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CHAPTER 8

A Copper Electrorefining Simulator for Inferential Constraint Control

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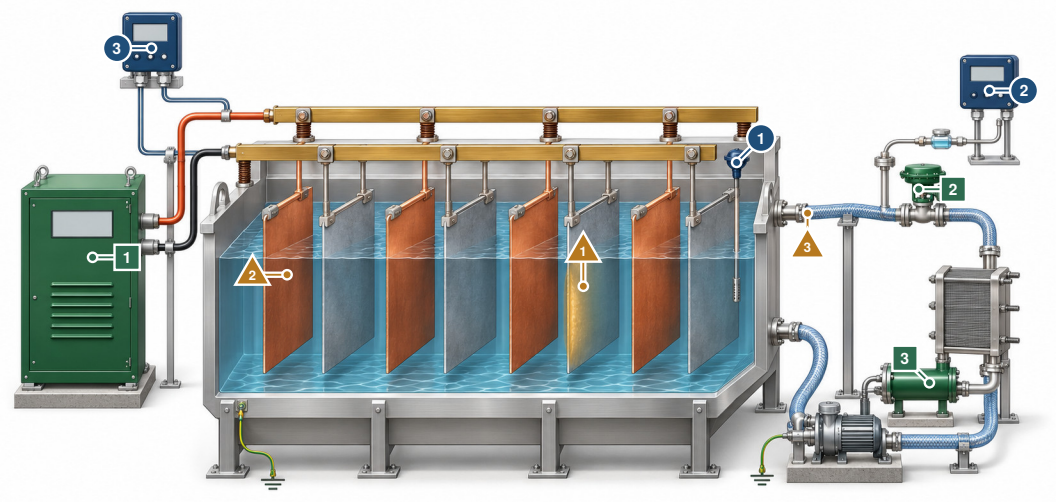
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CHAPTER 8

A Copper Electrorefining Simulator for Inferential Constraint Control



The controller moves 1 rectifier that sets the cell current 2 electrolyte flow valve 3 trim heater

The controller reads 1 electrolyte temperature 2 copper in the electrolyte, from a sample, two hours late 3 cell voltage

What disturbs it 1 solids and nodules at the cathode 2 anodes that passivate 3 leaner copper in the returning electrolyte

An electrorefining cell whose current is pushed toward a limit at the cathode surface that no instrument in the picture can read. Illustration, not to scale.

IN PLAIN TERMS **Pushing a Copper Cell Toward an Unseen Limit**

The main idea. A refining cell passes an electric current through an acid bath. The current carries copper from a rough slab onto a clean sheet, and more current makes more copper.

The challenge. Past a certain point, small bumps on the sheet grow faster the taller they get, and the sheet is spoiled. That point is worked out from book values and a late lab test, not read from a gauge. So a wrong guess goes unseen while every reading looks normal.

Why it matters. A safe margin costs a little copper, but crossing the true limit can spoil a whole crop.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	The first page, the history box of Section 8.2, the fact box of Section 8.3.2, the try-it and fact boxes of Sections 8.5.1 and 8.5.2, the try-it box of Section 8.5.5, the history box of Section 8.5.6, and the key points
Study	40 min	The whole chapter with its pause points and the twelve exercises
Practice	20 min	The first page, Section 8.5, Section 8.6, Section 8.7 and the open challenge after it

The control question A tankhouse makes more copper the harder it drives the rectifier, until nodules on the cathode start to compound past a stability limit, and the limiting current density that sets that limit is computed, not measured. How close to an inferred constraint should a constraint controller run, and what does a wrong estimate cost while every loop reports success?

Learning objectives

- Compute the limiting current density, the utilization and the stability limit at an operating point from the mass-transfer, growth and leveler laws.
- Explain why the copper loop is direct acting and how closing its valve lowers the production ceiling.
- Predict the true utilization a constraint controller holds when its estimate of the limit is biased, and size a back-off for it.
- Weigh a linear gain in copper against a compounding loss in grade over a cathode cycle, and choose a defensible utilization setpoint.
- Distinguish a disturbance the constraint controller defends from one it cannot see, and choose the remedy for each.

Prerequisites Sections 1.4.6 (inferential control and the back-off from an inferred limit), 1.4.5, 1.6.2 and 1.6.4.

The app https://apmonitor.com/apps/copper-electrorefining/

See also Volume 5, Chapter 10 and Volume 5, Chapter 11 for control on an inferred variable that no instrument reaches, and Volume 4, Chapter 2 and Volume 4, Chapter 6 for a production rate held against a limit, with an override behind it.

8.1 Introduction

An electrorefining cell dissolves an impure copper anode and deposits pure copper onto a cathode by passing current between them through an acidic copper sulfate electrolyte [1]. Faraday's law makes the production rate exactly proportional to the current, so the economic instruction to a tankhouse is to raise the rectifier current. Mass transfer sets the limit: copper ions reach the cathode surface by diffusion across a concentration boundary layer, and as the applied current density approaches the limiting current density the deposit roughens and the cathode is downgraded [2]. A protrusion on the cathode reaches further into the boundary layer than its surroundings, so it is better supplied and grows faster the taller it is, and the leveling agent in the electrolyte is what holds that instability back [3, 4].

No instrument on an industrial cell measures the limiting current density. The value is computed from a handbook diffusivity, a boundary-layer correlation and the assayed bulk concentration, so the controlled variable of the production loop is an inferred quantity. Inferential measurements are common in the process industries wherever a controlled variable has no online sensor [5, 6], and the loop inherits every error in the inference while its own performance statistics report only the error it can see. Two other plants of this volume hold inferred variables: the impact angle of Chapter 3 and the tailings grade of Chapter 7.

The app follows the browser and laboratory formats of the same series [7]. Its scenarios teach constraint control against an estimated limit, inferential measurement, an instability that compounds past a threshold, override protection and actuator saturation, which are standard topics in process control texts [8, 9]. Surveys of process control education ask for problems in which the economic decision and the control structure interact [10, 11]. Industry requires the same judgment, because the margin held between a constraint setpoint and an estimated limit is the routine economic decision on a tankhouse.

Section 8.2 places the cell in a tankhouse and in its control hierarchy. Section 8.3 gives the model in enough detail to replicate it: the mass-transfer limit, a nodule that grows past a stability limit set by the leveling agent, the cell voltage and the design point. Section 8.4 describes the two regulatory loops, the constraint controller and the voltage override. Section 8.5 reports the measured results

of the twelve scenarios: where the stability limit lies and what it is worth, how the additive and the temperature move it, two plant upsets, two errors in the estimate, dirty electrolyte and the graded crop. Section 8.6 names the hardware of a real tankhouse, and Sections 8.7 and 8.8 state the limitations and three observations that carry beyond this cell. The inferred limit and the back-off it needs are introduced in Section 1.4.6.

8.2 Background

A commercial tankhouse runs hundreds of cells in electrical series at current densities near 300 A/m^2 to 400 A/m^2, producing Grade A cathode at specific energies around 250 kWh/t [1, 12]. Electrorefining is a small share of the energy a copper operation uses compared with comminution [13], so the operating pressure on a tankhouse is throughput rather than power. Deposit quality is managed with leveling additives such as thiourea and glue, dosed to a concentration in the electrolyte [4], and anode passivation raises the cell voltage when the anodes are pushed too hard [14, 15]. The circulating electrolyte is filtered because suspended slime and fines that settle on the cathode become nodules [1]. Mass transfer at the cathode sets the achievable current density, and flow enhancements such as jetting between the electrodes change that limit substantially [16]. Impurity deportment through the purification circuit decides whether the cathode meets specification at least as often as physical roughness does [17].

> ⧖ HISTORY **Refining Copper by Current (1865)**
>
> Copper could be refined by electrolysis on a commercial scale only once the dynamo made large currents cheap. The eleventh edition of the *Encyclopaedia Britannica* credits Elkington, whose first patent is dated 1865, with the earliest serious attempt. Crude copper was cast into anode plates, and sheets of deposited copper served as cathodes, hung alternately in troughs of copper sulfate solution. A hundred or more troughs were coupled in series, the electrolyte was circulated through them once a day, and the current density was 5 or 6 amperes per square foot. Silver and the other insoluble impurities settled to the bottom of each trough. The encyclopedia judged the process identical in principle with the practice of its own day [18], and the cell of this chapter still works the same way.

