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CHAPTER 5

Three Arcs on One Floating Neutral: Electrode Regulation in an Arc Furnace

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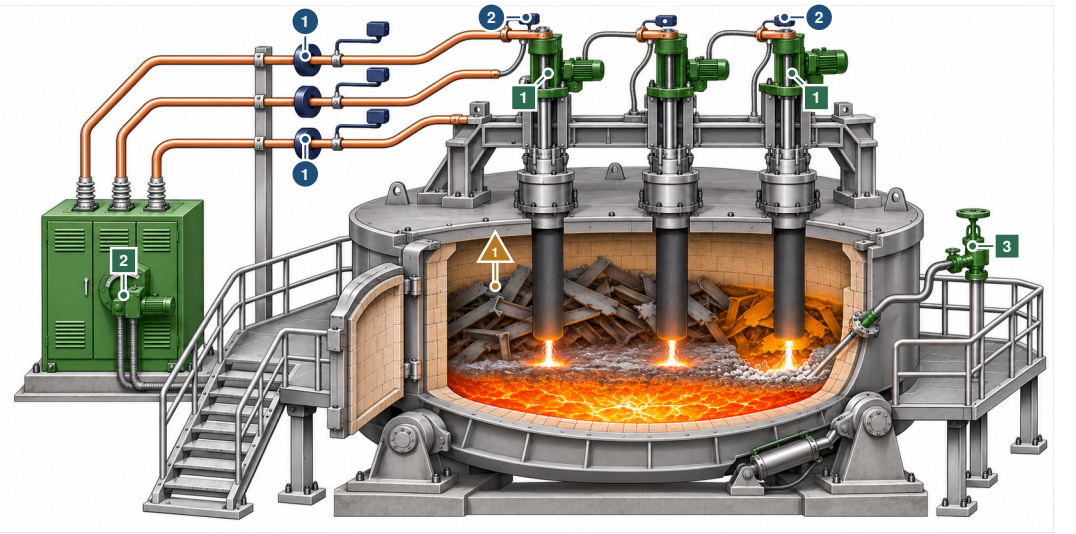
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CHAPTER 5

Three Arcs on One Floating Neutral: Electrode Regulation in an Arc Furnace



The controller moves 1 the three electrode masts 2 the transformer tap changer 3 the carbon injection that foams the slag

The controller reads 1 the current in each phase 2 the voltage from each electrode arm

What disturbs it 1 scrap that caves in under an electrode

Notice that the three arcs all return their current through the one steel bath, so moving one electrode's mast changes the current read on all three cables. Illustration, not to scale.

IN PLAIN TERMS **Three Arcs That Share One Pool**

The main idea. An arc furnace melts scrap steel with three huge electric arcs, glowing columns of hot gas that carry current from carbon rods down to the metal. Each rod moves up and down to keep its arc at the right length.

The challenge. All three arcs send their current back through the same pool of melted steel, so moving one rod changes the current in all three. Each rod's controller must therefore watch a reading that the other two rods barely disturb.

Why it matters. That choice decides how much power a melt uses, how long it takes and how fast the furnace walls wear.

THREE WAYS TO READ THIS CHAPTER

Explore	11 min	The first page, the fact box of Section 5.1, the history boxes of Section 5.2, the fact and try-it boxes of Section 5.6, and the key points
Study	45 min	The whole chapter with its pause points and the twelve exercises
Practice	15 min	The first page, Section 5.6, Section 5.8, Section 5.9 and the open challenge after it

The control question Three arcs return their currents through a steel bath that no wire ties to the transformer, so moving one electrode moves all three currents. Which variable should each phase's regulator hold, and which power program and slag practice bring a heat to tap within its energy, time and wear limits?

Learning objectives

- Explain from the floating neutral why lowering one electrode raises its neighbors' currents unequally.
- Compute the arc's extinction length and the circle-diagram maximum on a given tap.
- Evaluate the relative gain of current and impedance regulation from a circulant sensitivity matrix.
- Compare current and impedance regulators on the same step and through bore-in.
- Choose the regulator, power program and foamy slag that pass the graded heat, and say why one change alone does not.

Prerequisites Sections 1.6.2 (loop action and the velocity form), 1.4.7 (slow and fast parts joined) and 1.6.4 (scenarios and the scorecard).

The app https://apmonitor.com/apps/electric-arc-furnace/

See also Volume 6, Chapter 2 for loop pairing by the relative gain and the interaction it leaves. Volume 4, Chapter 2 for a protective override that acts only near its limit.

5.1 Introduction

At the top tap of the simulated 100 t furnace the transformer delivers 1000 V line to line, and three arcs 336 mm long each carry 60.1 kA and 26.6 MW. Lowering electrode 1 by 30 mm raises its own current to 63.6 kA, the current of phase 3 to 63.1 kA and that of phase 2 only to 60.8 kA. No electrode is wired to another, yet each moves the other two, because the three arc currents leave through the molten steel and the steel is not connected back to the transformer. A secondary circuit of this kind has a star point whose potential floats so that the three phasor currents add to zero, and an 80 MVA furnace modeled from plant data shows that point wandering inside the triangle of the phase voltages, reaching its edge when an arc goes out [1].

Arc furnaces that recycle scrap made about one third of the world's steel in 2011, at around 450 kWh/t of electricity [1], and a proposed fossil-free route reduces iron ore with hydrogen and makes steel from the product, with a variable share of scrap, for 3.48 MWh of electricity per tonne of liquid steel [2]. Every heat is melted under an electrode regulator, which moves three graphite columns on hydraulic masts to hold each arc between a short circuit and extinction while the scrap collapses under the tips. The problem has the shape of every decentralized multiloop system in which the loops share one node, such as parallel inverters on a common bus, zones of a heated slab or the tray loops of a column, and the relative gain of such a pairing measures how much one loop is changed by the others closing [3]. Volume 6, Chapter 2 pairs the loops of a reverse osmosis train by the same measure. Interaction between the phases of an arc furnace is documented as a source of unproductive electrode movement that industrial systems answer with dead zones [4], the floating-neutral circuit has been solved for proportional current control [5], and the current control of a three-phase submerged-arc furnace is a sibling problem [6].

> ✦ **DID YOU KNOW?** **Seven Tenths of American Steel**
>
> The United States made 81 million metric tons of raw steel in 2024, and 72 % of it came from electric arc furnaces, run by 49 companies at 104 minimills [7]. The converters of Chapter 4 made the rest. The simulated furnace of this chapter taps 100.45 t per heat.

The cases of this chapter are aimed at a belief that block diagrams invite, namely that three identical single-loop regulators on three identical phases act as three independent loops. In this model a current regulator tuned for phase 1 alone overshoots a 6 % step by 8 to 16 % and lies inside a quarter

of the step after 0.6 to 0.7 s, but with the other two phases also on automatic the same step overshoots 26 to 40 % and enters that band only after 2.0 to 2.6 s. The sign of the coupling is easy to guess wrong. The planning document from which the application was built expected a raised electrode to raise its neighbors' currents. A raised electrode lowers all three at the design point, and the second neighbor's sensitivity changes sign at shorter arcs.

The published treatments sit at either end of the problem. Furnace models are validated on plant data for operator training and optimization [1, 8], the arc's voltage characteristic is measured on pilot furnaces [9], and the control papers treat the electrode system without the heat around it [5, 10]. A student does not see, in one place, the circuit that couples the phases, a regulator choice that removes most of the coupling, and a heat whose energy, time and refractory depend on that choice. This chapter shows, from one heat of a simulated 100 t three-phase furnace in the browser, how current and impedance regulation, the power program and foamy slag decide the arcs' stability, the energy per tonne and the wear of the walls. The circuit, the arc law, the circle diagram and the relative gain are standard, and the analysis of Section 5.4, which places the change in sign of the neighbors' sensitivity where the neutral's displacement leaves a 30° cone, is elementary circuit algebra applied to the teaching model. The furnace's three currents add to zero to 10^{-6} of the short-circuit current at every step, a hand solve of the circuit agrees with the application to 0.1 %, and a measurement boundary keeps arc lengths, the bath between dips and the wear index away from the controllers. Nine cases run from one electrode moved by hand to a graded heat, each with a completion rule.

Section 5.2 follows the furnace and its regulators from ultra-high-power operation to hydrogen-reduced iron. Section 5.3 gives the model: the floating star, the arc and its extinction, the masts, the charge, bath, slag and wear, and the instruments. Section 5.4 derives the sign of the coupling, the relative gains of current and impedance regulation and the circle diagram that bounds the operating window. Section 5.5 describes the regulators, their overrides, the tap changer and the power program. Section 5.6 measures the nine cases, from one electrode moved by hand through the same step under both regulators, bore-in, melting, the flat bath, a tap step and a broken tip to the graded heat and a heat that no setting passes without foam. Section 5.7 separates code verification from agreement with published data, Section 5.8 describes the real instruments, final elements and controller platforms, Section 5.9 states the limitations, Section 5.10 concludes, and Section 5.11 holds the exercises.

5.2 Arc Furnaces and Their Electrode Regulators

How the electrode regulator grew from the push to ultra-high power into three loops on the arc resistance is told in the history box below. Two quantities can serve as the regulated variable, the arc current, which a current transformer gives directly, and the arc impedance or resistance, which needs the phase voltage measured against the furnace as well.

> ⧖ HISTORY **From Ultra-High Power to Three Loops (1968 to 2011)**
>
> The electrode regulator became the center of furnace automation when furnaces were pushed to the power their transformers could give. By 1968 an article in the Japanese electric-furnace literature was already titled as a trend toward ultra-high-power operation together with the automation of arc furnaces [11]. The analytical treatment followed within a decade. Billings and Nicholson modeled a three-phase furnace in 1977 to compare its control strategies [10], and a 1979 paper in *Automatica* combined the modeling and control of the electric arc furnace [12]. A model of an 80 MVA furnace published in 2011 controls the three electrode hydraulics through three univariable proportional integral derivative (PID) controllers, each on the relative error between a reference and the actual arc resistance [1]. That is the arrangement the simulator's impedance regulator follows.

Regulation then acquired a supervisory layer. A neural network electrode position system ran full time for five months in 1992, learning the relation between electrode position and the stability of furnace operation, and was reported to lower power consumption and furnace and electrode wear [13].

