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CHAPTER 6

Pricing Heat in a District Heating Network Whose Delay Follows the Flow

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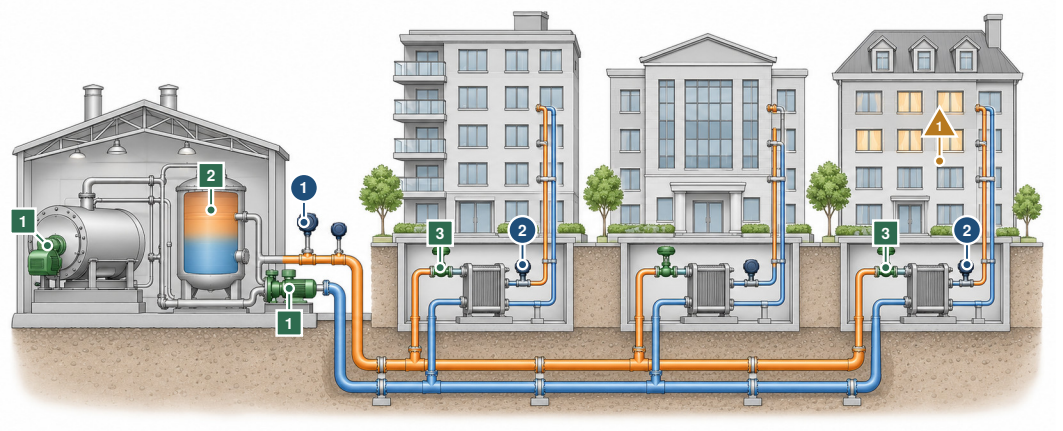
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CHAPTER 6

Pricing Heat in a District Heating Network Whose Delay Follows the Flow



The controller moves 1 heat from the plant and the pump's push 2 storage tank, charged or drawn down 3 each building's valve, setting its draw

The controller reads 1 supply temperature and pressure at the plant 2 each building's heat meter

What disturbs it 1 cold outdoors and each building's changing load

Hot water leaving the plant takes hours to reach the far building, and how many hours depends on how much the valves draw. Illustration, not to scale.

IN PLAIN TERMS **Sending Heat Ahead Through a Town's Pipes**

The main idea. A heat network pumps hot water from one plant through buried pipes to many buildings. The water takes hours to arrive, and how many hours depends on how much water the buildings draw.

The challenge. Heat sent early can be kept in the pipes, a tank and the buildings, but it pays only if cheap heat was spare when it was sent. A posted price for heat can share the work between the plant and the buildings, yet it stops working when the plant runs short.

Why it matters. Fuel bought at the wrong hour costs money, and a building left short of heat goes cold.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history in Section 6.2, the facts in Sections 6.4 and 6.6, the try-it segments in Section 6.6, and the key points
Study	50 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	20 min	the first page, the results (Section 6.6), Technology in Practice (Section 6.7), Limitations (Section 6.8) and the open challenges

The control question In a heat network the flow the consumers draw sets the transport delay, so heat sent ahead of a morning peak arrives when the valves allow. When does sending heat ahead pay, and can a posted price of heat coordinate a plant and six consumers as well as one central plan, even when the plant runs short?

Learning objectives

- Compute a consumer's transport delay as the sum of its segments' residences, each at its own flow, and explain why it is not the path's volume over the plant's flow.
- Explain when charging the pipes, the tank and the buildings ahead of a peak saves fuel, and why a boost that arrives on time can still cost more.
- Describe how dual decomposition splits the planning model into a plant problem and six consumer problems coordinated by a price per block.
- Judge from the imbalance and the duality gap whether a price has settled, and explain why a fixed price step fails when a capacity binds or the plant is short.
- Value the heat left in the stores at the end of a run and compare strategies on the graded cost.

Prerequisites Sections 1.6.2 (velocity form), 1.4.2 (prediction models), 1.4.6 (soft constraints), 1.4.8 (economic MPC and terminal value), 1.4.11 (plans coordinated by a price) and 1.6.4 (how an exercise names a run), and Chapter 5 for plans shared over a network with delays.

The app https://apmonitor.com/apps/district-heating/

See also Volume 5, Chapter 4 for a delay of hours carried as Lagrangian slugs and seen late by its analyzer. Volume 17, Chapter 6 for a solar field whose transit shows as a lag when its flow is set by hand, and Volume 17, Chapter 2 for one fuel valve whose heat and power arrive on different clocks.



6.1 Introduction

Water that leaves the simulated plant at 13:05 with 50 kg/s flowing reaches the hospital at the far end of the town 4.48 h later. The same 15 K step sent at 02:00, when the town's valves draw 78 kg/s, arrives in 2.83 h. The path between them is 5.95 km of pipe holding 598 m^3 of supply water, and neither delay equals that volume over the plant's flow (3.2 h at 50 kg/s), because less water passes each junction (35, 19 and 10 kg/s at 13:05) and every segment is crossed at its own flow. The controller sets the flow, and the flow sets the delay.

Two classes of control problem meet in this plant. The first is transport by plug flow through a line whose throughput the controller manipulates, so that the dead time a step test identifies at one throughput is wrong at the next. The second is the allocation of a limited resource among independent users, which a Lagrange multiplier separates into problems each user solves alone [1] and which a coordinating program can drive with prices [2]. Distributed and hierarchical model predictive control (MPC) gives the second class its control form [3, 4], and a heat network adds the first class to it, because the heat a price buys arrives hours after the price is posted.

Two beliefs about heat networks fail on this one. The first holds that a transport delay is a dead time, a property of the pipe. A boost of 15 K sent by hand at 04:00 reaches the hospital at 06:53, on time for the morning peak, and still costs $202 more than the plain supply curve, because the cheap source was already full and the charge was bought from the peak boiler. The second holds that a posted price coordinates as soon as it exists. At the shipped price step, 177 of the 216 control steps of a cold morning settle within 30 rounds, none settles at a step of 0.8, and after the plant's main source trips on the graded cold snap none of the 55 remaining steps settles, because the price that would ration the heat on the same planning problem is about $77,000 per MWh while the iteration reaches $984 at most.

A student meets these pieces in separate places. Network models for simulation and operation describe the delay and the loss [5–8], reviews of network control and predictive control of a network decide the supply temperature [9, 10], and distributed MPC is surveyed as an architecture and analyzed as an algorithm [3, 11], with no plant on which the supply curve, a centralized optimizer and

a price can be run against the same weather. This chapter compares how a heating curve, a hand-timed boost, a centralized MPC and a price-coordinated distributed MPC use a network whose delay follows its flow, and finds where the price stops coordinating, on one declared town in a browser simulator. The plug-flow pipe, the effectiveness relation, the two-capacity building, dual decomposition and the Hankel predictor are established methods. The results are measurements on one town and its seeded weather, among them that the price matches the centralized multiplier wherever it settles and cannot follow it when a capacity binds. The simulated network closes its energy balance to 6×10^{-7} of the heat produced, the planning model is linear with the delays frozen at the measured flows, and the dual iteration reports a gap that is the distance between a feasible plan and a proven lower bound. Each of the ten cases credits the decision it asks for rather than a state the town reaches (Section 1.6.4).

Section 6.2 traces the domain. Section 6.3 sets out the town: its pipes, hydraulics, substations, buildings, plant, weather and instruments. Section 6.4 explains why the flow sets the delay and why the network is a store. Section 6.5 gives the loops, the supply curve, the planning model, the centralized and distributed MPCs and the Hankel predictor. Section 6.6 reports the ten cases as measured, the graded challenge and the code verification, Section 6.7 describes the meters, pumps and control platforms of real networks, Section 6.8 states the limitations, and the exercises in Section 6.10 run the cases.

6.2 Hot Water Networks and the Price of Heat

District heating distributes heat from central sources to many buildings through insulated pipe. A 2014 statement of the fourth generation of the technology traces its earlier generations from steam systems through pressurized hot water to prefabricated pre-insulated pipe, and sets lower supply and return temperatures as the direction of the field [12]. An international review in 2017 surveyed where district heating and cooling are used and how much heat they deliver [13]. A lower supply leaves less margin over what the buildings need, and a lower return is worth fuel to the plant, so the simulated network carries both a return limit and comfort bands that the supply must meet.

> ⧖ HISTORY **1877: Steam Sold by the Meter**
>
> In a town in New York State, the inventor Birdsill Holly set out to avoid the inefficiency of heating every building with its own small boiler. His system, which the landmark dates to 1877, ran a large central boiler plant that sent steam at moderate pressure to the buildings around it through a loop of supply and return mains, heavily insulated to cut the heat lost on the way. Each customer paid for the steam used, found by metering the water that condensed. The idea succeeded, and before the end of the century district heating companies had formed, mainly in large cities. The landmark marks the site of the first district steam heating system in the United States [14]. In this chapter a price also decides when each building draws.

Running a network against fuel prices and cogeneration made its operation an optimization problem. Operational optimization of a district heating system was formulated in 1995 [5], and aggregated dynamic models that reduce a network to a few equivalent pipes for simulation followed in 2002 [6]. Temperature dynamics in a working network were modeled and compared with its measurements in 2007 [7], and an equation-based thermo-hydraulic pipe model for district heating and cooling was presented in 2017 [8]. Predictive control of a complex district heating network was reported in 2005 [10], and a 2018 review of control that unlocks the flexibility of heating and cooling networks treats the pipes and the connected buildings as stores a controller can charge [9]. The buildings' own mass had been evaluated as a store under optimal control in 2004 [15], and models of building heat dynamics with one or more thermal capacities were identified from data in 2011 [16].

Coordination by prices is older than the networks' control. The decomposition principle for linear programs of 1960 lets a coordinating program generate prices for the parts of a problem, which answer with proposals [2], and the generalized Lagrange multiplier method of 1963 allocates limited resources among independent activities [1]. Architectures for distributed and hierarchical MPC were

Table 6.1 Variable Classification

Role	Symbol	Meaning	Range or unit
Objective	\$	fuel cost, stores valued	dollars
CV	T_s	supply temperature	70 to 115 °C
	Δp_i	least differential pressure	SP 100 kPa
	$T_{z,i}$	zone of consumer i	its band
Constrained	$T_{pr,i}$	consumer's return	≤ 55 °C
	T_{rp}	plant's return (challenge)	≤ 55 °C
	T_s	firing high limit	≤ 120 °C
MV	P_c, P_b	CHP and boiler heat	$\leq \bar{C}$, $\leq \bar{B}$
	P_t	tank power, + out	$\lvert P_t \rvert \leq \bar{S}$
	H_p	pump head	$\leq \bar{H}_p$
	q_i	consumer's space heat	valve $m_i \leq \bar{m}_i$
DV	T_{out}	outdoor temperature	seeded
	g_i, $Q_{d,i}$	gains, hot-water draws	schedules
	events	CHP trip, burst, surgery, forecast error	
States	T_s, P_c, P_b	node, two source lags	°C, MW
	V_h, T_h, T_c	tank thermocline, layers	m^3, °C
	$T_{z,i}$, $T_{m,i}$	zone and mass, six each	°C
	parcels	18 pipes' contents	m^3, °C
Measured	T_s, T_{rp}, Q	plant	σ_T, σ_F
	$T_{ps,i}$, $T_{pr,i}$, m_i	each consumer	σ_T, σ_F
	$Q_{sub,i}$, $T_{z,i}$	meter, zone	σ_F, σ_T
	T_{out}	outdoor	0.2 K
Estimated	$\hat{T}_{z,i}$, $\hat{T}_{m,i}$	observer per consumer	°C

reviewed in 2009 [3], the coordination of several optimization-based controllers on one plant was set out as an opportunity and a challenge for process control in 2008 [4], and accelerated gradient methods were applied to dual decomposition in distributed MPC in 2013 [11]. On the modeling side, data-enabled predictive control replaced a parametric model by a Hankel matrix of recorded data in 2019 [17], resting on the 2005 result that a persistently exciting record of a linear system spans its trajectories [18].