The control structure on such a plant is regulatory PID at the base with supervisory logic above it [8, 19]. Where the supervisory layer pushes against a constraint that has no sensor, the constraint value is estimated from available measurements and model parameters, a structure the minerals industry uses widely for grades, recoveries and other unmeasured quantities [6]. The failure mode documented in this chapter, a constraint controller holding an estimate while the plant crosses the true limit, is the standard exposure of that structure.

8.3 Process Model

Figure 8.1 shows the simulated cell, its two regulatory loops and the supervisory constraint controller above them. The model integrates nine states: cell temperature, bulk copper concentration, rectifier current, electrolyte flow, heater duty, leveler concentration, nodule height, deposited mass and suspended solids. The thermal, copper, leveler and solids balances are conservation equations on a single well-mixed electrolyte volume with first-order actuator lags, so the model separates a fast electrical time scale (a rectifier lag of 0.02 h) from a slow thermal one (about 3 h) [9].

8.3.1 Mass Transfer Limit

Copper ions reach the cathode surface by diffusing across a boundary layer, so there is a current density above which the surface would be starved, the limiting current density

$$j_L = \frac{nFDC}{\delta}, \qquad \delta = \delta_0 \left(\frac{F_{\mathrm{ref}}}{F_e} \right)^{n_\delta} \tag{8.1}$$

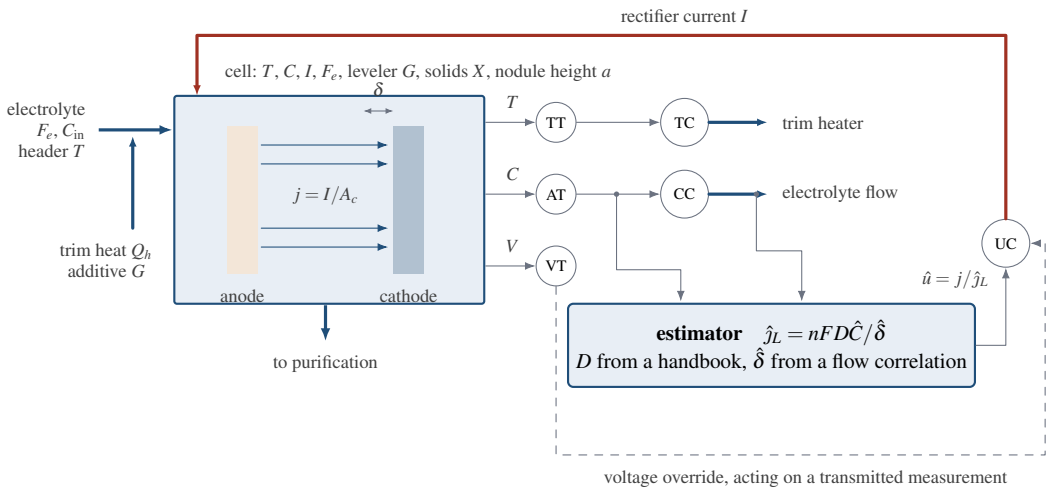


Figure 8.1 The simulated cell. Two regulatory loops hold cell temperature on a trim heater (TC) and bulk copper on the electrolyte flow (CC). A supervisory constraint controller (UC) holds current density utilization on the rectifier current, but the utilization it acts on is an estimate: the limiting current density is computed from a handbook diffusivity, a correlation boundary layer and the bulk assay, and no instrument on the cell measures it. Thick lines carry the manipulated variables (the heater and the flow valve on the right, the rectifier current across the top), thin arrows carry signals, and the dashed path is the cell voltage override, the only constraint in the structure that acts on a transmitted measurement.

where n is the number of electrons transferred, F the Faraday constant, D the diffusivity of the copper ion, C the bulk copper concentration, δ the boundary layer thickness, and δ_0, F_{ref} and n_δ correlate the layer to the electrolyte flow F_e. The controlled variable of the production loop is the fraction of that limit in use,

$$u = \frac{j}{j_L} = \frac{I/A_c}{j_L} \tag{8.2}$$

where j is the applied current density, I the rectifier current and A_c the cathode area. In the notation of Equation (1.9) u is the utilization η. Every term in the denominator of (8.2) is either a laboratory value or a correlation. The diffusivity comes from a handbook, the boundary layer from a flow correlation, and the surface concentration is not the bulk assay. No instrument on the cell measures j_L, and no instrument measures u. Figure 8.2 sets the path the controller closes beside the path the deposit follows.

🕮 **Pause.** Try Exercise 8.1 before reading on.

8.3.2 Deposit Quality

The cathode's quality is one physical quantity, the height a of a representative nodule, which follows a linearized morphological-stability law [3] written per unit of deposit thickness and multiplied by the plating velocity v,

$$\frac{da}{dt} = v\left[\frac{g\,(u-u_s)}{\delta}\,a + \beta X\right], \qquad v = \frac{\eta_c\, j\, M_{\text{Cu}}}{nF\rho_{\text{Cu}}} \tag{8.3}$$

where g is a tip amplification factor, u_s the stability limit, β a nucleation coefficient, X the suspended solids in the cell, η_c the cathode current efficiency, M_{Cu} the molar mass and ρ_{Cu} the density of copper. A bump that reaches a/δ into the boundary layer is better supplied by that fraction,

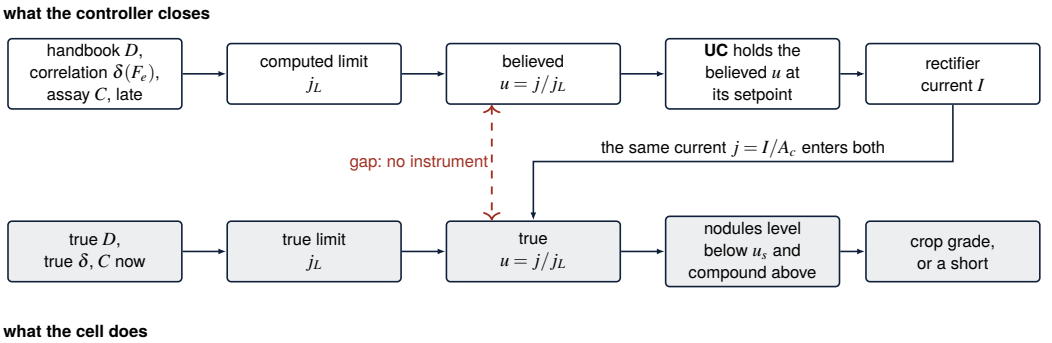


Figure 8.2 The two paths of the constraint loop. The controller closes the top path on a limit computed from a handbook value, a correlation and a late assay, and its integrated error measures only how well it holds that belief. The nodules follow the bottom path, set by the true limit through the same current, and the gap between the two rows reaches no instrument on the cell.

and with kinetics of first order in the surface concentration at a common electrode potential the local current answers its local limit with a sensitivity of exactly u, so the first term in the bracket is dimensionless, and $g = 2.2$ carries the focusing of the diffusion field onto a protrusion. Above u_s a nodule grows in proportion to its own height and its height compounds. Below u_s the same term is negative, the hollows fill faster than the bumps grow, and the plate levels itself. Particles that land on the cathode start new nodules at a rate proportional to X. Because every term carries $v \propto j$, nothing grows or levels while the current is off, whatever the temperature. A fresh blank starts at $a_0 = 5\,\mu\text{m}$, the plate advances 41 µm/h at design, a crop harvested with nodules above 2 mm is downgraded, and a nodule that reaches the 35 mm gap shorts the cell, which no leveling undoes.

> ✦ DID YOU KNOW? **A Week of Copper Is Under a Centimeter**
>
> Smooth copper plated at 350 A/m^2 for a week is only 7.8 mm thick, yet nodules grow across the 25 mm to 35 mm gap to the anode and short it. Such shorts are the largest cause of lost current efficiency, which runs at 93 % to 98 % in operation [20].

