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CHAPTER 6

A Mining Thickener Simulator for Teaching Override Selector Control

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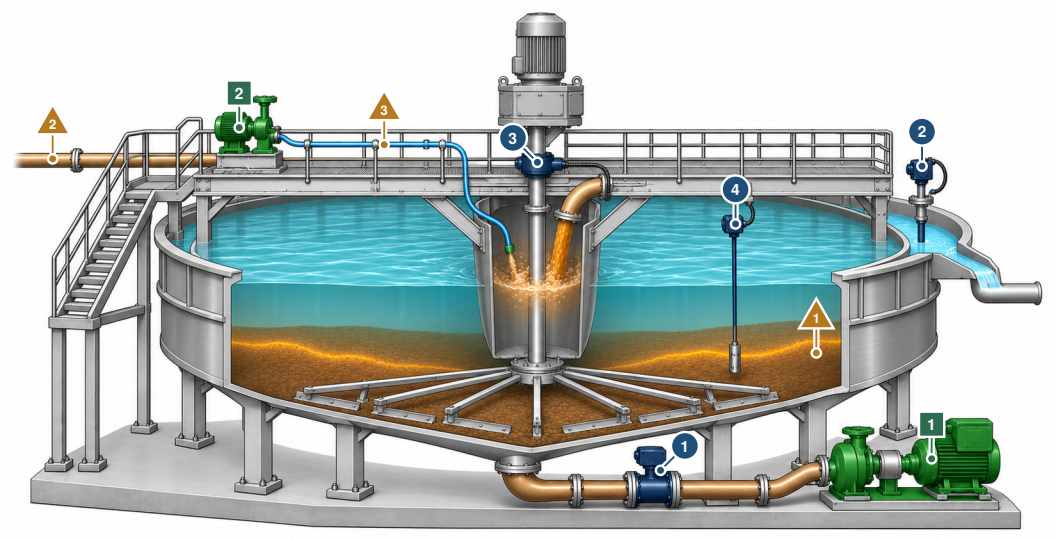
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CHAPTER 6

A Mining Thickener Simulator for Teaching Override Selector Control



The controller moves 1 the speed of the underflow pump 2 the flocculant dose

The controller reads 1 the density of the underflow 2 the clarity of the overflow water 3 the load on the rake 4 the depth of the settled bed

What disturbs it 1 finer ore that settles more slowly 2 more feed, or a thicker one 3 weaker flocculant from the make-up

Notice that the one pump at the bottom sets the underflow density, the bed depth and the load on the rake together. Illustration, not to scale.

IN PLAIN TERMS **Squeezing Water Out of Mine Waste**

The main idea. A thickener is a huge round tank where ground rock settles out of the water used to process it. Clear water spills over the rim to be used again, and a pump at the bottom draws off the thick mud.

The challenge. Slowing the pump keeps the mud in the tank longer, so it packs denser and less water is lost. But a deeper, heavier layer is harder for the slow rake to drag, so two safety controllers must be able to speed the pump up first.

Why it matters. Water that leaves with the mud must be replaced, and a stalled rake in a full tank takes days to recover.

THREE WAYS TO READ THIS CHAPTER

Explore	9 min	The first page, the two history boxes of Section 6.1, the fact box of Section 6.2.2, the try-it box of Section 6.4.1, the fact box of Section 6.4.4, and the key points
Study	35 min	The whole chapter with its pause points and the twelve exercises
Practice	10 min	The first page, Section 6.4, Section 6.5 and Section 6.6 with its open challenge

The control question Slowing the underflow pump is the only way to make a denser underflow, and it is also what deepens the bed the rake has to drag. How should a density controller and two protective controllers share that one pump, and what happens when the selector between them is wired the wrong way?

Learning objectives

- Compute the bed residence time, underflow density, bed depth and rake torque at an operating point from the compression and torque laws.
- Choose a selector's direction from the action of each controller and the direction that relieves each limit.
- Explain what external reset does to a losing controller and predict when a protection takes the pump.
- Compare the selector with the predictive controller on torque, density error and pump travel, and say what each protection costs.
- Choose a density setpoint the thickener can hold through a shift of disturbances.

Prerequisites Sections 1.4.3 (density against torque), 1.4.6 (selector direction and external reset), 1.6.2, 1.6.3 and 1.6.4.

The app https://apmonitor.com/apps/thickener/

See also Volume 4, Chapter 2 and Volume 4, Chapter 5 for protective overrides on other machines, and Volume 4, Chapter 4 for a low selector where less feed is the safe direction. Volume 2, Chapter 3 for back-calculation at an actuator bound.

6.1 Introduction

A mining thickener recovers process water by letting solids settle. The feed enters a large shallow tank through a central feedwell, the solids form a bed, the clarified water overflows a peripheral launder and is returned to the plant, and a slowly rotating rake drags the thickened solids to a central discharge cone where a variable-speed pump removes them.

> **HISTORY A Shallow Tank with Slow Scrapers (1907)**
>
> In 1907 John V. N. Dorr, of Lead, South Dakota, patented a decanting apparatus for the slimes of the cyanide process. The slimes had to be thickened until they carried one to three parts of liquid to one part of solids. In the cones used for this, solids collected on the slopes and then slid down suddenly, choking the discharge. Dorr's tank was of large diameter but shallow, 35 feet across and no more than 12 feet deep in the example he gave, and scrapers on slowly turning arms moved the thickened material gently and continuously to a central discharge without stirring the liquid above. The arms could be raised and lowered [1]. The vessel of this chapter, with its rake and central cone, keeps that form.

The economic objective is the densest underflow the unit can produce, because every cubic meter of water that leaves with the tailings has to be replaced. The limit on that density is mechanical rather than economic. The two collide directly because only one quantity sets the underflow density at steady state. Solids leave the bed at the concentration the compression zone has produced, and that concentration depends on how long the solids have been under the weight of the bed above them, as shown in Equation (6.1).

$$\begin{aligned} \tau_b &= \frac{M_s}{F_{\text{in}}}, \qquad X_u^* = X_{\min} + \Delta X \left(1 - \mathrm{e}^{-\tau_b/\tau_c}\right) \\ X_u &= X_u^* \, \sigma^{0.15} \max\left(0.80,\ 1 - 0.06 \max\left(0, \frac{D_f}{D_0} - 2.2\right)\right) \end{aligned} \tag{6.1}$$

where M_s is the solids inventory in tonnes, F_{in} the solids feed rate in tonnes per hour, τ_b the bed residence time in hours, X_u the underflow solids concentration in kg/m^3, $X_{\min}$ and ΔX the uncompressed density and the compression span, τ_c the compression time constant, σ the ore settleability as a fraction of the design value, D_f the flocculant dose and D_0 the reference dose. The second line is the overdose penalty, because a floc built at more than 2.2 times the reference dose

Table 6.1 Steady-state cost of underflow density at design feed. The rake trip is at 92 %.

Density (%)	Pump (%)	τ_b (h)	Bed (m)	Torque (%)
55	66.47	2.60	1.093	31.4
60	58.00	3.87	1.419	45.9
62	54.99	4.59	1.597	54.9
64	52.17	5.55	1.831	67.2
66	49.52	6.93	2.170	85.4
68	47.03	9.36	2.783	117.5

holds water and yields a thinner underflow at the same residence time. Every symbol carries the value listed in Table 6.4. Equation (6.1) is an inventory whose product quality depends on holdup, which is a structure that recurs wherever a residence time sets a conversion [2]. Slowing the underflow pump raises the inventory, which raises the residence time, which raises the density, and no second actuator has that effect. The inventory is also what the rake has to drag through, so the torque of Equation (6.2) rises with the depth and the density together.

$$T_q = T_{q0} + k_\tau h_b \left(\frac{X_u}{X_{\mathrm{ref}}} \right)^n \tag{6.2}$$

Here T_q is the rake torque as a percentage of the rated value, T_{q0} the torque with no bed, k_τ the torque per meter of bed, h_b the bed depth in meters, X_{ref} a reference density and n the density exponent.

Table 6.1 states the design problem in full. The relationship is convex, because the step from 66 to 68 % solids costs 32.1 points of rated torque while the whole span from 55 to 62 % costs 23.5. The 92 % trip is reached at 66.54 % solids, so a setpoint above that value is not aggressive but infeasible.

