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CHAPTER 9

A Building HVAC Simulator With a Comfort Band Instead of a Setpoint

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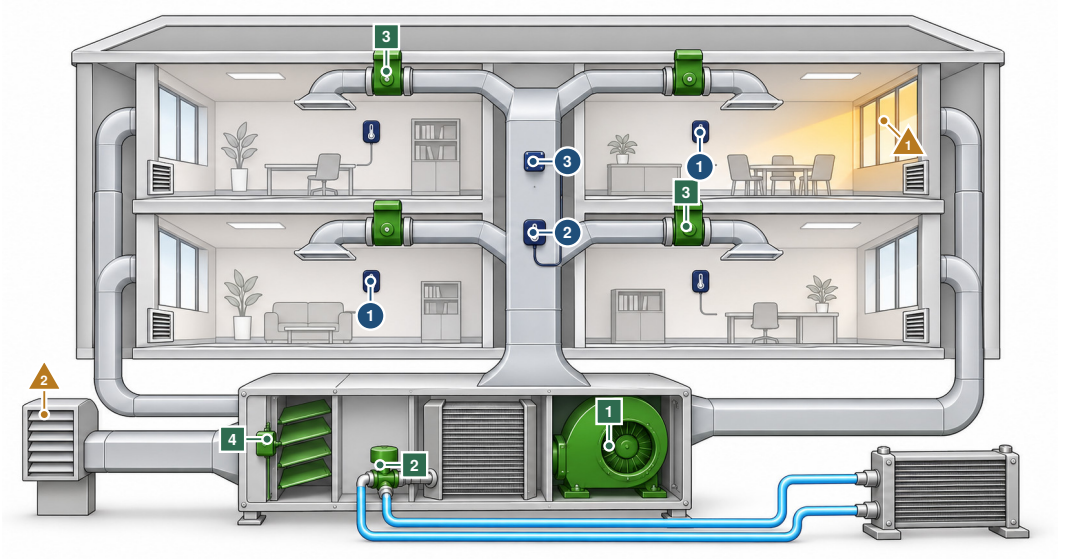
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CHAPTER 9

A Building HVAC Simulator With a Comfort Band Instead of a Setpoint



The controller moves 1 supply fan and its speed drive 2 chilled water valve at the coil 3 four terminal dampers, one per room 4 outdoor air damper

The controller reads 1 room temperatures, one per room 2 supply air temperature 3 duct static pressure

What disturbs it 1 sun on the glass 2 outdoor air temperature

One fan and one coil serve all four rooms while the sunlit room asks for the most air and the concrete floors hold the cooling bought early. Illustration, not to scale.

IN PLAIN TERMS **Spending the Freedom in a Comfort Range**

The main idea. Office workers accept any room temperature inside a range, not one exact number. The concrete floors soak up cooling bought early and give it back later. So a controller can cool hard on cheap, cool mornings and coast through the costly afternoon.

The challenge. The concrete takes hours to cool, so the choice is made long before anyone can tell whether it was right. A planner that looks too short a time ahead cools in a rush. It then sets a new and costly peak in power use.

Why it matters. On this chapter's main summer test day, using the range cuts the bill by 8.8 percent while every room stays comfortable.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history in Section 9.2, the facts in the Introduction and in Section 9.6.6, the try-it segments in Section 9.6, and the key points
Study	70 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	25 min	the first page, the results (Section 9.6), Technology in Practice (Section 9.7), Limitations (Section 9.8) and the open challenges

The control question An office is judged against a comfort band, so the setpoint inside it is free and the concrete behind it stores cooling. How should a controller spend that freedom against a tariff, a forecast and a curtailment, and what does a day-ahead plan add to a priced rule?

Learning objectives

- Compute the cooling a building's fabric stores across a comfort band.
- Explain how the tariff and the chiller curve set the cost of cooling and the supervisor's charge demand.
- Explain why a horizon shorter than the time to charge the store costs more than none.
- Measure what notice of a curtailment and the comfort band are each worth on the same event.
- Compare a receding-horizon plan with the priced rule, against a retuned baseline.

Prerequisites Sections 1.6.2, 1.4.3, 1.4.5, 1.4.6, 1.4.8 and 1.6.4, and Chapter 8 for a supervisor that chooses a cooling plant's setpoints.

The app https://apmonitor.com/apps/building-hvac/

See also Volume 17, Chapter 10 for an inventory dispatched inside its working band, Volume 17, Chapter 6 for a setpoint chosen rather than defended, and Volume 17, Chapter 11 for storage run by an inventory policy under a forecast bias.



9.1 Introduction

Nothing on this air handler is difficult to hold. The duct static pressure reaches 63 % of an 80 Pa step in 0.55 min, the supply air temperature reaches 63 % of a one degree step in 1.00 min, and the west zone reaches 63 % of a half degree step in 11.6 min, all three under ordinary proportional-integral control. What is hard is deciding what the zone number ought to be, when anywhere between 21.0 and 24.0 °C is a pass, when the concrete behind that number holds 856 kWh of cooling nobody is using, and when the decision has to be taken five hours before anybody can tell whether it was right.

That gap is the misconception the application corrects. A control course teaches a loop to hold a number, and a student who has only ever seen a block diagram concludes that the number was handed down by somebody else. In a commercial building it was not. The comfort specification is an interval. Comfort research states it as a range rather than a point [1], and the lease that turns that research into an obligation is written the same way. The width of the interval is the cheapest storage most buildings will ever have. Reading a comfort band as a setpoint plus a tolerance throws it away. Reading it as an interval turns the setpoint into a manipulated variable for a slower loop, and the plant then has four loops rather than three.

The class of problem is control against a specification that is a range, with the remaining freedom spent on an economic objective, and it is not confined to buildings. Industrial model predictive controllers routinely control to a zone rather than to a setpoint and give the leftover degrees of freedom to a steady-state economic optimization, which is the standard commercial architecture rather than a special case [2]. A blend property held between specification limits, a product moisture held above a giveaway floor and below a quality ceiling, a surge drum level held anywhere between its alarms, and a battery state of charge held inside a warranty window all have this shape, because each has an inventory whose charge and discharge cost different amounts at different times and a tolerance that pays for the arbitrage. The building version is the largest by installed capacity, because the fabric of the existing stock is storage already built and already paid for [3]. Volume 17, Chapter 10 dispatches the battery version against a feeder's net-load curve, and Volume 17, Chapter 6 chooses a solar field's outlet setpoint rather than defending a fixed one.

DID YOU KNOW? Buildings Draw Most of the Grid's Power

Buildings account for more than 70 percent of United States electricity use and a comparable share of peak demand [4]. In United States office buildings, heating, cooling and ventilation make up 51 percent of total energy use [5]. The modeled office puts about 40 % of its electricity into the HVAC plant (Section 9.8).

Two measurements make the building case quantitative. Four zones of slab and partition at 250 000 kJ/K each, plus their air, hold 855.8 kWh across a 3 K band, and 28.5 kWh for every tenth of a degree the band is widened. And cooling bought in the morning is cheaper than cooling bought in the afternoon before the tariff is read at all, because an air-cooled chiller rejects heat into the afternoon it is cooling against, so the machine modeled here runs at a coefficient of performance of 5.43 at five in the morning and 3.10 at four. A kilowatt-hour of cooling on the design day therefore costs 1.4 cents at five in the morning and 10.4 cents at three in the afternoon, a factor of 7.5 that is the tariff's 4.3 times the chiller's 1.8.

The building control literature reaches this decision through optimization formulations a first course cannot run [6–8], and the interactive material available to a student is either a thermal model with no controller or a controller with no economics. This chapter works with an interactive office building and its air handler, given in full so that either can be rebuilt, and prices a comfort band against a weather forecast on the same plant. The plant has seventeen states, and its rest condition is solved by one routine that the initializer, the manual balancing aid and the self-test all call. The four loops above it are transcribed from the executing code. The two-node zone model, the coil effectiveness relation and the precooling idea are standard. The central measurement is that on a three-hour utility curtailment two hours of notice reduces the energy drawn above the cap from 19.4 to 16.1 kWh while removing the comfort band raises it to 152.3 kWh, and that eight hours of notice still leaves 5.7 kWh because a ventilation minimum of 4.17 m^3/s puts a floor of 133 to 160 kW under the building. The graded challenge adds a smaller lesson: one of the four improvements it asks for turns out to be worth exactly zero dollars.

Section 9.2 traces how a building came to have a band to spend. Section 9.3 gives the seventeen-state model, its constants and its assumptions, and Section 9.4 its two time constants, its three process gains and the fan the zones share. Section 9.5 sets out the three regulatory loops, the supervisor that prices a kilowatt-hour of cooling, and the predictive controller of Section 9.5.4 that plans a day of cooling. Section 9.6 measures every day-long scenario: a band floated and a band used, the forecast horizon, a curtailment run three ways, a plant that runs short, the graded challenge, and the plan against the priced rule in Section 9.6.6. Section 9.7 describes the sensors, terminal boxes and controllers of real buildings. Sections 9.8 and 9.9 state what the model cannot settle and what the curtailment comparison means, and the exercises in Section 9.11 run the cases.

9.2 How a Building Came to Have a Band to Spend

The comfort band predates any decision to use it. A pneumatic room thermostat is a proportional device whose throttling range spans two or three degrees between a shut damper and an open one, so the first generation of automatic buildings held a range because the hardware could hold nothing narrower. Constant volume systems with terminal reheat met the worst zone by supplying cold air everywhere and burning it back where it was not wanted, and the cost of doing so is what ended the arrangement. Variable air volume replaced the reheat coil with a damper and made supply airflow the manipulated variable at the zone, which put a variable speed fan and a duct static pressure loop into a building that had previously had neither, and made resetting that pressure setpoint against the most open damper the standard efficiency measure on the resulting plant [9].

Direct digital control arrived next and quietly removed the band. A pneumatic thermostat has a throttling range and a digital one has a setpoint in a database, so the number a building is held to became a single value entered once at commissioning. The band survived in the specification and disappeared from the control system, which is the state most of the existing stock is still in.

Commercial buildings account for a large and well documented share of electricity use, and cooling is the part of it that peaks with the grid [10].

The first answers to the resulting peak were open loop. Night setback and optimum start recovered energy from the unoccupied hours, and demand limiting was duty cycling of equipment, which reduces the meter reading by taking cooling away from whichever zone is unlucky. Chilled water and ice tanks were the capital answer, and they work, at the cost of a plant room and a maintenance contract. The measurement that changed the argument was that the building itself is already such a tank. Preconditioning an office at night was shown to shift peak load by an amount set by the mass of the structure rather than by the size of the plant [11], and a sequence of studies then established what an optimal charge and discharge trajectory looks like, how much of the ideal a simple strategy recovers, and what the trajectory costs when the building is occupied throughout the discharge [12–14]. Demand limiting was reformulated on the same basis, as a model-based choice of a zone temperature trajectory rather than as equipment shedding [15], which is the form the strategy in Section 9.5 takes.

Three developments then made the strategy ordinary. Grey box models identified from a building's own operating data gave the resistance and capacitance parameters a controller needs and a design calculation never produces [16]. Weather forecasts reached the control system, and a controlled comparison on an occupied office building showed a predictive controller using them beating the rule-based sequence it replaced [6]. Utility programs made the economics explicit, because a demand charge and a curtailment penalty price a fifteen-minute average rather than a kilowatt-hour and that is what a thermal store arbitrages [17]. Predictive control of building cooling with storage then moved from simulation to demonstration on real plant [7], and the field is now reviewed as a mature subject with its own catalog of failure modes [8]. What remains unusual is not the algorithm. It is the willingness to write the comfort specification down as an interval and let a controller use it, because that is a term in a lease rather than a line in a control sequence.

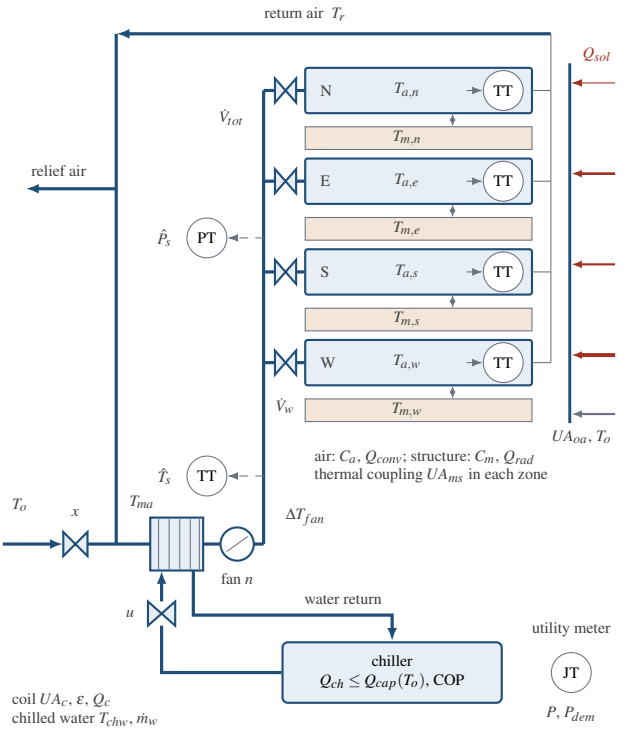


Figure 9.1 The floor plate in section with its air handler. Four perimeter zones carry a fast air node $T_{a,z}$ of capacitance C_a over a slow structural node $T_{m,z}$ of capacitance C_m, coupled by the surface conductance UA_{ms}, with convective gains Q_{conv} reaching the air and radiant gains Q_{rad} reaching the structure. Transmitted solar Q_{sol} enters through glazed area A_g and differs by orientation, shown by the weight of the four arrows at the design hour. Outdoor air enters at fraction x and mixes to T_{ma} ahead of the coil, and one coil, one fan and four pressure independent terminals serve all four zones with $\dot{V}_z = \min(\dot{V}_{z,\mathrm{SP}}, K_{box}\sqrt{\hat{P}_s})$. Three instruments close loops, the supply air thermistor $\hat{T}_s$, the duct static transmitter $\hat{P}_s$ and the zone sensors $\hat{T}_{a,z}$, and the utility meter reports the fifteen-minute demand P_{dem}.

> **HISTORY 2003: Precooling an Occupied Office**
>
> In October 2003 a national laboratory and a university tested precooling in an occupied office building of about 80,000 square feet with a medium-weight concrete structure. Its zones were normally held at a constant 72 °F. In eleven one-day tests the zone setpoints were held low through the morning, at 70 °F from 5 am, and then raised to 78 °F at 2 pm so that the rooms could float through the afternoon. From 2 to 5 pm the chiller power fell by 80 to 100 percent, without a thermal comfort complaint [18]. The report observed that conventional controls treat the building's thermal mass as an obstacle rather than an asset [18]. The supervisor of this chapter makes the same move with a price and a forecast in place of a clock.

