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CHAPTER 12

Thermostats That Fall Into Step on a Simulated Supermarket Refrigeration Pack

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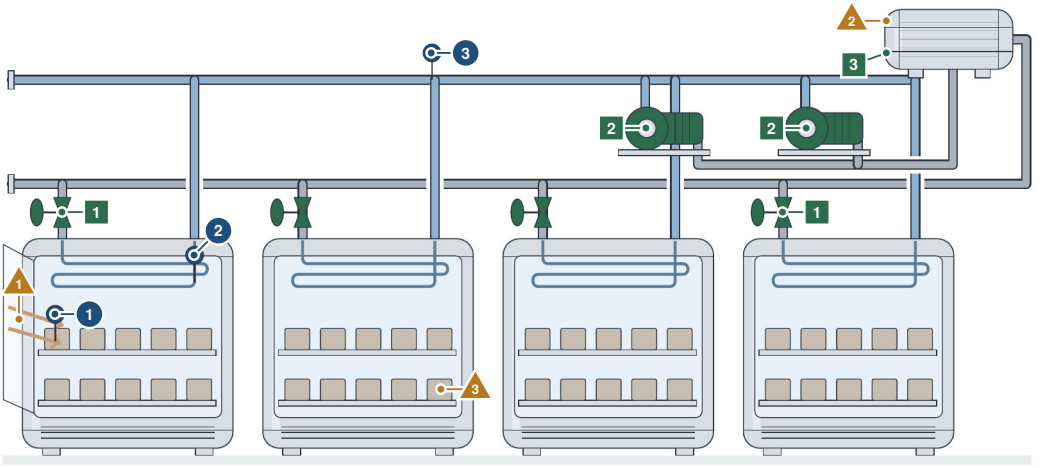
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CHAPTER 12

Thermostats That Fall Into Step on a Simulated Supermarket Refrigeration Pack



The controller moves 1 the valve into each case, opened and closed by its thermostat and its superheat loop 2 the compressors started and stopped on the rack 3 the speed of the condenser fans

The controller reads 1 the air in each case 2 the superheat at each coil outlet 3 the shared suction pressure

What disturbs it 1 a door held open 2 a hot afternoon at the condenser 3 warm goods restocked

Four display cases share one suction line and one rack of compressors (two of the chapter's three are drawn), so notice that the four valves open and close against one pressure. Illustration, not to scale.

IN PLAIN TERMS Display Cases That Fall Into Step

The main idea. A supermarket keeps four display cases cold with one shared set of compressors. Each case has a thermostat. It is a switch that lets cold liquid into the case when its air is too warm and shuts it off when the air is cold enough.

The challenge. The cases send their gas back through one shared pipe, so each case that switches slows or hurries the others. Within about an hour the four switch together, and the compressors must start and stop for all of them at once.

Why it matters. Every start wears a compressor, and a simple rule that keeps the cases apart cut three hours of starts from 45 to 8.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history in Section 12.2, the fact in Section 12.3, the fact in Section 12.5, the two try-its in Section 12.6, and the key points
Study	50 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	20 min	the first page, the results (Section 12.6), Technology in Practice (Section 12.7), Limitations (Section 12.8) and the open challenges

The control question Four display cases boil their refrigerant at one suction pressure, so thermostats that never exchange a signal still act on one another, and an expansion valve can hunt with its tuning unchanged. What pulls the cases into step, what keeps them apart, and what setpoint keeps a superheat loop stable?

Learning objectives

- Explain how a shared suction pressure pulls independent hysteresis thermostats into step, and why a larger suction volume does not prevent it.
- Compute the order parameter R from the thermostats' phases.
- Derive how much steeper the superheat answers the valve below the minimum stable superheat, and explain why a smaller gain is not the cure.
- Compare offsets, a scheduler and a profile MPC by compressor starts and goods temperature.
- Judge what floating pressures, a staggered defrost and head-pressure unloading can and cannot buy on a hot day.

Prerequisites Sections 1.4.7 (thermostats with hysteresis and dwell), 1.4.8 (integer actuators), 1.4.9 (cycle-to-cycle maps), 1.6.2 (the PI law as implemented) and 1.6.4 (scenarios and data). Section 1.6.3 is needed only for the predictive scheduler.

The app https://apmonitor.com/apps/refrigeration-pack/

See also Volume 7, Chapter 3 for a setpoint that chooses the plant, Volume 7, Chapter 14 for loops that settle alone and amplify together, and Volume 17, Chapter 8 for devices that hunt on one shared voltage.

12.1 Introduction

The major retail food outlets of the United Kingdom account for about 4.0 Mt of carbon dioxide a year through their energy use, according to a survey of 2570 stores that covered about 30 % of the stores there with more than 280 m^2 of sales area. The survey found a wide spread of electrical intensity, even within one chain, and estimated that bringing every store above the average down to it would save about 840 GWh a year, or 10 % [1]. The refrigeration and environmental control systems a store uses are among the factors it names for that spread. In the pack of this chapter, four display cases boil their refrigerant at one suction pressure that a rack of compressors holds, and a condenser on the roof rejects their heat together with the compressors' work. A thermostat on each case's air opens and closes that case's refrigerant supply, and an electronic expansion valve on each coil holds the superheat at its outlet. Unless a scheduler is switched on, nothing in the program connects one case's thermostat to another's.