That structure is what override control exists for. The application measures four properties of it: whether it works, what it costs, what external reset contributes, and what happens when the selector is wired backwards. The same structure, a protective controller and a quality controller sharing one actuator, protects a compressor near surge in Volume 4, Chapter 2 and the rotor and engine of a combine harvester in Volume 4, Chapter 4, where less feed is the safe direction and the selector is a low select.

> ⧖ HISTORY **Sizing a Thickener by Experiment (1916)**
>
> The first equation for the capacity of a thickener, a mass balance, was published by Mishler in 1912. In 1916 Coe and Clevenger divided a continuous thickener into four zones, from clear liquid at the top to a compression zone at the bottom, and sized the tank from laboratory settling tests. Where water has to be pressed out of the pulp, they held, the tank must keep the compressing pulp long enough to reach the required density. Used with a safety factor, their method was still the most popular in the mining industry in 1993, and the theory of sedimentation that Kynch published in 1952 opened a new field of research [3]. The compression law of Equation (6.1) rests on the same residence time.

The scenarios built on the six states cover a specific list of process dynamics and control principles, namely an integrating inventory whose holdup sets product quality, a slow dominant time constant against fast measurement filters and transport delays, actuator saturation at both ends of the pump and the reagent, override selection between competing controllers, external reset as the anti-windup mechanism of a selector, and a strongly nonlinear gain that steepens toward a constraint [2, 4, 5]. The physical fundamentals are a solids mass balance on the vessel, a compression rate law driven by residence time, a hindered settling velocity set by feed concentration and reagent dose, and a separation of time scales between a bed that turns over in four and a half hours and instruments that respond in minutes [6, 7]. Water recovery is the economic reason the industry cares, because

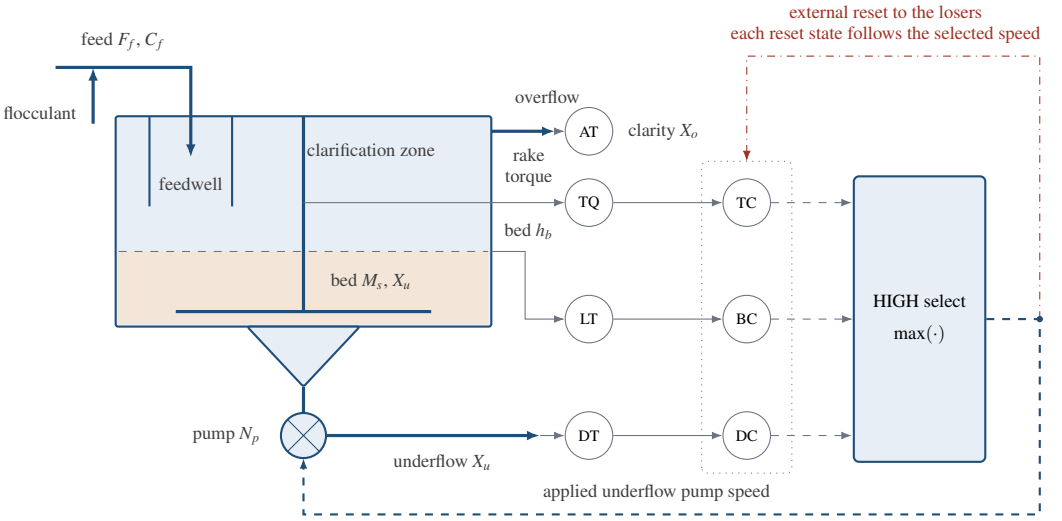


Figure 6.1 The simulated thickener. Only the underflow pump sets the underflow density at steady state, and it does so by setting the bed inventory, which is also what the rake has to drag. Three controllers ask for pump speed at once, the density controller for the plant's benefit and the torque and bed controllers for the machine's, and a high selector passes whichever wants the most pump. The dash-dotted path (red) is the external reset, which feeds the selected speed back to losing loops over their tracking times, and the dashed lines carry the three demands and the applied speed.

a concentrator that loses water to its tailings pays for the replacement, and a tripped rake in a full bed takes days to recover [8]. Section 1.4.3 places the thickener in the circuit of Chapter 1, and Section 1.4.6 states the rule for a selector's direction.

Section 6.2 gives the model: compression, clarity and flocculation, the bound on the inventory, the six state equations and every parameter. Section 6.3 describes the four PID loops, the high selector, its external reset and the optional predictive controller. Section 6.4 reports a pump move by hand, every scenario under the selector and under the MPC, the thickening-feed case in detail and the graded shift. Section 6.5 describes the real instruments, pump and control platform, Section 6.6 states what the model leaves out, and Sections 6.7 and 6.8 draw the lessons.

6.2 Process Model

Figure 6.1 shows the vessel, the three controllers that compete for the underflow pump and the selector between them.

6.2.1 Variables and Signal Roles

Table 6.2 classifies the quantities the operator and the four controllers act on. Six of them are integrated states and the rest are algebraic functions of those states, the parameters and the disturbances.

🕮 **Pause.** Try Exercise 6.2 before reading on.

6.2.2 Design Operating Point

The simulation clock runs in minutes. The vessel is 30 m in diameter, an area of 707 m^2, with the feedwell 3.4 m above the floor. At design it takes 1200 m^3/h of feed at 150 kg/m^3, which is 180 t/h of solids, with the pump at 55 % and the flocculant at 19.7 g/t. That holds 827 t of solids in a 1.597 m bed at a residence time of 4.59 h, discharges 177 m^3/h of underflow at 1017 kg/m^3, which is 61.99 % solids, returns 1023 m^3/h of water at 30.0 mg/L, and loads the rake to 54.9 %. The 19.7 g/t dose is

Table 6.2 Signal roles. Nominal values are the design steady state of Section 6.2.2. The range column gives the setpoint range for a controlled variable and the adjustable range for everything else.

Symbol	Role	Nominal	Range
X_u	CV, state	61.99 % solids	48–70 %
X_o	CV, state	30.0 mg/L	10–200 mg/L
T_q	override CV, state	54.9 %	55–90 %
h_b	override CV	1.597 m	1.00–3.20 m
N_p	MV, state	55.0 %	10–100 %
D_f	MV, state	19.7 g/t	0–70 g/t
M_s	state	827 t	—
F_f	DV	1200 m^3/h	500–2100 m^3/h
C_f	DV	150 kg/m^3	70–280 kg/m^3
σ	DV	1.00	0.35–1.60
s_f	DV	1.00	0.15–1.30
w_p	DV	1.00	0.60–1.10

the value the clarity loop settles at against its 30 mg/L setpoint, and the 100 mg/L recycle limit is therefore a factor of 3.3 above the design overflow.

> **DID YOU KNOW? Twelve Thickeners Behind One Plant**
>
> A 2017 survey of tailings thickeners lists a copper plant treating 32.5 million tonnes of ore a year with twelve paste thickeners, each 24 m across, and another of 14 million tonnes a year with two high-rate thickeners 45 m across [9]. The vessel of this chapter is 30 m across.

6.2.3 Overflow Clarity and Flocculation

The overflow carries whatever does not settle faster than the water rises. The application makes that an exponential in the ratio of the two,

$$v_r = \frac{F_f - F_u}{A}, \qquad X_o = X_{o,0}\exp\!\left(g\,\frac{v_r}{v_s}\right) \tag{6.3}$$

where v_r is the rise rate in m/h, F_f the feed rate, F_u the underflow rate, A the vessel area, X_o the overflow solids concentration, $X_{o,0}$ a clarity scale factor and g the clarity sensitivity. The exponential is flat while the ratio is small and steepens sharply as the rise rate approaches the settling velocity. At design the ratio is 0.257 and the overflow is 30.0 mg/L, and the ratio is clamped to 4 in the implementation so the exponential cannot overflow numerically.

Settling velocity depends on the flocculant through a Langmuir curve, so the reagent saturates,

$$v_s = v_0\,\sigma\,\frac{D_f/(D_f + K_d)}{D_0/(D_0 + K_d)}\,\mathrm{e}^{-k_h C_f} \tag{6.4}$$

where v_0 is the free settling velocity of flocculated solids, σ the ore settleability, D_f the flocculant dose, D_0 the reference dose, K_d the half-saturation dose, k_h the hindered settling coefficient and C_f the feed solids concentration in t/m^3. Table 6.3 gives the resulting operating map, and the graded shift of Section 6.4.4 shows what happens when the reagent runs out of authority.