The stability limit is set by the leveling agent,

$$u_s = u_0 + \Delta u_L\,\theta - \Delta u_{\text{org}}\,\frac{(G - G_{\text{org}})_+}{G_{\text{org}}}, \qquad \theta = \frac{G}{G + G_{50}(T)} \tag{8.4}$$

where $u_0 = 0.414$ is what a bare surface tolerates, θ the Langmuir coverage of the leveler at concentration G, $\Delta u_L = 0.17$ the lift at full coverage, $(x)_+$ the positive part, and the last term the collapse of the limit once the leveler is dosed above $G_{\text{org}} = 4.5\,\text{g/m}^3$ and occluded in the deposit instead [4]. At the design dose of 2 g/m^3 the limit is exactly 55 %. Adsorption is exothermic, so G_{50} rises with temperature with an Arrhenius factor of 60 kJ/mol about the design temperature $T_0 = 62\,^\circ\text{C}$.

The leveler balance carries the hot-electrolyte damage,

$$V_{\text{cell}}\,\dot{G} = \dot{Q}_{\text{add}} - \left[F_e + V_{\text{cell}}\left(k_a\,\frac{I}{I_0} + k_h(T)\right)\right] G \tag{8.5}$$

where k_a is the consumption at the electrodes at the design current I_0 and k_h an Arrhenius hydrolysis rate of 0.5/h at T_0 with an activation energy of 100 kJ/mol. The dosing recipe $\dot{Q}_{\text{add}}$ replaces the flow, consumption and hydrolysis losses at T_0, and nobody measures G online, so a hotter cell loses leveler faster than the recipe replaces it. Temperature therefore reaches the deposit only where it physically acts, through the diffusivity inside u and through the leveler, and there is no separate penalty for cold

or hot electrolyte. The suspended solids obey $V_{\text{cell}}\dot{X} = F_e(X_{\text{in}} - X) + \dot{m}_{\text{sl}} I/I_0 - V_{\text{cell}} k_s X$, the filtered return plus the anode slime kept in suspension less what settles, which gives 4.1 mg/L at design.

8.3.3 Cell Voltage

Cell voltage is contact plus activation plus ohmic plus a concentration overpotential that diverges as the current approaches its limit,

$$V = V_c + V_a\,\pi + \frac{R_g T}{nF}\ln\frac{1}{1-u} + I\,\frac{d\,\pi}{\kappa A_c} \tag{8.6}$$

where V_c is the contact and busbar drop, V_a the activation overpotential, π a passivation multiplier, R_g the gas constant, T the absolute temperature, d the interelectrode gap (written d as in the app's Show Model, since g is the tip factor of Equation (8.3)), κ the electrolyte conductivity and A_c the cathode area. The logarithm raises the energy cost sharply as u approaches 1. The voltage is also the second constraint: the app trips a limit at 0.40 V, which Section 8.5.4 reaches through anode passivation rather than through current.

8.3.4 Design Point

The simulation clock is in hours. At design the rectifier delivers 29 440 A, which is 320 A/m^2 over the 92 m^2 of cathode area, into electrolyte at 62.0 °C carrying 45.0 kg/m^3 of copper at 1.2 m^3/h. The boundary layer is 140 µm and the limiting current density is 586 A/m^2, so the design utilization is 54.6 %. The cell plates 33.68 kg/h at 0.285 V and 249 kWh/t, with the leveling additive at 2 g/m^3, dosed at 8.3 g/h, so the cell sits a hair under its 55 % stability limit and the nodules neither grow nor level. That specific energy is in the range real tankhouses report [12], and refining is a small share of the energy a copper operation uses compared with comminution [13], which is part of why the pressure on a tankhouse is throughput rather than power.

8.3.5 Initialization and Assumptions

Initialization solves the thermal, copper, leveler and solids balances at the given actuator settings, so the initial condition is an exact equilibrium. A manual-mode helper, Hold this utilization, sets the current that produces the utilization setpoint in steady state on the *true* limiting current density at the present flow and heater settings, which is the calculation the plant cannot make.

The cell is one lumped tank with a single cathode area and no cell-to-cell variation, where a real tankhouse has hundreds of cells in series and its worst cell sets the crop. Impurity deportment, short-circuit detection and stripping are absent, and anode slime enters only as suspended solids that seed nodules. The cathodes are harvested, and the run stops, at the end of the 168 h cycle (120 h in the graded crop), where the crop is graded once on the nodules it carries. Integration is fixed-step RK4 at 0.005 h with a 72 s controller scan (Section 1.6.1). The copper assay reaches the controllers 2 h after the sample is taken, and the cell temperature reaches them through a thermowell lag of 0.1 h.

8.4 Controller Design

The three loops run the velocity-form PID of Section 1.6.2, Equations (1.10) to (1.13), with the derivative acting on the measurement [21], executed every $\Delta t = 0.02$ h. The integral is seeded at reset with the initial actuator value, and the tracking time of the back-calculation is $T_t = \tau_I$. With that choice the integral settles on the clipped output itself, so a loop held against a limit for several tracking times leaves it on the scan at which its error changes sign. Only the temperature loop carries a derivative term, and every scenario ships it with $\tau_D = 0$.

Cell temperature is controlled with a trim heater (action +1, reverse acting in the convention of Section 1.6.2), bulk copper with the electrolyte flow (action −1, direct acting, because more flow carries the anode's surplus away), and above them the constraint controller moves the rectifier current against the estimated utilization of (8.2) (action +1, reverse acting). The app's controller notes and loop hints use the same names. In this chapter the symbol a is the nodule height, so the action is written

Table 8.1 Settled state against utilization setpoint with all three loops in service, read at the end of the 168 h cathode cycle of scenario 4 with the setpoint changed before the run. σv is the nodule growth exponent, a the nodule height at harvest, Down the hour at which a crosses the 2 mm downgrade line, and Value the app's cycle value, with the crop graded at harvest. The 70 % row is limited by the 42 kA rectifier and reads 69.9 %. Rows from 62 % short the cell, at 141.8, 88.9 and 64.1 h. Bold marks the most valuable crop.

u (%)	I (kA)	V (V)	σv (/h)	Rate (kg/h)	a (mm)	Down (h)	Value ($)
45	22.50	0.234	−0.046	25.74	0.01	–	37920
55	29.94	0.289	0.000	34.25	0.09	–	49113
58	32.28	0.306	0.022	36.92	1.07	–	52584
58.5	32.67	0.309	0.026	37.37	1.82	–	**53167**
59	33.06	0.312	0.030	37.83	3.18	152.6	34846
60	33.86	0.317	0.039	38.73	10.6	124.9	35608
62	35.46	0.329	0.057	40.57	35	91.5	37140
66	38.72	0.353	0.101	44.29	35	59.4	40242
70	42.00	0.378	0.152	48.05	35	43.9	43392

as its sign. The tuned set of scenarios 3 to 10 is close to the SIMC settings [22] on the two regulatory loops, 40 %/°C and 2 h on temperature against 47.7 %/°C and 2.1 h, and 0.35 (m^3/h)/(kg/m^3) and 8 h on copper against 0.33 and 10.5 h at the 0.4 m^3/h flow stop, where the process gain is nine times its design value and the 2 h assay delay dominates. The utilization loop, at 120 A per percent and 1 h, is integral dominated with a closed-loop time constant near 5.5 h, slower than the temperature loop that moves its limit, though faster than the assay-limited copper loop. Scenarios 1 and 2 and the graded cycle load cautious gains of 8 %/°C and 8 h, 0.3 and 15 h, and 25 A per percent and 4 h.

The distinction the app is built on is that the first two loops act on measurements and the third acts on an estimate. All three look identical on a control drawing, all three report an integrated error, and only two of them can be verified against an instrument.

A voltage override sits beside the constraint controller and acts after it. When the current the controller commands would put the cell voltage above 0.40 V at the present temperature, concentration and flow, the override bisects the command down to the largest current that keeps the voltage under the limit. The constraint controller's integral is then reset so that its output before limiting equals that current, as the framework does in Manual (Section 1.6.2), and the controller resumes from it without windup. The override acts in manual mode as well and does nothing while the commanded current keeps the cell under the limit, which at the design point it does by 115 mV, so in most scenarios switching it off changes nothing. It is the one protection on the cell that acts on a measurable quantity rather than an inferred one, like the protective override on measured flow in Volume 4, Chapter 2.

