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CHAPTER 6

Air Separation Simulator for Thermally Coupled Columns and Interacting Purity Loops

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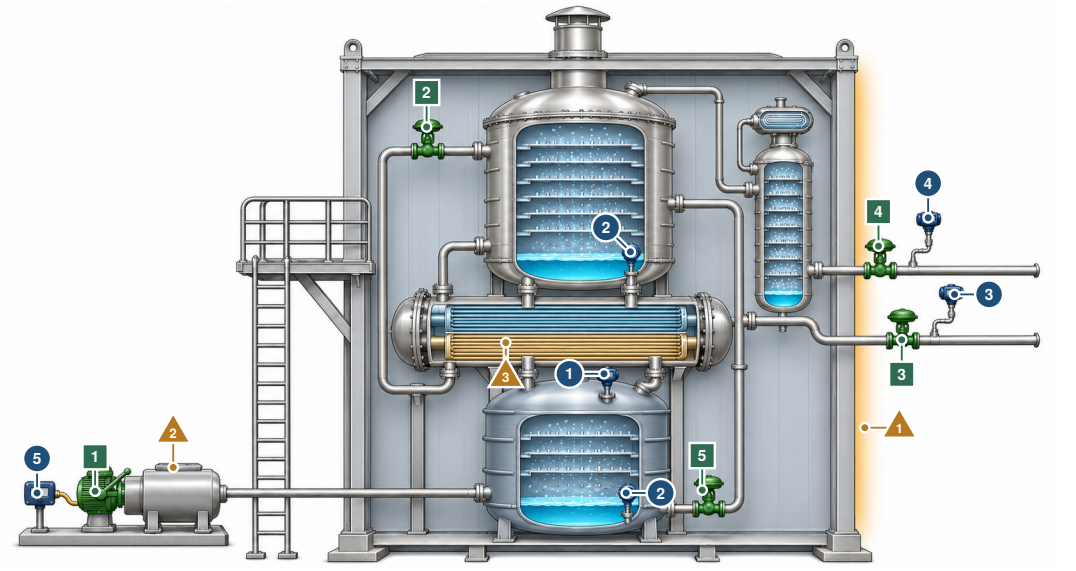
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CHAPTER 6

Air Separation Simulator for Thermally Coupled Columns and Interacting Purity Loops



The controller moves 1 compressor guide vanes 2 reflux split on the liquid nitrogen 3 liquid oxygen product valve 4 crude argon draw valve 5 rich liquid valve

The controller reads 1 high-pressure column pressure 2 levels in the two sumps 3 oxygen purity analyzer 4 crude argon analyzer 5 compressor power

What disturbs it 1 heat leak into the cold box 2 warm air at the compressor suction 3 fouling of the shared exchanger

Notice that the one exchanger between the two columns is the only condenser of the lower column and the only reboiler of the upper one, so a valve on either column moves both. Illustration, not to scale.

IN PLAIN TERMS **One Exchanger Shared by Two Columns**

The main idea. An air separation plant cools air until it turns to liquid, then boils it apart into oxygen and nitrogen in two tall columns. One heat exchanger between them cools the top of one column and heats the bottom of the other.

The challenge. So a valve on one column changes the pressure in the other and the power of the machine that squeezes the air. Some faults keep every reading on target while the plant quietly makes less.

Why it matters. Squeezing the air is nearly the whole energy bill, so a few percent of lost output decides whether a plant pays.

THREE WAYS TO READ THIS CHAPTER

Explore	13 min	the first page; the history box and the fact box of Section 6.2; the history box of Section 6.3; the try-it boxes of Sections 6.4 and 6.6; the fact box of Section 6.6; the key points
Study	65 min	the whole chapter with its pause points and the exercises
Practice	20 min	the first page; Section 6.6 with its result chart; Section 6.7; Section 6.8; the open challenge of Section 6.9

The control question One shell is the condenser of one column and the reboiler of the other, so all reflux and boil-up come from one duty. Which handle should each purity loop own when the strongest one belongs to the argon loop, and how are failures that hold every setpoint seen?

Learning objectives

- Explain why a valve on the low-pressure column moves the high-pressure column and the compressor at a fixed driving force.
- Compute the oxygen product from the sump balance, and the oxygen lost for each mole of liquid nitrogen sold.
- Rank the four handles by their effect on oxygen purity, and compute the relative gain array of the two purity loops.
- Tune the averaging level loop on the integrating sump by what it does to the product flow.
- Diagnose exchanger fouling and cold-box heat leak from the driving force, the recovery and the specific power.

Prerequisites Sections 1.4.5 (thermal coupling and Equation (1.9)), 1.4.4 (the Kremser relation), 1.6.2 (velocity form) and 1.6.4 (scenarios and data). Section 1.6.3 is needed only for the predictive controller.

The app https://apmonitor.com/apps/air-separation/

See also Volume 6, Chapter 2 for pairing by the relative gain array. Volume 6, Chapter 4 for units heated by one another's vapor. Volume 6, Chapter 8 for a plantwide control structure. Volume 9, Chapter 4 for offset-free predictive control with a disturbance estimator.

6.1 Introduction

A distillation column normally owns its reboiler and its condenser. Steam to one and cooling water to the other are utilities, and a utility can be turned up for one column without asking permission from another. A double-column air separation unit has neither. The nitrogen condensing at the top of the high-pressure column is what boils the oxygen in the sump of the low-pressure column, in the same shell and across the same tubes, and the exchanger transfers heat only because one column sits at 5.600 bar and the other at 1.400 bar, which buys 1.894 K of driving force and nothing more. That one temperature difference carries 546.0 kW. It condenses 109.2 mol/s of nitrogen, which is the reflux for both columns, and it boils 81.5 mol/s of oxygen, which is the only boil-up the low-pressure column has. There is no second condenser and no second reboiler, so nothing on the unit can be unloaded alone.

The consequence is a plant on which the unit boundary an engineer draws is fictitious. Pinching the waste nitrogen valve on top of the *low*-pressure column to 78 % raises the *high*-pressure column from 5.600 to 6.303 bar and the main air compressor from 722 to 767 kW, past its 752 kW motor rating, for exactly the same oxygen, with no pipe joining the two column tops and no controller involved. What the plant corrects is the belief that a valve belongs to the vessel it is bolted to, and behind that a second belief, that a badly behaved purity loop wants better tuning, when the crude argon draw valve moves the oxygen purity by 378 ppm per percent against the reflux split's 70 ppm and the strong handle is already spoken for.

The class of problem is wider than cryogenics. Any two units joined by one exchanger that is the only reboiler of one and the only condenser of the other inherit the same structure, and heat-integrated distillation trains, double-effect evaporators, reactors working against their own feed preheaters and refrigeration cascades are all built that way. Chapter 8 meets the other kind of thermal coupling, two column sections that share one condenser and one reboiler in a single shell. Dynamic comparisons of heat-integrated column configurations established that the integration changes the control problem and not merely the energy balance [1], interaction between candidate loops is measured with the relative gain array [2], and which variables a coupled plant should hold constant is the plantwide control question [3]. Inventories on such a plant are held by averaging loops rather than tight ones [4].

Cryogenic air separation is where these ideas earn money rather than marks. Air compression is essentially the whole energy bill of the unit, oxygen is worth so little per cubic meter that a few percent of specific power decides whether a site is competitive, and published assessments put the compression work at the center of the economics [5, 6]. Published control studies of this plant work on tray-by-tray models or on commercial simulator cases, and evaluate structures through steady-state loss or through model predictive control on reduced-order models [7–9]. However, none of those models can be opened, perturbed and re-run in a classroom hour, so a student meets the coupling as a claim rather than as an experiment. This chapter works with a browser-hosted double-column air separation unit with a crude argon sidearm, written out closely enough that its eighteen states, its shared exchanger and its five loops can be rebuilt from the text, and uses it to show that the interaction on this plant is a conservation law rather than a matrix of gains.

Section 6.2 traces how the double column and its operation developed. Section 6.3 sets out the model: the shared exchanger, the pressures and the compressor, a reduced-order column in which each lumped section is marched along its own operating line rather than treated as one equilibrium stage, the argon tracer and the sidearm, and every constant. Section 6.4 measures the coupling. The high-pressure column rises 3.11 bar for every bar of low-pressure column at constant duty, the duty setpoint meets the compressor rating at 5.699 bar and 84.10 mol/s of boil-up, and a relative gain array on the two purities returns $\lambda_{11} = 0.471$ for the pairing the plant actually uses. Section 6.5 describes the five loops, the compressor power override and the predictive controller. Section 6.6 produces the three failures that hold every setpoint, exchanger fouling, cold-box heat leak and a recovery pinch, diagnoses them from variables that are not controlled, and compares full cascade with model predictive control. Section 6.7 names the real devices behind the loops, and Sections 6.8 to 6.10 give the limitations, a discussion and the conclusion.

6.2 Background

Air separation began as a liquefaction problem and became a distillation problem. Commercial air liquefaction arrived at the end of the nineteenth century on the Joule-Thomson effect with regenerative heat exchange, and the expansion engine that followed cut the pressure and the work required, so that liquid air became a feedstock [10]. A single rectifying column answered the separation badly, because it can make pure nitrogen overhead or oxygen-enriched liquid in the sump but not both, the only reflux available being what its own condenser makes and the condenser needing a colder fluid than the column contains. The double column solved it by putting two columns at two pressures and letting the high-pressure overhead condense against the low-pressure bottoms, which supplies reflux to both from one duty. Exergy analyses show why that arrangement persists, since the temperature difference between the two ends is the entire thermodynamic penalty of the separation and the double column keeps it near the minimum a real exchanger allows [11]. Argon was added later as a sidearm hung off the low-pressure column where argon accumulates, and it has no reboiler either, because the vapor drawn from the argon belly is its boil-up.

> ⧖ HISTORY **Liquid Air and the First Rectifier, 1895 to 1905**
>
> In July 1895 Carl Linde, a professor in Munich, applied for a United States patent on a process for producing very low temperatures, liquefying air and separating its constituents. Air throttled through a valve from high to low pressure comes out colder, and Linde let the expanded air cool the incoming compressed air in a countercurrent coil about one hundred meters long until it liquefied [12]. His application of October 1903 added a rectifying column. Compressed air condensed in a coil in the liquid chamber at the column's foot, boiling the liquid there, and nitrogen drawn from the top was compressed again and condensed in a second coil to return as reflux, until pure nitrogen left the top and pure liquid oxygen collected below [13]. One stream condensing to boil another is the job the shared exchanger of Section 6.3 does between two columns.

Operation followed the hardware slowly. Early plants ran on fixed line-ups with the valve positions written on a card, a shift chemist who analyzed the product once or twice a day, and an operator who judged the column from pressure gauges and from the frost line on the cold box. The inventory is what made that workable, because twenty cubic meters of liquid oxygen and a sidearm holding two hundred kilomoles absorb hours of imbalance, so a plant lined up correctly stays lined up and a plant lined up incorrectly takes most of a shift to say so. What made it expensive was that nobody could see the compressor bill against the separation being bought with it.

Analysis of the columns themselves matured in the 1980s. Simulation of the upper column together with the crude argon sidearm as one connected problem was published in 1989 and established the belly draw and the sidearm reflux ratio as the quantities that decide argon recovery [14]. First-principles dynamic models of cryogenic distillation with multivariable control designed on them appeared around 2000 [7], and the obstacle to using such models online was removed by compartmental reduction, in which a column of many trays is collapsed to a few lumped sections whose dynamics reproduce the full model closely enough for control [8]. Nonlinear model predictive control on those reduced models followed on high-purity air separation columns [9], and that combination, regulatory loops for pressure and inventory with a multivariable controller above them, is the current standard answer.

The economics moved again in the last fifteen years. An air separation unit is a large, flexible electrical load with liquid storage attached, and the literature has since asked how much of its rate can be moved on an hourly electricity price given those inventories and dynamics [15, 16]. Both arguments rest on the physics this simulator carries, since what limits how fast the plant can be moved is the shared duty at one end and the liquid inventories at the other. The model of this chapter is in the spirit of the compartmental work, lumped where the control problem allows it and detailed where the coupling lives.

> ✦ DID YOU KNOW? **Electricity Is the Feedstock**
>
> In 2010 the industrial gas sector of the United States used 19.4 TW h of electricity, 2.5 percent of what all of manufacturing used [17]. A typical cryogenic unit uses more than 200 kW h of electricity per tonne of oxygen, against a thermodynamic minimum of around 51 kW h [18].

6.3 Air Separation Unit Model

The application simulates a double column taking 124.1 mol/s of air (10 014 m^3/h at normal conditions) through a main air compressor at 722.0 kW, with 12 % of it diverted to an expansion turbine and admitted to the low-pressure column as vapor. It delivers 23.70 mol/s of liquid oxygen (1912 m^3/h) at 99.622 %, 1.000 mol/s of crude argon (81 m^3/h) at 96.02 %, 2.18 mol/s of liquid nitrogen (176 m^3/h) and 97.2 mol/s of waste nitrogen carrying 2.53 % of the heavy key, for an oxygen recovery of 90.8 %, an argon recovery of 83.2 % and a specific power of 0.379 kW h per normal cubic meter of contained oxygen. Figure 6.1 carries the arrangement together with every symbol that appears below. Flows are in mol/s, hold-ups in mol, pressures in bar, temperatures in K and the clock runs in minutes, so every accumulation term carries a factor of 60.

