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CHAPTER 4

Hydrocyclone Classification Simulator for Cascade Control and Hidden Loop Coupling

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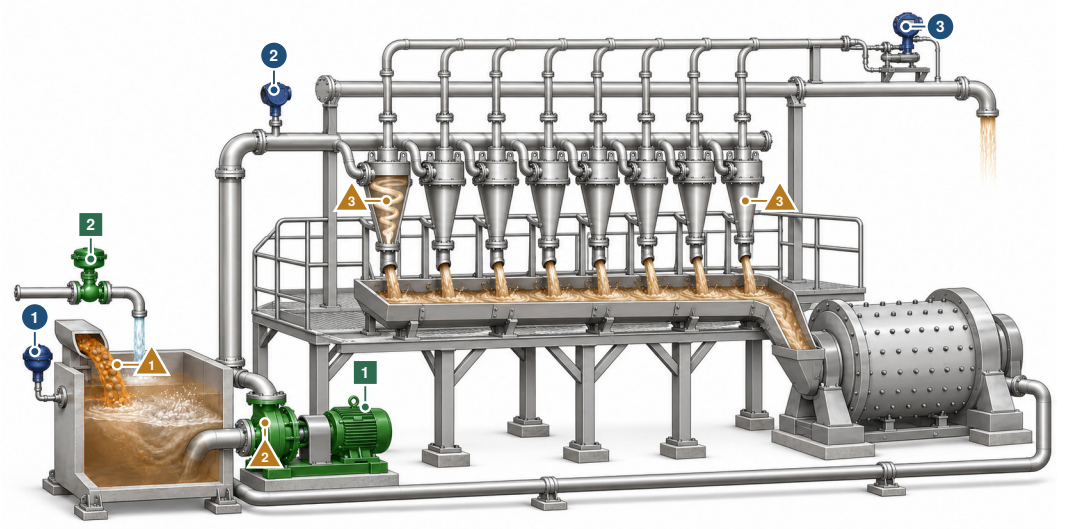
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CHAPTER 4

Hydrocyclone Classification Simulator for Cascade Control and Hidden Loop Coupling



The controller moves 1 the pump speed, through the feed pump's motor drive
2 the dilution water into the sump, through its valve

The controller reads 1 the sump level 2 the pressure of the cyclone feed
3 the fineness of the overflow, from a size analyzer on a sample line

What disturbs it 1 coarser new ore 2 a worn pump impeller 3 worn cyclone liners

Notice the water falling into the sump: the level loop that sets it also sets how thick the cyclone feed is, and with it most of the product size. Illustration, not to scale.

IN PLAIN TERMS **Sorting Ground Ore in a Spinning Cone**

The main idea. Ground ore and water are pumped into cones that spin them. Coarse grains go back to the mill, and fine grains leave as product.

The challenge. Pumping harder makes the product a little finer, but it also draws more water into the tank in front of the pump. That thins the mix, and a thinner mix sorts much finer, so the loop that only keeps the tank level steady quietly sets most of the product size.

Why it matters. A fix that steadies one reading can push the product off target through a loop that nobody watches for quality.

THREE WAYS TO READ THIS CHAPTER

Explore	8 min	The first page, the history and fact boxes of Section 4.2, the fact box and the try-it box of Section 4.5, and the key points
Study	45 min	The whole chapter with its pause points and the twelve exercises
Practice	15 min	The first page, Section 4.5, Section 4.6, Section 4.7 with its open challenge

The control question A size loop on the cyclone overflow sets the setpoint of a fast pressure loop on the feed pump, and the pressure is a weak handle on the cut. Which disturbances does that cascade take out of the product, and where does the outer loop's authority over the cut size actually come from?

Learning objectives

- Sort a disturbance by whether it enters the pressure measurement, and predict whether the cascade helps, hurts or does nothing.
- Split a change in cut size into a pressure part and a feed density part with the cut-size correlation.
- Compute the sump's integrating gain and hold-up and the circuit's recycle balance from the model.
- Explain why a tighter level loop holds the level better and the product no better.
- Commission the three loops from the fastest outward and judge the result against the challenge's four bands.

Prerequisites Sections 1.4.2, 1.4.6, 1.6.2 and 1.6.4.

The app https://apmonitor.com/apps/hydrocyclone/

See also Volume 3, Chapter 2 (disturbances sorted by where they enter). Volume 3, Chapter 3 (how fast an inner loop must be). Volume 6, Chapter 7 (a recycle that amplifies a small change).

4.1 Introduction

The product specification on a grinding circuit is written on the cyclone overflow, and the actuator nearest to it is the cyclone feed pump. Between the two sits an empirical correlation whose exponent on feed pressure is –0.225, so a 60 % increase in pressure returns a 10 % reduction in cut size. That is a weak handle, and the pump power required to obtain it rises with the cube of speed. The exponent alone does not justify placing the product size controller on the pump, yet that is the standard configuration in concentrators.

The justification is in the sump rather than in the correlation. The vessel in front of the pump holds 0.26 min of slurry at design flow, and its only spare volume is dilution water, so the level controller owns the single degree of freedom in the water balance. A larger draw on that sump lowers the level, the level controller adds water, the cyclone feed thins, and the cut-size correlation is exponential in feed solids. Stepping the pressure setpoint of the application from 90.20 kPa to 110.0 kPa moves the pump from 65.01 % to 72.32 %, the dilution water from 653.8 m^3/h to 753.0 m^3/h, the feed from 44.82 % to 43.13 % solids, and the cut size from 151.4 µm to 134.3 µm. Decomposing the logarithm of that cut-size ratio assigns 37.1 % to the pressure term and 62.9 % to the density term. Two thirds of the outer loop's steady-state authority therefore passes through the level controller, which appears on no control diagram as a product quality loop, and it arrives within half a minute rather than over the quarter hour the circulating load requires. The feed density covers 63 % of its final change 0.43 min after the step.

The first subject of this chapter is more conventional and is the reason the circuit is useful as a teaching plant. The feed pressure responds to pump speed with a gain of 2.59 kPa/% and a time constant of 7.5 s, while the mill recycle has a time constant of 5.1 min and reaches 95 % of its change in 15.8 min. A separation of 41 to 1 in dominant time constant, and of more than 100 to 1 in 95 % response time against the closed pressure loop, satisfies the standard condition for a cascade [1, 2] (Section 1.4.6), and the application quantifies the benefit. An impeller worn to 86 % of design delivery, applied identically under both architectures, produces 154.9 % min of integrated fineness error and 33.3 min off specification when the fineness controller drives the pump through a fixed characteristic, and 3.2 % min with 0.1 min off specification when a pressure loop is placed between them. A coarser new feed produces 53.7 % min under the single loop and 53.1 % min under the cascade, because ore coarseness does not appear in a pressure transmitter.

Classifying disturbances into those two groups before any tuning is performed is the analysis the plant is built to teach, and Volume 3, Chapter 2 makes the same sort on a level loop. On this

circuit the classification is not resolved by the block diagram alone. Worn cyclone liners appear in the pressure measurement and belong to the first group by the usual test, yet the cascade responds to them by moving the pump, which moves the density, which moves the product by 5.94 % of fineness at the peak against 1.97 % when the pump is left where a static characteristic put it. An inner loop that returns its own measurement to setpoint has rejected only the part of the disturbance that lived in that measurement.

The experiments the application supports are the standard content of a process dynamics and control course. The sump is an integrating process with 0.26 min of hold-up, the pressure loop is a first-order plant with a sampled measurement, the size analyzer contributes 2 min of dead time to the outer loop, the mill recycle is a positive feedback path that slows the circuit to a quarter hour and drives the size controller to its limit when the mill grinds poorly, and the three loops interact through a variable that is not measured in either of them. Cascade structure, integrating level control, anti-windup on a saturating actuator, dead time in a sampled loop, loop interaction and commissioning order are all exercised on one plant. The physical fundamentals are equally standard. The model is a set of conservation statements on water and on two solids classes in a perfectly mixed sump, closed by an empirical statement of the centrifugal separation and by a transport lag around the mill, and the time-scale separation between an algebraic cyclone, a sump that turns over in seconds and a quarter-hour recycle is what makes the reduction legitimate [1, 3].

This chapter works with a browser-hosted grinding circuit classifier, an eight-state circuit with a genuine recycle, three architectures and eleven scenarios, each with a completion requirement. It quantifies the cascade against six disturbances sorted by whether they enter the inner measurement, and it decomposes the outer loop's gain into the designed path and the path through the level controller. Two measured outcomes carry the analysis. The cascade divides the integrated fineness error by 49 for pump wear and by 1.01 for feed coarseness, and 63 % of the outer loop's steady-state authority is carried by the level controller. A third result is a commissioning order whose justification is a moving plant gain rather than a convention. The cut-size correlation, the partition curve and the cascade arithmetic are all standard.

Section 4.2 reviews how the cyclone displaced the classifiers it replaced and why its control rests on an instrument with a cycle time of minutes. Section 4.3 sets out the sump, pump, cyclone and mill model and its design point. Section 4.4 describes the three loops, the two architectures for the size controller and their tuning. Section 4.5 reports the disturbance comparison, the decomposition of the outer loop's gain, the effect of level tuning, the two unmeasured constraints and the graded challenge. Section 4.6 sets the loops on real instruments and platforms, Section 4.7 states what the model leaves out, Section 4.8 draws the general lesson for cascades, Section 4.9 summarizes, and the exercises in Section 4.10 work through the scenarios.