9.3 Process Model

The building is a 5000 m^2 office floor plate on four perimeter orientations, occupied by 417 people at design, with 40 kW of lighting and 50 kW of equipment at full output. One air handler serves all four zones through a chilled water coil on a modulating valve, a variable speed supply fan holding duct static pressure, four pressure independent variable air volume terminals and a dry-bulb economizer. An air-cooled chiller makes the water. Every quantity the equations below refer to is labeled on Figure 9.1.

Table 9.1 Classification of Variables. Nominal Values Are Those of the Design Point, 15:00 on the Design Day, at Which the Bench Scenarios Start.

Symbol	Role	Nominal	Range
$T_{a,z}$ (4)	CV, measured	23.00 °C	21.0–24.0 °C band
T_s	CV, measured	13.000 °C	10–18 °C SP
P_s	CV, measured	300.0 Pa	100–500 Pa SP
E	CV, inferred	285 kWh	0–856 kWh
$\dot{V}_{z,\mathrm{SP}}$ (4)	MV	3.80–6.25 m³/s	0–9.0 m³/s
u valve	MV	52.5 %	0–100 %
n fan	MV	86.2 %	25–100 %
$T_{z,\mathrm{SP}}$	MV of the supervisor	23.0 °C	T_{lo}–$(T_{hi}-0.3)$
T_o	DV, measured	35.0 °C	±10 °C shift
Q_{sol}	DV, measured	6.5–36.0 kW	0.15–1.15×
occupancy	DV, measured	1×	0.3–1.4×
$p(t)$ tariff	DV, known	0.320 $/kWh	0.5–3×
P_{cap}	DV, announced	none	0–400 kW
coil fouling	DV, unmeasured	1×	0.45–1.05×
filter loading	DV, unmeasured	1×	1–2.4×
chiller capacity	DV, unmeasured	1×	0.4–1.05×
$T_{a,z}$ (4)	state	23.00 °C	5–45 °C
$T_{m,z}$ (4)	state	24.19–25.84 °C	5–45 °C
$\dot{V}_z$ (4)	state	3.80–6.25 m³/s	0–9.0 m³/s
T_s	state	13.000 °C	2–40 °C
T_{chw}	state	6.00 °C	2–30 °C
n, u	states	86.2, 52.5 %	0–100 %
P_{dem}	state	217.8 kW	0–5000 kW

Table 9.1 sorts the variables by the part they play. The clock runs in minutes, because a day is 1440 of them and the structural time constant is 333. Every duty is in kW, every capacitance in kJ/K, and the factor of 60 between them is applied in one place rather than scattered through the balances. Airflow is volumetric in m³/s throughout against one volumetric heat capacity ρc_p, so no mass flow of air appears anywhere.

9.3.1 Two Thermal Masses per Zone

Each zone carries an air node and a structural node. The air node holds the air and the light furnishings, sees the supply air through the volumetric heat capacity of whatever flow its terminal is passing, and takes the convective share of the internal and solar gains, as shown in Equation (9.1).

$$\frac{C_a}{60}\frac{\mathrm{d}T_{a,z}}{\mathrm{d}t} = \dot{V}_z \rho c_p (T_s - T_{a,z}) + UA_{ms}(T_{m,z} - T_{a,z}) + UA_{oa}(T_o - T_{a,z}) + Q_{conv,z} \quad (9.1)$$

where C_a is in kJ/K, t is in minutes, every duty is in kW and the 60 converts between them. The structural node holds the slab, the columns and the partitions, exchanges with the air through the same surface conductance, and takes the radiant share of the gains, as shown in Equation (9.2).

$$\frac{C_m}{60}\frac{\mathrm{d}T_{m,z}}{\mathrm{d}t} = UA_{ms}(T_{a,z} - T_{m,z}) + Q_{rad,z} \quad (9.2)$$

Nothing but UA_{ms} connects the structure to the rest of the plant, which is what makes the store slow and the only route into and out of it. Two consequences are used later. The structural node has one time constant, $C_m/(60\,UA_{ms}) = 333\,\mathrm{min}$, that no actuator changes, and at rest (9.2) gives $T_{m,z} = T_{a,z} + Q_{rad,z}/UA_{ms}$, so the slab always sits above its zone air by the radiant gains over the surface conductance, 2.84 K in the west zone at the design point and 1.19 K in the north.

Gains are split between the two nodes by fixed radiant fractions, as shown in Equation (9.3).

$$\begin{aligned} Q_{rad,z} &= f_{sol} q_{s,z} + f_p q_p + f_l q_l + f_e q_e \\ Q_{conv,z} &= (1 - f_{sol}) q_{s,z} + (1 - f_p) q_p \\ &\quad + (1 - f_l) q_l + (1 - f_e) q_e \end{aligned} \quad (9.3)$$

where $q_{s,z} = A_g\,\mathrm{SHGC}\,I_{v,z}(1-f_b)/1000$ is the transmitted solar in kW for glazed area A_g in m^2 and vertical irradiance $I_{v,z}$ in $\mathrm{W/m}^2$ with a blind fraction f_b, and q_p, q_l and q_e are the per-zone people, lighting and equipment gains, each the whole-building value times its schedule divided by four. Vertical irradiance comes from the solar geometry at latitude 38° and declination 20°, with beam, half the sky diffuse and half the ground-reflected component on each face. At the design point the four faces take 114, 114, 224 and 636 $\mathrm{W/m}^2$, which is 6.5, 6.5, 12.7 and 36.0 kW of transmitted gain, and 70 % of each goes straight into concrete.

9.3.2 Terminals, Fan and Duct

A pressure independent terminal is a flow controller with a damper, so it delivers what it is asked for until its damper is wide open and then delivers whatever the static pressure will pass, as shown in Equation (9.4).

$$\tau_{box}\frac{\mathrm{d}\dot{V}_z}{\mathrm{d}t} = \min\left(\dot{V}_{z,\mathrm{SP}},\, K_{box}\sqrt{P_s}\right) - \dot{V}_z \tag{9.4}$$

The fan curve and the duct upstream of the pressure sensor are collapsed into one quadratic, as shown in Equation (9.5).

$$P_s = \max\left(0,\; P_0 s^2 - (R_f + R_{up} f_{filt})\dot{V}_{tot}^2\right), \qquad s = \frac{n}{100} \tag{9.5}$$

where P_0 is the shut-off pressure at full speed, R_f is the fan curve droop, R_{up} is the resistance of filter, coil and main duct, f_{filt} is a filter loading multiplier and $\dot{V}_{tot} = \sum_z \dot{V}_z$. Duct pressure dynamics are seconds rather than minutes, so P_s is algebraic and every dynamic in the pressure loop comes from the drive ramp and the dampers. Fan power and the fan heat that follows it are given by Equation (9.6).

$$P_{fan} = \frac{\dot{V}_{tot}\max(0, P_0 s^2 - R_f \dot{V}_{tot}^2)}{1000\,\eta_{fan}}, \qquad \Delta T_{fan} = \frac{P_{fan}}{\dot{V}_{tot}\rho c_p} \tag{9.6}$$

The motor sits in the airstream, so every watt of fan power arrives at the terminals as a temperature rise the coil has to remove again. At the design point that is 1.012 K for 22.19 kW of fan into 18.18 m^3/s of air.

9.3.3 Mixing, Coil and Chiller

Return air is the flow-weighted mixture of the four zones, $T_r = \sum_z \dot{V}_z T_{a,z}/\dot{V}_{tot}$. Outdoor air enters at fraction x, which is the ventilation minimum unless the economizer opens it further, as shown in Equation (9.7).

$$\begin{aligned} \dot{V}_{oa} &= \dot{V}_{oa,des}\,(0.25 + 0.75\,\phi_{occ}), \qquad x_{min} = \frac{\dot{V}_{oa}}{\dot{V}_{tot}} \\ x &= \begin{cases} \mathrm{clamp}\left(\dfrac{T_r - T_c^*}{\max(0.5,\, T_r - T_o)}, x_{min}, 1\right) & \text{eco} \\ x_{min} & \text{otherwise} \end{cases} \end{aligned} \tag{9.7}$$

where ϕ_{occ} is the occupied fraction, $T_c^* = T_{s,\mathrm{SP}} - \Delta T_{fan}$ is the coil-leaving target, and the economizer branch is taken only when the economizer is enabled, the outdoor air is at least 0.5 K below the return, and T_o is below the dry-bulb high limit of 18 °C. Opening only as far as the coil-leaving target requires keeps the coil from being asked to reheat air the damper over-cooled. The mixed air is $T_{ma} = xT_o + (1-x)T_r$, and x_{min} is floored at 5 % and clamped to 1.

The coil is a counter-flow effectiveness model whose conductance scales with both flows, as shown in Equation (9.8).

$$\begin{aligned} UA_c &= UA_{c,0}\left(\frac{\dot{V}_{tot}}{\dot{V}_0}\right)^{0.5}\left(\frac{\dot{m}_w}{\dot{m}_{w,ref}}\right)^{0.8} f_{foul} \\ \mathrm{NTU} &= \frac{UA_c}{C_{min}}, \qquad C_r = \frac{C_{min}}{C_{max}} \\ \varepsilon &= \frac{1-\mathrm{e}^{-\mathrm{NTU}(1-C_r)}}{1-C_r\mathrm{e}^{-\mathrm{NTU}(1-C_r)}} \\ Q_c &= \max\left(0,\ \varepsilon\, C_{min}\,(T_{ma}-T_{chw})\right) \\ T_s^{\dagger} &= T_{ma} - \frac{Q_c}{\dot{V}_{tot}\rho c_p} + \Delta T_{fan} \end{aligned} \tag{9.8}$$

where C_{min} and C_{max} order the air capacity rate $\dot{V}_{tot}\rho c_p$ and the water capacity rate $\dot{m}_w c_{pw}$, with $\dot{m}_w = \dot{m}_{w,\max} u/100$, and $T_s^{\dagger}$ is the supply air the coil is currently making. The removable singularity at $C_r \to 1$ uses $\varepsilon = \mathrm{NTU}/(1+\mathrm{NTU})$. Clipping Q_c at zero matters, because a cooling coil cannot heat. At the design point the coil is 69.7 % effective and removes 301.7 kW with 9.18 kg/s of water through a valve 52.5 % open.

Supply air, chilled water and the two actuators are first-order lags, as shown in Equation (9.9).

$$\begin{aligned} \tau_{sup}\frac{\mathrm{d}T_s}{\mathrm{d}t} &= T_s^{\dagger} - T_s, & \frac{C_w}{60}\frac{\mathrm{d}T_{chw}}{\mathrm{d}t} &= Q_c - Q_{ch} \\ \tau_{fan}\frac{\mathrm{d}n}{\mathrm{d}t} &= n_{cmd} - n, & \tau_{val}\frac{\mathrm{d}u}{\mathrm{d}t} &= u_{cmd} - u \\ \tau_{dem}\frac{\mathrm{d}P_{dem}}{\mathrm{d}t} &= P - P_{dem} \end{aligned} \tag{9.9}$$

The chilled water state is the one that reports a short plant. The machine takes the coil duty plus whatever is needed to pull the loop back to setpoint, up to its capacity, as shown in Equation (9.10).

$$\begin{aligned} Q_{want} &= Q_c + \frac{C_w}{60}\frac{T_{chw}-T_{chw,\mathrm{SP}}}{\tau_{chw}} \\ Q_{ch} &= \mathrm{clamp}(Q_{want},\,0,\,Q_{cap}), \quad \mathrm{PLR} = \frac{Q_{ch}}{\max(1, Q_{cap})} \\ Q_{cap} &= \max\left(20,\ Q_{cap,0}\,(1-\beta_{cap}(T_o-30))\right) f_{chl} \end{aligned} \tag{9.10}$$

Above capacity the chilled water temperature walks up rather than the coil duty being clamped, which is what a short plant does and what takes authority away from every valve in the building at the same instant. The coefficient of performance is a fixed fraction of Carnot with a condenser approach that grows with load, as shown in Equation (9.11).

$$\begin{aligned} T_{cond} &= T_o + \Delta T_{c,a} + \Delta T_{c,b}\,\mathrm{PLR} \\ T_{evap} &= T_{chw} - \Delta T_{evap} \\ \mathrm{COP} &= \eta_C \frac{T_{evap}+273.15}{\max(3,\, T_{cond}-T_{evap})} f(\mathrm{PLR}) \\ f(\mathrm{PLR}) &= \mathrm{clamp}\big(1+0.28\nu-0.55\nu^2,\,0.55,\,1.12\big) \end{aligned} \tag{9.11}$$

with $\nu = 1-\mathrm{PLR}$. Equation (9.11) contains half of the case for precooling, because T_{cond} follows the outdoor air. On the design day it gives 5.43 at five in the morning, 4.70 at eight, 3.94 at ten, 3.39

at noon and 3.10 at four in the afternoon, a loss of more than two fifths of the machine's efficiency to nothing but the time of day. Electrical demand is the sum in Equation (9.12).

$$P = \underbrace{\frac{Q_{ch}}{\mathrm{COP}} + P_{pmp,0}\left(\frac{\dot{m}_w}{\dot{m}_{w,\max}}\right)^3 + P_{pmp,b} + P_{fan}}_{\text{HVAC}} + \underbrace{P_{lgt} + P_{plg} + P_{base}}_{\text{house load}} \tag{9.12}$$

The chiller and pump terms are taken as zero when $Q_{ch} \le 2\,\mathrm{kW}$. At the design point the four terms are 98.5, 3.6, 22.2 and 93.5 kW for 217.8 kW at the meter, and the 93.5 kW of house load is the part no control decision can move.

9.3.4 What Is Stored, and What the Day Costs

Stored cooling is measured against the building this building would be if it were floating at the top of its comfort band, as shown in Equation (9.13).

$$E = \sum_z \frac{C_m\left(T_{hi} + \dfrac{Q_{rad,z}}{UA_{ms}} - T_{m,z}\right) + C_a\left(T_{hi} - T_{a,z}\right)}{3600}$$
$$E_{\max} = \frac{4(C_m + C_a)(T_{hi} - T_{lo})}{3600} \tag{9.13}$$

The structural reference is the band top plus the radiant gains over the surface conductance, not the band top itself, because (9.2) puts the slab above the air at rest by exactly that amount. Referencing it to the air ceiling instead reports a well-run building as 468 kWh in debt, and a supervisor asked to hold zero charge would then cool hard all day chasing a state that does not exist. At the design point $E = 285\,\mathrm{kWh}$ of $E_{\max} = 855.8\,\mathrm{kWh}$, and $E_{\max}$ grows by 28.5 kWh for every tenth of a degree of band width.

