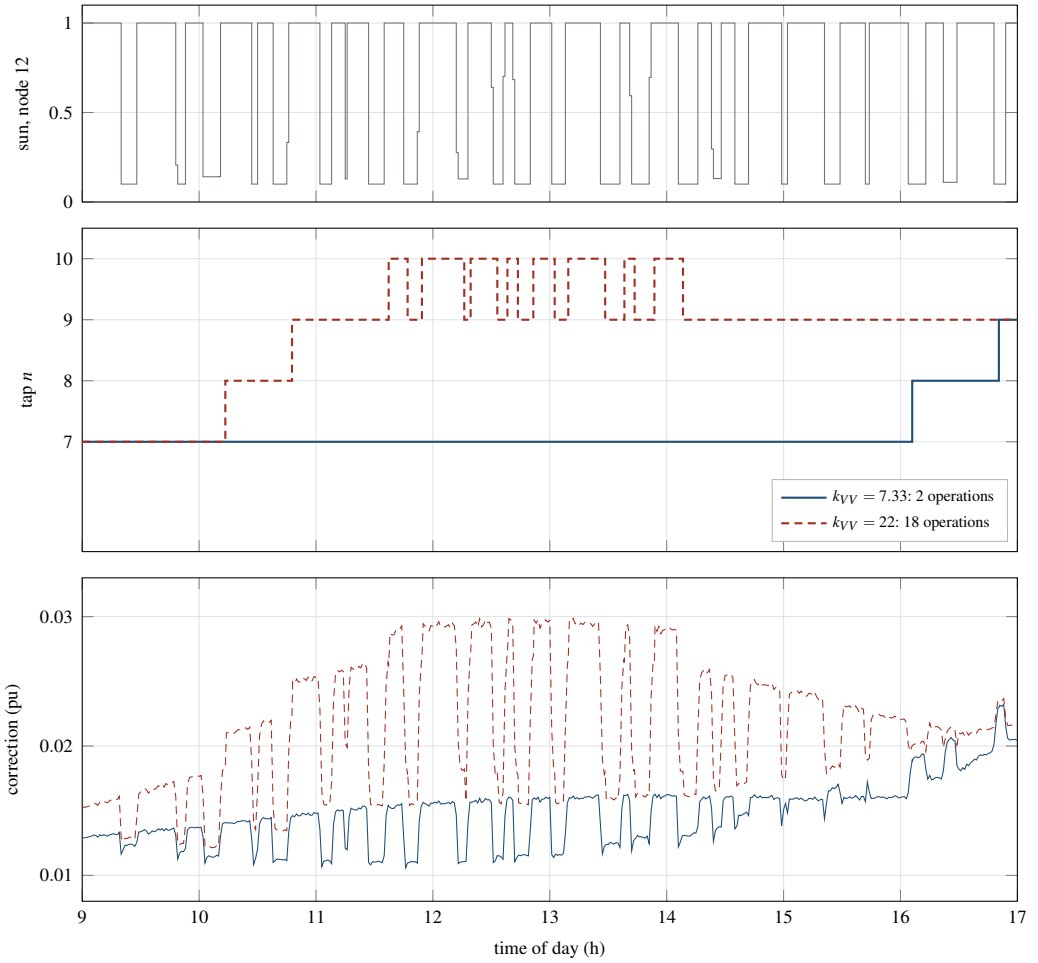


Figure 8.3 Case 5 from Reset on one seed, 09:00 to 17:00: the sunlight share at node 12 (teaching truth, the same at both slopes), the tap position and the compensator's correction c at the default Volt-VAR slope (solid) and at 22 (dashed). At 22 the correction swings close to the band's width with each shadow and the tap hunts.

operations with the capacitor out. A 20 s tap delay gives 5 and 2, a 120 s capacitor delay 5 and 3, a wider rule of 0.99 and 1.03 pu 4 and 0, and a schedule 6 and 2.

8.6.7 Conservation Voltage Reduction

Case 7 runs 16:00 to 22:00 without solar. The baseline delivers 24 053 kWh with the lowest node at 0.9731 pu. With conservation voltage reduction, which holds the inferred lowest node at the floor plus its margins, the feeder delivers 23 863 kWh, 190 kWh or 0.79 % less, with the lowest node at 0.9633 pu and 8 operations, and with the central program as well 23 720 kWh, 1.39 % less, at 0.9560 pu and 12 operations. The loads' energy falls by 192.9 kWh while the losses rise from 812.6 to 816.0 kWh. The load-weighted mean voltage falls 1.04 %, so the feeder's measured factor is 0.76, a little under the loads' 0.8, and the saving lies inside the national evaluation's 0.5 to 4 % [10]. With constant-power loads the same reduction delivers 16.9 kWh more than its own baseline as the losses rise from 818 to 835 kWh.

Table 8.5 The Challenge on Six Seeds (Default and 1–5)

Policy	pass	ops	out (min)	nodes, default seed
shipped: tap alone, Forward 0.01 + *j*0.15, 1.025, 1.67 %, 45 s	0/6	5	264.9–319.1	0.9754–1.0902
suggested: curves 7.33, Rev.-aware 0.06 + *j*0.06, 1.00, 2.5 %, 60 s	6/6	6–8	0	0.9522–1.0442
suggested, Forward compensator	6/6	8–10	0	0.9522–1.0419
suggested without the curves	0/6	6–8	102.0–160.4	0.9521–1.0597
suggested with 1.67 %, 45 s	0/6	23–29	0	0.9581–1.0442
suggested + central program	6/6	7–13	0	0.9557–1.0442
tap alone, Forward 0.09 + *j*0.09, 0.995, 4 %, 60 s	6/6	11–15	0	0.9579–1.0485

+ DID YOU KNOW? Voltage Reduction at National Scale

A 2010 national evaluation simulated conservation voltage reduction on 24 prototypical feeders and extrapolated the result. Deployed on every distribution feeder in the United States, it would cut annual energy consumption by 3.04 %, and deployed on the 40 % of feeders with the highest value, by 2.4 % [10].