4.2 Background

Wet classification decides the size at which ore leaves a grinding circuit, and before the hydrocyclone it was performed by rake and spiral classifiers, which are large, mechanically complex and limited to relatively coarse separations. The cyclone displaced them because it has no moving parts, occupies a fraction of the floor area, and separates finer. A modern circuit runs a cluster of them in parallel on a common manifold, with individual cyclones valved in and out to match the throughput, and the assembly is described by a correlation rather than by a mechanism [4, 5].

The correlations have a long history in the mineral processing literature. Empirical forms relating the cut size to feed pressure, spigot and vortex finder diameters and feed solids concentration were fitted to plant and pilot data through the 1970s and remain the working tools of circuit design, and their coefficients have been refitted and compared repeatedly as the data sets have grown [6]. A parallel line of work builds the separation from the fluid mechanics of the swirling flow and the settling of individual particles in it, which gives a mechanistic account of the shape of the correlation

but does not replace it for design [7]. Both traditions agree on the exponents that matter here, which are small on pressure and large on the geometry and the feed density.

> ⧖ HISTORY **A Cone That Purified Water (1891)**
>
> A United States patent of 26 May 1891, filed in November 1890 by Eugene Bretney of Indianapolis, describes a water purifier that frees water of impurities by centrifugal force. Water under pressure enters the top of a tank shaped as an inverted cone through a pipe set at one side, so that the inflow is tangential and the water whirls. The impurities driven to the wall descend to an adjustable annular opening at the point of the cone, and the tapering form, the patent explains, leads them there in a nearly straight line. The purified water leaves through a pipe that starts on the axis just below the cover and passes up through it [8]. The opening at the point is what this chapter calls the apex.

The physical content of the model is elementary. Mass is conserved separately on water and on the solids in each size class, the sump is an integrator because its outflow is set by a pump rather than by its own inventory, and the driving force for separation is the centrifugal field that the pressure drop creates inside a cyclone with no moving parts. Textbook treatments of process dynamics develop exactly these elements, integrating inventory, first-order actuator dynamics, recycle and the conditions under which a fast loop may be treated as an algebraic gain by a slower one [1, 3, 9]. What the circuit adds is that all of them appear together in a plant small enough to run in a browser.

Comminution accounts for a large share of the energy used in a concentrator. Estimates place the comminution share of total mine site energy in the tens of percent, with the associated emissions following [10]. Because the classifier sets the size at which material leaves the circuit, it also sets how much of that energy is spent regrinding material that was already fine enough, and it sets the size distribution presented to flotation or leaching. That is the industrial reason the classification loops are among the most valuable regulatory loops in the plant.

> ✦ DID YOU KNOW? **Dense Grains Ground Twice**
>
> Plant sampling of an iron ore grinding circuit, reported to the United States Department of Energy in 2005, found that the largest single source of overgrinding was the return of dense material to the mills by the hydrocyclones. Simulations indicated about 50 percent more throughput from the same number of mills once the circulating load was reduced [11].

The control literature on grinding circuits reports the difficulty rather than the value. Surveys of milling circuit control find that most industrial circuits run on single-loop regulatory control with an operator supplying the supervisory layer, and that the economic case for advanced control is routinely made and less routinely realized [12]. The measurement problem is a large part of the difficulty. Particle size is measured on stream by instruments with cycle times of minutes, ore properties are not measured at all, and the states a model-based controller would use are not observable [13]. Model development for these circuits has concentrated on obtaining a representation simple enough to be identified from plant data and rich enough to hold the recycle, and validation against operating data remains the hard part [14]. Supervisory schemes that manipulate sump water and cyclone feed rate against inferred size have been reported for ball mill circuits since the 1990s [15], and feedforward from measured feed size distribution is an active line of work for exactly the disturbance a cascade cannot see [16].

However, a cascade presented in a textbook is a pair of transfer functions and a rule about time-scale separation. That presentation does not show a cascade that is correctly structured, correctly tuned, and delivering most of its steady-state authority through a variable that appears in neither of its two loops. The application of this chapter provides a plant on which that distinction is measurable.

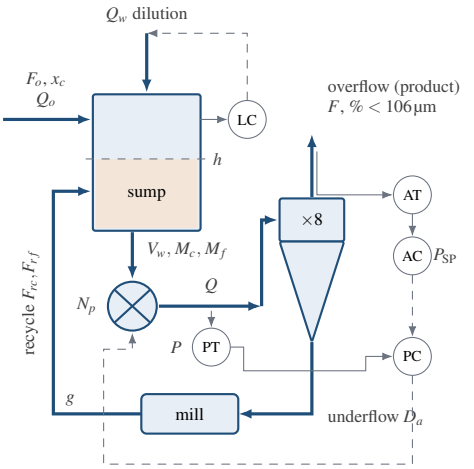


Figure 4.1 Closed grinding circuit as implemented. Sump inventory V_w, M_c, M_f under level control LC manipulating dilution water Q_w, a variable-speed pump N_p delivering Q to eight cyclones, a pressure transmitter PT and pressure controller PC forming the inner loop, and a particle size analyzer AT with controller AC writing the pressure setpoint P_{SP}. Underflow of apex diameter D_a returns to the mill, where a fraction g of the coarse is ground, and re-enters the sump as F_{rc} and F_{rf}.

4.3 Process Model

The plant is a closed grinding circuit reduced to the two units that decide the product. New ore and water enter a sump. A variable-speed centrifugal pump draws from the sump and feeds a manifold of eight identical hydrocyclones. The cyclone overflow is the product and leaves the circuit. The underflow returns to a ball mill, is ground, and comes back to the sump. Dilution water is added to the sump under level control. The arrangement, the instruments and the symbols used in the equations are drawn in Figure 4.1.

4.3.1 Working Units and the Design Point

The clock is in minutes, solids in t/h, slurry in m^3/h, pressure in kPa and size in µm. At the design condition the circuit takes 250 t/h of new feed at 85 % coarse with 110 m^3/h of water, the pump runs at 65 % and delivers 1380 m^3/h to eight cyclones at 90.19 kPa, the level controller holds 50 % with 653.8 m^3/h of dilution water, and the cyclone feed is 862 t/h of solids at 44.82 % by weight, which is 23.13 % by volume. The cut size is 151.4 µm, the overflow is 81.44 % passing 106 µm, the underflow is 67.3 % solids and the circulating load is 245 % of new feed.

4.3.2 Sump Inventory

The sump is treated as perfectly mixed in three quantities, water volume and two solids classes:

$$\frac{\mathrm{d}V_w}{\mathrm{d}t} = \tfrac{1}{60}\left(Q_w + Q_u + Q_o - Q\,\tfrac{V_w}{V}\right) \tag{4.1}$$

$$\frac{\mathrm{d}M_c}{\mathrm{d}t} = \tfrac{1}{60}\left(F_o x_c + F_{rc} - \tfrac{60 M_c}{\tau_s}\right) \tag{4.2}$$

$$\frac{\mathrm{d}M_f}{\mathrm{d}t} = \tfrac{1}{60}\left(F_o(1-x_c) + F_{rf} - \tfrac{60 M_f}{\tau_s}\right) \tag{4.3}$$

with total volume, hold-up time and level

$$V = V_w + \frac{1000\,(M_c + M_f)}{\rho_s}, \quad \tau_s = \frac{V}{Q/60}, \quad h = \frac{100\,V}{A_s H_s}. \tag{4.4}$$

Table 4.1 Controlled, Manipulated and Disturbance Variables

Symbol	Meaning	Unit	Range
Controlled variables			
F	overflow passing 106 µm	%	60–95
h	sump level	%	0–100
P	cyclone feed pressure	kPa	0–190
Manipulated variables			
N_p	feed pump speed	%	25–95
Q_w	dilution water	m³/h	50–1800
P_{SP}	pressure setpoint (cascade output)	kPa	40–170
Disturbance variables			
F_o	new feed to the circuit	t/h	120–420
x_c	coarse fraction of the new feed	–	0.65–0.96
Q_o	water with the new feed	m³/h	30–320
n	cyclones open	–	5–12
w_p	pump delivery multiplier	–	0.70–1.05
w_c	cyclone liner condition	–	0.65–1.10
r_o	ore classification multiplier	–	0.80–1.30
g_r	mill grinding effectiveness	–	0.40–1.30
States			
V_w	sump water volume	m³	—
M_c, M_f	sump coarse and fine solids	t	—
F_{rc}, F_{rf}	mill recycle coarse and fine	t/h	—
N_p, Q_w, D_a	actuator positions	—	—

Here Q_u is the water returning with the mill discharge and ρ_s is the solids density. Equations (4.1) to (4.3) are statements of conservation of mass on three components that share one vessel, and each carries the factor $\frac{1}{60}$ because the clock is in minutes while the streams are in hours. The pump is a linear characteristic in speed,

$$Q = k_p N_p w_p, \tag{4.5}$$

which is the relation that the impeller wear disturbance degrades and that the single-loop architecture inverts to obtain a speed from a pressure setpoint.

