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CHAPTER 11

Water Distribution Simulator That Schedules Integer Pumps Under a Pressure Floor

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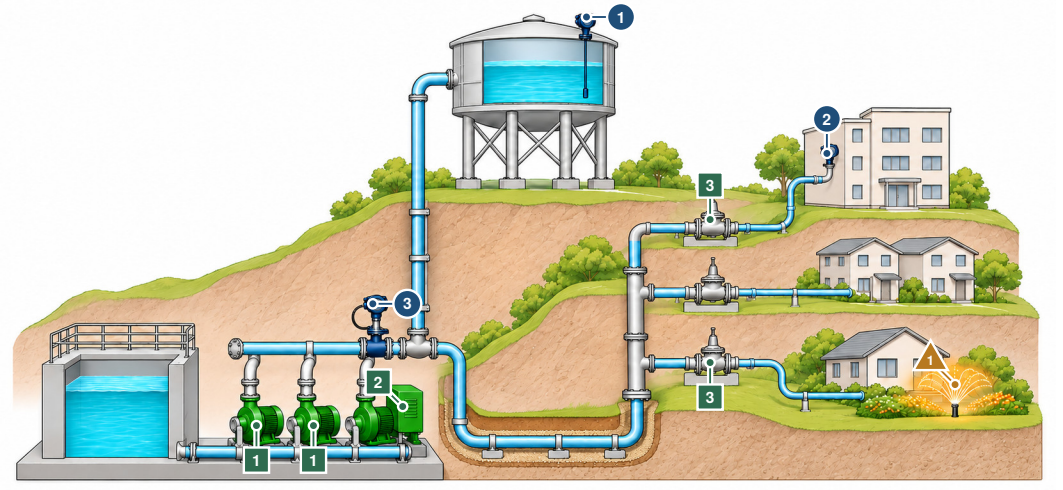
The app for this chapter
https://apmonitor.com/apps/water-network-pumping/

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CHAPTER 11

Water Distribution Simulator That Schedules Integer Pumps Under a Pressure Floor



The controller moves 1 two fixed-speed pumps, on or off 2 pump on a variable-speed drive 3 pressure-reducing valves

The controller reads 1 tank level 2 pressure at the highest customer 3 station flow

What disturbs it 1 customer demand

The pumps lift water to the hilltop tank, which feeds the zones through their valves, and the hour the pumps run decides both the energy bill and the pressure at the highest house. Illustration, not to scale.

IN PLAIN TERMS When the Pumps Run Decides the Bill

The main idea. A town's pumps lift water up to a tank on a hill, and the tank feeds the homes below. Power costs more in the evening, so a planner decides each hour which pumps to run.

The challenge. Each pump is either on or off, so the planner must choose among a huge number of patterns. The tank must stay high enough to give the top house its pressure, but higher pressure pushes more water out through leaks.

Why it matters. The usual rule, starting a pump when the tank runs low, pays far more for power than the planner and still lets the top house lose pressure.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history in Section 11.2, the try-it and fact segments in Sections 11.4 and 11.6, and the key points
Study	45 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 11.6), Technology in Practice (Section 11.7), Limitations (Section 11.8) and the open challenge

The control question A town's pumps are on or off, its tank is the only store, and the same tank level and valve settings decide the pumping bill, the pressure at the highest house and the water lost through leaks. When should the pumps run, and what does an integer schedule re-solved every hour add to a level band and to pressure management?

Learning objectives

- Compute a pump's operating point, power and least working speed from its curve and the affinity laws.
- Explain why a level band starts its pumps at the wrong hour and lets the hill lose its pressure floor first.
- Interpret an LP relaxation, its fractional pumps and the gap that branch and bound proves.
- Measure what a re-solve after a pump trip and modulated valves are each worth on the same day.
- Recognize a floor that no schedule and no valve setting can hold.

Prerequisites Sections 1.4.3, 1.4.6, 1.4.8, 1.4.10, 1.6.2 and 1.6.4, and Chapter 9 for storage bought against a tariff.

The app https://apmonitor.com/apps/water-network-pumping/

See also Volume 8, Chapter 14 for integer servers ordered ahead of a boot delay, and Volume 8, Chapter 12 for on-off compressors staged on one header and the starts they cost.

11.1 Introduction

At midnight a hilltop tank 600 m^2 in plan holds 4.0 m of water on a floor 68 m above the clearwell that feeds it. On the same day, the same town and the same pumps, a rule that starts a pump when the tank falls to 2.0 m and stops it at 6.0 m spends \$241.9 on energy and buys 571 kWh of it at the \$0.20/kWh evening price, and it lets the highest customer fall under a 25 m pressure floor at 06:20, thirteen minutes before the lead pump starts. An hourly mixed-integer schedule spends \$128.1, buys nothing at the peak price and holds the floor. The tank is the same. What differs is when the pumps run.

The electricity for pumping is a large part of the operating cost of a water supply network [1], and scheduling it against a tariff has been framed as an optimization problem since the 1970s, solved by dynamic programming [2, 3], linear programming [1] and, more recently, branch and bound over integer pump states [4], and reviewed as a whole for its models, forecasts and methods [5, 6]. Pressure completes the picture, since leaks discharge more as the pressure rises [7, 8], and reducing excess pressure through valve settings is an established route to lower losses [9–11]. Predictive and real-time control of urban water networks gathers both in one controller [12, 13]. The broader class is any plant whose actuators are on or off, whose inventory buffers a demand against a time-varying price, and whose losses depend on a state the controller chooses. Unit commitment of generators, chiller and compressor staging and batch scheduling have that shape. The refrigeration pack of Volume 8, Chapter 12 stages two fixed-speed compressors and one on a drive, the same arrangement as the pumps here, and the branch and bound of Land and Doig was written for exactly such discrete programs [14].

The plant targets one belief in particular, that a full tank is a safe tank and that a pump should run whenever the level is low. In this model a full tank at the wrong hour raises the energy bill and the leakage together, while a tank filled at night and drawn through the evening peak holds the floor at a lower cost. However, the literature a student meets first divides the problem in two. Scheduling studies fix the hydraulics and minimize cost [1, 4], pressure management studies fix the pumping and minimize leakage [10], and the public-domain network solver of the U.S. Environmental Protection Agency simulates the hydraulics of a schedule it is given rather than choosing one [15]. This chapter works with a browser simulator of a small town's water system in which the integer schedule, the valves, the pressure floor and the pressure-dependent leakage are coupled in one run. It measures what a level band, an hourly branch and bound, modulated valves and a re-solve after a pump trip achieve on the same day. The steady network equations, the leakage law, the linear program, the branch and

bound and the proportional-integral loops are all established methods. What the simulator adds for study is a quasi-static network whose node and tank balances close to the integrator's precision, a branch and bound on the framework's shared simplex that reports its relaxation, tree and gap to the learner, and a strict split between instruments and truth, so that no controller or estimator reads the simulated state. Ten cases with completion rules run from pumps worked by hand to a two-pump loss that no schedule can survive and a hot-day challenge.

Section 11.2 traces the domain. Section 11.3 sets out the network, its instruments and its parameters, and Section 11.4 shows why the hour a pump runs decides both the hill's pressure and the leakage. Section 11.5 describes the level band, the hourly schedule and its branch and bound, the flow and valve loops and the two estimators. Section 11.6 measures the ten cases and the challenge and verifies the code, Section 11.7 describes the instruments, pumps, valves and control platforms of real networks, Section 11.8 states the limitations, and the exercises in Section 11.10 run the cases.

11.2 From Pumping Costs to Pressure Management

A dynamic programming solution for the pumping costs of a water supply network was published by 1975 [2], and progressive optimality, an iterative dynamic program over the hours of the day and the subsystems of a network, scheduled eight reservoirs and seven pumping stations against a 24-hour demand forecast and a varying energy cost in 1989 [3]. Linear programming entered with a 24-hour minimum-cost schedule that accounted for pump efficiencies, the tariff structure and the network's linearized hydraulics, solved quickly enough for real-time use [1]. A 1994 review classified the methods of the time by system type, hydraulic model, demand model and optimization method, linear, dynamic or nonlinear programming [5]. Branch and bound formulations with piecewise linear component models later found schedules whose performance in a full hydraulic simulation lay within the solver's optimality gap and the uncertainty of the demand forecast [4]. Reviews in 2017 and 2019 collected the operational optimization and real-time control literature that grew from these roots [6, 13], and model predictive control of drinking water and drainage networks was reported as an application of control in 2013 [12].

Leakage followed a separate path. Network models that included pressure- dependent demand and leakage terms were proposed in 1985 [7], and excess pressure was minimized through valve settings to reduce leakage in 1989 [9]. Optimal settings of flow reduction valves followed in 1998 [10], and open- and closed-loop pressure control for leakage reduction in 2000 [11]. The power law between leak flow and pressure carries an exponent N_1 that is 0.5 for a fixed orifice and rises toward 1.5 when the leak opening itself grows with pressure, the fixed and variable area discharges (FAVAD) concept [16], and an analysis of elastically deforming leaks showed the exponent tending to 0.5 at low and 1.5 at high pressure [8]. The minimum service pressure itself is set by design standards. The recommended standards of a board of state and provincial health and environmental managers ask for at least 20 psi (140 kPa) at ground level at every point under all conditions of flow and a normal working pressure not under 35 psi (240 kPa) [17]. The simulator joins the two paths, because the hour a pump runs decides the tank level, and the tank level and the valves decide both the pressure at the hill and the leakage of the town.