8.5 Results

8.5.1 Locating the Stability Limit

Scenario 2 asks the reader to find the stability limit in manual mode. Table 8.1 shows why that is difficult: below the limit the growth exponent σv, the bracket of (8.3) times v, is negative or zero, the self-amplifying growth levels or stops, while particles still add height, and there is no other symptom. The cell voltage rises smoothly through the limit, from 0.289 to 0.306 V between 55 and 58 %, a change of six percent on an instrument with comparable noise. Above it the nodule trend sits on the floor for most of the crop and then takes off, because a nodule that doubles every day grows half of its final height in the final day. The first evidence is a rough plate at the end of a cycle.

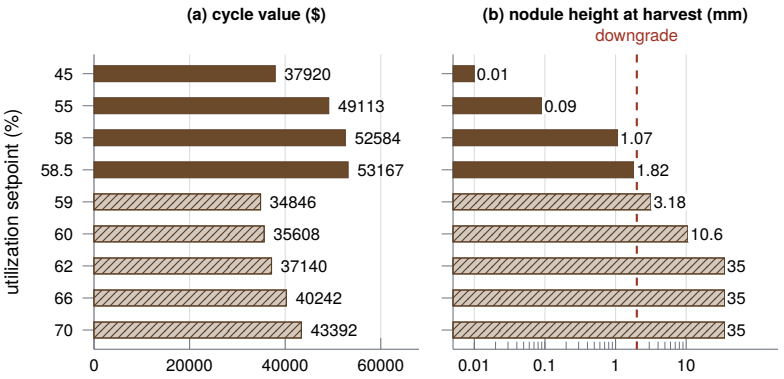


Figure 8.3 The utilization sweep of Table 8.1 as a chart. Solid bars are crops harvested below the downgrade line, hatched bars are downgraded. The value climbs to its peak at 58.5 % and then falls sharply at 59 %, the first setpoint whose nodules cross the 2 mm line, while every setpoint above it plates more copper and earns less.

Table 8.1 also prices the operating point over the 168 h cathode cycle. At 55 % the cell plates 5740 kg worth $49 113, at 58.5 % it plates 6213 kg worth $53 167 and is harvested with nodules of 1.82 mm, and at 66 % it plates 7250 kg worth $40 242. The 66 % cycle plates 17 % more copper than the 58.5 % cycle and earns 24 % less, because the nodules double every 6.9 h and cross the downgrade line at 59.4 h. A downgrade sells the whole crop at 65 % of the price, so the 59 % cycle, which crosses the line at 152.6 h, is worth $18 321 less than the 58.5 % cycle. The most valuable setpoint therefore sits 3.5 points above the stability limit, on a variable that is a ratio to an unmeasured quantity, and the sustainable ceiling over the cycle is 58.8 %, where a nodule doubles about once a day. Figure 8.3 draws the same sweep as a chart.

Scenarios 5 and 6 run the cycle at 59 and 66 %. The first is harvested at 3.18 mm, past the line it crosses at 152.6 h. The second is downgraded at 59.4 h and shorts at 88.9 h. Both crops sell downgraded, so the second earns $5396 more for its 969 kg of extra metal. The growth law also runs backwards. Lowering the 66 % setpoint to 55 %, the limit itself, stops the compounding, and the nodules then gain height only slowly, from the particles that keep seeding them, while lowering it to 50 % halves them every 25 h. Lowered at 59 h, with nodules of 1.92 mm, the crop is back under the line at 74.5 h, is harvested Grade A at 0.15 mm and is worth $50 524, $10 282 more than running on to the short. Lowered at 80 h, with 14.6 mm nodules, it is back under the line at 167.3 h, and at 85 h it is too late.

> **TRY IT IN THE APP** **6 · Push It to 66 %, the Cathode Is Scrap**
>
> Load the scenario and press Run. Watch the cathode nodule trend. It sits near the floor for days, then takes off, crosses the 2 mm downgrade line at 59.4 h and reaches the gap at 88.9 h, when the cell shorts. The crop plates more copper than at 58.5 % and still sells downgraded.

8.5.2 Additive Concentration

The leveling additive is the operator's defense against nodules, and its effect is not monotone.

Between 4.5 and 5 g/m^3 the stability limit in Table 8.2 falls eleven points, because the occlusion term of (8.4) overtakes the coverage term, and a crop that is harvested Grade A at 4.5 g/m^3 is downgraded at 61.4 h at 5 g/m^3. The coverage is still rising at 8 g/m^3 while the deposit is ruined, which is the behavior real tankhouses manage with thiourea and glue and is the reason additive is dosed to a concentration rather than to a rate [4]. Losing the additive is as costly in the other direction: with the pump stopped the limit falls toward 41.4 % and a crop at the design point is downgraded

Table 8.2 Additive setpoint at 58 % utilization over the 168 h cycle, read as in Table 8.1. θ is the leveler coverage and u_s the stability limit of (8.4). The setpoint slider stops at 8 g/m^3, and every downgraded row also shorts the cell.

G (g/m^3)	θ	u_s (%)	σv (/h)	a or Down	Value ($)
0.5	0.500	49.90	0.059	87.4 h	34104
2	0.800	55.00	0.022	1.07 mm	52584
4	0.889	56.51	0.011	0.28 mm	52563
4.5	0.900	56.70	**0.009**	0.24 mm	52558
5	0.909	45.74	0.089	61.4 h	34058
6	0.923	23.76	0.249	24.4 h	34048
8	0.941	−20.4	0.570	11.2 h	34028

within three days. At 58 % raising the dose from 2 to 4 g/m^3 halves the growth exponent and lowers the harvested nodule height from 1.07 to 0.28 mm, and changes the cycle value by $–21, the extra additive, because the crop is Grade A either way. The margin it buys pays only at a higher utilization or a dirtier electrolyte, which the graded cycle and scenario 11 use.

> **✦ DID YOU KNOW? The Most Leveler, the Fastest Ruin**
>
> From the simulator: in Table 8.2 the 8 g/m^3 dose gives the highest leveler coverage of any row, 0.941, yet its stability limit is −20.4 % and the crop is downgraded at 11.2 h. At 4.5 g/m^3 the same cell is harvested with nodules of 0.24 mm.

8.5.3 Effect of Temperature on the Limiting Current Density

Raising the cell temperature raises the diffusivity and therefore the ceiling of (8.1). At 58 % utilization over the 168 h cycle the limiting current density rises from 562.5 A/m^2 at 60 °C to 669.9 A/m^2 at 65 °C, and the rectifier follows it from 30.02 to 35.74 kA. The leveler balance (8.5) charges for it. The recipe was set at 62 °C, so the steady leveler falls from 2.21 to 1.71 g/m^3 over the same range, its coverage falls further because a warmer surface adsorbs less leveler at the same concentration, and the stability limit drops from 55.59 to 53.95 %, about 0.35 points per degree. The cycle value rises from 49 298 to $54 240 at 63 °C, where the crop is still harvested Grade A at 1.68 mm. One more degree reverses the result: at 64 °C the nodules double every 24 h instead of 27.5, the crop is downgraded at 156.5 h and the value falls to $36 223. At 68 °C, with the limit at 736.4 A/m^2 and the voltage still only 0.324 V, the crop is downgraded at 106.1 h and the cell shorts. Below the design point the heater runs out first, and the cell's own Joule heat holds it at 57.1 °C with the heater off against a 56 °C setpoint, where the recipe now runs the leveler rich and the nodules grow more slowly.

The temperature setpoint is therefore an optimum, with an edge that falls just above 63 °C at the design dose and moves with the dose, rather than a direction, and a supervisory optimizer built only on the limiting current would push past it. Lead anode corrosion and evaporation, which also accelerate with temperature, are outside the model.