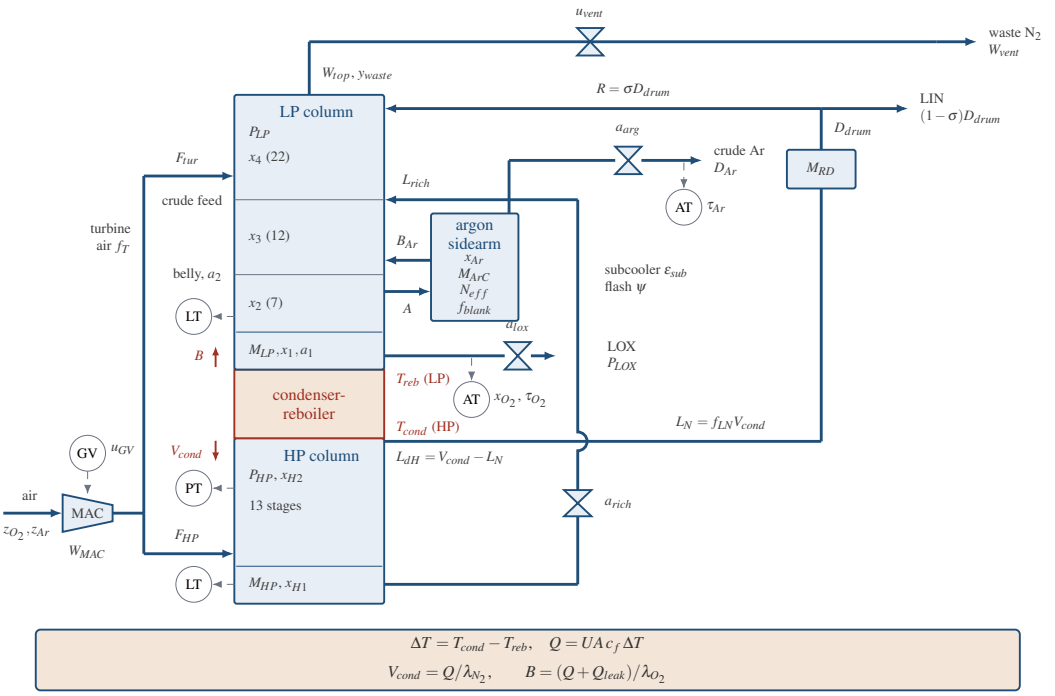


Figure 6.1 Double column with the condenser-reboiler drawn as the single wall between the two columns. The shared exchanger condenses V_{cond} of nitrogen on the high-pressure side and boils B of oxygen on the low-pressure side from one duty Q across one driving force $\Delta T = T_{cond} - T_{reb}$. The low-pressure column carries three marched sections of 22, 12 and 7 theoretical stages with the crude argon sidearm hung off its belly, and the high-pressure column is one 13-stage rectifying section solved as an absorber.

6.3.1 Variable Classification

Every signal a loop or the scorecard reads is classified in Table 6.1. The reflux split σ never touches the compressor, so it moves purity, recovery and liquid nitrogen production at constant power, and the argon in the liquid oxygen a_1 is the oxygen purity written from the other end, because the nitrogen left in the sump is 14 ppb.

6.3.2 The Shared Exchanger

One duty and one driving force set every internal flow on the plant, as shown in Equation (6.1).

$$\begin{aligned} Q &= UA\, c_f\, (T_{cond} - T_{reb}), \\ V_{cond} &= \frac{Q}{\lambda_{N_2}}, \qquad B = \frac{Q}{\lambda_{O_2}} + \frac{Q_{leak}}{\lambda_{O_2}} \end{aligned} \tag{6.1}$$

where UA is the exchanger conductance in W/K, c_f is a fouling multiplier, λ_{N_2} and λ_{O_2} are the latent heats in J/mol, Q_{leak} is cold-box heat leak in W taken from a slider calibrated in kW, and V_{cond} and B are the nitrogen condensed and the oxygen boiled in mol/s, with the duty floored at zero. Heat leak appears in B and not in V_{cond}, which is why a warm cold box costs product rather than energy.

Both saturation temperatures are Raoult bubble points of the actual local compositions at the actual local pressures, as shown in Equation (6.2).

$$\begin{aligned} T_{cond} &= T_{bub}(P_{HP}, x_{H2}, 0), \\ T_{reb} &= T_{bub}(P_{LP}, x_1 - a_1, a_1) \end{aligned} \tag{6.2}$$

Table 6.1 Signal Classification. Nominal Values Belong to the Design Line-Up Every Run Starts From.

Symbol	Role	Nominal	Range
x_{O_2}	CV, measured	99.622 %	98.5–99.9 % SP
P_{HP}	CV, measured	5.600 bar	4.6–6.6 bar SP
L (LOX sump)	CV, measured	50.0 %	20–80 % SP
x_{Ar}	CV, measured	96.02 %	85–99 % SP
L_{HP}	CV, measured	50.0 %	30–70 % SP
W_{MAC}	constraint	722.0 kW	cap at 752 kW
u_{GV}	MV	100.0 %	45–110 %
σ	MV	95.0 %	55–98 %
u_{lox}	MV	52.66 %	0–100 %
u_{arg}	MV	40.0 %	5–80 %
u_{rich}	MV	53.29 %	5–100 %
f_T	DV, set	12.0 %	0–25 %
u_{vent}	DV, set	100 %	40–100 %
u_A	DV, set	100 %	50–130 %
T_{suc}	DV, measured	30.0 °C	5–50 °C
c_f, c_a	DV, unmeas.	1.00×	0.50–1.05, 0.80–1.02
Q_{leak}	DV, unmeas.	0 kW	0–60 kW
ε_{sub} factor	DV, unmeas.	1.00×	0.40–1.00
P_{HP}, P_{LP}	states, bar	5.600, 1.400	1.05–12, 1.02–6
M_{HP}, M_{LP}, M_{RD}	states, mol	120000, 700000, 30000	0–240000, 0–1400000
x_{H1}, x_{H2}	states	0.3646, 4.68×10^{-5}	0–1
x_1–x_4	states	1.000, 1.000, 0.99995, 0.5365	0–1
a_1, a_2, x_{Ar}	states	0.003778, 0.10457, 0.9602	0–0.5, 0–0.6, 0–1
a_{GV}, a_{rich}, a_{lox}, a_{arg}	actuators	as MV	as MV
ΔT, V_{cond}, B	outputs	1.894, 109.2, 81.5	K, mol/s
a_1 in product	output	3778 ppm	—
y_{N_2} at belly	output	143 ppm	alarm 320 ppm
w_{spec}	output	0.379	kW h/m^3

where $T_{bub}(P,x_{O_2},x_{Ar})$ solves $x_{N_2}p^{sat}_{N_2}+x_{O_2}p^{sat}_{O_2}+x_{Ar}p^{sat}_{Ar}=P$ with $x_{N_2}=1-x_{O_2}-x_{Ar}$ and Antoine vapor pressures $\log_{10}(p^{sat}/\mathrm{bar})=A-B/(T+C)$ taken from the NIST WebBook, with (A,B,C) equal to $(3.7362,264.651,-6.788)$ for nitrogen, $(3.9523,340.024,-4.144)$ for oxygen and $(3.29555,215.24,-22.233)$ for argon. The solve is exactly ten Newton iterations with an analytic derivative and no early exit, because the duty is proportional to the difference of two of these temperatures that are only 1.894 K apart. Equation (6.2) makes the coupling run in both directions, since a purer sump boils hotter, closes the driving force and takes away its own boil-up.

The ratio of the two latent heats decides what the plant can make. Nitrogen at 5.6 bar gives up 5000 J/mol and oxygen at 1.4 bar needs 6700 J/mol, so one mole condensed boils only 0.746 of a mole and the rest has to leave the low-pressure column as liquid. The steady sump balance $L_2=B+P_{LOX}$, written per mole of air fed to the high-pressure column, is Equation (6.3).

$$\frac{P_{LOX}}{F_{HP}}=\sigma f_{LN}(1-\psi_R)+(1-f_{LN})(1-\psi)+\frac{B_{Ar}}{F_{HP}} -\frac{\lambda_{N_2}}{\lambda_{O_2}}-\frac{Q_{leak}}{\lambda_{O_2}F_{HP}} \tag{6.3}$$

where σ is the reflux split, f_{LN} is the fraction of the high-pressure condensate drawn off as liquid, ψ and ψ_R are the flash fractions of the two liquid feeds and B_{Ar} is the sidearm bottoms returned to the low-pressure column. At the design line-up the five terms are 0.37052, 0.49200, 0.10073, −0.74627 and zero, which sum to 0.21697 against the 0.2095 of oxygen the air contains, and multiplying by $F_{HP}=109.21\,\mathrm{mol/s}$ recovers the 23.70 mol/s of product. Equation (6.3) reproduces the model's own liquid oxygen draw to better than 2×10^{-9} over every control and disturbance setting the application

allows, and two of its terms are the whole of the plant's economics. Every mole of liquid nitrogen sold instead of returned as reflux costs very nearly a mole of oxygen, and the last term is cold-box heat leak, which subtracts product without touching any other term.

> ⧖ HISTORY **Reboiler Explosions, 1964 and 1997**
>
> The exchanger at the center of this chapter is also where an air separation unit is most dangerous. Hydrocarbons and other contaminants in the intake air boil higher than oxygen and collect in the oxygen-rich reboilers, above all in the sump of the low-pressure column. An industry code records three violent explosions that began in a reboiler, one in 1964 and two in 1997, and one of the 1997 explosions destroyed its unit and seriously damaged the neighboring plant that used the oxygen. All three are thought to have run the same course: contaminants accumulated in the exchanger passages ignited, the aluminum of the exchanger itself burned, the liquid oxygen flashed to vapor, and the column and the cold box burst. The code concludes that any hydrocarbon accumulation in a liquid oxygen reboiler must be prevented [19].

6.3.3 Pressures, Air and the Compressor

Each column pressure is a vapor inventory that integrates a flow imbalance, as shown in Equation (6.4).

$$\begin{aligned} C_{HP}\,\frac{\mathrm{d}P_{HP}}{\mathrm{d}t} &= 60\,(F_{HP} - V_{cond}) \\ C_{LP}\,\frac{\mathrm{d}P_{LP}}{\mathrm{d}t} &= 60\,(W_{top} - W_{vent}) \end{aligned} \tag{6.4}$$

where C_{HP} and C_{LP} are vapor capacitances in mol/bar, W_{top} is the vapor leaving the top of the low-pressure column and W_{vent} is what the waste valve passes. The high-pressure column therefore pressurizes until the nitrogen at its top condenses as fast as the air arrives, which fixes the condensing temperature, the driving force and the boil-up in the other column, so putting P_{HP} on the guide vanes puts that boil-up on the guide vanes. Differentiating V_{cond} of (6.1) with respect to P_{HP} gives 132.6 mol/s per bar, so (6.4) is a lag of $C_{HP}/(60 \times 132.6) = 2.51$ min on its own and about 11 min once the low-pressure column has moved with it. The waste valve is a square-root device, as shown in Equation (6.5).

$$W_{vent} = C_{vent}\,\frac{u_{vent}}{100}\,\sqrt{\max(0,\, P_{LP} - P_{vent})} \tag{6.5}$$

Air is a volumetric flow through the guide vanes of the main air compressor, so a warm morning is fewer moles through the same opening and more work for each of them, as shown in Equation (6.6).

$$\begin{aligned} F_{air} &= F_{air,0}\,\frac{a_{GV}}{100}\,\frac{T_{suc,0}}{T_{suc}}\,c_a, \qquad F_{tur} = F_{air}\,\frac{f_T}{100}, \\ F_{HP} &= F_{air} - F_{tur}, \\ W_{MAC} &= \frac{F_{air}\,R\,T_{suc}}{\eta_{MAC}}\,\ln\max\left(1.02,\; \frac{P_{HP} + \Delta P_{mac}}{P_{amb}}\right) \end{aligned} \tag{6.6}$$

where a_{GV} is the vane position in percent, T_{suc} is the suction temperature in K, c_a is the filter and molecular sieve condition, f_T is the turbine share of the air, R is the gas constant, η_{MAC} is the isothermal efficiency and ΔP_{mac} is the exchanger and piping drop above the column. The shaft power of (6.6) comes out in watts and is reported in kilowatts. Both ambient effects act the same way, which is why a twelve-degree afternoon costs 29 kW at a fixed duty.

6.3.4 Sections Marched Along Their Own Operating Lines

Relative volatilities are referred to the heavy key, which is oxygen and argon together, and both fall as pressure rises, as shown in Equation (6.7).

$$\alpha_{N_2}(P) = \alpha_{N_2}^{ref}\left(\frac{P_{ref}}{P}\right)^{n_{N_2}}, \quad \alpha_{Ar}(P) = \alpha_{Ar}^{ref}\left(\frac{P_{ref}}{P}\right)^{n_{Ar}} \tag{6.7}$$

At the design pressures α_{N_2} is 3.100 in the high-pressure column and 3.450 in the low-pressure column, and α_{Ar} is 1.105. Equilibrium between a liquid at heavy-key fraction x and its vapor is Equation (6.8).

$$y = \frac{x}{\alpha - (\alpha - 1)x}, \qquad x = \frac{\alpha y}{1 + (\alpha - 1)y} \tag{6.8}$$

Within a lumped section the flows are constant, so the net heavy key traveling downward is the same across every tray in it, and the section can be marched one stage at a time instead of solved as a boundary value problem, as shown in Equation (6.9).

$$\begin{aligned} W &= Lx - Vy \quad \text{(constant in a section)}, \\ y_k &= \max\left(\frac{Lx_k - W}{V},\ \frac{x_k}{\alpha - (\alpha - 1)x_k}\right), \\ x_{k+1} &= \frac{\alpha y_k}{1 + (\alpha - 1)y_k} \end{aligned} \tag{6.9}$$

where L and V are the liquid and vapor rates in the section, W is the net heavy key in mol/s and the march starts from the liquid entering the section's top. Equation (6.9) is exact under constant molar overflow and constant relative volatility and costs one multiply per stage. One equilibrium stage per lump reaches 97 % oxygen and stops, and α^n reaches ppm purities but only because that is the total-reflux answer, whereas this column runs at L/V near unity. The equilibrium floor in the second line of (6.9) states that liquid must get richer in the heavy key at every stage going down, so a section asked to send down more heavy key than its inlet liquid carries pinches and passes its inlet composition through instead of driving the operating line negative and producing pure nitrogen in one step.

The top of the high-pressure column is the exception, because its heavy key is 46.8 ppm and marching it would need the difference of two numbers near 24 mol/s whose answer is near 0.003 mol/s. It is solved as an absorber by the Kremser relation instead, the relation of Section 1.4.4 written for the heavy key, as shown in Equation (6.10).

$$\begin{aligned} A &= \frac{\alpha L}{V}, \qquad \phi = \frac{A - 1}{A^{n+1} - 1}, \\ y_{top} &= \frac{x_{in}}{\alpha} + \left(g_{in} - \frac{x_{in}}{\alpha}\right)\phi, \\ x_{bot} &= x_{in} + \frac{V}{L}\left(g_{in} - y_{top}\right) \end{aligned} \tag{6.10}$$

where g_{in} is the heavy key in the vapor entering the bottom of the section, n is the stage count and ϕ is replaced by $1/(n+1)$ when A is within 10^{-6} of unity.