4.3.3 Cyclone Pressure and Cut Size

Feed pressure follows the square of the flow through each individual cyclone and scales with the slurry density,

$$P = k_h \left(\frac{Q}{n} \right)^2 \frac{\rho_{sl}}{1395\,\mathrm{kg/m^3}}\, w_c, \tag{4.6}$$

so opening a cyclone lowers the pressure at fixed total flow and worn liners lower it at fixed flow and density. The cut size uses a reduced form of the established empirical correlation [6],

$$d_{50} = d_{50,0} \left(\frac{P}{P_0} \right)^{-\beta} \left(\frac{D_a}{D_{a,0}} \right)^{-\delta} \mathrm{e}^{\,\gamma(\phi_v - \phi_{v,0})}\, r_o, \tag{4.7}$$

with $\beta = 0.225$ on pressure, $\delta = 0.71$ on apex diameter, $\gamma = 0.063/\%$ on the volumetric feed solids ϕ_v and r_o an ore classification multiplier that is unity at the design condition. All three exponents

carry the same sign convention. Raising the pressure, opening the apex or thinning the feed each produces a finer cut. The magnitudes are unequal by construction. Pressure is weak, apex diameter is a little over three times stronger, and the feed solids term is exponential rather than a power law, which is why the 1.20 % change in ϕ_v of Table 4.5 moves the cut size further than the 22 % change in pressure that produced it.

The split of each size class between overflow and underflow is a corrected partition curve of logistic form with the water recovery added as a bypass,

$$E_i = R_f + (1 - R_f)\,\frac{1}{1 + (d_{50}/d_i)^m}, \tag{4.8}$$

evaluated at a representative coarse size $d_c = 300\,\mu\text{m}$ and fine size $d_f = 55\,\mu\text{m}$, with m the sharpness. The corrected term is one half at $d_i = d_{50}$, which is the definition of the cut size. The water recovery to underflow follows the apex,

$$R_f = R_{f,0}\left(\frac{D_a}{D_{a,0}}\right)^{a_{Rf}}, \tag{4.9}$$

so closing the apex sends less water to the underflow and thickens it.

Roping is represented by blending the partition of (4.8) toward the total solids mass split S_u as the underflow density rises,

$$E_i^* = \sigma E_i + (1 - \sigma) S_u, \quad \sigma = \text{sat}\left(\frac{\rho_{rope} - \rho_u}{\rho_{rope} - \rho_{soft}}, 0, 1\right), \tag{4.10}$$

where ρ_u is the underflow percent solids and σ runs from unity at a healthy apex to zero at a rope. At $\sigma = 0$ every size class reports to the underflow in the proportion in which it arrived, which is the definition of no separation, so (4.10) is the mechanism by which a roped cyclone stops classifying rather than classifying differently. Because S_u and ρ_u depend on the split they determine, (4.8) to (4.10) are solved by fixed-point iteration at each evaluation.

4.3.4 Circulating Load and Mill Recycle

Underflow solids go to the mill, where a fraction is ground from coarse to fine and the whole stream returns after a transport lag:

$$\frac{\mathrm{d}F_{rc}}{\mathrm{d}t} = \frac{U_c\,(1 - g_e) - F_{rc}}{\tau_m}, \qquad \frac{\mathrm{d}F_{rf}}{\mathrm{d}t} = \frac{U_f + U_c g_e - F_{rf}}{\tau_m}, \tag{4.11}$$

$$g_e = \min\left(1,\ \frac{F_{mill,\max}}{U_c + U_f}\right) g\, g_r. \tag{4.12}$$

Equations (4.11) make the circulating load a genuine positive feedback path with loop gain $E_c(1 - g_e)$ rather than a fitted transfer function. This is the recycle gain g of Equation (1.1), with the share g_e ground per pass in the place of Chapter 1's b, and the same positive feedback loads a reactor's recycle in Volume 6, Chapter 7. At the design condition $E_c = 0.918$ and $g_e = 0.32$, so the coarse recirculation gain is 0.624 and the total solids recirculation ratio is 0.71. The pressure setpoint step of Table 4.5, which raises the cyclone feed rate by 47 t/h, moves the recycle streams of (4.11) with a 63 % response time of 5.1 min and a 95 % response time of 15.8 min, so a 3 min transport lag becomes a quarter hour of circuit dynamics. Lowering the mill grinding effectiveness g_r raises that loop gain toward unity, where the circuit would have no stable operating point. Within the range the application offers it stops short of unity. At $g_r = 0.4$ with the size controller saturated the gain settles at 0.80 and the circulating load at 412 %.

Table 4.2 Model Parameters. Those Marked A Are Adjustable

Symbol	Meaning	Value	Unit
A_s	sump cross-section	4.0	m^2
H_s	sump depth	3.0	m
k_p	pump capacity per unit speed	21.23	$m^3/(h\,\%)$
τ_{pu}	drive lag	0.10	min
τ_w	dilution water valve lag	0.15	min
τ_a	apex actuator travel time	1.2	min
n	cyclones open	8	—
k_h	cyclone resistance coefficient	3.035×10^{-3}	$kPa\,h^2/m^6$
$d_{50,0}$	reference cut size (A)	150	µm
P_0	reference pressure	90	kPa
β	exponent on pressure (A)	0.225	—
δ	exponent on apex diameter (A)	0.71	—
γ	sensitivity to feed solids (A)	0.063	1/%
$\phi_{v,0}$	reference feed solids	22.97	% vol
$D_{a,0}$	reference apex diameter	100	mm
m	partition sharpness (A)	3.0	—
$R_{f,0}$	water recovery at reference apex	0.28	—
a_{Rf}	water split exponent on apex	1.8	—
d_c	representative coarse size	300	µm
d_f	representative fine size	55	µm
d_{spec}	product specification mesh	106	µm
g	coarse ground to fine per pass (A)	0.32	—
τ_m	mill transport lag (A)	3.0	min
$F_{mill,\max}$	mill throughput limit	1400	t/h
ρ_{rope}	underflow density at roping	80	% solids
ρ_{soft}	density at loss of sharpness	72	% solids
$P_{\max}$	feed pressure limit	190	kPa
τ_{psa}	analyzer averaging	0.4	min
θ_{psa}	analyzer dead time (A)	2.0	min
τ_{fh}	level transmitter filter	0.03	min
τ_{fP}	pressure transmitter filter	0.02	min
Δt_c	controller scan	0.02	min
ρ_s	solids density	2700	kg/m^3

4.3.5 Initialization

The application is initialized from a converged steady state rather than from an assumed one. At reset, the dilution water that closes the sump water balance at the operator's pump speed is found by direct calculation, the recycle streams are converged by damped substitution of (4.11) until neither moves by more than 1×10^{-11}, and the sump inventory is split between water and the two solids classes at the cyclone feed density implied by that converged state. The residual on every derivative at $t = 0$ is below 1×10^{-3} in relative terms, the remaining error being the difference between the exactly balanced water rate, 653.759 m^3/h, and the rounded value carried as the operator input, 653.8 m^3/h,

which moves the level by less than a fifth of a percent in half an hour. The pressure transmitter and the size analyzer start on the pressure and fineness of that state, so a case that starts off the design point, or a setting chosen before the run, is measured correctly from the first scan.

4.3.6 Assumptions

1. **Two size classes.** Solids are coarse or fine at two representative sizes rather than a distribution. The specification, percent passing 106 µm, is computed from the split of those two classes. A real circuit is described by a full distribution and the specification is one point on it, so the application cannot show a bimodal feed or the shape change that a roping cyclone actually produces.

2. **The cyclone is algebraic.** Its residence time is a fraction of a second against a 0.26 min sump, so (4.6) through (4.10) are evaluated instantaneously. This breaks down only for transients faster than the pressure loop itself.

3. **The correlation is reduced.** Vortex finder diameter, inlet area, cone angle and cyclone diameter are fixed and absorbed into $d_{50,0}$ and k_h. Fitted constants for a real installation differ from one another by more than the range this application explores [6].

4. **Roping is smooth.** The sharpness degrades continuously with underflow density. Real roping is abrupt, hysteretic and often intermittent between cyclones on the same manifold, and an operator observes it as a change in the shape of the discharge spray rather than as a number.

5. **The mill is one efficiency and one lag.** There is no ball charge, no power draw and no relationship between the distribution entering the mill and the one leaving it. Mill dynamics are the subject of Chapter 3.

6. **The sump is perfectly mixed.** Real sumps stratify, and a nuclear level gauge reads a density-weighted level that moves when the density moves, which would couple the level measurement to the density path directly.

7. **No rheology.** Water and solids balances only. Slurry viscosity is one of the physical reasons a dense cyclone feed classifies badly and it is absent here, so the exponential density term of (4.7) carries the whole of that effect.

8. **Fixed-step integration.** Classical fourth-order Runge–Kutta at 0.005 min with a 0.02 min controller scan (Section 1.6.1). Every run reproduces bit for bit on any machine, which a grinding circuit does not.

4.4 Controller Design

Cascade structure and the conditions under which it pays are standard material [1, 2] (Section 1.4.6), and the arrangement here is the conventional one. The application implements three loops and offers three architectures. The level and pressure loops are always available. The fineness controller can be switched off, wired to the pump through a fixed characteristic, or cascaded onto the pressure setpoint, and the comparison between the last two is the subject of Section 4.5.