The saturation is what limits the reagent as a handle. Doubling the dose from 30 to 60 g/t raises the settling velocity from 6.459 to 7.536 m/h, an increase of 17 %, and cuts the overflow from 22 to only 17 mg/L, while the rake torque rises from 54.9 to 56.7 % because the overdose term of (6.1) forces a 1.664 m bed to hold the same underflow density that a 1.597 m bed holds at the design dose. Cutting the dose is the expensive direction, because 5 g/t puts the overflow at 396 mg/L, four times the recycle limit, at a torque that is 0.4 % lower than the design value. Flocculant is therefore a trim rather than a lever.

Table 6.3 Flocculant dose against settling velocity and overflow at design feed. The dose is bounded at 70 g/t.

Dose (g/t)	v_s (m/h)	Overflow (mg/L)	Torque (%)
5	2.660	396	54.5
10	4.110	70	54.9
19.7	5.620	30	54.9
30	6.459	22	54.9
60	7.536	17	56.7

Pause. Try Exercise 6.3 before reading on.

6.2.4 Upper Bound on the Solids Inventory

If the pump cannot remove solids as fast as they arrive, the bed grows. It cannot grow forever, and the mechanism that stops it is the one that matters operationally. Once the bed reaches the feedwell there is no clarification zone left, the feed short-circuits to the launder, and the solids leave over the weir instead of through the pump. The application makes that explicit by blending (6.3) into a solids balance above $h_b = 2.9\,\mathrm{m}$, which both bounds the inventory and gives the steady-state solver a root to bisect. It is also what makes the failure mode in Section 6.4.4 so expensive, because an overflow of tens of thousands of milligrams per liter is not a clarity excursion but the tailings going to the process water tank.

6.2.5 State Equations

The six integrated states are the solids inventory, the underflow density, the pump speed, the flocculant dose, the overflow solids and the rake torque, as given by Equation (6.5).

$$\begin{aligned} \frac{\mathrm{d}M_s}{\mathrm{d}t} &= \frac{1}{60}\left[F_{\mathrm{in}} - \frac{F_u X_u}{1000} - \frac{F_o X_o}{1000}\right] \\ \tau_{\mathrm{comp}}\frac{\mathrm{d}X_u}{\mathrm{d}t} &= X_u^{\mathrm{ss}} - X_u, \qquad \tau_p \frac{\mathrm{d}N_p}{\mathrm{d}t} = N_p^{\mathrm{cmd}} w_p - N_p \\ \tau_f \frac{\mathrm{d}D_f}{\mathrm{d}t} &= D_f^{\mathrm{cmd}} s_f - D_f, \qquad \tau_{\mathrm{clar}}\frac{\mathrm{d}X_o}{\mathrm{d}t} = X_o^{\mathrm{ss}} - X_o \\ \tau_q \frac{\mathrm{d}T_q}{\mathrm{d}t} &= T_q^{\mathrm{ss}} - T_q \end{aligned} \tag{6.5}$$

with the algebraic relations of Equation (6.6).

$$\begin{aligned} F_u &= \min(k_p N_p, F_f), \qquad F_o = \max(0, F_f - F_u) \\ h_b &= \frac{1000\,M_s}{A\,\phi_b X_u} \end{aligned} \tag{6.6}$$

Here t is in minutes, M_s in tonnes, the flows in m^3/h, $F_{\mathrm{in}} = F_f C_f$ in t/h, and the factor of 60 converts the hourly solids terms to the minute clock. The superscript ss denotes the steady-state expressions of (6.1), (6.3) and (6.2), each of which the corresponding state approaches through a first-order lag. The mean bed concentration is a fixed fraction ϕ_b of the underflow concentration, which is what turns an inventory into a depth. Pump wear w_p and flocculant make-up strength s_f are operator-set disturbances that scale the commanded values, and both are unity at design.

6.2.6 Parameters, Initialization and Assumptions

Table 6.4 lists every constant with its value and unit.

Table 6.4 Model parameters and their design values. The pump is bounded to 10–100 % and the flocculant dose to 0–70 g/t.

Symbol	Meaning	Value	Unit
A	vessel area	707	m^2
k_p	pump capacity	3.218	m^3/h per %
ϕ_b	bed to underflow concentration ratio	0.72	—
$h_{b,\max}$	bed depth reaching the feedwell	3.4	m
$X_{\min}$	uncompressed underflow density	400	kg/m^3
ΔX	compression density span	850	kg/m^3
τ_c	compression time constant	3.55	h
τ_{comp}	underflow density lag	25	min
v_0	free settling velocity	12	m/h
k_h	hindered settling coefficient	4.5	m^3/t
K_d	flocculant half-saturation dose	12	g/t
D_0	reference flocculant dose	25	g/t
$X_{o,0}$	clarity scale factor	2.963	mg/L
g	clarity sensitivity to rise rate	9	—
τ_{clar}	clarification zone lag	12	min
T_{q0}	rake torque with no bed	12	%
k_τ	torque per meter of bed	26.875	%/m
n	torque exponent on density	2.2	—
X_{ref}	torque reference density	1017	kg/m^3
τ_q	rake torque lag	4	min
τ_p	pump lag	0.5	min
τ_f	feedwell mixing lag	1.5	min
θ_d	density gauge delay	1.2	min
θ_c	turbidity analyzer delay	2.5	min
τ_{bm}	bed level probe filter	3.0	min
τ_{qm}	rake torque signal filter	3.0	min
—	rake torque alarm and trip	75, 92	%
—	recycle water clarity limit	100	mg/L
—	integration step	0.02	min
—	controller scan	0.2	min

Initialization bisects the solids inventory at which the solids balance closes, which is monotone because a larger inventory means a denser underflow at the same volumetric rate. A manual-mode helper inverts the same balance to return the pump speed that would hold a chosen density.

The bed is one lumped inventory with an empirical compression law rather than a solved settling-velocity partial differential equation, which is the standard modeling framework for these units [6, 10] and is deliberately not used here. The mean bed concentration is assumed to be a fixed fraction of the underflow concentration, the rake torque is assumed algebraic in depth and density, and the four instruments are assumed to be first-order filters or pure delays with no drift. The engine of Section 1.6.1 integrates with fixed-step RK4 at 0.02 min and a 12 s controller scan.

6.3 Controller Design

All four loops run the velocity-form PID of Section 1.6.2, Equations (1.10) to (1.13), with the derivative taken on the measurement [11]. The scan is $\Delta t_c = 0.2$ min. The output is clipped to the actuator range, 10 to 100 % on the pump and 0 to 70 g/t on the flocculant. The bias is zero, and the integral starts at the output in force at $t = 0$. Anti-windup is back-calculation with a tracking time $T_t = \tau_I$, so while the output is clipped the integral moves toward the value that reproduces it by $\Delta t_c / \tau_I$ of the excess each scan ($1/1500$ on the density loop at $\tau_I = 300$ min), as on the blending tank of Volume 2, Chapter 3. The running sum of proportional and derivative increments is never truncated. The tracking time affects the reported results. In a counterfactual run in which the integral absorbs

Table 6.5 Controller tunings. The protection loops are the same in both sets. Derivative time is zero on every loop. The last four rows apply only with the pump on the predictive controller of Section 6.3.3.

Loop	Gain unit	Default	Faster
Density, K_c	% pump per % solids	3	6
Density, τ_I	min	600	300
Clarity, K_c	g/t per mg/L	0.25	0.90
Clarity, τ_I	min	60	15
Rake torque, K_c	% pump per % torque	2.2	
Rake torque, τ_I	min	120	
Bed level, K_c	% pump per m	150	
Bed level, τ_I	min	100	
MPC, T_s and horizon	min	10, 1200	
MPC move price, λ	—	30	
MPC filter intensity, ρ	—	1	
MPC limit margins	% torque, m bed	0.5, 0.02	

the whole excess in one scan, the backwards selector of scenario 11 peaks at 100.9 % rather than 109.9 %, and the recommended configuration of Section 6.4.4 misses its density band. The values measured with the shipped tracking time are in Tables 6.6 and 6.8.

All four loops set $a = -1$, because the process gain is negative in every case. A faster pump gives a thinner underflow, a shallower bed and less torque, and more flocculant gives a cleaner overflow, so a measurement above setpoint has to be answered with more of the manipulated variable. The controller output therefore rises as the measurement rises, which makes all four direct acting in the convention of Section 1.6.2, the one used in the process control literature [2, 11]. The application names them direct acting as well. It is the common sign across the three pump loops that makes a single selector correct for all of them.