8.6.8 The Unmeasured Customer

Case 8 runs a cloudy morning with the curves on and the meters failing at 12:00. Before the outage the true worst voltage lies inside the inferred worst node's margin at 23 of 24 fits, with a median margin of 0.0176 pu, although at 3 of those fits the true worst is a different node. The miss, at 11:50, is an error of 0.0057 pu against a margin of 0.0043 pu under a shadow's edge. After the outage all 24 fits hold the truth, with a median margin of 0.0191 pu, because the substation's P and Q alone fix the two scales and a stale meter adds 0.01 pu. The margin is conservative. Fit by fit it is a median of 44 times the error, whose own median is 0.0003 pu, and on seeds 1 to 3 every one of 48 fits holds the truth. The worst customer is inferred from measurements elsewhere on the plant, as the brick core temperature of Volume 5, Chapter 11 is.

🕮 **Pause.** Try Exercise 8.3 before reading on.

8.6.9 The Challenge

The challenge runs a day from midnight at 125 % solar with a cloud field of 1.5, graded on the truth: every node in 0.95 pu to 1.05 pu at every second, at most 20 tap operations and at most 2 % of the available solar curtailed, with the feeder, weather and meters as shipped. Table 8.5 lists the policies on six seeds from the application's benchmark file. The shipped tap changer alone spends 265 to 319 min out of band. Turning the curves on with the shipped compensator makes 67 operations and still leaves 222.9 min out, case 5's mechanism at the default slope on a sunnier feeder. The suggested settings coordinate locally, with the curves at the default slope, the compensator matched to the line, the setpoint at 1.00 pu and a slower, wider band, and pass on all six seeds with 6 to 8 operations.

A tap changer alone can just pass this day. Of about 1,300 tap-alone settings swept, 17 pass, every one with the compensator weighted like the line (X/R 1 to 2) and a slow or wide band, and the steadiest passes on all six seeds with 11 to 15 operations by letting its bus wander across a 4 % band. The infeasible variant is the same day at 150 % solar, the slider's top, where none of 1,536 tap-alone settings of setpoint, band, delay and compensator holds the band within 20 operations (the closest is 4.7 min out with 39 operations), while the curves at slope 11 with the suggested tap settings pass on all six seeds with 4 to 6. The clear-to-shadow spread of the profile, 0.075 pu at 125 % and 0.089 pu at 150 %, stays under the 0.10 pu band until about 170 %, so the tap changer fails there on operations and timing rather than on a static bound.

The central program also passes, with 7 to 13 operations, its five-minute estimate a step behind the clouds. From 09:00 to 17:00 its margins have a median of 0.054 pu and exceed half the band at 52 of 96 fits, up to 0.124 pu, so it holds at 56 of its 96 calls. As shipped, at 13:00, the inferred far end carries ±0.159 pu. The margins are conservative, a median of 6.4 times the error fit by fit, but too

Table 8.6 Central Structures on the Same Seed, Instruments and Noise: Minutes Any Node Is Out of Band · Tap Operations

Case	Local only	Program	Program, 5 min ahead	Program, 2 h ahead	Program, no half step	MPC	MPC, preview Off
3, reverse flow, 09:00 + 24 h	202.7 · 6	0 · 8	0 · 8	0 · 14	0 · 7	0 · 9	0 · 8
4, cloud bank, 12:00 + 24 h	7.5 · 9	0 · 19	0 · 19	0 · 18	0 · 15	0 · 15	0 · 13
7, CVR on, 6 h	0 · 8	0 · 12	0 · 12	0 · 6	0 · 11	0 · 13	0 · 13
energy (MWh)	23.86	23.72	23.73	23.80	23.66	23.71	23.71
8, meters out at 12:00, 24 h	0 · 13	0 · 31	0 · 25	0 · 28	0 · 37	0 · 21	0 · 19
challenge as shipped (curves Off)	264.9 · 5	30.1 · 21	30.1 · 21	10.2 · 21	65.2 · 19	11.1 · 21	30.1 · 21
challenge, suggested	0 · 6	0 · 11	0 · 13	0 · 9	0 · 11	0 · 11	0 · 11
challenge, suggested, clear sky	0 · 6	0 · 5	0 · 5	0 · 7	0 · 5	0 · 7	0 · 7
9, 150 % solar, default seed	46.2 · 6	21.2 · 17	50.5 · 19	30.0 · 11	17.2 · 13	0 · 15	15.9 · 11
9, six seeds summed	443.7 · 36	117.0 · 78	92.0 · 78	353.1 · 66	186.9 · 63	42.9 · 84	65.6 · 68

Local only: the case's tap changer, capacitor and curves with the central controller Off. The three program variants are measurement scripts, not settings: the program judged against the worse of now and the declared shapes 5 min or 2 h ahead, and the program without its half-step margin, the MPC's own rule. Curtailment is 0.00 % in every run. Periods the program decided after a rejected plan, MPC: 0, 0, 0, 6, 1, 3, 0, 8 and 39 over the seeds.

wide on this day to steer by. No passing run curtails any solar, because Volt-Watt starts at 1.06 pu, above the band, so the 2 % limit binds only on runs that already fail (0.43 % with the curves on and the shipped compensator).