4.4.1 Velocity Form PID Algorithm

All three loops run the velocity-form algorithm of Section 1.6.2 [17], Equations (1.10) to (1.13), with the output bias carried in the integral term, which starts at the actuator position. The scan period $\Delta t_c = 0.02\,\text{min}$ (1.2 s) is common to the three loops. The pressure and fineness loops are proportional-integral. The level loop also carries a derivative term, zero by default, taken on the measurement after a first-order filter of $\tau_f = 0.05\,\text{min}$, so that derivative action added to the level loop does not amplify transmitter noise. The limit acts on the output alone, and back-calculation with

tracking time $T_t = \tau_I$ corrects the integral, so each scan removes $\Delta t_c/\tau_I$ of the clipped excess, one three-hundredth for the fineness loop at $\tau_I = 6\,\text{min}$. Manual operation, and a loop that the chosen architecture leaves idle, track the actuator value in force, and in automatic mode the manual actuator values follow the controller outputs after every scan, so transfer in either direction is bumpless. The actuator spans are 25 % to 95 % on the pump, 50 m^3/h to 1800 m^3/h on the dilution water and 40 kPa to 170 kPa on the pressure setpoint written by the cascade, and no output carries a rate limit. All three loops are reverse acting in the ISA sense, action $a = +1$ in Chapter 1's convention, because more pump speed raises the pressure, more pressure raises the fineness and more dilution water raises the level, so a measurement above setpoint has to reduce the output. With the error written as SP − PV, that is what a positive K_c delivers.

4.4.2 Level Loop and Its Coupling to Feed Density

The level controller manipulates dilution water against a sump holding 0.26 min. The plant is integrating, and the volume-balance gain follows directly from (4.4):

$$\left.\frac{\mathrm{d}h}{\mathrm{d}t}\right|_{Q_w} = \frac{100}{60 A_s H_s} = 0.139\,\%/\text{min per m}^3/\text{h}. \tag{4.13}$$

Equation (4.13) is exact at the instant of a step in Q_w, because the composition terms in (4.1) to (4.4) cancel and leave the total volume responding only to the difference between the volumetric inflow and outflow. The pump removes a fixed total volume whatever the composition of the sump, so the relation holds closely after the step as well. The slope measured on the application in manual mode between 0.35 and 0.85 min after a step of 50 m^3/h is 0.141 %/min per m^3/h, which is 1.4 % larger, and it rises to 0.145 between 0.85 and 1.5 min as the underflow returns part of the extra water to the sump. For tuning, the sump is an integrator with the gain of (4.13). Default tuning is $K_c = 4\,(\text{m}^3/\text{h})/\%$ and $\tau_I = 8\,\text{min}$, and the tuning the application suggests after commissioning is $K_c = 18\,(\text{m}^3/\text{h})/\%$ with $\tau_I = 1.5\,\text{min}$.

4.4.3 Inner Pressure Loop and SIMC Tuning

The inner loop manipulates pump speed against feed pressure. Perturbing (4.5) and (4.6) about the design point, with the water balance closed, gives 2.59 kPa/%. The 63 % response time of the transmitted pressure to a manual pump step from 65 % to 66 % is 7.5 s, set by the 0.10 min drive lag and the 0.02 min transmitter filter with no dead time worth reporting. Applying the SIMC rules of Skogestad [18] to a first-order plant with a closed-loop time constant set equal to the open-loop one, $\tau_c = \tau_p = 0.125\,\text{min}$ and $\theta_p = 0$, gives

$$K_c = \frac{\tau_p}{K_p\,(\tau_c + \theta_p)} = \frac{1}{K_p} = 0.39\,\%/\text{kPa}, \qquad \tau_I = \tau_p. \tag{4.14}$$

The shipped default is $K_c = 0.15\,\%/\text{kPa}$ with $\tau_I = 2\,\text{min}$, deliberately conservative so that the effect of tightening the loop can be measured against a slack starting point, and the tuning the application suggests is $K_c = 0.8\,\%/\text{kPa}$ with $\tau_I = 0.15\,\text{min}$. How much faster than the outer loop an inner loop must be is measured directly in Volume 3, Chapter 3, where the lags lie within a factor of two of one another.

4.4.4 Outer Fineness Loop and the Two Architectures

The single-loop architecture inverts a calibrated pump characteristic. Given a demanded pressure it returns the speed that produced that pressure on a clean pump, at nominal density, with eight cyclones open:

$$N_p = \mathcal{N}^{-1}(P_{\text{SP}})_{\,w_p=1,\; w_c=1,\; n=8,\; \phi_v=\phi_{v,0}}\,. \tag{4.15}$$

Every disturbance that invalidates one of those four subscripts passes straight to the product. The cascade replaces (4.15) with the pressure loop, so the fineness controller writes P_{SP} and the inner loop finds whatever speed that requires.

Table 4.3 The Eleven Scenarios and the Mechanism Each Isolates

#	Title	Mechanism
1	Sump hold-up (manual)	0.26 min of inventory sets the fastest loop
2	What moves the cut size	the three exponents of (4.7)
3	Level control	feed rate raises level, water is removed, feed thickens
4	Inner loop alone	2.59 kPa/%, 7.5 s
5	Fineness without a cascade	pump wear through a static characteristic
6	The same wear, cascaded	pump wear inside the inner measurement
7	Where the authority is	fineness target step, split between the two paths
8	Circulating load runaway	recycle gain $E_c(1-g_e)$ raised
9	Roping	closed apex, then more feed, collapses the partition
10	Liners and a ninth cyclone	disturbances the inner loop hides in part
11	Challenge	four disturbances and noisy instruments over 130 min

Manual mode offers two helper buttons that perform these calculations by hand. Balance the sump returns the dilution water that closes the sump water balance at the present pump speed, which is the operation the level controller performs continuously and which makes the density coupling explicit. Hold this pressure returns the pump speed and dilution water that produce the pressure setpoint at steady state for the pump, cyclones and feed as they stand. It plays the role of (4.15) without its fixed calibration, and a wear disturbance applied afterward shows why such a characteristic ages.

The outer loop's plant gain is not the pressure exponent of (4.7). Measured on the closed-loop circuit with the level controller in service it is 0.275 %/kPa of fineness at the design point, falling to 0.231 %/kPa as a secant across the 19.8 kPa step of Table 4.5, and 63 % of it arrives through the density path. That path is also the fast one. After the step of Table 4.5 the feed density covers 63 % of its final change in 0.43 min, against the 5.1 min of the recycle, and it overshoots, reaching 41.80 % solids one minute after the step before the returning solids bring it back to 43.13 %. The shipped default tuning is $K_c = 0.3\,\mathrm{kPa/\%}$ with $\tau_I = 40\,\mathrm{min}$, and the scenarios that measure performance use $K_c = 1.2\,\mathrm{kPa/\%}$ with $\tau_I = 6\,\mathrm{min}$, obtained by treating the density path rather than the recycle as the dominant dynamic. The challenge scenario suggests a tighter $K_c = 2.0\,\mathrm{kPa/\%}$ with $\tau_I = 4\,\mathrm{min}$. A tuning derived from the recycle, $K_c = 7\,\mathrm{kPa/\%}$, drives the output between its limits of 40 kPa and 170 kPa with a period of 6.0 min in scenario 7.

4.5 Results

Table 4.3 lists the eleven scenarios and the mechanism each isolates. They run on the framework of Section 1.6.4, with a briefing, a completion rule, a scorecard and a CSV export for each, and every IAE quoted below is the scorecard's, integrated from the start of the run. Scenarios 1 and 2 operate the sump and the cyclone by hand, scenarios 3 and 4 close the level and pressure loops one at a time, which is the commissioning order the plant later justifies, scenarios 5 to 7 make the cascade argument and its decomposition, scenarios 8 and 9 are the two constraints that no instrument reports,

Table 4.4 Integrated Absolute Fineness Error Over 120 min and Peak Deviation, Same Disturbance Applied Under Both Architectures

	single loop		cascade	
Disturbance	IAE % min	peak %	IAE % min	peak %
Impeller wear to 0.86	154.9	13.08	**3.2**	2.81
Liner condition to 0.80	25.1	1.97	73.1	5.94
Ninth cyclone opened	26.6	2.08	76.8	6.20
Coarse feed 0.85 to 0.91	53.7	1.61	53.1	1.63
New feed 250 to 320 t/h	199.0	7.13	210.1	8.46
Grinding effectiveness 0.6	432.5	7.88	433.8	8.03

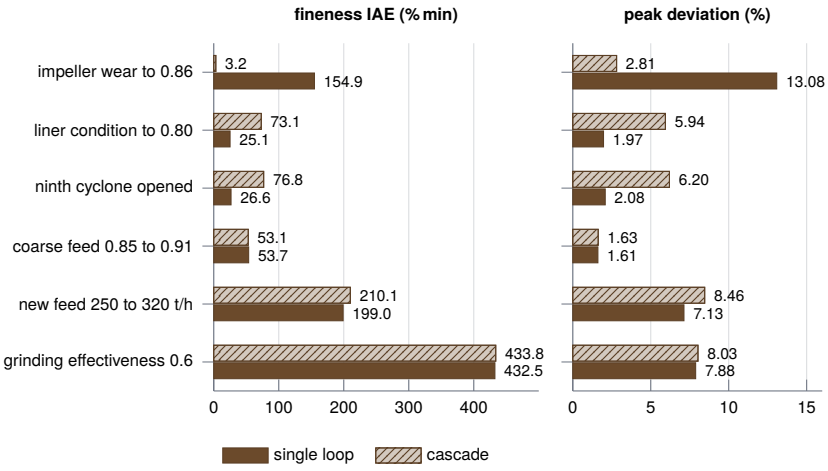


Figure 4.2 Table 4.4 as bars, single loop solid and cascade hatched. Notice the three groups: the cascade wins only on impeller wear, loses on the liners and the ninth cyclone, and changes nothing on the three disturbances that never reach the pressure transmitter.

scenario 10 shows a cascade making two resistance disturbances worse, and scenario 11 grades the whole commissioning sequence against four bands at once.