Two tunings ship with the application and both appear in the results. Table 6.5 lists them. The default set is deliberately slow, which is what the manual and exploratory scenarios use, and a faster set drives the automatic scenarios of Section 6.4. The integral times follow from the SIMC rules [12] applied to gains measured on the application, and the protection loops keep one tuning throughout.

6.3.1 Override Selector

Three controllers compete for one pump,

$$N_p = \max\left(N_{\text{dens}},\, N_{\text{torque}},\, N_{\text{bed}}\right) \tag{6.7}$$

where N_{dens}, N_{torque} and N_{bed} are the three pump demands. The direction of (6.7) is the design decision that matters [2, 5]. Both protecting controllers relieve their constraint by speeding the pump up, so the safe action is more pump and the selector has to pass the largest demand. A switching margin of 0.75 % of pump speed is applied, so the incumbent keeps the actuator until another demand beats it by that margin, which stops three controllers whose outputs external reset draws toward the same speed from trading the pump back and forth on measurement noise. The application offers the same wiring as a low select in scenario 11, whose results are in Section 6.4.2.

🕮 **Pause.** Try Exercise 6.1 before reading on.

6.3.2 External Reset

After each loop's ordinary PID update, external reset adds a correction from its locally clamped request u_j to the selected command u_{sel}:

$$I_{j,k+1} = I^{local}_{j,k+1} + \frac{\Delta t_c}{T_{t,j}}\left(u_{sel,k} - u_{j,k}\right). \tag{6.8}$$

Here I_j is the integral term of controller j, the I of Equation (1.12), $I_{j,k+1}^{local}$ is its value after the ordinary update and Δt_c is the controller scan. The local update already includes the loop's own output-clamp back-calculation, so the additional term counts only the selector mismatch. The tracking time is $T_{t,j} = \tau_{I,j}$. A finite tracking time matters: assigning the entire selected command to every losing integral makes its next request jump by its proportional term, and alternating losers can ratchet the pump to its limit. With finite reset feedback, ownership and pump motion follow the controller dynamics rather than that scan artifact. The winner needs no selector correction. A switching margin of 0.75 % suppresses small noisy ownership changes. Turning reset off is a windup experiment. Manual transfer retains the command after protection, including when MPC supplies the competing request.

6.3.3 Model Predictive Controller

A model predictive controller (MPC) can take the density controller's place on the pump. The density controller remains the default in the application and in every scenario except scenario 12, and it is the fallback. Under the MPC the pump command is still the output of the high selector, as shown in Equation (6.9).

$$N_p^{\mathrm{cmd}} = \max\left(N_{\mathrm{MPC}},\, N_{\mathrm{torque}},\, N_{\mathrm{bed}}\right) \tag{6.9}$$

The torque and bed controllers keep their tunings, the switching margin and the external reset of Equation (6.8), so the override structure of Figure 6.1 stays in service above the plan as a backstop, and the density controller tracks the applied speed exactly on every scan so that a return to it is bumpless. Every $T_s = 10\,\mathrm{min}$, or 50 scans, one quadratic program plans the pump over $H = 120$ steps, a horizon of 20 h, in twelve move blocks, short near the present and long toward the end, the first four of one step each and the last of 75 steps. Only the first move is carried out, and it holds until the plan is renewed [13], which is the receding-horizon form of Section 1.6.3, used in industrial practice [14].

The controller's model is not identified. It is the balance of Equations (6.1) to (6.6) restricted to the four states the pump moves, evaluated with the declared values of Table 6.4 and the design feed, as shown in Equation (6.10).

$$\begin{aligned} \frac{\mathrm{d}M_s}{\mathrm{d}t} &= \frac{1}{60}\left[F_{\mathrm{in}}^0 + d_1 - \frac{F_u X_u}{1000} - \frac{F_o X_o}{1000}\right] \\ \tau_{\mathrm{comp}}\frac{\mathrm{d}X_u}{\mathrm{d}t} &= X_u^{\mathrm{ss}}(M_s) + d_2 - X_u \\ \tau_p\frac{\mathrm{d}N_p}{\mathrm{d}t} &= N_p^{\mathrm{cmd}} - N_p, \qquad \tau_q\frac{\mathrm{d}T_q}{\mathrm{d}t} = T_q^{\mathrm{ss}}(h_b, X_u) - T_q \end{aligned} \tag{6.10}$$

Here $F_{\mathrm{in}}^0 = 180\,\mathrm{t/h}$ is the design solids feed of $1200\,\mathrm{m^3/h}$ at $150\,\mathrm{kg/m^3}$, the settleability is one, the dose is the reference dose D_0, the pump delivers its command, and d_1 in t/h and d_2 in $\mathrm{kg/m^3}$ are disturbances the model cannot explain. A change to a plant parameter, the feed or the ore therefore reaches the plant and not the model, and appears to the controller as mismatch. The flocculant is left out, because the delivered dose is the command times a make-up strength s_f that the controller is never told, and the overdose penalty of Equation (6.1), when it occurs, lands in d_2. At the design point the model's steady state equals the plant's at 61.9945 % solids, 54.905 % torque and a 1.5969 m bed, and its steady gains for a pump step from 55 to 56 % are −0.6796 points of solids, −3.3519 points of torque and −0.065 23 m of bed per point of pump, against −0.6796, −3.3517 and −0.0652 measured on the plant. Its slow eigenvalue corresponds to a time constant of 903 min, the bed, and its fast one to 25.7 min, the compression lag.

At each decision the four states are linearized at the present estimate and the speed applied now by central differences and sampled with an exact zero-order hold that carries the affine remainder, so the steep dependence of torque on depth and density in Equation (6.2) enters at the local operating point. The decisions are block moves ΔN_j of the pump command, the first measured from the speed

applied now, and each plan minimizes Equation (6.11).

$$\begin{aligned} \mathscr{J} = &\sum_{k=1}^{H} \left(\frac{\hat{w}_k - w_{\mathrm{SP}}}{0.25\,\%} \right)^2 + \lambda \sum_{j=1}^{12} \left(\frac{\Delta N_j}{1\,\%} \right)^2 \\ &+ \kappa_T \varepsilon_T^2 + \nu_T \varepsilon_T + \kappa_h \varepsilon_h^2 + \nu_h \varepsilon_h \\ \text{s.t. } &\hat{T}_{q,k} \le \bar{T}_k + \varepsilon_T, \quad \hat{h}_{b,k} \le \bar{h}_k + \varepsilon_h, \quad \varepsilon_T, \varepsilon_h \ge 0 \\ &10\,\% \le N_j \le 100\,\% \end{aligned} \tag{6.11}$$

Here $\hat{w}_k$, $\hat{T}_{q,k}$ and $\hat{h}_{b,k}$ are the predicted underflow solids in percent by mass, rake torque and bed depth k steps ahead, w_{SP} is the density setpoint, j counts the blocks, N_j is the command in block j, and ε_T and ε_h are slacks. With $\lambda = 30$, a move of one point of pump at a block boundary costs as much as a density error of 1.37 points held for one step. The limits are $\bar{T} = T_{\mathrm{SP}} - 0.5\,\%$ and $\bar{h} = h_{\mathrm{SP}} - 0.02\,\mathrm{m}$, under the setpoints of the two override controllers and read where those controllers read them, so a plan that settles on a limit leaves the overrides idle. The bed limit is a ceiling because the bed controller only ever asks for more pump. Both limits are soft. Each output carries a single slack over the whole horizon, so each slack is the deepest predicted crossing, priced at $\kappa_T = 10^5$ per $\%^2$ and $\nu_T = 10^4$ per % of torque and $\kappa_h = 10^7$ per m^2 and $\nu_h = 10^6$ per m of bed. A limit the estimate has already crossed is re-entered along a 60 min funnel. The command range is the pump range every loop clamps to. A dual active-set method [15] solves the program, and a plan that is not reported optimal, is not finite or leaves its range is rejected. With one block, density alone and $\lambda = 30$, the solver's first move for a one-point setpoint step is −2.367 033 736 % against −2.367 033 739 % by hand.

A steady-state Kalman filter estimates ten states, the four of Equation (6.10), the density gauge with its 1.2 min delay written as a first-order lag of the same length, the torque and bed signal filters, and the random-walk disturbances d_1, d_2 and a torque offset d_3 added to the predicted torque. Its gain is computed once from the model linearized at the design point, with measurement variances of $(0.25\,\%)^2$, $(1.2\,\%)^2$ and $(0.03\,\mathrm{m})^2$, the noise levels of the graded shift, and process variances per decision of 4ρ $(\mathrm{t/h})^2$, 25ρ $(\mathrm{kg/m^3})^2$ and 0.04ρ $\%^2$ on the three disturbances with $\rho = 1$. Each decision it integrates the nonlinear model over the period with the mean command applied and corrects with the density gauge, the torque signal and the bed probe, the three signals the PID loops read. It runs under both pump options, and at $t = 0$ it anchors on the measurements with the plant taken to be at rest. The estimated disturbances make the plan offset-free [16].