6.3.5 Internal Flows and the Net Heavy Key

The liquid streams follow from the valve positions and the condensate, as shown in Equation (6.11).

$$\begin{aligned} L_{rich} &= C_{rich}\frac{a_{rich}}{100}\sqrt{\max(0.02,\Delta P)}\min\left(1,\frac{M_{HP}}{8000}\right), \\ L_N &= f_{LN}V_{cond}, \qquad L_{dH} = V_{cond} - L_N, \\ D_{drum} &= k_{drum}\frac{M_{RD}}{M_{RD,0}}, \\ R &= \sigma D_{drum}, \qquad P_{LIN} = (1-\sigma)D_{drum}, \\ P_{LOX} &= C_{lox}\frac{a_{lox}}{100}\min\left(1,\frac{M_{LP}}{20000}\right), \\ D_{Ar} &= D_{Ar}^{max}\frac{a_{arg}}{100}, \\ A &= A_0\frac{u_A}{100}, \qquad B_{Ar} = \max(0, A - D_{Ar}) \end{aligned} \tag{6.11}$$

where A is the argon belly vapor draw and the two min terms starve a valve whose vessel has emptied. Both liquid feeds are throttled from 5.6 to 1.4 bar and would flash without the subcooler, so the flash fractions of Equation (6.12) carry its effectiveness.

$$\begin{aligned} \psi &= \frac{c_{pL}\left(T_{rich} - T_{rich}^{LP}\right)}{\lambda_{O_2}}\left(1 - \varepsilon_{sub}c_{sub}\right) + \frac{B_{Ar}\lambda_{Ar}}{\lambda_{O_2}L_{rich}}, \\ \psi_R &= \frac{c_{pL}\left(T_{cond} - T_{LN}^{LP}\right)}{\lambda_{N_2}}\left(1 - \varepsilon_{sub}c_{sub}\right) \end{aligned} \tag{6.12}$$

where $\Delta P = P_{HP} - P_{LP}$, c_{pL} is the liquid heat capacity, c_{sub} is the subcooler condition disturbance, the temperatures are bubble points of the same liquid at the two pressures, and the second term of ψ folds in the crude argon condenser, which has no coolant of its own and boils against the rich liquid. Each of the three terms is clipped to $[0, 0.45]$ and their sum to $[0, 0.70]$, and at design $\psi = 0.1800$ and $\psi_R = 0.0250$. The internal traffic then follows as shown in Equation (6.13).

$$\begin{aligned} R_l &= R(1-\psi_R), \qquad L_f = L_{rich}(1-\psi), \\ L_3 &= R_l + L_f, \qquad L_2 = L_3 + B_{Ar}, \\ V_3 &= \max(0, B - A), \qquad V_4 = V_3 + L_{rich}\psi + F_{tur}, \\ W_{top} &= V_4 + R\,\psi_R \end{aligned} \tag{6.13}$$

with design values $R_l = 40.46\,\text{mol/s}$, $L_3 = 94.19\,\text{mol/s}$, $L_2 = 105.19\,\text{mol/s}$, $V_3 = 69.50\,\text{mol/s}$, $V_4 = 96.19\,\text{mol/s}$ and $W_{top} = 97.2\,\text{mol/s}$.

The net heavy key crossing each section plane is written as what leaves below that plane plus what accumulates below it, which is what makes the whole column conserve, as shown in Equation (6.14).

$$\begin{aligned} W_2 &= (L_2 - B)x_1, \qquad W_3 = W_2 + D_{Ar}, \\ W_4 &= W_3 - L_{rich}x_{H1} - F_{tur}(z_{O_2} + z_{Ar}) \end{aligned} \tag{6.14}$$

Below the argon belly the only inventory is the sump, so the accumulation term collapses the product draw out of W_2 and at steady state $W_2 = P_{LOX}x_1$. That form is a conditioning choice. The net nitrogen rising between the crude feed and the belly is $V_3 - L_3 + W_3$, terms of 69.50, -94.19 and 24.70 mol/s whose design value is -3.4×10^{-7} mol/s, the 14 ppb of nitrogen in the product, and (6.14) telescopes it analytically to $-(L_2 - B)(1 - x_1)$ so that nothing is left to cancel. In the product-draw form a liquid oxygen valve 0.01 points from balance would add 0.0045 mol/s to it, as much as the nitrogen crossing

the belly in each direction. At design $W_4 = -2.456\,\mathrm{mol/s}$, so above the crude feed the heavy key travels net upward, and dividing that by W_{top} gives the 2.53 % of oxygen the waste nitrogen carries away. No stage count appears in it, which is why recovery is decided at the reflux split and not at the top of the column.

The three low-pressure sections are marched downward by (6.9) from the liquid entering each of them, using $(x_R, W_4, R_l, V_4, \alpha_{N_2}, 22)$ above the crude feed, $(x_{in,3}, W_3, L_3, V_3, \alpha_{N_2}, 12)$ between the crude feed and the belly, and $(x_{in,2}, W_2, L_2, B, \alpha_{N_2}, 7)$ below the belly, with the mixed inlets $x_{in,3} = (R_l x_4 + L_f x_{fl})/L_3$ and $x_{in,2} = (L_3 x_3 + B_{Ar} x_{B_{Ar}})/L_2$ and the flashed feed compositions $x_{fl} = (x_{H1} - \psi y(x_{H1}))/(1-\psi)$ and $x_R = (x_{H2} - \psi_R y(x_{H2}))/(1-\psi_R)$. When the top section's operating line starts below the equilibrium curve the march has nothing to say, so the model reports a drain rate $\Gamma = \max(0, W_4 - R_l x_R)$ instead and lets the section lose heavy key from its own hold-up at exactly the rate the balance is short by.

6.3.6 Argon as a Tracer and the Sidearm

Argon rides on the heavy key rather than appearing as a rigorous third component, and it leaves the belly three ways, to the sidearm, downward into the oxygen product and upward with the waste nitrogen. The two slips are set by the argon stripping factor in the section concerned, as shown in Equation (6.15).

$$\begin{aligned} S_{dn} &= \frac{\alpha_{Ar} B}{L_2}, \qquad S_{up} = \frac{\alpha_{Ar} V_4}{R_l}, \\ k_{dn} &= \frac{k_{dn,0}}{S_{dn}^{m_{dn}}}, \qquad k_{up} = k_{up,0} S_{up}^{m_{up}} \end{aligned} \tag{6.15}$$

with S_{dn} clamped to $[0.30, 3]$ and S_{up} to $[0.30, 8]$. At the design line-up $S_{dn} = 0.856$ and $S_{up} = 2.627$. The sixth power on S_{dn} is what makes anything that raises the liquid traffic below the belly put argon into the product, which is the mechanism behind the reflux split's effect on purity.

The sidearm has no reboiler, so the belly vapor is its boil-up and whatever is not drawn off condenses and returns as reflux, as shown in Equation (6.16).

$$\begin{aligned} f_{blank} &= \frac{1}{1 + \left(y_{N_2}/y^*_{N_2}\right)^4}, \quad R_{Ar} = f_{blank} \frac{B_{Ar}}{\max(0.02, D_{Ar})}, \\ N_{eff} &= N_{Ar} \frac{R_{Ar}}{R_{Ar} + R_{1/2}}, \qquad \frac{x^*_{Ar}}{1 - x^*_{Ar}} = \alpha_{Ar}^{N_{eff}} \frac{z^F_{Ar}}{1 - z^F_{Ar}} \end{aligned} \tag{6.16}$$

where y_{N_2} is the nitrogen in the belly vapor, $y^*_{N_2}$ is the blanketing threshold, N_{Ar} is the stage count at total reflux, $R_{1/2}$ is the reflux ratio at which the column has half its stages and z^F_{Ar} is the argon fraction of the argon-plus-oxygen entering the sidearm. Nitrogen will not condense at the argon condenser temperature, so it collects in the top of the shell and covers the tubes, and the fourth power makes that a threshold rather than a trend. At design $R_{Ar} = 10.58$, $N_{eff} = 52.4$, $f_{blank} = 0.9615$ and the belly nitrogen is 143 ppm against a threshold of 320 ppm. The sidearm relaxes toward x^*_{Ar} with the hold-up time constant $\tau_{Ar} = M_{ArC}/(60A)$, which is 277.8 min at the design belly draw.

6.3.7 State Equations

Eighteen states are integrated, as shown in Equations (6.4), (6.17), (6.18), (6.19) and (6.20). The three liquid inventories are plain balances.

$$\begin{aligned} \frac{\mathrm{d}M_{HP}}{\mathrm{d}t} &= 60\,(L_{dH} - L_{rich})\,, \\ \frac{\mathrm{d}M_{RD}}{\mathrm{d}t} &= 60\,(L_N - D_{drum})\,, \\ \frac{\mathrm{d}M_{LP}}{\mathrm{d}t} &= 60\,(L_2 - B - P_{LOX}) \end{aligned} \tag{6.17}$$

The liquid oxygen sump is the integrator the level loop lives on. Its inventory at 50 % is 700 000 mol, which is 19.6 m^3, and differentiating (6.17) against valve position gives an integrating gain of 0.001 93 %/min of level per percent of valve.

The compositions relax toward what each section's own profile asks for at the rate its liquid turns over, as shown in Equation (6.18).

$$\begin{aligned}
\frac{dx_{H1}}{dt} &= \frac{60\,L_{dH}\left(x_{bot}^{HP} - x_{H1}\right)}{\max(4000, M_{HP})},\\
\frac{dx_{H2}}{dt} &= \frac{60\,L_{dH}\left(y_{top}^{HP} - x_{H2}\right)}{M_{H2}},\\
\frac{dx_1}{dt} &= \frac{60\left[L_2\left(x_2 - x_1\right) - B\left(y(x_1) - x_1\right)\right]}{\max(20000, M_{LP})},\\
\frac{dx_2}{dt} &= \frac{60\,L_2\left(x_{bot,2} - x_2\right)}{M_{st}}, \quad \frac{dx_3}{dt} = \frac{60\,L_3\left(x_{bot,3} - x_3\right)}{M_{st}},\\
\frac{dx_4}{dt} &= \frac{60\left[\max(4, R_l)\left(x_{bot,4} - x_4\right) - \Gamma\right]}{M_{st}}
\end{aligned} \tag{6.18}$$

where $x_{bot,k}$ is the liquid leaving the bottom of section k from the march, frozen at the state itself when that section is infeasible. The sump is written as differences of compositions rather than of flows so that a sump which is filling or draining cannot make its own composition drift by arithmetic alone, and the argon states follow the same pattern, as shown in Equation (6.19).

$$\begin{aligned}
\frac{da_2}{dt} &= \frac{60\left[Ar_{in} - D_{Ar}x_{Ar} - P_{LOX}k_{dn}a_2 - W_{top}k_{up}a_2\right]}{M_{belly}},\\
\frac{da_1}{dt} &= \frac{60\left[L_2\left(k_{dn}a_2 - a_1\right) - B\left(\alpha_{Ar}a_1 - a_1\right)\right]}{\max(20000, M_{LP})},\\
\frac{dx_{Ar}}{dt} &= \frac{x_{Ar}^{*} - x_{Ar}}{\max(1, \tau_{Ar})}
\end{aligned} \tag{6.19}$$

where $Ar_{in} = F_{HP}z_{Ar}\eta_{Ar}^{HP} + F_{tur}z_{Ar}$ is the argon arriving at the low-pressure column. The exchange terms in the sump argon balance are L_2 and B rather than the product draw, which is why the argon in the liquid oxygen lines out in about 111 min rather than the eight hours the draw rate alone would suggest. The four actuator positions each lag their own command, as shown in Equation (6.20).

$$\tau_j \frac{da_j}{dt} = u_j - a_j, \qquad j \in \{GV,\ rich,\ lox,\ arg\} \tag{6.20}$$

The outputs the scorecard reports are the oxygen purity, the oxygen and argon recoveries and the specific power, as shown in Equation (6.21).

$$\begin{aligned}
x_{O_2} &= x_1 - a_1, \qquad & w_{spec} &= \frac{W_{MAC}}{\dot{V}_{O_2}x_{O_2}},\\
r_{O_2} &= \frac{P_{LOX}x_{O_2}}{F_{air}z_{O_2}}, \qquad & r_{Ar} &= \frac{D_{Ar}x_{Ar}}{F_{air}z_{Ar}}
\end{aligned} \tag{6.21}$$

where $\dot{V}_{O_2}$ is the product volumetric rate at normal conditions and r_{O_2} and r_{Ar} are the two recoveries.

6.3.8 Parameters, Initialization and Numerics

Every constant appears in Tables 6.2 and 6.3. Nine are hidden and never displayed, and the four stage counts are structure rather than parameters, so anybody who wants the volatilities, the air composition or the vessel spans has to infer them. The remainder are visible in the parameter panel and adjustable.