8.5.4 Passivation and Purification Disturbances

Scenario 7 runs at 59 % and passivates the anodes at 12 h, which multiplies the activation and ohmic terms of (8.6) by 1.45 [14, 15]. The voltage steps to 0.419 V at the next physics step (0.402 V in the 0.05 h log), the override cuts the current from 31.8 to 29.6 kA at that scan, and the rectifier lag returns the voltage to the 0.400 V limit within about 0.1 h. Until the anodes are cleaned at 80 h the cell rides the limit at 29.5 kA and 350 kWh/t, against 265 kWh/t before, at a utilization of 54.4 % against its 59 % setpoint. Passivation enters neither the true limit nor its estimate, so the constraint controller

Table 8.3 Model errors with all three loops in service at a 59 % setpoint. Rows are read at 150 h in scenario 9 and at the end of each phase of scenario 10 (99.95 h and 168 h), whose estimate runs 12 % optimistic from 8 to 100 h and then 12 % pessimistic. Gap is true less believed, and σv the nodule growth exponent at the true utilization.

	Believed u (%)	True u (%)	Gap (pts)	σv (/h)
Diffusivity 20 % low	58.99	73.74	+14.75	0.140
Estimate 12 % optimistic	58.92	65.99	+7.07	0.099
Estimate 12 % pessimistic	59.13	52.03	−7.10	−0.019

is blind to it. With the interlock off the cell runs at up to 0.435 V and 380 kWh/t and finishes the 168 h cycle downgraded at 152.6 h, exactly as scenario 5 does, because passivation does not touch the deposit. The protected crop is harvested Grade A at 0.64 mm, because the override held the cell at 54.4 %, below its stability limit, from 12 to 80 h as a side effect.

Scenario 8 runs at 58 % and leans the purification circuit from 44 to 37 kg/m^3 between 10 and 70 h. The copper loop is direct acting, so an assay below setpoint cuts the electrolyte flow to retain the anode's dissolved surplus in the cell. The assay arrives 2 h late, the flow command reaches its 0.4 m^3/h minimum at 13.4 h, and the bulk copper settles at 39.0 kg/m^3 against a setpoint of 45.0 kg/m^3. The limiting current density falls from 590.5 to 365.4 A/m^2, a 38 % cut of which the leaner copper explains 13 points and the boundary layer thickened by the closed valve, from 139 to 195 µm, the rest, and the plating rate follows from 35.7 to 22.3 kg/h. The valve moves the limit faster than the utilization loop moves the rectifier, so the true utilization overshoots to 78.0 % at 13.4 h, where the nodules double every 6.7 h, while the belief gap peaks at only 4.34 points. The overshoot costs almost nothing, because (8.3) multiplies the nodules already on the plate and at 10 h they are 0.011 mm high: they reach 0.023 mm by 18 h. The same overshoot near the end of a crop run close to its ceiling would double nodules a millimeter high. The copper loop leaves its stop at 90.84 h, when the assay returns to setpoint, as the back-calculation of Equation (1.13) with $T_i = \tau_I$ predicts. The crop is harvested Grade A at 0.29 mm and finishes with 1288 kg less copper and $11 030 less value than the same case without the upset, a loss to a change upstream that no loop on the cell was configured to see.

🕮 **Pause.** Try Exercise 8.3 before reading on.

8.5.5 Model Error and the Belief Gap

Scenarios 9 and 10 are the central scenarios.

In the first row of Table 8.3 the true diffusivity is 20 % below the handbook value, so (8.1) overstates the ceiling by a factor of 1.25 and the constraint controller pushes the current until its *estimate* reads 59 %. The cell is at 73.74 %. The nodules at that utilization double every 4.9 h against every 23 h in the same case with the handbook right. The cathode is downgraded at 45.3 h, the cell shorts at 66.2 h when a nodule reaches the 35 mm gap, and the crop is worth $34 843, within $3 of scenario 5, which also sells downgraded.

Figure 8.4 shows the operator's view of that condition. At 150 h the temperature loop has an integrated absolute error of 0.243 °C h and the copper loop 3.737 kg h/m^3, both identical to the run with the handbook right, because the current, and with it every measured variable, follows the estimate. Every trend is flat while the cell is shorted by nodules. The app scores the utilization loop against the true utilization, which no plant can do, and that statistic alone records the error, at 2093 % h against 30 % h with the handbook right.



Figure 8.4 Scenario 9 at 150 h, with the true copper diffusivity 20 % below the handbook value the estimator uses, cut to the plant view and the integrated errors of the three loops. (1) The cathode, where a nodule has reached the 35 mm gap and shorted the cell. (2) The true utilization, 73.7 %, against (3) the believed utilization of 59.0 % that the constraint controller holds. (4) The cell voltage, which the override reads. (5) The integrated absolute errors: the temperature and copper loops report 0.243 and 3.737, and the utilization loop, scored on the true value, reports 2093.

> **TRY IT IN THE APP** **9 · The Model Is Wrong, and Nothing Says So**
>
> Run the scenario to 150 h and compare the Learn panel with the plant view. The temperature and copper loops report integrated errors of 0.243 and 3.737, the same as with the handbook right. The constraint controller believes 59.0 % while the cell runs at 73.7 %, and only the utilization score, taken on the true value, reads 2093.

The second and third rows are the two phases of scenario 10, in which the limiting-current estimate is biased 12 % optimistic for 92 h and then 12 % pessimistic. The optimistic phase runs the cell at a true 65.99 % while the estimate reads 58.92 %, the nodules double every 7 h, the cathode is downgraded at 63.2 h, and the cell shorts at 92.5 h, before the bias reverses. The pessimistic phase leaves the cell at 52.03 % while the controller believes 59.13 %, which plates 2304 kg from 100 to 168 h against 2569 kg in scenario 5, a loss of 10.3 % that grows no nodules and is therefore never flagged. The widest instantaneous gap of the run, 11.09 points at the physics step (8.73 in the 0.05 h log), opens at the reversal, when the estimate jumps while the plant has not yet moved. The two phases quantify the asymmetry that sizes the operating margin: past the stability limit the damage compounds and ends in a short that nothing undoes, and short of it the penalty is linear and recoverable. The reversal was moved from 60 to 100 h for this reason, because a pessimistic phase that begins before the short levels the nodules back down and hides the asymmetry.

Pause. Try Exercise 8.2 before reading on.

8.5.6 Dirty Electrolyte

Scenario 11 runs at 58 % and bypasses the return filter from 24 to 72 h, raising the suspended solids in the returning electrolyte from 2 to 40 mg/L. The solids in the cell rise from 4.3 to 27.4 mg/L, so the nucleation term of (8.3) adds nodule height at 8 % of the plating rate instead of about 1 %. Nothing the control system computes moves: the true and believed utilization agree, both loops hold their setpoints, and the growth exponent is the same as in a clean run. The nodules are 0.35 mm high at 72 h, but at three points above the stability limit every seed then doubles every 33 h, and the crop

Table 8.4 The graded 120 h crop. Rows after the first use constraint control and the tuned gains unless marked, written as u_{SP} (%), T_{SP} (°C) and G_{SP} (g/m^3) and set before the run. A capital letter marks a clause met (temperature, nodules, voltage, copper).

Configuration	Clauses	Cu (kg)	a (mm)	Peak V (V)	$\lvert\Delta T\rvert$ (°C)	Value ($)
As loaded	`trVC`	4042	24.5	0.383	2.85	22396
60, 63, 4 (suggested)	`TRVC`	3857	0.33	0.368	1.17	32945
57, 63, 4	`TRVC`	3648	0.08	0.349	1.17	31153
62, 63, 4	`TRVC`	3998	1.04	0.381	1.17	34152
63, 63, 4	`TRVC`	4069	1.99	0.387	1.17	34758
64, 63, 4	`TrVC`	4140	3.93	0.393	1.17	22907
62, 63, 2	`TrVC`	3998	2.65	0.381	1.17	22131
62, 65, 4	`tRVC`	4176	1.57	0.386	3.00	**35661**
60, 63, 4, cautious	`trVC`	3971	2.38	0.363	3.16	21977
Fixed current, tuned	`TrVC`	4042	25.8	0.386	1.18	22391

is downgraded at 149.6 h, three days after the filter came back, and harvested at 3.0 mm against 1.07 mm for the same case without the bypass.