Each of these loops is a textbook loop, and the simulator is designed to correct a belief that loops which behave well alone keep behaving well together. Four identical thermostats started from seeded random phases fall into step after 65.2 min, because the suction pressure they share moves with each of them, and the rack then starts and stops against all four at once, 14 starts in the first hour and 30 in the next two. A superheat loop that holds (5.98 ± 0.16) K at a 6 K setpoint swings between 0.7 and 7.8 K at 2 K with the same gains, because below a minimum stable superheat the evaporator's gain and lag rise under the loop. Neither effect appears in the transfer function of one loop. The first belongs to the utility, since a suction side twenty times larger still locks the cases, sooner, in 52.6 min, and only a pressure that no case can move leaves them apart, and the second belongs to the operating point. Practical and simulation studies of supermarket systems have reported the first, valve actions that coincide under distributed hysteresis control and a compressor that periodically works hard at low efficiency and high wear [2].

The class of problem is a population of on-off or limit-cycling loads coupled through a supply they share. Thermostatically controlled loads on an electric grid are one example, and in heterogeneous populations of them only a spread in the loads' characteristic frequencies damps the aggregate power, whose free response also beats [3]. In a population of coupled limit-cycle oscillators some lock their phases once the coupling passes a threshold, and a review of 25 years of work on the Kuramoto

model traces that transition [4]. Inside the vapor-compression cycle a relative gain analysis has shown couplings between inputs and outputs strong enough that independent single-loop control is inadequate [5]. Compressors on one air header, chillers on one chilled-water loop, boilers on one steam header and heat pumps on one district loop share the shape, a common pressure or temperature that each unit moves and every other unit feels. In a line of cars in Volume 7, Chapter 14 each follower settles well alone and the line can still pass a disturbance back larger, and on the solar feeder of Volume 17, Chapter 8 a tap changer and the inverters act on one voltage while two of its timers chase each other.

However, the supermarket literature poses these questions on benchmark and plant models and answers them with optimization [6–9], and its evidence for synchrony comes from field data and simulations that a student cannot rerun after changing one setting [2]. This chapter maps where a superheat loop loses its stability and how a shared suction pressure pulls independent thermostats into step, on a browser simulation of a four-case pack described completely enough for a reader to build it again. The thermostats, the superheat loop, the sequencer and the floating pressures follow established practice. Measured on the model, the time four cases take to lock moves less with the suction volume and the rack's tuning than with where the cases start (52.6 to 65.2 min from 5000 to 250 L and 54.7 to 78.9 min across the suction loop's gain and integral time, against 31.9 to 94.0 min over five other seeds), and the opening at which a fixed valve meets the stable line moves 1.76 points as the case air cools by 0.28 K and frost builds. The model has 62 states, its charge and case energies close to 4.9×10^{-15} of their throughput, and its controller reads only its instruments. Ten cases and a graded day each isolate one decision.

Section 12.2 follows the history of expansion valves, racks and synchrony. Section 12.3 gives the model of the cases, the suction side, the rack and the condenser, and Section 12.4 works out what the stable line and the shared pressure imply. Section 12.5 sets out the thermostats, the superheat, suction and condenser loops and the supervisor as programmed. Section 12.6 reports the cases, from a valve opened by hand to the summer day, and Section 12.6.10 adds a hybrid predictive scheduler. Section 12.7 describes the real sensors, valves, compressors and controllers of a store, and Sections 12.8 and 12.9 give the limitations and the conclusions, and the exercises in Section 12.10 run the cases.

12.2 Expansion Valves, Compressor Racks and Cases in Step

The stability of the expansion valve was already a modeling subject in 1985, when a lumped model of an evaporator, built from its balance equations for enthalpy, mass and momentum, explained the random fluctuations of the liquid dry-out point, showed that a conventional thermostatic expansion valve often operates unstable, and showed with a process computer connected on line that a controller adapted to the evaporation process raised the evaporator's performance, and therefore its efficiency, by at least 5 % [10]. The thermostatic valve meters liquid on the signal of a remote bulb charged with a refrigerant liquid and vapor mixture, and mathematical models of that sensor were used in 1987 to find the simplest representation of the valve that remained adequate [11].

Electronic valves then took over the metering. A stepper-motor valve on a machine of less than 6 kW, tested at evaporating temperatures from −20 to 10 °C and compressor speeds from 1000 to 3000 rpm, was run under proportional-derivative and qualitative optimal regulation and held its superheat at the set value in steady operation, in a study that names the valve's primary task as metering the evaporator's flow without hunting [12]. In 2002 a control system built on the minimum stable superheat was investigated experimentally on an evaporator [13]. A later controller with a single outlet temperature sensor used the variance of that temperature, which grows at low superheat, to drive the superheat low enough for close to optimal filling of the evaporator [14]. The stable line of Section 12.3 is a teaching version of that boundary, and a real one is found on its evaporator by experiment or from the outlet temperature's variance.