6.3 A Plant, Three Trunks and Six Substations

The simulated town is drawn in Figure 6.1. A plant on the left holds a CHP unit, a peak boiler and a 150 m^3 storage tank above a hall whose mixing node sets the supply temperature T_s. A pump sends the water down a trunk of three segments, T1, T2 and T3 (2.0, 2.0 and 1.5 km, 5.5 km in all), which feeds six service branches of 400 to 450 m at junctions J1, J2 and J3, every pipe laid twice as supply and return. Six consumers hang at the branch ends: three apartment blocks, two office blocks and a hospital, each a substation feeding a lumped building. Every length, bore, loss coefficient, building constant, price and limit is a teaching value for a small town, chosen to be of plausible order and taken from no utility's data. The variables are classified in Table 6.1.

6.3.1 Plug Flow Keyed by Volume

Each pipe segment s, of length L_s, bore D_s, cross-section A_s and volume $V_s = A_s L_s$, holds its water as a first-in, first-out queue of parcels. The kraft digester of Volume 5, Chapter 4 carries its chip column the same way, as Lagrangian slugs. A parcel that entered at time t_0 at temperature T_0 leaves at the time t_1 at which the volume pumped in behind it equals the pipe's volume, and its excess over the

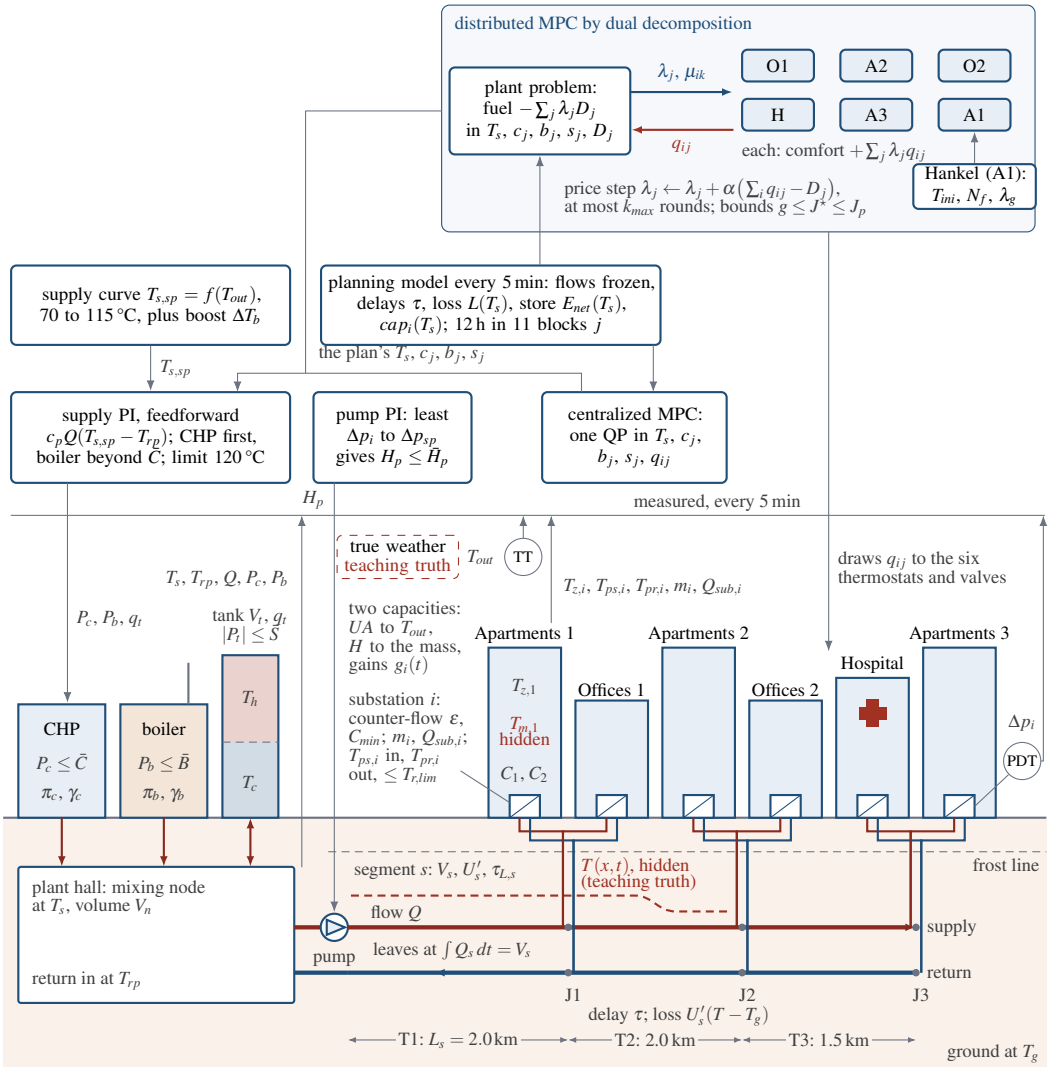


Figure 6.1 The town's heat network as a section with its control layers above it. The plant's units feed the hall's mixing node, the pump sends the water down trunk T1 below the frost line, and branches rise from J1 to J3 to the six substations and their buildings, with the symbols written once on Apartments 1. Every layer reads the measurements on the gray line, every five minutes. In the distributed MPC, drawn as a row of its own above the planning model, the plant problem posts prices and the six consumer problems answer with draws. The supply and return pipes are labeled at the trunk's end. The water's true temperature along the pipe, the buildings' mass and the true weather are teaching truth, drawn dashed or marked hidden, and feed no controller.

ground decays at the pipe's own rate on the way, as shown in Equation (6.1).

$$\int_{t_0}^{t_1} Q_s\,dt = V_s, \qquad \tau_{L,s} = \frac{\rho c_p A_s}{U'_s}$$
$$T(t_1) = T_g + (T_0 - T_g)\,\mathrm{e}^{-(t_1 - t_0)/\tau_{L,s}} \tag{6.1}$$

where Q_s is the segment's flow, U'_s the loss of the insulated pipe per meter and kelvin and $T_g = 5\,^\circ\mathrm{C}$ the ground. At a constant flow the residence is V_s/Q_s and the outlet follows the familiar decay of

Equation (6.2).

$$T_{out} - T_g = (T_{in} - T_g)\exp\big(-U_s' L_s/(\rho c_p Q_s)\big) \tag{6.2}$$

Each step pushes $Q_s \Delta t$ at the inlet and pops the same volume at the outlet, whole parcels and a share of the last, so nothing is interpolated and the energy that leaves is what entered less each parcel's decay. Trunk T1, 2.0 km of 0.45 m bore, holds 318 m^3 with $\tau_L = 224$ h. The segment flows are the sums of the consumers' flows they carry, $Q_s = \sum_{i \in s} q_i$, so the delay to a consumer is the sum of its segments' residences, each at its own flow. The true temperature profile $T(x,t)$ along every pipe is teaching truth that no instrument reads.

6.3.2 Pressure, Valves and the Least Consumer

The pump sets a head H_p. Each segment drops $k_s Q_s |Q_s|$ with $k_s = f L_s \rho/(2 D_s A_s^2)$ and a Darcy factor $f = 0.018$, and a consumer sees what is left after its supply and return paths, as shown in Equation (6.3).

$$\begin{aligned} \Delta p_i &= H_p - \textstyle\sum_{s \in \mathscr{P}_i} 2 k_s Q_s |Q_s| \\ m_i &\le \bar{m}_i \sqrt{\Delta p_i / \Delta p_{ref}} \end{aligned} \tag{6.3}$$

where $\mathscr{P}_i$ is the path from the plant to consumer i and $\bar{m}_i$ is 1.6 times its design primary flow at $\Delta p_{ref} = 100$ kPa, a warm-up margin. A valve passes the flow it is asked for up to that limit, as a pressure-independent valve does. Because every drop depends on every flow, each step finds the flows by a damped fixed point (relaxation 0.4, at most 300 rounds, to 1×10^{-5} kg/s).

6.3.3 Substations Rated by Effectiveness

Each substation is a counter-flow plate exchanger between the primary water and a radiator loop of fixed capacity flow C_s, whose radiators of conductance UA_r face the room. Eliminating the two secondary temperatures gives the space heat as one conductance from the arriving supply to the zone, as shown in Equation (6.4).

$$\begin{aligned} Q_{sub,i} &= G(C_p)\,(T_{ps,i} - T_{z,i}), \quad T_{pr,i} = T_{ps,i} - \frac{Q_{sub,i}}{C_p} \\ G &= \frac{\varepsilon C_{min}}{1 + \varepsilon C_{min}(1/\varepsilon_r - 1)/C_s} \\ \varepsilon &= \frac{1 - \mathrm{e}^{-N(1-C_r)}}{1 - C_r\,\mathrm{e}^{-N(1-C_r)}} \end{aligned} \tag{6.4}$$

where $C_p = m_i c_p$ is the primary capacity flow of the space-heating share, C_{min} and C_r the smaller capacity flow and the ratio of the two, $N = UA_h/C_{min}$ and $\varepsilon_r = 1 - \mathrm{e}^{-UA_r/C_s}$. The conductances are sized so that the design duty passes with a 95 °C supply, a 50 °C primary return and a 70/45 °C radiator loop. G rises with the flow and saturates, so a consumer short of heat at a low supply temperature opens its valve and its return rises. Above the return limit $T_{r,lim} = 55$ °C a limiter closes the valve until the return is back at the limit, giving up duty. A hot-water exchanger on the same connection meets the group's draw $Q_{d,i}$ and returns its primary water at 25 °C, with flow $Q_{d,i}/(c_p(T_{ps,i} - 25\,^\circ\mathrm{C}))$.

