INTERACTIVE CONTROL ENGINEERING
Principles and Applications

Volume 9 Model Predictive Control and Economic Optimization

CHAPTER 10

Wastewater Aeration Simulator for Supervisory Setpoint Selection Under a Discharge Permit

Pages 234–264

Edited by John D. Hedengren

How to cite this chapter. "Wastewater Aeration Simulator for Supervisory Setpoint Selection Under a Discharge Permit," in *Model Predictive Control and Economic Optimization*, John D. Hedengren, Ed., vol. 9 of *Interactive Control Engineering: Principles and Applications*. Advanced Process Solutions, LLC, 2026, ch. 10, pp. 234–264.

The page numbers in this file are those of the printed volume.

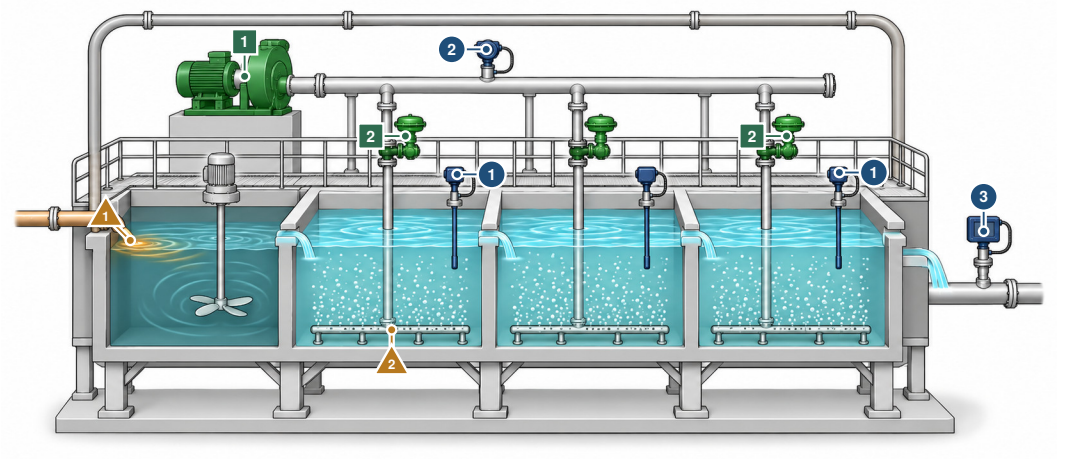
The app for this chapter
https://apmonitor.com/apps/wastewater-aeration/

The volume online: contents, other chapters and citation records
https://apmonitor.com/apps/books/v09-mpc-optimization/#wastewater-aeration

Copyright © 2026 Advanced Process Solutions, LLC. All rights reserved. Produced by Advanced Process Solutions, LLC.

CHAPTER 10

Wastewater Aeration Simulator for Supervisory Setpoint Selection Under a Discharge Permit



The controller moves 1 blower and its speed drive 2 three air valves, one per tank

The controller reads 1 dissolved oxygen, one probe per tank 2 air header pressure 3 effluent ammonium analyzer

What disturbs it 1 sewage load that follows the day 2 diffuser fouling

One blower and one header serve three tanks in series, while the ammonia that the permit counts leaves the last tank hours after the air that removed it. Illustration, not to scale.

IN PLAIN TERMS Buying Only the Air the Permit Needs

The main idea. Bacteria in large tanks eat the ammonia in sewage, and blowers feed them air. The plant is judged on the ammonia that leaves, averaged over eight hours. The air is a large part of its power bill.

The challenge. The ammonia that leaves answers a change of air only hours later. So the air has to be chosen ahead of time. A planner that knows the usual shape of the day saves air, but it lets out more ammonia.

Why it matters. On a settled day the planner uses 6.5 percent less power than a simple feedback controller, at about the same average ammonia.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history in Section 10.2, the facts in the Introduction and in Section 10.5.7, the try-it segments in Section 10.5, and the key points
Study	70 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	25 min	the first page, the results (Section 10.5), Technology in Practice (Section 10.6), Limitations (Section 10.7) and the open challenge

The control question An activated sludge plant is judged on an eight-hour mean of effluent ammonia that answers the air in about three hours, while its control system holds a dissolved oxygen setpoint that appears in no permit. Where should that setpoint sit hour by hour, and what is moving it worth at equal compliance, on the first day and on a settled day?

Learning objectives

- Compute the oxygen saturation, the nitrifier population and the header time constant of the works from its model equations.
- Explain why a cold snap moves the effluent ammonia and not the nitrifier population, and why the sludge age is the slow handle.
- Compare a fixed setpoint, the ammonia supervisor and most-open-valve control at equal compliance, and say why the first day overstates the supervisor.
- Evaluate the day-ahead MPC against the supervisor on energy, worst rolling mean and nitrogen discharged, with and without the load preview.
- Justify a supervisory investment on this plant in dollars a year.

Prerequisites Sections 1.6.2 (velocity form), 1.4.3 (receding horizon), 1.4.6 (soft constraints), 1.4.7 (back-off), 1.4.8 (economic MPC) and 1.6.4 (how an exercise names a run), and Volume 3, Chapter 3 for a dissolved oxygen cascade.

The app https://apmonitor.com/apps/wastewater-aeration/

See also Volume 4, Chapter 9 for valve-position control on a shared supply and Volume 4, Chapter 2 for the surge line and its override. Volume 19, Chapter 11 and Volume 19, Chapter 8 for supervisory loops that act on slow, sampled measurements of a living population.

10.1 Introduction

Every regulatory loop on the simulated works of this chapter behaves itself. The air header pressure settles in 17.3 s, the dissolved oxygen in each aerated tank settles in 6.20 min, and both are ordinary proportional integral loops that a first course covers [1]. The hard question is not how to hold a dissolved oxygen setpoint. It is what that setpoint ought to be, hour by hour, when the quantity that can be measured in 45 s carries no legal force and the quantity the discharge permit is written about responds in about three hours and is reported as an eight-hour rolling mean.

That gap is what makes the plant worth simulating rather than drawing. A block diagram of a cascade shows an outer loop writing an inner setpoint and says nothing about whether the outer loop is worth installing, because the answer depends on the size of a bill, the shape of a load curve and the wording of a limit. The misconception this application corrects is quantitative rather than structural. Aeration is 50 to 60 percent of a treatment works' electricity [2], so a control change that touches aeration sounds as though it should be worth tens of percent. At 5 m of diffuser submergence the oxygen saturation the bubbles work against is 12.39 mg/L, so a 2 mg/L dissolved oxygen setpoint removes only 16.1 % of the transfer driving force, and the entire practical setpoint range from 1.2 to 3.0 mg/L spans 32 % of a day's aeration energy and no more. Everything a setpoint policy can win has to come out of that third. Read at equal compliance, load-following the setpoint against an ammonia analyzer is worth 194 kWh out of 4446 kWh on the first day of operation and 66 kWh out of 4355 kWh on a settled day, while a most-open-valve controller on the same air header is worth 267 kWh out of 4335 kWh and gives up nothing at all.

DID YOU KNOW? Water, Wastewater and the Aeration Share

Drinking water and wastewater systems account for about 3 to 4 percent of United States energy use, and the energy of water and wastewater utilities is 35 percent of a typical municipal energy budget [3]. Inside a treatment works, aeration can take from 25 to as much as 60 percent of total plant energy [4].

The class of problem is general. Whenever a plant carries a cheap fast measurement that is easy to control and an expensive slow measurement that decides whether the plant is compliant or profitable, the engineering question moves from regulation to the choice of the controlled variable and its setpoint, which is the subject of self-optimizing control [5]. Blend ratio against a laboratory property, furnace excess oxygen against a stack analyzer, reactor temperature against a product specification and coagulant dose against a settled turbidity (Volume 3, Chapter 6) all have this shape, and in each case a supervisory loop with an integral time measured in hours sits above a regulatory loop measured in minutes [1]. The second structure this plant teaches is equally portable. Three throttled users share one air header, so the header pressure is a free variable that no quality measurement constrains, and walking it down until the busiest valve is nearly open removes a pressure drop that was being manufactured and then destroyed. Valve position control of that kind applies to chilled water, instrument air and fuel gas headers with no change of form [6].

However, the published case for ammonia-based aeration control is usually made on a trial lasting days to weeks, and a supervisory loop with an eight-hour integral time takes about a day to forget the setpoint it was commissioned from. The application measures both numbers. This chapter prices a supervisory dissolved oxygen setpoint on a simulated activated sludge plant against a legal limit and an electricity bill, so that the value of load-following aeration is measured at equal compliance and on a settled day rather than quoted from a first-day trial. The equal-compliance accounting finds 4 % on day one and 1.5 % on day three from a configuration nobody touched in between, and it finds the header loop nobody markets ahead of the marketed analyzer loop by a factor of four. The kinetics, the transfer correlation and the tuning arithmetic are all standard.

Section 10.2 traces the history that produced the current answer. Section 10.3 sets out a sixteen-state reduced activated sludge model with its air system, its blower surge line, its instrument chain and its rolling-mean permit, closely enough to rebuild. Section 10.4 describes the four proportional integral loops and the day-ahead predictive supervisor that can replace the ammonia loop. Section 10.5 measures fifteen scenarios, from three valves in manual to a graded February day. Section 10.6 describes the probes, blowers and controllers of real works, Section 10.7 states what the model leaves out, Section 10.8 separates the three decisions the plant carries, and the exercises of Section 10.10 run the same scenarios.

10.2 Where Aeration Control Came From

The activated sludge process was reported in 1914 as a laboratory observation that sewage aerated in the presence of accumulated solids nitrified within a few hours instead of a few days, and the same paper recorded that the accumulated sludge had to be retained between batches for the effect to survive [7]. Retention of solids is the whole of the mechanism, and it is the reason the plant of this chapter has a state with a time constant of days.

Air was originally supplied at whatever rate the blowers ran. The first control action available to an operator was the number of machines in service, and the operating rule was a visual one, because there was no instrument in the tank. Nitrification design became quantitative when growth kinetics were attached to the population. The Monod expression for a specific growth rate limited by a single substrate [8] supplied the form, and the recognition that the sludge age fixes the average growth rate a population is permitted to have converted nitrification from an observation into a design calculation with a safety factor on solids retention time [9]. The consolidation of that work into the activated sludge model family gave a common structure for the kinetics, the stoichiometry and the temperature corrections used in every simulator since [10].

Dissolved oxygen control arrived with the membrane probe. Once a probe could report a tank concentration continuously, the airflow could be trimmed against a setpoint rather than staged, and the whole subsequent history of instrumentation, control and automation on treatment works is organized around which measurement became available when [11]. Two consequences followed quickly. The first is that a dissolved oxygen setpoint is a proxy rather than an objective, because nothing in a discharge permit mentions it. The second is that holding a proxy perfectly can hide a fault, since a controller that opens a valve to compensate for fouled diffusers reports no error at all. Measurement of the alpha factor and its collapse over a diffuser grid's service life made the size of that hidden cost explicit and put oxygen transfer efficiency, rather than dissolved oxygen, at the center of aeration energy work [12, 13]. Modeling the air side as a system of blowers, headers and valves rather than as an ideal source of oxygen followed for the same reason, because the pressure the blower makes and the valves throttle away is invisible on a dissolved oxygen trend [14].

Ammonium analyzers reaching the reliability needed for closed-loop service is the change that produced the current answer. Once effluent or end-of-train ammonia could be read every few minutes, the dissolved oxygen setpoint could be moved against the variable the permit is written about rather than held at a value chosen for the worst hour, and aeration control reviews from the last fifteen years treat that supervisory layer as the standard modern structure [15]. Optimization studies over a diurnal load place the achievable saving in the single digits of percent rather than the tens once compliance is held constant [16], and practice guidance distinguishes the feedback form used here from a feedforward form that anticipates the load [17]. Energy benchmarking of operating works supplies the other half of the argument, because the specific electricity consumption that distinguishes a good plant from a poor one is dominated by equipment condition and load factor rather than by setpoint policy [2].

> HISTORY **2003: Smaller Blowers and Most-Open-Valve Logic**
>
> A federal survey of energy measures at treatment works tells of a medium-sized city's activated sludge plant whose organic load had fallen well below its design. With only one of its inlet-throttled centrifugal blowers running, the dissolved oxygen still sat between 4.5 and 8.0 mg/L, because the machine could not turn down far enough. In July 2003 the plant replaced two of its blowers with units of half the power, fitted new membrane oxygen probes, and replaced single-loop oxygen control with direct airflow control carrying most-open-valve logic. The change saved about 1,000,000 kWh a year, about 9 percent of the plant's energy, and paid back in about three years [4]. Scenario 10 of this chapter meets the same turndown limit at night, and scenario 9 prices the same header logic.

10.3 Process Model

The simulated works treats 20 000 m^3/d of separately sewered municipal sewage carrying 600 kg/d of ammoniacal nitrogen. Screened influent enters an anoxic zone of 1500 m^3 stirred by 15 kW of mixers, passes through three aerated tanks of 1700 m^3 each and leaves by a final channel carrying the ammonium analyzer. Mixed liquor is returned to the head of the anoxic zone at 3.00 times the influent flow from the end of the aerobic train and at 0.75 times it as return activated sludge, so the anoxic zone and each aerated tank see 65.97 m^3/min while the sewage itself crosses the basin in 7.92 h. Air is blown into the three tanks through fine-bubble diffusers 5 m under water, one butterfly valve per tank, from a common header of 250 m^3 fed by a single variable-speed centrifugal blower. The arrangement, the instrument tags and each symbol the equations use are drawn in Figure 10.1.

The clock is in minutes throughout, which is the only base in which a 17.3 s header transient and a four-day biomass recovery are both legible. Every kinetic rate is quoted per day on its slider and converted once, in the parameter declaration, by a factor of 1440. Concentrations are in g/m^3, which equals mg/L, flows in m^3/min, air in standard cubic meters per minute at 1.20 kg/m^3, pressures in kPa gauge except inside the blower work expression, and power in kW. Table 10.1 sorts the controlled,

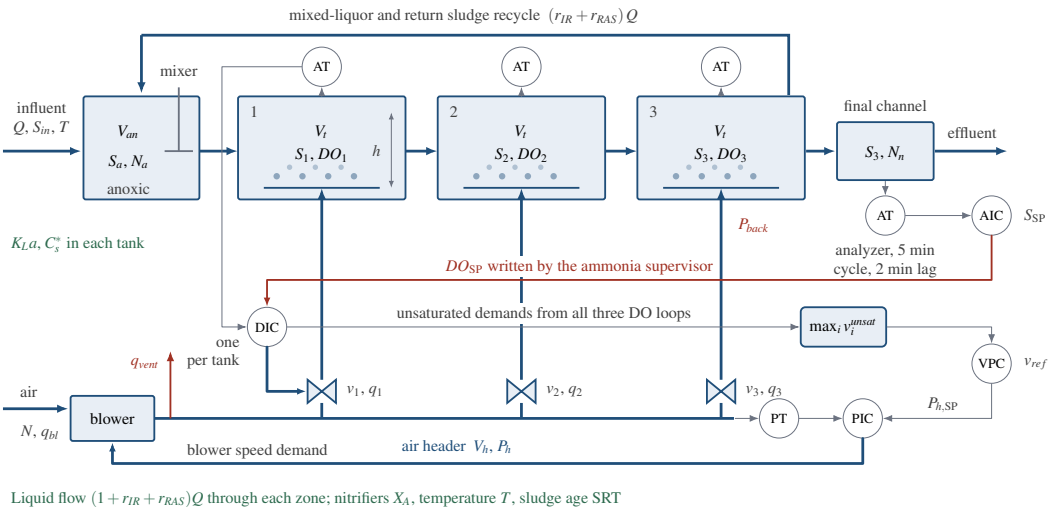


Figure 10.1 The simulated works and every symbol used in the equations. Influent Q at strength S_{in} joins a recycle of $(r_{IR}+r_{RAS})Q$ in the anoxic zone, then crosses three aerated tanks in series. Each tank has its own diffuser grid h below the surface, its own butterfly valve v_i passing q_i from the header at gauge pressure P_h, and its own dissolved oxygen probe. The blower speed N holds the header against the diffuser back pressure P_{back}, and q_{vent} is the air blown off to keep the machine off its surge line. The ammonium analyzer on the final channel measures S_3 and drives the dissolved oxygen setpoint written to all three tank controllers.