4.5.1 Disturbance Rejection Under Both Architectures

Table 4.4 applies six disturbances at $t = 6$ min under both architectures with identical outer-loop tuning and reports the integrated absolute fineness error over 120 min, read from the application's scorecard. Each run starts from scenario 6, with the architecture chosen before the run and the disturbance moved at 6 min. The ordering differs from the ordering a block diagram predicts.

Three groups appear, and Figure 4.2 draws them. Impeller wear is the case the textbook describes. It enters the inner measurement, the inner loop returns the pressure to within 2 kPa of setpoint in 10 s, and the integrated error falls by a factor of 49. Feed coarseness, feed rate and grinding effectiveness are invisible to a pressure transmitter, and the two architectures are within 6 % of one another on all three.

🕮 **Pause.** Try Exercise 4.3 before reading on.

Table 4.5 Pressure Setpoint Step From 90.20 kPa to 110.0 kPa With the Level Loop in Service

Quantity	before	after	change
Pump speed N_p (%)	65.01	72.32	+7.31
Dilution water Q_w (m³/h)	653.8	753.0	+99.2
Feed solids (% wt)	44.82	43.13	−1.69
Feed solids ϕ_v (% vol)	23.13	21.93	−1.20
Cut size d_{50} (µm)	151.43	134.28	−17.15
Fineness F (%)	81.44	86.02	+4.58
Decomposition of the change in $\ln d_{50}$			
Pressure factor $(P/P')^{-\beta}$		0.95633	37.1 %
Density factor $e^{\gamma\Delta\phi_v}$		0.92721	62.9 %

The middle group carries the result of interest. Worn liners and an extra cyclone both lower the resistance and therefore both appear in the pressure measurement, which places them in the first group by the usual test. The cascade makes them worse by a factor of three in integrated error. The reason is the density path. A single loop leaves the pump speed where the static characteristic put it, so the flow does not change, the density barely changes, and the cut size moves only through the weak pressure term, from 90.20 kPa to 71.86 kPa and the overflow from 81.44 % to 80.16 % passing. The cascade restores the pressure by raising the pump speed from 65.0 % to 73.4 %, which draws 118 m³/h more dilution water into the sump, thins the feed from 44.82 % to 42.46 % solids and takes the overflow to 85.49 % passing, an excursion three times the size of the one it prevented. The excursion is on the fine side and the outer loop brings the fineness back within 0.5 % of target 39 min after the wear, so no product is lost, but the disturbance the inner loop rejected was carried into the product by the correction itself.

🕮 **Pause.** Try Exercise 4.2 before reading on.

4.5.2 Decomposition of the Outer Loop Gain

Table 4.5 reports the steady states before and after a pressure setpoint step from 90.20 kPa to 110.0 kPa, made at 5 min in scenario 6 with the size controller switched off, which is the step scenario 4 scripts at 4 min. The run reaches the tabulated values within 60 min, and they agree with the steady-state solution of the model to the digits shown. The decomposition uses (4.7) directly, since the pressure and density terms are separable in the logarithm, and Figure 4.3 draws the two paths it separates. Scenario 7 reaches the same move from the outer loop, stepping the fineness target from 81.4 % to 86 % under the cascade. After the hour its completion rule asks for, the pressure has risen to 107.3 kPa and the same decomposition assigns 36.6 % to pressure and 63.4 % to density.

Two consequences follow. The outer loop's plant gain is not the one an engineer reading (4.7) alone would compute, and is larger by a factor of 2.7, so a tuning derived from the correlation puts more gain around the loop than intended, by the same factor, and a loop more aggressive than its design. The level controller, which has no product variable anywhere in its configuration, is also a quality controller, and widening its proportional band on the grounds that it is only a level loop detunes the product size.

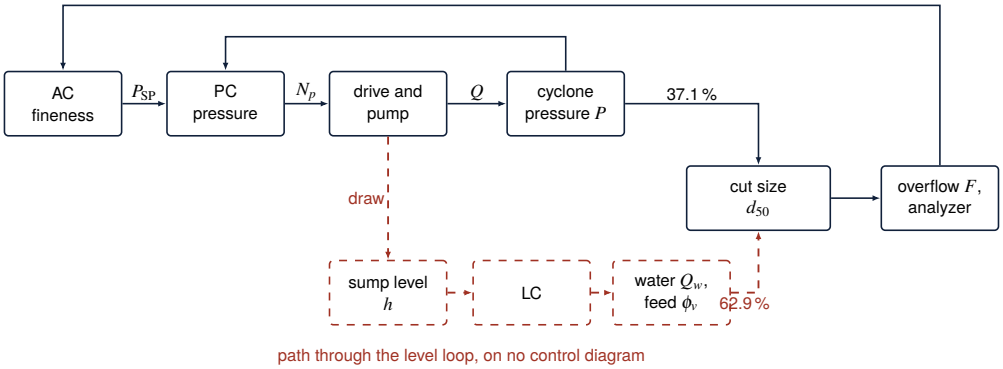


Figure 4.3 The two paths from the size controller AC to the cut size. The designed path runs through the pressure loop PC. The dashed path runs from the pump's draw on the sump through the level loop LC and the dilution water to the feed density. Notice that the dashed path carries the larger share of the steady-state change, 62.9 % in Table 4.5.

Table 4.6 Challenge Scenario Over 130 min With Three Level Tunings and the Suggested Cascade Tuning, Everything Else Unchanged

K_c ((m^3/h)/%)	τ_I (min)	level IAE (% min)	water travel (m^3/h)	fineness IAE (% min)
4	8	524.7	3465	138.9
8	4	168.7	6919	142.1
18	1.5	66.9	15682	149.4

✦ DID YOU KNOW? The Level Loop Moves the Cut

From the simulator: in Table 4.5 a pressure step from 90.20 to 110.0 kPa moves the cut size from 151.43 to 134.28 µm. Of that change, 62.9 % comes through the feed density that the level loop sets and only 37.1 % through the pressure itself.

The coupling also runs in the other direction. Scenario 3 raises the new feed from 250 t/h to 320 t/h at 3 min with the level and pressure loops closed and the size controller off, and returns it at 18 min. The level rises by at most 1.6 %, the controller removes water from 653.8 m^3/h to 603.8 m^3/h, the feed thickens from 44.82 % to 49.50 % solids, and by 18 min the overflow has fallen from 81.44 % to 70.71 % passing, within 0.03 % of its new steady state. A level loop holding its setpoint and a pressure loop holding 90.2 kPa have together moved the product by 10.7 % of fineness.

TRY IT IN THE APP **3 · Level Control on Dilution Water**

Load the scenario and press Run. The level and pressure loops are closed and the size loop is off. At 3 min the new feed rises from 250 to 320 t/h. Watch the level barely move while the dilution water falls, then watch the fineness slide from 81.44 to 70.71 % passing by 18 min, with the pressure on its setpoint throughout.

4.5.3 Effect of Level Loop Tuning

Because the level controller is coupled to the product, a natural question is whether tightening it improves the product. Table 4.6 answers that question on the challenge scenario, and the improvement does not appear.

Level control improves by a factor of 7.8 and costs a factor of 4.5 in valve travel, while the fineness error rises by 7.6 %. Only the tightest tuning keeps the level inside the challenge's ±4 % band, with largest deviations of 10.8 %, 7.2 % and 3.4 % for the three rows. The challenge carries transmitter noise, and the rise in fineness error with the tight tuning holds on each of six noise seeds, by 3 % to 12 %. The tighter loop passes level noise and every level deviation into the dilution water, and the feed density it delivers travels three times as far over the run. The product error in this scenario is set by the two feed rate steps, which the cascade cannot see, and no amount of level tuning addresses them. The coupling established in Table 4.5 is a steady-state authority rather than a disturbance rejection path, and the distinction between the two is what this measurement establishes.

4.5.4 Unmeasured Constraints: Roping and Circulating Load

Two limits bind in this circuit and neither is measured on most plants. Scenario 9 closes the apex from 100 mm to 68 mm at 5 min and raises the new feed from 250 t/h to 420 t/h at 40 min. The closed apex alone takes the underflow from 67.3 % to 73.4 % solids, with σ in (4.10) at 0.83. The feed step takes it to 77.9 % against the 80 % at which σ reaches zero, the roping alarm ($\sigma < 0.35$) sounds 0.77 min after the step, and the circuit settles with the underflow at 77.0 % and $\sigma = 0.38$. The coarse and fine recoveries converge from 0.918 and 0.313 to 0.725 and 0.455, the partition curve flattens, and the overflow drops to 53.9 % passing, with 74.4 min of the 80 min run off specification. The outer loop drives the pressure setpoint to its 170 kPa limit by 42.8 min, and the pressure loop holds it with an integrated error of 7.40 kPa min over the run. Scenario 8 lowers the grinding effectiveness to 0.55 and asks for 92 % fineness. The recycle gain rises, the circulating load climbs from 288 % to 386 % and the cyclone feed from 971 t/h to 1215 t/h, the outer loop holds its output at the 170 kPa limit for 79.7 % of the 90 min run, and the fineness settles at 76.5 %.