The design first proposed was $T_s = 1\,\mathrm{min}$ with 120 steps. After a one-point pump step at the design point the plant takes 892, 850 and 832 min to reach 63 % of its density, torque and bed change, so a 2 h horizon sees 10.0 to 14.1 % of the change and a 20 h horizon 74.5 to 76.4 %. Before the reset correction, the weights were set once, on scenario 12, from 40 pairs of λ and ρ scored by the mean of the density error, torque excess and bed excess integrals relative to the selector's, admitting only pairs that moved the pump no more than the selector. Twelve were admitted, and the largest λ within 10 % of the best score is $\lambda = 30$ with $\rho = 1$. At $T_s = 1\,\mathrm{min}$ none was admitted, the least travel being 147.3 against 123.1 %, and without that condition the rule picks $\lambda = 30$ with $\rho = 0.1$. Section 6.4.3 reports both periods.

A rejected plan hands the pump to the density controller, which has been tracking it, and control returns at the next accepted plan. Three rejections forced at 300 min in scenario 12 move the pump by 0.0065 % on the hand-over scan, and the MPC is back at 330 min. Every measured run had zero rejected plans. The controller reads the three measured signals, the speed command, the setpoints and its declared model, and nothing of the plant's state, parameters or disturbances. A test that feeds the recorded signals of a plant with fifteen parameters, the feed, the ore, the reagent and the pump changed to a separate controller reproduces all of its 60 estimates and plans. A plan takes 384 to 481 µs in Chrome, 0.77 to 0.96 % of one core at 200 times real time, and the application runs at 59.6 frames per second under both pump options.

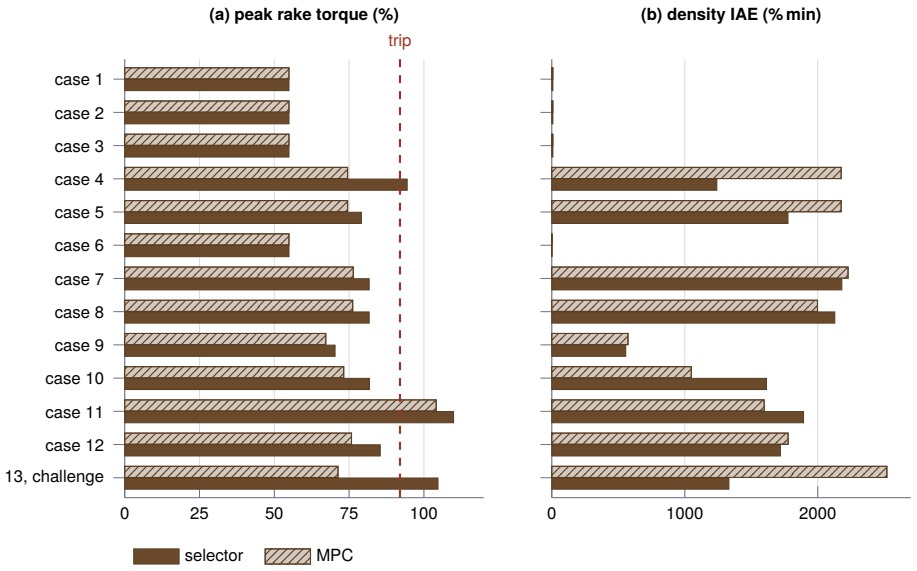


Figure 6.2 The selector and the MPC in every scenario, from Table 6.6: (a) peak rake torque against the 92 % trip (dashed line) and (b) density IAE. Solid bars are the selector and hatched bars the MPC. Notice cases 4, 12 and 13, where the MPC holds the rake lower at the cost of more density error, and case 11, where the wrong selector lets the rake pass its trip under both.

6.4 Results

6.4.1 A Pump Move by Hand

In scenario 1 the thickener runs in Manual for one minute before the pump is slowed from 55 to 51 %. Six hours later the bed has risen from 1.60 to 1.69 m, the density from 61.99 to 62.81 % and the torque from 54.9 to 59.6 %. The bed moves first, then the density and then the torque, and most of the change is still to come. Choosing 51 before Run instead starts the plant at the equilibrium of that speed and shows no response.

> **TRY IT IN THE APP** **1 · Operate the Thickener by Hand (MANUAL)**
>
> Load the scenario, press Run in Manual for one minute, then pause and slow the underflow pump from 55 to 51 %. Run on for six hours and watch the bed depth rise first, then the underflow density and then the rake torque. After six hours the bed has risen only from 1.60 to 1.69 m, and most of the change is still to come.

6.4.2 Selector and Predictive Comparisons

The results that follow use the external reset of Equation (6.8). All comparisons use the same seed, instruments, events and case settings. The selector is the structure each case loads with. Only the pump-control option changes for the MPC comparison. Each case runs for 1440 min, except the 900 min challenge. Density IAE is the integral of absolute true underflow-density error in percent-minutes. Torque is percent of rating. The complete-run measurements are listed in Table 6.6, and Figure 6.2 draws its torque and density columns as bars.

The low selector of case 11 blocks the protective demands when more pump is needed, under either pump option. A correct external reset does not repair a reversed selection. Turning reset off, as in case 10, instead lets a losing controller integrate an error while its command is unused. These are separate experiments from prediction. A protection must retain authority under both pump-control options.

Table 6.6 Selector and MPC in Every Scenario. Each Pair Is Selector / MPC

Case	Density IAE	Torque peak (%)	Bed peak (m)
1	8 / 8	54.9 / 54.9	1.597 / 1.597
2	8 / 8	54.9 / 54.9	1.597 / 1.597
3	8 / 8	54.9 / 54.9	1.597 / 1.597
4	1241 / 2176	94.4 / 74.5	2.338 / 1.966
5	1776 / 2176	79.1 / 74.5	2.055 / 1.966
6	1 / 1	54.9 / 54.9	1.598 / 1.597
7	2181 / 2228	81.7 / 76.4	2.560 / 2.391
8	2128 / 1998	81.7 / 76.2	2.497 / 2.335
9	556 / 573	70.3 / 67.2	1.894 / 1.831
10	1614 / 1049	81.8 / 73.2	2.062 / 1.898
11	1893 / 1597	109.9 / 104.1	2.635 / 2.529
12	1719 / 1777	85.4 / 75.8	2.451 / 2.239
13	1332 / 2520	104.7 / 71.3	3.280 / 2.590

Table 6.7 Case 12 Over 24 Hours

Configuration	IAE	Peak torque (%)	Torque override (min)
Selector	1719	85.45	416
MPC	1777	75.77	0
MPC, plan limits off	1459	81.50	241
MPC, selector off	1777	75.77	0
MPC, both off	1403	88.37	0
MPC, 1 min period	1182	75.42	296

Case 11 is deliberately extreme. Under the low selector the rake passes its trip and stays above it for the rest of a 600 min run. A physical machine would have stopped at the trip long before that, but the case shows what a selector wired in the wrong direction removes, which a real plant reveals only once.

6.4.3 Thickening Feed and the Cost of Protection

In case 12 the feed thickens from 150 to 200 kg/m^3 between 60 and 480 min, at a 64.5 % density request. The finite-reset selector peaks at 85.45 % torque, against 75.77 % for MPC. Over 24 h, density IAE is 1719 for the selector and 1777 for MPC, so the selector has slightly less density error while MPC has the smaller torque peak. Pump travel is 116.3 against 114.1 %. The selector's torque override owns the pump for 416 min, and the nominal MPC needs no torque takeover in this case. This does not establish that the protective selector is redundant: with MPC's plan limits left out, the override acts for 241 min and limits the torque peak to 81.50 %, while removing both plan limits and selector lets it reach 88.37 %. Table 6.7 separates these effects.

The declared MPC model, ten-minute period, twenty-hour horizon, move price and estimator have not been retuned. A one-minute period changes both the horizon and travel, so that row is a counterfactual rather than the shipped law. Compare torque, quality, clarity and movement together.