6.3.4 Two-Capacity Buildings

Each consumer is a block of buildings lumped into a zone at $T_{z,i}$ and a heavy mass at $T_{m,i}$, as shown in Equation (6.5).

$$\begin{aligned} C_1 \frac{\mathrm{d}T_{z,i}}{\mathrm{d}t} &= Q_{sub,i} + g_i(t) - UA\,(T_{z,i} - T_{out}) \\ &\quad - H\,(T_{z,i} - T_{m,i}) \\ C_2 \frac{\mathrm{d}T_{m,i}}{\mathrm{d}t} &= H\,(T_{z,i} - T_{m,i}) \end{aligned} \tag{6.5}$$

where $UA = Q_{des}/36\,\mathrm{K}$ carries the design duty at −15 °C outside and 21 °C inside, $H = r_H UA$ links the room to the structure, $C_1 = \tau_z(UA + H)$, $C_2 = \tau_{slow}/(1/UA + 1/H)$ and g_i is the internal gain on its schedule plus the sun on the glass. The fast time constants are 0.48 to 0.94 h and the slow ones 26 to 53 h. The slow one makes a building a store that a price can move. Chapter 9 spends a comfort band against the same kind of slow store, an office's slab. The apartments' bands are 20 to 23 °C, the offices' 20 to 24 °C from 07:00 to 18:00 and 16 to 26 °C otherwise, and the hospital's 21 to 23 °C. A block is judged in breach when its zone lies more than 0.5 K outside its band.

6.3.5 The Plant, Its Tank and Its Fuel

Return water at T_{rp} enters the hall's mixing node of volume V_n, where the CHP's and the boiler's heat are added, and a two-node stratified tank hangs between supply and return. The node, the thermocline and the sources follow Equation (6.6).

$$\begin{aligned} \rho c_p V_n \frac{\mathrm{d}T_s}{\mathrm{d}t} &= \rho c_p \left[(Q - q_t^+)(T_{rp} - T_s) + q_t^+ (T_h - T_s) \right. \\ &\qquad \left. + q_t^- (T_c - T_s) \right] + P_c + P_b \\ \frac{\mathrm{d}V_h}{\mathrm{d}t} &= q_t^- - q_t^+, \qquad \tau_c \frac{\mathrm{d}P_c}{\mathrm{d}t} = \min(P_{c,sp}, \bar{C}) - P_c \end{aligned} \tag{6.6}$$

where Q is the network flow, q_t^+ the tank's discharge (hot water from its top into the node, return water into its bottom) and q_t^- its charge, V_h the hot layer's volume at T_h above the cold layer at T_c, and the boiler follows its setpoint likewise with τ_b and $\bar{B}$. The hot layer obeys $V_h \dot{T}_h = q_t^- (T_s - T_h)$ and the cold layer $(V_t - V_h)\dot{T}_c = q_t^+ (T_{rp} - T_c)$, each less its share of a shell loss of 1500 W/K to 5 °C. A tank power P_t is turned into a flow over the layers' temperature difference, within 3 and 97 % of the volume. The fuel cost rate is Equation (6.7).

$$\dot{\$} = f(T_{rp})\,(\pi_c P_c + \pi_b P_b) + \tfrac{1}{2}\gamma_c P_c^2 + \tfrac{1}{2}\gamma_b P_b^2 \tag{6.7}$$

with P in MW, where $f = 1 + 0.1\,[1 - \mathrm{clip}((57 - T_{rp})/17, 0, 1)]$ charges up to 10 % more fuel as the return rises from 40 to 57 °C, the credit of a flue-gas condenser fading above its dew point. The CHP's price is its fuel less the power it sells, and its marginal cost $\pi_c + \gamma_c P_c$ reaches about the boiler's \$52 per MWh at its full 14 MW. Volume 17, Chapter 2 treats the other side of such a unit, one fuel valve whose electricity and heat arrive on different clocks.

6.3.6 Weather, Forecast and Instruments

A case declares its days by a daily mean and a swing with its minimum at 06:00 and maximum at 15:00. The true outdoor temperature adds colored noise of standard deviation 0.8 K and correlation time 3 h, drawn once from the case's seed, and the controllers' forecast corrects the declared weather by the latest measurement, as shown in Equation (6.8).

$$\hat{T}_{out}(t + \ell) = T_{dec}(t + \ell) + \big(T_{out,m}(t) - T_{dec}(t)\big)\,\mathrm{e}^{-\ell/3\,\mathrm{h}} \tag{6.8}$$

Gains and hot-water draws carry 10 % colored noise. Every five minutes the controllers receive one packet: at the plant T_s, T_{rp}, the flow Q, the pump head, the source meters and the tank's layers, at each consumer $T_{ps,i}$, $T_{pr,i}$, m_i, Δp_i, $Q_{sub,i}$, the hot-water meter and the zone $T_{z,i}$, and T_{out}, with white noise of $\sigma_T = 0.1$ K on temperatures (0.2 K outdoors), $\sigma_F = 0.5$ % on flows and meters and 2 kPa on pressures. The pipe profiles, the buildings' mass, the true weather and the load noise never enter the packet, and the app's tests search every controller file for a read of any of them.

6.3.7 Constants, Line-Out and Numerics

Table 6.2 lists the parameters. Each case starts lined out at the hour its weather declares: every zone at its preferred temperature with the heat that holds it there, the flows and arrival temperatures iterated

Table 6.2 Parameters, Defaults and Their Standing

Symbol	Meaning	Value	Unit, standing
ρ, c_p	water near 80 °C	970, 4190	SI
L_s, D_s	trunks T1, T2, T3	2.0, 2.0, 1.5	km
	bores	0.45, 0.35, 0.25	m, T
U'_s	trunk loss per pipe	0.80, 0.70, 0.60	W/(m K), T
	branch loss per pipe	0.40 to 0.45	W/(m K), T
f, T_g	Darcy factor, ground	0.018, 5	°C, T, A
Q_{des}	design duty A1, O1, A2	4.0, 3.0, 4.0	MW, T
	O2, H, A3	3.0, 3.5, 3.5	MW, T
τ_z, τ_{slow}	apartments	0.75, 40	h, T
	offices	0.5, 25	h, T
	hospital	1.0, 50	h, T
r_H	H/UA	3, 2.5, 3	T
$\bar{m}_i$	valve rating	1.6	× design, T
$\bar{C}$, $\bar{B}$	CHP, boiler capacity	14, 16	MW, T, A
V_t, $\bar{S}$	tank, its power	150, 8	m^3, MW, T, A
V_n	mixing node	40	m^3, H
τ_c, τ_b	CHP and boiler lags	0.15, 0.10	h, T, A
π_c, γ_c	CHP heat price	22, 2.1	$/MWh, /MW, A
π_b, γ_b	boiler heat price	52, 1.5	$/MWh, /MW, A
$\bar{H}_p$, Δp_{sp}	pump limit, setpoint	600, 100	kPa, T, A, C
$T_{r,lim}$	return limit	55	°C, T, A
$T_{s,des}$	curve at −15 °C	95	°C, C
σ_T, σ_F	instrument noise	0.1, 0.005	K, relative, A
K_T, $\tau_{I,T}$	supply loop	0.4, 1.0	MW/K, h, C
K_p, $\tau_{I,p}$	pump loop	0.5, 0.25	kPa/kPa, h, C
K_z, $\tau_{I,z}$	thermostats	1.0, 2	MW/K, h, C
α, k_{max}	price step, round cap	0.5, 30	$/MWh/MW, C
T_{ini}, N_f, λ_g	Hankel predictor	4, 12, 0.001	slots, C

T teaching value, A adjustable in the Model dialog, H hidden, C a control on the faceplates. The Model dialog also scales the pipe loss, the exchangers, the buildings' mass and Apartments 2's radiators. The challenge and case 10 ship a 15 MW boiler.

twice, every pipe filled at its inlet temperature, and the tank half charged. The engine's base is the hour (Section 1.6.1). A step of 10 s pops and pushes the pipes, then integrates the node, the tank, the sources, the buildings and eighteen ledgers by the fourth-order Runge–Kutta method with the step's boundary values held, and then enters the plant's outflow and each consumer's return into their pipes at the temperatures the ledgers integrated. The controllers run every 5 min, and the historian logs every minute.

6.3.8 Assumptions

The fronts stay sharp because the pipe has no mixing or diffusion, while a real front smears over a few hundred meters. The ground holds one temperature, and the loss coefficients are teaching values rather than a manufacturer's series. Each consumer is one zone and one mass with one radiator loop at a fixed flow and one hot-water exchanger, so a block's average temperature stands for its occupants'. The hydraulics are algebraic at every step, with no pressure waves, and pumping costs nothing. The CHP has no minimum load and its electrical side enters only through its net heat price. The weather is one declared record with seeded noise, and the forecast never sees the noise's future.

6.4 Why the Flow Sets the Delay

The delay a consumer sees is a sum of residences at flows the consumers choose. In case 1 the 13:05 step left at 50 kg/s while T2, T3 and the hospital's branch carried 34.9, 19.2 and 9.9 kg/s. Integrating

each segment's own flow until it has passed the segment's volume puts the front at the hospital 4.43 h after the setpoint jumped (1.69, 1.48, 0.92 and 0.35 h for T1, T2, T3 and the branch), or 4.58 h after the plant's own supply had risen by half the step 0.13 h later, and these bracket the measured 4.48 h. Frozen at the departure flows the sum is 4.62 h, and the path's 598 m^3 over the plant's whole flow would be 3.23 h. At 02:00, with 78.2 kg/s leaving, the integral gives 2.81 h against 2.83 h measured. The two delays stand in the ratio 1.58 while the plant's flows stand in the ratio 1.57, because the flow divides almost the same way at both hours.

Pause. Try Exercise 6.1 before reading on.

A slow delay makes the network a store. Raising T_s fills the pipes with hotter water that the consumers draw hours later, while the hotter pipe loses more on the way by Equation (6.2). The buildings hold more, since a block warmed a fraction of a kelvin in the evening still carries the heat at the morning peak when its slow time constant is 26 to 53 h. Timing either store needs the delay, which the flow sets, and the price of the charging heat, which the dispatch sets. A consumer whose radiators are too small opens its valve, raising its flow and its return together, and the plant pays through the condenser's credit of Equation (6.7).

DID YOU KNOW? Tanks That Bridge a Winter Peak

In one geothermal network, winter peaks call for about 18 400 m^3/h of hot water for space heating, while the geothermal sources typically give 7200 m^3/h. Beside an oil-fired peak plant, 101 000 m^3 of water tanks were built to bridge those peaks [9]. The tank of this chapter's town holds 150 m^3 (Table 6.2).

6.5 Loops, Planners and a Price

Every loop is the velocity-form proportional-integral (PI) law of Section 1.6.2, with back-calculation on the integral and the clamp on the emitted output, run every $\Delta t_c = 5$ min on the packet of measurements. In Manual the plant's heat and the pump's head come from hand stations, and the PI laws track them so that a return to Auto is bumpless.