TRY IT IN THE APP **9 · A Roping Apex Stops Classifying**

Load the scenario and run it for 80 min. The apex closes to 68 mm at 5 min and the new feed rises to 420 t/h at 40 min. Watch the underflow density climb, the roping alarm sound and the partition curve flatten while the pressure and level loops stay on their setpoints. The overflow ends at 53.9 % passing.

In both cases the level and pressure loops hold their setpoints while the size controller sits at its limit. The variables that failed, underflow density and circulating load, are computed rather than measured, and the operator's cue for the first of them is the shape of the underflow discharge.

Pause. Try Exercise 4.1 before reading on.

Figure 4.4 shows the circuit at the end of the roping scenario.

4.5.5 Graded Challenge Scenario

Scenario 11 applies four disturbances over two hours: new feed to 305 t/h at 8 min, impeller wear to 0.88 at 30 min, coarse feed to 0.91 at 55 min and feed back to 255 t/h at 85 min, with noise of 0.5 %, 1.2 kPa and 0.5 % (one standard deviation) on the level transmitter, the pressure transmitter and the size analyzer. It opens with the fineness controller off and the cautious tunings of scenario 1. The case

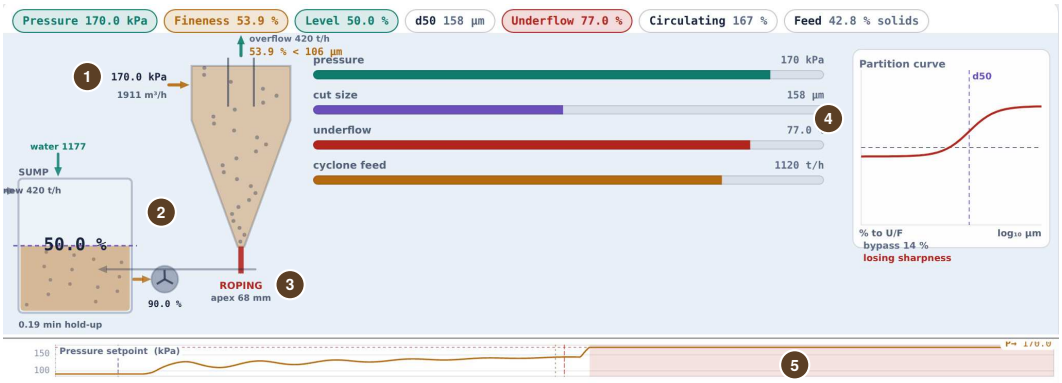


Figure 4.4 Scenario 9 at $t = 80$ min (1.33 h on the application clock), cropped to the plant view and the pressure setpoint strip, 40 min after the new feed was raised from 250 t/h to 420 t/h with the apex already closed to 68 mm. The pressure (1) and level (2) loops are holding 170.0 kPa and 50.0 %, while the underflow is at 77.0 % solids with the roping alarm on (3), the partition curve has flattened (4) and the overflow has fallen to 53.9 % passing 106 µm. The size controller's output is at its 170 kPa limit (5) and the product is off specification.

is passed when, over the full 130 min, the feed pressure stays within ±4 kPa of its setpoint outside the first minute after each upset, the level stays within ±4 %, the fineness returns within ±1.5 % of target after each of the four upsets, and the underflow never exceeds 76 % solids. Left as shipped it fails, with pressure deviations to 13.3 kPa, level deviations to 13.6 %, the fineness back after none of the upsets, 964.9 % min of fineness error and 121.9 min off specification. Commissioned from the fastest loop outward with the cascade in service and the tunings the application suggests, level $K_c = 18\,(\mathrm{m}^3/\mathrm{h})/\%$ with $\tau_I = 1.5$ min, pressure $K_c = 0.8\,\%/\mathrm{kPa}$ with $\tau_I = 0.15$ min and fineness $K_c = 2.0\,\mathrm{kPa}/\%$ with $\tau_I = 4$ min, the same run passes with 3.1 kPa, 3.4 %, four recoveries out of four and 69.5 % solids, and gives 149.4 % min and 14.9 min, a factor of 6.5 in integrated error. The verdict is the same on six noise seeds, with 149.4 % min to 164.9 % min for the suggested tunings. The scenarios' fineness tuning of $K_c = 1.2\,\mathrm{kPa}/\%$ with $\tau_I = 6$ min gives 284.0 % min and fails, because the fineness is still 2.7 % below target when the impeller wears at 30 min. Of the remaining 149.4 % min, 108.6 % min follows the two feed rate steps, which reach the product through the feed density and which no architecture in this application can detect before the analyzer reports them.

4.6 Technology in Practice

Figure 4.5 draws the three loops of Section 4.4 with their hardware.

What Measures It

The model reduces the pressure transmitter to a 0.02 min filter and the level transmitter to a 0.03 min filter, both with optional noise, and those filters stand for the sensing element and its seal.

> **TECHNOLOGY** **Sensor: A Pressure Transmitter With a Remote Seal**
>
> In most modern pressure transmitters a flexible metal isolating diaphragm passes the process pressure through a fill fluid to a silicon diaphragm. Its strain gauges unbalance a bridge circuit, and the voltage becomes a 4 to 20 mA or fieldbus signal. Where sludge would clog an impulse line, the diaphragm is set at the pipe as a remote seal joined to the transmitter by a filled capillary. The fill must stay free of gas, since bubbles absorb the diaphragm's motion and ruin the accuracy, and a height difference between seal and transmitter adds a constant offset to the reading [19].

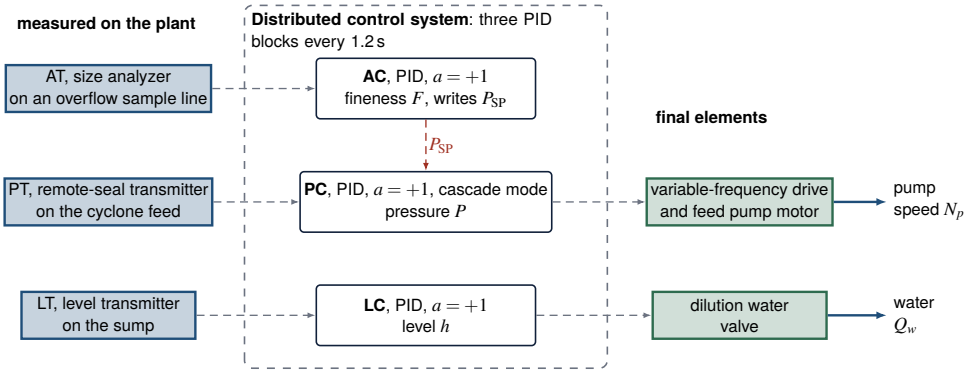


Figure 4.5 The three loops of the simulated circuit drawn with their hardware. The level transmitter LT, the pressure transmitter PT and the size analyzer AT feed three PID blocks, all reverse acting with $a = +1$. LC moves the dilution water valve, PC moves the variable-frequency drive of the feed pump, and AC writes the setpoint of PC, which runs in cascade mode. Notice that no block on the figure reads the feed density, which carries most of AC's authority.

The product measurement is slower. Particle size is measured on stream by instruments with cycle times of minutes [13], which the model represents as a 2.0 min dead time followed by a 0.4 min average. The feed density, the underflow density and the circulating load have no instrument at all.

What Moves It

The model's pump drive is a first-order lag of 0.10 min over a span of 25 to 95 % with no rate limit, the dilution water valve a lag of 0.15 min, and the apex actuator a travel time of 1.2 min.

> **TECHNOLOGY Final Element: A Variable-Frequency Drive on the Feed Pump**
>
> The cyclone feed pump is centrifugal, the most common pump design in the process industries, and its discharge pressure rises as its flow falls. Its speed is set by a variable-frequency drive, which rectifies the line power to direct current, smooths it, and inverts it back to alternating current at the frequency the commanded speed needs. A fixed-speed motor then delivers useful power over a wide range of speeds, and the drive can ramp the speed to spare the mechanism [19]. A worn impeller shows no fault at the drive: the speed is right and the flow is short.

The delivery multiplier w_p of Equation (4.5) stands for that short flow at the right speed.

Where the Controller Runs

Control loops in a plant are often nested or cascaded, the setpoint of one loop being based on the variable measured by another, and supervisory and lower-level loops run with cycle times from milliseconds to minutes. A distributed control system uses a supervisory loop to mediate a group of localized controllers [20], and programmable logic controllers run PID loops as often as they switch discrete devices [19]. The app integrates every 0.005 min and runs its three PID blocks on a 0.02 min (1.2 s) scan. On such a platform the chapter's structure is three velocity-form PID blocks with back-calculation, the output of the fineness block wired to the remote setpoint of the pressure block.