Models for control came alongside. A lumped moving-interface model of the evaporator and condenser predicted where the two-phase region ends, which the superheat depends on, and a relative gain analysis of its linearization found cross-couplings strong enough to indicate that independent single loops are inadequate [5]. A first-principles model of a transcritical air-conditioning system of 11 states was reduced to 5 by singular perturbation once its time scales had been separated [15].

The supermarket pack puts many evaporators on one rack. How its cases were found to fall into step, and how it became a benchmark for hybrid control, is told in the history box below.

> ⧖ HISTORY **2005 to 2008: Cases in Step Become a Benchmark**
>
> A doctoral thesis on the control of refrigeration systems, completed in 2005, set out the traditional supermarket pack: parallel compressors on one suction manifold, a suction pressure controller that switches compressors only outside a dead band, and on each display case a hysteresis controller that opens and closes the case's inlet valve [16]. It reported that in practice the cases tend to synchronize, which loads the central plant periodically and raises its energy use and wear. Hysteresis stayed in use because a thermostatic expansion valve rules out a continuous cascade, and because the off periods let frost melt, so defrosts are needed less often [16]. A study published in 2008 records that the system had become a benchmark problem of the European Network of Excellence on Hybrid Control [17].

Optimization followed. A steady-state model of a simple refrigeration system was used to find the suboptimal condenser pressure that minimizes energy consumption [18]. Nonlinear predictive control parameterized the on/off actions of the valves and compressors by the times at which they switch, so that the hybrid problem became a nonlinear program [7], predictive hybrid control was applied to the benchmark process itself [8], and an economic predictive controller used the thermal capacity of the goods to move the load against the daily outdoor temperature and electricity price and to offer ancillary services to a balancing market [9]. The floating pressures, the scheduler and the staggered defrost of Section 12.5 are rule-based counterparts of these optimizers, and each can be switched off to measure what it earns.

12.3 The Pack as Modeled

Figure 12.1 shows the pack. Four multideck cases, named dairy, meat, deli and produce, take the store's heat in through their air curtains, walls and lights and give it to a coil in the base. A thermostat on each case's air enables that case's electronic expansion valve, which a superheat loop opens and closes. The vapor from the four coils passes an accumulator into one suction side, a rack of two fixed-speed compressors (C1 and C2) and one variable-speed compressor (C3) pumps it to a roof condenser with three fans, and the liquid returns through a high side to the valves. Of the 62 integrated states, 24 belong to the cases (six each), 4 to the pack and 34 are ledgers that integrate every heat and refrigerant flow so that the balances can be audited. The signals are classified in Table 12.1. Time in the equations below is in seconds, and the engine, which counts minutes, scales each rate by 60 in a single place.

12.3.1 A Display Case

Each case is three lumped heat capacities and a refrigerant charge. The air, together with the case's own metal, exchanges heat with the store, the goods and the coil, as shown in Equation (12.1).

$$\begin{aligned} C_a \frac{\mathrm{d}T_a}{\mathrm{d}t} &= Q_{load} - UA_g(T_a - T_g) - Q_{aw} - Q_{sh} \\ C_g \frac{\mathrm{d}T_g}{\mathrm{d}t} &= UA_g(T_a - T_g) \\ C_w \frac{\mathrm{d}T_w}{\mathrm{d}t} &= Q_{aw} - Q_{tp} + Q_{def} - Q_{melt} \end{aligned} \tag{12.1}$$