Table 5.1 Variables by Role, With Values at the Flat-Bath Design Point

Symbol	Role	Design	Range
$\lvert I_k \rvert$	CV of a current regulator, measured	60.1 kA	0–105
Z_k	CV of an impedance regulator, measured	7.48 mΩ	0.41–12
$v_{c,k}$	MV, mast speed command	0	±120 mm/s
V_{tap}	MV (discrete), tap voltage line to line	1000 V	600–1000 in 9
h_s^*	MV (on/off), slag foaming by injection	450 mm	80 or 450
—	DVs, cave-in, broken tip, tap steps	events	
λ_c	DV, cave-in frequency multiplier	1	0–3
z_k, v_k	states, tip height and mast speed	—	0.3–5.2 m
w_k, s_k	states, burn-back and surface under tip	0, h_b	—
m_s, m_b	states, solid charge and bath	0, 115.5 t	—
T_b, h_s	states, bath and slag height	1560 °C, 450 mm	—
L_k	truth, arc length	336 mm	$\leq L_{ext}$
V_N	truth, bath potential	0	—
W	truth, wear index	—	$\leq$ 0.30 kRWI·min
E, T_{dip}	measured, energy meter and dip	—	dip 10 s late

Dynamic models that describe melting, chemical reactions and energy additions were built so that the heat could be optimized as a whole [8], and heat-transfer models divided the arc's power among conduction to the steel, radiation to the surfaces and losses to the gas and the electrodes [14]. Foaming slag became the means of running long arcs in the flat bath. In a high-productivity furnace the foam height is usually limited by the amount of slag present, and it falls toward the end of the process as the carbon monoxide generation falls [15].

> ⧖ HISTORY **Minimills and the Race for Minutes (1970s to 2000)**
>
> The electric arc furnace was at first confined to specialty steel. In the 1970s, as capital costs rose and the profits of the integrated plants fell, entrepreneurs saw it, coupled with a billet caster and a bar or section mill, as a low-cost way to make steel. These minimills melted scrap, made long products such as reinforcing bar and broke into flat-rolled products in the 1990s. Key developments of the furnace included long arcs for electrical efficiency with foamy slags to protect the walls, computer controls and ever more powerful transformers. By 2000 many shops were racing to shave every possible minute from a heat, and the best had heat times of 40 to 45 minutes, with the power on for 30 to 35 of them [16].

The interaction between the phases has stayed a live control subject. A 2003 study solved the furnace circuit with a floating neutral and tried proportional current and power control on it [5], current control of a three-phase submerged-arc ferrosilicon furnace was reported in 2002 [6], and a 2023 study found that the mutual influence between phases causes unproductive electrode movements, built a matrix of mutual-influence coefficients that vary with the current increment, and proposed it as a replacement for the usual dead zone [4]. The newest change comes from the charge. The influence of direct-reduced iron on the furnace's energy balance has been quantified [17], and in a laboratory furnace an iron-oxide-rich slag from such pellets changed how the arc moved over the melt [18]. The simulator places these pieces in one heat and declares the iron it melts as the product of the hydrogen shaft furnace of Chapter 3.

5.3 The Furnace as Modeled

The simulated furnace has a 6 m shell, three electrodes of 0.6 m diameter on a 1.3 m pitch circle, nine transformer taps and one bucket of 85 t of scrap and 25 t of direct-reduced iron (DRI) charged onto a 15 t liquid heel at 1540 °C, as shown in Figure 5.1. The heat taps 100.45 t. The model is a declared teaching model assembled from public literature, not a port of any furnace supplier's or licensed simulation and not plant data. Table 5.1 classifies the signals.

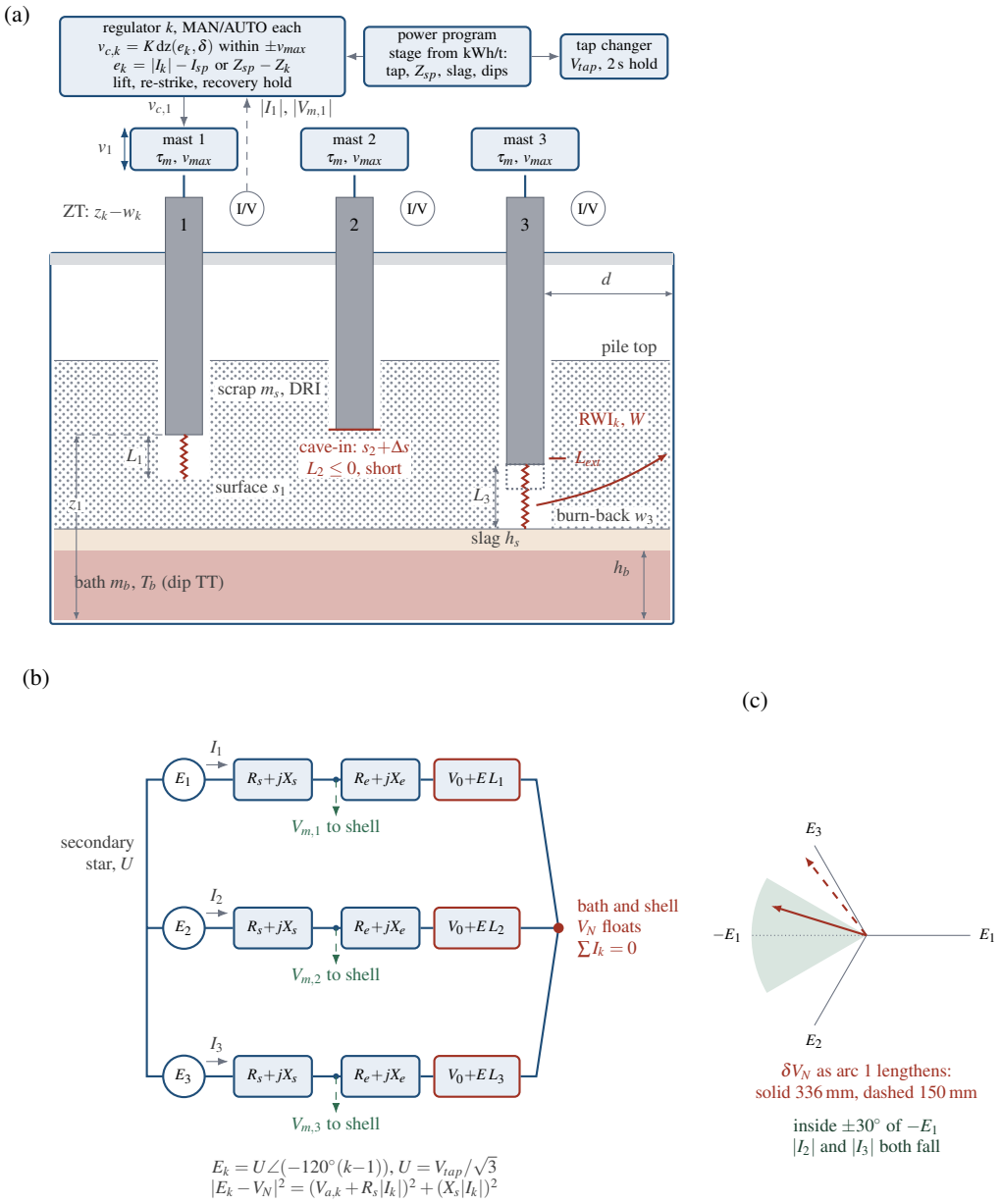


Figure 5.1 The furnace as modeled, after the application's Furnace view. (a) The section during bore-in. Masts of lag τ_m and rate limit v_{max} move tips at heights z_k (the encoders read $z_k - w_k$ before the burn-back w_k), each arc of length L_k ends on a surface s_k in its bore, a cave-in raises s_2 by Δs into a short, and arc 3 is drawn below its extinction length L_{ext}. The bath of mass m_b, level h_b and temperature T_b lies under a slag of height h_s, the bare part of an arc wears the wall at a distance d (RWI$_k$, integrated as W), and the regulator of each phase acts on its own measured current and voltage while the program sets taps and setpoints. (b) The secondary circuit, a star whose star point is the bath. Phase voltages E_k drive the series impedance $R_s + jX_s$, the column below the voltage tap $R_e + jX_e$ and an arc of voltage $V_0 + EL_k$, the regulator reads $V_{m,k}$ from the arm to the shell, and the bath potential V_N floats so that the currents add to zero. (c) The neutral's displacement when arc 1 lengthens, at the design length and at 150 mm, against the 30° cone about $-E_1$ inside which both neighbors' currents fall. Process model only. Not to scale.

5.3.1 A Star Whose Star Point Floats

The secondary is solved once per physics step as three root-mean-square phasors, a quasi-static description in which the arc's waveform within a cycle does not appear, as shown in Equation (5.1).

$$\begin{aligned} E_k &= U\angle\big(-120^\circ(k-1)\big), \qquad U = V_{tap}/\sqrt{3} \\ E_k - V_N &= (R_s + jX_s)\,I_k + V_{a,k}\,\frac{I_k}{|I_k|} \\ \sum_{k=1}^{3} I_k &= 0 \end{aligned} \tag{5.1}$$

where $V_{a,k}$ is the arc voltage of phase k, in phase with its current, and R_s and X_s lump the transformer, the flexible cables, the bus tubes and the electrode. The arc is a voltage set by its length rather than a resistance, so for a trial V_N each phase is explicit. Its magnitude obeys $(V_{a,k}+R_s|I_k|)^2+(X_s|I_k|)^2 = |E_k-V_N|^2$, a quadratic with one nonnegative root, and its angle follows from $I_k = (E_k-V_N)/(R_s + V_{a,k}/|I_k| + jX_s)$. A phase whose $|E_k-V_N|$ does not exceed $V_{a,k}$ carries nothing. The two real equations $\mathrm{Re},\mathrm{Im}\sum I_k(V_N) = 0$ are solved by a damped Newton iteration in the two components of V_N, started from the previous step's neutral, with a forward-difference Jacobian, a backtracking line search and a tolerance of 10^{-6} of the short-circuit current $U/|R_s+jX_s|$. Over whole heats it never needed more than four iterations, including steps with one phase out and another shorted. An extinguished phase carries no current, a short circuit ($L_k \le 0$) has $V_{a,k} = 0$, and with one or no phase able to conduct nothing flows.

The regulator measures each phase from its electrode arm to the furnace shell, which sits at the bath's potential, so the measured voltage includes the drop in the column below the measuring point, as shown in Equation (5.2).

$$V_{m,k} = V_{a,k}\frac{I_k}{|I_k|} + (R_e + jX_e)\,I_k, \qquad Z_k = \frac{|V_{m,k}|}{|I_k|} \tag{5.2}$$

A short circuit therefore reads $|R_e + jX_e| = 0.41\,\mathrm{m\Omega}$ rather than zero, which sets the lift's trip level.

5.3.2 The Arc, Its Extinction and Its Flicker

The arc voltage is linear in its length, with a seeded flicker on the length, as shown in Equation (5.3).

$$\begin{aligned} V_{a,k} &= V_0 + E\max(0, L_k + n_k), \qquad L_k = z_k - s_k \\ L_{ext} &= (k_{ext}U - V_0)/E \end{aligned} \tag{5.3}$$

where $V_0 = 40\,\mathrm{V}$ is the anode and cathode fall, $E = 1.2\,\mathrm{V/mm}$ the column gradient and n_k a colored noise of correlation time 0.1 s whose standard deviation is 14 mm while the arc burns on scrap and 3 mm on a flat bath, multiplied by $1 - 0.8\min(1, h_s/L_k)$ under foam. An arc longer than L_{ext} breaks and strikes again only when its tip comes within $L_{strike} = 25\,\mathrm{mm}$ of the charge. With $k_{ext} = 0.85$, L_{ext} is 212, 294 and 376 mm on taps 1, 5 and 9. The rule stands for an alternating arc that cannot reignite once the source voltage at current zero falls short of the arc's own, a teaching assumption without a measured source.