Table 6.2 Air, Exchanger, Pressure and Separation Parameters

Symbol	Meaning	Value	Unit	Visibility
$F_{air,0}$	air at 100 % vane	124.1	mol/s	adjustable
z_{O_2}	oxygen in air	0.2095	—	hidden
z_{Ar}	argon in air	0.0093	—	hidden
$T_{suc,0}$	reference suction	303.15	K	hidden
η_{MAC}	isothermal efficiency	0.767	—	adjustable
P_{amb}	atmospheric pressure	1.013	bar	hidden
ΔP_{mac}	drop above the column	0.35	bar	adjustable
W_{max}	compressor power limit	752	kW	adjustable
UA	exchanger conductance	288300	W/K	adjustable
λ_{N_2}, λ_{O_2}	latent heats, N_2 and O_2	5000, 6700	J/mol	adjustable
λ_{Ar}	argon latent heat	6400	J/mol	hidden
ΔT_{min}	unloading threshold	1.10	K	adjustable
C_{HP}, C_{LP}	vapor capacitances	20000, 25000	mol/bar	adjustable
C_{vent}	waste valve size	164.33	mol/s bar$^{-1/2}$	adjustable
P_{vent}	downstream of the vent	1.05	bar	adjustable
C_{rich}	rich liquid valve size	60.0	mol/s bar$^{-1/2}$	adjustable
$\alpha_{N_2}^{ref}$, α_{Ar}^{ref}	volatilities at 1.4 bar	3.45, 1.105	—	adjustable
P_{ref}	volatility reference	1.4	bar	hidden
n_{N_2}, n_{Ar}	pressure exponents	0.0771, 0.060	—	adjustable
f_{LN}	condensate drawn as liquid	0.400	—	adjustable
ε_{sub}	subcooler effectiveness	0.85	—	adjustable
c_{pL}	liquid heat capacity	56	J/mol K	hidden

Table 6.3 Inventory, Argon, Actuator, Instrument and Economic Parameters

Symbol	Meaning	Value	Unit	Visibility
$M_{HP,0}$, $M_{LP,0}$	sumps at 50 %	120000, 700000	mol	adjustable
M_{HP}^{max}, M_{LP}^{max}	sumps at 100 %	240000, 1400000	mol	hidden
$M_{RD,0}$	reflux drum hold-up	30000	mol	adjustable
M_{st}	hold-up per LP section	70000	mol	adjustable
M_{H2}	HP rectifying hold-up	45000	mol	adjustable
M_{belly}	argon belly hold-up	30000	mol	adjustable
M_{ArC}	sidearm hold-up	200000	mol	adjustable
A_0	belly draw at 100 %	12.0	mol/s	adjustable
D_{Ar}^{max}	argon draw, valve open	2.5	mol/s	adjustable
N_{Ar}	sidearm stages at total reflux	67	—	adjustable
$R_{1/2}$	reflux ratio for half the stages	2.95	—	adjustable
$y_{N_2}^*$	half-blanketing nitrogen	3.2×10^{-4}	—	adjustable
$k_{dn,0}$	downward argon slip	0.01538	—	adjustable
$k_{up,0}$	upward argon slip	1.240×10^{-3}	—	adjustable
m_{dn}, m_{up}	slip exponents	6.0, 2.0	—	adjustable
η_{Ar}^{HP}	air argon into the crude liquid	0.99	—	adjustable
N_{HP}, $N_{2..4}$	stages per section	13, 7, 12, 22	—	hidden
C_{lox}	LOX draw, valve open	45.0	mol/s	adjustable
k_{drum}	drum drain at normal level	44.0	mol/s	adjustable
τ_{GV}, τ_{rich}, τ_{lox}, τ_{arg}	actuator travel	0.30, 0.25, 0.35, 0.40	min	adjustable
τ_{O_2}, τ_{Ar}^m	the two analyzers	6.0, 9.0	min	adjustable
τ_{Pm}, τ_{Lm}	transmitter response	0.10, 0.50	min	adjustable
$x_{O_2}^{spec}$, x_{Ar}^{spec}	product specifications	0.995, 0.93	—	adjustable
L_{lo}, L_{hi}	LOX level alarms	25, 80	%	adjustable
p_W	electricity	0.062	\$/kW h	adjustable
p_{O_2}, p_{Ar}, p_{LIN}	product values	0.055, 1.20, 0.030	\$/m^3	adjustable

Every run opens on a genuine steady state. Two of the eighteen states are pure integrators, so the plant has no steady state unless the rich liquid valve and the liquid oxygen draw are chosen to close their balances. The initializer therefore runs the real derivative function forward at a fixed 0.4 min step for exactly 9000 iterations, with each of those two valves driven by a feedforward term asking for the flow that closes its own balance plus a small level trim, and it returns the state and the two positions. At the design controls it returns 53.29 % on the rich liquid valve and 52.66 % on the liquid oxygen valve, which is the line-up every quoted result starts from. Feedforward is used rather than a proportional level controller because any gain slow enough to be stable took longer to line out than the sidearm did, and any gain fast enough pushed the stripping section through its pinch.

The engine of Section 1.6.1 advances the state vector on a fixed 0.05 min (3 s) step under classical fourth-order Runge-Kutta, the five controllers execute on a 0.25 min (15 s) scan and four samples of every simulated minute reach the historian. The noise generator is seeded at 20260914, and the fastest speed is 60 minutes of plant time per second.

6.3.9 Assumptions

Detail is concentrated where the coupling and the inventories live and taken out everywhere else. Every simplification listed here has an operating region in which it stops being true.

Eighteen states stand in for several hundred. Each column section is a lump of theoretical stages marched along its own operating line, so there is no tray hydraulics, no flooding or weeping and no temperature profile of the kind an operator reads. A theoretical stage here is worth roughly two real trays, and the stage counts were calibrated against the design purities.

Relative volatility is corrected for pressure and not for composition. The real volatility of argon against oxygen runs from about 1.08 to 1.13 with temperature, and taking 1.105 throughout gets the difficulty of the separation right and the exact stage requirement wrong.

Argon is a tracer with two empirical slip coefficients. The exponents in (6.15) were calibrated to put 3778 ppm of argon in the liquid oxygen at design and to give the reflux split the sensitivity a real unit shows. The sidearm is one steady-state relation with a hold-up lag and its condenser blanketing is a fitted quartic sigmoid, so the shape of (6.16) is an argument and the 320 ppm threshold is a calibrated number.

The compressor is isothermal work at a constant efficiency. There is no surge line, no stage matching, no intercooler train and no real turndown limit, and the guide vane characteristic is linear in volume.

Refrigeration is not modeled. The expansion turbine appears only as air diverted from the high-pressure column and the cold-box heat leak is a slider, whereas on a real unit those two and the liquid production are one balance, and that balance is the most important slow constraint on the plant.

The arithmetic is too repeatable. One frozen step, one frozen scan and a bubble-point solve with no early exit hand back the identical run everywhere, whereas a real control system delivers scan jitter and an execution order that moves. The heavy key balance closes to machine precision except at the sidearm condenser vent, which carries under 0.002 mol/s of nitrogen at design.

6.4 Coupling and Interaction Analysis

6.4.1 A Valve on One Column Is a Valve on the Other

The waste nitrogen valve sits on top of the low-pressure column and touches no part of the high-pressure column, no compressor setting and no air rate. Pinching it from 100 to 78 % in scenario 2, with every handle in MANUAL, raises the low-pressure column from 1.400 to 1.626 bar by the time the plant has settled at 220 min, so the oxygen in its sump boils at 95.01 K instead of 93.47 K. Neither the duty nor the conductance has changed, so (6.1) still demands the same 1.894 K and the nitrogen at the top of the high-pressure column must condense at 96.91 K instead of 95.36 K, which it will only do at 6.303 bar. The ratio is the useful number, because the high-pressure column rises 3.11 bar for every bar of low-pressure column, the two bubble point curves having different slopes with a

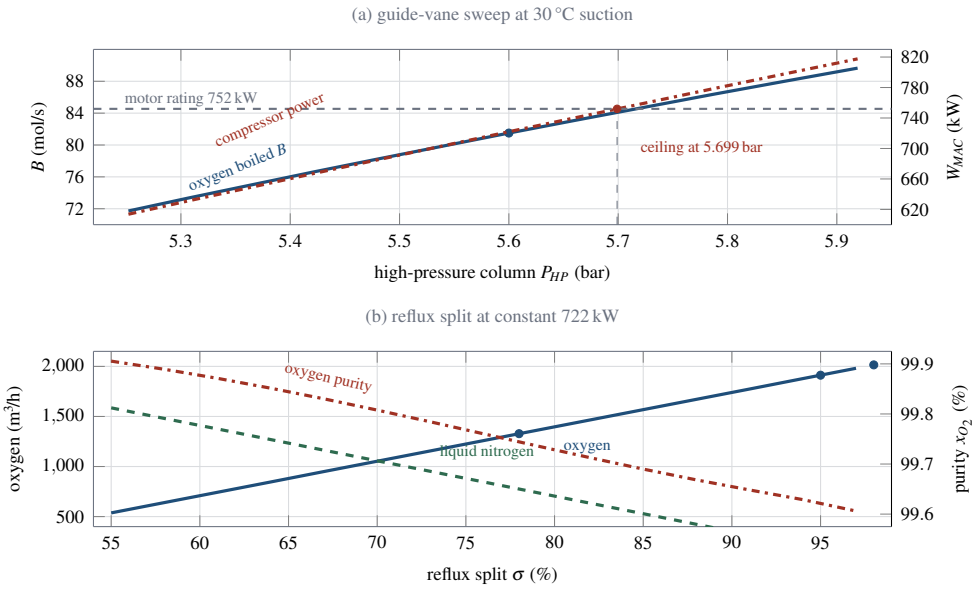


Figure 6.2 Two steady-state sweeps of the shipped model. In (a) the guide vane runs from 88 to 110 % and both low-pressure boil-up (solid, left axis) and compressor power (dash-dotted, right axis) are plotted against the pressure the loop holds, which crosses the motor rating (thin dashed line) at 5.699 bar. In (b) the reflux split runs from 55 to 97 %, with oxygen production solid, liquid nitrogen dashed and oxygen purity dash-dotted on the right axis, while the compressor power varies by less than half a kilowatt.

fixed driving force between them, so one tenth of a bar of low-pressure column is worth 20.2 kW continuously. The low-pressure column reaches 63 % of its move in 13 min and the high-pressure column 71 % of its own in 20 min, settling in about an hour, held back by nothing but the two vapor volumes of (6.4).

> **TRY IT IN THE APP** **2 · An LP-Column Valve Raises the HP-Column Pressure (MANUAL)**
>
> Load scenario 2 and press Run. At 40 min the waste nitrogen valve on top of the low-pressure column is pinched to 78 %. Watch the high-pressure column, which nobody touches: it settles at 6.303 bar instead of 5.600 bar, the gap across the condenser-reboiler stays at 1.894 K, and the compressor climbs to 767.5 kW, past its 752 kW rating.

Pause. Try Exercise 6.1 before reading on.

6.4.2 The Pressure Setpoint Is a Duty Setpoint With a Ceiling

Sweeping the guide vane at steady state produces Figure 6.2(a), which is the characteristic the pressure loop rides on. Boil-up in the low-pressure column is very nearly linear in the pressure of a column it does not enter, at 26.9 mol/s per bar over the range swept, and compressor power is linear at 306 kW per bar. Neither of those curves belongs to one unit. The motor rating of 752 kW is reached at 5.699 bar and 84.10 mol/s of boil-up, and that intersection is the ceiling on the duty setpoint. Asking for 5.780 bar instead needs the vane at 105.7 % and 776.3 kW, which is 54 kW on top of 722 and past the rating.

Dropping the setpoint instead is instructive because the two halves of the answer arrive on different clocks. In scenario 3 a move from 5.600 to 5.420 bar on the pressure and level loops brings the transmitter within 0.01 bar of the new setpoint 8 min after the step, undershoots to 5.385 bar and

stays within 0.01 bar from 24 min, and it takes the driving force from 1.894 to 1.780 K, the nitrogen condensed from 109.2 to 102.6 mol/s and the oxygen boiled from 81.5 to 76.6 mol/s. Less liquid arriving in the sump then lowers the level, the level controller closes the product valve, and the oxygen walks from 1912 to 1795 m^3/h five hours after the step, so the pressure loop settles in minutes and the plant in hours for a saving of 55 kW against 6.1 % of the production.

6.4.3 One Knob, Three Answers, at Constant Power

The reflux split decides how much of the liquid nitrogen drawn off the high-pressure column goes back as reflux and how much is sold, and Figure 6.2(b) is the trade it buys. At 78 % the plant makes 1328 m^3/h of oxygen at 99.744 % with 775 m^3/h of liquid nitrogen and 63.1 % recovery, at 95 % it makes 1912 at 99.622 % with 176 and 90.8 %, and at 98 % it makes 2015 at 99.594 % with 70 and 95.7 %, for 721.9, 722.0 and 722.0 kW. Scenario 5 makes the same two moves on the pressure and level loops, and its production reaches those settled values only through the sump level loop, undershooting to 1237 m^3/h six hours after the cut to 78 % and overshooting to 2126 m^3/h ten hours after the move to 98 %. The air, the duty and the driving force did not change, so the split is not a purity handle with a production side effect. It is a decision about how to divide a fixed amount of separation between two products and one specification.

6.4.4 Ranking the Handles, and What the Array Says

At steady state with the sump level held, three of the four manual handles move the oxygen purity, and only one of them was supposed to. Measured as central differences of one percent at the design line-up, the crude argon draw valve moves it by 378 ppm per percent, the guide vane by −174 ppm per percent and the reflux split by −70 ppm per percent. The argon valve wins because the nitrogen left in the liquid oxygen is 14 ppb, so the only impurity that matters is the 3778 ppm of argon the sidearm did not take out and the argon valve is the only handle that acts on it directly. Taking the two purities as controlled variables and the split and the argon draw as manipulated variables gives the steady-state gain matrix of Equation (6.22).

$$K = \begin{bmatrix} -0.0070 & 0.0378 \\ -0.1151 & -0.5536 \end{bmatrix} \%/\%, \qquad \lambda_{11} = 0.471 \tag{6.22}$$

where the rows are oxygen purity and crude argon purity and the columns are the reflux split and the argon valve. The relative gain array returns 0.471 for the diagonal pairing the application implements and 0.529 for the off-diagonal one, so it marginally prefers putting oxygen purity on the argon valve. The application pairs the other way anyway, for availability rather than sensitivity, because the guide vane belongs to the duty loop and the argon valve belongs to the argon specification, which leaves the reflux split as the only free handle. A λ_{11} of 0.471 is close enough to 0.5 that neither pairing decouples, and that is why both purity loops are tuned at 200 minutes of reset rather than made fast. The array is a steady-state measure, and Volume 6, Chapter 2 shows its other limit, a relative gain at the value for no interaction on two loops that still interact.

🕮 **Pause.** Try Exercise 6.3 before reading on.

6.5 Implemented Control Structure

6.5.1 Five Loops, Four Architectures and One Override

Five loops are implemented and a control architecture switch selects how many are in service. In MANUAL the four handles are whatever the operator typed and only the high-pressure sump level loop runs. The inner-loops setting closes column pressure on the guide vanes and sump level on the product valve, and leaves both purities to whatever the day gives, which is how a great many air separation units are actually run. Full cascade adds both purity loops and enables a compressor power override that caps the guide vane. The fourth setting hands the guide vanes, the product valve and the

reflux split to one model predictive controller, described in Section 6.5.5, while the argon loop, the override and the high-pressure sump loop run as they do in full cascade.