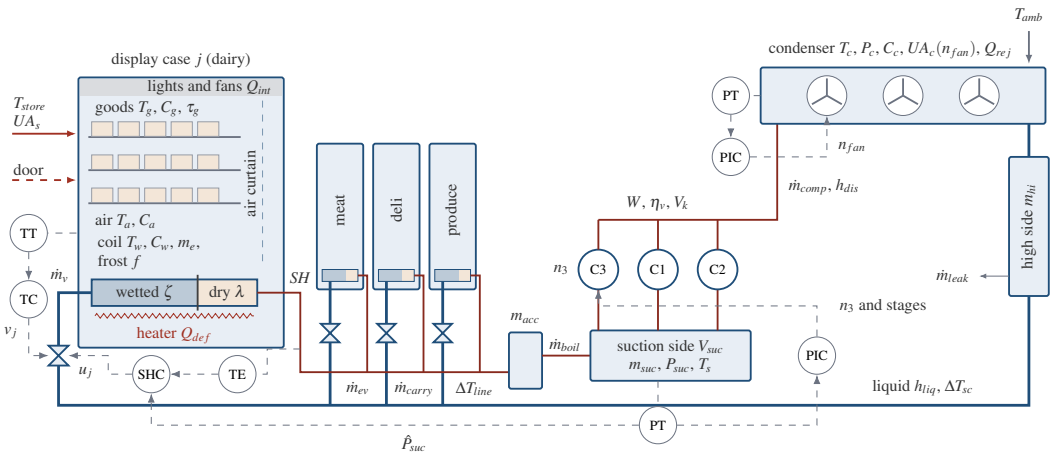


Figure 12.1 The pack, with the dairy case drawn in full as display case j and the meat, deli and produce cases as its copies on the same liquid and suction lines. The store heat enters through UA_s from T_{store}, the lights and fans add Q_{int} and an open door adds a conductance. The air T_a exchanges heat with the goods T_g and with the coil T_w, whose charge m_e wets a share ζ and leaves a dry length λ that sets the outlet superheat SH. A thermostat (TT, TC) gives each valve its call v_j and a superheat loop (TE, SHC, with the suction transmitter) its opening u_j. The coils' evaporation $\dot{m}_{ev}$ and any liquid carried over, $\dot{m}_{carry}$, reach the accumulator m_{acc} and the suction side m_{suc} at P_{suc}, where PIC sets C3's speed n_3 and stages C1 and C2. The compressors deliver $\dot{m}_{comp}$ at h_{dis} to the condenser T_c, whose fans n_{fan} PIC sets against T_{amb}, and the high side m_{hi} returns liquid at h_{liq} less any leak $\dot{m}_{leak}$. The liquid line is drawn heavier than the vapor lines, and the signal lines are dashed.

Table 12.1 Signals of the Pack by Role. Defaults Are the Reference Settings of Every Case

Symbol	Role	Default	Range
$T_{a,j}$	CV of TC-j, state	SP 2 °C, band 2 K	−25–10 °C, 0.5–6 K
v_j	MV of TC-j, valve call	thermostat	0, 1
SH_j	CV of SHC-j, state	SP 6 K	1–15 K
u_j	MV of SHC-j, valve opening	—	5–100 %, shut 0
T_s	CV of PIC-1, saturated suction	SP −10 °C, dead band 2 K	−30–0 °C
n_3, s_k	MVs of PIC-1, C3 speed and stages	—	30–100 %; on, off
T_c	CV of PIC-2, state	SP 40 °C	20–55 °C
n_{fan}	MV of PIC-2, fan speed	—	0–100 %
T_{amb}	DV, measured	22 °C ± 6 K, peak 15:00	±10 K, heat wave
UA_{door}	DV, a door held open	0	0.25 kW/K on one case
—	DV, restocking	—	40 % of goods at 10 °C
N_{out}	DV, condenser fans failed	0	0–3
$\dot{m}_{leak}$	DV, refrigerant leak	0	0–20 kg/h
—	DV, stuck valve, defrost that runs on	off	events
$T_{g,j}$	state, not measured	2 °C	—
$T_{w,j}$, $m_{e,j}$	states, coil and charge	equilibrium	—
f_j	state, frost	0	—
m_{suc}, m_{acc}	states, suction and accumulator	—	—
m_{hi}	state, high-side refrigerant	—	—
$\hat{T}_{a,j}$	measured air	lag 20 s	noise 0.05 K
$\widehat{SH}_j$	measured superheat	lag 6 s	noise 0.08 K
$\hat{P}_{suc}$, $\hat{P}_c$	measured pressures	no lag	noise 0.005 bar
R	monitor, thermostats' order parameter	—	0–1

where the case index j is dropped, $Q_{load} = (UA_s + UA_{door})(T_{store} - T_a) + Q_{int}$ is the store load, $UA_g = C_g/\tau_g = 0.194\,\text{kW/K}$ couples the goods to the air, $Q_{aw} = UA_{aw}(T_a - T_w)/(1 + f/f_{50})$ is the heat the coil takes from the air through a frost layer f, Q_{sh} is the heat that superheats the vapor, Q_{tp} is the boiling heat, Q_{def} is the defrost heater while it runs and $Q_{melt} = k_{melt} T_w \min(1, f/0.05\,\text{kg})$ melts

frost while $f > 0$ and $T_w > 0\,^{\circ}\mathrm{C}$. At 2 °C the store load is 4.91 kW a case, 19.64 kW for the four.

The coil's charge sets how much of it boils, as shown in Equation (12.2).

$$\begin{aligned} \frac{\mathrm{d}m_e}{\mathrm{d}t} &= \dot{m}_v - \dot{m}_{ev} - \dot{m}_{carry}, \qquad \zeta = \min(1, m_e/m_{max}) \\ Q_{tp} &= UA_{wr}\,\zeta'\,(T_w - T_s), \qquad \dot{m}_{ev} = Q_{tp}/\Delta h \\ \dot{m}_v &= u\,\dot{m}_{v,100}\sqrt{\frac{\rho_l(P_c - P_{suc})}{\rho_{ref}\Delta P_{ref}}}\,\min\left(1, \frac{m_{hi}}{m_{hi,min}}\right) \end{aligned} \tag{12.2}$$

where ζ is the wetted share of the coil, $\zeta' = \zeta$ when $T_w \geq T_s$ and $\max(\zeta, 0.3)$ when the coil is colder than saturation, so that a cold coil condenses vapor back and caps the suction pressure near the coldest coil while the rack is stopped, and $\Delta h = h_g(T_s) - h_{liq}$ while the coil boils and $h_g - h_f$ at T_s while it condenses. Liquid fed past a full coil leaves in about 2 s, $\dot{m}_{carry} = (m_e - m_{max})/2\,\mathrm{s}$ for $m_e > m_{max}$. In the valve law ρ_l is the liquid density at the valves, $\rho_{ref} = 1150\,\mathrm{kg/m^3}$ and $\Delta P_{ref} = 8\,\mathrm{bar}$, and the last factor starves the valves when a leak empties the high side. Frost grows at $r_f \min(1, \zeta/0.3)$ while $T_w < 0\,^{\circ}\mathrm{C}$ and $\zeta > 0.02$, and melts at Q_{melt}/L_f with $L_f = 334\,\mathrm{kJ/kg}$.