5.3.3 Masts, Surfaces and Electrode Consumption

Each mast is a hydraulic servo integrated exactly over a step for the speed command held through it, and the tip burns back in proportion to its current, as shown in Equation (5.4).

$$\begin{aligned} \tau_m\frac{\mathrm{d}v_k}{\mathrm{d}t} &= v_{c,k} - v_k, \qquad \frac{\mathrm{d}z_k}{\mathrm{d}t} = v_k + c_e|I_k| \\ \frac{\mathrm{d}w_k}{\mathrm{d}t} &= c_e|I_k|, \qquad P_k = V_{a,k}|I_k| \\ \frac{\mathrm{d}s_k}{\mathrm{d}t} &= -b\,P_k \quad \text{while } s_k > h_b \end{aligned} \tag{5.4}$$

where $\tau_m = 0.2\,\mathrm{s}$, $|v_{c,k}| \le v_{max} = 120\,\mathrm{mm/s}$, the stroke holds the tip between 0.3 and 5.2 m plus its burn-back, and the encoder reads $z_k - w_k$, the height a new electrode would have. The consumption c_e is declared from 1.5 kg of graphite per tonne of steel at 60 kA for 40 min, which burns 104.5 mm from each tip in a heat. An arc bores down through scrap at $b = 0.7\,\mathrm{mm/s}$ per megawatt of its power, and no surface stands above the pile top $h_b + m_s/(\rho A)$ with a bulk density $\rho = 1.2\,\mathrm{t/m^3}$ over $A = 28\,\mathrm{m^2}$, nor below the bath, whose level is $h_b = 0.145 + 0.007\,m_b$ in meters for m_b in tonnes.

A cave-in is a hard event in the post-step update. While more than 15 % of the charge is solid, an exponential clock with a mean of 30 s while any bore is open and 90 s once all are through picks an electrode at random and raises the surface under it by 100 to 300 mm, uniformly, never above the pile. A short arc is shorted by it, which is what the lift must clear. Arcs breaking and striking, shorts starting and ending, a broken tip, a tap step, a dip and the tap are also post-step events, each marked in the historian, so the derivative function stays pure.

5.3.4 Charge, Bath, Slag and Wear

The heat reaching the charge is the arcs' power weighted by how each arc is exposed, plus chemical heat, less the panels' losses, as shown in Equation (5.5).

$$\begin{aligned} Q &= \sum_k \eta_k P_k + P_{chem} - P_{loss} \\ \eta_k &= \eta_b + (\eta_{sc} - \eta_b)\,\chi_k \\ &\quad + (\eta_{sl} - \eta_b)\,c_k(1-\chi_k) \\ \frac{\mathrm{d}m_s}{\mathrm{d}t} &= -\frac{Q^+}{h_m}, \quad \frac{\mathrm{d}m_b}{\mathrm{d}t} = -Y\frac{\mathrm{d}m_s}{\mathrm{d}t} \quad (m_s > 0) \\ m_b c_p \frac{\mathrm{d}T_b}{\mathrm{d}t} &= Q \quad (m_s = 0), \qquad \tau_s \frac{\mathrm{d}h_s}{\mathrm{d}t} = h_s^* - h_s \end{aligned} \tag{5.5}$$

with the bath held at 1540 °C while $m_s > 0$, where $\chi_k = \mathrm{clip}_{[0,1]}\big((h_{top} - s_k)/L_k\big)$ is the share of the arc hidden by its bore's wall, $c_k = \min(1, h_s/L_k)$ the share under slag, η_{sc}, η_{sl} and η_b the efficiencies inside scrap, under foam and bare (0.90, 0.72 and 0.40), h_m the charge's melting energy (1.25 GJ/t for scrap and 1.42 GJ/t for DRI), Y its yield (0.92 and 0.89), $c_p = 0.82\,\mathrm{kJ/(kg\,K)}$ and $\tau_s = 60\,\mathrm{s}$. The efficiencies are a reduction by exposure of a heat-transfer model in which 15 to 20 % of the arc power reaches the steel by conduction and 75 to 85 % the furnace surfaces by radiation [14]. Burners and oxygen add $P_{chem} = 30\,\mathrm{MW}$ while scrap remains, up to a budget of 115 kWh/t of charge, so a slow heat cannot buy chemical energy by the minute, and $P_{loss} = 6 + 0.02(T_b - 1540)^+$ in megawatts. The slag foams toward $h_s^* = 450\,\mathrm{mm}$ when carbon is injected and less than 15 % of the charge is solid, and lies at 80 mm otherwise.

The wear of the walls is counted on the share of each arc that neither scrap nor slag covers, in the form of the refractory wear index named after Schwabe, as shown in Equation (5.6).

$$W = \int \max_k \Big(\frac{P_k V_{a,k}}{d^2}\,(1-\chi_k)(1-c_k) \Big)\,\mathrm{d}t \tag{5.6}$$

with P_k in megawatts, $d = 2.05\,\mathrm{m}$ from the electrode to the wall and W reported in kRWI·min. The energy meter integrates the arcs' power and the circuit's $R_s|I_k|^2$, which a high-current heat pays, and the energy per tonne is that meter divided by the tapped mass.

5.3.5 Instruments

Every controller works from one packet of measurements, which holds the three currents and the three arm-to-shell voltages, each with 0.3 % seeded noise and a 0.04 s transducer filter, $Z_k = V_k/I_k$ above 500 A, the metered power and energy, the three encoders, the tap indicator, the breaker, and the last thermocouple dip, which reads the true bath with $\sigma = 3\,^\circ\mathrm{C}$ and reports 10 s after it is taken. Arc lengths, the bath between dips, the scrap, the slag, the flicker and the wear are truth, logged in columns prefixed `truth_`, and the application's own tests fail if a controller file reads any of them.

Table 5.2 Model Parameters and Their Evidence. A Teaching Value Is Chosen for the Lesson, Not Measured on a Furnace

Symbol	Meaning	Value	Evidence
V_{tap}	taps, line to line	600–1000 V, 9	600–980 V in 18 [1]
R_s	circuit resistance	0.5 mΩ	0.37 mΩ + 0.12 mΩ [1]
X_s	operating reactance	5.5 mΩ	teaching value, cf. [19]
R_e, X_e	column below the voltage tap	0.1, 0.4 mΩ	teaching assumption
V_0	anode and cathode fall	40 V	common model value [9]
E	column gradient	1.2 V/mm	inside 0.5–3 V/mm [9]
k_{ext}	stability limit	0.85	teaching assumption
L_{strike}	re-strike gap	25 mm	teaching assumption
σ	flicker, scrap / flat	14 / 3 mm, 0.1 s	teaching value
m	scrap, DRI, heel	85, 25, 15 t	teaching value
f	supply frequency	60 Hz	declared (50 Hz in [1])
τ_m, v_{max}	mast lag, rate limit	0.2 s, 120 mm/s	teaching value (plan)
b	bore rate	0.7 mm/(s MW)	teaching value
—	cave-ins	30, 90 s, 100–300 mm	teaching value (plan)
c	electrode consumption	1.5 kg/t	teaching value (plan)
η_{sc}, η_{sl}, η_b	arc efficiency by exposure	0.90, 0.72, 0.40	reduction of [14]
P_{chem}, e_{chem}	chemical heat, budget	30 MW, 115 kWh/t	teaching value
P_{loss}	panels	$6 + 0.02\Delta T$ MW	teaching value
h_s^*, τ_s	foam height, time	450 mm, 60 s	teaching value
h_m	melting energy, scrap / DRI	1.25 / 1.42 GJ/t	teaching value (DRI table)
d	electrode to wall	2.05 m	geometry
W_{max}	wear budget	0.30 kRWI·min	teaching value (sweeps)

5.3.6 Parameters and the Declared Charge

Table 5.2 gives the parameters with the evidence for each. The circuit sits on a read source. The 80 MVA furnace of Logar et al. had a secondary of 600 to 980 V in 18 taps, rated secondary currents of 47 to 62 kA, arcs of 40 to 60 kA per phase, transformer and line resistances that add to 0.49 mΩ and a cathode drop of 30 to 40 V [1]. The arc law takes the 40 V that several furnace models use and a gradient inside the 500 to 3000 V/m that a non-linear model allows, while the pilot measurements themselves fit a non-linear relation [9]. The reactance is an operating value that includes the arc's own, a teaching value consistent with a circle-diagram analysis of a larger furnace whose operating reactance exceeds its short-circuit reactance [19]. The supply frequency is declared as 60 Hz, which enters only through the reactance and is 50 Hz in the read furnace.

The DRI is declared in the application's own file as the interface of the green steel line to a hydrogen shaft furnace. The table gives a metallization of 0.94, total iron 0.92, carbon 0, gangue 4.5 %, residual FeO 7.10 % from the metallization, a declared balance of 0.019 that closes the analysis, a charge temperature of 25 °C and a carbon demand of 12 kg/t to reduce the FeO. A review of the table found it plausible for hydrogen DRI, with a melting energy of 1.42 GJ/t against about 1.38 GJ/t by hand. The two apps are not coupled. Nothing is read from the shaft furnace, and a hot charge from a neighboring plant would be a mismatch for that app to report.

5.3.7 Initial State, Integration and Ledgers

A heat starts with a full bucket, the tips parked at 4.6 m over a pile whose top stands 3.5 m above the hearth, and every surface on that top. Other cases start lined out with the bores through and 60 % or 12 % of the charge left, or on a flat bath at 1560 °C, with balanced arcs of a stated length. The physics step is $\Delta t = 0.01$ s, the controller runs every 0.02 s and the historian logs at 5 Hz, the three clocks of Section 1.6.1. In place of the engine's Runge–Kutta step the application supplies its own update. Each step advances the flicker, solves the circuit at the step's start, holds the solution, integrates the masts exactly and the surfaces, charge, slag and ledgers explicitly, then applies the events. Six ledgers integrate the arc energy, the circuit losses, the net heat, the chemical heat, the panel losses and the wear.

5.3.8 Assumptions

Quasi-static phasors. The arc's waveform within a cycle, its harmonics and the flicker it puts on the supply network are absent, and the arc is a voltage in phase with its current. Intra-cycle arc models capture breakage and ignition that this description cannot [9].

A linear arc and a declared limit. The real voltage-length relation is non-linear and changes with the slag [9], and the stability limit k_{ext} and the re-strike gap are teaching values.

A declared charge. The pile, its bores and its cave-ins are a teaching model, not a melting simulation. The bath is held at 1540 °C while scrap remains, there is no chemistry, and the foam is a height that follows the injection, while the real foam height depends on the slag and its carbon monoxide supply [15].

One bucket, one heat. Charging, slag practice, sampling and the second bucket are out of scope, so tap to tap adds a declared 10 min to the power-on time.