6.4.4 Graded Fifteen-Hour Shift

The challenge, case 13, runs 900 min through four scripted disturbances. It asks for the worst density error from 594 min within ±1.5 % solids, the rake torque below 92 %, no more than 90 min above 100 mg/L and the bed below the 3.4 m feedwell, with the model parameters and disturbances as given. The case loads with the overrides off, the slow default tuning and a 63 % setpoint. Its worst density error is then 3.02 % solids, twice the band, the torque passes the trip, and the bed rises past the 2.9 m depth at which the feed starts to short-circuit to the launder. The overflow of 81 800 mg/L that follows is stylized, and it is the clearest available statement of what a bed near the feedwell costs. The suggested configuration puts the high selector across torque and bed with external reset, the faster tuning, and 58.5 % density and 30 mg/L clarity setpoints. It meets all four requirements. The flocculant reaches its 70 g/t stop after the finer ore and stays there, so some time above the recycle

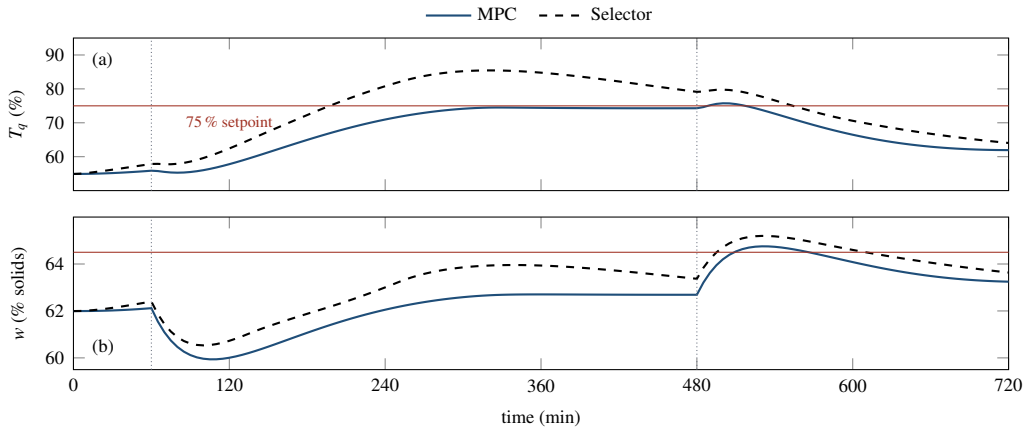


Figure 6.3 Case 12 over its first twelve hours: (a) rake torque and (b) underflow density under the selector (dashed black) and the MPC (solid blue). The thin horizontal lines (red) are the torque and density setpoints, and the dotted vertical lines mark the thick-feed interval. Both structures retain the same protective selector.

Table 6.8 Challenge Measurements

Configuration	Density IAE	Torque (%)	Bed (m)	Dirty (min)
As loaded	1332	104.7	3.280	350.8
As loaded, MPC	2520	71.3	2.590	0.0
Suggested	950	65.9	2.567	42.9
Suggested, MPC	1442	59.9	2.425	79.0

limit cannot be avoided. Table 6.8 gives complete-run measurements, rather than substituting density tracking for the separate torque, bed and recycle requirements.

With the suggested settings the MPC has more density error, 1442 against 950 percent-minutes, and more time above the recycle limit, 79.0 against 42.9 min. Selecting a feasible density setpoint, protecting the bed and maintaining clarity remain necessary, regardless of the pump controller. A transfer to Manual starts at the speed selected after protection and cannot release a protective override.

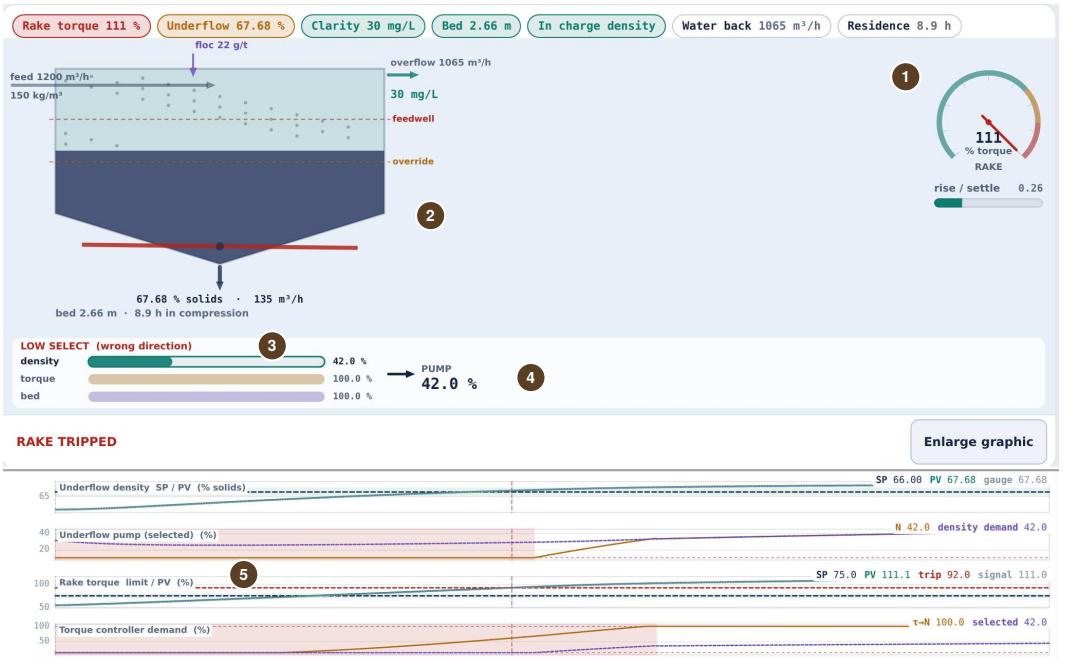


Figure 6.4 The application during scenario 11, under the low selector, with the plant view above the first four trends. (1) The rake gauge and the status badges read the torque past its trip, and the alarm line below the selector reports the rake tripped. (2) The vessel draws the bed, dark, under the clear zone, with dashed lines at the feedwell and at the bed override setpoint. (3) The selector panel names the wiring in force, a low select marked as the wrong direction, and draws the three pump demands. (4) The pump receives the smallest demand, the density controller's, while the torque and bed controllers ask for full speed. (5) The torque trend climbs past its limit and trip lines while the torque controller's demand rises and is never selected.

> ✦ **DID YOU KNOW? The Better Score Tripped the Rake**
>
> From the simulator: in the graded shift as loaded, with the overrides off, the PID run of Table 6.8 has less density error than the predictive controller, 1332 against 2520 percent-minutes. It also drives the rake to 104.7 %, past its trip, and keeps the overflow above the recycle limit for 350.8 min.

6.4.5 Application Access

The application runs in the browser with no installation and all computation is client-side. Manual and automatic modes are provided, and every run exports the full logged trajectory for offline analysis, as described in Section 1.6.4. The three competing pump demands and the selected output are displayed together with the wiring in force, which is what makes the comparison of Table 6.6 visible while it happens. Figure 6.4 shows that view during scenario 11. Manual mode supplies the inverse of the solids balance, returning the pump speed that would hold a chosen underflow density at steady state, so Table 6.1 can be built by hand and the ceiling found before a controller finds it. A second helper, Find the safe density, reports the densest underflow at which the steady torque reaches the torque override setpoint with the present feed.

6.5 Technology in Practice

Figure 6.5 draws the four loops of Section 6.3 with their hardware.

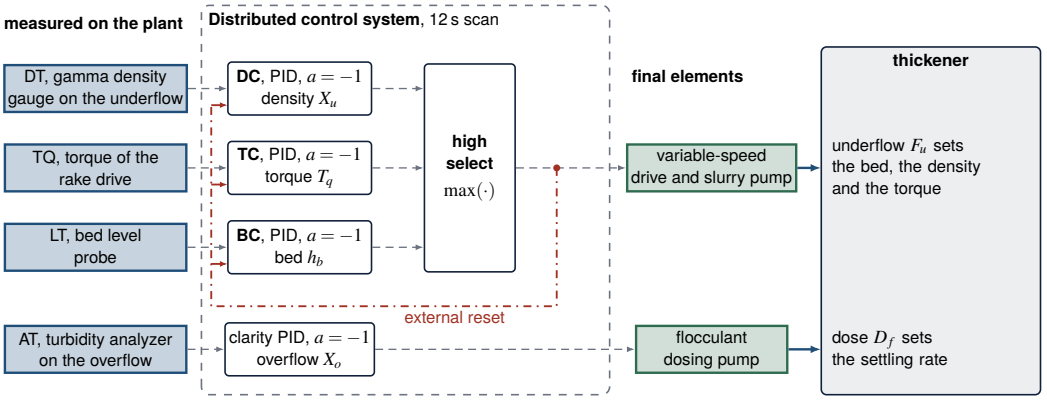


Figure 6.5 The four loops of the simulated thickener drawn with their hardware. The density gauge DT, the rake torque TQ and the bed level LT feed three PID blocks whose pump demands meet in a high select, and the selected speed goes to the drive of the underflow pump and back to all three blocks as their external reset (dash-dotted). The clarity loop on the turbidity analyzer AT owns the flocculant dosing pump alone.