> ⧖ HISTORY **1950: A Board of States Writes the Rules for Tanks and Pumps**
>
> In 1950 a board of state public health managers around the Great Lakes and the upper Mississippi created a Water Supply Committee, with one associate from each member state, to review water works practices, policies and procedures. A province joined in 1978. The committee's report was first published in 1953 and has been revised and republished many times since, as a guide to the design of public water supplies. Its chapter on storage asks that pumps be controlled from tank levels, with the signal sent by telemetry when appreciable head is lost between the source and the tank, and that level indication be provided at a central location. It also asks that the high and low levels of a tank that gives a system its pressure differ by no more than 30 feet [18]. The level band of Section 11.5 is a rule of that kind.

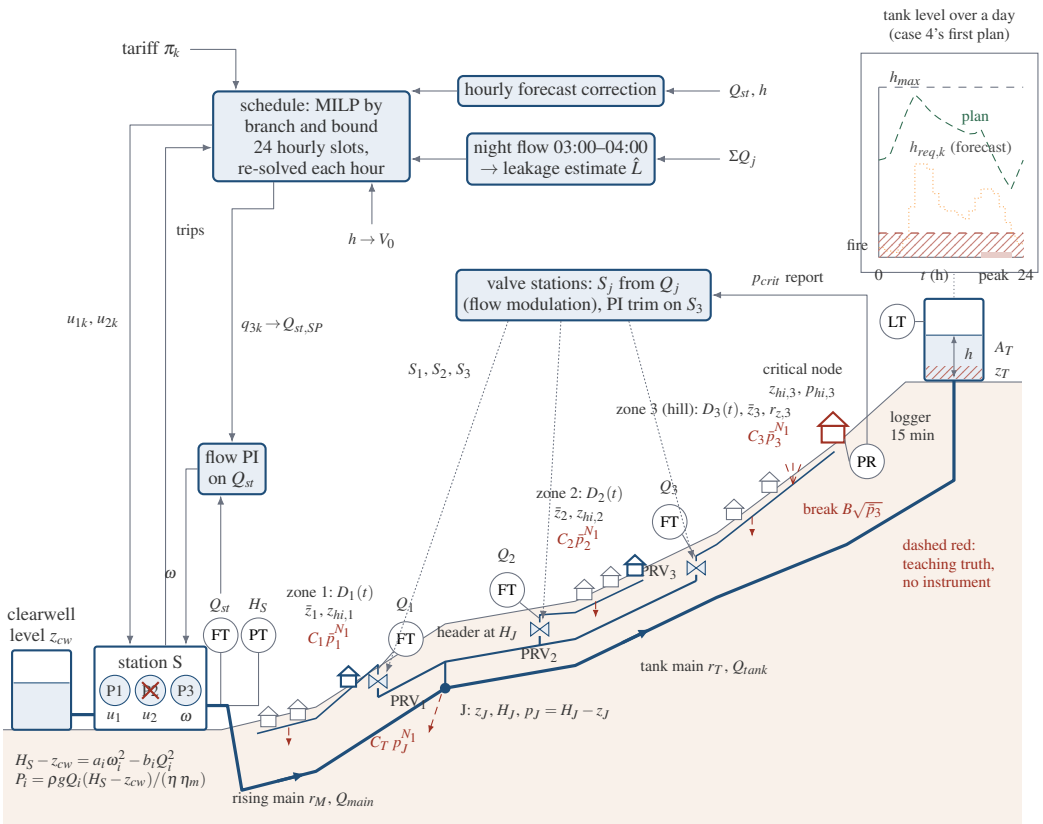


Figure 11.1 The town's water system in elevation, as the Network view draws it, with the control layer above. The station S lifts water from the clearwell at z_{cw} with pumps P1 and P2 on or off (u_1, u_2) and P3 at speed ω, up the rising main (r_M, Q_{main}) to the junction J. From J the tank main (r_T, Q_{tank}) fills the tank of area A_T on the summit, and the header at H_J feeds each zone through a pressure-reducing valve of loss k_v and outlet setting S_j. Each zone draws $D_j(t)$ through a main $r_{z,j}$ and leaks $C_j \bar{p}_j^{N_1}$ at its mean pressure, the trunk leaks $C_T p_J^{N_1}$, and a break in zone 3 discharges $B\sqrt{\bar{p}_3}$. Leakage and the break are teaching truth (dashed red) that no instrument reads. P2 is crossed as it is after the challenge's 18:00 trip. Signals arriving at a block are tagged by the instrument symbols (Q_{st}, h, Q_j) instead of drawn. The inset is case 4's first plan, made at midnight (dashed), against its floor rows $h_{req,k}$ (dotted), a forecast of the level the critical node needs with no pump running, not a measurement. Not to scale.

11.3 The Town as Modeled

The network is a declared teaching town, not any utility's, as shown in Figure 11.1. Water leaves a clearwell held at the treatment plant's level $z_{cw} = 100\,\text{m}$ through a station S of three pumps, two at fixed speed and one on a variable-speed drive, and climbs a 3 km rising main of resistance r_M to a junction J in town at $z_J = 125\,\text{m}$. From J a 1.2 km tank main of resistance r_T runs up to an elevated tank whose floor is at $z_T = 168\,\text{m}$, and a town header carries the junction's head H_J to three pressure-reducing valves, one at each zone. Zone 3 lies on the hill, and its highest customer, at $z_{hi,3} = 142\,\text{m}$, is the critical node at which the pressure floor is judged. Mean customer demand is 57 L/s (25, 20 and 12 L/s by zone). The engine integrates in hours with a step of 10 s.

Each variable of Table 11.1 carries its role. The tank's volume is the only hydraulic state, because the network is solved as steady flow at every step. Nine further states are ledgers integrated beside it

Table 11.1 Variables by Role, With Defaults

Variable	Role	Default	Range
$p_{hi,3}$	CV, critical node (true)	floor 25 m	15–35 m floor
h	CV, tank level	$h_0 = 4.0$ m	0–7 m
u_1, u_2	MV, fixed-speed pumps	off	0 or 1
ω	MV, pump 3 speed	stopped	0, 60–100 %
S_j	MV, valve outlet heads	34, 34, 40 m above valve	15–60 m
$D_j(t)$	DV, zone demand	25, 20, 12 L/s mean	noise 0–20 %
$\pi(t)$	DV, tariff	0.07, 0.10, $0.20/kWh	peak 17:00–22:00
events	DV, trips, break, hot day	—	as scripted
H_J	algebraic, junction head	solved	—
Q_i, Q_j	algebraic, flows	solved	—
$V = A_T h$	state, tank volume	2400 m^3	0–4200 m^3
E, C_e	state ledgers, kWh and $	0	—
water ledgers	state, m^3 (seven)	0	—

so that the balances can be checked: energy, its cost, and the water pumped, delivered, leaked, lost to a break, spilled over the top, not served from an empty tank and added by a teaching shortcut.

11.3.1 Pumps, Pipes and Valves

Each running pump lifts water from the clearwell to the station head H_S along a parabola, and the variable-speed pump's curve scales with its speed ratio by the affinity laws, as shown in Equation (11.1).

$$\begin{aligned} H_S - z_{cw} &= a_i \omega_i^2 - b_i Q_i^2, \qquad \omega_1 = \omega_2 = 1 \\ H_{up} - H_{down} &= r\,Q|Q|^{0.852}, \qquad r = \frac{10.67\,L}{C^{1.852} D^{4.871}} \\ H_{out,j} &= \min\left(S_j,\; H_J - k_v Q_j^2\right) \end{aligned} \tag{11.1}$$

where a check valve keeps any pump from running backward and a pump gives no flow past its shutoff head. The second line is the Hazen–Williams loss in meters and cubic meters per second. The source code of the EPANET 2 solver writes the same resistance as $4.727\,L C^{-1.852} d^{-4.871}$ in feet and cubic feet per second [15], which converts to 10.667, so the app's 10.67 is 0.03 % above it, a ratio of 1.00030 at 20, 60 and 120 L/s on the rising main. The third line is a pressure-reducing valve that holds its outlet head S_j or, when the junction cannot supply it, lies wide open with the loss $k_v Q_j^2$. The drive stops pump 3 below 60 % speed. By the affinity laws the pump cannot lift water into the tank at all below 78.6 % speed with the tank empty and 82.6 % with it full.

11.3.2 Zones, Leakage and the Junction

A zone's flow is its demand plus the leakage its mean pressure drives, and its highest customer sees the outlet head less the full loss of the zone main, as shown in Equation (11.2).

$$\begin{aligned} Q_j &= D_j(t) + C_j\,\bar{p}_j^{N_1} + \delta_{j3}\,B\sqrt{\bar{p}_3} \\ \bar{p}_j &= H_{out,j} - \bar{z}_j - \tfrac{1}{2} r_{z,j} Q_j^{1.852} \\ p_{hi,j} &= H_{out,j} - z_{hi,j} - r_{z,j} Q_j^{1.852} \\ Q_{main} &= \textstyle\sum_i Q_i = Q_{tank} + \sum_j Q_j + C_T\,p_J^{N_1} \end{aligned} \tag{11.2}$$

where $p_J = H_J - z_J$, leakage is zero at or below zero pressure, and the break term B is nonzero only after a main breaks in zone 3. The leakage law at $N_1 = 1.18$ is a teaching value inside the FAVAD range of 0.5 for fixed holes to 1.5 for flexible pipes [16]. At $N_1 = 0.5$ and 1.5 the law turns a halved pressure into 29.3 % and 64.6 % less leakage, where the article quotes 30 % and 65 %, and at 1.18 a tenth less pressure gives 11.7 % less leakage. The leakage coefficients are set so that leakage is about a fifth to a quarter of the water pumped.