6.5.1 The Regulatory Layer

The supply loop computes the heat the plant must add, a PI on the measured T_s whose bias is the feedforward $c_p Q_m (T_{s,sp} - T_{rp,m})$ from the measured flow and return. Its ceiling is the plant's capacity and a firing high limit that admits no more heat than brings the supply to $T_{hi} = 120\,°C$ within two control periods, $h_{hi} = c_p Q_m (T_{hi} - T_{rp,m}) + \rho c_p V_n (T_{hi} - T_{s,m})/(2\Delta t_c)$. The limit is a teaching value above the operating range, the continuous rating for which bonded pre-insulated pipe to the standard EN 253 is made, and with the sources' lags 40 MW by hand peaks at 126.7 °C. The dispatch then takes the tank's power P_t off the request, gives the CHP up to its available capacity and the boiler only what lies beyond it, so the boiler does not fire to cover the CHP's 9 min lag. Under the curve a tank rule charges on the CHP's spare heat and discharges to spare the boiler. The pump loop sets H_p from the measured path drops plus the setpoint Δp_{sp}, trimmed by a PI on the least measured Δp_i.

Each consumer runs its own thermostat, a PI on $T_{z,i}$ with its building's heat-loss feedforward, which sets its space heat q_i under the curve. The valve flow that delivers q_i is found by bisection on the monotone G of Equation (6.4), with the declared exchanger and radiators, at the measured supply and zone, and the return limiter cuts it when $T_{pr,i}$ would exceed $T_{r,lim}$. Under either MPC the plan's draw is applied and the thermostat becomes a guard that adds K_z times the shortfall when a zone comes within 0.3 K of its band's lower edge, a band read one hour ahead. The plan does not see the guard.

6.5.2 The Supply Curve and a Boost

The heating curve gives the setpoint of Equation (6.9), held between 70 and 115 °C.

$$T_{s,sp} = 70 + \frac{10 - T_{out,m}}{25}\,(T_{s,des} - 70) \tag{6.9}$$

with temperatures in °C, so the supply is 70 °C at 10 °C outside and $T_{s,des}$ at −15 °C. The manual boost adds ΔT_b for a set length from a clock hour every day. Neither looks ahead.

6.5.3 A Planning Model With Frozen Delays

Both MPCs solve one linear planning model, built every five minutes from the packet and the declared constants, a prediction model in the sense of Section 1.4.2. Twelve hours are split into 24 slots of $\Delta = 0.5$ h and eleven move blocks j (four of 0.5 h, four of 1 h, three of 2 h), with decisions $T_{s,j}$, c_j, b_j, s_j (CHP, boiler and tank, MW) and the six draws q_{ij}. The flows are frozen at their measured values, so each segment's residence $\tau_s = V_s/Q_s$, each consumer's delay d_i and its decay $\phi_i = \prod_s \mathrm{e}^{-\tau_s/\tau_{L,s}}$ are fixed for the solve and recomputed at the next. The supply pipes' stored heat E_{net} and their loss L are then affine in the plant's supply history, the controller's own record before now and the plan's blocks after it, and each block's heat balance and each slot's capacity are Equation (6.10).

$$\begin{aligned} n_j\Delta\Big(c_j + b_j + s_j - \textstyle\sum_i q_{ij}\Big) &= \textstyle\sum_{k\in j}\Delta\,(d_k + L_k(T_s)) \\ &\quad + E_{net}(t_{j+1}) - E_{net}(t_j) \\ q_{ik} &\le cap_i\big(T_{arr,ik}(T_s)\big) \end{aligned} \tag{6.10}$$

where n_j is the block's slot count, d_k the declared hot-water draws plus the return pipes' loss held at the measured returns, and $T_{arr,ik} = \phi_i \bar{T}_s(t_k - d_i) + (1-\phi_i)T_g$. The capacity is the tangent of the exchanger's wide-open duty (with the limiter's flow cut when it is on) at the measured arrival temperature. Each consumer's zone and mass follow Equation (6.5) discretized exactly at Δ from the declared constants, started at the observer's estimate (a steady-state Kalman filter per consumer on the same model, driven by its heat meter, $T_{out,m}$ and the declared gains, with process noise of 0.05 and 0.01 K per five minutes on zone and mass and the declared zone measurement noise, $\sigma_T = 0.1$ K). The objective is Equation (6.11).

$$\begin{aligned} J = {} & \textstyle\sum_k \Delta\big[\pi_c' c + \tfrac{1}{2}\gamma_c c^2 + \pi_b' b + \tfrac{1}{2}\gamma_b b^2 + \tfrac{1}{2}\gamma_s s^2\big]_k \\ & + \textstyle\sum_{i,k} w_i (T_{z,ik} - r_{ik})^2 + \sum_{i,j}(\rho_s \sigma_{ij}^2 + \rho_l \sigma_{ij}) \\ & + \textstyle\sum_j s_T (\Delta T_{s,j})^2 + \sum r_Q q^2 - \pi_T (E_{t,N} + E_{net,N}) \end{aligned} \tag{6.11}$$

where $\pi' = f(T_{rp,m})\pi$ freezes the condenser factor at the measured return, r_{ik} is the preferred temperature, w_i is 20, 10 and 100 \$ per K^2 and slot for apartments, offices and the hospital, σ_{ij} is the worst excursion of block j outside the band backed off by 0.5 K and met an hour early, priced at $\rho_s = 2\times10^4$ \$/K^2 and $\rho_l = 2\times10^3$ \$/K as the soft constraint of Equation (1.7), $\gamma_s = 0.5$, $s_T = 1$ \$/K^2 and $r_Q = 0.05$ \$/MW^2. The terminal price π_T values heat left in the tank and the pipes at the horizon's end, the terminal value of Section 1.4.8, at the boiler's base price if the forecast load then exceeds the CHP's capacity, and at the CHP's marginal cost otherwise. The CHP's and boiler's lags are not in the plan, and the supply loop dispatches.

6.5.4 Centralized MPC

The centralized MPC solves Equation (6.11) subject to Equation (6.10), the tank's charge inside its limits at every slot and the boxes on every input, as one quadratic program on the framework's linear MPC and its dense dual active-set solver. It has 176 variables (110 decisions and 66 comfort slacks) and 788 rows, and its Hessian is positive definite, with a smallest eigenvalue of 3.73×10^{-2}. The first block's supply temperature, tank power and draws are applied, and a failed solve keeps the previous plan.

6.5.5 Distributed MPC by Dual Decomposition

The distributed MPC solves the same problem by prices, the dual decomposition that Section 1.4.11 sets out on two tanks sharing a feeder. Chapter 5 divides a plan among gates that exchange plans

instead of prices. The plant keeps its own copy D_j of the heat it delivers in block j, and two couplings are relaxed: the balance $\sum_i q_{ij} = D_j$ with a price λ_j per block, and each capacity with a surcharge $\mu_{ik} \geq 0$ per consumer and slot. The Lagrangian then separates. Consumer i minimizes its comfort terms of Equation (6.11) plus $\sum_k \Delta(\lambda_{j(k)} + \mu_{ik}) q_{ik}$ on its own building model, and the plant minimizes its fuel, terminal and move terms less $\sum_j n_j \Delta \lambda_j D_j$ and the surcharges it earns by raising the capacity, subject to the balance in D_j, the tank and its boxes. The sum of the seven optimal values, less the surcharges' constant part, is the dual function $g(\lambda, \mu)$, a lower bound on the centralized optimum $J^\star$. Each round moves the prices on the imbalances, as shown in Equation (6.12).

$$\begin{aligned} \lambda_j &\leftarrow \lambda_j + \alpha \Big(\textstyle\sum_i q_{ij} - D_j \Big) \\ \mu_{ik} &\leftarrow \max\big(0,\ \mu_{ik} + 60\alpha\,(q_{ik} - cap_{ik}(T_s))\big) \end{aligned} \tag{6.12}$$

with α in \$/MWh per MW, warm-started from the previous control step. The balance update is Equation (1.12) with one price per block. A step settles when every block's imbalance and every capacity excess is under 0.1 MW and the gap below is under 1 % of the plan, or stops at $k_{max} = 30$ rounds. The primal plan is recovered by letting the plant solve once more with D_j pinned to the draws and rows that make its supply plan carry every draw. That plan meets every constraint of the centralized problem, so its cost J_p on the centralized objective is an upper bound and $g \leq J^\star \leq J_p$. The gap $J_p - g$ the app reports is therefore a real one. When no supply plan can carry the draws, each draw is first cut to the capacity of the plant's last plan, and when even that fails the cut draws and the plant's last plan are applied and the step is flagged NO PLAN FITS.

6.5.6 A Building Learned From Its Own Record

Apartments 1 can plan on a Hankel-matrix predictor built only from the run's record. Every 0.5 h slot the controller stores the block's metered heat, the measured T_{out} and the declared gain as u and the measured zone as y, and the last 72 slots (36 h) form block Hankel matrices of depth $T_{ini} + N_f = 16$, 57 columns wide. The predictor is Equation (6.13).

$$\begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix} \theta = \begin{bmatrix} u_{ini} \\ y_{ini} \\ u_f \end{bmatrix}, \quad y_f = Y_f \theta, \quad \theta = (\mathscr{H}^\top \mathscr{H} + \tilde{\lambda}_g I)^{-1} \mathscr{H}^\top b \tag{6.13}$$

where $\mathscr{H}$ stacks U_p, Y_p and U_f, b stacks the data on the right, and $\tilde{\lambda}_g = \lambda_g \,\mathrm{tr}(\mathscr{H}^\top \mathscr{H})/57$. The least-squares weights θ make the predicted zone affine in the future heat, which is the regularized form of data-enabled predictive control with no slack on the past outputs [17], a data-driven predictor of the kind Section 1.4.2 describes. It is rebuilt every hour, and the consumer's problem on it has eight free blocks and one band slack per slot.

6.5.7 Three Models and the Grade

The plant is the whole of Section 6.3. The planning model freezes the delays at the measured flows and the return pipes at the measured returns, and it carries the buildings' declared constants and a 5 °C ground, which are the plant's own until a case or the learner changes the plant, as case 6 halves a radiator: the valves, the observers and both MPCs keep the declared design, and only the return limiter sees the plant's. The Hankel predictor knows only the record. The fuel each run buys is graded with the heat it leaves in its stores valued, as shown in Equation (6.14).

$$\$_{graded} = \$ + \textstyle\sum_k p_k \big(E_k(0) - E_k(t_f) \big) \tag{6.14}$$

where k runs over the tank, the network (pipes and mixing node) and the buildings, and E_k is the heat each holds. In the challenge and case 10, which end with the CHP out, every MWh short or over is valued both ways at the boiler's marginal price at its cap, $p = \pi_b + \gamma_b \bar{B} = \$74.5$ per MWh, which equals comparing two runs with their end states matched. The other cases value deficits at π_b and surpluses at π_c, a conservative rule under which no run gains by emptying or filling its stores.