The day is billed by Equation (9.14).

$$\mathrm{Bill} = \int p\,P\,\mathrm{d}t + r_d \max_t P_{dem} + c_{DR}\int_{event} (P - P_{cap})_+\,\mathrm{d}t \tag{9.14}$$

where $p(t)$ is the time-of-use energy price, r_d is the demand charge on the highest fifteen-minute average the meter saw, and c_{DR} is the penalty on energy drawn above a curtailment cap while an event is running. The middle term is the one a fixed setpoint cannot touch. On the fixed-setpoint design day it is \$182 against \$434 of energy, set by one quarter of an hour in the whole month.

9.3.5 Weather, Schedules and the Tariff

The outdoor dry bulb is two half-cosines rather than one sinusoid, with the minimum at 05:00 and the maximum at 15:00, as shown in Equation (9.15).

$$T_o(c) = \begin{cases} T^o_{lo} + \dfrac{\Delta T_o}{2}\left[1 - \cos\dfrac{\pi\,u_r}{\Delta c_r}\right], & c_{min} \le c \le c_{max} \\ T^o_{hi} - \dfrac{\Delta T_o}{2}\left[1 - \cos\dfrac{\pi\,u_f}{\Delta c_f}\right], & \text{otherwise} \end{cases} \tag{9.15}$$

where c is the clock minute, $\Delta T_o = T^o_{hi} - T^o_{lo}$, the rising leg runs over $u_r = c - c_{min}$ and $\Delta c_r = c_{max} - c_{min}$, and the falling leg over $u_f = (c - c_{max}) \bmod 1440$ and $\Delta c_f = 1440 - \Delta c_r$. A single sinusoid would put the minimum twelve hours after the maximum and make the morning warm-up as slow as the evening cool-down, and the shape matters because the precooling

window is exactly the warming half. Occupancy, lighting and equipment follow smoothstep windows $w(c;a,b,e,f) = \min\left(S\left(\frac{c-a}{b-a}\right), 1-S\left(\frac{c-e}{f-e}\right)\right)$ with $S(\xi) = \xi^2(3-2\xi)$ clamped to $[0,1]$, so every gain term is continuously differentiable and the fixed-step integrator sees no kink. Occupancy is $0.03 + 0.97\,w(c;400,540,1000,1140)$ reduced by a quarter over a lunch window $w(c;700,730,790,820)$, lighting is $0.08 + 0.87\,w(c;380,440,1080,1200)$ and equipment is $0.35 + 0.60\,w(c;400,500,1020,1140)$. The building counts as occupied when occupancy exceeds 0.5, and the band tightens t_{lead} minutes before that, so a building floating to its 27 °C night setback is not still pulling down at half past nine.

Energy costs 7.5 cents a kilowatt-hour from 22:00 to 08:00, 11.0 cents from 08:00 to 14:00 and from 19:00 to 22:00, and 32.0 cents from 14:00 to 19:00. A curtailment is described by a cap P_{cap}, a start, an end, a notice period and a penalty rate. Three windows follow from those and they are deliberately not the same window. The event is active over $[c_{start}, c_{end})$, it is known to the controller only from $c_{start} - t_{notice}$, and it is priced as an event from $c_{start} - t_{DR}$ so that the plant has unloaded by the time the cap takes effect. Without that last lead the precool is still running flat out at the instant the cap bites and the building fails the first minutes of every event by having prepared for it too well.

9.3.6 Constants, Starting State and Numerics

Table 9.2 carries the constants. Twenty-two of the eighty-three never appear on screen, and those twenty-two are what a learner has to infer from a measurement rather than read off a panel. Post-step clipping holds the zone and structural temperatures between 5 and 45 °C, the terminal flows between 0 and $1.6\dot{V}_0$, the supply air between 2 and 40 °C, the chilled water between 2 and 30 °C and both actuators between 0 and 100 %.

Every scenario opens from a solved steady state. One routine evaluates the plant at rest for a given clock and a given set of actuator positions, and the initializer, the manual balancing aid and the verification suite all use it, so the design-point test and the operating point a student loads are the same point rather than two that look alike. It eliminates the structure with $T_{m,z} = T_{a,z} + Q_{rad,z}/UA_{ms}$, solves the terminal flows and duct static as a contracting fixed point, and closes the supply air loop by successive substitution whose loop gain is about 0.23, one minus the coil effectiveness times the recirculated fraction. Twelve day-long scenarios override the result with a measured overnight state, because a building whose plant has been off since six in the evening is at 23.3 to 23.9 °C of air over 23.9 to 24.7 °C of slab at four in the morning, and not at the free-float 28 °C it would reach if the sun never came up.

The engine of Section 1.6.1 advances the states under fourth-order Runge-Kutta at a frozen 0.05 min (3 s) step. Control executes on a 0.25 min (15 s) period, which is a building controller scan rather than a process control scan, and four samples of every logged column are kept per simulated minute. The sensor noise comes from a generator seeded at 20260914. The speeds are 1, 5, 20, 60 and 240 simulated minutes per second, and the twelve day-long scenarios need the top two.

9.3.7 Assumptions

Detail is spent where the behavior forces a controller design decision and withheld everywhere else. Each of the simplifications below fails somewhere.

Sensible heat only. There is no moisture anywhere in the model, so there is no latent coil load, no dehumidification, no reheat, and the economizer changeover is on dry bulb rather than on enthalpy. In a humid climate the latent load is often a third of the coil duty and a dry-bulb economizer can cost money rather than save it, so the coil sizing in (9.8) and the 18 °C limit in (9.7) belong to a dry climate and nowhere else.

Two thermal nodes per zone. A real slab has a temperature gradient through its depth and answers a step roughly as the square root of time rather than as the single exponential of (9.2). Two-node lumped models are the standard identifiable choice for control work and are adequate over a few

Table 9.2 Model Parameters. Values Are the Shipped Defaults. Hidden Constants Are Never Displayed in the Application and Are the Quantities a Student Has to Infer. Every Other Parameter Is Shown and Adjustable in the Model Panel.

Symbol	Meaning	Value	Unit	Visibility
Building and gains				
A	conditioned floor area	5000	m^2	shown
C_a	zone air capacitance	6750	kJ/K	shown
C_m	zone structural capacitance	250 000	kJ/K	shown
UA_{ms}	structure-to-air conductance	12.5	kW/K	shown
UA_{oa}	envelope plus infiltration, per zone	0.85	kW/K	shown
A_g	glazed area per zone	177	m^2	shown
SHGC	solar heat gain coefficient	0.32	—	shown
—	design occupancy	417	people	shown
—	sensible heat per person	0.075	kW	shown
—	lighting at full output	40	kW	shown
—	equipment at full output	50	kW	shown
P_{base}	lifts, hot water, other house load	8	kW	shown
f_{sol}	solar fraction reaching the structure	0.70	—	shown
f_p, f_l, f_e	radiant fractions, people/light/plug	0.45, 0.40, 0.25	—	hidden
Weather				
T^o_{lo}, T^o_{hi}	outdoor minimum and maximum	17, 35	°C	shown
c_{min}, c_{max}	clock minute of each	300, 900	min	hidden
φ, δ	latitude and solar declination	38, 20	°	shown
$I_{dn,0}$	extinction-free beam normal	950	W/m^2	hidden
τ_{atm}	clear-sky atmospheric extinction	0.14	—	hidden
$I_{dh,0}$	horizontal diffuse at the zenith	110	W/m^2	hidden
ρ_g	ground reflectance	0.20	—	hidden
c_0	clock minute the run starts	240	min	shown
—	freeze the clock, bench mode	0	—	hidden
Air side				
$\dot{V}_0$	design supply air flow	22.5	m^3/s	shown
ρc_p	volumetric heat capacity of air	1.206	kJ/(m^3 K)	hidden
P_0	fan shut-off pressure at full speed	1150	Pa	shown
R_f	fan curve droop coefficient	0.296	Pa s^2/m^6	hidden
R_{up}	resistance upstream of the sensor	1.383	Pa s^2/m^6	shown
η_{fan}	fan and drive total efficiency	0.62	—	shown
K_{box}	terminal coefficient, damper open	0.433	$m^3/(s\ Pa^{0.5})$	shown
—	terminal minimum, maximum flow	0.30, 1.60	of zone design	shown
$\dot{V}_{oa,des}$	outdoor air at design occupancy	4.17	m^3/s	shown
—	economizer dry-bulb high limit	18	°C	shown

Symbol	Meaning	Value	Unit	Visibility
Coil and plant				
$UA_{c,0}$	coil conductance at design flows	46.4	kW/K	shown
$\dot{m}_{w,\max}$	chilled water flow, valve open	17.5	kg/s	shown
$\dot{m}_{w,ref}$	flow the conductance is quoted at	11.4	kg/s	hidden
c_{pw}	water heat capacity	4.18	kJ/(kg K)	hidden
—	coil exponents on air, water flow	0.50, 0.80	—	hidden
$Q_{cap,0}$	chiller capacity at 30 °C	420	kW	shown
β_{cap}	capacity lost per K above 30 °C	0.0075	1/K	shown
η_C	fraction of Carnot achieved	0.45	—	shown
$\Delta T_{c,a}$	condensing approach at zero load	4.2	K	shown
$\Delta T_{c,b}$	extra approach at full load	7.8	K	shown
ΔT_{evap}	evaporating below chilled water	3.0	K	shown
C_w	chilled water loop capacitance	12 540	kJ/K	shown
$P_{pmp,0}$	pump at full flow	11	kW	shown
$P_{pmp,b}$	pump and controls standing load	2	kW	shown
Dynamics and instruments				
τ_{box}	terminal damper travel	0.35	min	shown
τ_{fan}	supply fan drive ramp	0.50	min	shown
τ_{val}	chilled water valve travel	0.50	min	shown
τ_{sup}	coil and supply duct transport	0.50	min	shown
τ_{chw}	chiller capacity control response	2.0	min	shown
τ_{dem}	utility demand meter window	15.0	min	shown
τ_{T_z}	zone temperature sensor	1.0	min	shown
τ_{T_s}	supply air thermistor	0.30	min	shown
τ_{P_s}	duct static transmitter	0.08	min	shown
Tariff, curtailment and schedules				
—	off, mid, on-peak energy price	0.075, 0.110, 0.320	$/kWh	shown
—	on-peak window	840–1140	min	hidden
r_d	demand charge	0.85	$/kW day	shown
c_{DR}	penalty above a curtailment cap	1.20	$/kWh	shown
t_{notice}	notice the utility gives	120	min	shown
t_{DR}	charging stops before the event	30	min	shown
w_{DR}	share of penalty the supervisor prices	0.50	—	shown
—	occupancy schedule breakpoints	400, 540, 1000, 1140	min	hidden
t_{lead}	optimum start, band tightens early	60	min	shown
T_{set}	unoccupied upper limit	27.0	°C	shown
Δ_{sp}	setpoint margin below the ceiling	0.30	°C	shown

hours [16], but this one reaches the last part of the store faster than a real building would, so every

storage figure reported here is optimistic by that amount.

Four perimeter zones and no core. A real 5000 m^2 floor plate has an interior zone with no solar gain and no envelope, which is easier to hold and would dilute the orientation spread the application exists to show.

The plant is reduced. The chiller of (9.11) is a Carnot fraction with a load curve rather than a compressor map, with no cycling at low load, no minimum unloading step, no staging and no condenser fouling, and below about ten percent load a real machine cycles and loses more efficiency than the curve shows. Equation (9.5) is a fan curve and a system curve collapsed into one quadratic, accurate near the design duty and increasingly approximate away from it, with no surge, no stall and no minimum flow. The demand meter in (9.9) is one first-order state where a utility meter is a fifteen-minute window whose boundaries are fixed by the clock, so a real peak that straddles two boundaries reads lower than this model shows.

The tariff is known exactly and the forecast errs only in level. The only weather forecast error available is a constant bias. Real precooling controllers spend most of their engineering effort on the forecast, and the interesting failures are the ones where it is wrong in shape. This model shows only the other kind, and Section 9.6 shows it to be worth about a dollar a day.

Occupant behavior is a schedule. Nobody opens a blind, complains at 24.1 °C or brings in a heater, and the comfort score counts zone minutes inside a band rather than anything a person would recognize as comfort. Adaptive comfort research puts the acceptable range wider and makes it depend on recent outdoor conditions [1], which would change the width of the store without changing what a controller does with it.

9.4 Dynamic Character

9.4.1 Two Time Constants in the Same Room

Timing one node at a time with the other pinned is what each time constant means, and the application measures both from its own derivative function rather than quoting them from the parameters. Stepping the supply air from 13 to 10 °C with the terminal flows and the structural nodes held, the west zone air reaches 63 % of its move in 5.45 min. Holding the zone air 3 K below where it was and releasing the structure, the west slab reaches 63 % in 335 min. The ratio is 61. Both agree with what (9.1) and (9.2) predict at the design duty, where the west terminal passes 6.25 m^3/s for an airflow conductance of 7.54 kW/K against 12.5 kW/K of surface and 0.85 kW/K of envelope, giving 5.39 min, and where $C_m/(60\,UA_{ms})$ gives 333 min.

The coupled pair has neither of those as an eigenvalue. Linearizing the two equations together at the design duty gives modes of 5.33 min and 838 min, and the slow one is the least useful of the three because it describes the building giving up rather than the store charging. What the separation means operationally is one experiment. With the terminals pinned and the supply air dropped three degrees, the west zone air falls 0.75 K in the first ten minutes and 1.02 K in twenty and then very nearly stops, because it has run into the slab, and twelve hours later the air is at 21.0 °C while the west slab has come from 25.84 to 24.30 °C. The first ten minutes are a temperature and are gone the moment the plant stops. Everything after is energy in storage and is still there afterward. The store cannot be filled in a hurry, and every disappointing result in Section 9.6 is that sentence being ignored.

9.4.2 Three Process Gains and a Shared Actuator

The three regulatory gains are measured at the design point. More valve is colder supply air at −0.123 °C/%. More fan speed is more duct static at +20.5 Pa/%, and it is the only one of the four loops with a positive process gain. More airflow into a zone makes it colder, and that gain has no single value because it depends on how long the measurement waits. With the supply air and duct static loops in service and the west terminal stepped from 6.25 to 7.35 m^3/s, it is −0.54 °C per m^3/s twenty minutes after the step, −0.59 at an hour while the slab has barely moved, and −1.24 once the slab has caught up, which takes the best part of two days. A loop tuned against the third number is

2.3 times too slow for the load it has to reject. With the valve and fan also held by hand the supply air warms as the flow rises, and the same kind of step reads only −0.27 at twenty minutes.