The series elements are eliminated so that one unknown is left, the junction head H_J. For a trial H_J each zone's flow is the root of an increasing function of Q_j, the station's flow the root of a decreasing one, and the junction's imbalance then falls strictly with H_J, so its root is unique. Every root is found by Newton's method safeguarded by a bracket to a declared tolerance of 10^{-7} m^3/s. Over every case the worst residual is 1.0×10^{-8} m^3/s, no step fails to converge and the junction needs at most 35 iterations.

11.3.3 Tank, Demand and Power

The tank integrates exactly the flow the solve returns, demand follows a daily pattern with colored noise, and each pump's electrical power follows its efficiency curve, as shown in Equation (11.3).

$$\begin{aligned} A_T \frac{\mathrm{d}h}{\mathrm{d}t} &= Q_{tank}, \qquad 0 \le h \le h_{max} \\ D_j(t) &= \bar{D}_j \, \phi(t) \, m_{hot}(t) \, (1 + \sigma_D n_j(t)) \\ P_i &= \frac{\rho g Q_i (H_S - z_{cw})}{\eta(Q_i) \, \eta_m \, \eta_d^{\delta_{i3}}} \\ \eta &= \eta_{max}(2x - x^2), \qquad x = \frac{Q_i}{\omega_i Q_{bep,i}} \\ \frac{\mathrm{d}E}{\mathrm{d}t} &= \textstyle\sum_i P_i, \qquad \frac{\mathrm{d}C_e}{\mathrm{d}t} = \pi(t) \textstyle\sum_i P_i \end{aligned} \tag{11.3}$$

where $\phi(t)$ interpolates 24 hourly multipliers normalized to a mean of one (from 0.458 at night to 1.627 at the 07:00 morning peak, with a second peak at 18:00), $m_{hot} = 1.12 + 0.25 \exp(-((t_h - 16)/3)^2)$ on a hot day and 1 otherwise, and n_j is first-order colored noise of correlation time 0.5 h drawn independently for each zone and held through each step. The tank spills at $h_{max} = 7$ m and pauses the run if it empties. The efficiency peaks at the best-efficiency flow, x is capped at 1.9, $\eta_m = 0.94$ is the motor and $\eta_d = 0.97$ the drive on pump 3. Energy is priced at \$0.07/kWh from 23:00 to 06:00, \$0.20/kWh in the five-hour evening peak from 17:00 and \$0.10/kWh otherwise, and an optional solar tariff adds \$0.04/kWh from 10:00 to 15:00. Leaked water is valued at \$0.30/m^3. All prices are teaching values.

11.3.4 Instruments

Every controller reads only the instruments. The tank level transmitter reads h with 1 cm of colored noise, the station flowmeter Q_{st} with 0.5 %, the station and valve pressure transmitters with 0.1 m, each zone meter Q_j at its valve with 1 %, and the energy meter with 1 %. A pressure logger at the critical house samples $p_{hi,3}$ every 15 min with 0.1 m of noise and holds each report until the next. Run contacts and trip relays report each pump, and the day's weather forecast is a declared input. Leakage, the break, the true demand and the true pressures are never measured, and the comma-separated (CSV) export of Section 1.6.4 carries them in columns prefixed `truth_`.

11.3.5 Parameters, Initialization and Numerics

Every parameter appears in Table 11.2, all of them teaching values and all adjustable in the Advanced panel. Each case starts at midnight with the tank at its case's level and the instruments lined out on the network as the case's pumps and valves leave it. Its noise starts from the case's seed, as Section 1.6.1 describes. The tank and ledgers are integrated by explicit Euler at 10 s, so the network is solved once per step and the tank integrates exactly the flow of that solve. Control decisions are taken once a minute, and the historian writes one row per simulated minute. With pump 1 alone filling the tank at 3 m, the solve gives 65.52 L/s at a head rise of 74.52 m, against a static lift of 71 m, and an independent bisection on the pump and system curves gives the same crossing.

Table 11.2 Model Parameters (Teaching Values, All Adjustable)

Symbol	Meaning	Value	Unit
z_{cw}, z_J, z_T	clearwell, junction, tank floor	100, 125, 168	m
A_T, h_{max}, h_{fire}	tank area, overflow, fire reserve	600, 7, 1	m^2, m, m
L, D	rising main; tank main	3000, 0.40; 1200, 0.40	m
$L_{z,j}$, $D_{z,j}$	zone mains 1, 2, 3	2500, 2000, 1500; 0.30, 0.30, 0.25	m m
C	Hazen–Williams coefficient	120	—
$\bar{z}_j$, $z_{hi,j}$	zone mean, highest customer	105, 118, 132; 112, 126, 142	m m
$z_{V,j}$	valve elevations	110, 122, 130	m
k_v	valve loss wide open	312	s^2/m^5
a_i	shutoff heads, pumps 1, 2, 3	110, 105, 110	m
b_i	curve coefficients	8264, 12500, 8264	s^2/m^5
$Q_{bep,i}$	best-efficiency flows	55, 40, 55	L/s
$\eta_{max,i}$	peak efficiencies	0.80, 0.78, 0.80	—
η_m, η_d, ω_{min}	motor, drive, least speed	0.94, 0.97, 0.6	—
$\bar{D}_j$	mean customer demand	25, 20, 12	L/s
σ_D, τ_D	demand noise, correlation	5, 0.5	%, h
N_1	leakage exponent	1.18	—
C_j, C_T	leakage, zones, trunk[a]	59.7, 57.9, 43.5; 29.7	—
B	break coefficient, zone 3[b]	1.27	—
π	night, day, peak, solar	0.07, 0.10, 0.20, 0.04	$/kWh
p_w, c_s	water value, wear per start	0.30, 2	$/$m^3$, $

[a] In 10^{-6} m^3/s per m^{N_1}. [b] In L/s per $\sqrt{m}$.

11.3.6 Assumptions

The hydraulics are quasi-static, so pipe transients and water hammer at a trip, which last seconds, are outside a model whose lesson takes hours. Each zone is lumped into a valve, a main to its highest customer and a mean pressure for leakage, while a real network has thousands of pipes and loops. Demand is driven, not pressure dependent, so customers draw their pattern whatever the pressure, and the run pauses rather than starve the town if the tank empties. The pressure floor of 25 m, about 35 psi, the fire reserve of 1 m and the leakage law are teaching values. A real floor comes from the utility's service standard and the state's design rules [17], a real fire reserve from the fire flow and duration the local authority requires, and a real exponent from a night step test on the zone. Pump wear is a declared cost per start, and demand charges and water quality are absent.

11.4 Why the Hour Matters

11.4.1 A Tank That Sets the Hill's Pressure

With every pump off the critical node is fed by the tank alone, so its pressure falls with the level and with the town's draw on the tank main. The declared model holds the critical node at the floor plus a 0.5 m margin with no pump running when the tank stands at 0.28 m at half the mean demand, 1.53 m at the mean and 3.73 m at 1.6 times the mean, near the morning peak. In case 1, with every pump off from a 2.5 m tank, the level falls to 1.20 m by 05:00 and the critical node goes under the floor at 05:35. The morning peak draws up to 108.6 L/s of demand and leakage, against 68 to 69 L/s from one pump, so pump 1 started at 05:00 lifts the hill from 25.97 to 26.73 m in a minute and still loses the floor for 120 min, lowest 24.55 m at 07:59. Both pumps from 04:00 hold it, lowest 26.11 m at 07:03. The tank is an integrator, and the pumps must lead the peak by hours, as the gates of Chapter 5 release water one travel time ahead of each published order.

> **TRY IT IN THE APP** **1 · Manual Pumps: Fill the Tank, Read the Hill**
>
> Load the scenario and press Run without touching a pump. Every pump stays off, so the tank alone feeds the town. Watch the tank level fall from 2.5 m to 1.20 m by 05:00 and the critical node go under the 25 m floor at 05:35.

11.4.2 A Tank That Also Sets the Leakage

The same level that protects the hill drives leakage everywhere upstream of a valve that is not holding its setting. Leakage rises by 0.17 L/s for every meter of tank level at mean demand in the declared model, so a tank held high all day pays for its margin in water. A pressure-reducing valve set for the peak gives too much at night. In case 3, with the schedule running, fixed valves leak 1252 m^3 of 5219 m^3 pumped, 24 %. A night cut of 2 m on every valve from 00:00 to 05:00 saves 13 m^3, while a 3 m cut breaks the floor for the whole window, lowest 24.53 m at 04:57, because a uniform cut is limited by the zone with the least margin. Modulated valves leak 1165 m^3 with the floor held, zone 1 falling from 377 to 331 m^3, zone 2 from 359 to 327 and zone 3 from 270 to 262, while the trunk's 245 m^3, upstream of every valve, does not move.

> **DID YOU KNOW?** **A Town That Lost Almost a Third of Its Water**
>
> A public water system in the southern United States serving 6,000 people found that about 30 percent of the water it produced went to apparent and real losses. With pressure monitors, a pressure regulating device and a lower operating pressure, its losses fell to 22 percent of the water produced [19].

11.5 The Control Layers as Programmed

The header MAN/AUTO runs the pump station. In Manual the learner switches pumps 1 and 2 and sets pump 3's speed. In Auto the station follows either the level band or the schedule, and the valves follow their own mode. Minimum run (2 h) and rest (1 h) are enforced in Auto, except that a tank within 0.15 m of overflow stops its pumps.