> ⧖ HISTORY **Particles, Jets and Nodules (Before 1911)**
>
> By 1911 refiners knew that a higher current density improves the deposit only up to a point, beyond which it turns dark, loose and spongy. Jets of electrolyte aimed at the cathode renewed the solution at its surface, and in Graham's experiments a jet allowed a good deposit at 280 amperes per square foot, ruined at once when the jet was checked. Swan showed that solid particles in the electrolyte are one of the most fruitful causes of the nodular growths on deposited copper, and jets were liable to carry anode mud onto the cathode. Refineries stayed at 7.5 to 18 amperes per square foot [18].

The two remedies are not equivalent. Lowering the setpoint to 56 % only while the filter is out barely saves the crop, at 1.83 mm and $51 860, because the seeds are amplified after the filter returns rather than while it is out. Raising the leveler to 4 g/m^3 from 25 h lifts the stability limit to 56.5 %, stretches the doubling time to 65 h, and harvests the crop Grade A at 0.82 mm and $52 566, a smoother plate than the clean crop at the design dose, for 1.24 kg of additive and no copper. The constraint controller defends the constraint it computes, and a disturbance that spoils the product by another route has to be met by the operator who knows the mechanism.

8.5.7 Graded Cycle

The challenge runs one 120 h crop through a header that cools to 52 °C at 18 h, a purification circuit that drops to 40 kg/m^3 at 45 h, anodes that passivate at 72 h and recoveries at 96 and 104 h. It asks for the temperature within ±1.5 °C of setpoint throughout, nodules below the 2 mm line at harvest, a logged voltage never above 0.40 V and more than 3400 kg of copper on the case as given, and it loads on fixed current at 29.44 kA with the cautious gains.

Table 8.4 is a four-constraint problem in which the profit maximum sits outside the feasible set, which is the ordinary situation on a real plant and a rare one in a textbook.

As loaded the crop fails twice. Fixed current carries the cell across its stability limit when the header cools and the copper falls, so the cathode is downgraded at 87.3 h, and the cautious temperature loop lets the cell drift 2.41 °C low at 20.5 h and 2.85 °C high at 53.65 h with the heater inside its range. Tuning alone does not rescue it, because fixed current with the tuned gains holds the temperature within 1.18 °C and is still downgraded, at 86.0 h, and the suggested setpoints with the cautious gains

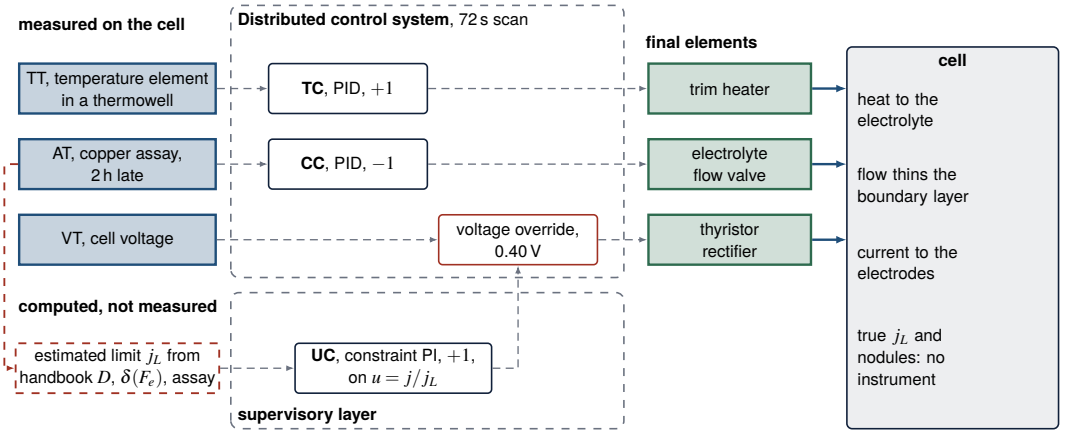


Figure 8.5 The loops of the simulated cell drawn with their hardware. The temperature element TT and the copper assay AT feed two PID blocks that move the trim heater and the electrolyte flow valve. In the supervisory layer the constraint controller UC acts on a limiting current density computed from a handbook diffusivity, a boundary-layer correlation and the assay (red dashes), and its command to the rectifier passes the voltage override, the one protection that reads a measurement.

fail both clauses. The suggested configuration, constraint control at 60 % and 63 °C with 4 g/m^3 of additive and the tuned gains, meets all four clauses. It leaves 1.67 mm of nodule margin, 32 mV of voltage margin and 0.33 °C of temperature margin, and revealing the suggested settings before Run or carrying them through Reset gives the same run.

The nodule clause sets the utilization. A setpoint of 63 % still passes at 1.99 mm, and 64 % is downgraded at 91.3 h. The additive moves that edge, because 62 % finishes at 1.04 mm with 4 g/m^3 and is downgraded at 104.2 h with the shipped 2 g/m^3. The 120 h ceiling is several points higher than the 168 h one, because the nodules have less time to compound. The temperature clause is scored from the first sample, so it limits how far the setpoint may move from the 62 °C at which the cell starts, and any setpoint at or above 64 °C fails at $t = 0$. The voltage clause never binds: the peak is 0.393 V at 64 %, the interlock never acts in any configuration tested, and switching it off leaves the suggested run unchanged.

The most valuable configuration tested, 62 % at 65 °C, is worth $35 661 and misses only the temperature clause. The best feasible one, 63 % at 63 °C, is worth 2.5 % less with 0.01 mm of nodule margin, and the suggested configuration is worth 7.6 % less with five sixths of the nodule allowance unspent, which is the margin the earlier scenarios argue for against an estimate that may be wrong.

8.6 Technology in Practice

Figure 8.5 draws the loops of Section 8.4 with their hardware.

What Measures It

The two regulatory loops act on ordinary measurements. The model reads the cell temperature through a thermowell lag of 0.1 h and the bulk copper from an assay that reaches the controllers 2 h after the sample is taken. The variable the constraint controller holds has no instrument at all. Its limit is computed, and the nodules it guards against are seen in a real tankhouse only once they short a cell.

TECHNOLOGY Sensor: Finding Shorts by Their Field and Heat

No instrument reads the limiting current density, but the damage done past it is found cell by cell. A shorted cathode carries abnormal current, and the traditional check is a hand-held gauss meter that senses the magnetic field around the cathode, often carried over the cells on a pole. Because shorts generate heat, infrared cameras with image processing are also used, though, like the gauss meter, they find short circuits and not open ones. A short is cleared by probing between the electrodes with a stainless steel rod and breaking off the nodule, often after lifting the cathode out, and many shorts are found only after significant damage has been done [23].

The model carries no short detection, so its scenarios run a shorted cell to the end of the cycle, where an operator would have found and cleared the short long before (Section 8.7).

What Moves It

The trim heater and the flow valve are first-order actuators, the flow held above its 0.4 m^3/h stop, and the rectifier is a lag of 0.02 h on the commanded current.

TECHNOLOGY Final Element: A Thyristor Rectifier

The rectifier turns the plant's alternating supply into the direct current that drives the cells in series. Large rectifiers deliver controlled currents in the kiloampere range, and thyristor phase-controlled units are the common type [24]. A controlled rectifier uses silicon-controlled rectifiers in place of diodes and sets its output by the moment in each cycle of the supply at which each one is triggered: the earlier the trigger, the longer it conducts [25]. Such units are judged on quality of control, harmonics, power factor, losses and efficiency [24]. A rectifier at its rating holds the current there, as the 42 kA rectifier of Table 8.1 does at 70 %.

The override lowers the command and never the measurement, so in scenario 7 the rectifier rides the 0.40 V limit at 29.5 kA until the anodes are cleaned.

Where the Controller Runs

Control loops in a plant are often nested or cascaded, with cycle times from milliseconds to minutes, and a distributed control system uses a supervisory loop to mediate a group of localized controllers [26]. Programmable logic controllers run PID loops as well, and the voltages of hundreds of electrolytic pots in the aluminum industry have been gathered to a central control computer [25]. The app runs its three PID blocks every 72 s, against a rectifier lag of 0.02 h and a thermal time scale of about 3 h. On such a platform TC and CC are PID function blocks, the limiting current density is a calculation block fed by the assay, and UC with the override is a supervisory application that writes the current setpoint of the rectifier.