5.4 How the Neutral Couples the Phases

5.4.1 The Sign of the Coupling

Raising electrode 1 lengthens arc 1, which lowers $|I_1|$ through its own equation and moves V_N. The neighbors' arc voltages do not change, and the magnitude relation under Equation (5.1) makes $|I_k|$ an increasing function of $|E_k - V_N|$ alone. To first order, therefore, a displacement δV_N changes phase k's current with the sign of

$$\delta|E_k - V_N| = -|\delta V_N| \cos\big(\arg \delta V_N - \arg E_k\big) \tag{5.7}$$

at a balanced point, where $V_N = 0$. Both neighbors' currents fall when δV_N lies within 90° of E_2 at −120° and of E_3 at +120° at once, which is the cone of ±30° about $-E_1$ drawn in Figure 5.1(c). A longer arc 1 drops more voltage, so its own current falls and the bath potential is pushed away from E_1. The push is not exactly opposite E_1, because the change in phase 1's current follows its own lag, 34.9° behind E_1 at the design point. At 336 mm the neutral moves 0.556 V per millimeter at 17.2° from $-E_1$ toward E_3, and $|E_k - V_N|$ changes by +0.531, −0.124 and −0.408 V/mm for phases 1, 2 and 3. All three currents fall, by 0.127, 0.035 and 0.114 kA/mm. Phase 3, which leads phase 1 by 120°, falls 0.90 as much as phase 1 and phase 2 only 0.27 as much. A star whose neutral were tied to the transformer would leave both neighbors unchanged. Over balanced arc lengths from 40 mm to extinction on taps 5 and 9 the push stays between 10 and 72° of $-E_1$, so the two neighbors never rise together, and the plan's expectation that a raised electrode raises the other two currents was a sign error, corrected during the build.

🕮 **Pause.** Try Exercise 5.1 before reading on.

5.4.2 Where the Second Neighbor Changes Sign

A shorter arc carries more current at a lower power factor, so its current lags further and the push rotates further from $-E_1$. At tap 9 the lag is 48.9 and 62.5° at 250 and 150 mm, the push sits 33.5 and 52.0° from $-E_1$, and phase 2's current rises by 6.6 and 31.6 A/mm when electrode 1 is raised, as shown in Figure 5.2(b). The sign of phase 2's sensitivity and the 30° crossing of the push were located independently on the application's solver. Both fall at 268.6 mm on tap 9 and 208.2 mm on tap 5, as Equation (5.7) requires. The program's stage points straddle the change. Melting at 317 mm and the flat bath at 336 mm on tap 9 lie on the side where every current falls, while the bore-in point, 200 mm on tap 5, lies on the other side, where phase 2 rises by 3.7 A/mm as electrode 1 goes up.

5.4.3 Interaction Measured as a Relative Gain

The sensitivity matrices of the three currents and the three measured impedances to the three tip heights are circulant at a balanced point. At the design point their diagonals are −0.1271 kA/mm and 0.0355 mΩ/mm, the current matrix's off-diagonal ratios are 0.90 and 0.27 and the impedance matrix's

0.39 and 0.12. The diagonal relative gain [3] of current regulation is 0.745 and that of impedance regulation 1.034. At the melting point the two are 0.639 and 1.007, and at the bore-in point 0.440 and 0.975. Divided by the diagonal, the current matrix has eigenvalues 2.17 and $0.68\angle\pm53°$, and the impedance matrix 1.51 and $0.78\angle\pm18°$. The complex pair belongs to the two modes in which the phases move against each other, and its 53° rotation is what turns three proportional current loops into a loop that rings. A phase's impedance is mostly its own arc's voltage over its own current, so the impedance matrix is nearly diagonal in the sense of the relative gain, although its off-diagonal ratio of 0.39 is not small.

5.4.4 The Circle Diagram and the Stable Window

For one balanced phase on one tap the arc power is $P(I) = I\big(\sqrt{U^2 - X_s^2 I^2} - R_s I\big)$. It is zero for an arc too long to carry current and for a short circuit at $U/|R_s + jX_s|$, and its maximum lies where the arc's resistance equals $|Z_s|$, as shown in Equation (5.8).

$$\begin{aligned} P^* &= \frac{U^2}{2\big(R_s + |Z_s|\big)} \le \frac{U^2}{2X_s} \\ I^* &= \frac{U}{\sqrt{2|Z_s|\big(R_s + |Z_s|\big)}}, \qquad Z_s = R_s + jX_s \end{aligned} \tag{5.8}$$

With $R_s = 0$ this is the circle of radius $U^2/(2X_s)$ with its maximum at a power factor of 0.707, the form used in a modified circle-diagram analysis of an AC furnace [19]. The per-phase maxima are 9.96, 17.71 and 27.67 MW at 42.47, 56.63 and 70.79 kA on taps 1, 5 and 9, as shown in Figure 5.2(a), and the circuit's resistance puts the top tap's maximum 8.7 % below the 30.30 MW of the lossless circle. The plan's bound $V_{tap}^2/(2X_s)$, 90.9 MW at the top tap, is the three-phase total of the same circle and is never exceeded. On tap 9 the arc is stable above 47.61 kA, where it reaches L_{ext}, so the program's setpoints of 60 and 65 kA sit on the long-arc side of the maximum, 40 and 58 mm short of extinction. The plan's 400 mm extinction length at the lowest tap would need an arc voltage of 520 V against a phase voltage of 346 V, so the limit was redefined as in Equation (5.3).

5.5 Electrode Regulation as Implemented

The controller runs every 0.02 s from the measurement packet. Its regulators are written in the application itself, not on the shared velocity-form PID of Section 1.6.2, because the regulated output is a mast speed and the mast already integrates it.

5.5.1 Current and Impedance Regulators

Each phase has its own regulator and its own MAN/AUTO switch, as shown in Equation (5.9).

$$\begin{aligned} v_{c,k} &= \mathrm{sat}_{v_{max}}\Big(K\,\mathrm{dz}(e_k,\delta) + \frac{K}{T_i}\int \mathrm{dz}(e_k,\delta)\,\mathrm{d}t\Big) \\ e_k &= |I_k| - I_{sp}\ \text{(kA, current)} \\ e_k &= Z_{sp} - Z_k\ \text{(m}\Omega\text{, impedance)} \\ \mathrm{dz}(e,\delta) &= e - \delta\,\mathrm{sgn}\,e \ \text{for}\ |e| > \delta,\ \text{else } 0 \end{aligned} \tag{5.9}$$

where K is K_I in mm/s per kA or K_Z in mm/s per mΩ, a positive e_k raises the electrode, the dead zone dz leaves errors inside $\pm\delta$ unanswered, the optional integral ($T_i = 0$ removes it) accumulates only while the output is not saturated, and it restarts from zero on a switch of regulator type, whose units differ, and on MAN. In MAN a mast drives toward its position station at 4/s times the gap between the station and the encoder reading $z_k - w_k$, clipped at the rate limit, and the station follows the encoder in AUTO, so transfers are bumpless. The header's MANUAL/AUTO throws all three phases. A raised electrode lowers its current and raises its impedance, so in the convention of Section 1.6.2

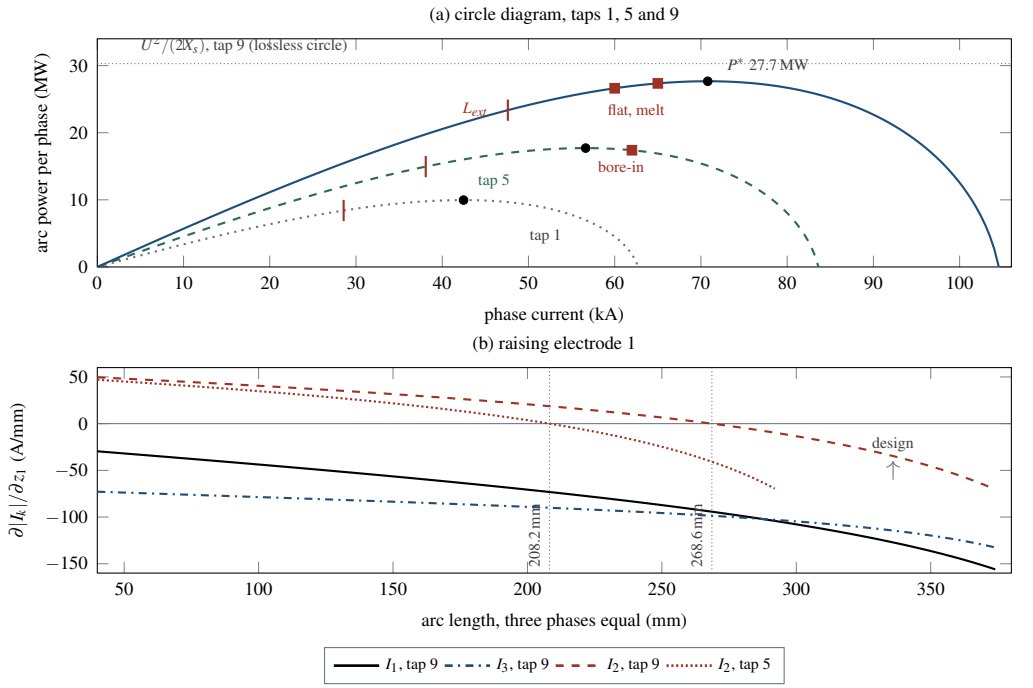


Figure 5.2 The operating window and the coupling, from the application's circuit functions. (a) Arc power per phase against current on taps 1 (dotted), 5 (dashed) and 9 (solid), with the maxima (dots), the stability limits at L_{ext} (bars) and the program's stage points (squares). (b) The change in each current per millimeter that electrode 1 is raised, with the three arcs of equal length: I_1 solid, I_3 dash-dotted, I_2 dashed on tap 9 and dotted on tap 5. Phase 2 changes sign at 268.6 mm on tap 9 and 208.2 mm on tap 5, where the neutral's push leaves the 30° cone.

the current regulator is direct acting, $a = -1$ with $e_k = a(I_{sp} - |I_k|)$, and the impedance regulator is reverse acting, $a = +1$. The mast integrates the speed command, so a steady burn-back r leaves an error of only r/K, 0.0011 mΩ for 0.044 mm/s at $K_Z = 40$.

5.5.2 Overrides Without Latches

Three overrides act only while their condition holds, like the protective override of Volume 4, Chapter 2, whose output rests on its lower clamp throughout healthy operation. The lift raises at v_{max} while a phase reads below $Z_{trip} = 1.0\,\text{m}\Omega$. The impedance rather than the current detects a short, because with a floating neutral a shorted phase's neighbor can carry more current than the shorted phase. On tap 4 with arc 1 shorted and the others at 200 mm the hand solve gives 69.3 kA in the short, 76.8 kA in the phase that leads it and 48.7 kA in the third. The re-strike lowers at v_{max} while a phase carries less than 1 kA. With the re-strike on, a recovery hold keeps a healthy phase still while another phase is out, shorted, or below half its own impedance setpoint, because two arcs in series across a line voltage read a high impedance that would drive them onto the charge just as the third strikes. A phase that is itself below half its setpoint never holds.