What the three share is one fan and one duct, and the sharing is what fails. Asking all four zones for 22.5 °C at the design hour works cleanly, and every zone lands on 22.49 °C with the fan peaking at 95.7 %. Asking all four for 22.0 °C does not. The terminals open together, total flow reaches 23.2 m³/s, the fan saturates and duct static falls to 168 Pa. North, east and south settle on 21.98 °C within two hours while the west zone is still at 22.57 °C at two hours and 22.15 °C at five. A shared resource does not run out equally, and the zone that runs out first is the one asking for the most, which here is the west terminal already passing 111.2 % of its 5.625 m³/s design flow against the north terminal's 67.6 %.

The same failure without the ambiguity is every terminal asked for 115 % at once. The fan reaches 100 %, static settles at 191 Pa rather than 300, each terminal passes 5.98 m³/s of the 6.47 m³/s it asked for, and 37.8 kW of fan is buying 23.91 m³/s. Lowering the static setpoint changes nothing, because the loop is already saturated in the direction that would need more pressure. Bringing the four demands back to 90 % holds 300 Pa at 92.7 % speed on 28.3 kW, a quarter less fan power for 15 % less air (20.25 against 23.91 m³/s), which is the argument for static pressure reset [9] in one comparison. Chapter 10 closes the same kind of reset as a loop, from the most-open air valve to the header pressure setpoint.

9.5 Control Structure as Implemented

9.5.1 The Shared PID Algorithm

One algorithm serves all four loops. It is the velocity form of Section 1.6.2, written here in this chapter's symbols in Equations (9.16) and (9.17).

$$e_k = a\,(\mathrm{SP}_k - \mathrm{PV}_k), \quad P_k = K_c e_k, \quad D_k = -K_c \tau_D\, a\, \dot{\mathrm{PV}}^{f}_{k} \tag{9.16}$$

$$\begin{aligned} \Sigma_k &= \Sigma_{k-1} + (P_k - P_{k-1}) + (D_k - D_{k-1}) \\ u^{uns}_k &= u_{bias} + \Sigma_k + I_{k-1} \\ u_k &= \mathrm{clamp}(u^{uns}_k, u_{\min}, u_{\max}) \\ I_k &= I_{k-1} + \left[\frac{K_c}{\tau_I} e_k + \frac{u_k - u^{uns}_k}{T_t} \right] \Delta t_c \end{aligned} \tag{9.17}$$

where $a = \pm 1$ is the action flag, $\Delta t_c = 0.25$ min is the scan period, Σ_k is the running sum S_k of Equation (1.15), u^{uns}_k is the output before limiting (v_k of Equation (1.16)) and the tracking constant T_t is τ_I for all four loops. None of the four loops sets a rate in the shared algorithm, so u_k is the clamped value. Section 1.6.2 explains why the sum is never clamped and why the integral alone is wound back at a limit. Under manual control I is set so that $u_{bias} + \Sigma_k + I_k$ equals the operator's actuator position exactly, which makes the transfer bumpless in both directions. All four loops run with $\tau_D = 0$, so (9.16) contributes no derivative term and the derivative filter is inactive. The one rate limit in the application is applied outside the controller and in one direction only, for the reason given in Section 9.5.3.

9.5.2 The Three Regulatory Loops

Table 9.3 lists the four loops as they execute. The three regulatory loops are paired in the obvious way, and the signs are worth stating because three of the four have a negative process gain and one does not. The zone, supply air and storage loops are direct acting, $a = -1$, because more of the manipulated variable makes the measurement smaller. The duct static loop is reverse acting, $a = +1$.

Manual operation hands the operator the chilled water valve, the supply fan and all four terminal flows, with the setpoint sliders inert, so the three process gains can be measured one actuator at a time. The zone architecture setting is separate from the mode, and selecting Manual there in Auto leaves

Table 9.3 The Four Loops as Executed. All Four Run the Algorithm of (9.16) and (9.17) With $\tau_D = 0$ at a 0.25 min Scan.

Loop	CV → MV	K_c	τ_I	Output limits
Duct static	$\hat{P}_s \to n$	0.045	0.60 min	25–100 %
Supply air	$\hat{T}_s \to u$	8.0	2.5 min	0–100 %
Zone (4)	$\hat{T}_{a,z} \to \dot{V}_{z,\mathrm{SP}}$	1.30	12 min	$\dot{V}_{z,\min}$–9.0 m³/s
Storage	$E/E_{\max} \to T_{z,\mathrm{SP}}$	0.045	60 min	T_{lo}–$(T_{hi} - \Delta_{sp})$

Units: K_c in %/Pa, %/°C, m³/(s °C) and °C/%
Action: $a = +1$ on duct static, $a = -1$ on the other three

the four flow stations to the operator while the air handler keeps holding supply air temperature and duct static, which is the commissioning case rather than a mode.

Three details of the regulatory layer are not generic. The zone loop's lower output limit is the ventilation minimum $\dot{V}_{z,\min} = 0.30\dot{V}_0/4 = 1.69\,\mathrm{m}^3/\mathrm{s}$ while the building is occupied and zero when it is not, so a zone controller with nothing to do still passes outdoor air. The air handler runs whenever the building is occupied or expected to be within t_{lead}, or whenever any zone measurement is more than 0.2 °C above what the controller asks of it, and that second clause is what lets the supervisor start the plant at four in the morning without a schedule, with both air-side loops tracking zero while it is off. The duct static measurement comes from a consistent evaluation of (9.5) rather than from a state, because the pressure is algebraic, and its filter is seeded from the plant rather than from zero, because the priming scan runs at a full 15 s step and a 0.08 min transmitter reaches 96 % of the real pressure in one call, so every automatic scenario would otherwise step its fan on the first scan.

9.5.3 The Supervisory Loop

The supervisor decides where inside the comfort band the zones should sit. The band itself is not constant. Its floor is always T_{lo}, and its ceiling is T_{hi} while the building is occupied or about to be, T_{DR} during a curtailment, and the night setback T_{set} otherwise. Cooling below T_{lo} is not storage, because it is money spent filling a tank whose extra volume the occupants would reject the moment they arrived.

What the supervisor compares is the marginal cost of a kilowatt-hour of cooling rather than of electricity, as shown in Equation (9.18).

$$c(t+\tau) = \frac{p(t+\tau) + w_{DR} c_{DR} \mathbb{1}[\text{priced event}]}{\max\left(0.5,\ \mathrm{COP}\left(T_o(t+\tau) + b_f,\ T_{chw,\mathrm{SP}},\ 0.7\right)\right)} \tag{9.18}$$

where b_f is the forecast bias, the part load ratio is fixed at 0.7 because the question is what the machine will be like in three hours rather than what it is doing this second, and the indicator is one only when the notice has arrived and the evaluated hour lies inside the priced event window. That single term is the entire difference between a controller that answers a curtailment and one that does not. Three models are in play at this point and they are not the same model. The seventeen-state plant of Section 9.3 is what the building does. The gains and time constants of Section 9.4 are what a loop is tuned against. And (9.18) is everything the supervisor itself knows, a price and a chiller curve with no thermal capacitance in it anywhere, which is why the storage loop has to close around a measured state of charge rather than predict one. The decision follows from the ratio of the worst cost inside the horizon to the cost now, as shown in Equation (9.19).

$$\begin{aligned} c_{worst} &= \max_{\tau = 0, 15, \ldots, H} c(t+\tau) \\ \sigma &= \mathrm{clamp}\left(K_{pre} \left(\frac{c_{worst}}{c_{now}} - 1 \right), 0, 1 \right) \end{aligned} \tag{9.19}$$

The charge demand σ is the setpoint of the storage loop as a fraction of $E_{\max}$, and the storage PI controller in Table 9.3 drives the zone setpoint until the measured $E/E_{\max}$ from (9.13) matches it.

Writing the excess of the ratio over one rather than the fractional saving $1 - c_{now}/c_{worst}$ matters for one case and it is the important one, because the fractional saving is bounded above by K_{pre}, so a curtailment penalty making the afternoon eight times dearer would commit barely more of the band than a tariff peak making it three times dearer and the event would get the same half-hearted precool as a Tuesday.

Two things make this a controller rather than a schedule. It responds to the price and the weather in front of it, so abolishing the on-peak rate changes what it does with no rule rewritten. And it discharges on its own, because the moment c_{now} catches up with c_{worst} the ratio collapses and σ goes to zero. When $\sigma \leq 0.02$ the loop is bypassed, the setpoint is placed at $T_{ceil} - \Delta_{sp}$, where T_{ceil} is whichever of T_{hi}, T_{DR} and T_{set} is the ceiling in force, and the controller tracks, so the building floats rather than regulating against a target of zero.

The one rate limit in the application is applied to the supervisor's output, outside the controller, and downward only, as shown in Equation (9.20).

$$T_{z,\mathrm{SP}} = \mathrm{clamp}\Big(\max\Big(T^{-}_{z,\mathrm{SP}} - \tfrac{r\Delta t_c}{60},\, u_{sup} \Big),\; T_{lo},\, T_{ceil} - \Delta_{sp} \Big) \tag{9.20}$$

where u_{sup} is the PI output, $T^{-}_{z,\mathrm{SP}}$ is the previous setpoint and $r = 1.0\,^{\circ}\mathrm{C/h}$ is the ramp limit, with I back-calculated by the whole amount the pacing removed, in one scan, so the integral carries what was applied. The limit is one-sided on purpose. Charging has to be paced, because the charge demand steps the moment the horizon reaches the peak window and an unrestrained PI answers that step by taking every zone to the floor of the band at once, which opens four terminals together and sets a new demand peak two hours before the one it was avoiding. Releasing is ceasing to buy and must be instant. A symmetric limit of the same rate inside the controller would get this wrong twice. It would take 2.7 hours to let a building precooled to the 21.0 °C floor back up to its 23.7 °C ceiling, so the plant would keep cooling through most of a three-hour event it had prepared for. It would also stop the setpoint following the band, so the drop from the 26.7 °C night ceiling to 23.7 °C when the band tightens at 06:49 would not be complete until 09:49, two hours after the building is occupied.

9.5.4 Model Predictive Control

The zone control has a fourth setting, which no shipped scenario carries as its default and only scenario 15 pins, in which a model predictive controller (MPC) writes the four zone setpoints in place of the storage loop of Table 9.3. Every $T_p = 15\,\mathrm{min}$ (sixty scans) it plans the cooling each zone should take over the next 24 h, $N = 96$ steps, in seventeen move blocks that run quarter-hourly for the first half hour, half-hourly for the next, hourly to ten hours ahead, two-hourly to sixteen and four-hourly to the end (102 variables with the two load slacks per block), and only the first block reaches the plant before the next decision repeats the computation, the receding horizon of Section 1.4.3 [2, 19]. The regulatory loops are untouched, the storage PI tracks the applied setpoint so that a rejected plan hands it back without a bump, and where the supervisor writes one setpoint for four zones the plan writes one for each.

The controller's model is Equations (9.1) and (9.2) with the airflow term written as the cooling $Q_z = \dot{V}_z \rho c_p (T_{a,z} - T_s)$ the terminal delivers, in which the balance is exactly linear, plus an unmeasured load d_z per zone, as shown in Equation (9.21).

$$\begin{aligned} \frac{C_a}{60}\frac{\mathrm{d}T_{a,z}}{\mathrm{d}t} &= -Q_z + UA_{ms}(T_{m,z} - T_{a,z}) + Q_{conv,z} + d_z \\ &\quad + UA_{oa}(T_o - T_{a,z}) \\ \frac{C_m}{60}\frac{\mathrm{d}T_{m,z}}{\mathrm{d}t} &= UA_{ms}(T_{a,z} - T_{m,z}) + Q_{rad,z} \end{aligned} \tag{9.21}$$

Every constant is the shipped default of Table 9.2 rather than the value in force on the run, so a learner who changes the building makes the model wrong in exactly that way, and $Q_{conv,z}$ and $Q_{rad,z}$

come from the schedules and the clear-sky geometry of Section 9.3 at their nominal values. The uncoupled time constants are 8.43 min for the air and 333 min for the slab. Stepped as the plant was in Section 9.4, the west terminal by 20 % with the air handler's loops closed, the model reads −0.541, −0.586 and −1.221 °C per m³/s at twenty minutes, an hour and two days against the plant's −0.537, −0.585 and −1.221, within 0.9 % with no fitted parameter.

The electrical side is a static map on the predicted cooling. At step k one kilowatt of zone cooling costs $\alpha_k = f_k(1+a_1)/\mathrm{COP}_k + a_1$ electric kilowatts linearly and $\beta_k = a_2(1+f_k/\mathrm{COP}_k)$ per kilowatt squared, where COP_k is (9.11) at the forecast ambient and the 0.7 part load ratio that (9.18) also prices, f_k is the share of the coil duty the chiller carries (one unless the economizer of (9.7) is open), and $a_1 = 0.0422$ and $a_2 = 3.09\times10^{-4}$/kW are the tangent at zero flow of the fan law of (9.6) at the static setpoint and the least-squares quadratic of its cube over 0 to 250 kW. The outdoor-air load is $\dot{V}_{oa}\rho c_p(T_o - \bar{T}_a)/\mathrm{COP}_k$ on the mean zone air the plan itself predicts, and the house load of (9.12) is added as it is. The map prices the energy of scenario 8's realized day at \$401.55 under the supervisor and \$402.62 under the plan, where the plant charged \$401.46 and \$402.47, and the flat-tariff day within 0.3 %.

A steady-state Kalman filter carries the estimate, on the twelve states $[T_{a,z}, T_{m,z}, d_z]$ sampled at the 0.25 min scan with d_z a random walk, corrected from the five signals the shipped controllers already read, the four filtered zone sensors $\hat{T}_{a,z}$ and the stored cooling E of (9.13), which fixes the slabs' sum. The per-scan variances are 10^{-4} and 10^{-5} K² on the air and the slab, 0.05 kW² on the load, 0.01 K² on each sensor and 25 kWh² on the stored cooling, and the gain is designed once at the declared defaults and frozen. One integrating load per measured zone makes the tracking offset-free, the input-disturbance estimate of Section 1.4.5 [20], and a 60 % plug-load step the model does not know lands each $\hat{d}_z$ within 30 % of the 7.5 kW it adds inside an hour.