Table 6.3 The Ten Cases

#	Start, weather	Shipped settings, events	Run
1	12:00, W1	boost +15 K at 13:00 for 2 h	18 h
2	18:00, W1	supply curve	18 h
3	as case 2	boost +15 K at 04:00 for 3 h	18 h
4	as case 2	centralized MPC	18 h
5	as case 2	distributed MPC, α 0.5	18 h
6	as case 2	curve, A2 radiators $\times$0.5, limiter off	18 h
7	as case 2	distributed MPC, CHP trip at 07:00	18 h
8	00:00, W2	curve with A1 excited, then distributed MPC	48 h
9	12:00, W3	challenge: curve, boiler 15 MW, CHP trip at 07:30 on the second morning	48 h
10	12:00, W3	distributed MPC, boiler 15 MW, CHP trip at 00:00 (infeasible)	48 h

Declared daily means, °C: W1 −2, −4, W2 −3, −5, −4, W3, the snap, −4, −7, −10, each with a 4 K swing, lowest at 06:00. Seeds: 11 (case 1), 12 (cases 2 to 7), 13 (case 8), 14 (cases 9 and 10).

Table 6.4 Measured Results of Cases 2 to 8

Case	Setting	Boiler (MWh)	Valued ($)	Worst (K)	Measured
2	curve	28.2	10 282	0	peak 10.9 MW, 06:25
2	curve, tank idle	36.2	—	—	
3	boost 04:00	35.6	10 484	0	hospital at 06:53
4	centralized MPC	19.4	9 953	0	guard 1.5 h
5	distributed, α 0.5	17.8	9 901	0.46	177 of 216 settle
5	distributed, α 0.8	—	9 909	—	none settles
5	distributed, α 1.5	25.3	9 936	—	none settles
6	limiter off	29.9	10 487	0	A2 return 68.7 °C
6	limiter on	—	—	1.15	A2 7.3 h out
7	distributed, trip	78.8	11 530	0.46	curve $12 088
8	distributed	59.2	26 168	0.17	238 of 288 settle
8	A1 on Hankel	—	+136	0.20	170 of 288 settle

Valued: the fuel cost with deficits in the stores at π_b and surpluses at π_e. Worst: the largest distance of any zone outside its declared band (case 6 on: Apartments 2, case 8 Hankel: Apartments 1). Case 8 Hankel: valued cost above the two-capacity run. A dash: not measured.

6.6 What the Cases Measure

The ten cases are listed in Table 6.3 and their outcomes in Table 6.4. Each number in this section was taken from the application's engine stepped without a browser, each case on its own seed and over its own run, with a named setting entered before Run. The same scripts find all 168 numbers that the app's own texts quote, with none missing.

6.6.1 Arrival at Two Flows

Case 1 starts at noon and sends one step. Under the shipped boost the step left at 13:05 and reached Apartments 1 in 2.06 h, Apartments 2 in 3.49 h and the hospital in 4.48 h, as shown in Figure 6.2a. With the boost moved to 02:00 and the case reset, the step left with 78 kg/s flowing and reached the same three in 1.34, 2.21 and 2.83 h. A step is counted as arrived when the supply reaching a consumer

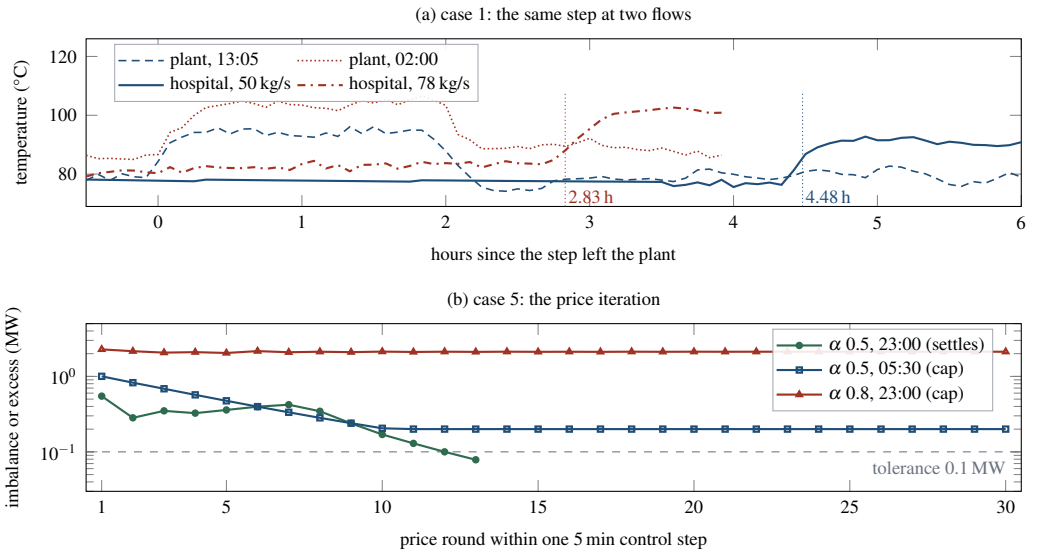


Figure 6.2 The two halves of the lesson, from the engine. (a) Case 1, the true supply temperature leaving the plant (dashed for the 13:05 step, dotted for the 02:00 step) and arriving at the hospital (solid and dash-dotted) after the 15 K step, at 13:05 with 50 kg/s and at 02:00 with 78 kg/s (the second run ends at 06:00). (b) Case 5, the larger of the imbalance and the capacity excess in each round of one control step, for a step that settles at 23:00 (circles), one that stops at the round cap at 05:30 (squares), and the same 23:00 step with $\alpha = 0.8$ (triangles).

has risen by half the step. The flow is lower in the afternoon, when the declared swing is near its daily maximum and the buildings need less heat, and higher at 02:00, when the night is colder.

> **TRY IT IN THE APP** **1 · The Delay at Two Flows**
>
> Load the scenario and press Run. Watch the 15 K step that leaves the plant at 13:05 with 50 kg/s flowing. It reaches Apartments 1 after 2.06 h, Apartments 2 after 3.49 h and the hospital only after 4.48 h, as the front crosses segments that carry less and less water.

6.6.2 Sending Heat Ahead by Hand and by Plan

The cold morning of case 2 drives the supply curve past the CHP's 14 MW from 01:17, and the boiler fires to 10.9 MW at 06:25, making 28.2 MWh over 18 hours for a valued cost of $10,282. The tank, charged on the CHP's spare heat in the evening, spares the boiler 8.0 MWh, for with the tank idle the boiler makes 36.2 MWh. Case 3's boost leaves at 04:00 with 79 kg/s flowing and reaches the hospital 2.88 h later, at 06:53, as the peak arrives. It still raises the boiler's energy to 35.6 MWh and the valued cost by $202, because the CHP was already at its capacity all night and the boost's heat came from the boiler, while the hotter pipe lost more to the ground. A charge pays only with cheaper heat to spare when it is made. The centralized MPC of case 4 finds that heat in the evening. It raises the supply while the CHP has room and lets it fall at the peak, so the consumers draw down the pipes, the tank and their own mass. The boiler makes 19.4 MWh, and the run costs $9,953, $329 or 3.2 % less than the curve, with the offices on their thermostats' guard for 1.5 h of the 18.

Pause. Try Exercise 6.2 before reading on.

6.6.3 A Price That Settles and One That Swings

At the shipped step $\alpha = 0.5$ the price of case 5 settles in 177 of 216 control steps, in 9.8 rounds on average, and the run costs \$9,901, within \$52 of the centralized MPC. The 39 steps that stop at the cap fall between 03:35 and 07:55, when the offices' warm-up meets a consumer's capacity. At 23:00 the imbalance falls below 0.1 MW in 13 rounds with a gap of 0.10 % of the plan, as shown in Figure 6.2b. At 05:30 the imbalance is under 0.1 MW from round 5 and under 0.01 MW from round 11, but one consumer's capacity excess stays at 0.20 MW from round 11 on, while its surcharge climbs by only $60\alpha \times 0.20\,\text{MW} = 6\,\$/\text{MWh}$ a round, and the recovered plan shows a gap of 73 %. At $\alpha = 0.8$ the same 23:00 step keeps 2.1 MW of imbalance through all 30 rounds while the price alternates, and no step of the run settles. At 1.5 none settles either, and the boiler makes 25.3 MWh instead of 17.8 for \$9,936. The recovery keeps every applied plan feasible, but it cannot make an unsettled price a good one.

> **TRY IT IN THE APP** **5 · The Price Converging, and a Step That Does Not**
>
> Load the scenario and run its 18 hours with the shipped Price step of 0.5. The price settles in 177 of the 216 control steps, and the run costs \$9 901. Reset, set Price step to 0.8 before Run and repeat: no step settles, and the run costs \$9 909.

Where the price settles it is the right price. At every one of case 5's 177 settled steps the first block's price equals the multiplier of the same block's balance in the centralized solution of the same planning problem to within 0.71 \$/MWh (median 0.11), and any block's to within 1.00 \$/MWh. The bracket $g \le J^\star \le J_p$ holds at all 216 recovered steps, and the recovered plan exceeds the centralized optimum by at most 0.91 % where the step settled. At the round cap the reported gap is an upper bound and can be a loose one. The recovered plan exceeded the optimum at that step by a median 13 % and at most 162 % in case 5, and by at most 604 % in the challenge.

6.6.4 A Hot Return, a Trip and a Learned Building

Case 6 gives Apartments 2 radiators half the size its load needs, with the limiter off. Its valve opens wide and its return reaches 68.7 °C, lifting the plant's mean return from 35.4 to 42.8 °C and the valued cost by \$202. With the limiter on, the plant is spared but the block spends 7.3 h more than 0.5 K below its band, at worst 1.15 K. In case 7 the CHP trips at 07:00 as the offices open, and the price rises from about 57 to 73 \$/MWh as the boiler becomes the marginal source. At that price the offices cut their draws by about a quarter and settle 0.1 to 0.9 K above their 20 °C edge, while the hospital, whose comfort weighs ten times as much, keeps 21.9 °C. The run costs \$11,530 against the curve's \$12,088.

Case 8 records the first day under the curve while Apartments 1's thermostat is moved ±0.5 K at random every two hours, then runs the distributed MPC. On the second day the Hankel predictor's 6 h prediction of the block's zone has an error of 0.43 K rms in open loop and 0.51 K in the loop, against 0.13 K for the two-capacity model, which has the plant's own structure and constants and is a bound rather than a fair rival. Planning on the predictor, which predicts the block colder than it is, the block asks for more heat than its exchanger passes. The price settles at 170 of 288 steps (238 with the two-capacity model), the block's guard adds heat for 4.5 h of the day, and the block stays within 0.20 K of its band for \$136 more over the two days. No step uses data recorded after it.