🕮 **Pause.** Try Exercise 5.3 before reading on.

That last rule corrects one of two controller defects that a review of the application found and fixed. The hold had latched, since a phase held whenever any other phase read low, including when it was itself low, so two phases recovering together held each other. On a flat bath with two tips broken at once all three arcs stayed at 61 to 75 mm, 100 kA and 35 to 38 MW for good. At the start of a heat the three fresh strikes held each other short, which had let the impedance regulator on the top tap

alone pass 3 of the build's six seeds, with a wear index of 0.18 to 0.42. With the latch removed it passes none of the six and none of 21 seeds, at 0.36 to 1.05 over the 21. The same latch had produced an 86 s short in a bore-in at $K_Z = 10$ with the lift off, now 2.04 s. The second defect was the tap changer's hold, which returned a zero speed before the lift was tested, so a cave-in during a tap step stayed shorted until the tap had stepped. On the challenge's seed this was a 2.06 s short. With the lift acting through the hold it lasts 0.53 s, and 1.00 s at most over 21 seeds.

5.5.3 Tap Changer and Power Program

The tap changer makes one step of 50 V per operation. The regulators hold the electrodes for 2 s, the voltage steps, and the next operation may start 3 s later. The program switches its stage on the metered energy per tonne of charge, and each stage names a current at a tap, from which the impedance setpoint is that of the balanced operating point on the declared circuit, so both regulators are asked for the same arcs. Bore-in runs tap 5 at 62 kA with 200 mm arcs to 45 kWh/t, melting tap 9 at 65 kA with 317 mm arcs to 300 kWh/t, and the flat bath tap 9 at 60 kA under foam, or tap 6 at 66 kA with 214 mm arcs if the slag is left flat. Dips start at 340 kWh/t and repeat every minute. Between dips the program carries the last reading forward by the metered energy at a declared net heating efficiency, 0.62 under foam and 0.32 bare, into the declared bath, and calls the tap when that estimate reaches 1630 °C once a dip has shown the bath above 1560 °C. A Fixed tap mode keeps the stages' setpoints on one tap.

5.5.4 Three Models, and Why No Optimizer

The simulation model is Section 5.3. The application fits no model to its own data, and the identified models a student builds are the sensitivity matrices of Section 5.4, measured by moving one electrode at a time. The controller's internal models are the declared circuit that turns a stage's current into an impedance setpoint and the program's declared bath mass, heat capacity and efficiencies. An operating-point optimizer, optional in the application's plan, was not built, since the wear term it would trade against power needs the slag height, which nothing measures, and on the declared values it would return the stage table.

5.6 Nine Cases on the Running Furnace

The cases appear in Table 5.3, and each carries the briefing and completion rule of Section 1.6.4. Each number in this section was read off a headless copy of the engine, advanced in the same 10 ms steps that the page uses, on the seed that the case declares, and a change that a sentence mentions is made after Run whenever the case's own text asks for it then. The review script that measures every case, rerun for this chapter, reproduced its earlier record line for line. Cases 2 and 3 are quoted as ranges over 31 step instants from 2 to 8 s after Run, because the arcs' flicker at the instant of the step moves the result, and Figures 5.3 and 5.4 quote one instant, 2.0 s. The first eight cases are summarized in Table 5.4.

> ✦ DID YOU KNOW? **A Wider Deadband That Moved No Less**
>
> From the simulator: in case 5 of Table 5.4, a 0.5 mΩ deadband cuts the masts' travel from 7.3 to 1.4 mm/s. Doubling it to 1.0 mΩ leaves the travel at 1.5 mm/s, and one arc wanders out to 376 mm and breaks.

5.6.1 One Electrode by Hand

In case 1 the currents read 60.0, 60.1 and 60.0 kA at 5 s, and with Phase 1 position lowered 30 mm they read 63.6, 60.8 and 63.1 kA at 10 s, with the bath potential 16 V off center. Raised 30 mm above its start, the same electrode takes all three down, to 56.1, 58.7 and 56.5 kA. The independent hand solve of the same states gives 63.80, 61.02 and 63.57 kA lowered and 56.16, 58.93 and 56.71 kA raised, without the noise and flicker of the running case.

Table 5.3 The Nine Cases

#	Start and settings	Teaching point
1	flat bath, 336 mm, all three on MAN	the coupling by hand
2	flat bath, current, $K_I = 20$, phase 1 offset +6 %	three loops ring
3	flat bath, impedance, $K_Z = 80$, offset −6 %	the same step settles
4	full bucket, stage taps, impedance $K_Z = 40$ (seed 4)	cave-ins, the lift
5	60 % of charge left, tap 9, 317 mm, deadband	extinction margin
6	13 t left, slag off, stage taps, Slag on	foam hides a long arc
7	mid-melt, tap 8, $Z_{sp} = 6.01$ mΩ, Tap up	a discrete step, held
8	flat bath, impedance, a 200 mm broken tip	burn-back, re-strike
9	full heat, current $K_I = 30$, Fixed tap 9 (seed 5)	graded heat

Table 5.4 Measured Results of Cases 1 to 8 on Their Own Seeds. Ranges in Cases 2 and 3 Span 31 Step Instants

#	Configuration	Measured
1	Phase 1 position −30 mm	60.0, 60.1, 60.0 → 63.6, 60.8, 63.1 kA, $\lvert V_N \rvert$ 16 V
2	$K_I = 20$, three on AUTO	overshoot 26–40 %, in at 2.0–2.6 s, I_3 swing 2.9–3.8 kA
	$K_I = 20$, phase 1 alone	8–16 %, in at 0.6–0.7 s
	$K_I = 30$, three on AUTO	41–222 %, in at 4.2–8.0 s, 0–2 arcs broken
3	$K_Z = 80$, three on AUTO	9–23 %, in at 0.5–0.7 s, I_3 swing 1.2–1.9 kA
	$K_Z = 80$, phase 1 alone	13–25 %, in at 0.5–0.7 s
4	impedance, $K_Z = 40$	10 cave-ins, 6 shorts (0.76 s), no arc broken, melting at 318 s
	current, $K_I = 30$	3 arcs broken, 7.7 s out, melting at 326 s
5	deadband 0, 0.5, 1.0 mΩ	masts 7.3, 1.4, 1.5 mm/s, longest arc 344, 360, 376 mm, 0, 0, 1 broken
6	slag off, 10 min	tap 6, 59 MW, wear 6.38
	Slag on at 2.5 min	TAP at 1629 °C after 390 s, wear 0.79
7	Tap up, tap 8 to 9	step 2.01 s after the button, 74.2 → 82.9 MW
8	200 mm tip broken	struck at 4.48 s, within 5 % 3.50 s later

5.6.2 Current Against Impedance on the Same Step

Case 2 asks phase 1 alone for 6 % more current, 3.6 kA, with each phase on a proportional current regulator at $K_I = 20$, the gain at which phase 1 by itself overshoots 8 to 16 % and is inside a quarter of the step at 0.6 to 0.7 s. With all three on automatic the step overshoots 26 to 40 %, settles inside a quarter of the step only after 2.0 to 2.6 s, and swings phase 3's current by 2.9 to 3.8 kA, as shown in Figure 5.3. The Furnace view two seconds after such a step is shown in Figure 5.4. At $K_I = 30$ the three loops go unstable at most instants, with overshoots of 41 to 222 %, phase 3 reaching 91.4 kA and up to two arcs broken. At $K_I = 10$ they ring less, 17 to 24 % with settling at 1.2 to 1.6 s. Case 3 makes the same request of impedance regulators at $K_Z = 80$, a 0.45 mΩ shorter arc. All three on automatic overshoot 9 to 23 % and settle at 0.5 to 0.7 s, the same as phase 1 alone (13 to 25 %, 0.5 to 0.7 s), and phase 3's current moves only 1.2 to 1.9 kA. Phase 3's current still changes when electrode 1 moves, but its impedance hardly does, so its regulator moves electrode 3 a little rather than passing the error round the phases.

🕮 **Pause.** Try Exercise 5.2 before reading on.

> **TRY IT IN THE APP** **2 · Current Regulation on All Three Phases**
>
> Load the scenario and press Run, then set Phase 1 offset to +6 %. Watch I3 on the Phase currents trend: phase 3's current swings by 2.9 to 3.8 kA although only phase 1 was asked to move, and the step overshoots 26 to 40 %. Then load scenario 3 and make the same request of the impedance regulators.

5.6.3 Bore-In, Melting and the Flat Bath

Case 4 bores a full bucket on tap 5. At the pause, when the program steps to melting, the impedance regulator had ridden 10 cave-ins and 6 short circuits, the longest 0.76 s, with no arc broken, and the program stepped up at 318 s. Current regulation at $K_I = 30$ on the same cave-ins broke 3 arcs, the

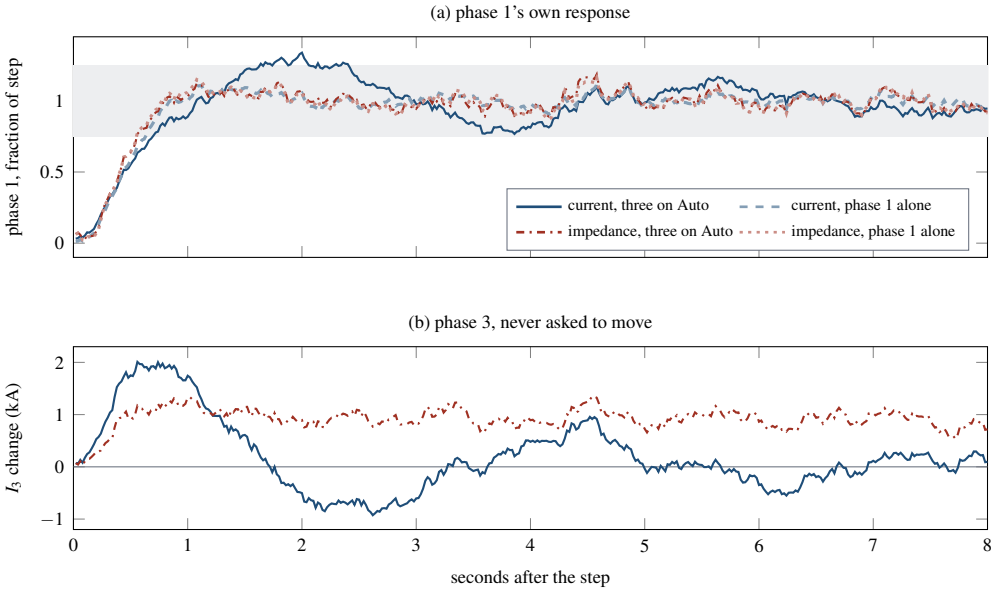


Figure 5.3 Phase 1 offset stepped 2.0 s after Run, measured signals from the running engine. (a) Phase 1's current (case 2, +6 %, $K_I = 20$) or impedance (case 3, −6 %, $K_Z = 80$) as a fraction of the step, with phases 2 and 3 on AUTO (current solid, impedance dash-dotted) or held on MAN from the start (current dashed, impedance dotted), and the band of a quarter of the step shaded. The current regulators overshoot 31.5 % together against 9.0 % alone and enter the band at 2.43 against 0.66 s. The impedance regulators overshoot 16.3 against 13.6 % and both enter at 0.60 s. (b) Phase 3's current with all three on AUTO, which swings 2.94 kA under current regulation (solid) and 1.41 kA under impedance regulation (dash-dotted).

longest out 2.6 s and 7.7 s in all, and paused at 326 s. Over six seeds the current regulator broke 2 to 5 arcs where the impedance regulator broke none. A short in one phase raises a neighbor's current by tens of kiloamperes, which a current regulator reads as its own error and answers by pulling its arc toward extinction. At $K_Z = 10$ the impedance regulator no longer reaches the mast rate on its own, and with the lift off as well the longest short grows from 0.98 to 2.04 s and three arcs break instead of one.