8.6.10 Predictive Control Against the Program

Two defects of the program were found and fixed while the MPC was added, and both change the comparison. First, the estimate $\hat{V}$ is fitted with the offsets in force, which the new offsets replace, yet the program had added the new ones to it and so counted the old ones twice. On the challenge with the suggested settings its summed offsets alternated 6 times in 23 acting calls from 10:00 to 14:00, for example -1.18, 0, -1.30 and 0 Mvar at 11:55 to 12:10. Starting from $\hat{V}^0$ in Equation (8.10) removed every alternation and moved the program on case 9 from 27.3 min out with 13 operations to 21.2 min with 17, and over case 9's six seeds from 155.5 to 117.0 min. The same fix took the peak per house and the power factor for the utility's model from their declared 4.6 kW and 0.95 instead of the run's values, so a heavier load is a model error that the load scale absorbs ($s_L = 1.43$ after 4 h with 6.0 kW per house and a power factor of 0.88, against 1.00 before). Second, with the curves Off the inverters run at unity power factor, yet the program, the MPC and the estimator modeled the offsets they sent. On the challenge as shipped the program asked for up to 2.43 Mvar over 10 500 s and the MPC for up to 1.58 Mvar over 6900 s, and the inverters delivered none. With no capability planned while the curves are Off, the challenge as shipped went from 143.7 to 30.1 min out for the program and from 159.7 to 11.1 min for the MPC, and cases 3 and 4 from 11.7 and 25.2 min to 0 for the program. No shipped case runs a central controller with the curves Off, and the program's numbers in Tables 8.4 and 8.5 are those after both fixes, the second moving none.

Table 8.6 sets the MPC beside the program and three stronger forms of it on the same seed, instruments and noise, the MPC at its shipped settings in every case. On case 9, the 150 % cloudy day with the suggested local settings, the MPC keeps every node between 0.9520 and 1.0498 pu all day with 15 operations, while the program's far end passes 1.05 pu at 10:56 and reaches 1.0529 pu at 11:26, 21.2 min out with 17 operations, as shown in Figure 8.5. The MPC has stepped the tap from 4 to 2 by 09:41, ahead of the noon rise, and at 11:26 its highest node stands at 1.0475 pu, while the program, holding its last target, keeps the tap at 4 until 11:28. Over the six seeds the MPC sums 42.9 min against 117.0 min for the program, with 84 operations against 78. Figure 8.4 draws case 9's two rows of the table as bars.

No classical row dominates, and neither of the MPC's two ingredients alone reproduces its case 9 result. The program without the half step, retuned by the MPC's own rule, spends 17.2 min on case 9,

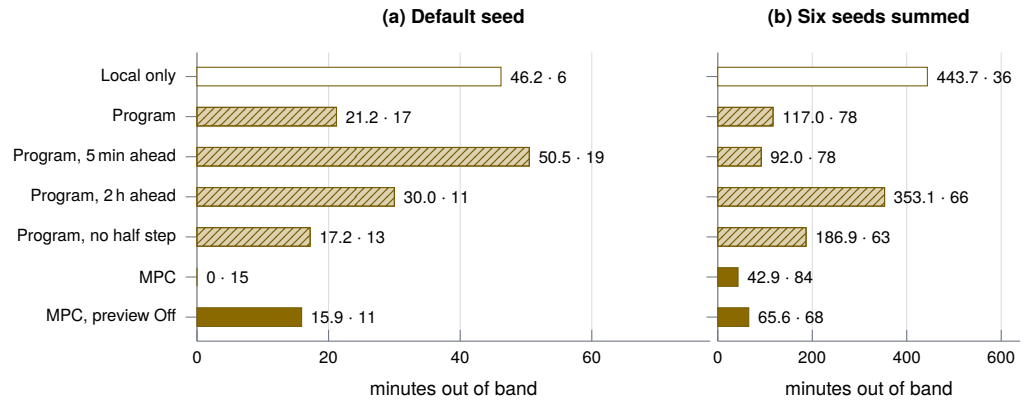


Figure 8.4 Case 9, the 150 % cloudy day with the suggested local settings, from Table 8.6: minutes any node is out of band (bars) and tap operations (after the dot), (a) on the default seed and (b) summed over six seeds. White: the local devices alone. Hatched: the program and its three stronger forms. Solid: the MPC. Notice that the MPC row is lowest in both panels, while the program's best variant changes from one panel to the other.