12.3.2 The Outlet Superheat and the Stable Line

The vapor leaving the wetted length is warmed by the case air over the dry length λ. Above the declared minimum stable superheat SH_{mss} that warming is the physics, and below it the superheat collapses G times faster with the dry length, which Equation (12.3) expresses.

$$SH^* = \begin{cases} \Delta T\left(1 - \mathrm{e}^{-a\lambda}\right), & \lambda \geq \lambda_{mss} \\ SH_{mss}\max\left(0,\ 1 - G\,\dfrac{\lambda_{mss} - \lambda}{\lambda_{mss}}\right), & \lambda < \lambda_{mss} \end{cases} \tag{12.3}$$

where $\Delta T = T_a - T_s$, $a = UA_{sh}/(\dot{m}_{ev}c_{pv})$, and $\lambda_{mss} = -\ln(1 - SH_{mss}/\Delta T)/a$ is the dry length at which the warming reaches the line, while a line at or above ΔT leaves the first branch in force. The superheat chases its target through a lag that also lengthens below the line, and the dry-out point wanders, as shown in Equation (12.4).

$$\begin{aligned} \tau_{sh}\left[1 + (D-1)\beta\right]\frac{\mathrm{d}SH}{\mathrm{d}t} &= SH^* - SH \\ \lambda &= 1 - \zeta + \sigma_\lambda\,(1 + A\beta)\,w\,\xi \\ w &= \min\left[1,\ (1-\zeta)/0.04\right] \end{aligned} \tag{12.4}$$

where $\beta = \min\{1, \max[0, (SH_{mss} - SH)/1\,\mathrm{K} + 0.5]\}$ blends from 0 above the line to 1 a kelvin under it, w removes the wander from a full coil, and ξ is a seeded noise of unit variance updated every 0.5 s as $\xi \leftarrow \alpha\xi + \sqrt{1-\alpha^2}\,\nu$, $\alpha = \mathrm{e}^{-0.5\,\mathrm{s}/\tau_\lambda}$, with ν a standard normal deviate. With $G = 8$, $D = 3$ and $A = 3$ the line is three declared factors on the gain, the lag and the wander. These are teaching values, a reduction of the empirical stability boundary of Section 12.2 rather than a measured evaporator, and a real line is found per evaporator by experiment [13] or on line [14].

12.3.3 Suction Side, Accumulator and Rack

The suction side is one volume whose vapor mass is a state, as shown in Equation (12.5).

$$\begin{aligned} \frac{\mathrm{d}m_{suc}}{\mathrm{d}t} &= \textstyle\sum_j \dot{m}_{ev,j} + \dot{m}_{boil} - \dot{m}_{comp} \\ \frac{\mathrm{d}m_{acc}}{\mathrm{d}t} &= \textstyle\sum_j \dot{m}_{carry,j} - \dot{m}_{boil} \\ \dot{m}_{boil} &= Q_{acc}\min\left(1, m_{acc}/0.05\,\mathrm{kg}\right)/(h_g - h_f) \end{aligned} \tag{12.5}$$

where the accumulator boils its liquid on $Q_{acc} = 1\,\mathrm{kW}$ of line heat, and the pressure is $P_{suc} = P(m_{suc}/V_{suc})$ from the vapor-density fit of Table 12.2, with $T_s = T_{sat}(P_{suc})$. The suction side is 250 L, a 50 L header with 200 L of suction lines. With the header alone the rack stops on its low-pressure switch 23 times in five hours of case 4.

The rack pumps and compresses the vapor as shown in Equation (12.6).

$$\begin{aligned} \dot{m}_{comp} &= \textstyle\sum_k \dot{m}_k = \sum_k s_k \eta_v V_k n_k \rho_1 \\ \eta_v &= 1 - c\left[(P_c/P_{suc})^{1/1.1} - 1\right] \\ W &= \textstyle\sum_k \dot{m}_k \Delta h_s / \eta_k \\ \Delta h_s &= c_s (T_1 + 273.15)\left[(P_c/P_{suc})^{n} - 1\right] \end{aligned} \tag{12.6}$$

where s_k is 1 while stage k runs, $n_k = 1$ for C1 and C2 and $n_3 \in [0.3, 1]$ for C3, $\rho_1 = \rho_{suc}(T_s + 273.15)/(T_1 + 273.15)$ is the inlet density at $T_1 = T_s + SH_1$, SH_1 is the coils' mixed outlet superheat plus $\Delta T_{line} = 4\,\mathrm{K}$ of line heat gain, η_v is clamped to $[0.05, 1]$, $\eta_k = \eta_{is}$ for C1 and C2 and $\eta_{is}[1 - 0.12(1 - n_3)^2]$ for C3, and the discharge leaves at $h_{dis} = h_1 + \Delta h_s/\eta_{is}$. Each displacement V_k is sized so that the compressor gives its rated cooling (10, 10 and 15 kW) at −10 °C suction, 10 K of total superheat, 40 °C condensing and 3 K of subcooling, which makes them 30.13, 30.13 and 45.20 $\mathrm{m^3/h}$ at full speed. The compressors start and stop at once.