Table 10.1 What the Control System Interacts With, at the Design Condition of 13 °C and Average Load. A Range Quoted Against a Controlled Variable Is the Travel of Its Setpoint.

Symbol	Role	Nominal	Range
DO_1	CV, measured	2.00 mg/L	0.3–4.0 mg/L
S_3	CV, measured	0.378 g N/m^3	0.3–4.0 g N/m^3
P_h	CV, measured	62.0 kPa	55–95 kPa
$v_{\max}$	CV, inferred	68.9 %	70–98 %
v_1, v_2, v_3	MV	68.9/50.0/31.7 %	0–100 %
N	MV	59.5 %	48–100 %
DO_{SP}	MV of AIC	2.00 mg/L	0.5–3.0 mg/L
$P_{h,SP}$	MV of VPC	62.0 kPa	55–95 kPa
T sewage	DV, measured	13.0 °C	6–24 °C
Q multiplier	DV, measured	1.00×	0.60–2.50×
S_{in} multiplier	DV, measured	1.00×	0.50–2.00×
diurnal swing	DV, measured	1.00×	0–1.50×
αF	DV, unmeasured	0.55	0.30–0.85
carbon load	DV, unmeasured	1.00×	0.60–1.60×
S_a	state	5.86 g N/m^3	≥ 0
S_1, S_2, S_3	state	3.39/1.40/0.378 g N/m^3	≥ 0
N_a, N_n	state	9.55/15.02 g N/m^3	≥ 0
DO_1, DO_2, DO_3	state	2.00/2.00/1.80 mg/L	0–20 mg/L
X_A	state	101.6 g VSS/m^3	≥ 0
P_h	state	62.0 kPa	0–200 kPa
N	state	59.5 %	0–100 %
v_1, v_2, v_3	state	68.9/50.0/31.7 %	0–100 %
T_w basin	state	13.0 °C	6–24 °C

manipulated, disturbance and state variables at the design condition every steady scenario opens from.

10.3.1 Load, Nitrogen and the Nitrifier Population

Influent flow and strength follow a measured diurnal shape rather than a sine, because the argument for load-following depends on the day being both predictable and lopsided. Twenty-four hourly factors are interpolated with a Catmull-Rom spline so that the load has no corner in it, the strength peaks about an hour after the flow does, and the two multiplied give an ammoniacal nitrogen load running from 222 kg/d at 03:53 to 1032 kg/d at 09:54 about a 600 kg/d mean. Ammonia is carried through four completely mixed zones in series, as shown in Equations (10.1) and (10.2).

$$V_{an}\frac{\mathrm{d}S_a}{\mathrm{d}t} = QS_{in}(1-f_{ass}) + Q_r S_3 - Q_{an}S_a \tag{10.1}$$

$$V_t\frac{\mathrm{d}S_i}{\mathrm{d}t} = Q_{an}(S_{i-1}-S_i) - \frac{\mu_A(T,S_i,DO_i)}{Y_A}X_A V_t \tag{10.2}$$

where Q is the influent flow in $\mathrm{m^3/min}$ at ammoniacal strength S_{in} in $\mathrm{g\,N/m^3}$, V_{an} and V_t are the anoxic and aerated tank volumes in $\mathrm{m^3}$, S_a is the anoxic ammonia, S_i is the ammonia in aerated tank i with $S_0 \equiv S_a$, $Q_r = (r_{IR}+r_{RAS})Q$ is the total recycle, $Q_{an} = Q + Q_r$ is the flow through every zone, f_{ass} is the influent nitrogen fraction assimilated into heterotrophic cells, Y_A is the autotroph yield in g VSS per g N and X_A is the nitrifier concentration in $\mathrm{g\,VSS/m^3}$. Both sides of (10.1) carry grams of nitrogen per minute. The specific growth rate is Monod in ammonia and in oxygen with an Arrhenius correction on the maximum, as shown in Equation (10.3).

$$\mu_A = \hat{\mu}_{A,20}\,\theta_A^{T-20}\,\frac{S}{K_{NH}+S}\,\frac{DO}{K_{O,A}+DO} \tag{10.3}$$

where $\hat{\mu}_{A,20}$ is the maximum specific growth rate at 20 °C in 1/min, θ_A is the temperature coefficient, K_{NH} is the ammonia half-saturation constant and $K_{O,A}$ is the oxygen half-saturation constant [8, 10]. The oxygen factor in (10.3) is the entire justification for holding a dissolved oxygen setpoint, and because it saturates the justification runs out. With $K_{O,A} = 0.5\,\mathrm{mg/L}$ the factor is 50 % at 0.5 mg/L, 67 % at 1 mg/L, 80 % at 2 mg/L, 86 % at 3 mg/L and 90 % at 4.5 mg/L.

The population is a state in its own right, growing at the mean of the three tank rates, decaying, and removed by wasting at the reciprocal of the solids retention time, as shown in Equation (10.4).

$$\frac{\mathrm{d}X_A}{\mathrm{d}t} = \left(\bar{\mu}_A - b_A(T) - \frac{1}{\mathrm{SRT}}\right)X_A, \qquad b_A = b_{A,20}\,\theta_B^{T-20} \tag{10.4}$$

where $\bar{\mu}_A$ is the mean of the three tank growth rates, $b_{A,20}$ is the decay rate at 20 °C with temperature coefficient θ_B, and SRT is the solids retention time in minutes. Setting the bracket in (10.4) to zero gives the least intuitive property of the plant. At steady state the wasting rate, not the temperature, fixes the average growth rate the nitrifiers are permitted to have, so $\bar{\mu}_A = b_A(T) + 1/\mathrm{SRT}$ moves only from 0.2411/d at 13 °C to 0.2305/d at 9 °C on a seven-day sludge age, against a maximum rate that falls from 0.4917 to 0.3723/d over the same interval. The steady population is therefore 101.6 g VSS/m³ at 13 °C and 102.2 g VSS/m³ at 9 °C, and the effluent ammonia carries the whole of the loss by rising from 0.378 to 1.378 g N/m³. Cutting the sludge age from 7 to 5 days is what actually moves the population, to 79.5 g VSS/m³ in the steady state and through 96.5, 92.6, 89.7, 87.4 and 85.7 g VSS/m³ on the first five days after the decision.

Nitrate is carried as two pools, one in the anoxic zone and one for the whole aerobic train, since nitrate is conserved along the train and nothing consumes it there. Denitrification in the anoxic zone is carbon-limited and scales with the raw sewage arriving rather than with the nitrate on offer, as shown in Equations (10.5) and (10.6).

$$V_{an}\frac{\mathrm{d}N_a}{\mathrm{d}t} = Q_r N_n - Q_{an}N_a - V_{an}r_{DN} \tag{10.5}$$

$$r_{DN} = k_{DN,20}\,\theta_{DN}^{T-20}\,\frac{N_a}{K_{NO3}+N_a}\,\frac{QS_{in}}{\bar{Q}\bar{S}_{in}} \tag{10.6}$$

where N_a and N_n are the anoxic and aerobic nitrate concentrations in g N/m^3, r_{DN} is the denitrification rate in g N/m^3 per minute, $k_{DN,20}$ is that rate at 20 °C with temperature coefficient θ_{DN}, K_{NO3} is the nitrate half-saturation constant and $\bar{Q}\bar{S}_{in}$ is the daily mean nitrogen load. The aerobic pool N_n is fed by $Q_{an}(N_a - N_n)/(3V_t)$ plus the mean nitrification rate of the three tanks. At the design condition the anoxic zone holds 9.55 g N/m^3 of nitrate and the aerobic train 15.02 g N/m^3.

10.3.2 Oxygen Transfer, the Air Header and the Blower

Dissolved oxygen in each tank is a balance of what the upstream zone brings, what the diffusers transfer and what the biology consumes, as shown in Equation (10.7).

$$\frac{\mathrm{d}DO_i}{\mathrm{d}t} = \frac{Q_{an}}{V_t}\,(DO_{i-1} - DO_i) + K_L a_i\,(C_s^{*} - DO_i) - R_i \tag{10.7}$$

where $DO_0 \equiv 0$ because liquor arrives from the anoxic zone with no oxygen in it, R_i is the total oxygen uptake rate in that tank in g/(m^3 min), and C_s^{*} is the saturation concentration the bubbles see. Omitting the recycle term in (10.7) understates the air tank 1 needs by 6.5 %, because at a combined recycle of 3.75 the oxygen-free stream entering tank 1 is nearly four times the influent. The transfer coefficient is a power law in diffuser flux and the saturation carries half the submergence as static head, as shown in Equations (10.8) and (10.9).

$$K_L a_i = \alpha F\, k_{La,0} \left(\frac{q_i}{V_t}\right)^{n} \tag{10.8}$$

$$C_s^{*} = \beta\, C_{s,w}(T) \left(1 + \frac{\rho g h}{2 P_{atm}}\right) \tag{10.9}$$

where q_i is the air to tank i in Nm^3/min, $n = 0.75$ is below unity because the bubbles coalesce and rise faster as the flux rises, αF lumps the mixed-liquor alpha factor with diffuser fouling, β is the salinity correction and $C_{s,w}$ is a cubic fit to the clean-water solubility tables accurate to 0.15 g/m^3 between 5 and 30 °C [12]. At 5 m and 13 °C, (10.9) returns 12.39 mg/L against 10.50 mg/L for the same water at the surface. Inverting (10.8) and (10.7) at steady state gives $q_i \propto [R_i/(C_s^{*} - DO_i)]^{1/n}$, so the air rises faster than linearly in the demand and faster than linearly in the dissolved oxygen demanded. At the design load, holding 3 mg/L instead of 2 mg/L costs 17.3 % more air and holding 1 mg/L saves 14.7 %, and at constant oxygen uptake the air per additional milligram is 1.97 times as steep at 3.5 mg/L as at 0.5 mg/L.

Each valve is an orifice in series with an inherent exponential characteristic, and the header is the only capacity in the air system, as shown in Equations (10.10) and (10.11).

$$q_i = C_v\, f(v_i) \sqrt{P_h - P_{back}}, \qquad f(v) = \frac{\mathrm{e}^{R_v v/100} - 1}{\mathrm{e}^{R_v} - 1} \tag{10.10}$$

$$\frac{V_h}{P_{atm}} \frac{\mathrm{d}P_h}{\mathrm{d}t} = q_{bl}(N, P_h) - \textstyle\sum_i q_i - q_{vent} \tag{10.11}$$

where C_v is the wide-open valve capacity, R_v is the rangeability exponent, ρ is the liquor density, g is gravitational acceleration, $P_{back} = \rho g h + \Delta P_{dif} = 53.05$ kPa is the pressure needed before any air reaches the water, and V_h is the header volume. With 250 m^3 of pipework, one Nm^3/min of imbalance moves the header 0.405 kPa/min. Both terms on the right of (10.11) push back on pressure, at −1.72 Nm^3/min per kPa from the blower curve and +6.82 Nm^3/min per kPa from the three valves at the design point, so the closed balance has a time constant of 17.3 s and a gain of 0.660 kPa per percent of blower speed. That is a factor of 21 faster than the 6.20 min of a tank, and the ratio is free rather than tuned. The fermenter of Volume 3, Chapter 3 is the opposite case, with its lags within a factor of two of one another, so there the speed of the inner loop has to be earned by tuning.

The blower is a single variable-speed centrifugal machine with a fitted quadratic head curve, a fan law and a surge line, as shown in Equations (10.12) and (10.13).

$$q_{bl} = \sqrt{\frac{a_0 (N/100)^2 - P_h}{a_2}} \ \text{when positive, otherwise 0} \tag{10.12}$$

$$q_{vent} = k_{vent} \max\left(0,\ \frac{N}{100} q_{surge} - q_{bl}\right) \tag{10.13}$$

where a_0 is the shut-off head at full speed, a_2 is the droop of the fitted head curve, q_{surge} is the surge flow at full speed and k_{vent} is the anti-surge valve gain. Equation (10.13) is a tight proportional anti-surge controller collapsed into the flow balance, and every standard cubic meter it discards has been compressed and paid for. Volume 4, Chapter 2 treats surge itself and the protective override that keeps a machine off it. Shaft and motor power follow the adiabatic compression of the delivered plus vented air, as shown in Equation (10.14).

$$W = \frac{\dot{m}\, c_p T_1}{\eta_{is}\, \eta_m} \left[\left(\frac{P_{atm} + P_h}{P_{atm} - \Delta P_f} \right)^{0.283} - 1 \right] \tag{10.14}$$

where W is the shaft and motor power in kW, $\dot{m} = \rho_{air}(q_{bl} + q_{vent})/60$ in kg/s, $c_p = 1005\,\mathrm{J/(kg\,K)}$ and T_1 is the inlet temperature. At the design condition (10.14) returns 158.9 kW for 122.1 $\mathrm{Nm^3/min}$ against 62 kPa, which is 1.64 kg of oxygen dissolved per kWh and 12.8 % transfer efficiency, both realistic field values for a five-meter fine-bubble grid in mixed liquor [13]. Adding the 15 kW of anoxic mixers and dividing by an aeration share of 0.55 gives a site draw of 316 kW, or 0.379 $\mathrm{kWh/m^3}$ treated.

10.3.3 The Permit, Which Is a Mean and Not a Fence

The discharge limit is an eight-hour rolling mean of effluent ammonia, held as a ring of 96 five-minute bins rather than as an exponentially weighted average, as shown in Equation (10.15).

$$\bar{S}_e(t) = \frac{1}{T_p} \int_{t-T_p}^{t} S_3(\tau)\, \mathrm{d}\tau \ \le\ S_{limit}, \qquad S_3 \le S_{max} \tag{10.15}$$

with $T_p = 480\,\mathrm{min}$, $S_{limit} = 2.5\,\mathrm{g\,N/m^3}$ and an absolute upper tier $S_{max} = 5.0\,\mathrm{g\,N/m^3}$ that is not averaged. A ring of real bins reproduces both halves of a rolling mean, because a morning excursion stays on the record until it falls out of the window and a quiet afternoon genuinely pays for it. That property is what makes a load-following setpoint legitimate rather than merely cheap, and the upper tier exists because a plant could otherwise average its way through anything.

10.3.4 Constants, the Opening State and the Numerics

The constants are collected in Table 10.2. Most can be reached from the parameter panel, five cannot, and that split matters because the hidden five are the ones a student has to infer from a test rather than read off a screen.