Pause. Try Exercise 6.3 before reading on.

6.6.5 The Cold Snap, Graded

The challenge runs two days of a cold snap from noon, to −14 °C at 06:00 on the second morning, with a 15 MW boiler, and trips the CHP at 07:30 that morning. A run passes with no block more than 0.5 K outside its declared band at any instant, the return at or below its limit for 95 % of the time, and a graded cost of at most \$31,553, 1.25 % below the supply curve's \$31,952, as listed in Table 6.5.

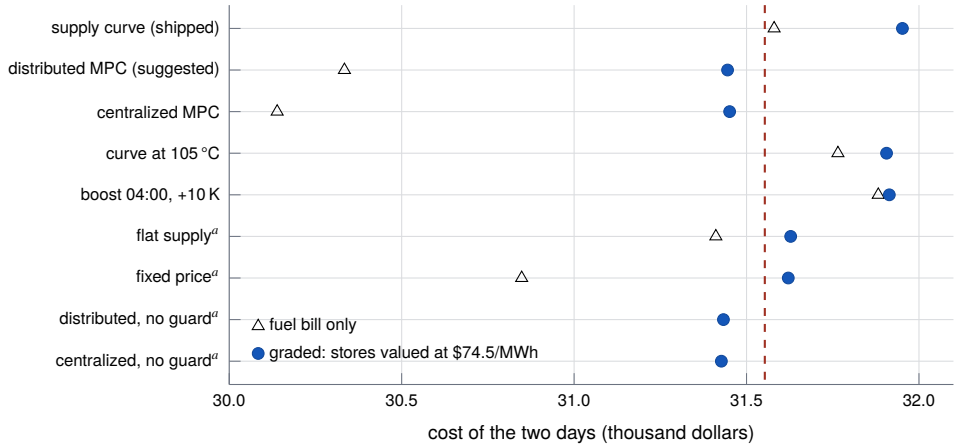


Figure 6.3 The challenge on seed 14 redrawn from Table 6.5: each policy's fuel bill (open triangles) and its graded cost with the stores valued (filled circles), with the $31 553 target as a dashed line. [a]Teaching ablations, not panel settings, as in the table. Notice that the two MPCs buy the least fuel but leave the most heat unpaid in their stores, so grading closes most of their lead, and that a run passes only if its circle lies left of the line and its comfort and return limits hold.

Table 6.5 The Challenge on Its Weather (Seed 14) and Seven Seeds

Policy	Bill ($)	Stores (MWh)	Graded ($)	Worst (K)	Verdict	Seeds
supply curve (shipped)	31 580	−5.0	31 952	0.20	fail	0 of 7
distributed MPC	**30 334**	**−14.9**	**31 445**	**0.37**	**pass**	**5 of 7**
centralized MPC	30 139	−17.6	31 451	0.16	pass	6 of 7
curve at 105 °C	31 765	−1.9	31 906	0	fail	0 of 7
boost 04:00, +10 K	31 882	−0.4	31 914	0	fail	0 of 7
flat supply[a]	31 411	−2.9	31 628	0	fail	0 of 7
fixed price[a]	30 848	−10.4	31 621	0	fail	0 of 7
distributed, no guard[a]	—	—	31 433	0.59	fail	—
centralized, no guard[a]	—	—	31 427	0.33	pass	—

Stores: the change of tank, network and buildings over the run, valued at $74.5/MWh. Seeds: passes against each weather draw's own supply curve. The distributed MPC (bold) is the suggested setting. [a]Teaching ablations in the app's tests, not settings on the panel.

Changing a model parameter or firing a disturbance by hand during the challenge leaves the attempt ungraded, and the app says so at once. The curve's bill is $31,580 with its stores 5.0 MWh short at the end. The suggested distributed MPC charges the network and the buildings in the evenings and draws them down at the peaks. Its bill is $30,334, 3.9 % less, but its stores end 14.9 MWh short, and graded at $74.5 per MWh it passes at $31,445, $108 inside the target, with the worst block 0.37 K out. Both halves of the strategy are needed. In two teaching ablations that no panel setting reaches, the distributed MPC held to the curve's supply temperature saves 1.01 %, and with a fixed price instead of a coordinating one 1.04 %, and both fail. The state a quarter hour after the trip is shown in Figure 6.4. Figure 6.3 draws each policy's bill beside its graded cost.

The verdict belongs to one weather. On seven draws, each against its own curve, the suggested setting saves 1.17 to 1.64 % and passes on five (one draw saves 1.17 %, another leaves a block 0.62 K out), the centralized MPC saves 1.15 to 1.62 % and passes on six, the two ablations save 0.48 to

1.15 % and pass on none, and the curve variants save 0.06 to 0.22 % and pass on none. The ranking holds on every draw. The valuation matters. With deficits charged at the boiler's $52 no-load price and surpluses credited at the CHP's $22, the asymmetric rule the other cases use, the verdict would turn on which store a run left empty, and valued both ways at one price the suggested setting reaches a 2 % margin only below 61.4 $/MWh. The suggested setting's comfort also depends on the consumers' guard, which holds Offices 1 and Offices 2 at their edges for 4.2 and 3.4 h over the two days. Without the guard, Offices 2 falls 0.59 K below its band and the attempt fails, while the centralized MPC passes without it at 0.33 K.

> **+ DID YOU KNOW? A Smaller Bill, a Larger Graded Cost**
>
> From the simulator: in the challenge the centralized MPC buys fuel for $30 139, less than the distributed MPC's $30 334, but leaves its stores 17.6 MWh short against 14.9 MWh. Graded with the stores valued, it costs $31 451 and the distributed one $31 445 (Table 6.5), so the bill alone ranks them wrongly.

After the trip the coordination does not converge. The boiler's 15 MW cannot carry every block's ask, and none of the 55 control steps after the trip settles. On 54 of them no plan fits, since the draws, cut to their capacities, still exceed what the plant can deliver in some block by a median of 2.2 MW and at most 3.3 MW. The price climbs to a median of $398 and at most $984 per MWh, while the multiplier of the first block's balance on the same planning problem is a median of $76,993 and at most $138,567, set by the comfort band's soft penalty. A fixed step of 0.5 cannot climb that far in 30 rounds. The plant gives what it can, the guard holds the offices, and the hospital keeps its heat. With no plan that fits, the tank charges only from spare heat while the supply is within 0.5 K of its setpoint, which holds or trims nine planned charges.

6.6.6 A Snap No Price Holds

Case 10 trips the CHP at midnight as the snap begins and keeps it out, so the 15 MW boiler and the tank carry two cold nights. Under the shipped distributed MPC the offices fall 4.67 K below their band, the apartments 2.76 K and the hospital 2.16 K. A logged sweep of fourteen settings a learner can reach (each strategy, the curve at 95 and 115 °C, an evening boost, an idle tank, two price steps, four band settings, Manual at the plant's limit, and the network charged in Manual before the trip) holds the hospital and every apartment block within 0.5 K in none. The best, Manual at the limit with raised edges and the quickest thermostats, leaves the apartments 1.52 K and the hospital 0.88 K out, with the offices 3.82 K below theirs. The claim is scoped to the shipped boiler and tank. At 17 MW the shipped distributed MPC holds both, at 0.45 and 0 K. Here the price after the trip sits at a median of $16,809 per MWh against a multiplier of $905,251.

6.6.7 Code Verification

Code verification tests the program against closed forms and conservation. A single pipe at constant flow matches Equation (6.2) at 0.02, 0.05 and 0.15 m^3/s on T1 and a branch with errors of 3.5×10^{-10} to 3.0×10^{-8} K. The parcel queue agrees with V/Q at constant flow and with $\int Q\,dt = V$ when the flow changes in transit to within one 10 s step, and the delays of case 1 agree with the segment-by-segment integral as reported in Section 6.4. Over the 18 hours of case 2 the plant produces 274.84 MWh, of which 257.64 MWh is delivered, 9.755 MWh is lost from the pipes (an integral of $U'(T - T_g)$ over every parcel, independent of the ledger), 1.174 MWh is lost from the tank and 6.268 MWh is stored, a residual of -1.65×10^{-4} MWh, or -6.0×10^{-7} of production. The plant's $\rho c_p Q(T_s - T_{rp})$ equals the sources' mean heat to 0.2 % in each of seven runs checked, the plant flow spans 36 to 191 kg/s, the fastest water moves at 1.53 m/s, and the pump reaches its 600 kPa limit only in case 10. The duty never exceeds C_{min} times the available temperature difference, at 38 880 substation-steps each of cases 2 and 6, where the largest excess is −4.67 W. Halving the step to 5 s moves the suggested challenge's bill from $30,334 to $30,331 and its saving from 1.59 to 1.58 %, with both verdicts unchanged. With the plant and its state wrapped during every controller call of cases 7, 8 and the

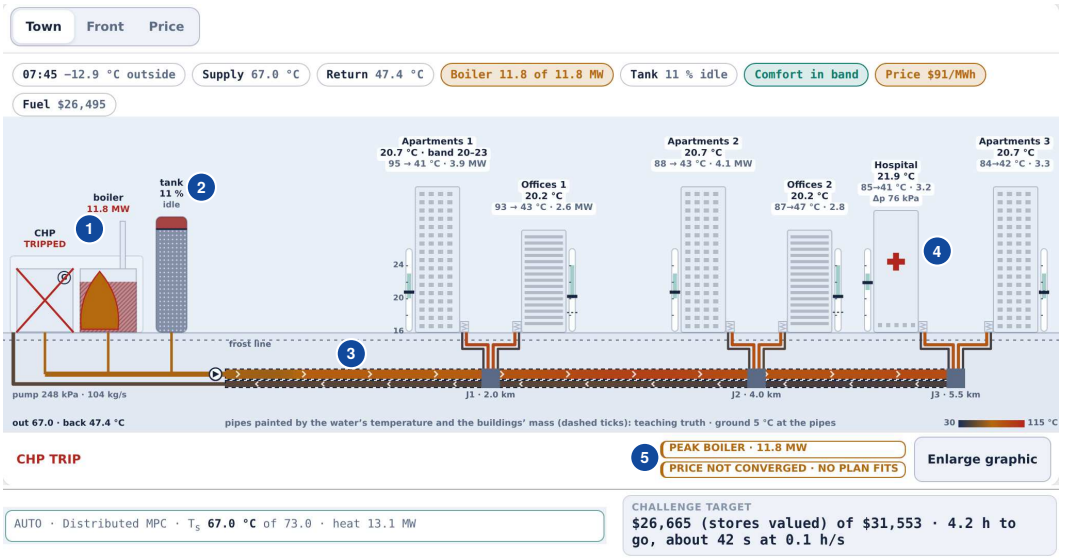


Figure 6.4 The challenge with the suggested distributed MPC ($\alpha = 0.5$) at $t = 43.75$ h, shown by the clock as 07:45 on the second morning, a quarter hour after the CHP trip, on the Town view. The figure joins two parts of the screen: the Town view above, and below it the supply faceplate's readout line and the challenge tile, with the header, the control panel, the trends and the rest of the Learn panel cut. (1) The CHP is crossed out, and the boiler, ramping toward its 15 MW cap (Table 6.3), gives 11.8 MW of the 13.1 MW asked. The supply leaves at 67.0 °C against a setpoint of 73.0 °C, so (2) the tank, 11 % full, stands idle although the plan, answering prices that did not settle, would by the record charge it at about 1.4 MW. (3) The trunk's water and the buildings' mass are drawn as teaching truth. The return is 47.4 °C at −12.9 °C outside, Offices 1 stands at 20.2 °C on its guard and (4) the hospital at 21.9 °C, and the first block's price is about 91 $/MWh. (5) The strip reads CHP TRIP, PEAK BOILER and PRICE NOT CONVERGED with NO PLAN FITS, and the challenge tile values the run at $26,665 against the $31,553 target with 4.2 h to go.