Case 5 melts on tap 9 at 65 kA, arcs of 317 mm against 376 mm to extinction, with 66.0 t of scrap left. With no deadband the masts travel 7.3 mm/s on average over 5 min and the longest arc reaches 344 mm. A 0.5 mΩ deadband, the size of the 14 mm flicker in impedance, cuts the travel to 1.4 mm/s while the longest arc rises only to 360 mm. At 1.0 mΩ the travel is no lower, 1.5 mm/s, the arcs wander to the band's edges and one reaches 376 mm and breaks.

Case 6 starts with 13.2 t of scrap in a 103 t bath and the slag flat, so the program runs tap 6 at 66 kA with 214 mm arcs and 59 MW. The wear index reads 0.15 at 2.5 min and 6.38 at 10 min, 21 times the challenge's budget. With Slag on at 2.5 min the program steps to tap 9 at 162 s, the bare long arcs wear the walls to 0.77 at 3.5 min while the foam is thin, and from 4 min, under 367 mm of foam rising to 443 mm, the index stops moving. The heat taps at 1629 °C after 390 s with the index at 0.79, and at 0.23 if the slag is started at 1 min.

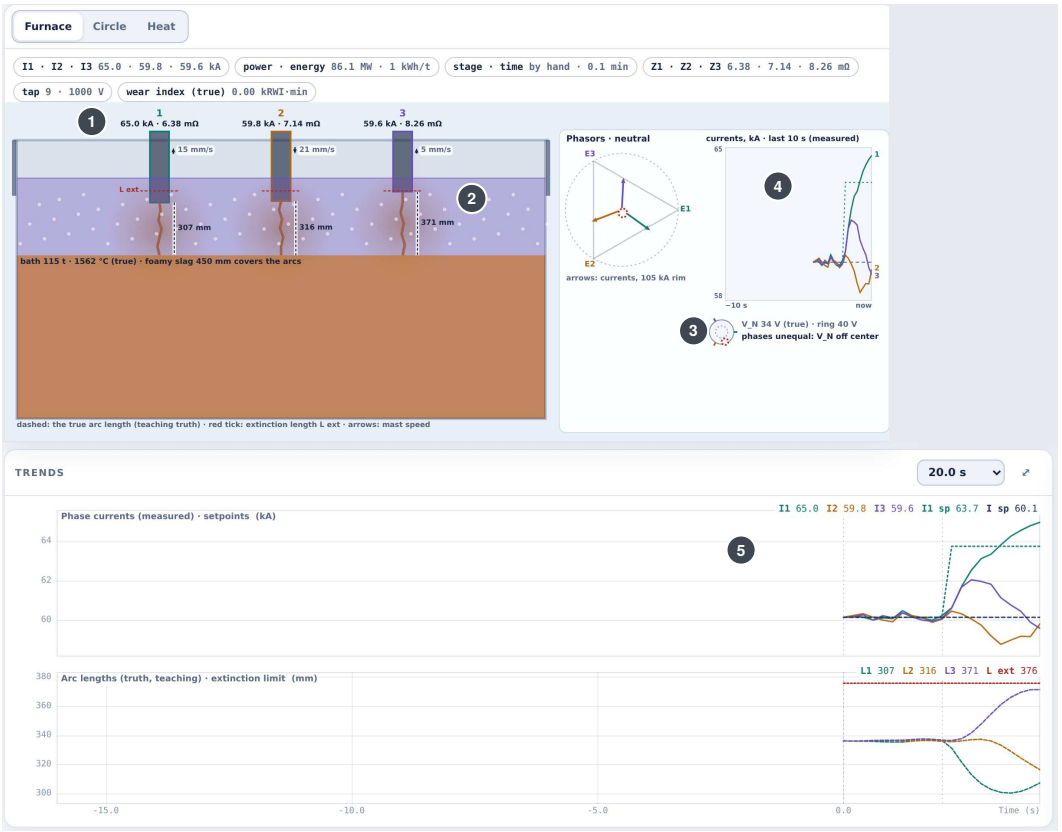


Figure 5.4 Case 2 on the Furnace view, 2.0 s after Phase 1 offset was set to +6 % with all three current regulators on AUTO ($t = 4.0$ s), cropped to the plant view and the trends. The badges (1) read 65.0, 59.8 and 59.6 kA and 6.38, 7.14 and 8.26 mΩ against a phase 1 setpoint of 63.7 kA and 60.1 kA for the others. Arc 1 has shortened to 307 mm, arc 3 (2) has drawn out to 371 mm, 5 mm from its extinction tick, the mast arrows show 15, 21 and 5 mm/s, and the bath potential sits 34 V off center on the magnified ring (3). The 10 s trace (4) and the trend (5) show I_1 past its dashed setpoint while I_3 overshoots and turns back.

> **TRY IT IN THE APP** **6 · The Flat Bath: Foamy Slag Hides a Long Arc**
>
> Load the scenario, press Run and watch the Wear index (truth) tile. With the slag flat it reads 0.15 at 2.5 min. Press Slag on then: the program steps to tap 9, the index climbs to 0.77 while the foam is thin and stops moving from about 4 min, and the heat taps at 1629 °C with the index at 0.79.

5.6.4 A Tap Step and a Broken Tip

Case 7 runs tap 8 at a fixed 6.01 mΩ, with currents of 62.7, 63.5 and 62.3 kA, arcs of 283 to 295 mm and 74.2 MW. Tap up steps the voltage 2.01 s after the button with the electrodes held. The currents jump to about 73 kA, and the regulators raise the electrodes until the impedance reads 6.01 mΩ again. Twenty seconds after the button the currents are 67.6, 67.5 and 65.9 kA, the arcs 303 to 309 mm and the power 82.9 MW. The same impedance on a higher tap is a longer arc at more current. In case 8 the tips burn 5.2 mm in 120 s and the encoders go down 5.2 mm while arc 1 stays at 336 mm. A 200 mm broken tip lengthens arc 1 to 536 mm, past extinction, the re-strike brings it down to strike 4.48 s later while the other two hold, and 3.50 s after the strike all three are within 5 % of 7.48 mΩ.

Table 5.5 The Graded Heat. Seed 5 in Detail, Then Passes Over the Build's Six Seeds and Over 21 Seeds

Configuration	T2T (min)	kWh/t	Wear	Out (s)	6	21
shipped, current $K_I = 30$, tap 9	40.3	411	0.64	195	0	0
impedance $K_Z = 40$, tap 9	38.6	412	0.63	0	0	0
current $K_I = 30$, stage taps	41.2	413	0.303	85	0	1
impedance, tap 7	43.6	405	0.40	0	4	12
current $K_I = 10$, stage taps	40.9	413	0.25	54	5	16
suggested, impedance, stage taps	40.5	414	0.23	0	6	19

Every configuration taps at 1630 °C on seed 5. T2T is tap to tap, Out the time with an arc out.

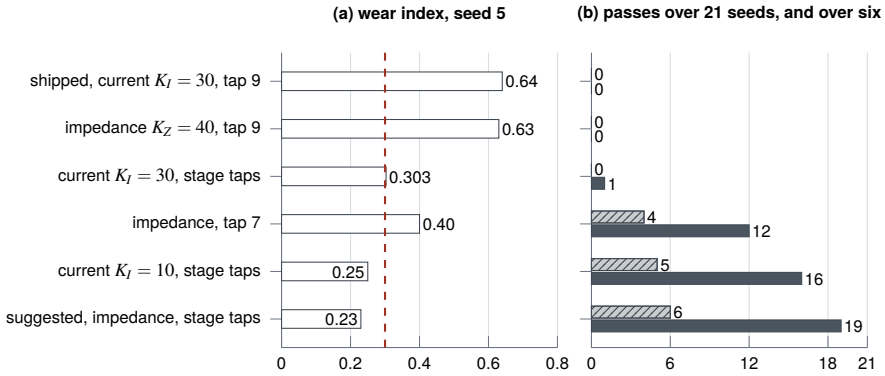


Figure 5.5 The graded heat of Table 5.5 as a chart: the wear index on seed 5 as open bars against the limit of 0.30 (dashed) in (a), and the passes over 21 seeds (solid) and over the build's six (hatched) in (b). Notice that either single change from the shipped settings, the impedance regulator or the stage taps, still misses the wear limit on seed 5 and passes at most once, while the two changes together pass 19 of 21.

5.6.5 The Graded Heat

The challenge runs one heat from a full bucket to the tap and asks for a tap at 1620 °C or hotter on the true bath, a tap-to-tap time of 48 min or less (power-on time plus the declared 10 min), at most 420 kWh/t on the meter, a wear index of at most 0.30, no arc out for more than 5 s at a time and at most 30 s of arcs out over the heat. The last criterion was added because a re-strike takes about 3.3 s, so the plan's 5 s per episode never binds, while a heat that breaks an arc every minute is not acceptable practice. Every limit is a teaching value. Real tap temperatures come from the ladle route and the caster, energy targets from a shop's benchmarks, wear limits from its refractory practice and time with arcs out from its supply contract. Table 5.5 gives the configurations on the case's seed and over 21 seeds, and Figure 5.5 draws their wear and passes.