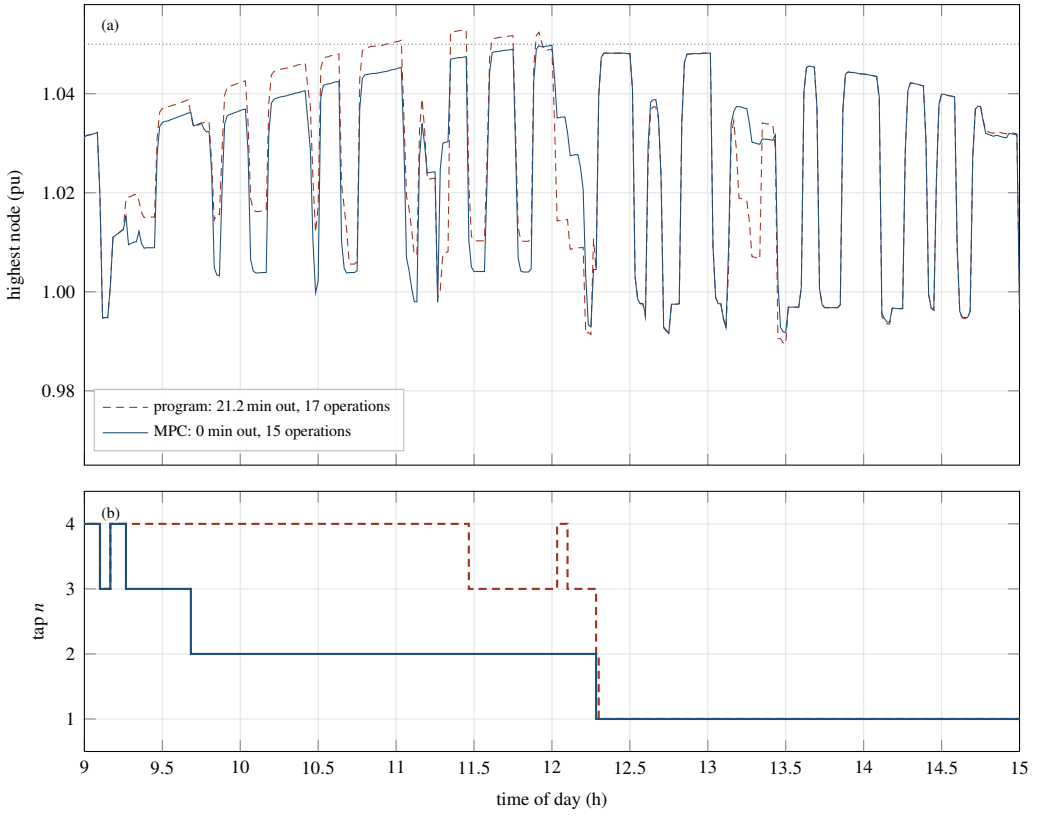


Figure 8.5 Case 9 on the default seed, 09:00 to 15:00, under the program (dashed) and the MPC (solid). (a) The highest true node voltage in each minute against the 1.05 pu limit. (b) The tap. The MPC steps the tap to 2 by 09:41, while the program keeps tap 4 until 11:28 and is out of band from 10:56.

Table 8.7 Worst Cases: Minutes Out · Tap Operations

Case	Local	Program	Prog. 5 min	MPC
3, reverse flow, 09:00 + 24 h	202.7 · 6	0 · 8	0 · 8	0 · 9
9, 150 %, cloudy	46.2 · 6	21.2 · 17	50.5 · 19	0 · 15
150 %, clear, suggested	80.6 · 6	0 · 5	0 · 5	0 · 13
challenge, suggested, shadows at 120 m/s	0 · 6	0 · 9	0 · 13	0 · 11
2, cold-load pickup at 19:00	40.2 · 9	0 · 5	0 · 5	0 · 5
6, capacitor out at 19:00	0 · 12	0 · 2	0 · 3	0 · 2
2, transmission 0.97 pu at 19:00	1.3 · 12	2.3 · 8	2.3 · 8	2.3 · 8
challenge, suggested, 1.03 pu at 12:00	3.0 · 11	1.0 · 18	1.0 · 18	1.0 · 16
challenge, suggested, meters out at 10:00	0 · 6	27.3 · 11	0 · 11	0 · 13

186.9 min over the seeds and 65.2 min on the challenge as shipped. Given the MPC's 2 h of declared shapes, the program spends fewer minutes outside the band than the MPC on the challenge as shipped, 10.2 min against 11.1 min, although neither meets its band requirement. It spends 30.0 min on case 9 and 353.1 min over the seeds, because one bus move must then hold the band against the worst of the next 2 h. With 5 min ahead it is the strongest classical row over the seeds, 92.0 min, and it holds the band on seeds 3 and 5, where the MPC spends 22.9 and 4.6 min. Without its preview the MPC still sums 65.6 min over the seeds and 15.9 min on case 9, below every program row, and the preview of the clear-sky rise takes it to 42.9 and 0. It spends fewer operations than the program on cases 4, 8 and 9 (15, 21 and 15 against 19, 31 and 17) and more on the clear days (7 against 5 at 125 %, 13 against 5 at 150 %).

The largest remaining losses come from the hold rule that the MPC shares with the program. Of the MPC's 42.9 min over the six seeds, 36.7 min fall while it holds (13.7 on seed 2 and 22.9 on seed 3), 6.2 min while a plan acts and none in a period the program decided. On seed 3 its last plan before a cloud set the bus target at 1.0070 pu with the tap at 2, and the hold kept them from 11:55 to 12:35, where the program's hold kept 0.9949 pu at tap 1 and the program spends 0.9 min out. The program loses 21.17 of its 21.2 min on case 9 while it holds. A program that keeps its offsets through a hold, a change to the shipped rule rather than a tuning, sums 14.5 min over the six seeds with 54 operations, 0 on the default seed with 7, below the MPC's 42.9 with 84, while the same change moves the MPC to 44.8 min, so the rule is left as shipped. On case 9 the MPC plans at 200 of its 288 calls, holds at 80 and hands 8 periods to the program, every one on the band-row cap, and no plan in any run needed more than 51 nodes. Without the caps the seeds sum 43.4 min and the largest solve grows from about 0.16 to about 1.5 s in Node, and holding the MPC's own last plan instead of handing over sums 27.3 min over the seeds but 29.9 min on the challenge as shipped, so the program remains the fallback.