12.3.4 Condenser, High Side and the Day

The condensing temperature and the high side's refrigerant follow Equation (12.7).

$$\begin{aligned} C_c \frac{\mathrm{d}T_c}{\mathrm{d}t} &= \dot{m}_{comp}(h_{dis} - h_{liq}) - UA_c(n_{fan})(T_c - T_{amb}) \\ \frac{\mathrm{d}m_{hi}}{\mathrm{d}t} &= \dot{m}_{comp} - \textstyle\sum_j \dot{m}_{v,j} - \dot{m}_{leak} \end{aligned} \tag{12.7}$$

where $UA_c(n) = UA_{c0} + (UA_c - UA_{c0})[n(N_f - N_{out})/N_f]^{0.8}$ for $N_f = 3$ fans with N_{out} failed, $P_c = P_{sat}(T_c)$, the liquid reaches the valves at $h_{liq} = h_f(T_c - \Delta T_{sc})$ with $\Delta T_{sc} = \min[\varphi_{sc}\max(0, T_c - T_{amb}), 5\,\mathrm{K}]$, and the fans draw $(N_f - N_{out})P_{fan}n^3$. The ambient is $T_{amb} = \bar{T} + A_T \sin[2\pi(t_{clock} - 9\,\mathrm{h})/24\,\mathrm{h}] + \delta$ with a heat wave adding 10 K over an hour, holding it and removing it over two. A high-pressure cut-out trips every compressor at $P_c \geq P_{sat}(56\,^\circ\mathrm{C}) = 15.30\,\mathrm{bar}$ and locks the rack out for 10 min and until P_c has fallen 2 bar, and a low-pressure switch stops it at $P_{suc} \leq P_{sat}(-25\,^\circ\mathrm{C}) = 1.06\,\mathrm{bar}$ for a minute and until $P_{suc} > P_{sat}(-22\,^\circ\mathrm{C})$. Both are teaching values. The application notes that real cut-outs are set below the system's maximum allowable pressure under EN 378-2 and the compressor maker's limits. Energy is the integral of W, the fan power and the defrost heaters, and the specific energy divides it by the integrated case load $\sum_j Q_{load,j}$.

> ✦ DID YOU KNOW? **An Industry Leak Rate of One Quarter**
>
> When a federal partnership with food retailers was launched in 2007, its agency developed an industry average leak rate of 25 % for supermarket refrigerant. The partners' average emissions rate was 12.9 % in 2021, about half of that estimate [19].

12.3.5 Refrigerant Properties and Instruments

Every property comes from a declared fit, listed in Table 12.2, and not from an equation of state. Against the seven-point R134a table that the application's checks carry, from −40 to 60 °C, the fits hold the saturation pressure within 0.32 %, the enthalpies within 0.18 kJ/kg and the vapor density within 0.28 %. Real properties come from the international standard formulation for R134a, a Helmholtz equation of state that represents the vapor pressure to about 0.02 % and the density to about 0.05 % [20]. R404A and R290 are offered as approximate alternatives, and no case uses them.

Table 12.2 Declared R134a Fits, With T in °C, P in bar, h in kJ/kg on the IIR Reference and ρ in kg/m^3. Each Column Is a Cubic $c_0 + c_1 x + c_2 x^2 + c_3 x^3$

x	h_f T	h_g T	ρ_f T	$\ln P$ $\ln \rho_g$
c_0	199.9849	398.617	1294.945	−1.700271
c_1	1.338574	0.5925403	−3.230513	0.9737556
c_2	1.453863×10^{-3}	-1.158092×10^{-3}	-8.782384×10^{-3}	0.04937264
c_3	1.089485×10^{-5}	-1.643745×10^{-5}	-8.253043×10^{-5}	−0.009170624

Saturation $\ln P_{sat} = A - B/(T + C)$ with $A = 9.939613$, $B = 2163.556$ and $C = 244.0$. Vapor heat capacity $c_{pv} = 0.897 + 0.00436\,T_s$ kJ/(kg K). Isentrope $c_s = 0.604$ kJ/(kg K) and $n = 0.1185$. The vapor density $\rho_g(P)$ inverts the $\ln P$ cubic by Newton's method.