11.5.1 The Level Band

The level-band rule every utility starts with starts the lead pump when the measured level falls to $h_{low} = 2.0\,\mathrm{m}$, the lag pump at $h_{low} - 0.5\,\mathrm{m}$, and stops both at $h_{high} = 6.0\,\mathrm{m}$, swapping lead and lag at every stop. It reads neither the tariff nor the hill's pressure.

11.5.2 The Hourly Schedule

The schedule is a mixed-integer linear program over the next $K = 24$ slots, the first running from now to the next whole hour. Its decisions in slot k are the fixed pumps $u_{1k}, u_{2k} \in \{0,1\}$, pump 3's flow $q_{3k} \in [0, \bar{q}_3]$ and start indicators $s_{ik} \in [0,1]$, and it is stated in Equation (11.4).

$$
\begin{aligned}
\min\ & \textstyle\sum_k \pi_k \Delta_k (P_1 u_{1k} + P_2 u_{2k} + e_3 q_{3k}) \\
& \quad + c_V \textstyle\sum_{k=1}^{K-1} \Delta_k V_k + c_s \textstyle\sum_{i,k} s_{ik} \\
\text{s.t. } & V_k = V_0 + \textstyle\sum_{m<k} (Q_1 u_{1m} + Q_2 u_{2m} + q_{3m} - \hat{D}_m)\Delta_m \\
& A_T h_{fire} \le V_k \le A_T (h_{max} - 0.3), \quad V_K \ge V_{term} \\
& V_{k+1}/A_T + \beta_k (Q_1 u_{1k} + Q_2 u_{2k} + q_{3k}) \ge h_{req,k} \\
& s_{ik} \ge u_{ik} - u_{i,k-1}, \qquad \textstyle\sum_k s_{ik} \le N_s - n_i \\
& u_{ik} - u_{i,k-1} \le u_{i,k+\tau}, \quad \tau = 1, \dots, R_{run} - 1 \\
& u_{i,k-1} - u_{ik} + u_{i,k+\tau} \le 1, \quad \tau = 1, \dots, R_{rest} - 1
\end{aligned}
\tag{11.4}
$$

where V_0 is the measured level times A_T, Δ_k the slot length in hours, π_k the slot's mean price, $N_s = 4$ the starts limit split at the next midnight with n_i the starts already made today, and $R_{run} = 2$ and

$R_{rest} = 1$ slots, so the rest rows are vacuous at hourly slots. A tripped pump is fixed off, a pump inside its minimum run or rest is fixed for the leading slots, and after a switch from Manual the operator's pumps are fixed for the first slot so that the transfer has no bump. The program is linear because the pump flows and powers are the declared model's values at a nominal head, with the tank at mid-level and three pumps running: $Q_1 = 55.29\,\mathrm{L/s}$ at $P_1 = 61.12\,\mathrm{kW}$, $Q_2 = 40.26\,\mathrm{L/s}$ at 45.65 kW, and pump 3 up to $\bar{q}_3 = 55.29\,\mathrm{L/s}$ at $e_3 = 1.140\,\mathrm{kW}$ per L/s. The leakage term charges the stored volume at $c_V = p_w\,(\partial L/\partial h)/A_T = 3.0 \times 10^{-4}\,\$/(\mathrm{m^3\,h})$, with $\partial L/\partial h = 0.167\,\mathrm{L/s}$ per meter from the declared model. At midnight the program has 120 variables, 48 of them binary, and 144 rows.

Three rows carry the physics. The forecast $\hat{D}_k$ is the declared pattern's mean over the slot times the base demand, the hot-day multiplier when the weather forecast says so, an optional forecast error and a measured correction, plus the leakage the night flow last reported. The floor row asks the level at the end of each slot, plus a secant credit β_k per unit of station flow, to reach $h_{req,k}$, the level at which the declared network, with valve 3 wide open and no pump running, holds the critical node at the floor plus 0.5 m at the slot's peak forecast demand. A table of h_{req} and β over demand multipliers from 0.2 to 2.6 is built once by bisection on the network. The terminal V_{term} is the level the day's first plan had at the clock hour the horizon now ends, so solving again every hour cannot spend the tank a little lower each time. The first plan uses $V_K \geq V_0$. Section 1.4.8 explains why a plan needs a value or a constraint on what it leaves at the end of its horizon.

The program is solved by a depth-first branch and bound [14], the method of Section 1.4.10, written for this application on the framework's shared bounded-variable simplex and since moved to the framework's core, where the power-aware bed schedule of Volume 6, Chapter 10 uses it too. A node is pruned when its relaxation is infeasible or not below the incumbent by more than \$0.01 or 10^{-4} of it, it becomes the incumbent when every binary is within 10^{-6} of an integer, and otherwise it branches on the binary whose fraction is nearest one half into a child with the binary at 0 and one at 1, the side nearer the fraction explored first. The previous plan, shifted one slot, is offered as a first incumbent and accepted only if it satisfies every row. The search stops when the tree is exhausted, with the incumbent proved optimal, or at a node limit of 200, where it returns the incumbent with the gap to the best open bound. If the program has no feasible point it is solved again with a slack on every floor, fire-reserve and terminal row priced at \$2000 per meter of level per slot, and the plan is flagged as unable to hold the floor. The slacks are the soft constraints of Section 1.4.6, used here only after the hard program has failed. Only the first slot is carried out, and the program is solved again at the start of every hour from the measured level and at once when a trip relay changes, the receding horizon of Section 1.4.3. An option keeps the plan made before a trip until it runs out, as a teaching comparison and not an operating mode.

11.5.3 Pump 3 on the Station Flowmeter

Pump 3's speed follows a proportional-integral loop that holds the slot's planned station flow, $Q_{st,SP} = Q_1 u_{1k} + Q_2 u_{2k} + q_{3k}$, against the metered station flow. It is the velocity-form PID with back-calculation anti-windup of Section 1.6.2, at $K_c = 0.2\,\%$ per L/s and $\tau_I = 3\,\mathrm{min}$, executed every minute. Its lower limit is the larger of the drive's 60 % and the shutoff speed $\sqrt{(H_{S,m} - z_{cw})/a_3}$ at the measured station head, and a stopped pump is started at the speed that gives q_{3k} alone, $\sqrt{(H_{S,m} - z_{cw} + b_3 q_{3k}^2)/a_3}$. When the plan asks the most pump 3 can give the pump runs at full speed with the loop tracking, and a planned flow under 4 L/s leaves it stopped.

11.5.4 Valves

In Fixed mode each valve holds an outlet pressure, 34, 34 and 40 m above valves at 110, 122 and 130 m. Time of day takes a night reduction, 2 m by default, off every valve from 00:00 to 05:00. Modulated mode sets each outlet head from its own zone meter so that the zone's highest customer

Table 11.3 The Cases (Each Starts at Midnight)

#	Case and settings	Teaching point
1	Manual, every pump off, tank 2.5 m, 12 h	the tank sets the hill
2	Level band, fixed valves	starts at the wrong hour
3	Schedule; valves fixed, time of day or modulated	leakage against the floor
4	Schedule, as case 2's day	relaxation and branching
5	Schedule once a day, forecast 20 % low, noise 10 %	receding horizon
6	Hot day, pump 1 trips at 07:00, plan kept	reconfiguration
7	Main break in zone 3 at 10:00, 30 h	the night flow
8	Solar tariff, 6 starts allowed, no wear cost	the starts limit
9	Challenge: hot day, peak, pump 2 trips at 18:00	the whole problem
10	Hot day, pumps 1 and 2 lost, tank 3 m	an infeasible floor

receives what it needs at the metered flow, as shown in Equation (11.5).

$$\begin{aligned} S_j &= z_{hi,j} + p_{floor} + 1 + r_{z,j} Q_{j,m}^{1.852}, \qquad j = 1,2 \\ S_3 &= z_{hi,3} + p_{crit,SP} + r_{z,3} Q_{3,m}^{1.852} + \Delta S_3 \end{aligned} \tag{11.5}$$

where $p_{crit,SP} = 26\,\text{m}$ and the trim ΔS_3, between -6 and $+12\,\text{m}$, is a PI on the logger at $K_c = 0.6$ and $\tau_I = 1\,\text{h}$, executed when a report arrives, its integral held while valve 3 lies wide open, since an open valve cannot add head. No separate lift of the setpoints is applied when the floor cannot be met, because in the infeasible case valve 3 already lies open and in the feasible cases the trim already raises it.

11.5.5 Two Estimators

Every hour the town's use, the station meter less the tank's gain on its transmitter, is divided by the forecast of the same hour, and the ratio corrects the forecast through an exponential filter of weight 0.5 held between 0.7 and 1.5. The night flow is the mean of the three zone meters over the quietest hour, 03:00 to 04:00, less the customers' use in that hour by the declared pattern, reported at 04:00 as the leakage $\hat{L}$. It sees only leakage downstream of the zone meters, so the trunk main's loss, about 3 L/s at night, is invisible to it. The leak detector of Volume 10, Chapter 12 is a mass balance of the same kind, metered flows against the change in inventory, and like this one it is only as good as its meters.

🕮 **Pause.** Try Exercise 11.2 before reading on.

11.5.6 Three Models

The plant is the network of Section 11.3 with its leakage multiplier, its break and its noise. The controller's model is the declared network with neither, reduced to nominal pump flows, a floor table and a leakage slope, and corrected only through the measured forecast ratio and the night flow. The identified quantities are those two estimates. A parameter a learner edits during a run rebuilds both the plant's network and the declared model, so the two never disagree about the network the learner drew.