6.5.2 The Discrete Algorithm

One implementation serves all five loops. It is the velocity form of Section 1.6.2, and one execution is Equation (6.23), in which k counts scans and $\Delta t_c = 0.25\,\text{min}$.

$$e_k = a\,(\text{SP}_k - \text{PV}_k)\,, \qquad P_k = K_c e_k \tag{6.23a}$$

$$D_k = -K_c \tau_D\, a \left(\frac{\text{dPV}^f}{\text{d}t} \right)_k$$

$$s_k = s_{k-1} + \Delta P_k + \Delta D_k \tag{6.23b}$$

$$c_k = u_{bias} + s_k + I_{k-1} \tag{6.23c}$$

$$u_k = \min(u_{max},\ \max(u_{min},\ c_k)) \tag{6.23d}$$

$$I_k = I_{k-1} + \left[\frac{K_c}{\tau_I} e_k + \frac{u_k - c_k}{T_t} \right] \Delta t_c \tag{6.23e}$$

Here $a = \pm 1$ is the action, $\Delta P_k = P_k - P_{k-1}$ and $\Delta D_k = D_k - D_{k-1}$ are the proportional and derivative increments, s_k accumulates them without limit, c_k is the demanded output and u_k is what reaches the actuator lag of (6.20). In the symbols of Equations (1.14) to (1.17), s_k is S_k, c_k is v_k and u_{bias} is u_b. The last term of (6.23e) is the back-calculation of Chapter 1, with tracking time $T_t = \tau_I$ on every loop [20], so the integral alone absorbs the clipping. Derivative action is switched off everywhere, since $\tau_D = 0$ on all five loops makes D_k vanish, and none of the loops carries a rate limit, the actuator lags already supplying one. MANUAL keeps every loop executing with $I_k = u_{man} - u_{bias} - s_k$, the bumpless transfer of Section 1.6.2. Output bias is zero throughout, and every initialization, whether a scenario load or a Reset, seeds each integral from the actuator position the initializer returns for the settings in force, so the opening execution hands back the line-up it started from.

The actions follow Chapter 1's convention, in which $a = +1$ marks a reverse-acting loop on which a rising measurement lowers the output and $a = -1$ a direct-acting one. Table 6.4 lists the five loops with their pairings, default tunings, actions and output limits. Only the pressure loop is reverse acting. The level, purity and sump loops are direct acting, because raising each of their outputs lowers its own measurement.

Table 6.4 Default Tuning of the Five Loops. Action +1 Is Reverse Acting.

CV	MV	K_c	τ_I (min)	a	Limits
P_{HP}	u_{GV}	40	3	+1	45–110 %
L	u_{lox}	3	300	−1	0–100 %
x_{O_2}	σ	30	200	−1	55–98 %
x_{Ar}	u_{arg}	2.5	200	−1	5–80 %
L_{HP}	u_{rich}	2.0	30	−1	5–100 %

Three of the five settings in Table 6.4 are worth explaining. The pressure loop carries 40 % of vane per bar with 3 min of reset against a process gain of 0.0306 bar per percent and about 11 min of coupled column dynamics, which makes it the fastest thing on the plant. The level loop carries 3 % of valve per percent with 300 min of reset, which looks slack and is the correct answer, because the process gain is 0.001 93 %/min per percent of valve and a loop tight enough to hold the level throws the whole flow imbalance into the product line [4]. That is averaging level control, and Volume 2, Chapter 5 tunes a loop on another integrating level from a measured slope. The oxygen purity loop carries 30 % of split per percent with 200 min of reset, slow because it faces 111 min of sump inventory behind a handle worth 70 ppm per percent, so the 6 min analyzer is not what makes it

slow. The argon loop faces 278 min of sidearm hold-up, so anything responsive on that trend is far too fast.

6.5.3 The Compressor Power Override

The power limit is enforced on the actuator rather than as a controlled variable of its own, as shown in Equation (6.24).

$$u_{max} = \mathrm{clamp}\big(u_{max} + K_W\,(W_{max} - \hat{W}_{MAC})\Delta t_c,\, 45,\, 110\big) \tag{6.24}$$

where $K_W = 0.06\,\%$ per kW min and $\hat{W}_{MAC}$ is the filtered power measurement. Equation (6.24) is an integral-only constraint controller whose state is a valve position, so it is seeded at the top of its range and falls only when the limit is real. Written as a high limit on the pressure loop's output rather than as a second controller into a low selector, the pressure loop's own back-calculation in (6.23e) does the anti-windup, so when the limit lets go the loop comes off it without a bump. The cap is deliberately slow, because the guide vane is also the boil-up handle for a column with twenty cubic meters of inventory underneath it, and snatching it back at the speed a power trip would prefer costs an hour of recovery on the argon. A protective override that must act in seconds, on a compressor near surge, is the subject of Volume 4, Chapter 2.

6.5.4 Instruments

The oxygen analyzer is a first-order filter of 6.0 min standing for a paramagnetic analyzer on a sample line and the crude argon chromatograph is a filter of 9.0 min, while the pressure transmitter and the power transducer are 0.10 min and the two level transmitters are 0.50 min. Optional colored noise is zero by default and is scaled by 1, 8 and 12 on the oxygen analyzer, the argon chromatograph and the level transmitters.

6.5.5 Model Predictive Control

The fourth architecture replaces the pressure, level and oxygen purity loops by a plan of the kind set out in Section 1.6.3. Every $T_s = 1$ min, four controller scans, a quadratic program chooses the guide vane u_{GV}, the product valve u_{lox} and the reflux split σ over $N = 120$ steps in twelve move blocks of $\{1,1,1,1,2,2,3,4,6,9,15,75\}$ steps, each a move from the input applied now, and the opening move is held for T_s [21, 22]. The argon loop keeps its valve, whose applied position enters the plan as a measured input held over the horizon, and the cap of (6.24) stays the vane's upper bound.

The controller's model is the plant of Section 6.3 at the declared parameter values and nominal disturbances (30 °C suction, $c_f = 1$, no heat leak), linearized by central differences at the design line-up (100 % vane, 95 % split, 40 % argon valve). The integral of the high-pressure sump loop at its shipped tuning, which runs under every architecture, and the 6.0 min analyzer lag join the eighteen plant states, and an exact zero-order hold over T_s gives Equation (6.25).

$$\begin{aligned} \tilde{x}_{k+1} &= A_d\,\tilde{x}_k + B_d\,(\tilde{u}_k + d_k) + E_d\,\tilde{u}_{arg,k}, \qquad d_{k+1} = d_k \\ \tilde{y}_k &= C\,\tilde{x}_k \end{aligned} \tag{6.25}$$

Here a tilde marks a deviation from the design point, $\tilde{x}$ holds the twenty states, $\tilde{u}$ the three decided inputs, d one unmeasured disturbance per decided input, $\tilde{u}_{arg}$ the applied argon valve and $\tilde{y}$ the outputs P_{HP}, L, x_{O_2}, W_{MAC} and the analyzer reading. Step-response fits were not used, because one percent of vane raises the purity 0.000 78 % at 15 min and lowers it 0.0191 % by 960 min, while one percent of split delivers 42, 65 and 94 percent of its −0.0096 % answer by 60, 120 and 480 min. Against the engine's mean response to ±1 percent of vane, ±1.5 of product valve, ±0.5 of split and ±2 of argon valve from scenario 1 without noise, the model errs by at most 0.73 % of the largest response between 15 and 960 min, the worst input being the argon valve, and by 0.20 percent for the split.

A steady-state Kalman filter estimates $\tilde{x}$ and d once a minute under every architecture from the readings the loops use, P_{HP}, L and the analyzer, with assumed noise of 0.002 bar, 0.05 % and 0.002 %. Each disturbance carries process noise $\rho\, r_m^2 / G_m^2$, with $\rho = 100$, r_m the noise of its measurement and G_m the effect of one percent of its input on it (0.0306 bar, 0.116 % of level in an hour, 0.0097 % of purity), which makes the prediction offset free [23]. Volume 9, Chapter 4 builds the same kind of estimator around an unmeasured fuel bias. The compressor power is corrected by its measured offset. A constant output bias instead of the filter, tried on a diagnostic copy outside the application, loses scenario 7 by an oxygen IAE of 21.25 against 16.38 % · min and 22.0 against 0.1 min above the motor rating.

The plan minimizes Equation (6.26).

$$
\begin{aligned}
J = \sum_{k=1}^{N} \Big[& q_P \Big(\frac{P_{HP,k} - P^{SP}}{0.05} \Big)^2 + q_L \Big(\frac{L_k - L^{SP}}{3} \Big)^2 \\
& + q_O \Big(\frac{x_{O_2,k} - x_{O_2}^{SP}}{0.05} \Big)^2 \Big] + \lambda \sum_{j=1}^{12} \sum_{m=1}^{3} \Big(\frac{\Delta u_{m,j}}{2} \Big)^2 \\
& + \sum_{i \in \{O,L,W\}} \left(\rho_s s_i^2 + \mu_s s_i \right)
\end{aligned}
\tag{6.26}
$$

Here pressure is in bar, level, purity and inputs in percent, $q_P = 1$, $q_L = q_O = 0.1$, $\lambda = 3$, $\Delta u_{m,j}$ is the move of input m in block j, and the slacks of the oxygen, level and power bands, in units of 0.05 %, 3 % and 10 kW, are priced at $\rho_s = 10^4$ and $\mu_s = 10^3$, so a reachable band costs nothing. The tracked purity is the model's true x_{O_2}, and its low weight makes the target a preference while the specification is a limit, as shown in Equation (6.27).

$$
\begin{aligned}
& 45 \le u_{GV} \le \min(110, u_{max}), \qquad 0 \le u_{lox} \le 100 \\
& 55 \le \sigma \le 98, \qquad x_{O_2,k} \ge 100\, x_{spec} + 0.01 - s_O \\
& L_{lo} + 4 - s_L \le L_k \le L_{hi} - 4 + s_L \\
& W_{MAC,k} \le W_{max} - 1 + s_W, \qquad s_O,\ s_L,\ s_W \ge 0
\end{aligned}
\tag{6.27}
$$

where u_{max} is the cap of (6.24), $x_{spec} = 0.995$, $L_{lo} = 25\,\%$ and $L_{hi} = 80\,\%$ are the level alarms and $W_{max} = 752$ kW, all read from the run. The input rows are hard, the output rows soft and judged at every second step, and a band already broken when a plan starts is regained along a first-order lag of 60 min (5 min for the power). The back-offs of 0.01 % and 1 kW keep a plan that rides a limit from crossing it on its estimate's error, and without the kilowatt a diagnostic copy counts 754.6 min of scenario 7 above the rating. Energy carries no price, because with the pressure on its setpoint the vane has no steady-state freedom and a price on kilowatts could buy only a pressure offset.

A rejected plan, not optimal, not finite or outside its bounds, hands the three handles without a bump to the full cascade loops, which track the applied commands while the plan acts. As Section 1.6.3 describes, the model never reads the run's parameters or the plant's states, so a learner's change to the plant reaches it only as a disturbance for the filter. A dual active-set method [24] solves a plan in a mean of 2.06 ms in a browser, 0.21 percent of one core at real time, and in 7.88 ms at a horizon of 480 min.

6.6 Results

The application carries fourteen scenarios arranged as a laboratory sequence, each of which loads a starting mode, a control architecture, eleven tuning constants, a set of disturbance positions and its own timed events. In order they are four handles on one plant (1), an LP-column valve that raises the HP-column pressure (2), the pressure loop as the duty loop (3), twenty cubic meters of memory in the sump (4), purity, recovery and liquid nitrogen on one knob (5), full cascade held steady (6), a

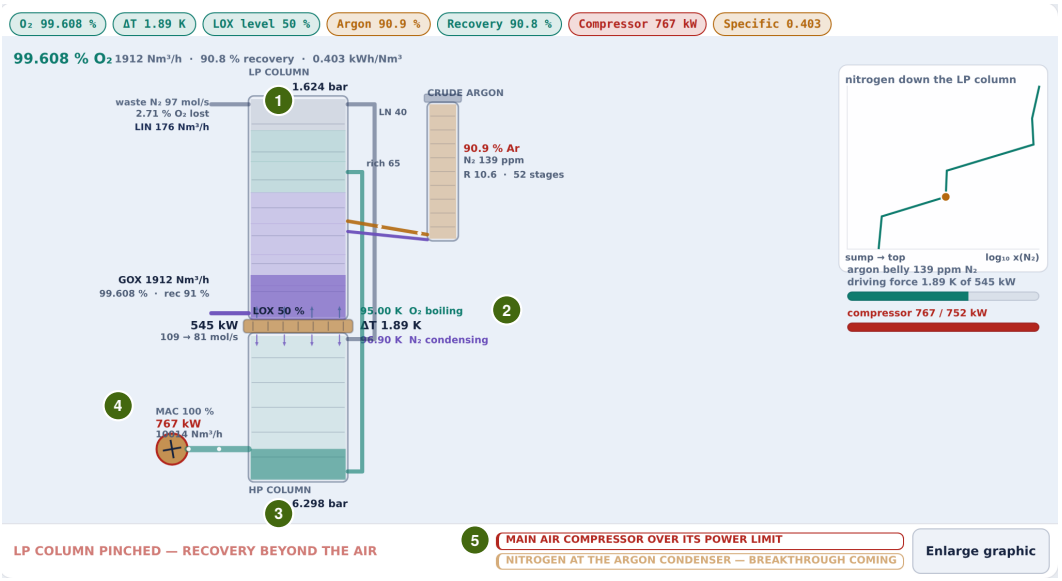


Figure 6.3 Scenario 2 at $t = 120$ min, eighty minutes after the waste nitrogen valve on top of the low-pressure column was pinched to 78 %, in MANUAL with only the high-pressure sump loop in service. (1) The low-pressure column sits at 1.624 bar. (2) Both saturation temperatures at the two ends of the condenser-reboiler are translated upward, 95.00 K boiling against 96.90 K condensing, with the gap between them still 1.89 K. (3) The high-pressure column has arrived at 6.298 bar. (4) The guide vane has not moved from 100 %, and the compressor takes 767 kW against its 752 kW rating for the same 1912 m^3/h of oxygen. (5) The limit banner names the overload.

warm afternoon and the power limit (7), the shared exchanger fouling (8), the argon lost and found (9), nitrogen at the argon condenser (10), the cold box warming (11), the last two percent of recovery (12), a purity under full cascade and under model predictive control (13) and the graded challenge, a shift on a badly lined-up plant (14). Each is credited by a completion rule of the kind described in Section 1.6.4, and the challenge is judged on the case as given, so changing a model parameter or a disturbance voids the attempt. Integrated absolute error, overshoot, settling time, actuator travel and saturated fraction appear on the scorecard for each loop, and the integrated error and the travel keep accruing after a transient has ended. Beside the loops, the scorecard carries the oxygen and argon purities and recoveries, the driving force, the specific power, the compressor power, the minutes off specification and the margin. One hundred and forty-five columns leave the historian, among them the two saturation temperatures, the driving force, the duty, the argon in the liquid oxygen, the belly nitrogen and the running specific power. An integrated error from a plant whose slowest loop has a 278 min time constant is meaningless without the window it was taken over, so every comparison below is made between two settled conditions, at a named instant, or as an elapsed time to a named fraction of the total move.