Table 8.7 runs every structure through the worst cases the application can pose. In none of these cases is the MPC out of band longer than the program, no run curtails solar, no two tap steps come closer than T_d, the tap stays within ± 16 and no capacitor recloses inside its lockout, because the band, the timer, the tap clamp, the lockout and Volt-Watt act below either central controller. The one case in which both central structures lose to the local devices is the transmission sag to 0.97 pu at 19:00 in case 2, 2.25 min out against 1.33 min. Neither holds in that run. Before the sag both run the feeder lower, the lowest node at 0.9666 pu against 0.9725 pu, inside their margined band, so the sag takes it to 0.9370 pu instead of 0.9426 pu and the tap changer needs three 45 s steps to bring it back over 0.95 pu instead of two. The size of that loss is set by the band's timer, which neither central controller overrides.

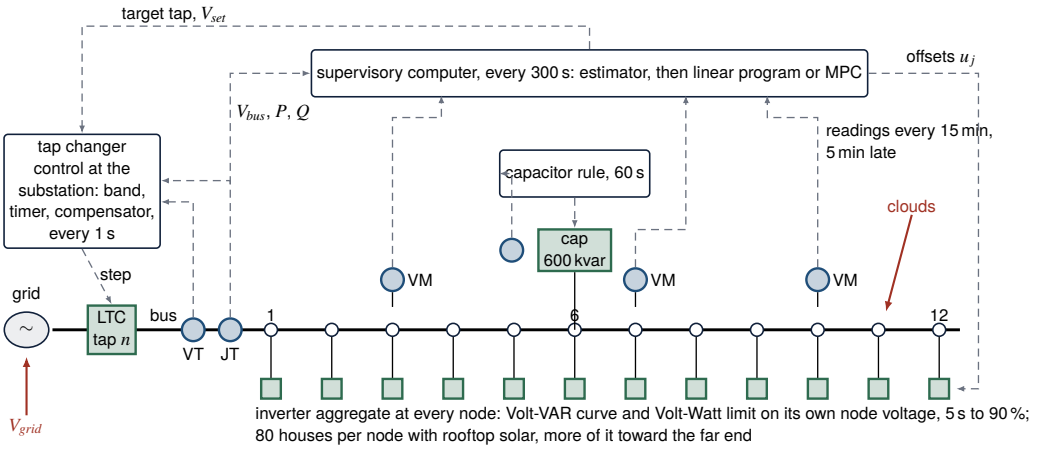


Figure 8.6 The feeder's loops drawn with their hardware. The tap changer's control at the substation reads only the voltage transformer VT and the power transducer JT on the bus and steps the load tap changer (LTC) after its timer. The capacitor at node 6 has its own voltage sensor and rule, and each node's inverter aggregate follows its curve on its own node voltage. The three smart meters VM reach only the supervisory computer, which sends a target tap and setpoint to the tap changer's control and offsets to the inverters. Notice that no device that moves the far end reads the far end except the inverters there.

> **DID YOU KNOW? When the Meters Fail, the Local Devices Win**
>
> From the simulator: on the challenge day with the suggested settings and the three meters out from 10:00, the central program spends 27.3 min out of band with 11 tap operations, while the local devices alone and the MPC spend none (Table 8.7).

8.6.11 Checks of the Code, Agreement and Limits of the Claims

Code verification tests the simulator against its own declared equations. Over the challenge day as shipped the ledger residual $E_{sub} - (E_{load} - E_{pv} + E_{loss})$ is 1.6×10^{-14} of the load energy, and the branch flows close node by node to 1.0×10^{-17} pu. One tap step is 0.6250 %, and the Volt-VAR function returns +0.44, 0, 0 and −0.44 at 0.92, 0.98, 1.02 and 1.08 pu, and −0.44 at 1.04 pu for slope 22. Every tap step in case 2 and on the hunting day follows T_d outside the band on its side, and none is decided inside it. Halving the step to 0.5 s leaves the operations of case 5 at both slopes and of case 2 unchanged (18, 2 and 7) and their energies equal to 10^{-4} MWh. The linear program's objective matches a second simplex at all 360 calls checked, within 1.9×10^{-11} relative. A day costs 0.70 s headless, and the challenge plays its 24 hours in about 24 s at 3600× in a browser.

Agreement with published relations is a separate claim. The Volt-VAR breakpoints are the category B defaults as stated in [16], the measured CVR saving of 0.79 % lies inside the simulated national range of 0.5 to 4 % [10], and the linearized flow stays within 0.46 % of the exact branch flow. The model was fitted to no feeder's records and is therefore not validated against one, and without a study of learners no learning gain is claimed.

8.7 Technology in Practice

The devices of Section 8.3 are, in the field, instrument transformers, a regulating transformer with its control, a switched capacitor bank, smart meters and inverters. Figure 8.6 draws the chapter's loops with that hardware in place of the schematic's symbols.

What Measures It

The tap changer's control reads the bus voltage and the current through the transformer, and the capacitor's control reads its own voltage. A customer's voltage is known only where a meter reports it. Smart meters give hourly demand for every customer, and engineers treat such measurements as the best basis for spreading load along a feeder, with billing data, load research statistics and transformer sizes filling the gaps [24]. Many voltage reduction schemes rely on remote measurements from a handful of locations and assume that they represent the whole system [10]. The chapter's three meters, read every 15 min and reported 5 min late, are such a handful.