No scenario opens on a transient. The opening state is solved for rather than approached, and the population comes from bisection on the residual $\bar{\mu}_A(X_A) - b_A - 1/\mathrm{SRT}$ at the daily average load, because a plant that has been running since March does not start the day at whatever the current hour happens to demand. The ammonia chain is then swept to convergence at the load actually arriving at $t = 0$, since the anoxic zone is fed by a recycle that comes from the end of the train and the loop has to be iterated rather than marched. Valve positions, header pressure and blower speed are then solved from the oxygen demand so that (10.7) and (10.11) are stationary. The application's test suite verifies this by evaluating every derivative at the opening state and requiring the largest relative value to be below 10^{-6}.

Table 10.2 Constants of the Model. The Five Marked Hidden Never Reach the Screen. Kinetic Rates Are Held per Minute and Presented per Day.

Symbol	Meaning	Value	Unit	Visibility
$\bar{Q}$	dry-weather average flow	13.889	m^3/min	adjustable
$\bar{S}_{in}$	average influent ammonia	30.0	g N/m^3	adjustable
f_{ass}	influent N assimilated	0.12	—	adjustable
V_{an}	anoxic zone volume	1500	m^3	adjustable
V_t	volume of each aerated tank	1700	m^3	adjustable
h	diffuser submergence	5.0	m	adjustable
r_{IR}	internal recycle ratio	3.00	$\times Q$	adjustable
r_{RAS}	return sludge ratio	0.75	$\times Q$	adjustable
$\hat{\mu}_{A,20}$	maximum autotroph growth	0.80	1/d	adjustable
$b_{A,20}$	autotroph decay	0.12	1/d	adjustable
θ_A	growth temperature coefficient	1.072	—	adjustable
K_{NH}	ammonia half-saturation	0.70	g N/m^3	adjustable
$K_{O,A}$	oxygen half-saturation	0.50	mg/L	adjustable
Y_A	autotroph yield	0.24	g VSS/g N	adjustable
SRT	solids retention time	7.0	d	adjustable
$k_{DN,20}$	anoxic denitrification rate	0.18	g N/m^3/min	adjustable
K_{NO3}	nitrate half-saturation	0.30	g N/m^3	adjustable
θ_{DN}	denitrification coefficient	1.08	—	adjustable
β	salinity correction	0.95	—	adjustable
$k_{La,0}$	transfer correlation coefficient	2.50	1/min	adjustable
n	airflow exponent	0.75	—	adjustable
$R_{c,0}$	carbonaceous demand at mean load	2656	g/min	adjustable
f_{endog}	endogenous share of that demand	0.25	—	adjustable
V_h	air header volume	250	m^3	adjustable
ΔP_{dif}	diffuser pressure loss	4.0	kPa	adjustable
C_v	valve capacity, wide open	55.0	$Nm^3/min/\sqrt{kPa}$	adjustable
R_v	valve rangeability exponent	2.5	—	adjustable
τ_v	air valve travel time	0.25	min	adjustable
a_0	blower shut-off head	275.0	kPa	adjustable
a_2	blower curve droop	0.00238	kPa min^2/Nm^6	adjustable
q_{surge}	surge flow at full speed	82	Nm^3/min	adjustable
$N_{\min}$	minimum blower speed	48	%	adjustable
τ_{bl}	blower speed ramp	0.20	min	adjustable
η_{is}	isentropic efficiency	0.72	—	adjustable
η_m	motor and drive efficiency	0.95	—	adjustable
ΔP_f	inlet filter and silencer loss	2.0	kPa	adjustable
W_{mix}	anoxic zone mixers	15	kW	adjustable
τ_{DO}	DO probe time constant	0.75	min	adjustable
τ_{Ph}	header transmitter lag	0.05	min	adjustable
t_{cyc}	analyzer cycle time	5.0	min	adjustable
θ_{NH}	analyzer transport lag	2.0	min	adjustable
τ_T	basin thermal lag	480	min	adjustable
T_p	permit averaging window	480	min	adjustable
S_{limit}	rolling-mean permit	2.5	g N/m^3	adjustable
S_{max}	absolute upper-tier limit	5.0	g N/m^3	adjustable
η_{aer}	aeration share of site power	0.55	—	adjustable
θ_B	decay temperature coefficient	1.029	—	hidden
$Y_{O/N}$	oxygen per unit N oxidized	4.33	g O_2/g N	hidden
k_{vent}	anti-surge valve gain	2.0	—	hidden
T_1	blower inlet temperature	293	K	hidden
ρ_{air}	standard air density	1.20	kg/m^3	hidden

The engine of Section 1.6.1 advances the physics under a fourth-order Runge-Kutta integrator whose step is frozen at 0.02 min (1.2 s). Controllers fire every 0.1 min (6 s), which is the scan a works distributed control system actually runs, and two samples of everything are logged per simulated minute, so one day of plant is 72 000 steps and 2880 records. The default seed is 20 260 914.

10.3.5 Assumptions

Four completely mixed zones and no clarifier. There is no dispersion model, no dead volume and no settling tank. Return activated sludge is a flow ratio and the mixed-liquor solids are a parameter, so the application cannot show sludge bulking, solids washout in a storm, or any of the settleability problems that actually close plants down.

One lumped autotroph population. Real nitrification is two organisms in series, ammonia to nitrite and nitrite to nitrate, and nitrite accumulation under transient low-oxygen conditions is a documented failure mode this structure cannot represent. Nitrous oxide, which is where a genuinely low dissolved oxygen setpoint stops being free, is not modeled at all [18]. The application enforces a setpoint floor as a control limit instead, which is what a real plant does but not why.

Heterotrophic oxygen demand is a forcing, not a state. The carbonaceous demand is scaled by the arriving load and split between the three tanks in fixed proportions of 0.50, 0.30 and 0.20, so it responds to the influent instantly where a real basin's stored substrate would smear it over an hour or two. This application's morning air peak is therefore slightly sharper than a real one.

Denitrification is carbon-limited and proportional to arrival. There is no readily biodegradable COD state and no separate anoxic biomass, so (10.6) gets the direction and the magnitude of the nitrate right and would get the response to a methanol dose wrong.

Oxygen transfer is one power law with one condition slider. Alpha, beta and fouling are lumped, there is no dependence on mixed-liquor solids and no flux distribution across the grid, and (10.9) uses the mid-depth approximation, which overstates transfer near the surface and understates it at the floor.

One blower standing in for a hall. There are no inlet guide vanes, no staging of multiple machines, no discharge temperature limit and no motor thermal model, so a_0 is the intercept of a fit rather than a measured head and q_{surge} is the minimum flow of the whole hall rather than of one impeller. The anti-surge controller is (10.13) inside the flow balance rather than a loop a student can detune.

The analyzer fails only by being slow. A real wet-chemistry instrument also drifts between calibrations, fails to a last good value that looks entirely plausible, and reports its own health. A supervisory loop that trusts a stuck reading is a more common industrial failure than one that is badly tuned, and this application cannot stage it.

Sludge age is a parameter rather than a wasting flow. The application can show what a solids retention time decision does to the population and cannot show the settlement-tank capacity or the sludge-treatment load that paying for it costs.

10.4 Controller Design

10.4.1 The Shared Algorithm

All six controllers, three dissolved oxygen loops and three single loops above them, run the velocity-form algorithm of Section 1.6.2, written here in the symbols the later equations use, as shown in Equations (10.16) and (10.17).

$$e_k = a\,(\mathrm{SP}_k - \mathrm{PV}_k) \tag{10.16}$$

$$P_k = K_c e_k, \qquad D_k = -K_c \tau_D\, a \left.\frac{\mathrm{d}\mathrm{PV}^f}{\mathrm{d}t}\right|_k$$

$$\Sigma_k = \Sigma_{k-1} + (P_k - P_{k-1}) + (D_k - D_{k-1}) \tag{10.17}$$

with $a = +1$ on the two reverse acting loops and $a = -1$ on the two direct acting ones. The running sum Σ_k collapses to $P_k + D_k$ under whatever gains are in force, and the filtered measurement PV^f advances by an exact zero-order-hold recursion. Every loop in this application uses $\tau_D = 0$, so the derivative term contributes nothing and the filter is inert. The output is formed and the integral is

Table 10.3 The Implemented Loops. All Four Are Proportional Integral With $\tau_D = 0$ and a 0.1 min Scan.

Loop	CV	MV	K_c	τ_I (min)	Action
PIC	P_h	N	1.20	0.5	reverse
DIC ×3	DO_i	v_i	8.0	6	reverse
VPC	$v_{\max}$	$P_{h,\mathrm{SP}}$	0.15	45	direct
AIC	S_3	DO_{SP}	1.20	480	direct

back-calculated against whatever was applied, as shown in Equation (10.18).

$$\begin{aligned} u_k^{raw} &= u_{bias} + \Sigma_k + I_{k-1} \\ u_k &= \min(u_{\max}, \max(u_{\min},\ u_k^{raw})) \\ I_k &= I_{k-1} + \left(\frac{K_c}{\tau_I} e_k + \frac{u_k - u_k^{raw}}{T_t} \right) \Delta t_c \end{aligned} \tag{10.18}$$

where the integral update is disabled when its gain is zero. T_t is the tracking time constant and defaults to τ_I, and $\Delta t_c = 0.1$ min is the scan period of every loop [1]. The running sum Σ_k is the S_k of Equation (1.15) and u_k^{raw} is the v_k of Equation (1.16). Anti-windup is back-calculation on the integral alone and Σ_k is never clamped, for the reason Section 1.6.2 gives. No loop carries a rate limit. In manual the integral of (10.18) is replaced by $I_k = u^{man} - u_{bias} - \Sigma_k$, which lands the output on the operator's number, so a transfer either way is bumpless.

10.4.2 Four Loops, Three Orders of Magnitude Apart

Table 10.3 lists what each loop measures, what it moves, and the tuning the application ships. Ordering the four by speed is the design decision, and the four signs are where an error is easiest to make.

More blower speed raises the header and more valve opening raises the tank oxygen, so both of those process gains are positive and both controllers are reverse acting with $a = +1$ in (10.16). More dissolved oxygen means less effluent ammonia, and raising the header pressure puts more differential across every valve so that each has to close, so both supervisory gains are negative and both controllers are direct acting with $a = -1$. Getting the ammonia loop's sign backward produces a controller that answers rising ammonia by turning the air down, which is stable for about an hour and then is not, and the application's test suite pins all four directions by nudging each loop away from the point it is already tracking.

The header pressure controller writes blower speed over 48 to 100 %. Its lower bound is not a motor limit, because below about 48 % the machine cannot raise the header above P_{back} at all, so the turndown is set by the depth of the tank. The three dissolved oxygen controllers write valve position over 0 to 100 % against setpoints in the ratio 1.00, 1.00 and 0.90, which is tapered aeration carried into the setpoints as well as into the diffuser density. A gain of 8 %/mg/L against a measured process gain of 0.180 mg/L per percent of valve and a 6.20 min lag is about half of what an internal model control rule would return, and the detuning is deliberate because the same three controllers see three different oxygen demands that all move together through the day.

10.4.3 Most-Open-Valve Control

The valve position controller holds the busiest air valve near a target opening by moving the header pressure setpoint, over 55 to 95 kPa with a default target of 90 %. Two details decide whether it works. It has to be slow, because it sits above three dissolved oxygen loops and becomes a disturbance to them if it moves faster than they settle, hence 45 min of integral time against their six. And it has to watch the demand rather than the shaft, as shown in Equation (10.19).

$$v_{\max} = \max_i\ u_i^{raw}, \qquad P_{h,\mathrm{SP}} = \mathrm{PI}\big(v_{ref},\ v_{\max}\big) \tag{10.19}$$

where u_i^{raw} is the unsaturated output of dissolved oxygen loop i from (10.18) rather than its clamped value. A valve pinned at 100 % cannot report that it wanted 130 %, so a controller reading valve position stops raising the header exactly when the plant most needs it raised [6]. The same structure returns load to an absorption chiller in Volume 4, Chapter 9.

Pause. Try Exercise 10.2 before reading on.

10.4.4 The Ammonia Supervisor and Its Instrument

The ammonium analyzer is modeled as a transport lag of 2 min from the final channel to the kiosk, followed by a sample and hold on a 5 min instrument cycle. Additive colored noise with a standard deviation of 0.05 g N/m^3 is applied before the delay and held along with the sample. What the supervisor reads is therefore a staircase that is already several minutes old. The supervisor writes the dissolved oxygen setpoint over a hard travel of 0.2 to 4.5 mg/L and an operator travel of 0.5 to 3.0 mg/L, the floor because a basin below about 0.5 mg/L goes anoxic inside the floc and starts producing nitrous oxide [18], and the ceiling because (10.3) is 86 % saturated at 3 mg/L and 90 % at 4.5 mg/L while the air still costs full price.

An integral time of 480 min on a loop whose measurement updates every five minutes looks absurd and is correct. A change in dissolved oxygen takes about three hours to reach the analyzer through the basin and the kinetics, and the quantity the loop is defending is itself an eight-hour mean by (10.15), so the supervisor is tuned to the plant it supervises rather than to the instrument it reads. Whenever a supervisor is out of service the corresponding controller is run in tracking mode against the operator's own number, so all four loops are seeded from the plant at $t = 0$ and none of them steps its actuator on the first scan.

10.4.5 A Day-Ahead Predictive Supervisor

A model predictive controller (MPC) is the fourth value of the aeration control option, after manual, fixed dissolved oxygen and ammonia-based control. Fixed dissolved oxygen remains the default, and only scenario 14 opens on the MPC. It moves the handle the ammonia supervisor moves, the dissolved oxygen setpoint of all three tanks, and everything beneath that setpoint acts as shipped, from the three dissolved oxygen loops and their taper to the header loop, most-open-valve control when selected, the blower's speed limits and its anti-surge vent. Every $T_s = 15$ min one optimization plans the setpoint over $H = 96$ steps, a full day of one load cycle and three permit windows, in 19 move blocks of 1, 1, 1, 1, 2, 2, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 12, 12 and 12 steps. Only the first block takes effect, staying in force until the next decision, which is the receding horizon of Section 1.4.3 [19, 20]. A case study of predictive control on an activated sludge process is reported in [21]. On every scan the setpoint is clamped to the supervisor's operator floor and ceiling, 0.5 and 3.0 mg/L by default.