6.6.1 Open-Loop Moves and Settled Comparisons

Scenario 1 makes the pairing argument with four open-loop moves, each held for an hour from the design line-up with only the high-pressure sump loop in service, and each measured from the controller scan at which it took effect. Within the hour five percent more guide vane is 0.157 bar in the high-pressure column, 47 kW at the compressor and 4.06 mol/s more boil-up in a column the vane does not enter, but only 25 ppm off the oxygen purity, which keeps falling to 917 ppm off at 12 h and 968 ppm at 24 h. Six points more liquid oxygen valve is 2117 m^3/h one minute later and 2130 an hour

later, after which the level falls at 0.69 % per hour and the purity creeps up only 173 ppm in 12 h, so what that valve really moves is inventory. With the product valve fixed, one percent more reflux split leaves the production at 1912 m³/h and takes 41 ppm off the purity in the hour and 103 ppm in a day, against the 70 ppm of the level-held steady state. One percent more argon valve moves the oxygen purity only 23 ppm in the hour but 370 ppm by 12 h, and takes 0.52 points off the crude argon in 12 h and 0.60 in a day. The strongest handle on the purity is therefore also the slowest to show it. Table 6.5 collects the closed-loop results.

Table 6.5 Measured Results. Each Block Compares the Condition Before a Move With the Settled Condition After It, Unless the Block Names an Instant.

Quantity	Before	After	#
Waste valve 100 to 78 %, MANUAL			
P_{LP} (bar)	1.400	1.626	2
P_{HP} (bar)	5.600	6.303	2
ΔT (K)	1.894	1.894	2
W_{MAC} (kW)	722.0	767.5	2
Exchanger conductance $1.00 \rightarrow 0.90 \rightarrow 0.78$*, inner loops*			
ΔT (K)	1.894	2.253	8
Guide vane (%)	100.0	92.8	8
Oxygen (m³/h)	1912	1762	8
W_{MAC} (kW)	722	670	8
w_{spec} (kW h/m³)	0.379	0.381	8
Then the setpoint raised to 5.780 bar			
ΔT (K)	2.253	2.386	8
Guide vane (%)	92.8	98.3	8
Oxygen (m³/h)	1762	1874	8
W_{MAC} (kW)	670	722	8
Heat leak 0 to 45 kW, full cascade, at 1000 min			
Oxygen purity (%)	99.622	99.763	11
Oxygen (m³/h)	1912	1295	11
Recovery (%)	90.8	64.9	11
w_{spec} (kW h/m³)	0.379	0.531	11
Argon valve 40 to 58 %, inner loops, at the argon minimum			
Sidearm reflux ratio	10.58	6.72	9
Crude argon (%)	96.02	68.8	9
Oxygen purity (%)	99.622	99.931	9
Argon in the product (ppm)	3778	690	9

6.6.2 A Failure That Holds Every Setpoint

Scenario 8 fouls the shared exchanger to 0.90 and then 0.78 with the pressure and level loops in service. The driving force widens from 1.894 to 2.253 K because the pressure controller allows it, but $Q = UA\,c_f\,\Delta T$ and the conductance has fallen further than the driving force has risen, so the condenser takes less nitrogen away and the controller holds 5.600 bar by *closing* the vane from 100.0 to 92.8 %. Oxygen production falls from 1912 to 1762 m³/h, 7.8 % of the plant, the compressor power goes *down* from 722 to 670 kW, specific power moves only from 0.379 to 0.381 kW h/m³ and the purity rises to 99.75 % by 1400 min because there is less to separate. Every controlled variable is on setpoint, nothing alarms, and the only trend that says the plant has become eight percent smaller is an exchanger temperature difference of two kelvin.

🕮 **Pause.** Try Exercise 6.2 before reading on.

Raising the pressure setpoint to 5.780 bar is what an operator watching that trend would have done hours earlier, and it widens the driving force to 2.386 K, brings the vane back to 98.3 % and the production to 1874 m³/h at 722 kW. That is where the bill arrives, because the same 52 kilowatts the fouling saved now buy back production that used to be free, which at 0.062 $/kWh is about twenty-eight thousand dollars a year.

TRY IT IN THE APP **8 · The Shared Exchanger Fouls, and the Plant Quietly Unloads**

Load scenario 8 and press Run. The exchanger fouls at 60 and 600 min. Watch the driving force widen from 1.894 to 2.253 K, the guide vane close from 100.0 to 92.8 % and the oxygen fall from 1912 to 1762 m^3/h with nothing alarming. At 1400 min the pressure setpoint goes to 5.780 bar and buys the production back to 1874 m^3/h.

6.6.3 Purity Is Not Performance

Scenario 11 puts 25 kW and then 45 kW of heat leak into the low-pressure sump in full cascade. Through (6.1) the 45 kW boils 6.7 mol/s more liquid oxygen and condenses nothing anywhere, which is the last term of (6.3) acting alone, so those moles leave with the waste nitrogen instead of as product. By 1000 min production has fallen from 1912 to 1295 m^3/h, recovery from 90.8 to 64.9 % and specific power has risen from 0.379 to 0.531 kW h/m^3, while the purity analyzer has moved from 99.622 to 99.763 % because what is left in the sump is cleaner. The purity loop cannot pull it back, because it has held the reflux split against its 98 % stop since 218 min, and production recovers only to 1373 m^3/h at 68.8 % recovery by 2880 min, once the level loop has refilled the sump. The two measurements that would have caught it within minutes, recovery and specific power, are not controlled variables.

Scenario 12 is the other blind spot. At a 98 % reflux split the plant already recovers 95.7 % of the oxygen in the air, and cutting the expansion turbine from 12 to 3 % sends more air to the high-pressure column and pushes for the rest. It does not arrive, and Equation (6.14) is the reason, because a column cannot send more of a component out of the bottom than came in at the feed, so when W_4 crosses zero the section above the crude feed runs out of heavy key to send upward. Its profile pinches, nitrogen floods down to the argon belly to a peak of 17 700 ppm, the crude argon falls from 95.3 to 38.7 % and the oxygen purity follows it to 98.81 %, bottoming 3.4 h behind the argon and 4.8 h *after* the turbine was put back. Both failures share a shape worth carrying away, because on a plant with several products any single quality measurement can be improved by making less of the product.

DID YOU KNOW? **A Warmer Cold Box, a Purer Product**

From the simulator: with 45 kW of heat leak under full cascade, the oxygen purity at 1000 min has risen from 99.622 to 99.763 %, while the production has fallen from 1912 to 1295 m^3/h and the recovery from 90.8 to 64.9 % (Table 6.5). The number the product is sold on improves as the product disappears.

6.6.4 Inventory and the Argon

Scenario 4 starts the liquid oxygen sump at 42 % instead of 50. The level controller closes the product valve by twenty-four points at once, from 52.7 to 28.7 %, and the oxygen production drops from 1912 to 1042 m^3/h with it. The level then returns on the inventory's own clock, reaching 44.6 % at one hour, 46.7 % at two, setpoint at four and a quarter, a peak of 51.8 % at 8.6 h and within 0.2 points of setpoint from 18 h. That is averaging level control working as designed, because the alternative throws the whole flow imbalance into a product line whose customer is a steel plant.

Scenario 9 opens the crude argon draw from 40 to 58 %, and the crude argon reaches a minimum of 68.8 % five hours later, then climbs back to 75.6 % by the time the valve closes at 600 min, close to the 75.7 % it would line out at. Argon recovery reads 120.5 % three minutes after the draw opens and 119 % an hour later, which is not a fault, because the sidearm is taking argon out faster than the air brings it in and the difference is the belly emptying. Coming back is slower than going, at 84.7 % three hours after the valve is closed, 93.5 % after nine and 95.5 % after fourteen, still short of where it started. Scenario 10 blankets the sidearm condenser instead by pinching the waste valve with a heavy belly draw, which takes the belly nitrogen from 267 to a peak of 3375 ppm 23 min after the pinch and settles it at 344 ppm over the 320 ppm threshold, collapsing the sidearm reflux ratio from 9.85 to 0.001 and the crude argon to a minimum of 79.9 % 67 min after the pinch.

6.6.5 What the Constraint Controller Decides

Scenario 7 warms the compressor suction from 30 to 42 °C. Guide vanes pass a volume, so the same opening passes four percent fewer moles, and the isothermal work of (6.6) is four percent higher because it is proportional to suction temperature, so holding 5.600 bar takes the vane from 100.0 to 104.0 % and the power from 722 to 751 kW, already touching the rating. Raising the pressure setpoint to 5.720 bar for more oxygen is the interesting half. On inner loops the pressure controller takes what it needs, driving the vane to 108.0 % and the compressor to 788 kW, thirty-six over the rating and continuously, while in full cascade Equation (6.24) caps the vane at 104.1 % and the trend sits at 5.605 bar against 5.720 bar for the rest of the afternoon. The cap moves at the slow rate the vane allows, so the request itself still takes the compressor to 786 kW one minute later and keeps it more than half a kilowatt above the rating for 9 min before the vane is held. A constraint controller does not solve the constraint, it decides which variable gives way, and here it is the duty and therefore the production, because the alternative is a tripped motor.

Cooling the suction to 28 °C in the evening lets the same 752 kW buy more moles, the cap relaxes to 103.0 % and the setpoint is met at 5.715 bar. What arrives is duty rather than oxygen, because boil-up rises from 81.5 to 84.5 mol/s and the purity controller spends every mole of it on liquid nitrogen, taking the reflux split from 95.0 to 92.7 % and the liquid nitrogen from 176 to 267 m³/h while the oxygen settles at 1898 m³/h, below the 1912 it started at. More separation is not more oxygen unless somebody decides it is.

6.6.6 The Graded Challenge

Scenario 14 hands over a plant that came off the night shift with the crude argon valve at 33 %, weak tunings loaded and the exchanger already at 0.88, and then warms the suction to 39 °C at 90 min, fouls the exchanger to 0.74 at 300 min and cools the suction again at 520 min, in a run judged over twenty hours. Every attempt starts from this prescribed handover state. Learner valve settings made before Run are actuator requests applied when control begins, rather than inputs to a new steady starting plant. Reset keeps those requests and restores the same handover state. The case passes when the oxygen stays within ±0.06 % of 99.62 % and the high-pressure column within ±0.05 bar of 5.60 bar over the last five hours, with the crude argon at or above its 93 % specification and the compressor at or below 780 kW throughout. At handover the oxygen reads 99.310 %, off the 99.5 % sales specification, while the crude argon analyzer reads a comfortable 98.32 % and the argon recovery is 70.3 %. Almost all of it is the argon valve, because at 33 % the sidearm draws 0.83 mol/s against 1.14 mol/s arriving, so 30 % of the argon in the air escapes the crude product, half of it into the liquid oxygen at 6902 ppm. Left as shipped, the plant still has the oxygen 0.191 % short of its setpoint during the last five hours and fails, against the 0.06 % allowed, while the other three terms pass, with the high-pressure column within 0.024 bar, the crude argon at least 98.13 % and a compressor peak of 727 kW.

The suggested configuration is full cascade with the tuned settings, the manual valves left at the handover positions. The argon loop opens the valve from 33 to 40 % within the hour and to a peak of 43.9 %, the oxygen is back on specification at 99.50 % after 159 min, and the argon recovery is 83.8 % at the end of the twenty hours with the crude argon never below 94.5 %. Holding 5.600 bar through a quarter of the exchanger’s conductance means closing the vane from 99.4 % two hours in to 90.5 %, so the power falls from 730 to 654 kW and never approaches the 780 kW the target forbids, and what the plant gives up instead is production, 1944 down to 1837 m³/h at the end of the run after a trough of 1544 m³/h at 400 min. The case passes, with the oxygen within 0.049 %, the high-pressure column within 0.000 03 bar, the crude argon at least 94.52 % and a compressor peak after the handover instant of 722 kW. Buying the duty back with the pressure setpoint is the wrong answer, because at 5.85 bar the compressor peaks at 786 kW and (6.24) is far too slow to catch it.

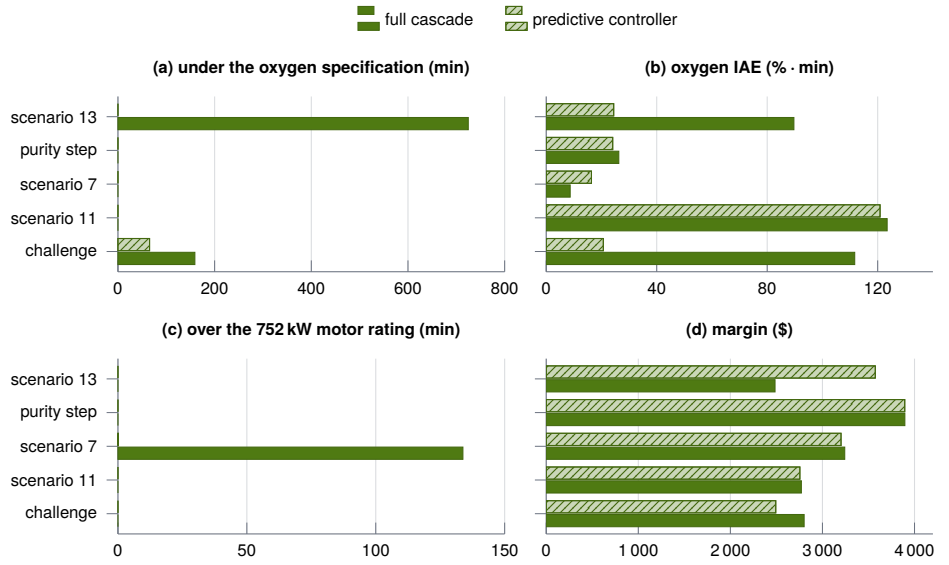


Figure 6.4 Four columns of Table 6.6 as bars, full cascade dark and the predictive controller hatched. Notice that the plan wins where time under the specification or over the motor rating is counted, in scenario 13, scenario 7 and the challenge, while the oxygen IAE of scenario 7 and the margins of the step and the heat leak barely move or go the other way.

6.6.7 Comparison With Model Predictive Control

Table 6.6 runs full cascade, with its shipped tuning and the suggested line-up on the challenge, against the predictive controller with the settings of scenario 13 on every case and only the architecture changed. Both face the same events, instruments and duration without noise, neither receives a preview, and the purity step is a scripted event on the line-up of scenario 6 because no shipped scenario contains one. Figure 6.4 draws four columns of the table as bars.