> **TECHNOLOGY Sensor: Instrument Transformers at the Regulator**
>
> A tap changer's control never touches the primary voltage. A potential transformer steps the line voltage down to a safe level, typically 120 V, and a current transformer steps the line current down, typically to 5 A at full load, for meters and protective relays [25]. The control combines that local voltage and current with an impedance setting, entered in volts, to estimate the drop along the circuit [24]. Feeder-head data may be logged every minute or every second, and bad data, from communication failures, outages and unusual configurations, are to be expected. The rate of change of a measurement helps to find them [24]. The chapter's VT and JT carry 0.05 % and 0.5 % noise.

What Moves It

Three kinds of final element act on the feeder. The regulating transformer steps the source for every customer. A switched capacitor bank adds reactive power at one node under voltage, time, temperature, time-biased, reactive power or current control [24]. The inverters inject or absorb reactive power along a Volt-VAR curve, which may carry hysteresis and a deadband, with a response time that can be programmed between 300 ms and several seconds [24]. The chapter's inverters reach 90 % of a change in 5 s, while its tap changer waits 45 s before each step and its capacitor 60 s.

> **TECHNOLOGY Final Element: The On-Load Tap Changer**
>
> A load tap changer is a power transformer that changes its turns ratio under load. Common ranges are plus or minus 5 or 10 % in 16 or 32 discrete steps, and a substation unit normally moves all three phases together on the feedback of one [24]. Its control holds a setpoint inside a bandwidth and waits out a time delay before each step. A unidirectional unit assumes that power flows away from the substation, while a bidirectional one senses the direction and switches to a second group of settings [24]. Photovoltaic swings that multiply the steps can shorten the device's life and add maintenance [24]. The chapter's tap changer has ± 16 steps of 0.625 % and no mechanism time.

Where the Controller Runs

A tap changer's control sits beside the transformer and runs the band, the timer and the compensator on its own measurements, and each capacitor and each inverter carries its own rule. Above them, the inverters' Volt-VAR control can be coordinated at the level of supervisory control and data acquisition with capacitor banks and voltage regulators [24]. Supervisory and local loops of this kind run with cycle times from milliseconds to minutes [26]. In the chapter the local devices execute every second, and the estimator, the linear program and the predictive controller every 300 s, as the supervisory layer that sends a tap target, a setpoint and the inverters' offsets.

What Is Modernizing It

Three developments touch these loops. The first is the smart inverter. Many inverters already in service can provide advanced functions with a software upgrade or new parameters, and an inverter works below its maximum current rating more than 95 % of the time, which leaves capacity for reactive power [17]. In 2014 these functions had to be set inside each inverter by hand, and setting them remotely was foreseen [17]. The second is communication. Ethernet, Internet Protocol and wireless

devices are replacing older proprietary technologies, which raises the likelihood of cybersecurity vulnerabilities and incidents in control systems [26], a concern for a program that sends offsets to every node. The third is central optimization over a forecast. The predictive controller of Section 8.5.5 is one form of it, and Section 8.6.10 measures what it buys on this feeder.

> **IN PRACTICE** **Regulator Settings on a Solar Feeder**
>
> Planners coordinate regulators in series by their time delays [24]. A compensator set on the impedance to the first customer raises the regulator's output with load current while that customer stays below 126 V on a 120 V base, the first house setting, and a line that uses a compensator should be monitored [24]. With variable photovoltaic output the bandwidth may need widening, and some utilities disable the compensator and move the setpoint instead. When devices hunt, a nonunity inverter power factor is tried first, then the setpoint, bandwidth and delays [24]. The voltage limits are those of ANSI C84.1 [24].

8.8 Limitations

Every value is synthetic. The feeder is balanced, single-phase and linearized, so it carries none of the unbalance, the single-phase laterals or the secondary drops that decide many real voltage complaints, and its far end reads up to 0.46 % high. The impedances, the solar layout and the load shape were chosen so that noon overvoltage exists, and the shipped compensator and the hunting window are teaching choices that the text labels. The ZIP shares are fixed and the inverters are ideal aggregates with continuous response. The Volt-Watt breakpoints are not confirmed against the standard, and the band and the curves are teaching values from ANSI C84.1 and IEEE 1547-2018 rather than any utility's practice. The estimator's margins are conservative by design, so the central program is weaker here than local coordination on a cloudy day, and a real distribution management system has more meters and a different model. A simulated pass is not a certification or a planning study.

The MPC inherits the estimator's errors because its model is the estimator's declared feeder. It forecasts no cloud, holds the sensitivities S and G fixed over 2 h, plans neither the band's own steps between decisions nor the capacitor, which stays on its shipped rule under both central controllers, and treats the tap as an integer over only the first 20 min, with a continuous ramp after. It shares the program's hold rule, which carries 36.7 min of its 42.9 min over case 9's six seeds, and the comparison rests on one feeder and six weather seeds.

> **OPEN CHALLENGE** **Thousands of Volt-VAR Curves on One Network**
>
> As inverters replace machines, reactive power and voltage control move into thousands of devices with local Volt-VAR curves. Time delays and filters that slow those curves are known to cause voltage oscillations and interactions between inverters and the voltage-regulation equipment on the same distribution network, and piecewise linear curves have inherent instability points [27]. A 2020 national laboratory roadmap asks how the curves should be coordinated with capacitor banks and other devices, how the instability points are avoided, and whether reactive power should be controlled at each inverter, by local groups or over communication networks [27]. Progress would be coordination rules that answer these questions for any feeder.