The controller's model is written from Section 10.3 with the declared values of Table 10.2 and the design disturbances (13 °C, $\alpha F = 0.55$, nominal carbon load), never with the run's own settings, so a learner's change to the plant appears to the controller as mismatch. It keeps the four ammonia balances and one lag for the dissolved oxygen loops, as shown in Equation (10.20).

$$
\begin{aligned}
V_{an}\frac{\mathrm{d}\hat{S}_a}{\mathrm{d}t} &= \hat{Q}\hat{S}_{in}(1-f_{ass}) + \hat{Q}_r\hat{S}_3 - \hat{Q}_{an}\hat{S}_a + V_{an}\,d \\
V_t\frac{\mathrm{d}\hat{S}_i}{\mathrm{d}t} &= \hat{Q}_{an}\left(\hat{S}_{i-1}-\hat{S}_i\right) - \frac{\mu_A\left(13,\hat{S}_i,\widehat{DO}_i\right)}{Y_A}X_A^d V_t \\
\tau_c\frac{\mathrm{d}\widehat{DO}_1}{\mathrm{d}t} &= DO_{\mathrm{SP}} - \widehat{DO}_1, \qquad \widehat{DO}_i = \gamma_i\widehat{DO}_1
\end{aligned}
\tag{10.20}
$$

where hats mark the controller's quantities, $i = 1,2,3$, $\hat{Q}$ and $\hat{S}_{in}$ come from the preview, μ_A is (10.3) at 13 °C, $X_A^d = 101.58\,\mathrm{g\,VSS/m^3}$ is the design-point population of (10.4) held constant, d is an unexplained ammonia load into the anoxic zone in g N/m^3 per minute, $\gamma_2 = 1.00$ and $\gamma_3 = 0.90$ are

the operator's taper ratios, and $\tau_c = 4.3\,\text{min}$ is the closed-loop answer of the dissolved oxygen loops, the time constant plus dead time fitted to ±0.3 mg/L setpoint steps at 2 mg/L on a flat, noise-free day (fits at 1 to 3 mg/L leave residuals of at most 2.2 %). The predicted power $\hat{W}_k$ applies (10.8) to (10.14) at the declared values to the air each tank needs for its forecast carbonaceous and nitrification uptake, at the measured header pressure held for the day, with the surge vent and the 15 kW of mixers. A setpoint step from 2.0 to 2.5 mg/L changes the effluent ammonia six hours later by $-0.0646\,\text{g N/m}^3$ on the plant and $-0.0641\,\text{g N/m}^3$ on the model, and a step to 1.5 mg/L by $+0.1337$ and $+0.1319\,\text{g N/m}^3$. Driven open loop through scenario 5's day by the probe readings, the model follows the effluent ammonia with an RMS error of $0.137\,\text{g N/m}^3$ and the power with 1.6 kW, 0.9 % of the mean.

The oxygen factor of (10.3) makes (10.20) strongly nonlinear in the variable the plan moves. At steady state the gain from dissolved oxygen to effluent ammonia is $-0.001\,\text{g N/m}^3$ per mg/L at 04:00 and 3 mg/L but $-18\,\text{g N/m}^3$ per mg/L at 20:00 and 0.5 mg/L, and along the plans actually chosen the sensitivity over one decision period still runs from -0.0069 to $-0.465\,\text{g N/m}^3$ per mg/L, a factor of 67. The framework's shared linear MPC class carries one fixed model and cannot follow that, so this application forms its own linear time-varying prediction and hands the condensed program to the framework's shared quadratic program solver, the linearization along the plan that Section 1.4.2 lists among the prediction models of this volume. At each decision the model is advanced by fourth-order Runge-Kutta in 2.5 min substeps along the current plan, linearized by finite differences at all 96 steps, and the sensitivity of every step's state to every block's setpoint u_j is accumulated. The plan minimizes Equation (10.21).

$$\begin{aligned} \mathscr{J} = &\sum_{k=1}^{H} \hat{W}_k T_s + \sum_{h=1}^{24} \left[\sigma_1 \left(\varepsilon_h + \eta_h \right) + \sigma_2 \left(\varepsilon_h^2 + \eta_h^2 \right) \right] \\ &+ \lambda \sum_{j=1}^{19} \left(u_j - u_{j-1} \right)^2 \\ \text{s.t. } & \hat{\bar{S}}_{e,k} \le \bar{S}_c + \varepsilon_h, \quad \hat{S}_{3,k} \le S_{max} - 0.5 + \eta_h \\ & \varepsilon_h, \eta_h \ge 0, \quad DO_{lo} \le u_j \le DO_{hi} \end{aligned} \tag{10.21}$$

Here h is the hour holding step k, T_s is in hours, u_0 is the setpoint applied now, DO_{lo} and DO_{hi} are the operator floor and ceiling, and ε_h and η_h are slacks. The predicted rolling mean $\hat{\bar{S}}_{e,k}$ is the window of (10.15) formed from the predicted $\hat{S}_3$ and, for the part already past, the controller's ring of the last 32 quarter-hour means of the analyzer reading. The plan's ceiling $\bar{S}_c = 2.2\,\text{g N/m}^3$ is the challenge's target, $0.3\,\text{g N/m}^3$ inside S_{limit}, a back-off in the sense of Section 1.4.7. Both ceilings are soft with one slack per hour, with the linear and quadratic prices of Equation (1.7), at $\sigma_1 = 2000$ kWh per g N/m^3 and $\sigma_2 = 2 \times 10^4$ kWh per (g N/m^3)^2, and the move price is $\lambda = 100$ kWh per (mg/L)^2. Energy is linear and nothing below the ceilings is priced. The program has 67 variables and 278 inequality rows, the smallest eigenvalue of its Hessian is 1.30, so it is strictly convex, and a dual active-set method [22] solves it.

Up to four such programs form one sequential quadratic program, warm started from the previous plan shifted by one decision as in real-time iteration [23]. Each pass relinearizes along the last plan, moves no block by more than 1.0 mg/L, and keeps the longest of the steps 1, 1/2, 1/4 and 1/8 that lowers the nonlinear objective, stopping when none does or no block would move 0.01 mg/L. Over 6530 plans, every scenario under the MPC and nine mismatched days, no program ended other than optimal and no plan was rejected, and 60 % used all four passes. A compass search over all 19 blocks of the same objective, started from the returned plan, lowers it by at most 0.002 % at five of six decisions examined and by 0.12 % at a run's first plan, whose first two passes hit the step limit.

A steady-state Kalman filter carries the four ammonia states and d. Every 0.1 min scan, under every aeration structure, it integrates (10.20) with the probe readings in place of $\widehat{DO}_i$, and at each decision it corrects all five states from the analyzer's held reading against the model's $\hat{S}_3$ of 4.5 min

earlier, the declared transport lag plus half the cycle. The gain belongs to the 15 min model at the design point, with process variances per decision of 10^{-4} (g N/m^3)^2 on each concentration and $10^{-4}\varphi$ on d, $\varphi = 1$, and a measurement variance of 0.0225 (g N/m^3)^2, the analyzer noise plus 0.02 for its staircase. The random walk d makes the plan offset-free, as an input-disturbance estimate does in Section 1.4.5 [24].

The preview is a switch. Typical day feeds the declared average flow and strength through the application's diurnal curve at the clock hour, a calendar rather than the day's measured load, which on the application's ordinary dry day is exactly the load that arrives, while any other day reaches the plan only through the analyzer. Off feeds the declared average all day, which is all the supervisor knows of the hours ahead, and is the like-for-like comparison. No perfect preview is offered. The prices λ and φ were set once, before any comparison, on scenario 7's day. Of 35 pairs, 23 kept the setpoint travel per day within the supervisor's 12.58 mg/L at 1.30 g N/m^3, the worst rolling mean at or below 2.25 g N/m^3 and no minute over the permit, and the largest λ within 1 % of the least energy was taken, $\lambda = 100$ with $\varphi = 1$ at 8023 kWh over days 2 and 3 against 8016 kWh. It lay on the grid's edge, and $\lambda = 300$ cost 1.3 % more.

The plan is rejected whenever the solver reports anything but optimal, a move is not finite or it leaves the floor and ceiling, and the ammonia supervisor, which tracks the applied setpoint on every scan, takes over until the next accepted plan. Under the MPC the dashed setpoint line on the ammonia trend still shows that fallback supervisor's 1.90 g N/m^3 aim, which is nothing the plan aims at. Three rejections forced at 600 min in scenario 14 with the noise off move the setpoint by 0 mg/L, and the plan is back at 645 min. The controller reads the probe readings, the analyzer's held reading, the measured header pressure, the clock hour, the applied setpoint, the operator's floor, ceiling and taper and its three panel settings, and never the plant's states, parameters or disturbances. A test feeds a second plant, different in 14 parameters, 7 disturbances and every state, the first plant's instrument readings and obtains the same 39 plans, so the plant, its instruments and (10.20) stay three separate models. In two runs of scenario 14 in Chrome at a speed of 200 simulated minutes per second a plan takes a mean of 13.1 and 13.2 ms, 17.4 and 17.6 % of one core, at 59.6 to 59.8 frames per second, and at a speed of 20 at most two frames in 15 s take 33.4 ms.

10.5 Results

Fifteen scenarios build up in a laboratory order, and Table 10.4 lists them. Each pins the opening mode, the structure in service, the four tunings, the disturbance settings and whatever events are timed into it, and declares its objective ahead of the run. All run results come from the app's engine at the default seed. Unless stated otherwise, settings are applied before Run, and Reset retains those settings and recreates the same initial plant and controller state (Section 1.6.1). Counterfactual model benchmarks are identified explicitly.

10.5.1 The Design Condition and the Two Fast Loops

Scenario 2 flattens the diurnal swing without changing its average and closes the two regulatory loops. Ammonia falls 5.86, 3.39, 1.40 and 0.378 g N/m^3 through the four zones on airflows of 67.7, 36.6 and 17.8 Nm^3/min through valves at 68.9, 50.0 and 31.7 %. The tanks are not three identical stages. At 3.39 g N/m^3 the ammonia factor in (10.3) is already 83 % of its maximum, so tank 1 nitrifies close to flat out and removes a fixed mass per minute, while tank 3 at 0.378 g N/m^3 sits at 35 % of the factor and works at a third of the rate on a tenth of the concentration.

Scenario 1 makes the header coupling visible by hand. After one minute of Run, opening valve 1 from 68.9 to 78.9 % and touching nothing else pulls the header from 62.0 to 59.7 kPa, and because (10.10) is a square root in the differential, tank 2 loses 4.93 Nm^3/min and tank 3 loses 2.4 Nm^3/min without either valve moving. Half an hour later tank 1 is at 2.73 mg/L, tank 2 at 1.63 mg/L and tank 3 at 1.17 mg/L on its way to 1.10 mg/L. Scenario 4 shows what that coupling costs when the inner loop is thrown away. With the header controller detuned to $K_c = 0.12$ % per kPa

Table 10.4 The Fifteen Scenarios and What Each One Isolates

#	Scenario	What it exercises
1	Three valves by hand	parallel users on one header
2	Flat load, loops closed	tapered aeration, Monod in ammonia
3	Dissolved oxygen steps	process gain that drifts under the loop
4	Header loop detuned then repaired	why the inner loop must be fast
5	A whole day at 2.0 mg/L	load, lag and the evidence gap
6	The same day at 1.2 mg/L	the fixed-setpoint frontier
7	Ammonia-based control	equal-compliance accounting
8	The supervisor at $\tau_I = 10$ min	tuning to the instrument
9	Most-open-valve control	removing an unused pressure drop
10	A dry Sunday night	blower surge line, reduced marginal saving
11	A three-day cold snap	the state with no fast handle
12	Liquors at 09:00 and at 01:00	timing, returned mass and preceding state
13	Diffusers foul over a year	a failure that holds every setpoint
14	The eight-hour mean, planned a day ahead (MPC)	planning the permit's own mean against the supervisor
15	Graded February day	the whole structure under a budget

and $\tau_I = 8$ min, a 1.2 mg/L setpoint step sags the header to 55.4 kPa and leaves all three tanks 0.68, 0.61 and 0.46 mg/L from setpoint fifteen minutes later, on errors none of them caused. Repairing the tuning mid-run and taking the same step back down keeps the header inside 61.9 to 64.9 kPa and the fifteen-minute errors at 0.03, 0.11 and 0.26 mg/L.

Scenario 3 measures the gain the dissolved oxygen loop is tuned against and shows it moving in two directions at once. The equal-percentage trim in (10.10) makes the valve gain rise as it opens while the 0.75 exponent in (10.8) makes the transfer gain fall as the air rises, and the two nearly cancel. Measured at fixed header pressure the gain is 0.198, 0.180 and 0.163 mg/L per percent of valve at setpoints of 1, 2 and 3 mg/L, a drift of 18 % across the whole range. Setting the rangeability exponent to zero for a linear trim gives 0.191, 0.144 and 0.110 mg/L per percent, a drift of 42 %, with the valve sitting at 41.2 % open instead of 68.9 %. That cancellation is why equal-percentage trim is specified on air valves, and it is one of the few places where the right answer is to change the plant rather than to schedule the controller.

10.5.2 A Whole Day, and What the Setpoint Range Is Worth

Scenario 5 runs midnight to midnight at a fixed 2.0 mg/L. The 24-hour mean effluent ammonia is 1.100 g N/m^3 and the peak is 3.27 g N/m^3 at 11:45, an hour and three-quarters after the load peaked at 09:54, because the sewage takes 7.92 h to cross the basin. The worst eight-hour rolling mean is 2.277 g N/m^3 against a 2.5 g N/m^3 limit, and the day costs 4335 kWh, which is 181 kW around the clock and 0.394 kWh per cubic meter treated.

Two facts in that run set up everything after it. The morning peak was entirely predictable, because the load starts climbing at 06:00 and peaks at 09:54 while the analyzer does not see the consequence until 11:45, and the same curve repeats every weekday. And at 04:00 the effluent ammonia is 0.011 g N/m^3, two orders of magnitude inside the limit, while the plant spends 47.4 Nm^3/min and 76.7 kW holding 2.0 mg/L of oxygen that nothing is using. Dropping the setpoint to 1.0 mg/L at that hour takes the ammonia to 0.023 g N/m^3 on 40.9 Nm^3/min and 68.3 kW.

Scenario 6 starts at 1.2 mg/L and costs 3813 kWh, with a worst rolling mean of 3.786 g N/m^3 and instantaneous peak 4.79 g N/m^3. Its initialization uses that lower oxygen profile. For a comparison that holds the opening plant state fixed, scenario 5 is loaded and run for one minute at its supplied setpoint before the target is changed. Each trial retains the same opening plant state and runs to 1440 minutes. Sweeping that common state gives the frontier this plant lives on, plotted in Figure 10.2 and tabulated in Table 10.5. The 1.2 mg/L benchmark sits at its far end, saving 516 kWh against

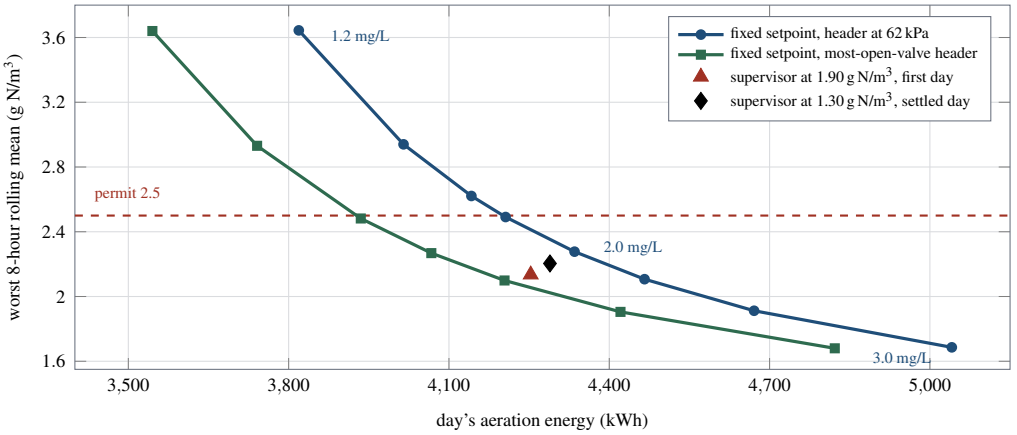


Figure 10.2 The fixed-setpoint frontier over one diurnal day, with and without most-open-valve control on the air header. Each marker on a curve is one fixed dissolved oxygen setpoint between 1.2 and 3.0 mg/L, and every run starts from the same equilibrium, with the DO target changed after one minute. The header loop moves the whole curve left by about 270 kWh at unchanged compliance, while the ammonia supervisor moves one point along the curve by 194 kWh on its first day and 66 kWh once settled. Circles mark the header at 62 kPa, squares the most-open-valve header, the triangle and the diamond the supervisor's first and settled days, and the dashed horizontal line the permit.