The plan is the quadratic program of Equation (9.22), in dollars over the horizon, an economic objective in the sense of Section 1.4.8, with $Q_{z,k}$ the cooling of zone z at step k held over its block b, p_k the tariff, P_k the metered load the map predicts and s the slacks.

$$
\begin{aligned}
\min\ & \sum_{k=1}^{N}\sum_{z} p_k\left(\alpha_k Q_{z,k} + \beta_k Q_{z,k}^2\right)\frac{T_p}{60} + r_d\, s_{pk} \\
& + c_{DR}\sum_{b} s_{DR,b}\frac{T_p}{60} + \sum_{z}\left(q_b s_z^2 + l_b s_z\right) \\
& + w_T\left(E_N - E_0\right)^2 + m\sum_{b}\sum_{z}\Delta Q_{z,b}^2 \\
\text{s.t. } & T_{lo} - s_z \le T_{a,z,k} \le T_{ceil,k} - \Delta_{sp} + s_z \\
& P_k \le P_{peak} + s_{pk}, \qquad P_k \le P_{cap} + s_{DR,b} \ \text{(notified)} \\
& \textstyle\sum_z f_k Q_{z,k} + Q_{vent,k} \le Q_{cap}(T_{o,k}) \\
& Q_{z,k} \ge \dot{V}_{z,\min}\rho c_p\left(T_{ceil,k} - T_s\right) \ \text{(occupied } k\text{)} \\
& T_{a,z,k} - T_{a,z,k'} \ge -\tfrac{r}{60}T_p\,(k-k'), \quad 0 \le Q_{z,k} \le Q_{box}
\end{aligned}
\tag{9.22}
$$

where r_d and c_{DR} are the demand charge and the penalty of (9.14), so the second term prices every kilowatt of the predicted peak above a ceiling P_{peak} (zero by default) and the third every kilowatt-hour above a notified cap, $q_b = 40\,\$/\mathrm{K}^2$ and $l_b = 4\,\$/\mathrm{K}$ price each zone's worst excursion from the band over the horizon, $w_T = 2\times10^{-3}\,\$/\mathrm{kWh}^2$ holds the heat content of the fabric at the horizon's end, E_N, to its value now, and $m = 10^{-3}\,\$/\mathrm{kW}^2$ times the move setting damps the moves. The band, the demand ceiling and the cap are soft, each with its slack, as in Section 1.4.6. Hard are the box between zero (the coil cannot heat) and Q_{box}, the ventilation minimum $\dot{V}_{z,\min}$ while the building is occupied, the chiller's declared capacity of (9.10) at the forecast ambient over every block with $Q_{vent,k}$ the outdoor-air duty, and the ramp limit r of (9.20) written on each zone's predicted air. The load slacks are decision variables in electric kilowatts, because a kilowatt of cooling costs a different number

Table 9.4 Measured Outcome of Every Day-Long Scenario, Read From the Application's Scorecard at $t =$ 1440 min. Comfort Counts Occupied Zone Minutes Inside the Band, Per Zone.

#	Day	kWh	Peak (kW)	Total ($)	Comf. (%)	Over cap (kWh)
5	design, fixed 23 °C	2388	214	615	100.0	—
6	mild, economizer at 09:00	1887	158	471	100.0	—
7	design, band, $H = 0$	2269	202	585	100.0	—
8	design, band, $H = 480$ min	2302	188	561	100.0	—
9	design, band, $H = 90$ min	2286	222	602	100.0	—
10	event, band, 120 min notice	2266	209	431	97.8	16.1
11	event, band, no notice	2258	213	437	97.7	19.4
12	event, fixed 23 °C	2388	214	614	100.0	152.3
13	heat wave, band, short plant	2800	232	684	100.0	—
13	heat wave, fixed 23 °C	2817	247	723	89.8	—
14	flat daytime tariff, band	2280	192	397	100.0	—
15	design, MPC, 24-hour plan	2356	183	558	100.0	—
16	challenge, as handed over	2508	247	953	36.1	220.9
16	challenge, suggested line-up	2631	235	621	100.0	0.2

of electric kilowatts at every hour, and a zone's ceiling is relaxed by a funnel with a 20 min time constant after every drop of the band, so that the one slack the solver carries per output over the horizon is not set by an unavoidable recovery and then licensed everywhere.

The setpoint written to each zone loop is the air the plan predicts at the end of its first step, clamped into the band in force and paced downward by the ramp limit of (9.20) exactly as the supervisor's command is, and a zone the plan leaves at its ventilation minimum floats at the ceiling. The preview is what the supervisor already reads and nothing more: the tariff, the forecast ambient with its bias b_f, the band schedule, and the curtailment cap from the minute the notice arrives, through the same test (9.18) uses, and never before it: with two hours of notice the first cap row appears in the 09:00 plan. With the preview switched off the present price, ambient and event state are held over the horizon, and that setting is a row of every comparison in Section 9.6.6.

Five conditions admit a plan: an optimal status from the solver, a first move equal to the plan's first entry, every planned cooling inside its box, no negative slack, and every prediction, the cost and the four setpoints finite. Anything else is rejected, the historian is marked, and the band supervisor, whose storage loop has been tracking the mean of the four setpoints at every scan, takes the zone setpoint until a plan succeeds. Five forced rejections in the verification suite each moved the supervisor's command by 0.0006 °C against a ramp step of 0.0042 °C, and the next valid plan took the setpoint back fifteen minutes later. A plan takes 15.6 ms on average in the browser (the largest 41 ms), which is 0.10 % of one core at real time and 25 % at the 240 times speed, so a frame guard stops the simulation's stepping after 25 ms of wall time in a frame while the MPC is chosen. At a three-hour horizon a plan takes 0.3 ms.

Four models are now in play rather than three: the plant of Section 9.3, the loop gains of Section 9.4, the price and chiller curve of (9.18) the supervisor knows, and Equation (9.21) with the load map the plan knows, with no coil, duct, sensor filter or chilled water loop in it.

9.6 Results

Sixteen scenarios run in order. The first four are frozen at 15:00 on the design day so that nothing drifts underneath a measurement: the four actuators by hand, every loop closed on the bench, the supply air step with the terminals pinned, and every terminal asked for 115 %. The other twelve run a full 1440-minute day. A scenario pins the opening mode, the zone architecture, the four tunings, the disturbance sliders and any timed events, and the completion rules and the challenge work as Section 1.6.4 describes. The challenge is judged on the case as given, so changing a model parameter or a disturbance voids the attempt. Table 9.4 collects the scorecard each day-long run ends on.

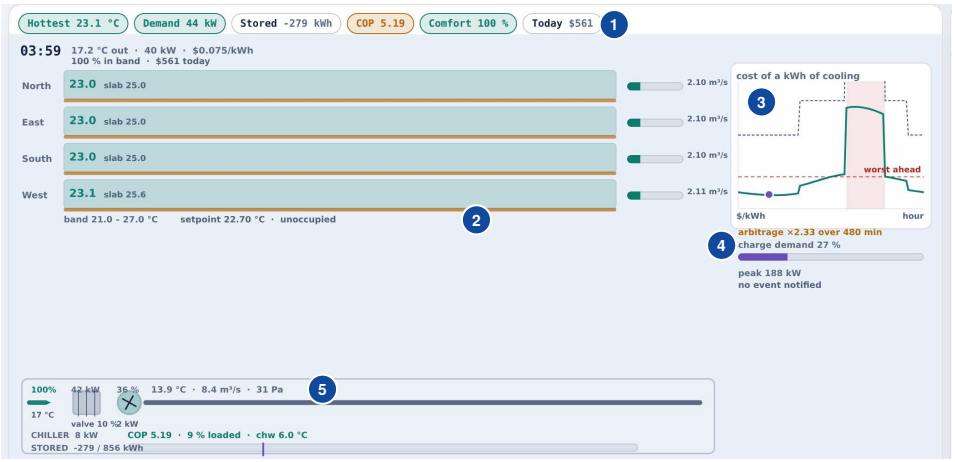


Figure 9.2 Scenario 8 at $t = 1440$ min, the end of its day, on the design weather with the supervisor given an eight-hour horizon, cropped to the application's plant view. During the day, as the trends recorded (they are not in the crop), the supervisor walked the west setpoint down to the 21.0 °C floor of the band by 07:42 and released it to the 26.7 °C night ceiling at 17:52, and the fabric filled to 68.4 % of its band, which is 585 kWh, by 08:44, and both regulatory loops held their setpoints whenever the fan ran. The frame catches the next morning's charge beginning at 04:00. (1) The chips: the day closes at 100 % comfort and $561. (2) The four zone strips, air over slab, with the band in force and the setpoint at 22.70 °C. (3) The cost inset: a kilowatt-hour of cooling now against the worst inside the horizon, the only pair of curves the supervisor reads, with the day's 188 kW peak below it. (4) The charge demand of 0.27. (5) The air handler strip with the fan restarting. At this minute the supply air was at 13.88 °C against 13.0 °C and duct static at 31 Pa against 300 Pa. The day used 2302 kWh (Table 9.4).

9.6.1 Floating a Band Is Not Using One

Scenario 5 is the building most of the stock actually is, a fixed 23 °C setpoint with night setback and optimum start, run competently. It holds every zone in band for 100 % of the occupied zone minutes and comes to 2388 kWh, $434 of energy and a 214 kW peak set by the quarter hour ending 15:27, which is $182 of demand charge for $615 all in. The orientation spread shows in the terminals rather than in the temperatures. East and west glass both peak at 705 W/m^2, while the east terminal tops out at 4.68 m^3/s at 10:33 and the west at 5.51 m^3/s at 16:56, because a zone integrates and because only one of those hours coincides with a 35 °C ambient, a full building and the on-peak rate.

Scenario 7 gives the same building a comfort band and a zero horizon, so the supervisor never sees a dearer hour and simply parks the zones just under the ceiling. The day costs $585 against $615, a saving of 4.9 %, and the peak falls only from 214 to 202 kW. What it is not is storage. The charge demand of (9.19) never leaves zero, the stored cooling of (9.13) peaks at 286 kWh at 08:40 on nothing but the slab's overnight coolth meeting a tightened band, and the afternoon load is met the moment it arrives. Widening a band saves energy, because a building rejects roughly in proportion to how far inside is from outside and seven tenths of a degree is not far. Using a band saves money, and the two are different operations.

🕮 **Pause.** Try Exercise 9.1 before reading on.

Scenario 8 gives the same supervisor an eight-hour horizon. It reaches the 21.0 °C floor by 07:42, the fabric is at its fullest at 585 kWh at 08:44, and the day costs $561 with every zone in band all day. Against the fixed setpoint that is 8.8 % off the bill. Two features of that saving are worth separating. Twelve dollars of it is energy and twelve is demand charge, because a 188 kW peak is fourteen kilowatts below a 202 kW one, so exactly half of the win is a term the fixed-setpoint building

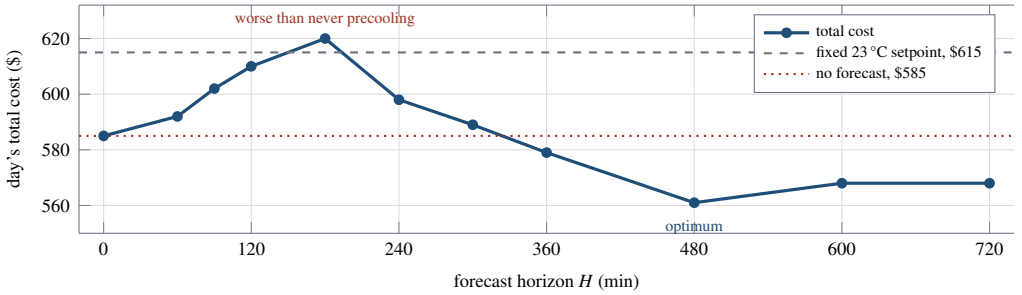


Figure 9.3 The day's bill against the supervisor's forecast horizon on scenario 8's weather, with everything else unchanged. Between about one and five hours a forecast is strictly worse than no forecast, and at three hours the building pays $620 against the $615 of a fixed 23 °C setpoint. Comfort is 100 % at every point on the curve, so the whole of the variation is money.

cannot reach. And the day uses more electricity than the floating case, 2302 kWh against 2269 kWh. Precooling is not an efficiency measure. It buys the same cooling at a better hour and accepts a little extra loss for doing it.

> **TRY IT IN THE APP** **8 · The Band, With Eight Hours of Forecast**
>
> Load the scenario and press Run in Auto. Watch the inset of what a kilowatt-hour of cooling costs, now against the worst ahead, and then the zone setpoint walk down to the 21.0 °C floor by 07:42. The stored cooling is fullest, at 585 kWh, at 08:44, and the day closes at $561 with every zone in band (Table 9.4).

The supervisor has no schedule, no clock and no rule naming a peak. Scenario 14 removes the on-peak rate, so electricity costs $0.110 per kilowatt-hour from 08:00 to 22:00 and the $0.075 overnight rate is the only step left in the tariff, and the building still precools. Floating costs $409 and the supervisor costs $397 with the peak down from 202 to 192 kW. Nearly two thirds of that saving is cooling bought before eight at the overnight rate. Raising the overnight rate to $0.110 as well, which leaves only the chiller curve of (9.11) and the demand meter, cuts the saving to $4.39 ($417.14 against $421.53) and the peak reduction to 202 against 198 kW. The same scenario measures the forecast, because an error of −6 °C gives $396 and +6 °C gives $398 against $397 with none. The forecast is worth about a dollar a day here, less than the horizon setting and very much less than the width of the band.

9.6.2 A Forecast Horizon That Fails at Both Ends

Scenario 9 cuts the horizon to ninety minutes and costs $602 against the $585 of a building that never precooled. Every cent of the difference is demand charge, because the energy bill is $413 either way while the peak goes from 202 to 222 kW. Sweeping the horizon over the same day gives Figure 9.3, over which the energy bill moves from 401 to $414 while the demand charge moves from 160 to $206, so the horizon is a demand-charge knob and very little else.

The shape has one rule behind it. The horizon has to cover the time it takes to charge the store, not the time until the peak. The floating building draws 137 to 163 kW between nine and eleven against a 202 kW afternoon peak, so a charge has 39 to 66 kW of headroom before it sets a new peak, and the slab takes up what it is given on a 335 min time constant. At eight hours the supervisor starts charging at four in the morning and the store is not at its fullest until 08:44. At three hours it first sees the 14:00 price rise at 11:00, takes the setpoint from 23.7 to 22.3 °C in the next two hours and sets the day's 242 kW peak at 14:00, the minute the on-peak rate begins. A horizon shorter than the charging time guarantees the charging happens in a rush at the worst possible hour. Comfort is 100 %

at every horizon in the sweep, which is what makes the failure invisible from the temperature trends and visible only on the demand meter.