11.6 Ten Cases, Measured

The cases appear in Table 11.3. Each number that follows comes from a headless run of the engine of Section 1.6.1 on the case's own seed, stepped exactly as in the browser, after the last change to the application's behavior. Times are the controller's, so a pump starts at the control step that switches it. A case yields an identical run when loaded first, when reached from the menu after another case, and when restarted with Reset.

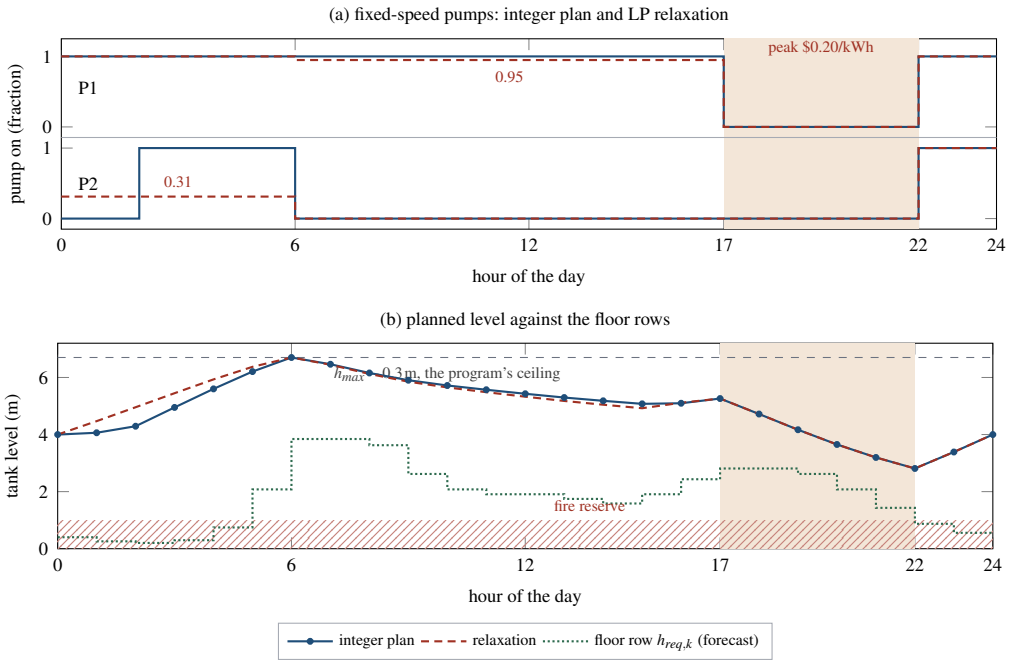


Figure 11.2 Case 4's first solve at midnight. (a) The fixed-speed pumps of the integer plan (solid) and of the LP relaxation (dashed), slot by slot, with the relaxation's fractions 0.95 and 0.31. (b) The planned tank level of both (integer plan solid with dots, relaxation dashed), which differ by at most 0.67 m, against each slot's floor row before its credit for station flow (dotted). Shaded, the evening peak. Hatched, the fire reserve.

11.6.1 The Band Against the Schedule

The band in case 2 waits for the tank to reach 2.0 m, which it does in the morning peak, so the lead pump starts at 06:33, after the hill has dipped under the floor at 06:20 for 12.8 min to 24.43 m. The lag pump follows at 09:03 and both run to midnight, through the evening peak, for $241.9 of energy (1881 kWh). Case 4 runs the same day on the schedule. The first plan fills the tank on night power to 6.70 m by 06:00, coasts through the day on pump 1 with pump 3 topping it up from 15:00, runs nothing in the evening peak and refills from 22:00, as shown in Figure 11.2. The day costs $128.1 (1577 kWh), none of it at the peak price.

Pause. Try Exercise 11.1 before reading on.

The relaxation of that first program sets pump 1 at 0.95 from 06:00 to 17:00 and pump 2 at 0.31 through the night, 17 fractional slots in all, at $194.91. Branch and bound needs 29 nodes, 15 of them branched, to a depth of 7, and the incumbent improves three times, to $195.97, 195.95 and finally $195.90, proved optimal $1.00 (0.51 %) above the relaxation. The two are close because pump 3's continuous flow fills most of the gaps the binaries leave, and the search still has to branch to turn the fractions into runs that respect the two-hour minimum. Sixteen of the day's 24 solves branch.

11.6.2 Pressure, Valves and the Floor

Case 3's comparison of valve modes is given in Section 11.4. Lowering the valves of zones 1 and 2 by hand saves leakage only until those zones' own highest customers lose the floor, which happens below 32.6 m at valve 1 and 31.9 m at valve 2, so the verdict holds every zone's highest customer to the floor and not only the critical node.

11.6.3 Receding Horizon, a Trip and a Break

Case 5 forecasts demand 20 % low with 10 % noise. Solved once at midnight and followed blind, the plan ends the day with the tank 0.67 m below a plan that no longer describes the day, for \$519.8. Solved again every slot, each plan starts from the measured level, the forecast correction reaches $\times 1.32$, the tank stays within 0.11 m of the plan in force and the day costs \$499.7.

Case 6 trips pump 1 at 07:00 on a hot day. Kept, the plan made before the trip counts on pump 1 to fill the tank through the day, so the tank never fills, the critical node falls under the floor at 17:00 for 6 h, lowest 23.23 m, and the tank empties at 22:24. Re-solved at 07:01 with pump 1 fixed off, in 11 nodes with 5 branched, the program moves the refill to pumps 2 and 3, pump 2 running from 07:01 to 17:00, and the floor holds at a lowest 25.70 m for \$559.0, against \$545.0 without the trip.

Case 7 breaks a main in zone 3 at 10:00. The break discharges 557 m^3 by 30 h. The forecast correction, $\times 1.05$ when the main breaks, reaches $\times 1.17$ by 16:00, so the schedule pumps for the lost water before anyone knows it is lost, and the floor holds at a lowest 25.23 m. The night flow tells how much of it is loss. The zone meters' 03:00–04:00 mean, less the pattern's 27.5 L/s of night use, reports 11.2 L/s the first night (38.7 metered) and 19.2 L/s the second (46.8 metered). The truth downstream of the meters is 11.8 L/s and then 11.8 plus the break's 7.8 L/s, while the trunk main's 3.0 L/s sits upstream of every meter and is never seen.

> **TRY IT IN THE APP** **7 · A Main Breaks: The Night Flow Finds It**
>
> Load the scenario and run its 30 h. A main breaks in zone 3 at 10:00, and the forecast correction climbs from $\times 1.05$ to $\times 1.17$ by 16:00. Read the night flow reported at 04:00 each morning: 11.2 L/s after the first night and 19.2 L/s after the second. The trunk main's 3.0 L/s never appears.

11.6.4 Starts, and an Infeasible Day

On the solar tariff of case 8, six starts a day give each fixed pump three starts for a day of \$462.9, and one start each costs \$469.0. The first program's own cost rises from \$138.39 in 9 nodes to \$148.11 in 7, with a limit of two needing 158 nodes for \$142.52. Only the program's own cost is ordered by the limit. The realized day is not (\$462.4 at three starts, \$464.1 at two), because the day's demand is not the forecast the plan was made on.

Case 10 loses pumps 1 and 2 at midnight on a hot day. Pump 3 alone gives 66 to 68 L/s, and the town's draw of demand and leakage exceeds 67 L/s from 05:16 to 21:31 without a break. Every one of the 24 solves after the trips is the relaxed program, flagged as unable to hold the floor, and the critical node falls under it at 17:12, for 96 min in all, lowest 24.56 m, with the tank under its fire reserve at 19:48. A logged sweep of twelve runs, valves modulated, by time of day and fixed at 30, 40, 50 and 60 m, each scheduled and by hand, breaks the floor in all twelve, for 96 to 117 min, or all day with valves fixed at 30 m, where the valve itself is under the floor. A valve can take head away but cannot add it.

Pause. Try Exercise 11.3 before reading on.

11.6.5 The Challenge

The challenge runs a hot day with the evening peak and trips pump 2 at 18:00. It passes at 24 h if the critical node and each zone's highest customer never fall below 25 m, the tank never falls under its fire reserve or spills, neither fixed-speed pump starts more than four times and the energy cost plus the leaked water is at most \$530, with every parameter as shipped. Changing a parameter, the floor, the tariff peak, the starting level or the starts limit marks the challenge as not graded. The five policies of Table 11.4 share the plant, the events and the instruments. The shipped level band with fixed valves fails on every seed. It breaks the floor in the morning peak, for 55 min from 06:05 on the case's seed, and buys 384 kWh at the peak price. The suggested schedule with modulated valves passes on all eleven seeds and buys under 1 kWh at the peak price. The trip at 18:00 is absorbed by a

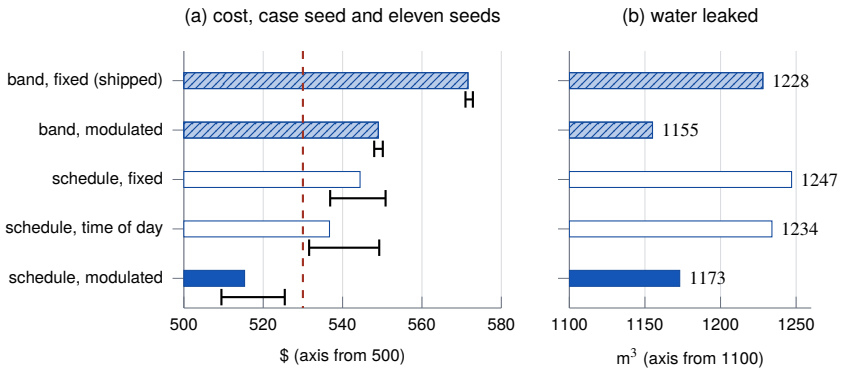


Figure 11.3 The challenge of Table 11.4: (a) the graded cost on the case's seed (bars) with its range over eleven seeds (the bracket under each bar), and (b) the water leaked on the case's seed, for the level band (hatched bars), the schedule with fixed or time-of-day valves (open bars) and the schedule with modulated valves (dark bar). The red dashed line is the $530 target. Notice that only the dark bar and its whole bracket lie below the target, and that the modulated valves give the least leakage under either pump policy.