Table 10.5 Fixed-Setpoint Frontier Over One Diurnal Day, Read After the First Eight Hours So That the Rolling-Mean Ring Is Full. Every Run Starts From the Same 2.0 mg/L Equilibrium, With Targets Changed After One Minute.

DO_{SP}	header at 62 kPa		most-open valve	
(mg/L)	energy (kWh)	$\bar{S}_e$ (g N/m^3)	energy (kWh)	$\bar{S}_e$ (g N/m^3)
1.2	3819	3.644	3545	3.640
1.5	4015	2.940	3741	2.931
1.8	4206	2.491	3935	2.482
2.0	4335	2.277	4067	2.268
2.2	4466	2.107	4204	2.099
2.5	4671	1.912	4421	1.905
3.0	5041	1.686	4822	1.680

the 2.0 mg/L day while the worst rolling mean reaches 3.644 g N/m^3 and the instantaneous peak 4.66 g N/m^3. The curve is steep at the bottom and flat at the top, because the half milligram from 1.2 to 1.7 mg/L buys 1.02 g N/m^3 of rolling mean for 323 kWh while the half milligram from 2.5 to 3.0 mg/L buys 0.22 g N/m^3 for 370 kWh. End to end the whole practical range is 1222 kWh, 32 % of the cheapest day on it, which is the arithmetic that bounds every claim made for a setpoint policy on this plant.

10.5.3 What Load-Following Is Worth, Read Honestly

The fine matching values 2.02 mg/L and 2.17 mg/L below are counterfactual model benchmarks, between the interface's 0.05 mg/L setpoint increments. The other fixed-setpoint runs use the exposed grid.

Scenario 7 puts the analyzer in charge of the dissolved oxygen setpoint, aiming at 1.90 g N/m^3 with $K_c = 1.20\,\mathrm{mg/L}$ per g N/m^3, $\tau_I = 480\,\mathrm{min}$ and a travel limit of 0.5 to 3.0 mg/L. The setpoint starts where the operator left it and walks down all night, reaching its 0.5 mg/L floor at 05:07, its

Table 10.6 The First Day Is Not the Settled Day. Identical Configurations Run for Three Consecutive Diurnal Days, Energy and Worst Rolling Mean per Day. Each row is a fresh run. Fixed targets other than 2.00 mg/L change after one minute. Supervisor aims are applied before Run. The last four rows run scenario 14's settings, the MPC of Section 10.4.5 three ways and the ammonia supervisor tuned with the day known (aim 1.40 g N/m^3, ceiling 2.5 mg/L, $K_c = 4.8$, $\tau_I = 900$ min).

	day 1		day 2		day 3	
Configuration	kWh	$\bar{S}_e$	kWh	$\bar{S}_e$	kWh	$\bar{S}_e$
Fixed 2.00 mg/L	4335	2.277	4341	2.245	4342	2.221
Fixed 2.02 mg/L	4348	2.258	4354	2.226	4355	2.201
Fixed 2.17 mg/L	4446	2.130	4452	2.094	4453	2.066
Supervisor at 1.90 g N/m^3	4253	2.135	4109	2.563	4105	2.598
Supervisor at 1.30 g N/m^3	4471	1.919	4307	2.183	4289	2.204
MPC, typical day	4018	2.211	4010	2.219	4011	2.207
MPC, preview off	4306	2.597	4309	2.630	4306	2.604
MPC, ceiling 2.15 g N/m^3	4044	2.162	4032	2.169	4032	2.157
Supervisor in hindsight	4352	1.995	4230	2.175	4212	2.213

3.0 mg/L ceiling at 10:05 and 1.3 mg/L again by late afternoon. Between midnight and 07:00 the plant spends 564 kWh against the fixed setpoint's 646 kWh, and from 07:00 to midnight the two spend 3689 kWh each to the kilowatt-hour. It buys the night outright and a cleaner morning for nothing.

Over that first day the supervisor finishes at a worst rolling mean of 2.135 g N/m^3 on 4253 kWh against the fixed setpoint's 2.277 g N/m^3 on 4335 kWh, which is better on both counts and is something no point on the frontier of Table 10.5 manages against any other. Comparing at nearby compliance rather than at equal setpoint, the fixed setpoint that reaches 2.135 g N/m^3 is 2.17 mg/L and costs 4446 kWh, so the saving is 194 kWh, or 4.4 %.

Running the identical configuration for a second and a third day is what changes the answer, and Table 10.6 reports it. The supervisor settles at 4109 and 4105 kWh on rolling means of 2.563 and 2.598 g N/m^3, cheaper than the fixed setpoint and outside the 2.5 g N/m^3 limit, while the fixed setpoint barely moves at 4341 and 4342 kWh. Nothing broke. An eight-hour integral takes about a day to forget the 2.0 mg/L setpoint it was seeded from, and while it is forgetting it runs richer than its own aim asks for. Aiming at 1.30 g N/m^3 instead gives a settled day of 4289 kWh at 2.204 g N/m^3 against the fixed setpoint's 4342 kWh at 2.221 g N/m^3, and at nearby compliance a fixed 2.02 mg/L reaches 2.201 g N/m^3 on 4355 kWh, so the settled saving is 66 kWh, or 1.5 %. Four percent on the day of commissioning and one and a half percent for the rest of the loop's life is the honest pair, and the reason the first number is the one usually quoted is that most trials run for a week.

🕮 **Pause.** Try Exercise 10.3 before reading on.

Scenario 8 is scenario 7 with the supervisor's integral time at 10 min, which is a sensible number for a flow loop. Over the day the fast supervisor finishes at 2.440 g N/m^3 on 4206 kWh against the slow one's 2.135 g N/m^3 on 4253 kWh and the fixed setpoint's 2.277 g N/m^3 on 4335 kWh, so it has made the effluent worse than having no supervisor at all while moving the setpoint 16.8 mg/L against 15.8 mg/L to do it. The fast loop saves 47 kWh against the slow one while using more of the compliance margin. A flat-load comparison with the analyzer noise enabled makes the activity visible. Starting from scenario 2, enabling the ammonia supervisor and 0.05 g N/m^3 analyzer noise, and measuring the final ten hours of a three-day run gives 10.91 mg/L of setpoint travel over a 0.70 mg/L band for the fast loop, against 6.72 mg/L over 0.27 mg/L for the slow one. The input load is constant throughout, and the staircase, transport delay and sampled noise still drive controller activity.

10.5.4 The Loop Nobody Sells

Scenario 9 switches most-open-valve control in at a flat load. Comparing means over the opening hour and minutes 300–360, the header comes down from 62.00 to 56.03 kPa, the busiest valve opens

from 68.94 to 87.74 %, the blower slows from 59.54 to 57.65 %, and total aeration power falls from 173.93 to 160.96 kW, a reduction of 7.5 %. The largest dissolved-oxygen excursion is 0.113 mg/L in tank 1. Oxygen transferred per kilowatt-hour rises from 1.64 to 1.78 kg. Nothing was given up anywhere, because the 6.0 kPa that disappeared was being manufactured by the blower and destroyed across three butterfly valves. Over a full diurnal day the same loop takes the fixed-setpoint plant from 4335 to 4067 kWh, which is 267 kWh or 6.2 %, with the rolling mean unmoved at 2.268 g N/m^3 against 2.277 g N/m^3. The two loops add, since the supervisor with the header loop underneath it finishes its first day at 4025 kWh.

> **TRY IT IN THE APP 9 · Most-Open-Valve Control**
>
> Load the scenario, press Run and let it pass the switch at minute 60. Watch the header setpoint walk down while the three dissolved oxygen traces hardly move. Between the opening hour and minutes 300 to 360 the header falls from 62.00 to 56.03 kPa and the aeration power from 173.93 to 160.96 kW.

10.5.5 Two Losses Hidden by Successful Regulation

Scenario 13 takes the diffuser condition from 0.55 to 0.48 to 0.38, which is about a year of service on membrane diffusers, with the dissolved oxygen loops in service throughout. The dissolved oxygen returns near its target after each step, although tank 1 falls to 1.37 mg/L during the last transition. Effluent ammonia briefly reaches 0.408 g N/m^3 and remains below the permit. No consent alarm fires. What moves is everything nobody watches. Airflow goes from 122 to 146 to 200 Nm^3/min, the valves from 69/50/32 to 86/65/43 %, the blower from 59.5 to 75.7 %, the power from 174 to 205 to 276 kW, and oxygen per kilowatt-hour from 1.64 to 1.37 to 1.00 kg. That last hundred kilowatts is $106,000 a year at twelve cents. There is no dissolved oxygen alarm that will ever fire, because holding the dissolved oxygen is exactly what the loops are doing, and the only way to see the loss is to monitor the air it takes to hold it [2, 13].

Scenario 10 is a dry Sunday night at three quarters of the usual flow and nine tenths of the usual strength. By 04:10 the tanks want 31.3 Nm^3/min while the machine is passing 37.1 Nm^3/min against a surge line at 40.0 Nm^3/min, so the blow-off valve of (10.13) vents 5.8 Nm^3/min and 1002 standard cubic meters over the seven hours of the run. The dissolved oxygen stays within 0.13 mg/L of setpoint the whole time. What the surge line takes is not the power, it is the saving. Taking the setpoint from 2.0 to 0.5 mg/L at that load cuts the air the tanks receive from 33.7 to 27.2 Nm^3/min, which is 19 %, while the machine turns down only from 37.9 to 35.6 Nm^3/min and the plant draws 61.4 kW against 64.3 kW, which is 4.5 %. The same setpoint change at the daily average load moves the plant from 173.9 to 133.2 kW, or 23 %. Every cubic meter the tanks stop taking is made up by the blow-off valve instead, so the air discarded over the comparison rises from 584 to 2134 standard cubic meters while the meter falls only from 545 to 477 kWh. Both comparison runs start at the supplied 1.5 mg/L, change target after one minute and continue for seven more hours. Selecting 0.5 mg/L before Run instead initializes a long-term washed-out equilibrium, rather than the operating plant. Neither failure has a tuning answer, and both have capital or scheduling answers that need exactly the number the scorecard is showing.

10.5.6 Cold, and a Disturbance That Should Not Have Arrived

Scenario 11 drops the sewage from 13 to 9.5 °C on the first morning of three days and recovers it on the third. The nitrifier population does not move, wandering between 97.6 and 101.9 g VSS/m^3 with the diurnal load and ending where it began, exactly as (10.4) requires. The effluent ammonia carries the whole loss, tripling from 0.378 to 1.19 g N/m^3 at 9.5 °C, with the rolling mean reaching 4.47 g N/m^3, the supervisor pinned on its 3.0 mg/L ceiling for twenty-three hours of the three days and the limit exceeded for eighteen. Air buys back about half, because at 9 °C raising the setpoint from 2.0 to 3.5 mg/L takes the ammonia from 1.378 to 0.901 g N/m^3 against 0.378 g N/m^3 in the warm. The other half is not available at any price, and the handle that would deliver it is the sludge

age, which moves the population by 5.1 g VSS/m^3 on the first day after the decision and is still moving on the fifth.

Pause. Try Exercise 10.1 before reading on.

Scenario 12 represents dewatering liquors by multiplying influent nitrogen strength by 1.45 for four hours, without adding a separate hydraulic stream. The first pulse starts at 09:00 on day 1. Effluent reaches 7.95 g N/m^3, the rolling mean reaches 5.26 g N/m^3, and the day costs 4890 kWh. The second pulse starts at 01:00 on day 2, whose peak effluent is 3.22 g N/m^3, worst rolling mean 2.43 g N/m^3 and energy 4153 kWh. Those consecutive days differ in their preceding state and in added mass: the pulses add 71.21 kg and 18.56 kg of nitrogen respectively above the same diurnal baseline. They are not an equal-mass timing comparison.

A counterfactual run applying only the 01:00 pulse on the first day, from the same opening state as the morning run, reaches 2.92 g N/m^3 peak effluent and 2.086 g N/m^3 worst rolling mean on 4336 kWh. The 554 kWh difference from the morning pulse therefore combines timing and the smaller injected mass. The native lesson shows the benefit of the nighttime operating pattern, but does not isolate its value per kilogram returned.

10.5.7 Planning the Eight-Hour Mean a Day Ahead

Scenario 14 runs scenario 7's ordinary day three times under the MPC of Section 10.4.5, then asks for the supervisor at 1.30 g N/m^3 on the same day. Each comparison changes only the named setting over three days on the default seed unless ten seeds are stated, and reads worst means from the logged half-minute record, 0.001 to 0.002 g N/m^3 below the every-step reading of Table 10.6.

> **TRY IT IN THE APP** **14 · The Eight-Hour Mean, Planned a Day Ahead (MPC)**
>
> Load the scenario and run three days on MPC with the typical-day preview, at 200 simulated minutes per second. On the third day watch the dissolved oxygen setpoint: the plan never asks for more than 2.49 mg/L, and by 20:00 it has let it fall to 0.86 mg/L while the effluent climbs to 3.22 g N/m^3. The three days cost 4018, 4010 and 4011 kWh (Table 10.6).

The MPC spends 4018, 4010 and 4011 kWh on its three days at worst means of 2.210, 2.218 and 2.206 g N/m^3 with no minute over the permit. The supervisor at 1.30 g N/m^3, the aim that settles at nearly the same compliance, spends 4471, 4307 and 4289 kWh at 1.918, 2.182 and 2.203 g N/m^3, and interpolated across its aims to the MPC's third-day worst mean it needs 4288 kWh, so the plan uses 277 kWh, or 6.5 %, less. That saving needs the calendar. With the preview off the MPC knows what the supervisor knows and loses on both counts, at 4306 kWh and 2.602 g N/m^3 on the third day with 362 min over the permit in three days, and at all ten seeds of Table 10.7. The best supervisor setting found with the day known, from 1638 combinations of aim, floor, ceiling, gain and integral time, spends 8442 kWh over days 2 and 3 at 2.212 g N/m^3 with four times the shipped gain and the maximum integral time of 900 min, and 268 more combinations around it reach only 8434 kWh at 2.218 g N/m^3, so the plan's 8021 kWh sits 4.9 % below that hindsight bound. Figure 10.3 draws the third day of Table 10.6.

The third day, shown in Figure 10.4, says where the energy goes. The supervisor spends 2.9 hours on its 3.0 mg/L ceiling around the morning peak, while the plan never sets more than 2.49 mg/L. At 20:00 the supervisor still asks for 2.23 mg/L and holds the effluent under 1.61 g N/m^3, while the plan has let the setpoint fall to 0.86 mg/L and lets the effluent reach 3.22 g N/m^3 at 20:37, since that ammonia will be averaged with the empty hours after midnight, and its rolling mean stays between 2.01 and 2.21 g N/m^3 from 15:00 to midnight. It transfers 6272 against 6333 kg of oxygen a day at lower dissolved oxygen, so on less air. Its night costs 469 against 443 kWh.