> **IN PRACTICE** **Commissioning a Cascade From the Inside Out**
>
> The secondary controller of a cascade is tuned first. It does not depend on the primary, while the primary can exert no control at all if the secondary sits in manual. The secondary then has three modes: manual, automatic with a local setpoint, and cascade, in which the primary writes its setpoint. A secondary left in automatic defeats the cascade as surely as a controller left in manual [19]. In this circuit the pressure loop is that secondary, and the level loop comes before both.

What Is Modernizing It

The first development is seeing inside the cyclone. Its internal flow is difficult or impossible to observe directly, because of the reinforced walls and the opaque slurry, and electrical impedance tomography estimates the conductivity field inside from electrodes outside. It collects several hundred frames a second, enough for real-time monitoring, and Bayesian methods now carry the numerical error of the reconstruction into its uncertainty [21]. The second is control above the cascade. Model-based supervision of the sump water and the cyclone feed rate [15] and feedforward from the measured feed size distribution [16], both met in Section 4.2, reach disturbances that no pressure transmitter sees. The third is a classification model that accounts for density. A Department of Energy project on iron ore circuits set out to model the mechanisms by which hydrocyclones retain fine, dense particles until they are overground, and to improve existing models to account for that behavior [11].

4.7 Limitations

The two size classes are the largest simplification. A real classifier is judged on a distribution, and the specification used here is a scalar computed from the split of two representative sizes. That is adequate for the control questions the application asks and inadequate for any statement about product distribution, because it cannot represent the shape change that a degraded partition curve actually produces.

> **OPEN CHALLENGE** **Grains That Differ in Density**
>
> A hydrocyclone sorts by density as well as by size. In an iron ore circuit, a cut set so that light quartz leaves only below its liberation size sends the dense magnetite to an even finer size, and the cyclone returns it to the mill to be overground. Earlier models could not handle such mixed-density feeds, nor the fish-hook in the measured efficiency curve, where some finer particles are more likely to report to the underflow than coarser ones [22]. This chapter has one solids density. Progress would be a classification model simple enough for control that keeps a partition curve for each mineral.

The cut-size correlation is reduced to three variables and its constants are fitted rather than measured. Vortex finder diameter, inlet geometry and cone angle are absorbed into two constants, and published comparisons of these correlations show that fitted coefficients differ between installations by more than the ranges explored here [6]. Nothing in this application supports a numerical claim about a particular cyclone. Mechanistic models built from the swirling flow field give a physical account of the same behavior and are correspondingly harder to identify [7].

The mill is a single grinding efficiency and a transport lag, with no power draw, no charge and no size distribution. That is sufficient to make the circulating load a genuine recycle with the right time constant and insufficient to say anything about grinding. Slurry rheology is absent entirely, so the whole effect of a dense feed on classification is carried by the exponential term of (4.7). Roping is smooth where a real apex is abrupt and hysteretic. The particle size analyzer is a continuous dead time equal to its nominal cycle time followed by a 0.4 min average, with optional noise. Unlike the sampled assays of Section 1.4.5, it does not hold its reading between cycles, and it has none of the calibration drift, sample line plugging or distribution-dependent bias that make on-stream sizing difficult in practice.

4.8 Discussion

The instruction to commission a control system from the fastest loop outward is usually given as a convention. This circuit supplies a reason. Until the level loop is holding, the cyclone feed density wanders, so through (4.6) the pressure loop's plant gain wanders, so the fineness loop's plant is not stationary. Tuning any loop against a moving plant produces a tuning valid at one condition, and the order in which the loops are closed is what makes each plant stationary in turn.

The same reasoning identifies what a cascade can and cannot be expected to do, and Table 4.4 shows that the usual test is incomplete. The question is not only whether a disturbance appears in the inner measurement, but also whether the inner loop's correction moves a variable that the outer loop cares about. Worn liners satisfy the first condition and fail the second. On this circuit that is a benign failure, because the excursion is on the fine side, but the general form of it is not benign. Any cascade whose inner loop manipulates a flow into a vessel under level control is coupled to the composition of that vessel, and the coupling is invisible in both loops.

Feedforward is the remaining option for the disturbances that no cascade reaches. Feed size distribution is measurable, and feedforward from it has been demonstrated for grinding circuits [16]. The application does not implement it, and the coarse feed row of Table 4.4, 53.1 % min of integrated error that the cascade does not remove, is the available improvement.

4.9 Conclusion

A grinding circuit classifier holds two time scales a factor of 41 apart in one manifold, and the resulting cascade is worth a factor of 49 in integrated product error against the disturbance it was designed for and nothing at all against three of the six disturbances tested. The third group in between is the result this chapter is built around, where an inner loop rejects a disturbance in its own measurement and carries it into the product through the level controller, and where the outer loop's steady-state authority is 63 % density and 37 % pressure. Both effects run through a loop that appears on no control diagram as a quality loop, and both are measurable in the browser in under two minutes. The commissioning order that follows, level first and outward, is then a statement about which plant gains are stationary when rather than a convention. On a plant with a genuine recycle, the application lets a reader measure cascade structure, loop interaction and commissioning order.

KEY POINTS

- The product size depends more on how thick the cyclone feed is than on the feed pressure, and the level loop sets that thickness.
- The cascade divides the integrated fineness error by 49 for pump wear and by 1.01 for feed coarseness.
- Worn liners and an extra cyclone appear in the pressure, yet the cascade makes them worse by a factor of three in integrated error.
- A tighter level loop holds the level 7.8 times better and the product no better.
- Commissioning from the level loop outward makes each loop's plant stationary in turn.

Where else this applies. The same disturbance sort on a level loop in Volume 3, Chapter 2, an inner loop's speed in Volume 3, Chapter 3, a recycle that amplifies in Volume 6, Chapter 7.

References

[1] D. E. Seborg, T. F. Edgar, D. A. Mellichamp, and F. J. Doyle, III, *Process Dynamics and Control*, 4th ed. Hoboken, NJ: John Wiley & Sons, 2016.

[2] F. G. Shinskey, *Process Control Systems: Application, Design, and Tuning*, 4th ed. New York: McGraw-Hill, 1996.

[3] B. W. Bequette, *Process Control: Modeling, Design, and Simulation*. Upper Saddle River, NJ: Prentice Hall, 2003.

[4] B. A. Wills and J. A. Finch, *Wills' Mineral Processing Technology: An Introduction to the Practical Aspects of Ore Treatment and Mineral Recovery*, 8th ed. Oxford: Butterworth-Heinemann, 2015.

[5] T. J. Napier-Munn, S. Morrell, R. D. Morrison, and T. Kojovic, *Mineral Comminution Circuits: Their Operation and Optimisation*. Brisbane: Julius Kruttschnitt Mineral Research Centre, University of Queensland, 1996.

[6] K. Nageswararao, D. M. Wiseman, and T. J. Napier-Munn, "Two empirical hydrocyclone models revisited," *Minerals Engineering*, vol. 17, no. 5, pp. 671–687, 2004.

[7] K. T. Hsieh and R. K. Rajamani, "Mathematical model of the hydrocyclone based on physics of fluid flow," *AIChE Journal*, vol. 37, no. 5, pp. 735–746, 1991.

[8] E. Bretney, "Water-purifier," U.S. Patent 453,105, 1891, application filed November 3, 1890; patented May 26, 1891.

[9] K. J. Åström and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2nd ed. Princeton, NJ: Princeton University Press, 2021.

[10] J. Jeswiet and A. Szekeres, "Energy consumption in mining comminution," *Procedia CIRP*, vol. 48, pp. 140–145, 2016.

[11] S. K. Kawatra, T. C. Eisele, T. Weldum, D. Larsen, R. Mariani, and J. Pletka, "Optimization of comminution circuit throughput and product size distribution by simulation and control," Michigan Technological University, Quarterly Technical Progress Report, U.S. DOE award DE-FC26-01NT41062, 2005.

[12] D. Wei and I. K. Craig, "Grinding mill circuits—a survey of control and economic concerns," *International Journal of Mineral Processing*, vol. 90, no. 1–4, pp. 56–66, 2009.

[13] D. Hodouin, "Methods for automatic control, observation, and optimization in mineral processing plants," *Journal of Process Control*, vol. 21, no. 2, pp. 211–225, 2011.

[14] J. D. le Roux, I. K. Craig, D. G. Hulbert, and A. L. Hinde, "Analysis and validation of a run-of-mine ore grinding mill circuit model for process control," *Minerals Engineering*, vol. 43–44, pp. 121–134, 2013.

[15] V. R. Radhakrishnan, "Model based supervisory control of a ball mill grinding circuit," *Journal of Process Control*, vol. 9, no. 3, pp. 195–211, 1999.

[16] M. G. Maritz, J. D. le Roux, and I. K. Craig, "Feed size distribution feedforward control for a grinding mill circuit," *IFAC-PapersOnLine*, vol. 52, no. 14, pp. 7–12, 2019.

[17] K. J. Åström and T. Hägglund, *Advanced PID Control*. Research Triangle Park, NC: ISA – The Instrumentation, Systems, and Automation Society, 2006.

[18] S. Skogestad, "Simple analytic rules for model reduction and PID controller tuning," *Journal of Process Control*, vol. 13, no. 4, pp. 291–309, 2003.

[19] T. R. Kuphaldt, *Lessons in Industrial Instrumentation*, version 2.33 ed. Open textbook, Creative Commons Attribution 4.0 license, ibiblio.org/kuphaldt/socratic/sinst, 2024.