🕮 **Pause.** Try Exercise 9.2 before reading on.

Raising the aggressiveness K_{pre} at a long horizon produces the same failure by a different route, because the charge demand saturates the moment the peak comes into view, the setpoint dives and four terminals open together. At $K_{pre} = 0.8$ and $H = 480$ min the day costs \$613 at a 243.5 kW peak and 98.5 % comfort, against \$561 and 188 kW at the shipped 0.20. The ramp limit of (9.20) is what stands between a storage controller and its own demand charge.

9.6.3 A Curtailment, Run Three Ways

Scenarios 10 to 12 are the same day three times. The tariff is flat from 08:00 to 22:00, so the only price signal in the working day is the event itself, a 140 kW cap on the whole building from eleven until two with \$1.20 per kilowatt-hour charged on everything above it, against 93.5 kW of lights, equipment and house load.

With two hours of notice nothing in the controller changes except one term of (9.18). At nine o'clock the forecast cost of a kilowatt-hour of cooling inside the event window goes from 3.6 to 22.6 cents, the ratio in (9.19) goes from 1.40 to 8.85 and saturates the charge demand, and the setpoint walks down at its 1 °C/h limit. The plant runs to 190 kW by half past ten and stops charging half an hour before eleven so the chiller has unloaded when the cap takes effect. Through the event the zones drift up to 25.3 °C, under the 25.5 °C ceiling the tenants agreed to, and the building draws 16.1 kWh above the cap, \$19 of penalty on a \$431 day.

Taking the notice away gives 19.4 kWh and \$437. That is the honest size of two hours of warning on this plant, a sixth of the overage and six dollars, and it is much smaller than intuition suggests. The reason is arithmetic rather than tuning. Two hours of charging on a store whose time constant is 335 min moves the west slab from 24.47 to 24.30 °C, a difference of 0.17 K, and puts 327 kWh in the fabric at eleven o'clock instead of 278 kWh. Eight hours of notice instead of two still leaves 5.7 kWh above the cap with 686 kWh stored.

Running the same event on a building holding 23.0 °C because that is what it was told to hold gives 152.3 kWh above the cap, \$183 of penalty and \$614 for a day the band building did for \$431. It is also the most comfortable of the three on paper, at 100 % of occupied zone minutes inside the band against 97.8 % and 97.7 %, because it never let a zone move, and it paid for that at \$1.20 per kilowatt-hour. The storage was present in all three buildings, because it is the same slab and the same 856 kWh. What the first two had and the third did not was permission to use it.

Where the remaining 16.1 kWh sits explains why more notice cannot reach zero, and it is at the end of the event rather than the start, with 0.1, 2.5 and 13.4 kWh drawn in its three hours. From the first scan of the event every terminal is on its ventilation minimum, because the relaxed ceiling lifts the setpoint to 25.2 °C at once, total flow falls from 11.6 to 6.75 m³/s, and (9.7) still asks for up to 4.17 m³/s of outdoor air, so the outdoor air fraction is 0.62 as the event begins and still 0.46 over lunch at one o'clock, and the coil cools most of a summer afternoon from scratch. Whole-building demand bottoms out at 133 kW at 11:02 and climbs to 160 kW by two o'clock as the outdoor air warms, and at one o'clock the 146 kW on the meter is 93.5 kW of house load, 46.1 kW of chiller, 4.5 kW of fan and 2.3 kW of pump. Once the terminals are on their minimum the store buys comfort rather than kilowatts. Precooling buys the first hour of a curtailment and the ventilation rate decides the third, and the ventilation rate is not a control setting.

🕮 **Pause.** Try Exercise 9.3 before reading on.

The precool does not set a new peak here. The draw jumps to 211 kW at three minutes past two as the plant restarts, but the day's 209 kW demand peak is set at 15:38 by the ordinary afternoon load, and it is below the 214 kW of a building that did nothing. The event also does not end when the event ends, because the zones are at the relaxed ceiling at two o'clock and the ordinary 24 °C band returns the same minute, so every one of the lost comfort minutes falls after the curtailment finished.

9.6.4 A Plant That Runs Short

Scenario 13 raises the whole diurnal curve by 6 °C and removes a fifth of the chiller. On a fixed 23 °C setpoint the machine gives out first and takes every other loop with it, because the chilled water walks to 14.3 °C, the supply air setpoint becomes unreachable and every coil valve loses authority at the same instant, so each zone spends 61 min above the band for 89.8 % comfort and the day costs $723 at a 247 kW peak. The same plant under the band architecture never leaves setpoint. Chilled water holds 6.00 °C all day, comfort is 100 %, the peak is 232 kW and the day costs $684, on a plant that is no larger. What precooling buys on a day like this is capacity that did not have to be installed.

9.6.5 The Graded Challenge

The challenge is a day three degrees above design with ten percent more people and a three-hour 165 kW curtailment from three o'clock notified at one, handed over on a fixed setpoint with soft tuning and the economizer switched off. The day passes when every zone is in band for at least 99 % of the occupied zone minutes, less than 5 kWh is drawn above the cap and the bill is under $750 all in. As handed over it costs $953, holds the band for 36.1 % of the occupied zone minutes and draws 220.9 kWh above the cap. The suggested configuration does it for $621 with 100.0 % comfort and 0.2 kWh above the cap, margins of 1.0 percentage point, 4.8 kWh and $129, whether the settings are applied before Run or carried through Reset. It departs from the tuning the other scenarios ship in one supervisory value, a precool aggressiveness K_{pre} of 0.30 rather than 0.20. The supervisor is in service from the priming scan at 04:00 and charges from the 23.0 °C operator setpoint, and at 0.20 it commits too little of the band, holds 342 kWh at three o'clock and lets the day draw 6.9 kWh above the cap, which fails. At 0.30 it holds 539 kWh.

Disabling one layer at a time from that configuration ranks the improvements the scenario asks for, and the ranking is the result. Restoring the handed-over zone loop tuning of $K_c = 0.30$ and $\tau_I = 55$ min alone takes the day to $840 and 112 kWh above the cap, and restoring the handed-over duct static tuning alone drops comfort to 95.3 %. The handed-over supply air tuning of $K_c = 1.5$ and $\tau_I = 18$ min costs $20 and the day still passes. Returning to a fixed setpoint takes the day to $909 and 188.9 kWh, and keeping the band on a 30 min horizon takes it to $681 and 31.2 kWh, because the supervisor does not see the curtailment until half past two. The economizer is worth exactly nothing. Switched on or off the day costs $621 either way to the dollar and draws 0.2 kWh above the cap either way, because on a day three degrees above design the outdoor air never falls below 20.0 °C and so never gets under the 18 °C high limit of (9.7). The damper never opens past its ventilation minimum. The scenario's briefing asks for the economizer to be switched on, and it was worth checking rather than assuming.

The band edges are the part with no tuning rule at all. Every tenth of a degree of width is 28.5 kWh of storage by (9.13), so narrowing the band to 22 to 24 °C takes the same day from 0.2 to 5.9 kWh above the cap and dropping the event ceiling from 25.5 to 24.5 °C takes it to 28.0 kWh and $665. Both fail, and neither is an engineering decision.

One thing the suggested answer does not fix is worth stating rather than hiding. The day's 235 kW demand peak is set at 14:00 by the charge before the event. It is below the 247 kW the untouched building reached, so against the day as handed over the energy bill falls $57, the penalty $265 and the demand charge $11, but Equation (9.18) prices energy and a penalty and knows nothing about the demand charge, and a supervisor carrying that term would ramp more gently and start earlier. The predictive controller of Section 9.5.4 carries it, and Section 9.6.6 measures what that is worth on this day.

Table 9.4 carries one caution. Every figure in it is a whole-day total over the same 1440-minute window, so the comparisons between rows are sound, and none of them is an integrated error that would keep accruing after a transient ended. The comfort column is the one that misleads on its own. Row 12 is the most comfortable and by $183 the most expensive of the three curtailment days, and rows 10 and 11 lose their comfort minutes after the curtailment they survived had already finished.

Table 9.5 The Plan Against the Band Supervisor Over Full Days With the Same Seed, Instruments and Forecast. Excursion Is the Band Overshoot Integrated Over Occupied Zone Minutes and Travel the Supervisor's Setpoint Travel.

Case	Structure	Bill ($)	Energy ($)	Demand ($)	Penalty ($)	Peak (kW)	Over cap (kWh)	Comfort (%)	Excursion (°C min)	Travel (°C)
8, design day	supervisor, $H = 480$ min (shipped)	561.19	401.46	159.73	0	187.9	—	100.0	0.0	13.29
	supervisor retuned once, $K_{pre} = 0.25$	552.74	398.96	153.78	0	180.9	—	100.0	0.0	12.59
	MPC, 24 h (scenario 15)	557.61	402.47	155.14	0	182.5	—	100.0	0.0	9.34
	MPC, 3 h	578.65	408.85	169.80	0	199.8	—	100.0	0.0	10.58
	MPC, 3 h, demand charge not priced	650.89	436.93	213.96	0	251.7	—	100.0	0.0	16.30
	MPC, 24 h, demand charge not priced	600.28	407.78	192.50	0	226.5	—	100.0	0.5	10.35
	MPC, 24 h, preview off	566.96	403.53	163.43	0	192.3	—	100.0	0.0	7.12
10, curtailment, 120 min notice	supervisor (shipped)	431.16	233.99	177.90	19.27	209.3	16.1	97.8	25.1	15.55
	supervisor retuned once	427.83	233.23	176.76	17.83	208.0	14.9	97.8	24.7	15.79
	MPC, 24 h	421.43	237.37	164.46	19.60	193.5	16.3	100.0	0.0	12.86
	MPC, 3 h	449.36	235.88	164.93	48.55	194.0	40.5	100.0	0.1	16.50
	MPC, 24 h, preview off	422.20	229.99	175.53	16.68	206.5	13.9	97.8	24.6	9.66
11, curtailment, no notice	supervisor (shipped)	437.05	233.08	180.65	23.31	212.5	19.4	97.7	25.6	12.55
	supervisor retuned once	433.50	232.42	179.53	21.56	211.2	18.0	97.8	25.4	12.79
	MPC, 24 h	421.69	236.38	164.82	20.50	193.9	17.1	100.0	0.0	12.39
	MPC, 3 h	451.19	236.44	163.91	50.84	192.8	42.4	100.0	0.1	15.74
13, heat wave, chiller at 0.8	supervisor (shipped)	684.30	487.09	197.21	0	232.0	—	100.0	0.0	13.74
	supervisor retuned once	668.77	480.54	188.22	0	221.4	—	100.0	0.0	12.94
	MPC, 24 h	682.12	487.09	195.03	0	229.4	—	100.0	0.0	8.70
	MPC, 24 h, preview off	693.38	491.42	201.95	0	237.6	—	100.0	0.0	8.06
14, flat daytime tariff	supervisor, $K_{pre} = 0.5$ (shipped)	396.65	233.22	163.43	0	192.3	—	100.0	0.0	11.76
	supervisor retuned once, $K_{pre} = 0.25$	403.18	235.18	168.00	0	197.7	—	100.0	0.0	9.79
	MPC, 24 h	398.41	243.21	155.20	0	182.6	—	100.0	0.0	8.78
	MPC, 24 h, preview off	398.66	235.05	163.61	0	192.5	—	100.0	0.0	7.45
16, challenge	as handed over (fixed setpoint, soft loops)	952.77	477.34	210.37	265.07	247.5	220.9	36.1	937.1	8.00
	suggested (supervisor, $K_{pre} = 0.30$, tuned loops)	620.53	420.71	199.59	0.23	234.8	0.2	100.0	0.0	14.25
	suggested loops, supervisor retuned once	616.52	415.19	200.94	0.38	236.4	0.3	100.0	0.0	14.82
	suggested loops, MPC, 24 h	589.68	414.28	173.71	1.69	204.4	1.4	100.0	0.0	8.40
	suggested loops, MPC, preview off	612.13	412.71	181.89	17.53	214.0	14.6	100.0	0.0	5.51
	handed loops, MPC, 24 h	761.27	440.45	185.34	135.47	218.0	112.9	28.4	1633.6	6.03

Every MPC row makes 96 plans in a day, one every fifteen minutes from the priming scan, none rejected, at 15 ms a plan (0.3 ms at three hours). The supervisor retuned once is $K_{pre} = 0.25$ at $H = 480$ min on every case. On scenario 11 the preview-off plan gives scenario 10's preview-off row, because a plan that holds the present event state learns of the curtailment only when it starts.

9.6.6 A Receding-Horizon Plan Against the Priced Rule

Scenario 15 runs scenario 8's day under the MPC of Section 9.5.4 with the preview on, a 24-hour horizon, the demand ceiling at zero and the move setting at one, and those settings are carried to every other case with no retuning. Table 9.5 sets the plan beside the band supervisor at its pinned tuning on the day-long cases that exercise the supervisor and on the challenge, each a full day with the same seed, instruments, noise, tariff, forecast and notice for both. The third row of each case is the supervisor retuned once, at $K_{pre} = 0.25$ and the eight-hour horizon, the single best setting over scenarios 8, 10 and 14 together on a grid of 72, and the remaining rows are the plan at three hours, without the demand charge in its objective, and with its preview off. Every number in Table 9.4 is the same run as before.

> **TRY IT IN THE APP** **15 · The Same Day, Planned: A 24 h MPC Against the Priced Rule**
>
> Load the scenario and run the day on MPC at speed 240. Watch the MPC plan inset and the four zone setpoints: the plan never takes a zone to the floor, and at noon the west sits at 21.91 °C against the north's 22.91 °C. Then read Today and Peak demand and set them beside $557.61 and 182.5 kW (Table 9.5).