Table 10.7 takes the saving apart one switch at a time at the MPC's third-day worst mean of 2.206 g N/m^3. The day's shape is worth about 247 kWh, from 4429 kWh to 4182 kWh for a plan whose ceiling sits on the instantaneous ammonia, and planning the permit's own mean a further

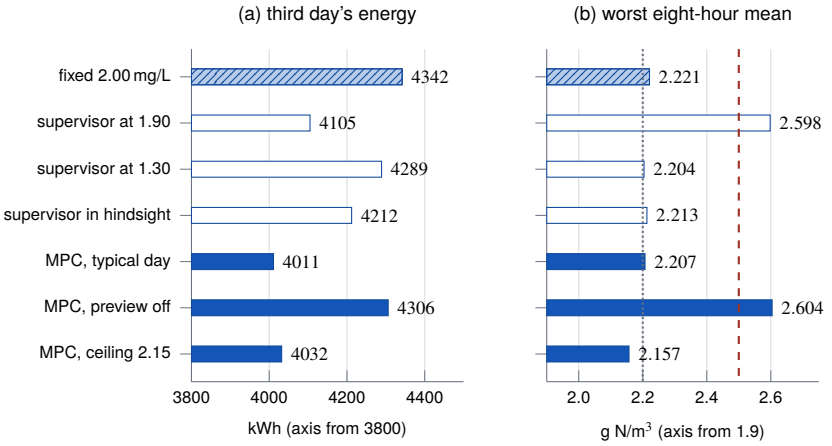


Figure 10.3 The third day of Table 10.6: (a) the energy and (b) the worst eight-hour rolling mean, for the fixed setpoint (hatched bar), the ammonia supervisor (open bars) and the MPC (dark bars). The red dashed line is the 2.5 g N/m^3 permit and the gray dotted line the plan's 2.2 g N/m^3 ceiling. Notice that the two bars past the permit are the supervisor aimed at 1.90 g N/m^3 and the plan with its preview off, while the plan with the typical day is the shortest bar in (a) near the ceiling in (b).

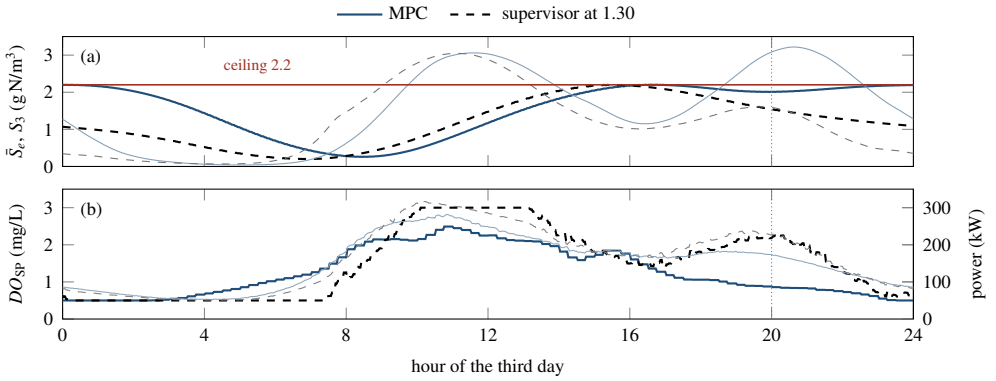


Figure 10.4 Scenario 14's third day under the MPC (blue, solid) and the ammonia supervisor at 1.30 g N/m^3 (black, dashed). (a) Eight-hour rolling mean (thick) against the plan's 2.2 g N/m^3 ceiling, and effluent ammonia (thin). (b) Dissolved oxygen setpoint (thick, left axis) and total aeration power (thin, right axis). The dotted line marks 20:00.

171 kWh. The hour's price of air is worth nothing measurable, because a plan pricing every milligram-hour of dissolved oxygen at one flat rate lands on the same 4011 kWh. Feedback cannot reproduce the result either, since a review copy of the supervisor reading the analyzer's eight-hour mean, not shipped, exceeded the permit at all 63 settings tried, with aims of 1.9 to 2.3 g N/m^3, gains of 0.15 to 4.8 and integral times of 240 to 900 min.

Table 10.7 Where the Plan's Saving Comes From, and Ten Noise Seeds. Upper Block: Third-Day Energy Interpolated to a Worst Eight-Hour Mean of 2.206 g N/m^3. Lower Block: Days 2 and 3, Mean and Range Over Seeds 1 to 10 Past the Default. Worst Means Read From the Logged Half-Minute Record.

Configuration (third day)	Energy at a worst mean of 2.206 g N/m^3 (kWh)
Supervisor at 1.30 g N/m^3 (worst 2.203)	4289, interpolated 4288
MPC, preview off, ceiling on the instantaneous ammonia	about 4429
MPC, preview off, ceiling on the eight-hour mean	not reached (ceiling 1.0 gives 4662 at 2.240)
MPC, typical day, ceiling on the instantaneous ammonia	about 4182
MPC, typical day, ceiling on the eight-hour mean (shipped)	4011
Same, every milligram-hour of oxygen at one flat price	4011
Supervisor reading the eight-hour mean (review copy)	over the permit at all 63 settings, lowest worst mean 2.716

Configuration (ten seeds)	energy (kWh)	worst mean (g N/m^3)	minutes above 2.20	permit minutes	N, day 3 (kg)
MPC, typical day	8028 [8017, 8038]	2.218 [2.214, 2.222]	165 [87, 212]	0	35.9
MPC, preview off	8622 [8613, 8631]	2.607 [2.570, 2.625]	759 [740, 780]	351	34.4
Supervisor at 1.30 g N/m^3	8596 [8590, 8600]	2.205 [2.200, 2.212]	11 [0, 31]	0	29.0
Supervisor in hindsight	8443 [8437, 8448]	2.218 [2.210, 2.224]	57 [32, 72]	0	30.8
MPC, ceiling 2.15 g N/m^3	8073 [8061, 8084]	2.168 [2.163, 2.172]	0	0	35.1

+ DID YOU KNOW? A Plan Without Its Forecast Loses

From the simulator: on the third day of scenario 14 the plan with the typical-day preview spends 4011 kWh at a worst rolling mean of 2.207 g N/m^3. With the preview off it spends 4306 kWh at 2.604 g N/m^3, over the permit, while the ammonia supervisor aimed at 1.30 g N/m^3 spends 4289 kWh at 2.204 g N/m^3 (Table 10.6).

The plan rides its ceiling. Its plans made at 0, 1440, 2040 and 2640 min predict a peak eight-hour mean of 2.192, 2.200, 2.200 and 2.200 g N/m^3 over the day ahead, and the following 24 hours realize 2.210, 2.218, 2.218 and 2.206 g N/m^3 through its model error. Over ten seeds its worst mean exceeds 2.20 g N/m^3 on all 30 seed-days, for 87 to 212 min in three days and by at most 0.022 g N/m^3, and ends 0.013 g N/m^3 above the supervisor's on average and higher at every seed, so the headline is energy at a compliance 0.013 g N/m^3 worse. For information only, a ceiling of 2.15 g N/m^3, which any learner can set, stays under 2.20 g N/m^3 on every seed-day for 45 kWh more over two days, still 523 kWh below the supervisor with a worst mean no higher than the supervisor's at all ten seeds.

What the plan spends is ammonia. On the third day it discharges 35.9 kg of nitrogen against 29.0 kg, 6.9 kg or 24 % more, because a plan told only a ceiling spends all of it, and it takes every margin the shipped settings leave. In scenarios 2, 3, 4, 9 and 13 the worst mean rises from 0.38 to 0.44 to between 2.09 and 2.13 g N/m^3 and the nitrogen discharged from 7.4 to 7.9 to between 28.0 and 28.8 kg a day, for 7.9 to 8.3 % less energy over three days, and in scenario 10 the worst mean rises from at most 0.95 to 2.21 g N/m^3 for 11.5 % less. On heavier days than its model it also breaks the permit, 80 to 948 min in three days against the supervisor's 499 to 1207 min. Scenario 11's cold snap costs it 12 007 against 11 738 kWh, 2.3 % more, for a worst mean of 3.948 against 4.468 g N/m^3 and no upper-tier minute against 170, and scenario 12's two days of liquors cost 9195 against 9043 kWh, 1.7 % more, with both structures over the permit.

10.5.8 Graded February Day

Scenario 15 is one ordinary February day at 12.8 °C with no event of any kind on it, and every gain an order of magnitude out. The target asks for a rolling mean never above 2.20 g N/m^3 after the first eight hours, nothing above the 5.0 g N/m^3 upper tier, and under 4180 kWh on the meter. As

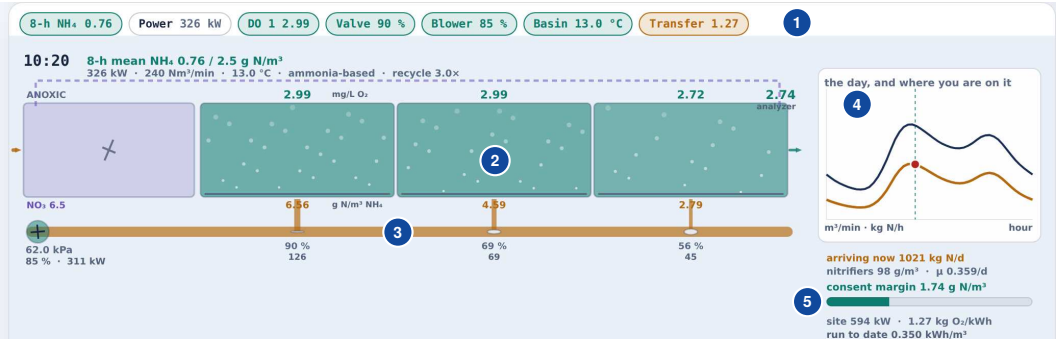


Figure 10.5 Scenario 7 in progress at 10:20 on the first day of ammonia-based aeration control, cropped to the plant view. (1) The chips: rolling mean, power, tank 1 oxygen, busiest valve, blower speed, basin temperature and oxygen transferred per kilowatt-hour. (2) The three aerated tanks with their oxygen and ammonia readings. (3) The air header and its three valves, with the header pressure at the blower end. (4) The load inset, whose marker sits just past the 09:54 crest of the day, beside the 1021 kg/d arriving now. (5) The consent margin under the permit. By that minute the ammonia supervisor had driven the dissolved oxygen setpoint from 0.5 mg/L overnight to its 3.0 mg/L ceiling fifteen minutes earlier (Section 10.5), the three dissolved oxygen loops were tracking it through their air valves, and the effluent ammonia was still climbing toward a peak the basin would not deliver until 11:15. The rolling mean at that minute was 0.754 g N/m^3, which the scene prints rounded, still under the 2.5 g N/m^3 permit line.

handed over the day finishes at a rolling mean of 10.5 g N/m^3, 10.465 g N/m^3 at every step, with 527.56 min above the upper tier on 3832 kWh, so it fails on ammonia while it is inside the energy budget. The inner-to-outer repair sequence is appropriate because the header loop is underneath everything and the dissolved oxygen loops cannot even be judged until it is right. Repairing the two regulatory loops alone reaches 4298 kWh at 2.39 g N/m^3, which is close on energy and fails the limit. Adding most-open-valve control at a 92 % target saves 286 kWh, leaving 4011 kWh with a worst rolling mean of 2.384 g N/m^3. Energy now meets the budget but ammonia still fails. The suggested supervisor aims at 1.65 g N/m^3 with a 3.2 mg/L ceiling. It finishes at 4123 kWh and a worst rolling mean of 2.062 g N/m^3, with no upper-tier time, meeting every clause with about 57 kWh to spare. Applying these settings before Run or retaining them through Reset gives the same result. An aim of 1.30 g N/m^3, which settles scenario 7, over-aerates this day.

The arithmetic at the end of that day is the point of the challenge. A fixed setpoint hand-picked at 2.25 mg/L, with the header optimized at a 94 % target, also clears the target at 4159 kWh and 2.160 g N/m^3. Thirty-seven kilowatt-hours, under one percent, separates the two answers on this particular day. The difference is that 2.25 mg/L is a number somebody chose for this day, and on the same works two degrees warmer it is spending money on oxygen nothing will use, while nobody goes back and lowers it in June. The supervisor spends that four percent only on the days that need it.

The MPC does not meet this target either. On the day as handed over it plans above the detuned loops and ends at 9.764 g N/m^3. In place of the suggested supervisor, eleven seeds end at 3772 to 3790 kWh and worst means of 2.206 to 2.219 g N/m^3, failing the 2.20 g N/m^3 clause at all eleven by its model error, and 2.790 g N/m^3 with the preview off. With its ceiling at 2.15 g N/m^3 it meets every clause at all eleven seeds on 3800 to 3821 kWh, against the suggested supervisor's 4118 to 4128 kWh.

10.6 Technology in Practice

What Measures It

> **TECHNOLOGY** **Sensor: The Dissolved Oxygen Probe**
>
> Aeration basins use three kinds of oxygen probe. A membrane electrode, long the most common, lets oxygen pass a membrane to a cathode that reduces it, and the resulting signal is correlated with the concentration. It needs calibration weekly or monthly and a new membrane about every quarter. A galvanic electrode holds its accuracy longer. An optical probe reads how dissolved oxygen quenches the glow of a dye, needs less calibration, and has only a sensor cap to replace, about once a year. In mixed liquor every probe gathers biological growth or grease that makes its reading inaccurate, so probes are wiped weekly to monthly or cleaned by air blasts, water sprays or wipers [4]. No open, vendor-neutral source gave a typical accuracy. The chapter's probe is a 0.75 min lag (Table 10.2).

Automated aeration control takes its input from oxygen probes in the basins, read by a process controller [4]. The quantity the permit names is measured another way. The federal reference method for ammonia nitrogen in wastes is wet chemistry: alkaline phenol and hypochlorite react with the ammonia to form indophenol blue, whose color is read by a colorimetric detector on automated continuous-flow equipment. Its applicable range is 0.01 to 2.0 mg/L as nitrogen, higher concentrations are diluted first, and about 60 samples an hour can be analyzed [25]. In this chapter the oxygen probes are 0.75 min lags, the header transmitter a 0.05 min lag, and the ammonium analyzer a 2 min transport lag followed by a 5 min sample and hold (Table 10.2).

What Moves It

The total air is set at the blowers: positive displacement machines through variable-frequency drives, multistage centrifugal machines often by inlet throttling, and newer single-stage machines by variable speed, inlet guide vanes and variable discharge diffusers. The air is then divided among the basins by valves, by hand in small plants and by automatic valves in larger ones. Most-open-valve control adjusts the header pressure so that the most open valve stays essentially full open, and it is now commonly integrated into new aeration control systems [4]. The chapter's three butterfly valves are equal-percentage orifices with a 0.25 min travel time, and its blower follows a 0.20 min speed ramp (Table 10.2).