Table 11.4 The Challenge at 24 h: Case Seed and Eleven Seeds

Policy	Energy ($)	Leaked (m^3)	Cost ($)	11 seeds ($)	Pass
band, fixed (shipped)	203.2	1228	571.6	570.7–573.1	0/11
band, modulated	202.4	1155	549.0	547.7–550.4	0/11
schedule, fixed	170.4	1247	544.4	536.6–551.1	0/11
schedule, time of day	166.6	1234	536.7	531.3–549.5	0/11
schedule, modulated (sugg.)	163.5	1173	515.3	509.2–525.7	11/11

The two band policies break the floor on every seed (lowest 24.08 and 23.98 m). The scheduled ones hold it on every seed.

re-solve at 18:01 whose relaxation is already integral, one node. The two mixed scheduled policies hold the floor on every seed and fail only on cost, so both levers are needed. Figure 11.3 draws the five policies against the target.

The $530 target sits in a narrow gap. Over the case's seed and seeds 1 to 60 the suggested settings cost $509.0 to 527.4 and pass 60 of 61, and the schedule with time-of-day valves costs $531.3 to 549.5 and passes none. On seed 27 the suggested settings put the critical node 8 mm under the floor for one 10 s step at 17:52. The plan had kept the tank 0.54 m above that hour's floor row, but zone 3's own demand ran 18 % over its pattern, a 3.7-standard-deviation swing of the 5 % noise, while valve 3 lay open and no pump ran, and the logger's 15 min samples never saw the dip. That is the residual of an hourly plan made on a town-wide forecast against fifteen-minute noise concentrated in one zone, not a fault of the floor rows, and the scorecard counts it as a hard limit.

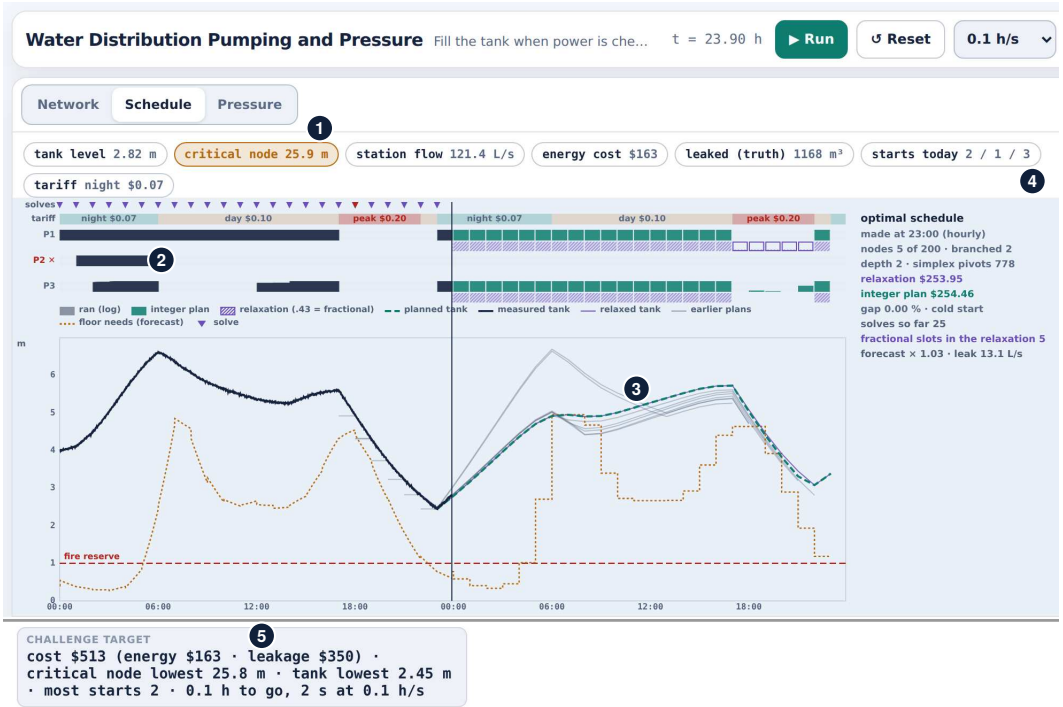


Figure 11.4 The challenge with the suggested settings (schedule, modulated valves) at $t = 23.9$ h. The figure joins two parts of the screen, the header with the Schedule view (top) and the challenge tile of the Learn panel (bottom), and leaves out the control panel, the trends and the rest of the Learn panel. (1) The badges read the tank at 2.82 m, the logger at 25.9 m (the run's record puts the true critical node at 26.03 m), the station at 121.4 L/s, $163 of energy and 1168 m^3 leaked, and the night price of $0.07/kWh. (2) The pump rows show what ran against the tariff strip, nothing in the evening peak, pump 2 crossed after its 18:00 trip and the mark of the trip's re-solve at 18:01 among the hourly solve marks. (3) Ahead of the clock the integer plan made at 23:00 stands over the relaxation's sub-rows, with the planned tank dashed against the measured, the floor rows and the fire reserve. (4) The side panel reports that plan optimal in 5 nodes with 2 branched, a relaxation of $253.95 against $254.46, 25 solves so far, a forecast correction of $\times 1.03$ and a night-flow leakage of 13.1 L/s. (5) The challenge tile reads $513 ($163 of energy and $350 of leakage) with a lowest critical node of 25.8 m and a lowest tank of 2.45 m.

> ✦ **DID YOU KNOW? Valves That Lower the Energy Bill**
>
> From the simulator: in the challenge, the schedule with fixed valves spends $170.4 on energy and leaks 1247 m^3, while the same schedule with modulated valves spends $163.5 and leaks 1173 m^3. The valves move no pump, yet the energy falls too, and only the modulated policy passes on all eleven seeds (Table 11.4).

The application at that instant of the case's seed is shown in Figure 11.4.

11.6.6 Code Verification

Code verification and agreement with selected literature are kept apart here, and neither is a validation against a real network. Over case 7's 30 h the tank's gain of 1300.423 m^3 equals the integrated junction balance of the solves to 1.6×10^{-4} m^3, and the water ledger closes to 2.3×10^{-4} m^3 of 9059.9 m^3 pumped. On every one of 250 solves over eleven runs, 166 of them branched, the relaxation is no higher than the plan, and every plan and every realized day keeps the minimum run, the minimum

rest and the starts limit. The branch and bound matches enumeration on two instances written for the check: a unit commitment with 12 binaries, optimal at $348.10 in 21 nodes against all 4096 assignments solved through a different simplex, and the application's own program over six slots with 9 free binaries, $31.581448 in 17 nodes against 512 assignments. Stopped at six nodes, the first instance returns its incumbent with a bound of $338.70 and a gap of 2.70 % that brackets the optimum. On the shipped application every one of the 286 solves of the suggested challenge over eleven seeds and the 209 solves of the other scheduled cases reached an incumbent and proved it optimal, at most 29 and 106 nodes against the limit of 200, so the two-hour slots held in reserve were not needed.

Halving the step to 5 s with every noise off moves the challenge's day energy by at most 0.009 % for all five policies, with identical pump starts. With the instruments' noise on, a new step is also a new draw of that noise, the hourly plan can move an hour of pumping, and the day's energy moves by up to 2.3 % for the scheduled policies with fixed or time-of-day valves, and by up to 3.2 % as shipped with the 5 % demand noise also redrawn, while no verdict and no ranking changes in either comparison. In agreement with the literature, the affinity laws hold to round-off on the pump curve, the Hazen–Williams resistance matches EPANET's to a ratio of 1.00030, and the leakage law reproduces the FAVAD figures quoted in the previous section. In a desktop browser at the top speed of 0.5 h/s the challenge's day takes under 50 s of wall time (48.0 s in six runs), every one of 531 solves made during play takes under 250 ms (slowest 196.6 ms, median 30.0 ms), and the first solve, made while a case loads and before the clock moves, takes about half a second in case 5 (521 ms at most over three loads), whose first program needs 106 nodes. These are wall-clock bounds on one machine, not properties of the method.

11.7 Technology in Practice

What Measures It

Design standards ask for controls that hold the levels of distribution storage, for level indication at a central location, and for overflow and low-level alarms placed where they are watched around the clock [18]. A common level measurement on an open tank is a differential pressure transmitter connected at the bottom with its low side vented to the air, so that the difference it reads is the hydrostatic head of the water. The lowest level it can measure is set by the point where it joins the tank, and the density of the fluid, the ambient temperature and the humidity affect its accuracy [20]. In this chapter the tank transmitter reads h with 1 cm of colored noise, the station flowmeter Q_{st} with 0.5 % and each zone meter with 1 % (Section 11.3).