challenge, the controller read only the measurement packet. Stepped headless, a control step takes a median of 8.9 ms on case 5 (87 ms at the 95th percentile, 101 ms at most) and the centralized QP a median of 31.5 ms. In a desktop browser at the top speed of 0.25 h per second, an 18-hour case takes about 72 s. The challenge's two days take 196 s of wall time with every price round solved (a median of 9.5 ms per control step), or 6.6 s for the Fast-forward helper that runs the first day's same steps and 99 s for the second day.

6.6.8 Agreement With Published Relations

Agreement with published relations is a separate matter. The counter-flow effectiveness of Equation (6.4) agrees with the framework's implementation and with a marched exchanger of 20 000 cells at three points (0.377589 against 0.377592 at $N = 0.5$, $C_r = 0.25$, and 0.750000 three ways at $N = 3$, $C_r = 1$), and every substation gives its design duty with a 50.00 °C return at 95 °C and the design flow. The pipe loss coefficients are teaching values, but conduction through a polyurethane shell and the soil gives 0.43 to 0.58 W/(m K) per pipe for these bores, matching the branches' 0.40 to 0.45 and below the trunks' 0.60 to 0.80, as older pipe would be. The network loses 3.55 % of the heat produced in case 2. At a medium-weight construction's 165 kJ/(m^2 K) of floor, the buildings' heavy capacities imply 32.5 to 79.5 thousand m^2 and design loads of 44 to 92 W/m^2, the order of a block of existing buildings. The cases have not been evaluated with students, so this chapter makes no claim about learning.

6.7 Technology in Practice

What Measures It

At the plant the supply and return temperatures and the flow give the heat sent out, and at each building a heat meter gives the heat taken. Such meters are inspected monthly for leaks, loud running, blocked strainers and, on an ultrasonic meter, the seat of its transducers and the state of the coupling gel, and they are calibrated each year or when the trended data show a drift [19]. A meter reports by a 4 to 20 mA signal, a fieldbus or calibrated pulses, and a pulse rate set too high can saturate the data system and lose counts [19]. The chapter's packet stands for these instruments: every five minutes it brings each consumer's supply, return, flow, pressure difference, heat meter and zone, with 0.1 K of noise on the temperatures and 0.5 % on the flows and meters.

> **TECHNOLOGY Sensor: The Heat Meter**
>
> A heat meter finds the energy delivered as the water's mass flow times the temperature difference between supply and return. A flow computer, built into the meter or separate, combines the flow reading with the two temperatures and integrates the energy over each interval. The flow element may be a differential pressure, turbine, vortex or ultrasonic meter. A transit-time ultrasonic meter times sound pulses sent with and against the flow, suits clean water and covers a flow range of about 20 to 1. Its coupling gel must be rated for the pipe's temperature or it can melt away and weaken the signal, and a meter on hot water must stay below saturation so that it does not cavitate [19]. The chapter's meters carry 0.5 % noise and never fail.

What Moves It

The plant's final elements are the fuel to the cogeneration unit and the boiler, the charge and discharge of the storage tank, and the circulating pump, while each building moves its own substation valve. A substation valve is a throttle, and a throttle valve controls flow by raising the back pressure and dissipating the fluid's energy, which a pump on a variable speed drive avoids by slowing [20]. In the chapter the sources follow their setpoints through first-order lags, the tank's power is turned into a flow, the pump's head is set by a loop on the least measured pressure difference, and each valve passes the flow it is asked for up to its limit, as a pressure-independent valve does.

> **TECHNOLOGY Final Element: The Variable-Speed Circulation Pump**
>
> A pumped loop's flow can be set by a bypass line, a throttle valve, several pumps or the pump's speed. A bypass usually wastes the most energy, and a throttle valve raises the pump's back pressure and dissipates energy. Speed is the most efficient. A variable frequency drive, by far the most popular adjustable speed drive, changes the frequency of the motor's supply, and the pump's head and power curves drop down and to the left as it slows. Where static head is a large part of the total, a pump slowed too far can vibrate as it does against its shutoff head [20]. The chapter's pump sets a head of up to 600 kPa (Table 6.2) and costs nothing to run.

Where the Controller Runs

A network spread over a town is run by supervisory control and data acquisition. A central control server polls its field devices at defined intervals, for example every 5 or 60 s, sends them new set points and watches for alarms, and supervisory and lower-level loops run with cycle times from milliseconds to minutes [21]. The chapter's plant steps every 10 s and its loops and planners every 5 min (Figure 6.5). The planners belong in the supervisory layer, and a price sent to each building is one of the simplest forms of distributed control in a heat network. Multi-agent schemes let consumer agents sell load shedding to producer agents within their comfort bounds [9].

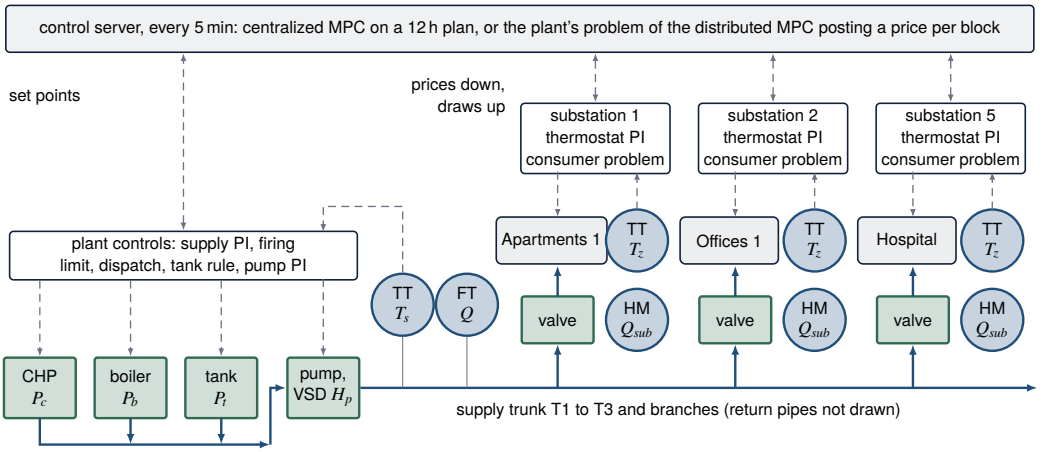


Figure 6.5 The town's loops on field hardware, drawn from Sections 6.3 and 6.5. Green boxes are final elements, blue circles transmitters (TT temperature, FT flow, HM heat meter), dashed arrows signals, and three of the six substations are drawn. Notice that the price travels to the substations at once, while the heat it buys travels through the trunk for hours.

> **IN PRACTICE** **Four Loops of Classic Network Control**
>
> Classic network control has four parts. The circulating pump holds the pressure difference between supply and return high enough for flow at the network's far edge. Each building's heat demand control meets its comfort and hot-water needs, and flow control opens the substation valve far enough to pass that heat. The plant holds the supply temperature on a curve of the outdoor temperature. Pressure and supply control are central, and the other two sit in the substations. This control is robust and easy to extend, but in a heat deficit the consumers near the plant take what they need and the far ones go short [9]. The chapter's regulatory layer and supply curve are these four loops.

What Is Modernizing It

Three developments bear on the chapter's method. Large water tanks beside a cogeneration plant cut the peak and let the plant run at its best ratio of heat to power, and one simulated network saved up to 12 % of its primary energy that way. Hybrid control, in which a central optimization sets the load profile and agents in the buildings carry it out, is gaining interest and was judged very promising. Smart meters installed for electricity that also measure heat use and room temperatures could share the cost of the communication network that an advanced controller needs [9]. The chapter's centralized and distributed MPCs are the two ends of that range.

6.8 Limitations

The pipes, buildings and prices are teaching values. The loss coefficients are of the order conduction predicts and the buildings of a plausible order, but neither is fitted to a network, so the delays and costs are those of a declared town. Both MPCs plan with the delays frozen at the measured flows and the return pipes held, so a front can arrive sooner or later than planned when the flow changes, which the next solve five minutes later corrects. The CHP's lag is not in the plan. The comfort the MPC runs report includes the consumers' guard, which the plan does not see, and the suggested setting's pass in the challenge depends on it. The planning model carries the buildings' declared constants, which are the plant's in every case but case 6, so model error appears only through the frozen network, the Hankel predictor and a plant that a case or the learner changes.

OPEN CHALLENGE Sensing a Whole Network

An advanced network controller needs the valve positions, the flows and temperatures in the pipes, and the state of the buildings, the plants and the stores, possibly several times an hour. A network can span a street or a region, so the sensors and the communication network cost much to install and to run. Estimators and hierarchical agents can cut the data sent, and the fewest sensors that reveal a network's state, and where to place them, remain open questions [9]. This chapter's controllers read every consumer every five minutes. Progress would be plans that keep their quality on a few well-placed meters.

The fixed-step price iteration settles at most steps of a cold morning but does not converge where a capacity binds hard or the plant is short. At the round cap the reported gap is only an upper bound, and the recovered plan exceeded the optimum by up to 604 % at one step. When no plan fits, the applied plan is the plant's last answer to unsettled prices, kept from charging the tank at the consumers' expense by a rule, not a plan. The graded margin rests on one weather draw (passed on five of seven) and on a valuation rule, both ways at \$74.5 per MWh, that the other cases do not use. Their rule values deficits at the boiler's price and surpluses at the CHP's, so their valued costs are conservative rather than neutral to the end state. A simulated pass certifies no network, contract or plant.