> **TECHNOLOGY** **Final Element: The Variable-Speed Centrifugal Blower**
>
> Aeration blowers are low-pressure, high-volume compressors that push air to the basins at typically 5 to 14 psi. A positive displacement machine gives a constant volume over a wide range of pressure, while a centrifugal machine gives a wide range of flow over a narrow range of pressure. A federal survey puts a variable-speed multistage centrifugal blower at 60 to 70 percent nominal efficiency with a turndown to 50 percent of rated flow, and reports variable-speed operation 15 to 20 percent more efficient than throttling. Surge does more damage to a single-stage machine, which typically comes with its own surge protection [4]. The chapter's blower is one variable-speed machine with a surge line and a blow-off vent (Equations (10.12) and (10.13)).

Where the Controller Runs

Most aeration control systems are programmable logic controllers networked under a supervisory control and data acquisition system, and larger facilities sometimes use a distributed control system [4]. Control loops may be nested, with one loop's setpoint taken from another, and supervisory and lower-level loops run continuously with cycle times from milliseconds to minutes [26]. The chapter's six proportional integral controllers scan every 0.1 min, and its MPC remakes a 96-step plan every 15 min above the oxygen loops, in the place the ammonia supervisor would hold, as Figure 10.6 draws them.

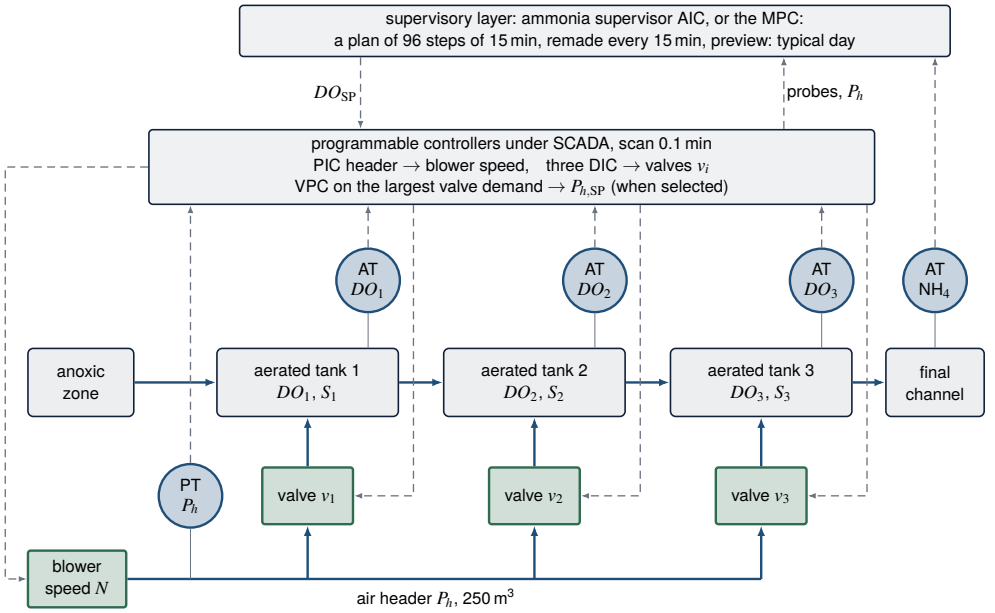


Figure 10.6 The chapter's loops on field hardware, drawn from Sections 10.3 and 10.4. Green boxes are the final elements (blower speed and the three butterfly valves), blue circles the sensors (header pressure, one oxygen probe per tank and the ammonium analyzer on the final channel), dashed arrows signals. Notice that the supervisory layer moves no device: it writes one oxygen setpoint, and the three oxygen loops and the header loop turn it into air.

> **IN PRACTICE** **Hunting, and the Cascade Under the Oxygen Loop**
>
> The common automated arrangement is a cascade. Each basin's oxygen loop sets an airflow controller on its valve, and a pressure transducer on the main header drives the blowers, so a valve that opens pulls the header down and the pressure loop calls for more air. Oxygen takes minutes or longer to settle after the air changes, so a quick loop overshoots and moves the valve again, which operators call hunting. Conservative tuning stops it but leaves the loop slow, and the curved relation between oxygen and airflow makes tuning harder [4]. This chapter's oxygen loops move their valves directly, a variant the survey also names (Table 10.3).

What Is Modernizing It

Three developments bear on this chapter's method. Optical probes are the first: they need less calibration and maintenance, and a more reliable probe could pave the way for more automated oxygen control. Model-based setpoints are the second. One commercial scheme runs an on-line process simulation from measured temperature, ammonia, nitrate and influent flow, with an ammonia analyzer in the anoxic zone and a nitrate analyzer at the end of the aerobic zones, and returns oxygen setpoints and the internal recycle flow in real time [4]. This chapter's MPC takes the same feedforward step with a calendar preview. The third is a direct reading of transfer: a low-cost monitor of oxygen transfer efficiency helps predict when diffusers need cleaning, and a study estimated an average energy efficiency improvement of 15 percent [4]. That is the quantity scenario 13 shows a probe cannot see.

10.7 Limitations

A single day's scorecard is not a steady state, and this is the limitation that most affects how the results above should be read. The supervisor is seeded bumplessly from whatever dissolved oxygen setpoint

was in force, and with an eight-hour integral time it takes about a day to forget it, which is why scenario 7 is worth 4.4 % at equal compliance on its first day and 1.5 % on its third. That behavior is a real property of slow supervisory loops rather than a numerical artifact, and the application exists partly to make it visible, but any figure quoted from one simulated day has to say which day it was.

The biology is thinner than the air system. One lumped autotroph population with Monod kinetics cannot show nitrite accumulation, and nitrous oxide is absent altogether, which removes the mechanism that makes a very low setpoint genuinely expensive rather than merely risky [18]. Sludge age is a parameter rather than the consequence of a wasting flow, so the application shows what a solids retention time decision does and not what it costs in settlement capacity. There is no clarifier at all, so the failure modes that actually take plants out of service are outside the model. The heterotrophic demand is a forcing rather than a state, which sharpens the morning air peak.

The air system is more complete and still simplified. One machine with a fitted head curve stands in for a hall of three, so the turndown behavior of scenario 10 is the turndown of a staged hall represented by a single surge line rather than the turndown of one impeller, and no answer involving staging can be tested. Oxygen transfer is one power law with alpha, beta and fouling lumped into a single slider, and the mid-depth approximation in (10.9) is a standard simplification rather than a profile [12]. The valve model has no stiction, no hysteresis and no finite stroking rate beyond its 0.25 min travel time, any of which would add a limit cycle that tuning cannot remove.

The instrument chain fails only by being slow. Drift, a stuck reading that looks plausible and an instrument health signal are all absent, and a supervisory loop that trusts a frozen analyzer is a more common industrial failure than one that is badly tuned. The numerics are also more repeatable than a control system, because a fixed integration step and a fixed scan reproduce one trajectory on every machine while a distributed control system does no such thing.

Three scenarios are stylized on purpose. A diffuser condition that falls from 0.55 to 0.38 in five hours compresses a year of fouling into one run, which no physical grid does, but it isolates a cost that is invisible on the controlled variable. The returned liquor stream arrives as a step, because a step is the disturbance whose response can be read off a trend, while a real dewatering run ramps. The cold snap moves the sewage three and a half degrees in a few hours rather than over a week, so that the population and the ammonia can be watched together inside a three-day run.

The MPC's model of (10.20) is thinner again. It fixes the nitrifier population, the basin at 13 °C, the diffusers at $\alpha F = 0.55$ and the carbonaceous load, omits the nitrate pools and the limits of the loops and valves beneath it, and holds the header pressure for a day. Open loop on scenario 7's day its ammonia error is 0.145 g N/m^3 RMS, but 1.93 g N/m^3 at 9.5 °C and 4.16 g N/m^3 at a 1.45 strength, and at $\alpha F = 0.38$ its power error is 46 % of the mean. Its preview is a calendar exact only on the application's ordinary day, its ceiling is chosen by hand, and since nothing in its objective values the nitrogen discharged it spends every margin.

> ⚠ **OPEN CHALLENGE** **Savings Claims Without Independent Data**
>
> A federal survey found that control of nitrification on ammonia, nitrate and nitrite measurements was growing overseas but had few full-scale applications in the United States. For one commercial scheme it reported savings claimed by the supplier that no independent full-scale data had verified, and it warned that savings are site specific and depend on the control in place before the upgrade [4]. This chapter's first and settled days show how far one answer moves with the day and the reference chosen. Progress would be the independent full-scale data the survey found missing, read at equal compliance as Section 10.5 reads its own runs.

10.8 Discussion

The plant carries three decisions with different answers, and separating them is the purpose of the application.

The first is where a setpoint should sit, and it is an economic question with a legal constraint rather than a control question. Table 10.5 is a frontier and not an optimum, and the shape of it comes from two saturating mechanisms working in opposite directions. What a milligram of dissolved oxygen buys falls because (10.3) is a Monod term, and what it costs rises because the air needed goes as $(C_s^* - DO)^{-1/n}$. The product is a cost per unit of ammonia removed that is 56.6 times larger between 2.5 and 3.5 mg/L than it is between 0.5 and 1.5 mg/L. Choosing a point on such a curve is exactly the self-optimizing control question of which variable to hold and at what value [5], and the answer here is that no fixed point on it is right for both February and June.

The second is whether a supervisory loop earns its instrument, and the honest answer on this works is that it does not earn it on electricity. A settled saving of 66 kWh a day at twelve cents is about $2,900 a year against an ammonium analyzer costing several times that in capital and a few thousand a year in reagent, and the accounting only gets worse if the comparison is made at equal setpoint rather than at equal compliance [16]. What the loop actually buys is the February margin. On the graded day at 12.8 °C a fixed 2.0 mg/L finishes at 2.39 g N/m^3 and needs raising to 2.25 mg/L to hold 2.160 g N/m^3, which is 3995 against 4159 kWh, four percent more every day of the year because nobody lowers it again in June. A supervisor spends that four percent only on the days that need it, and the general shape of that argument, a small continuous prize against a large occasional one, recurs in most supervisory control business cases [15, 17].

The third is the one with no vendor. Most-open-valve control removes about 6 kPa of manufactured and destroyed pressure for 267 kWh a day and moves no controlled variable by more than 0.113 mg/L, which is four times what the analyzer loop returns on a settled day. Its absence from most plants is not a technical matter, because the header pressure is a commissioning number that nobody feels entitled to change, and the same structure and the same reticence appear wherever throttled users share a supply header [6]. Diffuser fouling belongs in the same category, since a hundred kilowatts of drift is invisible on every alarm list and visible immediately in oxygen transferred per kilowatt-hour [13].

Beyond the four loops implemented, the same equations carry an optimization problem. A sixteen-state balance with actuator limits, a surge constraint and a rolling-mean permit suits model predictive control, because the permit of (10.15) is a constraint on a moving average rather than a setpoint and an optimizer can hold it directly instead of through a margin chosen by hand [27]. The predictability of the load curve makes the same formulation a feedforward one, which is the distinction practice guidance draws within ammonia-based control [17]. An estimator running against the same balances recovers the diffuser condition and the nitrifier population from airflow, power and the analyzer, which turns scenario 13 from a lesson into a soft sensor. Section 10.4.5 builds the first two of these ideas, as an economic MPC in the sense of Section 1.4.8, but not the third, and still needs a ceiling chosen by hand 0.3 g N/m^3 inside the permit, of which its model error spends up to 0.018 g N/m^3.

10.9 Conclusion

This chapter studied an interactive activated sludge works whose regulatory loops are easy and whose setpoint is not. The simulator integrates sixteen states covering ammonia and nitrate through four zones, dissolved oxygen in three aerated tanks, one nitrifier population, an air header, a blower and three valves, with a 17.3 s pressure loop under a 6.20 min oxygen loop under an ammonia measurement that answers in three hours and is judged as an eight-hour mean. One physical number sizes every result, because 5 m of submergence puts the saturation the bubbles see at 12.39 mg/L, so a 2 mg/L setpoint is 16.1 % of the driving force and the whole practical setpoint range spans 32 % of a day's aeration energy. Inside that range, ammonia-based control read at equal compliance is worth 194 kWh of 4446 kWh on its first day and 66 kWh of 4355 kWh once its eight-hour integral has settled, while most-open-valve control on the header is worth 267 kWh of 4335 kWh with the rolling mean unmoved at 2.27 g N/m^3. A supervisor tuned like a regulatory loop at 10 min of integral time ends the day dirtier than no supervisor at all, at 2.440 g N/m^3 against 2.277 g N/m^3. A cold snap leaves the nitrifier population at 102 g VSS/m^3 and puts the entire loss on the effluent, because a seven-day

sludge age fixes the growth rate the population is permitted to have. A model predictive option planning the eight-hour mean a day ahead uses 278 kWh less than the supervisor at 1.30 g N/m^3 on the settled day, a saving owed to the typical load curve and to the permit's own measure, which it loses without that curve and pays for with 24 % more nitrogen discharged.

A fuller model would add a two-step nitrification with nitrite, a nitrous oxide term that prices a low setpoint properly, a clarifier with a solids inventory, and a staged blower hall so that the turndown answer can be designed rather than diagnosed. As it stands, the simulator is a place to decide where a supervisory setpoint belongs and to quote honestly what moving it is worth.

KEY POINTS

- The works is judged on an eight-hour mean of effluent ammonia, while its loops hold an oxygen setpoint no permit names.
- Read at equal compliance, the ammonia supervisor is worth 194 kWh on its first day and 66 kWh on a settled day, and most-open-valve control 267 kWh with nothing given up.
- The day-ahead MPC that plans the permit's own mean uses 277 kWh, or 6.5 %, less than the supervisor on the settled day, and loses that saving without the typical-day preview.
- The plan pays with 24 % more nitrogen discharged.

Where else this applies. A town's water network (Chapter 11) also plans a day of pumping against a tariff, and there the band rule breaks the pressure floor on every seed while the scheduled plan with modulated valves passes all eleven (Table 11.4).

References

[1] D. E. Seborg, T. F. Edgar, D. A. Mellichamp, and F. J. Doyle, III, *Process Dynamics and Control*, 4th ed. Hoboken, NJ: John Wiley & Sons, 2016.

[2] S. Longo, B. M. d'Antoni, M. Bongards, A. Chaparro, A. Cronrath, F. Fatone, J. M. Lema, M. Mauricio-Iglesias, A. Soares, and A. Hospido, "Monitoring and diagnosis of energy consumption in wastewater treatment plants. A state of the art and proposals for improvement," *Applied Energy*, vol. 179, pp. 1251–1268, 2016.

[3] U.S. Environmental Protection Agency, "Energy efficiency in water and wastewater facilities: A guide to developing and implementing greenhouse gas reduction programs," U.S. Environmental Protection Agency, Washington, DC, Local Government Climate and Energy Strategy Series EPA-430-R-09-038, 2013.

[4] ——, "Evaluation of energy conservation measures for wastewater treatment facilities," U.S. Environmental Protection Agency, Office of Wastewater Management, Washington, DC, Tech. Rep. EPA 832-R-10-005, Sep. 2010.

[5] S. Skogestad, "Plantwide control: the search for the self-optimizing control structure," *Journal of Process Control*, vol. 10, no. 5, pp. 487–507, 2000.

[6] C. L. Smith, "Valve position control," in *Advanced Process Control: Beyond Single-Loop Control*. Hoboken, NJ: John Wiley & Sons, 2010, pp. 179–197.