> **TECHNOLOGY** **Sensor: Pressure Monitors at the Highest Customers**
>
> A federal fact sheet on pressure management advises placing pressure monitors at the highest and lowest elevations of each pressure zone and wherever pressure tends to run low. The logging interval can be set to seconds, keeping the minimum, maximum and average of each interval, and low-pressure alarms let operators react quickly. A drop in pressure may last a few seconds or many hours, and a surge swings between very high and very low pressure [19]. Many transducers let the pressure strain a fine wire grid whose change in resistance gives the signal, and they lose accuracy when the air pressure around them varies [20]. No open, vendor-neutral source gave an accuracy. The chapter's logger reports every 15 min, and on seed 27 it missed a dip of one 10 s step (Section 11.6).

What Moves It

Fixed-speed pumps make up the majority of the pumps utilities have installed, while variable-speed pumps are more popular in new installations [4]. Design standards ask each booster pumping station for at least two pumps, sized so that peak demand can be met with the largest one out of service, and for a flow rate indicator and totalizer. Where static pressure exceeds 100 psi, pressure reducing devices are required on mains or on individual service lines [18]. The chapter's station has two fixed

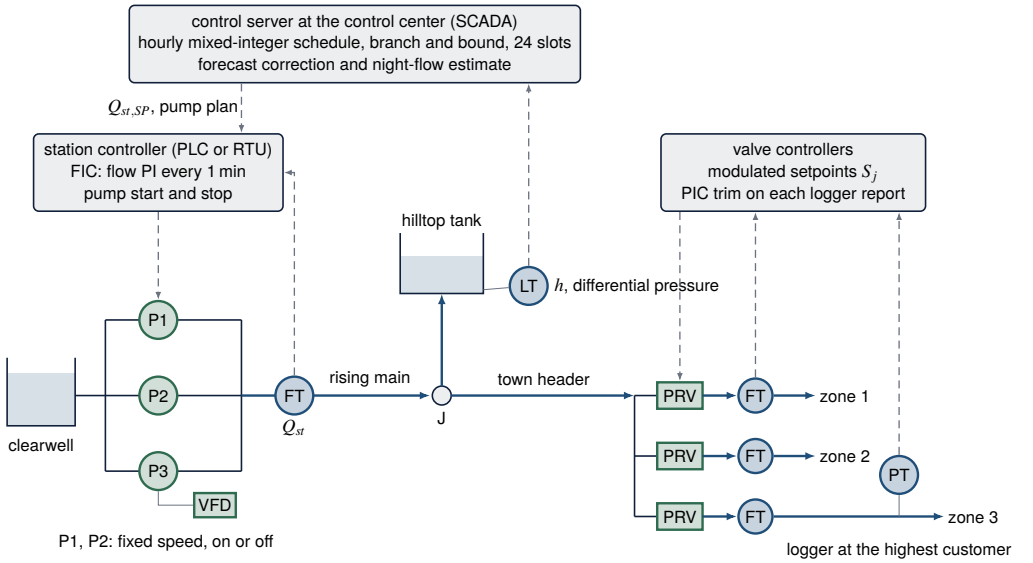


Figure 11.5 The chapter's control layers drawn with real devices. The control server solves the hourly schedule and sends the station a flow setpoint and a pump plan, and the station controller starts and stops the fixed-speed pumps P1 and P2 and holds the metered flow (FT, Q_{st}) with pump 3's variable-frequency drive. The tank level (LT) returns to the server, the three pressure-reducing valves follow their zone meters (FT), and the logger (PT) at the highest customer trims valve 3. Notice that the only measurement of the hill's pressure arrives every 15 min.

pumps and one on a drive, and its three zones each take water through a pressure-reducing valve. Case 10 removes both fixed pumps on a hot day, and pump 3 alone cannot hold the floor.

> **TECHNOLOGY Final Element: Fixed-Speed Pumps in Parallel**
>
> A station can meet a changing demand with several smaller pumps in parallel instead of one large one. Each pump started shifts the operating point to the right along the system curve, and where static head is high, switching pumps on and off keeps each running pump nearer its best efficiency point. Pumps of different sizes need care, because the largest can push the others below their minimum flow. Starts have a price: repeated starting and stopping wears motor controllers, seals and bearings, and large motors draw high starting currents [21]. Soft starts or variable-frequency drives limit the pressure swings a start causes [19]. The chapter's two fixed pumps differ in size, and the schedule prices each start and allows four a day (Table 11.2, Equation (11.4)).

Where the Controller Runs

Supervisory control and data acquisition systems are used in distribution systems such as water distribution. A control server at a control center stores and processes the data, field sites hold remote terminal units or programmable logic controllers that control the local process, and the server polls the field devices at defined intervals and sends new set points as required [22]. In this chapter the hourly schedule plays the server's part and sends the station a flow setpoint and a pump plan. The flow loop on pump 3 runs every minute and the valve trim at each 15 min logger report, the local loops of Figure 11.5.

> **IN PRACTICE** **A Full Tank Is Not a Fresh Tank**
>
> Storage tanks have often been kept full to be ready for emergencies, and a federal issue paper on finished water storage warns that a full but underused tank lets its water age, which favors microbial growth and chemical change. Operators manage water age by turning the stored water over and letting the level move. A complete turnover in three to five days has been recommended as a starting point, with a goal set for each tank. One large utility let its levels fall a further ten feet in normal operation and cut the mean residence time by two to three days [23]. The schedule of this chapter draws its tank down each day, but its model carries no water quality (Section 11.8).

What Is Modernizing It

Three developments bear on this chapter's method. The first is exact scheduling. Optimal pump scheduling has been reported to cut a network's energy cost by 10 to 20 percent, and branch and bound can return schedules with a bounded optimality gap where heuristic methods give no bound [4]. The second is the network model kept live. The public-domain solver EPANET can be used to optimize the operation of tanks and pumps, and one utility moved from a static model to a real-time one that imports data from its supervisory system [24]. The third is metering by area. District metered areas, parts of a network that can be isolated and metered separately, support pressure and supply optimization, especially with real-time control [19]. The night flow of Section 11.5 reads the zone meters in the same spirit.

11.8 Limitations

The town is five nodes with teaching sizes, curves and prices, so the numbers show a mechanism rather than predict any network. The hydraulics are quasi-static, demand ignores pressure, and transients, water quality, demand charges and pump wear beyond a price per start are absent. The leakage law has one exponent for every zone and a coefficient fitted to give leakage of about a fifth to a quarter of supply, and the floor, the fire reserve and the exponent are teaching numbers, and their real counterparts come from a service standard, a fire authority and a field test.

The schedule's program is linear in nominal pump flows and powers at mid-tank, its floor rows come from the declared network at each hour's forecast peak, and the hourly re-solve from the measured level absorbs what the linearization misses. It forecasts the town as a whole, so a swing of demand in one zone within an hour can still dip the floor, as seed 27 shows. The branch and bound re-solves every relaxation from the start because the shared simplex has no warm start, and the node limit bounds its time. The protection the scorecard applies is an educational verdict on the truth, not a compliance test.

> **OPEN CHALLENGE** **Loops Slow the Exact Search**
>
> Branch and bound gives a pump schedule with a bound on how far it can be from the best, which heuristic searches cannot. A study of such formulations found that loops in the network raise the effort sharply: over 24 time steps, one formulation took 1 s on a network without loops and 43 s on one with two. Smaller time steps improved the computed cost by 5 to 10 percent while the effort grew exponentially. Loops are common in real networks, and the study named two ways forward: solvers that exploit the structure of loops, and reductions to equivalent networks without them [4]. The town of this chapter is five nodes with no loop (Section 11.8).

11.9 Conclusion

The simulator of this chapter couples integer pump scheduling, pressure-reducing valves and pressure-dependent leakage on one quasi-static network whose balances close to round-off. On the same day the level band buys 571 kWh at the peak price and lets the hill dip under its floor, while the hourly schedule spends \$128.1 against \$241.9 and buys none. Branch and bound turns a relaxation 0.51 % cheaper than the plan into a plan proved optimal in 29 nodes, and every one of 495 solves on the

shipped cases finishes inside its node limit. Modulated valves save 87 m^3 of leakage a day where a uniform night cut of 3 m breaks the floor, a re-solve after a trip holds a floor that a kept plan loses for six hours, and the night flow reveals a break that the daytime forecast correction had already paid for. Two pumps lost on a hot day cannot be planned around, and the program says so from its first solve. The schedule forecasts the town rather than each zone and the network is lumped. A fuller model would add pressure-dependent demand, a looped network with an EPANET-class solver, demand charges and a zone-level forecast. The simulator shows that the cheapest safe day for a water network is decided by when the pumps run, not by how full the tank is kept.

KEY POINTS

- The cheapest safe day for a water network is decided by when the pumps run, not by how full the tank is kept.
- On the same day the level band buys 571 kWh at the peak price and lets the hill dip under its floor, while the hourly schedule spends $128.1 against $241.9 and buys none.
- Branch and bound turns a relaxation 0.51 % cheaper than the plan into a plan proved optimal in 29 nodes.
- Modulated valves save 87 m^3 of leakage a day, and a re-solve after a trip holds a floor that a kept plan loses for six hours.

Where else this applies. The refrigeration pack of Volume 8, Chapter 12 stages two fixed-speed compressors and one on a drive, the same arrangement as the pumps here.

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11.10 Exercises

11.1 Concept check · 2 min

In scenario 2 the level band starts the lead pump at 06:33, thirteen minutes after the critical node first falls under the 25 m floor. Why does the band let the hill dip before it acts?