[20] K. Stouffer, M. Pease, C. Tang, T. Zimmerman, V. Pillitteri, S. Lightman, A. Hahn, S. Saravia, A. Sherule, and M. Thompson, "Guide to operational technology (OT) security," National Institute of Standards and Technology, Gaithersburg, MD, Special Publication 800-82r3, 2023.

[21] C. J. Oates, J. Cockayne, R. G. Aykroyd, and M. Girolami, "Bayesian probabilistic numerical methods in time-dependent state estimation for industrial hydrocyclone equipment," *Journal of the American Statistical Association*, vol. 114, no. 528, pp. 1518–1531, 2019.

[22] H. J. Walqui, T. C. Eisele, and S. K. Kawatra, "Optimization of comminution circuit throughput and product size distribution by simulation and control," Michigan Technological University, Quarterly Technical Progress Report, U.S. DOE award DE-FC26-01NT41062, 2003.

4.10 Exercises

4.1 ○ Concept check · 1 min

In scenarios 8 and 9 the level and pressure loops hold their setpoints while the product fails. Which of these quantities does the circuit of Figure 4.1 measure with an instrument?

(a) The circulating load returned to the mill

(b) The underflow density at the apex

(c) The cyclone feed pressure

(d) The mill grinding effectiveness g_r

4.2 Concept check · 2 min

The cyclone liners wear, and the liner condition w_c falls from 1.00 to 0.80 at a fixed pump speed. Which statement agrees with Table 4.4?

(a) The cascade restores the pressure in seconds, so the fineness barely moves
(b) Neither architecture can see the wear, so both give the same integrated error
(c) The single loop is worse, because its pressure falls and nothing restores it
(d) The cascade is about three times worse, because restoring the pressure thins the feed

4.3 Concept check · 2 min

A coarser new feed, x_c from 0.85 to 0.91, gives 53.7 % min of integrated fineness error under the single loop and 53.1 % min under the cascade. Why does the cascade help so little?

(a) Ore coarseness does not appear in the pressure transmitter, so the inner loop never acts on it
(b) The analyzer's dead time is longer under the cascade, which cancels what the inner loop gains
(c) The size controller saturates at 170 kPa under both architectures
(d) The inner loop's correction thins the feed by as much as it restores the pressure

4.4 Theory and calculation · 6 min

Use the cut-size correlation, Equation (4.7), with $\beta = 0.225$ and $\gamma = 0.063/\%$.

(a) At a fixed apex and feed density, by what factor must the pressure rise for a cut 10 % finer? Give it to two decimals. *Checkpoint:* 1.60

(b) The pressure step of Table 4.5 takes the pressure from 90.20 to 110.0 kPa and the feed solids ϕ_v from 23.13 to 21.93 % by volume. Compute the pressure factor and the density factor to four decimals, and the share of the change in $\ln d_{50}$ that each carries, to 0.1 %. *Checkpoint:* 0.9563 and 0.9272; 37.1 % and 62.9 %

(c) An engineer computes the outer loop's plant gain from the pressure term alone and tunes the fineness controller for it. By what factor is the real gain larger, and is the tuned loop slower or faster than intended? *Checkpoint:* 2.7; faster

4.5 Theory and calculation · 6 min

The sump of Table 4.2 has a cross-section of 4.0 m^2 and a depth of 3.0 m. At the design point it runs at 50 % level and the pump draws 1380 m^3/h.

(a) Compute the slurry volume and the hold-up time, to 0.01 min. *Checkpoint:* 6.0 m^3; 0.26 min

(b) Compute the integrating gain of the level on the dilution water, in %/min per m^3/h, to three decimals. *Checkpoint:* 0.139

(c) Scenario 1 cuts the dilution water by 200 m^3/h in Manual. Ignoring the valve lag, estimate how long the level takes to fall from 50 to the 12 % suction alarm, to 0.01 min. *Checkpoint:* 1.37 min

4.6 Theory and calculation · 7 min

At the design point the new feed is 250 t/h at 85 % coarse, the cyclones send $E_c = 0.918$ of the coarse and $E_f = 0.313$ of the fine to the underflow, and the mill grinds $g_e = 0.32$ of the coarse it receives. Treat Equations (4.11) at steady state.

(a) Compute the coarse recirculation gain to four decimals and the coarse solids in the cyclone feed, to 0.1 t/h. *Checkpoint:* 0.6242; 565.5 t/h

(b) Compute the fine solids in the cyclone feed, to 0.1 t/h, and the circulating load, the underflow solids as a percentage of the new feed, to 1 %. *Checkpoint:* 296.4 t/h; 245 %

(c) With the grinding effectiveness at its lowest setting the gain settles at 0.80. By what factor does the coarse recycle now amplify the new coarse feed, and how many times the design amplification is that, to one decimal? *Checkpoint:* 5; 1.9

4.7 App challenge · 8 min

Open scenario 6, "The same disturbance, with the cascade", at the default seed.

(a) Run at 1 min per second and, when the clock reads between 5.5 and 7 min, press *Pump impeller wears (86 %)*. Switch to 20 min per second and run to 120 min. Read the fineness loop's IAE on the scorecard, within 0.1 % min, and the Off-spec time tile. *Checkpoint:* 3.2 % min; 0.1 min

(b) Reload the scenario with the reload button, set Size control to Single loop before Run, and repeat. Read the same two tiles, the IAE within 0.5 % min. *Checkpoint:* 154.9 % min; 33.3 min

(c) Compute the ratio of the two IAE values and explain it from where the disturbance enters. *Checkpoint:* about 49

4.8 App challenge · 9 min

Open scenario 7, "Where the outer loop's authority really comes from", as shipped at the default seed. The fineness target steps from 81.4 to 86 % at 5 min.

(a) Run at 20 min per second to at least 90 min (1.5 h), pause and export the CSV. The file's time column is in hours. From the last row before 5 min and the row nearest 1.000 h, read `press_kPa`, `feed_vol_pct` and `d50_um`, to 0.01 kPa, 0.001 % and 0.1 µm. *Checkpoint:* 90.15, 23.133, 151.5; 107.27, 22.059, 136.1

(b) Split the change in $\ln d_{50}$ into a pressure part and a density part with Equation (4.7), within 0.3 %, and check that the two factors reproduce the cut size at 1 h. *Checkpoint:* 36.6 % pressure, 63.4 % density

(c) Read `fine_pct` at 1.000 h and `press_kPa` at 1.500 h, within 0.1. Is the circuit at steady state after the hour, and how does the split compare with Table 4.5? *Checkpoint:* 85.5 %; 108.9 kPa

4.9 App challenge · 8 min

Open scenario 11, the challenge, at the default seed, and run each part for the full 130 min at 20 min per second.

(a) Run it as shipped. Read the challenge verdict and the fineness loop's IAE, within 1 % min. *Checkpoint:* fails: 13.3 kPa, 13.6 %, 0 of 4; 964.9 % min

(b) Reload, press Reveal suggested tuning and run again. Read the verdict, the IAE within 0.5 % min and the Off-spec time tile. *Checkpoint:* passes: 3.1 kPa, 3.4 %, 4 of 4, 69.5 %; 149.4 % min; 14.9 min

(c) Reload, reveal the suggested tuning, then set the fineness loop to $K_c = 1.20\,\mathrm{kPa/\%}$ and $\tau_I = 6\,\mathrm{min}$ before Run. Read the verdict and the IAE within 1 % min, and say which upset the fineness misses. *Checkpoint:* fails, 3 of 4; 284.0 % min; the feed step at 8 min

4.10 Application area · 4 min

Roping, Equation (4.10), sets the sharpness factor σ from the underflow density, with $\rho_{rope} = 80\,\%$ and $\rho_{soft} = 72\,\%$ solids.

(a) Compute σ at 73.4 and 77.0 % solids, to three decimals, and the underflow density above which the alarm at $\sigma < 0.35$ sounds, to 0.1 %. *Checkpoint:* 0.825, 0.375; 77.2 %

(b) On a real circuit nothing reads the underflow density. What does the operator watch, and name two remedies once the apex ropes. *Checkpoint:* the shape of the discharge; open the apex, cut the solids, open a cyclone

4.11 Application area · 4 min

The Introduction says the pressure is a weak handle on the cut and that pump power rises with the cube of speed. Hydraulic power is $P\,Q$.

(a) At a fixed feed density and eight cyclones, $P \propto Q^2$. By what factor must the flow and the hydraulic power rise for a cut 10 % finer by pressure alone? Give two decimals. *Checkpoint:* 1.26; 2.02

(b) In Table 4.5 the pump goes from 65.01 to 72.32 % with the pressure from 90.20 to 110.0 kPa, and Q is proportional to speed. Compute the rise in hydraulic power and the fall in cut size, both in percent, to one decimal. *Checkpoint:* 35.7 %; 11.3 %

4.12 Review the agent · 5 min

An AI assistant reviewing the challenge writes: "Table 4.5 shows that 63 % of the outer loop's authority passes through the level controller, so the level loop is the real quality loop. Raising the level gain from 4 to 18 $(m^3/h)/\%$ and cutting its integral time from 8 to 1.5 min will therefore cut the challenge's fineness error substantially." Find the flaw and say how to check it on the app.