IN PRACTICE What the Tankhouse Operator Watches

In practice the cell voltage of a copper refinery is normally held between 0.25 and 0.5 V, and a cell outside that range points to passivation of some of its anodes, which raises the energy used and lowers the cathode quality [27]. Short and open circuits are common in service. They come from poor anodes and cathodes, poor contact with the current supply, misaligned electrodes, and local differences in temperature, additive level or chemistry, so the cells are checked one by one and shorts are cleared by hand [23].

What Is Modernizing It

The first development is continuous monitoring of each electrode. Sensors that measure the magnetic field around the conductor carrying current to a cathode let a processor find open circuits as well as shorts, without anyone walking over the cells [23]. The second is the rectifier itself. Chopper rectifiers built from high-current insulated gate bipolar transistors are being considered as an alternative to the thyristor unit, and the pulse-width-modulated current-source rectifier of medium-voltage motor

drives is a future option [24]. The third is estimation. Inferential measurements are common wherever a controlled variable has no online sensor [5], and a moving horizon estimator that reconciled the voltage, the assay and the temperature could recover the diffusivity error of scenario 9 rather than absorb it, as Section 8.8 argues [28].

8.7 Limitations

There is one cell. A real tankhouse runs hundreds in electrical series, the current is common to all of them, and the crop is set by the worst cell rather than the average one. That single fact changes the control problem qualitatively, because the supervisory decision is about a distribution rather than a value, and nothing here represents it.

Impurity deportment is absent. Arsenic, antimony, bismuth and nickel decide whether the cathode passes a Grade A specification at least as often as physical roughness does, they interact with each other and with the additives, and their control is the principal reason a tankhouse has a purification circuit at all [17]. The app's quality variable is a nodule height and should be read as a physical proxy for grade.

Short-circuit detection and stripping are absent, and anode slime appears only as suspended solids with one nucleation coefficient for slime, fines and gypsum alike. Real short-circuit management is an infrared survey and a manual intervention, and the cell shorted in Figure 8.4 would have been found and cleared by an operator long before the cycle ended.

The boundary layer is a single power law in flow. Real mass transfer at a vertical cathode is driven by natural convection as well as forced flow, and the enhancement available from jetting or from air sparging is large enough to change the achievable current density substantially [16]. Equation (8.1) reproduces the right dependence and not the right number for any particular cell house.

The deposit law (8.3) is a linearized stability result for one representative nodule [3]. It holds for bumps smaller than the boundary layer and is stretched here all the way to the gap, so a real nodule much taller than the layer grows, and levels, more slowly than the model says, and a real cathode carries a population of nodules rather than one. The plate is graded on the nodules it carries at harvest, where a real overgrown nodule still holds the electrolyte it trapped, so the late rescues of scenario 6 are more generous than practice. The stability limit in a real cell depends on the additive package, the anode surface, the electrolyte impurities and the flow distribution, and locating it is an experimental campaign rather than a calculation [2].

> **OPEN CHALLENGE** **How a Nodule Crosses the Gap**
>
> Most studies of nodules describe their earliest stage, under 1 to 2 mm high. Experiments with seeded bumps found the height growing exponentially with time and suggested that only a nucleus of about 3 mm grows into a short within the operating time. Flakes of mold release agent from anode casting were found in nodules cut from shorted cells, but nodules without such a nucleus also form, for reasons not yet known [20]. Progress would be a measured account of what seeds the nodules that reach the anode, and a way to see them growing before they short.

8.8 Discussion

Three observations generalize beyond this cell.

The first is that an inferred controlled variable inherits every error in the inference, and the loop's own performance statistics say nothing about it. Table 8.3 has a controller with an excellent integrated error running a cell fifteen points above the utilization it believes. The remedy is not a better controller. It is either a measurement, or a periodic validation of the estimate against something that can be measured, or a margin large enough to absorb the estimate's uncertainty [5]. Validation against a late laboratory value is the bias update of Volume 5, Chapter 10 and Chapter 7. On this cell the only online candidate is the cell voltage, whose concentration overpotential in (8.6) makes it weakly informative about the true limit, and a moving horizon estimator that reconciled the voltage, the assay

and the temperature (Section 1.6.3) could recover the diffusivity error rather than absorb it [28]. The third remedy is priced by Table 8.1: the difference between 58.5 and 55 % utilization is $4054 per cycle, and that difference is the cost of the margin that protects against the estimate being wrong.

The second is that an instability that changes sign at a threshold turns a modest model error into a categorical outcome. Because the growth exponent of (8.3) is negative below the stability limit, a plant can operate for years with a badly calibrated estimator and discover it only when a change in flow, temperature or feed moves the true utilization across the line, after which the damage compounds out of sight until the last days of a crop. Constraints that are smooth degrade visibly, and constraints with a threshold do not. Scenario 11 adds the corollary that the threshold is not the only exposure: particles change how many seeds the instability has to amplify while every computed quantity stays where it was.

The third is that the profit maximum is often outside the feasible set. Table 8.4 shows the most valuable configuration violating a measured constraint, the best compliant one 2.5 % below it with almost no nodule margin, and the suggested one 7.6 % below it with five sixths of the nodule allowance unspent, which is the argument for putting the constraints into the optimization rather than leaving them to an override to discover [6, 19].

8.9 Conclusion

An electrorefining cell is controlled against a limiting current density that no instrument measures, and past a stability limit the nodules on the cathode grow in proportion to their own height, so the penalty for crossing it compounds. The app prices that shape over a 168 h cycle: 58.5 % of the limiting current density is worth $53 167 and finishes Grade A, 55 % is worth $49 113, and 66 % plates seventeen percent more copper and is worth twenty-four percent less. The central scenarios then make the estimate wrong rather than the controller. A handbook diffusivity twenty percent in error leaves a constraint controller holding a believed 58.99 % while the cell runs at 73.74 %, with two regulatory loops reporting integrated errors of 0.243 °C h and 3.737 kg h/m^3, unchanged from a run with the handbook right, and a cell shorted by nodules 58.2 h after the error appears. A bypassed filter downgrades a crop with no error in the estimate at all, and more leveler, not less current, saves it. The graded cycle places the profit maximum outside the feasible set, and the suggested configuration gives up 7.6 % of the most valuable one tested to meet all four constraints with margin. The app lets a reader make that trade, constraint control against an estimated limit, with realistic economics.

KEY POINTS

- More current makes more copper, but past a stability limit the nodules on the cathode grow in proportion to their own height.
- No instrument measures the limiting current density, so the constraint controller acts on an estimate and inherits every error in it.
- Over a 168 h cycle, 58.5 % of the limit finishes Grade A, while 66 % plates seventeen percent more copper and is worth twenty-four percent less.
- A handbook diffusivity twenty percent in error runs the cell at 73.74 % while the controller believes 58.99 %.
- A bypassed filter downgrades a crop with no error in the estimate, and more leveler saves it.

Where else this applies. Volume 5, Chapter 10 and Volume 5, Chapter 11 also control a variable that no instrument reaches, and Volume 4, Chapter 2 holds a rate against a limit with an override behind it.

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8.10 Exercises

8.1 Concept check · 2 min

Which of these quantities does no instrument on the cell of Figure 8.1 measure, so that the loop holding it acts on a computed value?

(a) The cell temperature T, read through a thermowell
(b) The bulk copper concentration C, from the laboratory assay
(c) The cell voltage V, from the voltage transmitter
(d) The current density utilization $u = j/j_L$

8.2 Concept check · 1 min

Scenario 10 runs the same cell with its limiting-current estimate 12 % optimistic and then 12 % pessimistic. Why does the chapter conclude that a constraint controller should hold a margin below its estimated ceiling?

(a) Past the limit the nodules compound toward a short, while short of it only copper is lost
(b) The pessimistic phase plates less copper and also grows nodules, so both errors cost the same
(c) The optimistic phase plates more copper, so it is the cheaper of the two errors
(d) The voltage override cuts the current whenever the true utilization passes its setpoint

8.3 Concept check · 2 min

In scenario 8 the returning electrolyte drops from 44 to 37 kg/m^3 of copper. The copper loop runs with action -1. What does it do, and what does that do to the limiting current density?