[7] E. Ardern and W. T. Lockett, "Experiments on the oxidation of sewage without the aid of filters," *Journal of the Society of Chemical Industry*, vol. 33, no. 10, pp. 523–539, 1914.

[8] J. Monod, "The growth of bacterial cultures," *Annual Review of Microbiology*, vol. 3, no. 1, pp. 371–394, 1949.

[9] G. Tchobanoglous, H. D. Stensel, R. Tsuchihashi, and F. L. Burton, *Metcalf and Eddy Wastewater Engineering: Treatment and Resource Recovery*, 5th ed. New York, NY: McGraw-Hill, 2014.

[10] M. Henze, C. P. L. Grady, W. Gujer, G. V. R. Marais, and T. Matsuo, "A general model for single-sludge wastewater treatment systems," *Water Research*, vol. 21, no. 5, pp. 505–515, 1987.

[11] G. Olsson, "ICA and me – A subjective review," *Water Research*, vol. 46, no. 6, pp. 1585–1624, 2012.

[12] M. K. Stenstrom and R. G. Gilbert, "Effects of alpha, beta and theta factor upon the design, specification and operation of aeration systems," *Water Research*, vol. 15, no. 6, pp. 643–654, 1981.

[13] D. Rosso, L. E. Larson, and M. K. Stenstrom, "Aeration of large-scale municipal wastewater treatment plants: state of the art," *Water Science and Technology*, vol. 57, no. 7, pp. 973–978, 2008.

[14] L. Rieger, J. Alex, W. Gujer, and H. Siegrist, "Modelling of aeration systems at wastewater treatment plants," *Water Science and Technology*, vol. 53, no. 4-5, pp. 439–447, 2006.

[15] L. Åmand, G. Olsson, and B. Carlsson, "Aeration control – a review," *Water Science and Technology*, vol. 67, no. 11, pp. 2374–2398, 2013.

[16] L. Åmand and B. Carlsson, "Optimal aeration control in a nitrifying activated sludge process," *Water Research*, vol. 46, no. 7, pp. 2101–2110, 2012.

[17] L. Rieger, R. M. Jones, P. L. Dold, and C. B. Bott, "Ammonia-based feedforward and feedback aeration control in activated sludge processes," *Water Environment Research*, vol. 86, no. 1, pp. 63–73, 2014.

[18] M. J. Kampschreur, H. Temmink, R. Kleerebezem, M. S. M. Jetten, and M. C. M. van Loosdrecht, "Nitrous oxide emission during wastewater treatment," *Water Research*, vol. 43, no. 17, pp. 4093–4103, 2009.

[19] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.

[20] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.

[21] M. O'Brien, J. Mack, B. Lennox, D. Lovett, and A. Wall, "Model predictive control of an activated sludge process: A case study," *Control Engineering Practice*, vol. 19, no. 1, pp. 54–61, 2011.

[22] D. Goldfarb and A. Idnani, "A numerically stable dual method for solving strictly convex quadratic programs," *Mathematical Programming*, vol. 27, no. 1, pp. 1–33, 1983.

[23] M. Diehl, H. G. Bock, and J. P. Schlöder, "A real-time iteration scheme for nonlinear optimization in optimal feedback control," *SIAM Journal on Control and Optimization*, vol. 43, no. 5, pp. 1714–1736, 2005.

[24] G. Pannocchia and J. B. Rawlings, "Disturbance models for offset-free model-predictive control," *AIChE Journal*, vol. 49, no. 2, pp. 426–437, 2003.

[25] J. W. O'Dell, "Method 350.1: Determination of ammonia nitrogen by semi-automated colorimetry, revision 2.0," U.S. Environmental Protection Agency, Environmental Monitoring Systems Laboratory, Cincinnati, OH, Tech. Rep., Aug. 1993.

[26] K. Stouffer, M. Pease, C. Tang, T. Zimmerman, V. Pillitteri, S. Lightman, A. Hahn, S. Saravia, A. Sherule, and M. Thompson, "Guide to operational technology (OT) security," National Institute of Standards and Technology, Gaithersburg, MD, NIST Special Publication 800-82r3, 2023.

[27] J. D. Hedengren, R. Asgharzadeh Shishavan, K. M. Powell, and T. F. Edgar, "Nonlinear modeling, estimation and predictive control in APMonitor," *Computers & Chemical Engineering*, vol. 70, pp. 133–148, 2014.

10.10 Exercises

10.1 Concept check · 2 min

In Scenario 11 the sewage drops from 13 to 9.5 °C for two days with the ammonia supervisor in service. The nitrifier population wanders between 97.6 and 101.9 g VSS/m^3 and ends where it began, while the effluent ammonia triples. Why does the population not fall?

(a) The supervisor raises the dissolved oxygen to its ceiling, which gives back the growth rate the cold took away

(b) Decay slows with the same temperature coefficient as growth, so the two effects cancel in the balance

(c) Wasting fixes the average growth rate at steady state, so the ammonia rises until the nitrifiers grow that fast again

(d) Three days is too short for a state with a time constant of days, so a longer run would show the loss

10.2 Concept check · 1 min

The most-open-valve controller reads the largest *unsaturated* output of the three dissolved oxygen loops, not the measured valve positions. Why?

(a) The unsaturated output is quieter than a stem position, so the slow loop can carry a higher gain

(b) The valves take a quarter of a minute to travel, a lag that would destabilize a 45-minute integral

(c) Reading the demand lets the loop act before the dissolved oxygen moves, as a feedforward would

(d) A valve pinned fully open cannot report that it wanted more, so a position reading stops raising the header

10.3 Concept check · 2 min

On its first day the ammonia supervisor of Scenario 7, aimed at 1.90 g N/m^3, saves 194 kWh against a fixed setpoint at nearby compliance. Run on with nothing changed, it settles at 4109 and 4105 kWh with worst rolling means of 2.563 and 2.598 g N/m^3, outside the permit. What made the first day look better than the loop is?

(a) Its eight-hour integral needs about a day to forget the 2.0 mg/L it was seeded from, so day one ran richer than its aim

(b) The nitrifier population declined at the low overnight oxygen, so the later days needed less ammonia removed

(c) Analyzer noise accumulated in the integral over three days and pushed the setpoint away from its aim

(d) The second and third days carried a lighter load, so the same aim let more ammonia through

10.4 Theory and calculation · 6 min

Equation (10.9) gives the saturation the bubbles see. Take $\beta = 0.95$, a clean-water solubility $C_{s,w} = 10.50\,\mathrm{mg/L}$ at 13 °C, $\rho = 1000\,\mathrm{kg/m^3}$, $g = 9.81\,\mathrm{m/s^2}$, $P_{atm} = 101.325\,\mathrm{kPa}$ and $h = 5\,\mathrm{m}$.

(a) Compute C_s^*. *Checkpoint:* 12.39 mg/L.

(b) What share of the transfer driving force does a 2 mg/L setpoint remove? *Checkpoint:* 16.1 %.

(c) At a fixed uptake R, the steady air is $q \propto [R/(C_s^* - DO)]^{1/n}$ with $n = 0.75$. Find how the air for one more mg/L depends on DO, and compute its ratio at 3.5 mg/L against 0.5 mg/L. *Checkpoint:* $\propto (C_s^* - DO)^{-(1/n+1)}$, a ratio of 1.97.

10.5 Theory and calculation · 7 min

At the flat average load the works treats 20 000 m^3/d at 30 g N/m^3, assimilates a fraction $f_{ass} = 0.12$ of it and discharges 0.378 g N/m^3. Use $V_t = 1700\,\mathrm{m^3}$, $Y_A = 0.24$, $b_{A,20} = 0.12/\mathrm{d}$, $\theta_B = 1.029$, 13 °C and a seven-day sludge age.

(a) Compute the steady mean growth rate $\bar{\mu}_A$ from Equation (10.4). *Checkpoint:* 0.2411/d.

(b) Summing Equation (10.2) over the three tanks, the nitrifiers remove $3\bar{\mu}_A X_A V_t / Y_A$ of ammonia a day. At steady state this equals the load less assimilation less the effluent. Compute X_A. *Checkpoint:* 520.4 kg N/d nitrified, $X_A = 101.6\,\mathrm{g\,VSS/m^3}$.

(c) Cut the sludge age to five days. Compute $\bar{\mu}_A$, and X_A if the nitrified mass did not change. The model's steady state is 79.5 g VSS/m^3. What effluent ammonia does that imply? *Checkpoint:* 0.2982/d, 82.1 g VSS/m^3, about 1.2 g N/m^3.

10.6 Theory and calculation · 7 min

The header balance of Equation (10.11) sets how fast the inner loop can be. At the design point the three valves pass 122.1 Nm^3/min from a header at 62.0 kPa, the back pressure is $P_{back} = 53.05\,\mathrm{kPa}$, the blower curve gives −1.72 Nm^3/min per kPa, and $V_h = 250\,\mathrm{m^3}$.

(a) From Equation (10.10), show that $\partial \sum q_i / \partial P_h = \sum q_i / (2(P_h - P_{back}))$ at fixed valve positions, and evaluate it. *Checkpoint:* 6.82 Nm^3/min per kPa.

(b) Linearize Equation (10.11) at fixed speed and compute the header time constant. Compare it with the 6.20 min of an aerated tank. *Checkpoint:* 0.289 min or 17.3 s, about 21 times faster.

(c) In Scenario 1 opening valve 1 pulls the header from 62.0 to 59.7 kPa. Valves 2 and 3 do not move and passed 36.6 and 17.8 Nm^3/min. Predict the flow each loses. *Checkpoint:* About 5.0 and 2.5 Nm^3/min (the run reads 4.9 and 2.4).

10.7 App challenge · 7 min

Load Scenario 5, *5 · A whole day at a fixed setpoint*, at the default seed and run it past 1440 min at a speed of 60 (60 simulated minutes per second, about 25 s). Export the CSV (its first lines start with #) and use the row at `time_min` = 1440.

(a) Read `metric_energy_kWh`, the largest `nh4_avg_gNm3` after 480 min, and the largest `nh4_gNm3` with its clock time (`clock_h`). *Checkpoint:* 4335 kWh (±1), 2.275 g N/m^3 (±0.003), 3.27 g N/m^3 at about 11:45.

(b) Load Scenario 6, *6 · The same day, cheaper, and over the consent*, as shipped (1.2 mg/L), and repeat. Also read `metric_consent_min` at 1440. What does the lower setpoint save, and what does it cost? *Checkpoint:* 3813 kWh, 3.784 g N/m^3, 4.79 g N/m^3, 603 min over the permit. A saving of 12 %.

(c) The frontier of Table 10.5 puts 1.2 mg/L at 3819 kWh and 3.644 g N/m^3. Why does Scenario 6 differ? *Checkpoint:* A different opening state.

10.8 App challenge · 9 min

Load Scenario 7, *7 · Ammonia-based aeration control*, at the default seed and run it past 4320 min at a speed of 200 (about 22 s). Export the CSV. A day's energy is the difference of `metric_energy_kWh` between the rows at its start and end, and its worst mean is the largest `nh4_avg_gNm3` from 480 min into the day to its end.

(a) Give each day's energy and worst mean, and `metric_consent_min` at 4320. Tolerances: 1 kWh and 0.003 g N/m^3. *Checkpoint:* 4253, 4109, 4105 kWh. 2.134, 2.562, 2.597 g N/m^3. 222 min.

(b) The fixed 2.0 mg/L setpoint of Scenario 5 spends 4335, 4341 and 4342 kWh on three days at worst means of 2.277, 2.245 and 2.221 g N/m^3 (Table 10.6). Which conclusion does day one support, and which does day three? *Checkpoint:* Day one: cheaper and cleaner. Day three: cheaper and illegal.

(c) Reload the scenario, set *Effluent ammonia setpoint* to 1.30 before Run and repeat (a). Compare the third day with the fixed 2.02 mg/L of Table 10.6, 4355 kWh at 2.201 g N/m^3. *Checkpoint:* 4471, 4307, 4289 kWh. 1.918, 2.182, 2.203 g N/m^3. 66 kWh or 1.5 % saved.

10.9 App challenge · 10 min

Load Scenario 14, *14 · The eight-hour mean, planned a day ahead (MPC)*, at the default seed and run it past 4320 min at a speed of 200. Read each day as in the previous exercise. Under the MPC the dashed setpoint line of the ammonia trend shows the fallback supervisor's 1.90 g N/m^3, not the plan's ceiling.

(a) Give each day's energy and worst mean, and the largest `nh4_gNm3` on day three with its clock time. *Checkpoint:* 4018, 4010, 4011 kWh. 2.210, 2.218, 2.206 g N/m^3. 3.22 g N/m^3 at about 20:37.

(b) Reload, set *MPC load preview* to *Off* before Run and repeat the energies and worst means. Read `metric_consent_min` at 4320. *Checkpoint:* 4306, 4309, 4306 kWh. 2.595, 2.628, 2.602 g N/m^3. 362 min.

(c) The supervisor at 1.30 g N/m^3 spends 4289 kWh on its third day and discharges 29.0 kg of nitrogen, the MPC 35.9 kg. What does the plan save, and how many kilowatt-hours does it buy with each extra kilogram of nitrogen? *Checkpoint:* 278 kWh, 6.5 %, about 40 kWh per kg.

10.10 Application area · 4 min

The chapter prices three changes to the aeration control of this works at twelve cents a kilowatt-hour: the settled ammonia supervisor saves 66 kWh a day, most-open-valve control 267 kWh a day, and a fixed setpoint chosen for the February day (2.25 mg/L, 4159 kWh) costs more than 2.0 mg/L with the same header loop (3995 kWh) on every day it is left in place.

(a) Put each in dollars a year. *Checkpoint:* About $2,900, $11,700 and $7,200.

(b) An ammonium analyzer costs several times the first figure in capital and a few thousand dollars a year in reagent. Which change would you make first, and what does the analyzer actually buy? *Checkpoint:* The header loop. The analyzer buys not having to set the worst-day setpoint all year.

10.11 Application area · 4 min

The works has one variable-speed blower whose surge flow at full speed is 82 Nm^3/min, falling in proportion to speed, with a minimum speed of 48 % and an anti-surge gain $k_{vent} = 2$. In Scenario 10, a dry Sunday night, the tanks take 31.3 Nm^3/min at 04:10 while the machine passes 37.1 Nm^3/min against a surge line of 40.0 Nm^3/min.

(a) What is the surge flow at the minimum speed, and what does Equation (10.13) vent at 04:10? Check the balance. *Checkpoint:* 39.4 Nm^3/min. 5.8 Nm^3/min, equal to $37.1 - 31.3$.

(b) At that load, taking the setpoint from 2.0 to 0.5 mg/L cuts the air to the tanks by 19 % and the power by 4.5 %. Explain, and name two equipment remedies and the number that sizes them. *Checkpoint:* The vent makes up the air the tanks stop taking. A smaller companion blower, wider turndown, or a basin taken offline, sized on the vented volume.

10.12 Review the agent · 5 min

An AI assistant reviews the graded challenge, Scenario 15, and writes: *"Press Reveal suggested tuning, then set Aeration control to MPC. The plan holds its 2.2 g N/m^3 ceiling on the eight-hour mean, which is exactly the challenge's 2.20 g N/m^3 clause, and it plans the day ahead, so the day passes with about 350 kWh more to spare than the suggested supervisor."* Find the flaw and say how you would check it in the app.