On the design day the plan closes at $557.61 and a 182.5 kW peak against the supervisor's $561.19 and 187.9 kW, both at 100 % comfort. The $3.58 is $4.59 less demand charge and $1.00 more energy, because both buy their cooling before eight at the overnight rate, and the plan then shapes the afternoon under the peak it has already set, as shown in Figure 9.5: its 182.5 kW peak is set at 10:15 by the charge itself and the meter stays under 181 kW through the on-peak window, where the supervisor's day peaks at 16:13. The plan never takes a zone to the 21.0 °C floor, and it releases the north first, to 22.91 °C by noon against the west's 21.91 °C, which one setpoint for four zones

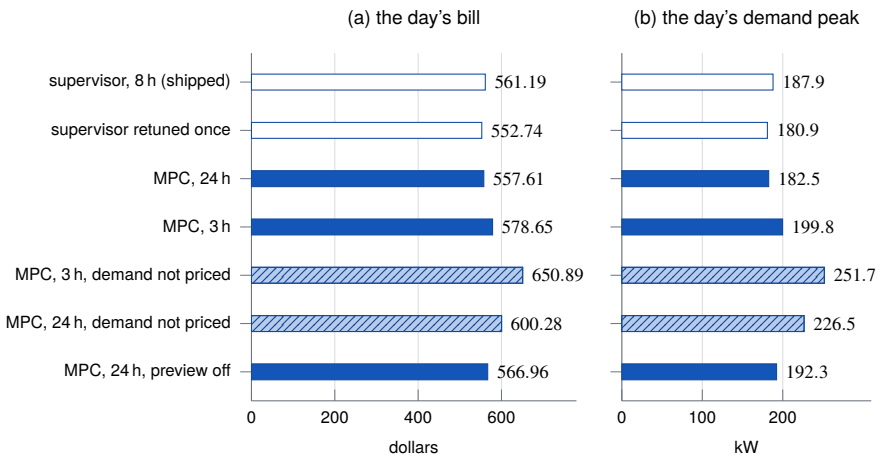


Figure 9.4 The design day of Table 9.5 (case 8) under the band supervisor (open bars), the plan (dark bars) and the plan with the demand charge left out of its objective (hatched bars): (a) the bill and (b) the demand peak. Notice that the hatched bars are the two longest in both panels, at either horizon, while the supervisor retuned once has the shortest.

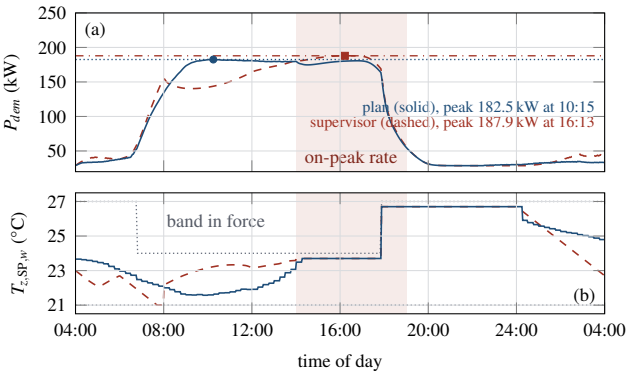


Figure 9.5 The design day under the plan (scenario 15, solid lines) and under the band supervisor (scenario 8, dashed lines). (a) The fifteen-minute demand: the plan's 182.5 kW peak (dotted line, circle) is set at 10:15 by its own charge and the meter stays under 181 kW through the on-peak window, shaded, where the supervisor's 187.9 kW peak (dash-dotted line, square) falls at 16:13. (b) The west zone setpoint: the plan's quarter-hour steps against the supervisor's 1 °C/h ramp to the 21.0 °C floor at 07:42, with the band in force as gray dotted lines.

cannot do. The win is against the shipped tuning. The supervisor at $K_{pre} = 0.25$ does the same day for \$552.74 at 180.9 kW, \$4.87 better than the plan, and beats it on the heat wave as well, \$668.77 against \$682.12, where the plan ties the shipped tuning's \$684.30 to within \$2.18. Figure 9.4 draws the design-day rows of Table 9.5.

The curtailment days are where the plan buys what no setting of the supervisor buys. With two hours of notice it costs \$421.43 against \$431.16 and draws 16.3 kWh above the cap against 16.1 kWh, at 100 % comfort where the supervisor loses 13.5 min and 25.1 °C min of excursion after the event, and no setting of the supervisor on the grid reaches 100 % on that day. Two thirds of the plan's overage, 10.6 kWh of the 16.3, falls in the event's last quarter hour, because between 13:45 and 14:00 the plan opens the terminals and brings the zones from 25.2 °C to the 24.0 °C ceiling that returns at

two o'clock, which the supervisor does not attempt. With no notice it costs $421.69 against $437.05 and 17.1 kWh against 19.4 kWh. With its preview off, the plan costs $422.20 on the notified day at 97.8 % comfort and 13.9 kWh, because a plan that holds the present price has nothing to arbitrage and sees the cap only once the event is running, so it does what the supervisor does.

The flat daytime tariff is the case the plan loses, by $1.76: $398.41 against $396.65, a 10 kW lower peak ($8.23 of demand charge) bought with $9.99 more energy. The loss is not the demand weight, which is the tariff's own $0.85 per kilowatt, because with the demand charge left out of the objective the day costs $399.35 and with a 190 kW ceiling $396.00. The load map prices both realized days' energy within 0.3 %, which is the size of the difference, so on a day with nothing to arbitrage but the chiller curve the two controllers are inside the model's resolution of each other, the dependence on the rate structure that the optimal-control studies of building thermal storage examine [21].

On the challenge the plan is given the suggested regulatory tuning and economizer with the zone control alone changed, and closes at $589.68 with 100 % comfort, 1.4 kWh above the cap and a 204.4 kW peak, against the suggested configuration's $620.53, 0.2 kWh and 234.8 kW. The demand term the supervisor omits is worth $25.88 on this day, and the charge before the event, which sets the suggested configuration's peak at two o'clock, is paced under a peak 30 kW lower.

The three-hour horizon of scenario 9 is the counterfactual scenario 15 asks for. The plan at three hours costs $578.65 at 199.8 kW, better than the supervisor at the same horizon ($619.98 and 242.2 kW) and $21.04 worse than at 24 hours, because it cannot value the cheap morning until the on-peak window is three hours away. It does not reproduce the supervisor's failure, and the reason is the demand term: the same three-hour plan with the demand charge left out of its objective costs $650.89 at 251.7 kW, the pathology of Figure 9.3 and worse, and at 24 hours without the demand charge $600.28 at 226.5 kW, worse than the supervisor at eight hours. A short horizon charges in a rush whether the charge is a rule or a plan, and what spares the plan is that it prices the peak it is about to set. The lesson of Figure 9.3 survives in a weaker form: the horizon that sees the peak but not the time to charge for it is still the expensive one.

DID YOU KNOW? A Plan Cannot Rescue Soft Loops

From the simulator: on the challenge day the 24-hour plan, set above the zone, supply air and duct static loops as handed over, closes at $761.27 with 28.4 % comfort and 112.9 kWh above the cap. Above the suggested loop tuning the same plan closes at $589.68 with 100.0 % comfort (Table 9.5, case 16).

9.7 Technology in Practice

What Measures It

TECHNOLOGY Sensor: The Zone Temperature Sensor

In a variable air volume system each zone has a temperature sensor that reports to the controller of its terminal box [22]. In a plain thermostat the sensor and the controller share one housing [23]. Its faults are a dead signal, a constant bias from lost calibration or a poor location, a drifting signal, and an intermittent or noisy one [22]. Space temperature sensors are among the points that most need calibration, and one out of calibration can waste energy for years unnoticed [23]. A precooling field test added sensors inside hollow spheres, which weigh the radiant temperature of the cooled surfaces with the air temperature and so track comfort better [18]. No open, vendor-neutral source gave a typical range or accuracy. The chapter's zone sensor is a 1.0 min lag with seeded noise (Table 9.2).

A building automation system gathers outdoor temperature and humidity, indoor temperature, humidity and carbon dioxide, and the state, temperatures and flows of its equipment [4]. Temperature, humidity, pressure, flow and power are its common sensors [23]. Occupancy is read more coarsely: motion sensors give presence only, while advanced HVAC control needs a head count, and carbon dioxide, often used as its proxy, drifts and needs frequent calibration [4]. In this chapter the zone

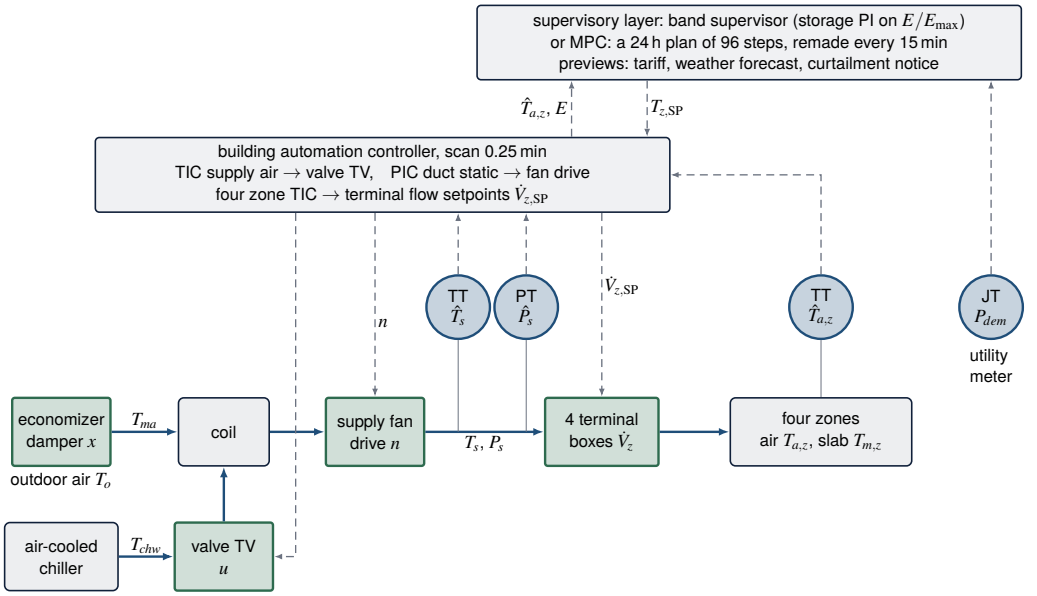


Figure 9.6 The chapter's loops on field hardware, drawn from Sections 9.3 and 9.5. Green boxes are the final elements (economizer damper, chilled water valve TV, supply fan drive and the four terminal boxes), blue circles the sensors (supply air, duct static, the four zone sensors drawn as one, and the utility meter), dashed arrows signals. Notice that the supervisory layer moves no device: it writes the zone setpoint, and the zone loops turn it into airflow.

sensors, the supply air thermistor and the duct static transmitter of Figure 9.1 are first-order lags of 1.0, 0.30 and 0.08 min, and the utility meter is a 15 min demand window (Table 9.2). Occupancy enters as a known schedule.

What Moves It

The devices a building controller drives are air dampers, mixing boxes, control valves and, in some cases, fans, pumps and motors [23]. The terminal dampers also steer the fan: a retrofit guide uses their positions to reset the duct static setpoint, raising it when a set number of dampers are wide open and lowering it when most are not [5]. The chapter's chilled water valve and fan drive are 0.50 min lags, the terminals 0.35 min lags, and the economizer damper opens only as far as the coil-leaving target needs (Equation (9.7)).

> **TECHNOLOGY** **Final Element: The VAV Terminal Box**
>
> A pressure-independent terminal box carries an airflow sensor and a damper. Its controller moves the damper to meet the zone's load, and a preset minimum airflow keeps ventilation air flowing when the zone needs no cooling [22]. A common practice sets that minimum at half the maximum, more than ventilation needs, and lowering it to each zone's real need saves fan energy [5]. Faults met in service include a damper stuck open, shut or part way from a failed motor, a broken linkage or corrosion, a wrong minimum or maximum flow setpoint, and a biased or drifting airflow sensor [22]. In the chapter each terminal is a 0.35 min lag capped by what the duct static will pass, with a minimum of 0.30 of zone design flow (Equation (9.4), Table 9.2).

Where the Controller Runs

Direct digital control measures a variable, compares it with a setpoint and signals a terminal device, using proportional-integral algorithms [23]. Building controls form a hierarchy: local controllers at

the devices answer setpoints, and supervisory levels above them carry precooling and the temperature and pressure resets, usually as rules [4]. Supervisory and lower loops run with cycle times from milliseconds to minutes [24]. For load shifting the report gives a predictive controller a horizon of typically a day and a re-evaluation interval between 5 and 15 min, since the thermal dynamics are slow [4]. The chapter's four loops scan every 0.25 min and its plan looks 24 h ahead and is remade every 15 min, as Figure 9.6 draws them.

> **IN PRACTICE** **Widen the Deadband, Then Keep It Wide**
>
> Many buildings run with little or no deadband, the gap between the heating and cooling setpoints. A federal retrofit guide for offices widens it from 71 and 73 °F to 69 and 75 °F, since most occupants are comfortable between 73 and 79 °F in summer. With direct digital control at the zones it is one global edit at the operator workstation [5]. A federal maintenance guide asks for the widest gap between the two setpoints, an unoccupied cooling setback 5 to 10 °F above the occupied setpoint, and overrides recorded and removed as soon as possible [23]. The chapter's 21 to 24 °C band and 27 °C night setback are such settings, owned by the operator, not the controller.

What Is Modernizing It

Three developments bear on this chapter's method. Predictive supervisory control is the first: early experiments at building sites reported energy savings of up to 17 to 65 percent against the installed control sequences [4]. Grid signals are the second. An open data schema exchanges electricity prices and demand response commands between buildings and the grid, and some programs use communicating thermostats to precool before a setback during peak hours, which avoids both discomfort and a recovery spike [4]. The third is cheaper sensing and diagnosis: wireless, battery-powered sensors make retrofits more economical, and newer automation systems can detect and diagnose equipment faults and advise the operator [4]. Each one feeds the plan a signal this chapter takes as known: the tariff, the curtailment notice, and a measured state of the fabric.

9.8 Limitations

The application is not a building energy model. With no moisture anywhere, two nodes per zone, no core zone and a Carnot chiller, nothing here settles a question about a real building's annual consumption, its plant sizing, or the load it could shed next August. Its figures are realistic for a dry-climate office of this size, at 856 kWh across a 3 K band, a coefficient of performance from 5.43 to 3.10 across one day, and about 40 % of the building's electricity in the HVAC plant, and none of them is a measurement of any particular building.

Three limits bear directly on the results reported here. The two-node zone of Section 9.3.7 reaches the last part of the store faster than a real slab would, so every storage figure is optimistic and the curtailment results are the ones most exposed to that. The demand meter is a first-order lag rather than a clock-aligned integrating window, so a real peak that straddles two window boundaries reads lower than Table 9.4 shows, and the demand charge is the term the horizon sweep of Figure 9.3 is entirely about. The supervisor prices energy and a curtailment penalty and does not price the demand charge at all, which is why the charge before the challenge's event still sets that day's 235 kW peak at two o'clock.