(a) It opens the flow to bring more copper in, which thins the boundary layer and raises j_L
(b) It cuts the flow to keep the anode's surplus in the cell, which thickens the boundary layer and lowers j_L
(c) It cuts the flow, and the richer copper it retains raises j_L above its design value
(d) It does nothing until the assay arrives, because the loop is reverse acting

8.4 Theory and calculation · 6 min

Use Equation (8.1) with $n = 2$, $F = 96\,485\,\mathrm{C/mol}$, $D = 6.0 \times 10^{-10}\,\mathrm{m^2/s}$ and a molar mass of copper of 0.063 546 kg/mol.

(a) At design the bulk copper is 45.0 kg/m^3 and the boundary layer 140 µm. Compute the copper concentration in mol/m^3 to 1, j_L to 1 A/m^2, and the utilization at 320 A/m^2 to 0.1 %. *Checkpoint:* 708 mol/m^3; 586 A/m^2; 54.6 %

(b) The boundary layer follows $\delta_0 (F_{\mathrm{ref}}/F_e)^{n_\delta}$ with $F_{\mathrm{ref}} = 1.2\,\mathrm{m^3/h}$ and $n_\delta = 0.30$. By what factor, to 0.01, does it thicken when the flow closes to 0.4 m^3/h? *Checkpoint:* 1.39

(c) In scenario 8, j_L was 590.5 A/m^2 with 45.0 kg/m^3 and 139 µm. The upset leaves 39.0 kg/m^3 and 195 µm at the same temperature. Compute the new j_L within 2 A/m^2, the total cut in percent to 1, and the share of the cut due to the copper alone. *Checkpoint:* 365 A/m^2; 38 %; 13 points

8.5 Theory and calculation · 6 min

Use Equation (8.4) at the design temperature with $u_0 = 0.414$, $\Delta u_L = 0.17$, $G_{50} = 0.5\,\mathrm{g/m^3}$, $G_{\mathrm{org}} = 4.5\,\mathrm{g/m^3}$ and $\Delta u_{\mathrm{org}} = 1.0$ (the app's Show Model lists the last two).

(a) Compute the coverage θ to 0.001 and the stability limit u_s to 0.1 % at $G = 2$, 4 and 5 g/m^3. *Checkpoint:* 0.800, 55.0 %; 0.889, 56.5 %; 0.909, 45.7 %

(b) The cell runs at $u = 58$ % with the same current and boundary layer at 2 and at 4 g/m^3. Using Equation (8.3), compute the ratio of the growth exponents to 0.01. Table 8.2 gives $\sigma v = 0.022\,/\mathrm{h}$ at 2 g/m^3. What are the two doubling times, to 1 h? *Checkpoint:* 0.50; 32 h and 63 h

8.6 Theory and calculation · 6 min

The utilization loop sees an almost static plant: the rectifier lag is 0.02 h, and with j_L fixed, $u = 100\,I/(A_c j_L)$ in percent. Take $A_c = 92\,\mathrm{m}^2$ and $j_L = 586\,\mathrm{A/m}^2$.

(a) Compute the plant gain K_p in percent per ampere, to three significant figures. *Checkpoint:* 0.001 85 %/A

(b) For a PI controller with gain K_c and integral time τ_I on a static gain K_p, show that a setpoint step settles with the time constant $\tau_{cl} = \tau_I(1+K)/K$, where $K = K_c K_p$. Evaluate it, to 0.1 h, for the tuned set, 120 A per percent and 1 h. *Checkpoint:* 5.5 h

(c) Evaluate it for the cautious set of the graded cycle, 25 A per percent and 4 h, to 1 h, and compare it with the 120 h crop. *Checkpoint:* 90 h, most of the crop

8.7 App challenge · 8 min

Open scenario 5, whose name begins “Push it to 59%”, at the default seed. Run each part at 20 h per second until the run stops at the harvest, 168 h (the clock reads 7d 00:00), and read the Nodules and Copper this cycle tiles and the value in the harvest note.

(a) Run it as shipped, at 59 %. Also read the hour of the CATHODE DOWNGRADED mark, or the first row of the exported CSV whose `nodule_mm` reaches 2 (column Time (h)), within 0.1 h. *Checkpoint:* 3.18 mm; 6281 kg; $34 846; 152.6 h

(b) Press Reset, set Current density utilization setpoint to 58.5 % before Run, and run to the harvest. *Checkpoint:* 1.82 mm, Grade A; 6213 kg; $53 167

(c) By what percentage, to 0.1, does the 59 % crop exceed the 58.5 % crop in copper, and by what percentage, to 1, does it fall short in value? Explain the difference in one sentence. *Checkpoint:* 1.1 % more copper; 34 % less value

8.8 App challenge · 8 min

Open scenario 9, “The model is wrong, and nothing says so”, at the default seed. The true diffusivity falls 20 % below the handbook value at 8 h.

(a) Run at 20 h per second and pause at 150 h (the clock reads 6d 06:00), within 0.3 h. Read the Model error and Nodules tiles and the IAE tiles of the three loops: temperature and copper within 0.002, utilization within 5. *Checkpoint:* +14.7 points; 35.00 mm, shorted; 0.243, 3.737, 2093

(b) Open scenario 5, which loads the same settings without the diffusivity change, and read the same tiles at 150 h. *Checkpoint:* +0.0 points; about 1.85 mm; 0.243, 3.737, 30

(c) Which of these numbers could a real tankhouse compute, and what do they say about the two runs? *Checkpoint:* only the first two IAEs, which are identical

8.9 App challenge · 9 min

Open the challenge scenario, “one cathode cycle, one crop”, at its default seed. Run each part to the harvest at 120 h at 20 h per second. Read the challenge result in the progress panel (copper within 1 kg, nodules within 0.01 mm, peak voltage within 0.001 V, worst temperature error within 0.01 °C) and the value in the harvest note within $5.

(a) Run it as loaded. *Checkpoint:* fails: 4042 kg, 24.49 mm, 0.383 V, 2.85 °C; $22 396

(b) Reload the scenario with its reload button, press Reveal suggested tuning and run. *Checkpoint:* passes: 3857 kg, 0.33 mm, 0.368 V, 1.17 °C; $32 945

(c) Reload, press Reveal suggested tuning, then set Current density utilization setpoint to 63 % before Run, and run. By what percentage, to 0.1, is this crop worth more than the suggested

one, and how much nodule margin does it leave? Which would you run on a real tankhouse? *Checkpoint:* passes: 4069 kg, 1.99 mm, 0.387 V, 1.17 °C; $34 758; 5.5 %; 0.01 mm

8.10 Application area · 4 min

A tankhouse runs 400 cells in electrical series on one rectifier, so every cell carries the same current. The estimator describes the average cell, but a survey shows the worst cell's limiting current density 6 % below the average. Take the ceiling over a 168 h cycle as 58.8 % utilization, as in Section 8.5.1.

(a) What utilization setpoint, computed on the average cell, keeps the worst cell at its ceiling? Give it to 0.1 % and the value on the slider's 0.5 % grid at or below it. *Checkpoint:* 55.3 %; 55.0 %

(b) What share of the copper does that give up against running the average cell at its ceiling, to 1 %? *Checkpoint:* 6 %

8.11 Application area · 3 min

The cell runs at 0.285 V at design and at 0.309 V at 58.5 % utilization (Table 8.1). Take a cathode current efficiency of 0.965, electricity at $0.085 per kW h and copper at $8.60 per kilogram, as in the app's Show Model.

(a) Using Faraday's law, compute the specific energy at each voltage, to 1 kW h/t. *Checkpoint:* 249 and 270 kW h/t

(b) What is the electricity cost per kilogram at 58.5 %, to $0.001, and its share of the copper value to 0.01 %? *Checkpoint:* $0.023 per kilogram; 0.27 %

8.12 Review the agent · 5 min

For scenario 11, an AI assistant writes: "The suspended solids only harm the cathode while the filter is bypassed, from 24 to 72 h. Lower the utilization setpoint from 58 to 56 % for exactly that window, which slows plating and therefore seeding. Raising the leveler is pointless, because the particles do not change the stability limit." Find the flaw and say how you would check it in the app.