Table 12.3 Constants of the Model. Sliders of the Show Model Dialog Are V, Hidden Parameters H and Code Constants C

Symbol	Meaning	Value	Class
Display cases (each)			
C_a, C_g, τ_g	air and metal, goods, goods lag	320, 1400 kJ/K, 120 min	V
UA_s, Q_{int}, T_{store}	store load, lights and fans, store	0.24 kW/K, 0.35 kW, 21 °C	V
UA_{aw}, UA_{wr}, UA_{sh}	air to coil, coil to boiling, air to vapor	1.4, 1.2, 0.21 kW/K	V
C_w, m_{max}	coil metal, charge of a full coil	25 kJ/K, 1.5 kg	V
$\dot{m}_{v,100}$	valve capacity at 8 bar	0.10 kg/s	V
τ_{sh}, SH_{mss}	superheat lag, stable line	18 s, 3 K	V
G, D, A	steepening, lag and wander factors	8, 3, 3	V
σ_λ, τ_λ	wander, its correlation time	0.012, 15 s	V, H
r_f, f_{50}	frost rate, frost halving UA_{aw}	0.5 kg/h, 5 kg	V
Q_{def}, k_{melt}	defrost heater, melting	2 kW, 0.8 kW/K	V, H
Suction side and rack			
V_{suc}	suction side volume	250 L	V
Q_{c1}, Q_{c2}, Q_{c3}	rated cooling of C1, C2, C3	10, 10, 15 kW	V
n_{min}, η_{is}, c	C3 least speed, efficiency, clearance	0.30, 0.65, 0.05	V, V, H
ΔT_{line}, Q_{acc}	line heat gain, accumulator heat	4 K, 1 kW	V, H
T_{LP}	low-pressure cut-out, saturated	−25 °C	H
Condenser and high side			
UA_c, UA_{c0}, C_c	full fans, fans stopped, capacity	2.7, 0.35 kW/K, 60 kJ/K	V
N_f, P_{fan}	fans, power of one at full speed	3, 0.55 kW	H, V
φ_{sc}, $\Delta T_{sc,max}$	subcooling share, largest	0.35, 5 K	V, H
m_{ch}, $m_{hi,min}$	liquid charge, flashing threshold	30, 6 kg	V, H
T_{HP}, t_{lock}	high-pressure cut-out, lockout	56 °C, 10 min	V
Weather			
$\bar{T}$, A_T	daily mean ambient, half range	22 °C, 6 K	V
Instruments			
—	air, outlet sensor lags and noise	20, 6 s; 0.05, 0.08 K	H
—	pressure transmitter noise	0.005 bar	H
Code constants			
UA_{door}, L_f	open door, latent heat of frost	0.25 kW/K, 334 kJ/kg	C
—	carry-over time, condensing floor	2 s, 0.3	C

The controller reads only its instruments. The case air sensors lag 20 s with 0.05 K of noise, the coil outlet sensors lag 6 s with 0.08 K, the two pressure transmitters carry 0.005 bar of noise and the ambient sensor 0.1 K, while the defrost termination reads the coil temperature directly. The goods are never measured.

12.3.6 Starting State and Numerics

Cases 1 and 2 start every coil at equilibrium, the air at the temperature where the store load equals what the coil takes at the setpoint superheat, which in case 2 is 2.14 °C with the valves at 32.31 % and the coils 82.1 % wet at −6 °C suction. The other cases place each thermostat at a phase of its cycle, seeded or pinned, on a triangle that cools from cut-in to cut-out over the first 60 % of the cycle. The condenser starts at the larger of its setpoint and the ambient plus 8 K, the rack with the stages and C3 speed that carry the cases cooling at $t = 0$, and the fans with the speed that holds that condensing temperature. The suction side starts with the saturated vapor density of its setpoint and the high side with the 30 kg charge less what the coils hold. The fourth-order Runge-Kutta method integrates the 62 states on a 0.5 s step, the controller scans every 2 s, the historian logs every 5 s, and

the wander has its own 0.5 s clock so that a finer step replays the same realization (Section 1.6.1). The seed is 20261010. Over the challenge day of Section 12.6 with the suggested settings, the charge and every case's energy close to 4.9×10^{-15} of their throughput. Halving the step changes the day's 183.51 kWh by less than one part in 10^7 and leaves its 108 compressor starts unchanged, because the controller scans on its own 2 s clock whatever the step, so every thermostat and stage switches on the same scan. The application passes its 36 checks.

12.3.7 Assumptions

Lumped cases. A case is three heat capacities and a charge, with no air curtain dynamics, no temperature profile along a shelf and no product core, so the goods follow the air through one two-hour lag. Frost is laid at a declared rate with no latent heat and no humidity balance, so only its melting appears in the coil's energy.

A declared stable line. The superheat is one exponential over one dry length, and its behavior below the line is three declared factors rather than a two-phase flow simulation. A real line is measured on its evaporator.

One suction volume. The suction side is one pressure, its vapor's density is the saturated value at that pressure, and there is no pressure drop between the cases and the rack.

Ideal machines. The compressors are displacements with a clearance efficiency and a fixed isentropic efficiency, with no oil return, motor heating or starting transient, and the condenser and receiver are one saturation temperature with a lumped capacity.

Fitted properties and teaching thresholds. The refrigerant comes from fits good to a fraction of a percent, and every threshold, from the 3 K line and the 56 °C cut-out to the goods band of 1 to 3 °C and the challenge's limits, is a teaching value set on this model.