OPEN CHALLENGE Charging the Pipes Ages the Pipes

Raising and lowering the water temperature stores a limited amount of heat in a network and shaves a peak. Cycling more often between high and low temperatures may speed fatigue in steel pipes, and cracks can grow until a pipe fails. A simple and accurate way to state how much flexibility a network and its buildings offer is still lacking [9]. The chapter's plan charges a supply move only 1 \$/K^2 and counts no fatigue. Progress would be a plan that weighs the fuel saved against the life of the pipe.

6.9 Conclusion

A town's heat network delays its heat by the flow its consumers draw, and this simulator measures what that does to four ways of running the plant. The same 15 K step reaches the far end in 4.48 h at 50 kg/s and 2.83 h at 78 kg/s, because every segment passes its volume at its own flow. A boost timed by hand arrives with the peak and still costs \$202 more than the curve, while a centralized MPC that charges the network on the CHP's spare evening heat saves 3.2 %. On the cold morning the price-coordinated run costs within \$52 of the centralized one, and where the price settles it equals the centralized multiplier, but it does not converge when the plant's capacity binds, and after a trip it stays two orders below the price that would ration the heat. On the graded cold snap the distributed MPC saves 1.59 % with its stores valued and passes on five of seven weathers. The model is declared and its numbers are teaching values. A fuller model would add front dispersion, pumping energy, a CHP with a minimum load, an accelerated or scheduled price step, and consumers whose models differ from the plant's. The town makes a price of heat something a class can post, watch settle, and watch fail.

KEY POINTS

- A town's heat network delays its heat by the flow its consumers draw.
- The same 15 K step reaches the far end in 4.48 h at 50 kg/s and 2.83 h at 78 kg/s.
- A boost timed by hand arrives with the peak and still costs \$202 more than the curve, while a centralized MPC that charges the network on the CHP's spare evening heat saves 3.2 %.
- Where the price settles it equals the centralized multiplier, but it does not converge when the plant's capacity binds.

Where else this applies. Plug flow keyed by volume also carries the chip column of Volume 5, Chapter 4 (Section 6.3), and plans shared over a network with delays also run the gates of Chapter 5 (Section 6.5).

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6.10 Exercises

6.1 Concept check · 2 min

In Scenario 1 the 15 K step leaves the plant at 13:05 with 50 kg/s flowing and reaches the hospital 4.48 h later, although the 598 m^3 of pipe on its path would pass at the plant's whole flow in about 3.2 h. Why does the step take longer?

(a) The pipe loses heat on the way, so the front cools and is slow to rise by half the step

(b) Less water passes each junction, so each later segment is crossed at its own smaller flow

(c) The pump's head falls in the afternoon, which slows the water in every segment by one factor

(d) The front smears along the path, so its middle arrives well after its leading edge

6.2 Concept check · 1 min

In Scenario 3 a 15 K boost leaves at 04:00 and reaches the hospital at 06:53, as the morning peak arrives, yet the valued cost rises $202 above the supply curve's. Why?

(a) The front arrives after the peak has passed, so the heat stored in the pipes is wasted

(b) The hotter supply pushes the returns past their limit, and the limiters cut the duty

(c) The CHP was already at its capacity, so the charge was bought from the peak boiler

(d) The boost switches off the tank's rule, so the tank no longer spares the boiler

6.3 Concept check · 2 min

In Scenario 8 the Hankel predictor, built only from Apartments 1's own record, predicts the block's zone 6 h ahead with an error of 0.43 K rms in open loop, against 0.13 K for the two-capacity model. Why does the chapter call the two-capacity result a bound rather than a fair rival?

(a) It is fitted to the same record with fewer parameters, so it cannot overfit the data

(b) It is rebuilt every hour from the last 36 h, so it always has the newest data

(c) It predicts with the true outdoor temperature, which the Hankel predictor never sees

(d) It has the plant's own structure and declared constants, so no fitted model can do better

6.4 Theory and calculation · 6 min

The hospital's path runs through trunks T1, T2 and T3 and its own branch, 598 m^3 in all. Use Table 6.2 ($\rho = 970\,\mathrm{kg/m^3}$) and the flows of case 1 at 13:05: 50 kg/s in T1, 34.9 in T2, 19.2 in T3 and 9.9 kg/s in the hospital's branch.

(a) Find the volumes of T1, T2 and T3 from their lengths and bores, and the branch's volume as what remains of the 598 m^3. *Checkpoint:* 318, 192, 74 and 14 m^3.

(b) Find each segment's residence at its 13:05 flow, held frozen, and their sum. *Checkpoint:* 1.71, 1.49, 1.03 and 0.38 h, 4.61 h in all.

(c) Find the path's volume over the plant's whole flow. Compare both results with the measured 4.48 h, and say why the frozen sum comes out late. *Checkpoint:* About 3.22 h.

6.5 Theory and calculation · 6 min

Trunk T1 is 2.0 km of 0.45 m bore with a loss of 0.80 W/(m K) per pipe, buried in ground at 5 °C.

(a) Find T1's loss time constant τ_L of Equation (6.1). *Checkpoint:* About 224 h.

(b) Water enters T1 at 95 °C and spends 1.71 h in it at 50 kg/s. How much does it cool on the way? Check the result with Equation (6.2). *Checkpoint:* About 0.69 K.

(c) Find the heat T1's supply pipe loses with water at 95 °C along its length, and at 110 °C. What does the difference amount to over a boost of three hours? *Checkpoint:* 144 and 168 kW, about 0.07 MWh more over three hours.

6.6 Theory and calculation · 7 min

Use the fuel prices of Table 6.2 with the condenser factor $f = 1$.

(a) Find the CHP's marginal cost $\pi_c + \gamma_c P_c$ at its full 14 MW and compare it with the boiler's base price. *Checkpoint:* 51.4 \$/MWh against 52 \$/MWh.

(b) Find the boiler's marginal cost at the challenge's 15 MW cap, and say why the challenge values every MWh its stores end short or over at this price. *Checkpoint:* 74.5 \$/MWh.

(c) After the challenge's trip the price sits at a median of \$398 per MWh while the centralized multiplier is a median of \$76,993. With $\alpha = 0.5$ and an imbalance of 2.2 MW in every round, how many rounds, how many control steps of 30 rounds and how many hours would the price need to climb that far? How far could it climb in the 55 control steps after the trip? *Checkpoint:* About 69 600 rounds, 2320 steps and 193 h. About 1800 \$/MWh.

6.7 App challenge · 8 min

Load Scenario 1, *The delay at two flows*, at its default seed and change nothing. Run at 0.25 h per second until the run pauses at 06:00 ($t = 18$ h, about 72 s) and read the *Supply step arrived* tile. Then set Boost start to 2, press Reset and run to the pause again.

(a) For each run give the hospital's arrival time (H) and the plant's flow when the step left, to 0.02 h and 1 kg/s. *Checkpoint:* 4.48 h at 50 kg/s. 2.83 h at 78 kg/s.

(b) Find the ratio of the two hospital delays and the ratio of the two flows. Scale the afternoon delay by the flow ratio and compare with the night run. *Checkpoint:* 1.58 and 1.56, 2.87 h against 2.83 h.

(c) Read Apartments 1 (A1) and Apartments 2 (A2) in both runs. Why does every consumer's delay shrink by nearly the same factor? *Checkpoint:* A1 2.06 and 1.34 h, A2 3.49 and 2.21 h.

6.8 App challenge · 9 min

Load Scenario 2, *The supply curve on a cold morning*, at its default seed and change nothing. Run at 0.25 h per second until the run pauses at 12:00 ($t = 18$ h, about 72 s) and read the tiles *Fuel cost*, *Fuel cost, stores valued* and *Peak boiler*. Do the same with Scenario 4, *Centralized MPC: when to send heat*, and Scenario 5, *The price converging, and a step that does not*, as loaded.

(a) Give case 2's boiler energy, its peak and its valued cost. *Checkpoint:* 28.2 MWh, 10.9 MW, \$10,282.

(b) Give the boiler energy and the valued cost of cases 4 and 5, and each one's saving against case 2 in dollars and percent. *Checkpoint:* Case 4 19.4 MWh and \$9,953, \$329 or 3.2 %. Case 5 17.8 MWh and \$9,901, \$381 or 3.7 %.

(c) From case 4's *Fuel cost* and the tank, pipes and buildings on its valued tile, rebuild the valued cost with Equation (6.14), deficits at the boiler's 52 \$/MWh and surpluses at the CHP's 22 \$/MWh. *Checkpoint:* \$9,696, tank −3.2, pipes −4.3, buildings +6.0 MWh: \$9,954.

6.9 App challenge · 10 min

Load the challenge, *Challenge: a two-day cold snap and a CHP trip*, at its default seed.

(a) Change nothing. Press Fast-forward day 1, then run at 0.25 h per second to the end at $t = 48$ h (about 100 s). Read the tiles *Challenge target*, *Fuel cost* and *Fuel cost, stores valued*. *Checkpoint:* Not met, \$31,952 of \$31,553. Bill \$31,580, stores −5.0 MWh.

(b) Set Strategy to Distributed MPC, leave Price step at 0.5, press Reset, press Fast-forward day 1 and run to the end again. Read the same tiles. *Checkpoint:* Met, \$31,445 of \$31,553. Bill \$30,334, stores −14.9 MWh.

(c) Show that each graded cost is the bill plus the stores' shortfall at 74.5 $/MWh. Give the distributed MPC's saving against the curve on the bill and on the graded cost, and say which one the target uses and why. *Checkpoint:* $31,952 and $31,444. 3.9 % on the bill, 1.59 % graded.

6.10 Application area · 4 min

In Scenario 6 Apartments 2 has radiators half the size its load needs, and its return limiter is off. Its return reaches 68.7 °C and lifts the plant's mean return from 35.4 to 42.8 °C.

(a) Evaluate the condenser factor $f(T_{rp})$ of Equation (6.7) at the two mean returns, and say what each kelvin of return above 40 °C costs. *Checkpoint:* 1.000 and 1.017. About 0.6 % more fuel per kelvin.

(b) With the limiter on, the plant is spared. What does Apartments 2 pay instead, and what cure would a network engineer ask the owner for? *Checkpoint:* 7.3 h more than 0.5 K below its band, at worst 1.15 K.

6.11 Application area · 3 min

In Scenario 10 the CHP trips at midnight as the cold snap begins. Under the shipped distributed MPC the offices fall 4.67 K below their band, the apartments 2.76 K and the hospital 2.16 K, and none of fourteen settings a learner can reach holds the hospital and every apartment block within 0.5 K. As the network's engineer, what would you propose, and what does the result not prove?

6.12 Review the agent · 5 min

An AI assistant reviews Scenario 5 and writes: *"At the shipped price step of 0.5, 39 of the 216 control steps stop at the 30-round cap, so the price moves too slowly. Raise the Price step to 1.5 and the price will reach its settled value in a third of the rounds, every step will converge and the run will cost less."* Find the flaw in the claim and say how you would check it in the app.