8.9 Conclusion

A distribution feeder with rooftop solar is a plant whose slow discrete actuator sits at one end while its disturbances and its worst customer sit at the other, and the simulator of this chapter puts that on one screen with a linearized branch flow, ZIP loads, seeded clouds, category B inverter curves, a capacitor and a tap changer with a band, a timer and a compensator. At solar noon the compensator holds the bus 0.0083 pu high while 1.08 MW flows back, because the reactive power still flows forward and X/R exceeds 1.68. A cloud moves the far end 0.031 pu and the bus 0.001 pu. A Volt-VAR slope of 22 against a reactance-weighted compensator turns 2 tap operations into 18 on a cloudy day, inside a window the text states. Conservation voltage reduction saves 0.79 % on the declared loads and costs

energy on constant-power ones, and three meters with the substation's telemetry bracket the worst customer at 47 of 48 fits in case 8. A tap changer alone just passes the cloudy challenge at 125 % solar and passes nowhere at 150 %, where the inverter curves still do. A mixed-integer predictive controller on the same estimate and weights holds the 150 % cloudy day that the program misses by 21.2 min, although on the shipped challenge, which neither holds, a program given a 2 h look-ahead spends fewer minutes out of band, 10.2 min against 11.1 min. The model is balanced and linear and its settings are declared. It leaves out unbalanced phases, line regulators with reverse-power modes, a second feeder on the bus, inverter ride-through and measured load shapes. The simulator shows that on a solar feeder the slow device should set the level and the fast ones should trim the spread.

KEY POINTS

- A tap step moves every customer with the bus, so only devices at the nodes can change the spread of voltage along the feeder.
- At solar noon the compensator holds the bus 0.0083 pu high while 1.08 MW flows back, because the reactive power still flows forward.
- A cloud moves the far end 0.031 pu and the bus 0.001 pu.
- A Volt-VAR slope of 22 against a reactance-weighted compensator turns 2 tap operations into 18 on a cloudy day.
- A mixed-integer predictive controller holds the 150 % cloudy day that the program misses by 21.2 min.

Where else this applies. Integer decisions found by branch and bound also schedule the pumps of Volume 9, Chapter 11, and a late measurement limits the composition loop of Volume 5, Chapter 3.

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8.10 Exercises

8.1 ⌕ Concept check · 2 min

At 12:00 in case 3 the substation exports 1.08 MW while 0.64 Mvar still flows in from the grid. The compensator is Forward with $R_{set} = 0.01$ pu and $X_{set} = 0.15$ pu. What does it do to the bus?

(a) It holds the bus up, because its reactive term outweighs the active one while X_{set}/R_{set} exceeds $-P/Q$

(b) It pulls the bus down, because the current through the transformer has reversed together with the active power

(c) It does nothing, because a Forward compensator drops its correction while the measured active power is negative

(d) It pulls the bus down by the whole line drop, because the far end is the high point when the solar is spread out

8.2 ⌕ Concept check · 2 min

In case 4 a cloud bank removes 60 % of the sunlight in 10 s and stays 25 min. With the inverter curves off, the far end falls from 1.0447 pu to 1.0137 pu by 12:40, the bus moves only from 1.0268 pu to 1.0258 pu, and the tap never moves. Why?

(a) The time delay of 45 s is longer than the fall, so any step would have come too late to help

(b) The drop happens along the line beyond the bus, so the regulated voltage never stays outside its band for 45 s

(c) The compensator cancels the cloud in the regulated voltage, so the regulator sees no change at all

(d) The tap changer is locked out after the cloud's first edge, as the capacitor is after it opens

8.3 Concept check · 2 min

The three meters of case 8 fail at 12:00. Afterward the inferred worst node's margin still holds the true worst voltage at all 24 fits, with a median margin of 0.0191 pu against 0.0176 pu before. What keeps the estimate useful without the meters?

(a) The meters' last readings are replayed at every fit, so the profile stays fixed at the one of 12:00

(b) The estimator reads the logged truth while the meters are stale, which the wider margin then reports

(c) The substation's P and Q alone fix the load and solar scales, and the stale flag widens the margin

(d) The central program holds its last target during the outage, so the voltage profile stops moving

8.4 Theory and calculation · 6 min

The feeder is 12.47 kV on $S_b = 10\,\text{MVA}$. Segments 1 to 6 are $0.306 + j0.385\,\Omega$/km, segments 7 to 12 are $0.55 + j0.42\,\Omega$/km, and each segment is 0.8 km long. One tap step is 0.625 %.

(a) Find the base impedance, the whole feeder's impedance in per unit and its R/X. *Checkpoint:* $15.55\,\Omega$, $0.2642 + j0.2485$ pu, $R/X = 1.06$.

(b) Express the shipped band of 1.67 % and the bands of 1.25 % and 1.0 % in tap steps, and say why a band must be wider than one step. *Checkpoint:* 2.67, 2.0 and 1.6 steps.

(c) A shadow edge sweeps from node 12 toward the bus at 60 m/s. How long after node 12 does it reach node 6 and the bus? *Checkpoint:* 80 s and 160 s.

8.5 Theory and calculation · 6 min

At 12:00 in case 3, $P = -1.081\,\text{MW}$, $Q = 0.645\,\text{Mvar}$ and $V_{bus} = 1.039\,\text{pu}$ on $S_b = 10\,\text{MVA}$. Use Equation (8.6).