Scenario 13 lowers the oxygen target from 99.62 to 99.55 % at 60 min, five hundredths above the sales specification, and raises the crude argon target from 96 to 97.5 % at 480 min. Both structures answer the first step by driving the split to its 98 % stop. The argon request then closes the argon valve from 38.4 to 33.8 % within the hour and leaves more argon in the liquid oxygen. Under full cascade the purity loop, on its 200 min reset, brings the split off its stop only as the error accumulates, to 97.1 % at 600 min and 91.1 at 900 min, so the oxygen is under specification from 663.0 to 1387.5 min, at lowest 99.441 %, and 21 400 m^3 of it is vented. The plan lowers the split as the sag begins, to 94.3 % at 540 min, 90.1 at 600 min and 81.0 at 961 min, and the purity never falls to the specification, its lowest value being 99.545 %, as shown in Figure 6.5. The margin is $3574 against $2485, paid for with 25.2 and 25.5 percent of split and product valve travel against 14.7 and 16.6.

The oxygen floor never binds on this case. With the floor switched off every logged variable except the slack, the solver iterations and the solve time is identical, and with it on the largest oxygen slack is 4.3×10^{-19} percent, so the plan's tracking of the 99.55 % target holds the specification. The floor binds only on the challenge, which starts at 99.310 %, where it shortens the time under specification from 71.4 to 65.4 min.

The purity step and the heat leak are draws. On the step the oxygen IAE is 24.07 against 26.27 % · min and the margin $3894 against $3895, with a pressure IAE of 0.228 against 0.007 bar · min. On scenario 11 the oxygen IAE is 120.86 against 123.42, the level IAE halves from 1518 to 725 % · min and the pressure IAE rises from 0.535 to 1.790 bar · min.

Scenario 7 is a loss. The plan holds the motor rating, with a peak of 752.0 kW against 786.4

Table 6.6 Full Cascade Against Model Predictive Control

Structure	Under spec (min)	Lowest x_{O_2} (%)	IAE x_{O_2} (% min)	IAE P_{HP} (bar min)	IAE L (% min)	Travel u_{GV}, u_{lox}, σ (%)	Peak W_{MAC} (kW)	Over rating (min)	Margin ($)	Plans, fall-backs
Scenario 13 (1440 min)										
full cascade	724.7	99.441	89.66	0.024	1122	0.3, 16.6, 14.7	722.5	0.0	2485	—
predictive	0.0	99.545	24.43	0.204	218	0.3, 25.5, 25.2	722.5	0.0	3574	1440, 0
Purity step (1440 min)										
full cascade	0.0	99.544	26.27	0.007	319	0.1, 4.7, 3.9	722.0	0.0	3895	—
predictive	0.0	99.549	24.07	0.228	153	0.1, 6.5, 5.1	722.1	0.0	3894	1440, 0
Scenario 7 (1200 min)										
full cascade	0.0	99.591	8.67	60.269	90	17.9, 2.4, 2.4	786.4	133.8	3242	—
predictive	0.0	99.578	16.38	63.614	244	5.6, 21.8, 16.6	752.0	0.1	3202	1200, 0
Scenario 11 (1200 min)										
full cascade	0.0	99.622	123.42	0.535	1518	7.6, 17.9, 2.9	722.0	0.0	2771	—
predictive	0.0	99.621	120.86	1.790	725	6.7, 29.1, 3.0	722.2	0.0	2755	1200, 0
Challenge (1200 min), full cascade on the suggested line-up										
full cascade	159.1	99.310	111.68	2.038	1218	25.9, 21.5, 14.3	724.6	0.0	2801	—
predictive	65.4	99.310	20.67	1.743	286	25.4, 75.7, 71.6	731.7	0.0	2493	1200, 0

Errors, travel and the minutes of true purity under the 99.5 % specification accumulate from $t = 0$ as on the scorecard, and the challenge's lowest purity is its starting value. Over rating counts minutes above 752 kW. Margin is the value of on-specification oxygen, crude argon and liquid nitrogen at the prices of Table 6.3 less the electricity. The purity step lowers the oxygen target from 99.62 to 99.55 % at 60 min.

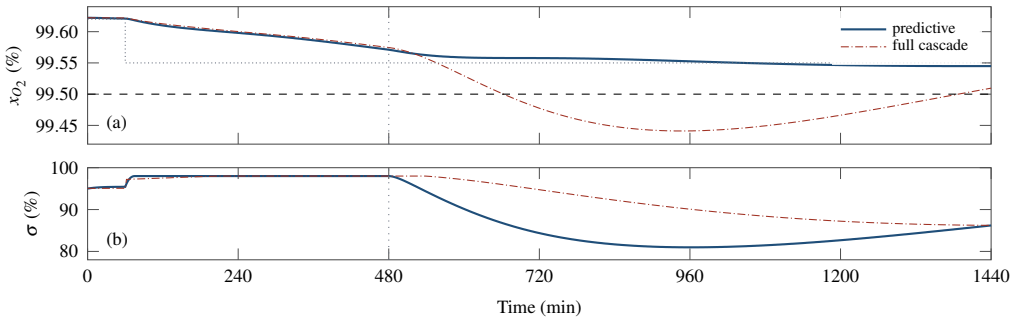


Figure 6.5 Scenario 13 under the predictive controller (thick solid line) and under full cascade (dash-dotted line). (a) True oxygen purity against the 99.5 % specification (dashed) and the target (dotted). (b) Reflux split. The vertical dotted line marks the argon request at 480 min.

and 0.1 min above it against 133.8, but its oxygen IAE is 16.38 against 8.67 % · min, its level IAE 244 against 90, its product valve and split travel 21.8 and 16.6 percent against 2.4 and 2.4, and its margin $3202 against $3242, because the 5.72 bar request is out of reach. Under full cascade the vane is within 0.05 percent of the override's cap on 3598 of 4801 logged rows and under the plan on none, since the plan's own ceiling 1 kW under the rating holds the compressor in 1152 of its 1200 decisions, and the quadratic objective spends the split, at its stop for 10.2 percent of the run, and the product valve on the hundredths of a bar they can still buy. A diagnostic copy outside the application that clips the pressure target to the present pressure whenever a plan is made within 0.5 kW of that ceiling reduces the oxygen IAE to 0.77 and the level IAE to 28, which confirms the mechanism. The application has no steady-state target calculation that would remove the loss.

The challenge is reported for information, because its scoring is unchanged. Both structures meet the target, full cascade with the suggested line-up holding the oxygen within 0.049 percent and the plan, with only the architecture switched, within 0.001 percent with a compressor peak of 732 kW.

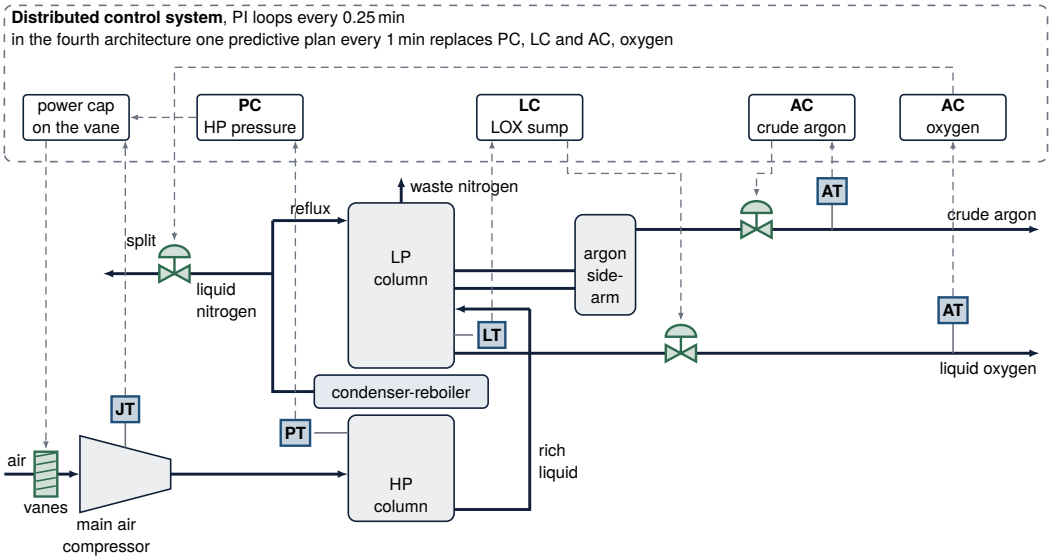


Figure 6.6 The loops of the air separation unit with their hardware. Lettered boxes are sensing devices: the pressure transmitter (PT) on the high-pressure column, the level transmitter (LT) on the low-pressure sump, the oxygen analyzer (AT) on the liquid oxygen, the chromatograph (AT) on the crude argon and the power transducer (JT) of the compressor motor. The vanes and the valve symbols are final elements, dashed arrows are signals. Notice that the pressure loop acts through the compressor's vanes, that the power cap limits the same vanes, and that the oxygen loop reaches across the plant to the split on the liquid nitrogen. The high-pressure sump loop on the rich liquid is left out. Not to scale, and not a piping diagram.

Unscored, the oxygen inherited at 99.310 % stays under specification for 159.1 min under full cascade and 65.4 min under the plan. No plan was rejected on any case of Table 6.6.

6.7 Technology in Practice

What Measures It

Figure 6.6 draws the five loops of Section 6.5 with their devices. The loop the chapter turns on reads the oxygen purity, and on a real unit that reading comes from an analyzer fed by a sample line. The product here is liquid, and a cryogenic liquid sample has to be taken so that it does not partly vaporize, which would misread heavy components such as hydrocarbons [19]. The pressure and the two levels are read by ordinary transmitters, and the level of the low-pressure sump matters for safety as well as for the product flow, because control of the reboiler's level is important in preventing dry boiling [19].

TECHNOLOGY Sensor: A Paramagnetic Oxygen Analyzer

A paramagnetic analyzer is selective because no other industrial gas is nearly as paramagnetic as oxygen, although nitric oxide interferes, and one that reads above the oxygen of ambient air is spanned with pure oxygen [25]. In the classic cell two nitrogen-filled glass spheres hang on a quartz thread in the field of a permanent magnet, and oxygen drawn into the field turns them out of equilibrium. Such cells answer in 3 to 10 s, limited by the gas exchange in the cell, need a small and stable sample flow and are sensitive to vibration [26]. A sample line that plugs leaves an analyzer reading old, stale sample with no way of knowing it [25]. The model's 6.0 min lag stands for the analyzer and its sample line together.

What Moves It

The pressure loop's final element is not a valve. It is the main air compressor's guide vane, which through the shared exchanger sets the boil-up of a column the air never enters (Section 6.4). The other handles are cryogenic valves on the product, the argon draw and the rich liquid, each lagging its command as in Equation (6.20), and the reflux split, which the model applies without a lag. The compressor power override writes a high limit on the vane rather than a second controller into a selector.

TECHNOLOGY Final Element: Compressor Inlet Guide Vanes

The main air compressor of an air separation unit delivers between 4 and 10 atm. On a centrifugal machine the capacity is typically set by variable inlet guide vanes on at least the first stage, which change it more efficiently than a throttling valve on the suction, and on an axial machine by rows of variable stator blades, whose actuating mechanism can bind from rust and dirt [27]. A control valve with excess friction moves in jerks, and integral action turns that into a cycle that detuning slows but does not remove [25]. Every dynamic compressor also needs an automatic anti-surge system [27]. The model's vane is linear in volume, moves between 45 and 110 % and meets no surge line.

Where the Controller Runs

An industry guideline divides the instrumented systems of an air separation unit into critical safety systems, operational safety systems and routine plant control. The critical systems may share components with the routine controls only if a failure of the routine system cannot compromise them [27]. Routine control runs in a distributed control system, in which a supervisory level sends set points to field controllers, loops may be nested, and cycle times range from milliseconds to minutes [28]. The app executes its five PI loops every 0.25 min and its predictive controller every 1 min, inside that range. The loops and the power override belong to the regulatory layer and the predictive controller to the layer above it. The shutdowns that protect a real reboiler belong to a safety layer the model does not have.

IN PRACTICE Keep the Reboiler Under Liquid

A bath-type main reboiler is kept submerged to the depth its designer specifies, or fully when that is unknown. Below it the liquid stops recirculating and evaporates completely in the passages, concentrating the heavier impurities by extremely high factors, and that accumulation cannot be detected by analyzing the sump. The level shown on the instruments is not the submergence, and level instruments and their alarms are verified at least once a year. A plant that makes little liquid oxygen purges 0.1 to 0.2 percent of its air as liquid, while one that makes more than 2 percent dilutes the contaminants by itself [19]. The chapter's plant sells all of its oxygen as liquid.

What Is Modernizing It

Three developments bear on this plant. Linear model predictive control is already the technology of the air separation industry, and nonlinear predictive control on a reduced compartmental model of a high-purity column has been solved within a 2 min sampling interval in closed-loop simulation [9].

Production scheduled on day-ahead electricity prices, with product stored in the cheap hours, cut the operating cost and the peak power demand of an air separation unit in simulation studies [17]. And computer-based control systems let such plants run unattended, with minimal staff or from a remote site, provided that every response an operator would give at an attended plant is designed into the controls [27].

6.8 Limitations

Nobody should size a column or an exchanger with this model. Lumped sections, constant relative volatilities, an argon tracer and an absent refrigeration balance together rule out any statement about the tray count, recovery, argon yield or turndown of a particular plant. Three lumped sections place every composition the model knows at a section boundary, so nothing can be said about where inside a section a pinch sits or what temperature profile an operator would read, and the argon results are calibrated in magnitude rather than predicted. Refrigeration is the most consequential omission, because on a real unit the expansion turbine, the cold-box heat leak and the liquid production are one balance and that balance decides whether a plant can choose its liquid make at all. Bit-exact repeatability is itself unlike practice. Two of the scenarios are stylized on purpose. Fouling that steps the conductance to 0.90 and then 0.78 in one move is faster than the weeks a real carbon dioxide film takes, but it separates the loss of conductance from every other cause. Cutting the expansion turbine to 3 % would not be done on a real unit, but it is the shortest path to a rectifying section with no heavy key left to send upward. What survived the simplification, one duty shared by two columns, two inventories measured in hours, an argon balance read from both ends and a power limit that binds on a warm afternoon, survived because each of them has to be designed around, and Section 6.3.9 bounds every simplification individually.