(a) The fixed-speed pumps take several minutes to run up to speed, so the hill keeps falling after the start command

(b) The band acts on the tank level alone, and in the morning peak the hill needs more level than the 2.0 m start point

(c) The one-hour minimum rest holds the lead pump off after its last stop, which delays the start past the low band

(d) The band waits for the next fifteen-minute logger report before it sees that the critical node has lost its floor

11.2 Concept check · 2 min

In scenario 7 the night flow reports 11.2 L/s of leakage on the first night, while the trunk main leaks another 3.0 L/s that the estimate never includes. Why does the estimate miss the trunk main?

(a) The trunk main leaks only by day, when the junction head is highest, so its loss is zero in the quiet hour

(b) The estimator subtracts the trunk main's leakage, computed on the declared model, before it reports the night flow

(c) The estimate is built from the three zone meters at the valves, and the trunk main leaks upstream of all of them

(d) The declared pattern counts the trunk main's loss as part of the 27.5 L/s of the customers' night use

11.3 Concept check · 2 min

In scenario 10 pumps 1 and 2 are out on a hot day. Twelve runs with the valves modulated, on time of day, and fixed at 30, 40, 50 and 60 m, each scheduled and by hand, all break the floor. Why can no valve setting hold the critical node?

(a) The trim on valve 3 winds up while the logger reads low and holds the valve partly shut through the evening

(b) The relaxed program prices the floor slack at $2000 per meter, so the schedule stops pump 3 to save energy

(c) The fire-reserve row forces the schedule to keep water in the tank instead of sending it up to the hill

(d) A pressure-reducing valve can only take head away, and pump 3 alone cannot keep the tank high enough for the hill

11.4 Theory and calculation · 6 min

Pump 1 has a shutoff head $a_1 = 110\,\mathrm{m}$, a curve coefficient $b_1 = 8264\,\mathrm{s^2/m^5}$, a best-efficiency flow of 55 L/s and a peak efficiency of 0.80, and its motor is 94 % efficient (Table 11.2). Use Equations (11.1) and (11.3) with $\rho = 1000\,\mathrm{kg/m^3}$ and $g = 9.81\,\mathrm{m/s^2}$.

(a) Pump 1 alone fills the tank at 3 m with 65.52 L/s. What head does it add? How much of it is lost to friction between the station and the tank? *Checkpoint:* 74.52 m; 3.52 m above a 71 m static lift

(b) The schedule's program uses pump 1 at its nominal 55.29 L/s. Compute the head, the efficiency and the electrical power there, and the energy per cubic meter lifted. *Checkpoint:* 84.74 m, 0.800, 61.12 kW, 0.307 kWh/m^3

(c) Pump 3 has the curve of pump 1 scaled by its speed ratio. Find the least speed at which it can lift water into the empty tank and into the full tank at 7 m, and say whether the drive's 60 % minimum matters. *Checkpoint:* 78.6 % and 82.6 %

11.5 Theory and calculation · 6 min

The leakage law of Equation (11.2) is $C_j \bar{p}_j^{N_1}$ with $N_1 = 1.18$.

(a) Compute the fraction by which leakage falls when the pressure is halved at $N_1 = 0.5$ and at $N_1 = 1.5$, and when it falls by a tenth at $N_1 = 1.18$. *Checkpoint:* 29.3 %, 64.6 %, 11.7 %

(b) In case 3 the three zones leak 377, 359 and 270 m^3 with fixed valves and 331, 327 and 262 m^3 with modulated ones. If the modulated valves had lowered every zone's mean pressure by the same fraction at every hour, what fraction would that be? Why does the trunk main's 245 m^3 not move? *Checkpoint:* about 7.3 %

(c) The schedule charges the stored volume at $c_V = p_w\,(\partial L/\partial h)/A_T$. Compute it from $p_w = 0.30\,\$/\mathrm{m^3}$, $\partial L/\partial h = 0.167\,\mathrm{L/s}$ per meter and $A_T = 600\,\mathrm{m^2}$. *Checkpoint:* 3.0×10^{-4} \$/(m^3 h)

11.6 Theory and calculation · 7 min

The schedule of Equation (11.4) has, in each of its 24 slots, the two pump states u_{1k} and u_{2k}, pump 3's flow q_{3k} and two start indicators s_{ik}.

(a) How many variables and how many binaries does the program have? How many on-off assignments of the binaries are there, and how many linear relaxations did case 4's first solve need to prove its plan optimal? *Checkpoint:* 120 variables, 48 binaries, $2^{48} \approx 2.8 \times 10^{14}$; 29

(b) The Schedule view's side panel reports that solve as a relaxation of \$194.91, an integer plan of \$195.90 and a gap of 0.00 %. How far is the plan above the relaxation, in percent? Why does the panel show a gap of zero? *Checkpoint:* 0.51 %

(c) With $R_{run} = 2$ and $R_{rest} = 1$, write the minimum-run and minimum-rest rows for pump 1 in slot k. If pump 1 is off in slot 4 and on in slot 5, what must it do in slot 6? *Checkpoint:* on in slot 6; no rest rows

11.7 App challenge · 8 min

Load scenario 2, "The level band across the tariff", at the default seed. Set the speed to 0.25 h/s and press Run. The case pauses at 24 h, about 96 seconds later.

(a) Read the Energy cost · leakage tile and the Critical node · worst tile. What did the day's energy cost, how many kWh did it take, and how long was the critical node below the floor? *Checkpoint:* \$242, 1881 kWh; 13 min, lowest 24.4 m

(b) Load scenario 4, "The schedule: relaxation and branch and bound", and run it to its pause at 24 h in the same way. Read the same tiles. *Checkpoint:* \$128, 1577 kWh; never below, lowest 26.1 m

(c) Export the CSV of each run and read `tank_m` in its last row. Compute the average price each day paid per kWh. Is the difference in kWh between the two days a fair measure of what the schedule saves? *Checkpoint:* 4.55 m and 2.41 m; \$0.129 and \$0.081 per kWh

Give money to \$1, energy to 5 kWh and levels to 0.02 m.

11.8 App challenge · 8 min

Load scenario 6, "A pump trips at the peak", at the default seed. It loads with On a trip at Keep the plan. Set the speed to 0.5 h/s and press Run.

(a) The run pauses when the tank empties. Read the clock and the Critical node · worst tile. When did the tank empty, and how low did the critical node fall? *Checkpoint:* 22:24; 23.2 m at 19:12

(b) Reload the scenario with the reload button, set On a trip to Re-solve on the Schedule card before pressing Run, and run to the pause at 24 h. Read the Critical node · worst and Energy cost · leakage tiles. In the exported CSV, read `sched_nodes` and `sched_branched` in the rows from 07:02 to 08:00. *Checkpoint:* 25.7 m, never below; \$559; 11 nodes, 5 branched

(c) The run of (a), continued by pressing Run once more after the pause, stops at 24 h with \$485 on the tile, less than the re-solved day. Why is that not a saving? *Checkpoint:* about 678 kWh not pumped

Give pressures to 0.1 m and money to \$1.

11.9 App challenge · 9 min

Load the challenge, "a hot day, the peak and a trip" (case 9 of Table 11.3), at the default seed, and set the speed to 0.5 h/s. Each run pauses at 24 h, 48 seconds after Run, with the verdict on the Challenge target tile.

(a) Run it as shipped. Read the verdict, the cost and its split, and the time the critical node spent below the floor. *Checkpoint:* not met; \$572 (energy \$203, leakage \$368); 55 min

(b) Set Pump control to Schedule and Valves to Modulated, press Reset, which keeps them, and run again. Read the tile. *Checkpoint:* met; \$515 (energy \$163, leakage \$352)

(c) Set Valves back to Fixed, keep the schedule, press Reset and run. Which part of the target fails now? What do (a) to (c) say about the two levers? *Checkpoint:* not met on cost, \$544; floor held

Give money to \$1.

11.10 Application area · 4 min

The chapter's pressure floor is 25 m of head at the highest customer.

(a) Convert it to kilopascals and to pounds per square inch (1 psi $= 6.895$ kPa), with $\rho = 1000\,\mathrm{kg/m^3}$ and $g = 9.81\,\mathrm{m/s^2}$. How many meters is the 20 psi minimum of the design standards cited in Section 11.2? *Checkpoint:* 245 kPa, 35.6 psi; 14.1 m

(b) The floor, the 1 m fire reserve and the leakage exponent are teaching values. Where would a utility take each of them from?

11.11 Application area · 5 min

An operator objects that filling the tank at night only to hold it high all day pays for its margin in leaked water. Use pump 1's nominal 55.29 L/s at 61.12 kW, the tank's 600 $\mathrm{m^2}$ and the program's leakage price $c_V = 3.0 \times 10^{-4}\,\$/(\mathrm{m^3\,h})$.

(a) How long does pump 1 take to lift one meter of tank, how much energy does that take, and what does buying it at the \$0.07 night price instead of the \$0.20 peak price save? *Checkpoint:* 3.01 h, 184 kWh, \$23.95

(b) What does holding that extra meter for a whole day cost in leaked water? Is the operator's objection right? *Checkpoint:* \$4.32

11.12 Review the agent · 5 min

An AI assistant looks at case 4's first solve (Figure 11.2) and writes: "Branch and bound spent 29 relaxations to gain one dollar. Skip it and round the relaxation instead. Pump 1 at 0.95 from 06:00 to 17:00 rounds to on, pump 2 at 0.31 through the six night slots rounds to off, and pump 3 keeps its relaxed flows. The rounded plan keeps the two-hour minimum run, and since rounding moves each slot by less than half a pump, it is as feasible as the relaxation and costs within a few dollars of \$194.91." Find the flaw, and say how you would check the rounded plan before using it.