As shipped the heat taps hot enough, after 30.3 min of power and at 411 kWh/t, but breaks 60 arcs, leaves arcs out for 195 s and wears the walls to 0.64. Each single change fails on this heat, the impedance regulator on the top tap on wear (0.63, its long arcs striking on top of a fresh pile) and the stage taps with the shipped current regulator on arcs out (85 s, 28 arcs broken). Together they tap at 1630 °C after 30.5 min of power, at 414 kWh/t with a wear index of 0.23 and no arc out. In that heat the program steps to melting at 5.29 min and to the flat bath at 24.42 min, takes three dips and taps at 30.47 min, through 43 cave-ins and 4 shorts, the longest 0.53 s. The claim is not universal. Over 21 seeds the suggested settings pass 19 times. Both misses are on wear, one from two cave-ins a second apart that left an arc burning above the pile at the top tap for half a minute, the other from wear in the first two minutes. The stage taps alone pass once and the shipped settings and the impedance regulator alone never. A current regulator detuned to $K_I = 10$ passes 16 of 21 with the stage taps,

Table 5.6 Code Verification and Agreement With Published Data

Check	Result
Code verification	
$\lvert\sum I_k\rvert$, suggested heat	$\le 1.00 \times 10^{-6}$ of I_{sc} on 182 843 steps
independent hand solve, Python	design point, case 1, short, open phase within 0.1 %
Newton against the plan's fixed point	agree to 0.1 A
circle maximum by sweep	Eq. (5.8) to 6×10^{-7} at every tap
melting and flat-bath energy	closed to 10^{-9}
whole heat, first law	0.036 % gap (6 MW before strike)
energy meter against trapezoid	0.006 %
halving Δt	suggested −0.001, shipped +0.089 %, verdicts kept on 8 seeds
load path (first, list, Reset)	9 of 9 cases identical
Agreement with published values	
currents at the program's points	60–66 kA at about 104 MVA, against arcs of 40–60 kA and a rated 47–62 kA at 80 MVA [1]
arc voltage at 100, 200, 336 mm	160, 280, 443 V, on the 40 V intercept and inside 0.5–3 V/mm [9]
energy per tonne	414 kWh/t against about 450 typical [1]
circle diagram, $R_s = 0$	radius $U^2/(2X_s)$, pf 0.707 [19]

while the impedance regulator with the stage taps passes all six of the build's seeds at gains of 20, 40, 80 and 120. The current regulator has to be detuned to live with the interaction, and the impedance regulator does not.

5.6.6 An Honestly Infeasible Heat

Without foamy slag no setting passes. Over 333 settings on three seeds, every tap program, both regulators and hand-set taps and arcs for the whole heat or from the flat bath on, and 286 more around the closest, none meets the challenge even when tapped by an oracle the instant the true bath reaches 1620 °C. An arc short enough to hide in the 80 mm slag runs at high current toward the short-circuit end of the circle, where the metered loss grows with the square of the current, and an arc long enough to carry the power wears the walls. The closest, 80 mm arcs on tap 5 from the flat bath on, misses 420 kWh/t by 0.36 kWh/t, 0.09 %, unchanged at half the time step. A margin that thin is stated beside the result, and a heat tapped on dips would miss by more. The plan's own infeasible variant, a current regulator at the shipped gain unable to ride the cave-ins, holds on the challenge's seed (none of 128 settings at $K_I = 30$ passes) but not on others (8 of 128 on one seed, 1 on another), so it is kept as a comparison and not offered as the proof.

5.7 Code Verification and Agreement With Published Data

Verification and agreement are kept apart in Table 5.6. The first block checks that the code solves its own equations. The second compares the teaching model's numbers with published ones, which supports plausibility and not validation, since no furnace was fitted.

The first-law gap of the whole heat is a declared consequence of the model, not an integration error. During the first 9 s, before the arcs strike, the panels' 6 MW is booked in the net-heat ledger while the bath, held at 1540 °C while scrap remains, does not pay it. In the suggested heat the arcs deliver 38.55 MWh and the circuit loses 3.06 MWh, a meter reading of 41.61 MWh or 414.2 kWh/t on 100.45 t, while the chemical heat spends 101.7 kWh of its 115 kWh per tonne of charge, at a mean arc power of 75.9 MW and an arc efficiency to the charge of 0.872. A heat costs 2.7 µs per physics step in Node, 0.50 s for the suggested heat, and in the browser the heat runs at the full 200× speed, 8.7 to 10.8 s of wall time in two separate sets of measurements.

5.8 Technology in Practice

Figure 5.6 draws the regulators of Section 5.5 with the hardware that measures and moves them, and the power program above them.

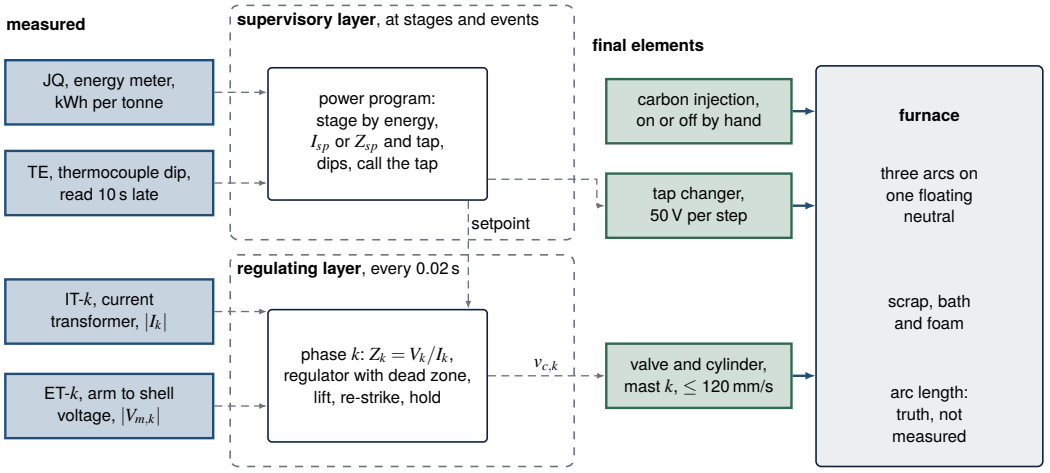


Figure 5.6 The control of the simulated furnace drawn with its hardware, for one phase k of three. The current transformer IT-k and the arm-to-shell voltage ET-k feed the phase's regulator, which commands the speed $v_{c,k}$ of the hydraulic mast. The power program reads the energy meter JQ and the dips TE and sets the regulators' setpoints and the tap, while the carbon injection that foams the slag is switched by hand. Notice that the regulator closes its loop on electrical signals alone: the arc length it aims at is never measured.

What Measures It

Both regulators of this chapter work from two signals per phase, the current in the phase's conductor and the voltage from its electrode arm to the furnace shell. In plant practice instrument transformers bring both down to levels a panel instrument can take and isolate the instrument from the high voltage of the power circuit, a current transformer for the current and a potential transformer for the voltage [20]. The model's 0.3 % noise and 0.04 s transducer filter on each current and voltage stand for this chain.

> **TECHNOLOGY** **Sensor: Current and Voltage Transformers**
>
> A current transformer is an iron ring around the power conductor, which passes through it as a primary of one turn, while many turns of secondary wire around the ring step the line current down to a standard 5 A for the instrument. A potential transformer steps the voltage down in the same way, with standard accuracy classes of 0.3, 0.6 and 1.2 % of its rated ratio. Driven through too large a burden, a current transformer's core saturates and its signal no longer follows the shape and size of the line current, and an open secondary develops a dangerous voltage [20]. The arc length itself is difficult to measure in an industrial furnace, so it is controlled through its dependence on the voltage [9].

The arc length that both regulators aim at is logged as truth in the model and read by no controller, as in a real furnace.

What Moves It

Each electrode hangs from an arm on a mast that a hydraulic cylinder raises and lowers. In the 80 MVA furnace that the chapter's circuit is drawn from, three pumps keep an accumulator of 7 m^3 at about 64 bar for the three cylinders [1]. The model's mast, with a lag of 0.2 s and a rate limit of 120 mm/s, stands for this servo without the valve's hysteresis or its leakage.

TECHNOLOGY Final Element: The Hydraulic Electrode Mast

In one 80 MVA furnace the regulator drives three control valves. Each sets the flow of oil from a pressurized accumulator to the cylinder that lifts one electrode assembly of arm, graphite column, cables and cooling water, about 17 000 kg. A command of ± 10 V takes the valve, through a hydraulic pre-amplifier, from full flow out of the cylinder to full flow in. The valve shows hysteresis, is non-linear within ± 20 % of opening and leaks about 0.5 % of its rated flow. The actuators must accelerate the assembly to 100 mm/s in about 100 to 500 ms [1].

The slower final elements are the tap changer, which steps the transformer 50 V at a time while the regulators hold the electrodes, and the carbon injection that foams the slag.

Where the Controller Runs

Control loops in a plant are often nested, and supervisory and lower-level loops run with cycle times from milliseconds to minutes [21]. In the 80 MVA furnace the three PID regulators send their outputs through an input and output card as voltage signals to the valves, and the operating data were logged over a one-second window [1]. The app runs its regulators every 0.02 s, a regulating layer that holds the three regulators, the lift, the re-strike and the recovery hold. The power program, the tap changer and the dips form a slower supervisory layer that sets the regulators' setpoints at each stage.

IN PRACTICE How a Shop Foams Its Slag

Slag foaming is critical for a productive arc furnace. The foam shields the refractories from the intense arc, which allows a high power input. Carbon is injected into the slag, where it reacts with iron oxide to form carbon monoxide, and the decarburization of the metal adds more gas. Foam made by reaction has small bubbles and can last several minutes after the gas stops, while gas injection alone gives larger bubbles and a less stable foam. Simulated furnace slags in the laboratory foamed highest at about 25 % iron oxide, although the gas generation kept rising at higher contents [15].

What Is Modernizing It

The first development is machine vision of the arc itself. On a pilot-scale alternating-current furnace the arc length was measured by image analysis and set against the furnace's electrical data, and the relation between arc length and voltage was found to be non-linear and dependent on the slag composition [9], the relation for which the linear arc law of Equation (5.3) stands in. The second is the validated furnace model. The 80 MVA furnace model was fitted to operating data so that it could serve control design, the optimization of energy consumption and an operator-training simulator [1].

5.9 Limitations

The circuit is quasi-static at 60 Hz, without the arc waveform, harmonics, supply flicker or the series reactor, and a direct-current furnace, whose single electrode has no phase coupling, is not modeled. The arc's voltage is linear in its length, while measured arcs are non-linear and slag dependent [9], and the extinction rule, the re-strike gap, the flicker, the cave-ins, the bore rate and the exposure efficiencies are teaching values. One bucket is melted, the bath is one temperature, there is no chemistry, and the foam is a height, so an iron-oxide-rich slag from hydrogen-reduced iron, which changed the arc's movement in a laboratory furnace [18], has no effect here. The DRI table is a declaration, not a product of the shaft furnace. The recovery hold is the build's design rather than a cited practice, and the regulators read perfect transducers apart from noise and a filter. Every limit of the challenge is a teaching value. The verdicts of single-change policies flip on knife-edge seeds, which is why the counts are given over 21 seeds, and the suggested settings miss 2 of them.

OPEN CHALLENGE **Melting Iron Reduced by Hydrogen**

Pellets reduced with hydrogen carry no carbon, unlike pellets reduced with natural gas, so carbon must be added in the furnace both to finish reducing the iron and to foam the slag. A changing slag composition and a larger amount of slag are among the identified challenges of the hydrogen route, and more slag dissolves more refractory and raises the power demand. Optical emission spectroscopy, already proven in pilot and industrial furnaces, offers a way to follow the arc, the residual hydrogen and the slag while such pellets melt [18]. Progress would be a slag and power practice for carbon-free charges set from measurements inside the furnace rather than from experience with scrap.