The plan's model leaves out what the plant has and the supervisor's model never needed: the sensor filters, the terminal and duct lags of (9.4), the chilled water loop of (9.9) that walks up when the plant is short, the pump, the part-load dependence of (9.11) and every disturbance slider, whose effect on the zones the estimated load carries while the capacity row keeps the declared capacity. The comparison of Section 9.6.6 is read at that model's resolution, 0.3 % of a day's energy, which is why the $1.76 flat-tariff loss is reported as a tie and not tuned away. The three-hour result differs from the supervisor's by one term rather than by the horizon, because a plan that does not price the demand charge fails at three hours as the supervisor does, at $650.89 against $619.98.

Repeatability is itself unrealistic. A frozen step, a frozen scan period and a frozen seed give the same trajectory everywhere, whereas a real building automation system jitters its scan, quantizes every analog point more coarsely and reorders what it reads. The four features that survived did so because each one forces a decision on whoever designs the controller, and they are the separation of 61 between the two thermal nodes, the shared fan that runs out unequally, the chiller whose efficiency follows the weather, and a controlled variable that is an interval.

> **OPEN CHALLENGE** **Why Predictive Control Is Still Rare in Buildings**
>
> A national report on building controls finds that predictive control has not been widely adopted in buildings despite field demonstrations. Legacy automation hardware has little memory and computation, cloud services bring latency and lost connections, accurate and robust models are costly to develop, train and calibrate, and operators are slow to accept the result. An optimizer's choices can also look like a black box, harder to trust and to tune by hand [4]. This chapter's plan takes its model from the declared constants with no fitted parameter. Progress would be models that update themselves from measured data and plans restated as rules an operator can check, both of which the report proposes [4].

9.9 Discussion

The comparison between scenarios 10, 11 and 12 does not come out the way the framing of most demand response programs suggests. Notice is what those programs are built around, and on this plant two hours of it takes 3.4 kWh, a sixth, off the overage and $6 off the day's bill. Permission takes off 136 kWh and $183 against the same cap on the same day, about forty times the energy and thirty times the money, and it is not a control setting. Buying notice and leaving the comfort specification alone optimizes the smaller of the two terms [17], and the same conclusion has been reached from the optimal-control side, where the strategies recovering most of the available reduction are the ones given the widest temperature trajectory [13, 15].

The model of Section 9.3 supports methods beyond the implemented loops. The seventeen-state plant with a stated objective, a hard ventilation floor and a band constraint is already in the shape a predictive controller wants, and the controller of Section 9.5.4 is that shape on two nodes per zone, carrying the demand charge this supervisor omits [6, 7] and an estimate of the structural temperature that nobody instruments. A field implementation on an occupied office building reports its results together with a cost-benefit account [25]. Whether any of it is worth doing is settled before the controller is specified, by whether the band exists on paper. Chapter 6 charges the same kind of store, buildings, pipes and a tank, ahead of a morning heat peak, and Chapter 8 selects the setpoints of a cooling plant whose optimum sits on a constraint that moves with the weather.

> **OPEN CHALLENGE** **Weights for Comfort, Cost and the Grid**
>
> The same report calls the balance among energy efficiency, grid benefits and occupant preferences a key challenge. Weights in one cost function are usually found by iterating in simulation, which means costly retuning once the controller is deployed, and occupant preferences are subjective and may change with time [4]. This chapter prices the band at fixed dollars per kelvin squared and carries one tuning of the plan to every case, and a supervisor retuned once beats it on the design day and the heat wave. Progress would be weights set from measured comfort and from preferences that occupants state, which the report lists as open research [4].

9.10 Conclusion

This chapter studied a commercial building and its air handler as a laboratory for control against a specification that is an interval. The simulator integrates seventeen states, four zone air temperatures over four structural temperatures, four terminal flows, the supply air, the chilled water, two actuators and the utility demand meter, with a counter-flow coil, a Carnot-fraction air-cooled chiller whose condensing temperature follows the weather, and a dry-bulb economizer. The two thermal nodes in

one room answer in 5.45 and 335 min, a ratio of 61, and the second holds 856 kWh across a 21.0 to 24.0 °C band. Three proportional-integral loops reach 63 % of a step in 0.55, 1.00 and 11.6 min, and a fourth prices a kilowatt-hour of cooling against the worst hour inside its horizon and spends the band on storage. Precooling takes the design day from $615 on a fixed setpoint to $561 at equal comfort, and half of the $24 it saves against a floating band is demand charge. The horizon that produces it is a real optimum with a range between one and five hours where a forecast costs more than none. On a three-hour curtailment two hours of notice moves the energy drawn above the cap from 19.4 to 16.1 kWh while removing the comfort band moves it to 152.3 kWh. A fourth zone control, a receding-horizon plan on the same two nodes per zone with the demand charge in its objective, closes the design day at $557.61 against $561.19, holds both curtailment days at 100 % comfort for $9.73 and $15.36 less, takes the challenge's peak from 234.8 to 204.4 kW, loses the flat-tariff day by $1.76, and is beaten on the design day and the heat wave by the supervisor retuned once. A fuller model would add a latent coil load and a distributed slab. The building shows what a control system can do with the freedom a specification leaves it.

KEY POINTS

- An office is judged against a comfort band, so the setpoint inside it is free and the concrete stores cooling.
- Floating in the band saves energy, and using it as a store saves money: $585 against $561 on the design day.
- A forecast horizon shorter than the time to charge the store costs more than no forecast at all.
- Two hours of notice cut a curtailment's overage from 19.4 to 16.1 kWh, while removing the band raises it to 152.3 kWh.
- A 24-hour plan that prices the demand charge beats the shipped rule, $557.61 against $561.19, and a rule retuned once beats the plan.

Where else this applies. A data hall's supervisory layer (Chapter 8) also chooses a setpoint that a loop below it holds, and through one day its lowest-cost chilled water setpoint stays between 15.6 and 16.9 °C (Table 8.6).

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9.11 Exercises

9.1 ○ Concept check · 2 min

Scenario 7 gives the building a 21 to 24 °C band and a forecast horizon of zero. The day costs \$585 against \$615 on a fixed 23 °C setpoint, and the peak falls only from 214 to 202 kW. Why is this saving not the use of the fabric as a store?

(a) The band's ceiling is too close to the setpoint, so the slab cannot warm far enough to hold any cooling

(b) The ramp limit stops the setpoint from falling fast enough to charge the slab before the afternoon

(c) With no horizon the worst cost ahead is the cost now, so the charge demand stays at zero and the afternoon load is met as it arrives

(d) The economizer fills the slab with cool night air, so the store is already full when the band tightens

9.2 Concept check · 2 min

On scenario 8's day the bill under the band supervisor is \$585 with no forecast, \$620 with a three-hour horizon and \$561 with an eight-hour one (Figure 9.3). Which rule explains the shape of that curve?

(a) The horizon only has to reach the start of the on-peak window, and anything longer is wasted
(b) A longer horizon always lowers the bill, because the supervisor sees more of the price curve
(c) The forecast error grows with the horizon, so the shortest horizon that sees any price rise is the cheapest
(d) The horizon has to cover the time it takes to charge the store, or the charge is bought in a rush that sets a new demand peak

9.3 Concept check · 2 min

In scenario 10 the building draws 16.1 kWh above the 140 kW cap, 13.4 kWh of it in the event's third hour, and eight hours of notice instead of two still leaves 5.7 kWh. Why can more notice not bring the overage to zero?

(a) With every terminal on its ventilation minimum the coil still cools the required outdoor air, and that load rises above the cap as the afternoon warms
(b) The supervisor stops charging half an hour before the event, so the store is empty by the third hour
(c) The demand meter lags by fifteen minutes, so energy drawn after the event is still counted against the cap
(d) The chiller loses capacity in the afternoon heat, so the chilled water walks up and every valve opens

9.4 Theory and calculation · 6 min

Use Equation (9.13) and the constants of Table 9.2.

(a) Compute $E_{\max}$ for the 21 to 24 °C band and the storage added by every tenth of a degree of band width. *Checkpoint:* 855.8 kWh, 28.5 kWh per 0.1 K.

(b) At the design point every zone's air is at 23.00 °C and the slabs are at 24.19, 24.19, 24.53 and 25.84 °C. If the slab reference in Equation (9.13) were the band top itself, by how many kilowatt-hours would E fall at this state? *Checkpoint:* About 469 kWh.

(c) A tenant asks for the band to be narrowed to 22 to 24 °C. How much storage does the building lose? *Checkpoint:* About 285 kWh.

9.5 Theory and calculation · 6 min

The west zone at the design duty passes 6.25 m³/s with $\rho c_p = 1.206\,\mathrm{kJ/(m^3\,K)}$.

(a) From Equations (9.1) and (9.2), compute the west air node's time constant with the slab held, the slab's time constant with the air held, and their ratio. *Checkpoint:* 5.39 min, 333 min, about 62.

(b) The predictive controller's model, Equation (9.21), takes the cooling Q_z as its input instead of the airflow. What air time constant does that model have, and why is it longer? *Checkpoint:* 8.43 min.

(c) Treat the slab as a single lag with the 335 min the chapter measures. What fraction of a step in the air does the slab follow in two hours of charging, and in eight? *Checkpoint:* 30 % and 76 %.

9.6 Theory and calculation · 7 min

The supervisor charges in proportion to $\sigma = \mathrm{clamp}(K_{pre}(c_{worst}/c_{now} - 1), 0, 1)$, Equation (9.19), with $K_{pre} = 0.20$.

(a) The chiller's coefficient of performance on the design day is 5.43 at 05:00 and 3.10 at 16:00, when the tariff is \$0.075 and \$0.320 per kilowatt-hour. Compute the cost of a kilowatt-hour

of cooling at both hours, their ratio, and how much of the ratio is the tariff and how much the chiller. *Checkpoint:* 1.38 and 10.3 cents, 7.5 = 4.27 × 1.75.

(b) In scenario 10 the ratio c_{worst}/c_{now} is 1.40 just before the notice arrives at 09:00 and 8.85 just after. Compute σ at both. *Checkpoint:* 0.08 and 1 (clamped from 1.57).

(c) The fractional saving $K_{pre}(1 - c_{now}/c_{worst})$ was the other candidate. Compute it for the curtailment's 8.85 and for a tariff peak with a ratio of 3, and say why the chapter rejects it. *Checkpoint:* 0.177 and 0.133.

9.7 App challenge · 8 min

Load scenario 9, "Ninety minutes of forecast, which is worse than none", default seed, as loaded (Band + precooling, Forecast horizon 90 min). Run each day to its end at speed 240 (240 min per second, so a day takes 6 s).

(a) Read the Today, Peak demand and Comfort tiles at $t = 1440$ min. *Checkpoint:* \$602, 222 kW, 100.0 %.

(b) Set Forecast horizon to 0, then 180, then 480 min, each before Run or with Reset, and record Today and Peak demand for each. *Checkpoint:* \$585 and 202, \$620 and 242, \$561 and 188 kW.

(c) The demand charge is \$0.85 per kilowatt of the day's peak. Split each of the four bills into demand charge and energy, and say which part the horizon moves. *Checkpoint:* In the order 90, 0, 180 and 480 min: energy about \$413, 413, 414 and 401, and demand 189, 172, 206 and 160.

Read the bill to ±\$1 and the peak to ±1 kW. The exported CSV columns `metric_energy_usd` and `metric_demand_usd` give the split directly.

9.8 App challenge · 9 min

Run the curtailment day three ways, default seed, at speed 240, each for the whole 1440 min, and read the Over the cap, Today and Comfort tiles at the end.

(a) Scenario 10, "A curtailment, with two hours of notice", as loaded, then scenario 11, "The same curtailment, with no notice at all", as loaded. *Checkpoint:* 16.1 kWh, \$431 and 97.8 %, then 19.4 kWh, \$437 and 97.7 %.

(b) In scenario 11 open Show Model, set Notice the utility gives (under Parameters) to 480 min, press Reset and run the day again. *Checkpoint:* 5.7 kWh, \$421, 98.5 %.

(c) Scenario 12, "And the same curtailment with no band at all", as loaded. Compare what two hours of notice and what the band are each worth, in kilowatt-hours above the cap and in dollars. *Checkpoint:* 152.3 kWh, \$614, 100.0 %.

Read the overage to ±0.1 kWh, the bill to ±\$1 and comfort to ±0.1 %.

9.9 App challenge · 9 min

Load scenario 15, "The same day, planned: a 24 h MPC against the priced rule", default seed, as loaded (Zone control MPC, Plan horizon 24 h, Preview On). Run each day to $t = 1440$ min at speed 240.

(a) Read Today, Peak demand and Comfort, and the last row of the exported CSV columns `metric_energy_usd` and `metric_demand_usd`. *Checkpoint:* \$558, 183 kW and 100.0 %, with \$402.47 and \$155.14 in the file.

(b) Press Reset, set Zone control to Band + precooling and run the day. Read the same five values and say where the plan's \$3.58 comes from. *Checkpoint:* \$561, 188 kW and 100.0 %, with \$401.46 and \$159.73 in the file.

(c) Set Zone control back to MPC, set Plan horizon (under Predictive controller in Advanced) to 3 h, press Reset and run. Compare with the supervisor's \$620 and 242 kW at a three-hour horizon. *Checkpoint:* \$579, 200 kW.

Read the bill to $\pm$\$1, the peak to $\pm$1 kW and the CSV values to $\pm$\$0.05.

9.10 Application area · 4 min

A facilities manager receives complaints about warm afternoons and proposes two changes to the lease: narrow the comfort band from 21–24 to 22–24 °C, and lower the ceiling allowed during a utility event from 25.5 to 24.5 °C.

(a) What does each change cost on the graded challenge day, as the chapter measures it from the suggested configuration? *Checkpoint:* 5.9 kWh above the cap, then 28.0 kWh and \$665.

(b) Whose decision is the band width, and what should the engineer bring to that decision? *Checkpoint:* The tenants' and the owner's, in the lease.

9.11 Application area · 3 min

The challenge asks for the dry-bulb economizer to be switched on.

(a) Why is it worth exactly nothing on that day? *Checkpoint:* The outdoor air never falls below 20.0 °C.

(b) The model has no moisture. Why would the same 18 °C dry-bulb changeover be a poor choice in a humid climate? *Checkpoint:* Latent load, so an enthalpy changeover.

9.12 Review the agent · 5 min

An AI assistant summarizes scenario 15: "The 24-hour MPC closes the design day at \$557.61 against \$561.19 for the band supervisor, so a plan beats a priced rule on this building. Replace the supervisor with the MPC and the bill will fall on every day like this one." Find the flaw and say how you would check it in the app.