The predictive controller’s model is linear at one line-up and at the declared parameter values. It omits every gain that moves with the operating point, such as the pinch near the 98 % split stop, the nitrogen blanketing of the argon condenser, the vane’s moles and kilowatts at another suction temperature and the exchanger conductance after fouling, it holds the argon valve over the horizon although the argon loop moves it, and it has no steady-state target calculation, which is why it loses scenario 7.

6.9 Discussion

The economic argument on an air separation unit is a ratio rather than a rate. Fouling moves the production by eight percent while leaving specific power at 0.381 kW h/m^3, and heat leak moves it to 0.531 kW h/m^3 while the purity analyzer improves, so neither failure is visible on a controlled variable. Fouling shows in the driving force and heat leak in recovery and specific power, which is why studies of air separation operation evaluate candidate structures by steady-state loss rather than by closed-loop response [3, 15]. The same equations carry methods beyond the linear predictive controller of Section 6.5.5, which plans three handles from one design point and leaves the argon to its own loop. Eighteen states with an active compressor constraint, two purity specifications and two large inventories are a natural nonlinear model predictive control problem, which is where the published work on this plant went [8, 9], and a nonlinear controller of that kind with a steady-state target calculation would trade oxygen purity, argon recovery and compressor power directly instead of leaning on a relative gain array of 0.471 to pick a pairing that cannot decouple. The inventories are the second opportunity, because twenty cubic meters of liquid oxygen and a sidearm holding two hundred kilomoles are the storage that makes an air separation unit a demand-response asset [16], and the same equations with an electricity price attached become a scheduling problem. Rebuilding the eighteen equations in the Gekko Optimization Suite, with the nine hidden constants declared as parameters to be estimated [29], is the shortest route to either.

OPEN CHALLENGE Scheduling the Plant on the Price of Power

Electricity is in effect the only paid feedstock of an air separation unit, and products stored as liquid let it make more when power is cheap and less when it is dear [17, 30]. Prices can change from every several hours to every five minutes, the time scale of the plant's own dynamics, so a schedule has to carry a model of them, and such models are large, nonlinear and stiff [17]. Linear models fitted to a year of industrial data served over a scheduling horizon, but their accuracy drifted unpredictably and had to be updated online [30]. Progress would be a schedule that knows how far the sump, the sidearm and the compressor rating let the plant move as the plant changes.

6.10 Conclusion

This chapter studied a double-column cryogenic air separation unit as an interactive laboratory for thermal coupling and the loop interaction that follows from it. The simulator integrates eighteen states covering two column pressures, three inventories, six composition lumps solved by marching each section along its own operating line, two argon tracers, a sidearm relaxing with a 278 min hold-up constant and four actuator positions, all driven by one duty of 546.0 kW across 1.894 K that condenses 109.2 mol/s of nitrogen and boils 81.5 mol/s of oxygen. Pinching a valve on the low-pressure column raises the high-pressure column by 3.11 bar per bar at constant driving force and costs 45.5 kW for the same oxygen. The duty setpoint meets the 752 kW motor rating at 5.699 bar and 84.10 mol/s of boil-up, and the reflux split moves production from 1328 to 2015 m³/h and purity from 99.744 to 99.594 % while the compressor stays within 0.2 kW of 722 kW. The crude argon valve moves the oxygen purity by 378 ppm per percent against the reflux split's 70 ppm, and the relative gain array of the two purity loops returns 0.471 for the pairing the plant is obliged to use. Three failures hold every setpoint, with fouling taking eight percent of the production while the compressor power falls, 45 kW of heat leak taking recovery from 90.8 to 64.9 % while the purity analyzer improves, and a recovery pinch that gives no warning until the belly nitrogen reaches 17 700 ppm. A linear model predictive controller on the vanes, the product valve and the split keeps the oxygen above its specification through a day that full cascade spends 724.7 min under, and loses the oxygen and level comparison when the pressure setpoint lies beyond the motor rating. A fuller model would add a refrigeration balance so that liquid make becomes a constraint rather than a slider, a rigorous third component in place of the argon tracer, and a steady-state target calculation that keeps the predictive controller from pursuing an unreachable setpoint. The app puts a plant on which nothing can be moved alone within reach of anyone with a browser.

KEY POINTS

- One exchanger is the only condenser of the high-pressure column and the only reboiler of the low-pressure column, so nothing on the plant can be moved alone.
- A valve on the low-pressure column raises the high-pressure column by 3.11 bar per bar at constant driving force, and the duty setpoint meets the 752 kW motor rating at 5.699 bar.
- The crude argon valve moves the oxygen purity by 378 ppm per percent against the reflux split's 70 ppm, and the relative gain of the pairing the plant uses is 0.471.
- Fouling, heat leak and a recovery pinch hold every setpoint, and show only in variables nobody controls.

Where else this applies. Volume 6, Chapter 4 heats units with one another's vapor, and Chapter 8 shares one condenser and one reboiler between two column sections.

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6.11 Exercises

6.1 Concept check · 2 min

In scenario 2 the waste nitrogen valve on top of the low-pressure column is pinched to 78 % with every handle in Manual, and the high-pressure column rises from 5.600 to 6.303 bar. No pipe joins the two column tops. Why does the high-pressure column rise?

(a) Waste nitrogen backs up through the main exchanger into the high-pressure column

(b) The pressure loop opens the guide vanes to keep the oxygen production up

(c) The condenser-reboiler duty rises, so more nitrogen condenses at a higher pressure

(d) The sump oxygen boils hotter, and the same duty needs the same driving force

6.2 Concept check · 1 min

In scenario 8 the condenser-reboiler loses 22 % of its conductance while the pressure and level loops hold their setpoints. What does the main air compressor power do?

(a) It falls, because the pressure loop closes the vanes as less nitrogen condenses

(b) It rises, because the pressure loop opens the vanes to restore the lost duty

(c) It stays at 722 kW, because the pressure and so the pressure ratio are held

(d) It rises, because the wider driving force needs a higher column pressure

6.3 Concept check · 2 min

At steady state with the sump level held, which handle moves the oxygen purity most per percent?

(a) The reflux split, the handle the oxygen purity loop is paired with

(b) The crude argon draw valve, since the impurity left in the product is argon

(c) The guide vanes, because they set the boil-up of the low-pressure column

(d) The liquid oxygen valve, because it sets how fast the product leaves the sump

6.4 Theory and calculation · 8 min

Equation (6.3) gives the oxygen product per mole of air fed to the high-pressure column. At design $F_{HP} = 109.21\,\text{mol/s}$, $f_{LN} = 0.400$, $\psi_R = 0.0250$, and one mol/s of product is 80.69 m³/h at normal conditions.

(a) The five terms are 0.37052, 0.49200, 0.10073, −0.74627 and zero. Compute P_{LOX}/F_{HP} and P_{LOX}, and say where the fourth term comes from. *Checkpoint:* 0.21698 and 23.70 mol/s; $-5000/6700$.

(b) Hold every other term and raise the reflux split by one percentage point. Compute the gain in oxygen product in mol/s and in m³/h, and the oxygen lost for each mole of liquid nitrogen sold instead of returned. Give three significant figures. *Checkpoint:* 0.426 mol/s, 34.4 m³/h; 0.975 of a mole.

(c) Compute the last term and the product lost for 45 kW of cold-box heat leak, in mol/s and m³/h. *Checkpoint:* 0.0615; 6.72 mol/s, 542 m³/h.

6.5 Theory and calculation · 6 min

Equation (6.22) gives the steady-state gains of the two purities on the reflux split and the argon valve: $K_{11} = -0.0070$, $K_{12} = 0.0378$, $K_{21} = -0.1151$ and $K_{22} = -0.5536$ (%/%).

(a) Compute $\lambda_{11} = 1/(1 - K_{12}K_{21}/(K_{11}K_{22}))$ and λ_{12} to three decimals. *Checkpoint:* 0.471 and 0.529.

(b) Compute the gain of the reflux split on the oxygen purity when the argon loop is closed and holds the crude argon purity, and its ratio to K_{11}. Give three significant figures. *Checkpoint:* –0.0149 %/%, 2.12 times.

(c) Say what (b) means for tuning the oxygen purity loop with the argon loop in Manual and then in Auto. *Checkpoint:* The loop gain doubles when the argon loop closes.

6.6 Theory and calculation · 8 min

The liquid oxygen sump holds 1 400 000 mol at 100 % level, and the product valve passes $C_{lox} = 45.0$ mol/s fully open (Equation (6.11)). The level loop has action -1 (direct acting) and the shipped tuning of Table 6.4.

(a) Compute the integrating gain of the level on the valve, in percent of level per minute per percent of valve, to three significant figures. *Checkpoint:* 0.001 93 %/min per percent.

(b) Scenario 4 starts the sump at 42 % against a 50 % setpoint with the valve at 52.66 %, and the integral is seeded so that the first execution asks for that position plus the proportional term. Use Equation (6.23) to find the valve it asks for at $K_c = 3$ and at $K_c = 9$. *Checkpoint:* 28.66 %; -19.34, held at 0 %.

(c) Ignore the integral. Compute the closed-loop time constant $1/(K_cK)$ for both gains, in minutes, and the level after 60 min at $K_c = 3$, to one decimal. *Checkpoint:* 173 min and 58 min; 44.3 %.

6.7 App challenge · 8 min

Load scenario 2, An LP-column valve raises the HP-column pressure (MANUAL), and change no setting. The waste nitrogen valve is pinched to 78 % at 40 min. Use the default seed, run at 20 min/s to 215 min and export the CSV file. Its time column is in hours.

(a) In the rows at 0.5 and 3.5 h, read `hp_bar`, `lp_bar`, `Tcond_K`, `Treb_K` and `mac_kW`. Accept 0.002 bar, 0.02 K and 0.2 kW. *Checkpoint:* 5.600, 1.400, 95.36, 93.47 and 722.0 kW; 6.303, 1.626, 96.91, 95.01 and 767.5 kW.

(b) Compute the rise of the high-pressure column per bar of low-pressure column and the driving force before and after. Compare the ratio with the 2.87 that Equation (1.9) gives. *Checkpoint:* 3.11; 1.894 K both times.

(c) The vane stays at 100 %, so $F_{air} = 124.1$ mol/s. With Equation (6.6), $T_{suc} = 303.15$ K, $\eta_{MAC} = 0.767$, $\Delta P_{mac} = 0.35$ bar, $P_{amb} = 1.013$ bar and $R = 8.314$ J/(mol K), compute W_{MAC} at both high-pressure column pressures. *Checkpoint:* 722.0 and 767.5 kW, 45.5 kW apart.

6.8 App challenge · 8 min

Load scenario 8, The shared exchanger fouls, and the plant quietly unloads, and change no setting. The conductance falls to 0.90 at 60 min and to 0.78 at 600 min, and the pressure setpoint goes to 5.78 bar at 1400 min. Use the default seed, run at 60 min/s to 1700 min and export the CSV file. Its time column is in hours.

(a) In the rows at 0.5 and 23.3 h, read the guide vane, the oxygen, the compressor, the driving force and the specific power, columns `gv_pct`, `gox_Nm3h`, `mac_kW`, `dTcr_K` and `spec_power_`

`kWh_Nm3`. Accept 0.2 %, 3 m³/h, 0.5 kW, 0.005 K and 0.001. *Checkpoint:* 100.0, 1912, 722.0, 1.894, 0.379; 92.8, 1762, 669.8, 2.253, 0.381.

(b) In the row at 28.3 h, read the same columns. Accept 0.2 %, 5 m³/h, 0.5 kW, 0.005 K and 0.001. Compute the power the setpoint move costs and its price over a year of 8760 h at 0.062 $/kWh. Accept $600. *Checkpoint:* 98.3, 1877, 721.4, 2.386, 0.386; 51.6 kW, $28 000.

(c) Say which controlled variable showed the fouling before 1400 min, and which trend did. *Checkpoint:* None; the driving force.

6.9 App challenge · 8 min

Load scenario 4, Twenty cubic meters of memory, and change no setting. The sump starts at 42 % against a 50 % setpoint. Use the default seed, run at 20 min/s to 480 min and export the CSV file. Its time column is in hours.

(a) Read `lox_cmd_pct` in the first row, the lowest `gox_Nm3h`, and `lox_level_pct` at 1 and 8 h. Accept 0.05 %, 2 m³/h and 0.05 %. *Checkpoint:* 28.66, 1042, 44.56 and 51.79.

(b) Pause, press Reset, set LOX level loop gain to 9 before Run, run to 480 min again and export. Read `lox_cmd_pct` in the first row, from which row to which `gox_Nm3h` stays below 1 m³/h (accept 0.01 h), and the level at 1 h. *Checkpoint:* 0; from 0.046 to 0.450 h; 47.37 %.

(c) Compare the two runs as the steel plant that buys the oxygen would. *Checkpoint:* The shipped gain keeps the product flowing.

6.10 Application area · 4 min

A plant manager wants an alarm that would have caught both the exchanger fouling of scenario 8 and the cold-box heat leak of scenario 11 while every loop held its setpoint.

(a) From the chapter's measured values, say what each failure does to the oxygen purity, the production and the specific power. *Checkpoint:* Purity up in both; production down in both; specific power flat for fouling, up for heat leak.

(b) Propose what to trend and alarm, and say why the oxygen analyzer cannot be one of them. *Checkpoint:* Driving force, recovery and specific power.

6.11 Application area · 4 min

Table 6.3 prices oxygen at 0.055 $ and liquid nitrogen at 0.030 $ per normal cubic meter. At a 95 % reflux split the plant makes 1912 of oxygen and 176 of liquid nitrogen (m³/h), and at 78 % it makes 1328 and 775, with the compressor unchanged.

(a) Compute the hourly value of the two products at each split and the difference. *Checkpoint:* $110.44 and $96.29 an hour; $14.15 less at 78 %.

(b) Find the liquid nitrogen price at which the two splits are worth the same, and relate it to part (b) of the first theory exercise. *Checkpoint:* 0.0536 $ per cubic meter.

(c) Name the limitation of the model that makes selling liquid nitrogen look cheaper here than on a real unit. *Checkpoint:* No refrigeration balance.

6.12 Review the agent · 5 min

An AI assistant proposes a plan for scenario 14, the challenge A shift on a badly lined-up plant: "Press Reveal suggested tuning, then raise the HP column pressure setpoint to 5.85 bar before Run to buy back the duty the fouling takes, and return it to 5.60 bar before 900 min. The compressor power override caps the guide vanes at the 752 kW rating, so the compressor can never reach the 780 kW the target forbids." Find the flaw and say how you would check it in the app.