5.10 Conclusion

The arc furnace is a plant whose three loops share one floating node, and the simulator logs that node beside the currents, impedances and true arc lengths of one heat. Lowering one electrode 30 mm raises all three currents, phase 3 0.90 as much as phase 1 and phase 2 0.27 as much, and the second neighbor changes sign exactly where the neutral's push leaves a 30° cone, at 268.6 mm on the top tap. The relative gain is 0.745 for current regulation and 1.034 for impedance regulation, and on the same 6 % step three current regulators overshoot 26 to 40 % where three impedance regulators overshoot 9 to 23 % and settle as fast as one. Through bore-in the current regulator breaks 2 to 5 arcs on six seeds and the impedance regulator none, foam is the difference between a wear index of 6.38 in ten minutes and one that stops, and without foam no setting meets the heat, the closest by 0.09 %. The suggested impedance regulator with the stage taps passes the graded heat at 414 kWh/t on 19 of 21 seeds. The model is quasi-static and its charge declared. A fuller model would add the arc's waveform and its flicker on the supply, a non-linear arc law fitted to measurements, slag chemistry with a foam that follows it, a second bucket and the shaft furnace's product read rather than declared. The furnace shows why the choice of what a loop measures can matter more than how it is tuned.

KEY POINTS

- Three arcs close their circuit through one steel bath that no wire ties back to the transformer, so moving one electrode moves all three currents.
- Lowering one electrode raises all three currents, phase 3 0.90 as much as phase 1 and phase 2 0.27 as much.
- The relative gain is 0.745 for current regulation and 1.034 for impedance regulation, and on the same step three impedance regulators settle as fast as one.
- Without foamy slag no setting meets the graded heat, and the suggested settings pass it on 19 of 21 seeds.
- The choice of what a loop measures can matter more than how it is tuned.

Where else this applies. Volume 6, Chapter 2 pairs its loops by the same relative gain, and Volume 4, Chapter 2 has a protective override that acts only near its limit.

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5.11 Exercises

5.1 ⌕ Concept check · 2 min

In case 1 all three phases are on MAN on a flat bath with arcs of 336 mm, and Phase 1 position is lowered by 30 mm. What happens to the currents of phases 2 and 3?

(a) Neither changes, because no wire connects one electrode to another

(b) Both rise, phase 3 nearly as much as phase 1 and phase 2 only a little

(c) Both fall, because the three currents must add to zero when phase 1 rises

(d) Both rise by as much as phase 1, because the bath potential stays at zero

5.2 ⌕ Concept check · 2 min

In case 2 three current regulators ring on a step of phase 1's setpoint, while in case 3 three impedance regulators settle the same kind of step as fast as phase 1 alone. What explains the difference?

(a) The impedance regulators run at four times the gain, so they act before the neighbors move
(b) The impedance regulators use a deadband that ignores the neighbors' small errors
(c) Each phase's impedance is mostly its own arc's voltage over its own current
(d) The impedance step is too small to move the neighbors' currents at all

5.3 Concept check · 2 min

The short-circuit lift raises an electrode at full rate while its phase reads an impedance below $Z_{trip} = 1.0\,\mathrm{m\Omega}$. Why is a short detected by a low impedance and not by a high current?

(a) The current transformers saturate in a short, so the current reading is lost
(b) A shorted phase reads exactly zero impedance, so any positive trip level is safe
(c) A short circuit draws less current than a normal arc on the top tap
(d) With a floating neutral a neighbor of the shorted phase can carry more current than the shorted phase

5.4 Theory and calculation · 5 min

The arc of Equation (5.3) has $V_0 = 40\,\mathrm{V}$, $E = 1.2\,\mathrm{V/mm}$ and $k_{ext} = 0.85$. Taps 1, 5 and 9 give 600, 800 and 1000 V line to line.

(a) Find the arc voltage of a 336 mm arc. *Checkpoint:* 443 V.

(b) Find the phase voltage U and the extinction length L_{ext} on taps 1, 5 and 9, to the nearest millimeter. *Checkpoint:* 212, 294 and 376 mm.

(c) The application's plan set a 400 mm extinction length on the lowest tap. Show that no arc of that length can burn there. *Checkpoint:* 520 V against a phase voltage of 346 V.

5.5 Theory and calculation · 6 min

One balanced phase on tap 9 has $U = 577.4\,\mathrm{V}$, $R_s = 0.5\,\mathrm{m\Omega}$ and $X_s = 5.5\,\mathrm{m\Omega}$. Use Equation (5.8).

(a) Find $|Z_s|$, the maximum arc power P^* per phase and the current I^* at which it occurs. *Checkpoint:* 5.523 mΩ, 27.67 MW, 70.79 kA.

(b) Find the lossless bound $U^2/(2X_s)$ and how far below it P^* lies, in percent of the bound. *Checkpoint:* 30.30 MW, 8.7 %.

(c) Find the short-circuit current $U/|Z_s|$. The arc is stable above 47.61 kA. On which side of the maximum do the program's setpoints of 60 and 65 kA lie, and why is that the side to run on? *Checkpoint:* 104.5 kA, the long-arc side.

5.6 Theory and calculation · 7 min

At a balanced point the sensitivity of the three currents to the three tip heights, divided by its diagonal, is the circulant matrix

$$G = \begin{bmatrix} 1 & a & b \\ b & 1 & a \\ a & b & 1 \end{bmatrix},$$

with $a = 0.90$ and $b = 0.27$ for the currents and $a = 0.39$ and $b = 0.12$ for the impedances. The eigenvalues of a circulant matrix are $\mu_m = 1 + a\,\omega^m + b\,\omega^{2m}$ with $\omega = e^{j2\pi/3}$ and $m = 0, 1, 2$, and every diagonal element of its inverse is $\frac{1}{3}\sum_m 1/\mu_m$.

(a) Find the eigenvalues of the current matrix as magnitude and angle. *Checkpoint:* 2.17 and $0.685\angle \pm 52.7°$.

(b) Find the diagonal relative gain $\lambda = g_{11}\,(G^{-1})_{11}$ of current regulation, to three decimals. *Checkpoint:* 0.742.

(c) Repeat (a) and (b) for impedance regulation and say what the two results mean for three loops tuned one phase at a time. The chapter's values, from unrounded sensitivities, are 0.745 and 1.034. *Checkpoint:* 1.51 and $0.781 \angle \pm 17.4°$, then 1.035.

5.7 App challenge · 8 min

Load scenario 2, "Current regulation on all three phases", at the default seed with no changes, and set the speed to $1\times$. Press Run and, at any time between 2 and 8 s, set Phase 1 offset to $+6\,\%$. Run at least 10 s more and read the Phase 1 step response tile. Then load scenario 3, "Impedance regulation on the same step", and repeat with an offset of $-6\,\%$.

(a) Give the overshoot, the time after which the response stayed inside a quarter of the step, and the I3 swing for each case. Because the arcs' flicker moves them with the instant of the step, give each as a range that holds across the window. *Checkpoint:* Case 2: 26 to 40 %, 2.0 to 2.6 s, 2.9 to 3.8 kA. Case 3: 9 to 23 %, 0.5 to 0.7 s, 1.2 to 1.9 kA.

(b) Phase 1 alone, with phases 2 and 3 on Man, settles inside a quarter of the step in 0.6 to 0.7 s at $K_I = 20$ and 0.5 to 0.7 s at $K_Z = 80$. What has the interaction cost each regulator? *Checkpoint:* Current: about three to four times the settling time. Impedance: nothing measurable.

5.8 App challenge · 7 min

Load scenario 5, "Melting at the top tap: extinction and the deadband", at the default seed and set the speed to $20\times$. Run until the case pauses at 5 min and read the Mast travel tile (average mast speed and longest arc) and the Arcs broken · longest out tile. Set Impedance deadband to 0.50 mΩ, press Reset and run to the pause again. Repeat with 1.00 mΩ. Give speeds to ± 0.1 mm/s and lengths to ± 1 mm.

(a) Tabulate the three runs. *Checkpoint:* 7.3, 1.4 and 1.5 mm/s, then 344, 360 and 376 mm, then 0, 0 and 1 broken.

(b) Express each deadband in millimeters of arc with the diagonal sensitivity of 0.0355 mΩ/mm, and explain why the larger deadband buys no less travel. *Checkpoint:* About 14 and 28 mm.

5.9 App challenge · 9 min

Load the challenge, "Challenge · the full heat" (case 9), at its own seed with no changes, set the speed to $200\times$ and run until the heat pauses at TAP (about 10 s). Read the verdict, the Heat tile, the Wear index (truth) tile and the Arcs broken · longest out tile. Repeat three times after Reset: with Regulator set to Impedance alone, with Power program set to Stage taps alone, and with both.

(a) Tabulate the energy per tonne, the tap-to-tap time, the wear index, the arcs broken and the verdict for the four heats. *Checkpoint:* Only both changes together pass, at 414 kWh/t and a wear of 0.23.

(b) Say what each single change fixes and what it leaves, in terms of the chapter's cases 4 and 6. *Checkpoint:* Impedance fixes the arcs out. Stage taps hold the wear near the budget.

5.10 Application area · 3 min

In case 6 the last 13.2 t of scrap melts with the slag left flat. The program then runs tap 6 at 66 kA with 214 mm arcs and 59 MW, and the wear index reaches 6.38 at 10 min. With Slag on at 2.5 min the program moves to tap 9 and the heat taps at 1629 °C after 390 s with the index at 0.79. Why does a foamy slag allow both more power and less wall wear, and why can no setting pass the challenge without it?

5.11 Application area · 3 min

Case 7 runs tap 8 at a fixed impedance setpoint of 6.01 mΩ with about 63 kA, arcs of 283 to 295 mm and 74.2 MW. Tap up steps the voltage 2.01 s after the button with the electrodes held, and 20 s after the button the furnace runs at about 67 kA, arcs of 303 to 309 mm and 82.9 MW.

(a) Explain why the same impedance on a higher tap gives a longer arc at more current and more power. *Checkpoint:* The tap moves the circle. The impedance fixes the ratio $|V_m|/|I|$.

(b) The tap changer holds the regulators for 2 s. Why must the short-circuit lift still act during that hold? *Checkpoint:* A cave-in during a tap step stayed shorted 2.06 s. With the lift acting, 0.53 s.

5.12 Review the agent · 5 min

An AI assistant tuning the current regulators writes: "With phases 2 and 3 on Man, phase 1 at $K_I = 20$ overshoots a 6 % step by only 8 to 16 % and settles in under 0.7 s. The three phases are identical, so the same tuning holds for all three. To settle faster, raise all three to $K_I = 30$, which the single-phase test shows is still well damped." Find the flaw, and say how you would check it in the app before anyone uses the gain.