What Measures It

The model reduces the underflow density gauge to a 1.2 min delay with no drift.

> **TECHNOLOGY** **Sensor: A Gamma Density Gauge on the Underflow**
>
> A radiation gauge sends gamma rays from a shielded source through the pipe or vessel to a detector on the far side. The rays are attenuated by the material they cross, so a denser slurry lets fewer reach the detector. A cesium-137 source, with a half-life of 30 years, drifts less in calibration than cobalt-60 at 5.3 years, and the source is made safe for maintenance by closing a lead shutter [17]. Gamma density meters on slurry pipelines are in wide industrial use [18]. The accuracy of radiation gauges varies with coating on the vessel wall, source decay and detector drift [17]. Such errors build slowly, and a density loop holds a wrong reading as faithfully as a right one.

The other three signals are the rake torque and the bed level, each filtered over 3.0 min, and an overflow turbidity analyzer 2.5 min late. A plant study lists rake torque, underflow density, overflow turbidity, the solids interface level and the solids inventory, read as a bed pressure, among the measured variables of a thickener [19], and full-scale profiles of a thickener bed have been taken with a gamma-ray gauge [20].

What Moves It

The model runs the pump from 10 to 100 % of speed through a 0.5 min lag, with the delivered speed scaled by the wear factor, and the flocculant from 0 to 70 g/t through a 1.5 min mixing lag.

TECHNOLOGY Final Element: A Variable-Speed Slurry Pump

The underflow is drawn off by a pump whose motor runs on a variable-frequency drive. Changing the speed of the machine that moves a fluid wastes less energy than throttling its outlet, but a variable-speed machine often cannot change the flow as quickly as a control valve, and it offers few options for a fail-safe state. Pumps in slurry service wear quickly and are rebuilt or replaced at regular intervals [17]. A worn pump delivers less at the same speed, which is the pump wear w_p of Table 6.2, so the speed that holds a given density creeps upward between overhauls.

The same plant study names the flocculant rate and the underflow rate as the two variables normally used to control a thickener, with the feed rate touched only in an emergency [19], the division of handles in Table 6.2.

Where the Controller Runs

Control loops in a plant are often nested or cascaded, with cycle times from milliseconds to minutes, and a distributed control system uses a supervisory loop to mediate a group of localized controllers [21]. Programmable logic controllers run PID loops as well [17]. The app runs its four PID blocks on a 0.2 min (12 s) scan, inside that range and short against a bed that turns over in four and a half hours. On such a platform the chapter's structure is three PID blocks feeding a high-select block whose output returns to each block as its external reset, and the predictive controller is a supervisory application that writes a pump request into the same selector every 10 min.

IN PRACTICE Watching the Bed Through Pressure

On one operating thickener, pressure sensors at several heights give the average bed density between each pair of points, and that profile is used to control the bed volume. A high pressure near the top of the wall raises an alarm for an over-expanded bed and, a little higher, drives the flocculant to its maximum. Comparing the bed density in the cone with the density actually pumped shows how well the rake delivers [22]. A pressure tap at the bottom of a settling vessel is kept clear of sediment by a small purge of clean water through a partly open valve [17].

What Is Modernizing It

The first development is predictive control with the rake inside the plan. Published work on paste thickeners has added a rake torque constraint to a predictive controller to prevent rake lifting and pumping problems, and has used a sampling interval that is shorter near the present than far ahead [19], as the move blocks of Section 6.3.3 are. The second is data-driven models: one study fitted regression models of the paste line pressure difference and of the rake torque to a year of one-minute data from 73 measurements [19]. The third is online rheology: an automated capillary rheometer on a side stream reports how the underflow flows without a laboratory sample [23], a property that this chapter's model does not carry.

6.6 Limitations

The bed is one lumped inventory with an exponential compression law. The established description of a continuous thickener is a conservation law with a flux function whose shape produces discontinuities, and the analysis of those solutions is a substantial literature in its own right [6, 10]. Nothing here reproduces a settling front, a rising interface or the discontinuous behavior at the feed level, and the regulator designs that follow from those solutions [24] have no counterpart in this model. The origin of a thickener's steady states is treated properly in that literature and not here.

Compression is a residence-time exponential rather than a compressive yield stress, so the application cannot say what the underflow will be for a particular material, only how it responds to a change in inventory [7].

⚠ OPEN CHALLENGE A Sedimentation Model That Predicts

Published sedimentation models of thickeners are not fully predictive. Each holds for one flocculation state, and none accounts for the shear that raking applies to the aggregates, so they substantially under-predict the underflow density. A twenty-year industrial research program on thickeners found its other sub-models well advanced and named the understanding of sedimentation and dewatering as the main barrier [20]. Progress would be a model that predicts the underflow from the unit's dimensions, its flows, the feed and the flocculation.

Rake torque is an algebraic function of depth and density. Real torque depends on rake geometry, on the yield stress of the bed at the depth the arms run, on whether the arms are raised, and on the presence of islands and rat-holes that form and collapse [25]. The trip in this application is an alarm rather than a mechanism, and the bed that a stalled rake sits in consolidates far more than this model represents.

Flocculant is a single dose with a single Langmuir response. Real flocculation depends on the mixing energy in the feedwell, the polymer molecular weight and charge, the dilution water and the shear history, and the difference between a good and a bad feedwell is larger than the difference between a good and a bad dose. Underflow rheology is absent, so the application cannot represent the transition from a pumpable to an unpumpable underflow that limits real deep cone units.

The predictive model uses the design-feed balance with the reagent at its reference dose and no clarification zone. Feed, ore, reagent strength and pump wear are represented only through three estimated disturbances. Its limits are soft, its ten-minute period delays its response between decisions, and its pump command can change discretely. The protection loops remain active under both pump options, so an apparent advantage of prediction in case 12 cannot be attributed to a delayed reset. The suggested challenge retains density and clarity losses under the MPC, as reported in Table 6.8.

Finally, the plant has one thickener. Real circuits have several in series or in parallel with recycle, and the water balance across them is where the economic argument actually lives [8].

6.7 Discussion

Selector direction determines whether protection can reach the actuator. A correct reset law cannot compensate for the wrong selector direction. The reset law also shapes the comparison with predictive control. Assigning the selected command to every losing integral at each scan ratchets the pump, and the finite reset feedback of Equation (6.8) removes that artifact and changes the quality tradeoff. Comparisons must use the same instruments and protections, and must include density and clarity alongside torque. A quality-only score can prefer a run that violates a mechanical constraint, which motivates separate physical criteria rather than one scalar score [26, 27].

6.8 Conclusion

A thickener makes denser underflow by retaining a heavier bed, so quality and mechanical protection compete for the same pump. The high selector passes protective demands in the safe direction. Its losing reset states follow the selected command, and Manual retains that command after protection. In the thickening-feed comparison, the selector and MPC torque peaks are 85.45 and 75.77 %, with density IAE 1719 and 1777 percent-minutes. Under the suggested challenge settings, MPC instead loses density tracking and clarity. The scenarios therefore distinguish physical protection, prediction and the selection of a feasible operating point without attributing an inherited reset defect to the optimizer.

KEY POINTS

- A thickener makes a denser underflow by holding a heavier bed, so the plant's water and the rake's safety compete for one pump.
- The steady torque reaches the 92 % trip at 66.54 % solids, so a higher setpoint is infeasible.
- Both protections relieve their limits with more pump, so the selector is a high select, and external reset cannot repair a reversed one.
- In case 12 the MPC peaks at 75.77 % torque against 85.45 % for the selector, with density IAE 1777 against 1719.
- A feasible setpoint, bed protection and clarity remain necessary whichever controller drives the pump.