(a) Find the correction c with the shipped dials $R_{set} = 0.01\,\text{pu}$, $X_{set} = 0.15\,\text{pu}$, and the dial ratio above which c is positive. *Checkpoint:* $c = 0.008\,27\,\text{pu}$, ratio 1.68.

(b) Find c for a compensator matched to the line, $R_{set} = X_{set} = 0.06\,\text{pu}$. *Checkpoint:* $c = -0.0025\,\text{pu}$.

(c) On forward flow at a steady power factor of 0.95, the correction is proportional to $R_{set} + X_{set}\tan\varphi$. Evaluate it for the shipped dials and for $R_{set} = X_{set} = 0.045\,\text{pu}$. *Checkpoint:* 0.0593 and 0.0598.

8.6 Theory and calculation · 6 min

The category B Volt-VAR curve has a deadband $\delta = 0.02\,\text{pu}$ and saturates at 44 % of the rating. The installed solar is 75 % of the 4.416 MW evening peak, on $S_b = 10\,\text{MVA}$, and the compensator has $X_{set} = 0.15\,\text{pu}$.

(a) Find the default slope that reaches saturation at 1.08 pu, and the voltage at which the steepest allowed slope, 22, saturates above nominal. *Checkpoint:* 7.33 and 1.04 pu.

(b) The open-loop response time is $T_{ol} = 5\,\text{s}$ to 90 %. Find the time constant τ of the first-order lag. *Checkpoint:* 2.17 s.

(c) Find the coupling factor $X_{set} k_{VV} \sum_j S_j$ by which a common change of the node voltages inside the slope moves the compensator's correction, at both slopes. *Checkpoint:* 0.36 and 1.09.

8.7 App challenge · 6 min

Load case 2, *An evening without solar: the band and the timer*, with the default seed. It runs from 12:00 and pauses at 24:00. At 3600× that takes about 12 s. Read the scorecard tile *Customers in band (truth) · tap operations* at the pause.

(a) Run as loaded. Give the tap operations and the lowest node voltage. *Checkpoint:* 7 operations, lowest node 0.972 pu.

(b) Set Bandwidth to 2.50 %, press Reset, and run to the pause again. *Checkpoint:* 5 operations, lowest node 0.970 pu.

(c) Press the reload button to restore the case, set Time delay to 120 s before Run, and run to the pause. *Checkpoint:* 5 operations, lowest node 0.972 pu.

8.8 App challenge · 7 min

Load case 5, *The slope: a steep curve that hunts*, with the default seed. It runs a cloudy day from 09:00 with the curves on and pauses at 17:00, about 8 s at 3600×. Read the tap operations on the scorecard tile *Customers in band (truth) · tap operations* at the pause, and watch the status line of the scene.

(a) Run as loaded, at the default Volt-VAR slope of 7.33. *Checkpoint:* 2 operations.

(b) Set Volt-VAR slope to 22.00, press Reset, and run to the pause. Does the status line read HUNTING? *Checkpoint:* 18 operations, and yes, first shortly before noon.

(c) Keep the slope at 22.00, set Compensator X (Advanced) to 0.120 pu, press Reset and run to the pause. *Checkpoint:* 2 operations.

8.9 App challenge · 8 min

Load case 9, *150 % solar: the MPC against the linear program*, with the default seed. It runs a cloudy day from midnight with the suggested local settings and Central controller on MPC, and pauses at 24:00, about 24 s at 3600×. Read the tile *Customers in band (truth) · tap operations* at the pause.

(a) Run as loaded. *Checkpoint:* In band, nodes 0.952 to 1.050 pu, 15 operations.

(b) Press the reload button, set Central controller to On (the linear program) before Run, and run to the pause. *Checkpoint:* 21.2 min out, highest node 1.053 pu, 17 operations.

(c) Press the reload button, set MPC preview (Advanced) to Off before Run, and run to the pause. What share of the program's minutes out does the MPC remove without its preview, and what share with it? *Checkpoint:* 15.9 min out, 11 operations, about 25 % without the preview and 100 % with it, within 1 point.

8.10 Application area · 4 min

A utility considers conservation voltage reduction on a residential feeder and has the case 7 results of this chapter: over 16:00 to 22:00 the baseline delivers 24 053 kWh and the run with conservation voltage reduction 23 863 kWh, while the load-weighted mean voltage falls 1.04 %. The loads are 30 % constant impedance, 20 % constant current and 50 % constant power.

(a) Find the saving in kWh and in percent, the feeder's measured factor and the loads' own factor $2a_Z + a_I$. *Checkpoint:* 190 kWh, 0.79 %, 0.76 against 0.8.

(b) Explain why the measured factor is lower than the loads' factor, and what the result would be on a feeder of constant-power loads.

8.11 Application area · 4 min

A distribution engineer wants to stop the noon overvoltage of case 3 by switching the regulator's compensator from Forward to Reverse-aware, keeping the shipped dials $R_{set} = 0.01$ pu, $X_{set} = 0.15$ pu.

(a) State what the change fixes at noon and what it costs on a cloudy day, with the measured counts of case 5 at the default slope.

(b) Name the local settings that the chapter shows passing the cloudy 125 % challenge on all six seeds, and the dial ratio they use.

8.12 Review the agent · 5 min

An AI assistant reviews case 5 and writes: “At slope 22 the tap changer makes 18 operations against 2 at 7.33, so the inverters’ Volt-VAR loop is unstable above a slope of about 20. The stability condition on the sampled update in the Volt-VAR literature confirms it. Cap the slope at 7.33 on every feeder.” Find the flaw and say how you would check it in the application.