Where else this applies. One actuator shared by a quality loop and a protection in Volume 4, Chapter 2 and Volume 4, Chapter 4.

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6.9 Exercises

6.1 Concept check · 1 min

In Figure 6.1 the torque and bed controllers both relieve their limits by speeding the pump up. Which selector belongs between them and the density controller?

(a) A high select, because more pump is the safe direction for both limits

(b) A low select, because the two protecting controllers are direct acting

(c) A high select, but only while external reset is switched off

(d) Either direction, as long as external reset is switched on

6.2 Concept check · 2 min

Which of these quantities does no instrument on the thickener of Figure 6.1 measure, so that every loop sees it only through its effects?

(a) The underflow density X_u

(b) The ore settleability σ

(c) The rake torque T_q

(d) The overflow clarity X_o

6.3 Concept check · 2 min

At design feed, Table 6.3 doubles the flocculant dose from 30 to 60 g/t. Which statement agrees with the table?

(a) The underflow gets denser, because faster settling shortens compression

(b) The overflow falls by more than half, so the clarity loop gains authority

(c) The overflow falls only from 22 to 17 mg/L and the torque rises slightly

(d) Neither the overflow nor the torque moves, because the reagent is saturated

6.4 Theory and calculation · 6 min

At the design point the thickener holds $M_s = 827\,\mathrm{t}$ of solids and takes $1200\,\mathrm{m^3/h}$ of feed at $150\,\mathrm{kg/m^3}$. Use Equations (6.1), (6.6) and (6.2) with the values of Table 6.4, $\sigma = 1$ and no overdose penalty.

(a) Compute the solids feed, the bed residence time to 0.01 h and the compressed underflow concentration X_u^* to 1 kg/m^3. *Checkpoint:* 180 t/h; 4.59 h; 1017 kg/m^3

(b) Compute the bed depth h_b to 0.001 m and the rake torque to 0.1 %. *Checkpoint:* 1.597 m; 54.9 %

(c) An underflow of 66 % solids is $X_u = 1129.3\,\mathrm{kg/m^3}$. What residence time gives it, to 0.01 h, and what inventory, to 1 t? *Checkpoint:* 6.93 h; 1247 t

6.5 Theory and calculation · 5 min

Use Equation (6.2) and Table 6.1.

(a) At 66 % solids the bed is 2.170 m deep and $X_u = 1129.3\,\mathrm{kg/m^3}$. Compute the torque to 0.1 %. *Checkpoint:* 85.4 %

(b) Compute the torque cost per point of solids from 55 to 60 % and from 66 to 68 %, to 0.1, and their ratio to one decimal. *Checkpoint:* 2.9 and 16.1; 5.5

(c) Interpolate linearly between the 66 and 68 % rows for the density at which the torque reaches the 92 % trip, to 0.01 %. Is the estimate above or below the model's 66.54 %, and is that the safe side? *Checkpoint:* 66.41 %; below, the safe side

6.6 Theory and calculation · 6 min

The torque loop loses a high selector to a density demand of 55 % pump speed. Its measured torque is 76 %, setpoint 75 %, proportional gain 2.2, integral time and tracking time 120 min, scan 0.2 min, zero bias and no derivative. The integral carried into this scan is 50 %. Neither local output limit binds.

(a) Find the torque loop's emitted request on this scan and say whether it beats the incumbent by the 0.75 % switching margin. *Checkpoint:* 52.2 %; no.

(b) Include ordinary integration and the selector back-calculation of Equation (6.8). Find the next integral and, with unchanged measurements, the next request. *Checkpoint:* 50.00833 and 52.208 33 %.

(c) Explain why assigning $I = 55$ after every losing scan would give a different handover, and why a constant one-point torque violation can eventually take the pump with finite reset feedback. *Checkpoint:* Instant assignment jumps the next request to 57.2; finite feedback moves the integral toward the selection over its tracking time.

6.7 App challenge · 8 min

Open scenario 1, "Operate the thickener by hand (MANUAL)", at the default seed and a speed of 60 (60 min per second).

(a) Run for one minute, pause, set Underflow pump speed to 51.0 % and run again. Pause when the clock reads 361 min (6.02 h). Read the Underflow and Bed residence tiles, then export the CSV and read `bed_m` and `tq_pct` in its last row. Tolerances: 0.02 %, 0.01 h, 0.002 m and 0.1 %. *Checkpoint:* 62.81 %, 4.97 h; 1.691 m, 59.6 %

(b) Reload the scenario with the reload button, set the pump to 51.0 % before Run, run 30 min and read the same four values. *Checkpoint:* 64.87 %, 6.08 h; 1.960 m, 74.2 %, unchanged from $t = 0$

(c) Run (b) is the steady state at 51 %. What fraction of the steady change in density had arrived in (a) after six hours, to 0.01? Compare it with $1 - \mathrm{e}^{-6/15}$ for a 15 h time constant and explain the difference. *Checkpoint:* 0.28 against 0.33

6.8 App challenge · 8 min

Open scenario 11, "The wrong selector", at the default seed and a speed of 200 (200 min per second). The CSV time column of these runs is in hours.

(a) Run it as loaded, with Override selector on Low select, to 10 h and export the CSV. Find the first row with `tq_pct` at or above 92, within 0.02 h, the largest `tq_pct`, within 0.2, and the controller `override_who` names and the `pump_cmd_pct` during the first three hours. *Checkpoint:* 3.68 h; 109.9 %; bed, 10.0 %

(b) Press Reset, set Override selector to Torque + bed before Run, and run to 10 h. Read the largest `tq_pct`, within 0.2, and the first row in which `override_who` reads torque, within 0.02 h. *Checkpoint:* 79.1 %; 4.19 h

(c) Press Reset, set Override selector to None and run to 10 h. Read `tq_pct` in the row at 10.000 h, within 0.2. Rank the three wirings by peak torque and explain why the low select is worse than none. *Checkpoint:* 91.2 %; low, none, high

6.9 App challenge · 8 min

Open the challenge, "Challenge · sit on the torque limit for a shift", at the default seed and a speed of 200, and run each part for the full 900 min. Read the result line of the progress checklist, which gives the worst density error, the peak torque, the minutes above 100 mg/L and the largest bed.

(a) Run it as loaded. Read the four values and say which requirements fail. *Checkpoint:* 3.02 %, 104.7 %, 351 min, 3.28 m; density, torque and clarity fail

(b) Reload, press Reveal suggested tuning and run again. Read the four values. *Checkpoint:* 1.30 %, 65.9 %, 43 min, 2.57 m; passes

(c) Reload, press Reveal suggested tuning, set External reset to Off before Run and run again. Read the four values, say which requirements fail, and explain how the missing reset causes it. *Checkpoint:* 1.67 %, 71.5 %, 94 min, 2.72 m; density and clarity fail

6.10 Application area · 4 min

Table 6.1 runs the pump at 54.99 % for 62 % solids and at 49.52 % for 66 %. The pump capacity is $k_p = 3.218\,\mathrm{m^3/h}$ per %, and the application values recovered water at $0.45 per cubic meter.

(a) Compute the underflow at both densities and the extra water recovered at 66 %, to 0.1 $\mathrm{m^3/h}$, and its value per hour, to the cent. *Checkpoint:* 177.0 $\mathrm{m^3/h}$ and 159.4 $\mathrm{m^3/h}$; 17.6 $\mathrm{m^3/h}$; $7.92 per hour

(b) Find the safe density reports 64.96 % solids at 50.9 % pump for the 75 % torque setpoint. What share of the gain in (a) does that point keep, to 1 %, and what does the plant buy with the rest? *Checkpoint:* 75 %

6.11 Application area · 3 min

The rake trips at 92 % and the torque override ships with a 75 % setpoint. In the graded shift the torque signal carries noise with a standard deviation of 1.2 %.

(a) How far past the setpoint did the torque go under the selector in cases 5 and 12 of Table 6.6? *Checkpoint:* 4.1 and 10.4 points

(b) Add three standard deviations of the noise to the larger overshoot. How many points are left below the trip, to one decimal, and why should an engineer not raise the setpoint to 88 % for a denser underflow? *Checkpoint:* 3.0 points

6.12 Review the agent · 5 min

An AI assistant reviewing Table 6.7 writes: "With the selector off the MPC peaks at 75.77 % torque, against 75.77 % with it, and the torque override held the pump for zero minutes. The overrides add nothing once the MPC is in charge, so remove them and save the commissioning effort." Find the flaw and say how to check it on the app.