12.4 What the Line and the Shared Pressure Imply

12.4.1 A Steep Stretch That Moves

Figure 12.2(a) draws the dairy coil's steady superheat in case 1 against the valve opening, holding the case air, the suction and the frost at their values in the first and fifth steps of the case's manual sequence. Above the line the superheat falls 0.65 K for each percent of opening at the first step's air of 2.36 °C, and past the line it loses the remaining 3 K within 0.17 points of opening, 18 K per percent. That steep stretch is the stable line seen from the valve. Its position is not a property of the valve, because the opening at which a fixed valve meets the line moves from 35.63 % to 33.87 % between the two steps as the air cools to 2.08 °C and 0.20 kg of frost builds. A setpoint chosen as an opening would cross the line by itself.

An open-loop step of the valves from 31.5 to 32.5 % on case 2's conditions, in a model run with every noise off and the case's scripted setpoint steps removed, gives a process gain of −1.03 K per percent on the true outlet superheat, and a two-point fit at 28.3 and 63.2 % of the change gives a lag of about 143 s and 7 s of apparent dead time. The model holds no transport delay, so that dead time is the coil's charge and the 18 s superheat lag acting in series. The superheat loop's gain of 1.2 %/K therefore makes a loop gain of 1.24 at 6 K, and with the noise still off the loop steps from 6 to 8 K to reach 7.8 K after 186 s. Below the line the same opening moves the superheat more than ten times as far, three times as slowly and with three times the wander, so the identical proportional-integral law sees a different plant. The setpoint chooses the plant in the same way in Volume 7, Chapter 3. At a 2 K setpoint it settles into the limit cycle of Figure 12.2(b), whose troughs under 1.5 K come 2.1 to 5.1 min apart.

12.4.2 One Pressure for Four Cases

Every case boils at the saturation temperature of one pressure, so its boiling heat $UA_{wr}\zeta(T_w - T_s)$ falls whenever another case raises T_s. Over the first three hours of case 4, in the 30 seconds after a cut-in with no other valve or stage switching from 10 seconds before to 30 seconds after, the suction

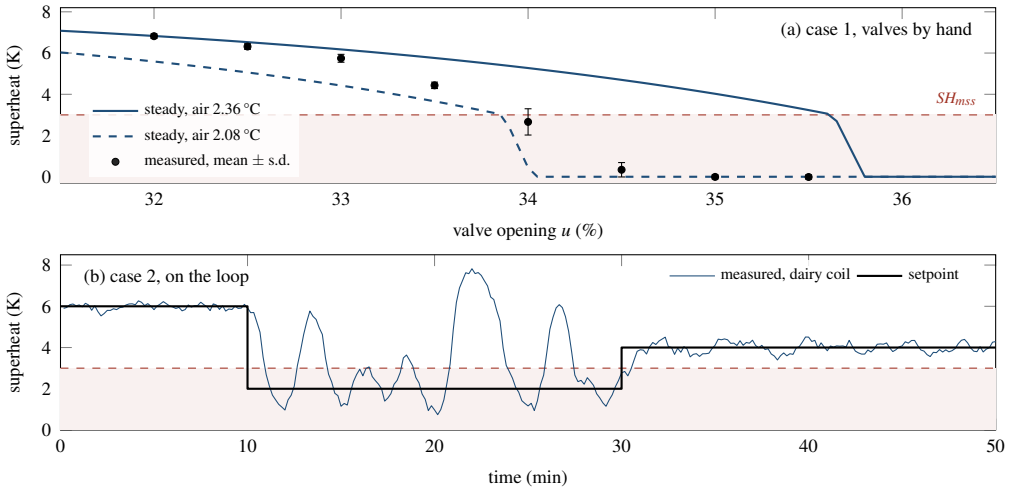


Figure 12.2 The superheat and its stable line, from the engine. (a) The dairy coil's steady superheat in case 1 against the valve opening, at the air and frost of the case's first and fifth steps, with the measured superheat of each six-minute step as a mean and standard deviation over its last four minutes. (b) The dairy coil's measured superheat in case 2, on its loop, as the setpoint steps from 6 to 2 K at 10 min and to 4 K at 30 min. The shaded band lies below the declared 3 K line.

rose a median 1.02 K (11 events) and the cases already cooling lost a median 11.0 % of their boiling heat. A cut-out lowered the suction 0.72 K. The rack's staging answers in the opposite sense and by more, since a compressor start lowered the suction a median 2.38 K within 20 seconds (32 events) and lifted the cooling cases' boiling heat 14.3 %, and a stop raised it 1.86 K and cut that heat by 14.4 %. A case ahead in its cycle is slowed by the cases that open after it, and the timing of every stage change is set by the cases together. Over many cycles the phases draw together.

🕮 **Pause.** Try Exercise 12.1 before reading on.

The pull is strong, and what carries it matters less than where the cases start. From the same seeded phases the four cases are in step after 65.2 min on 250 L, 53.1 on 1000 L and 52.6 on 5000 L, each volume set before the run. Halving or doubling the suction loop's gain gives 56.2 and 62.6 min and its largest setting of 60 %/K gives 78.9 min, and integral times of 15, 60 and 120 s give 67.2, 77.0 and 54.7 min. A 4 K dead band locks them sooner, at 47.9 min, with the measured suction spanning −14.0 to −3.3 °C between its 1st and 99th percentiles from the first half hour to the third hour against −12.1 to −4.2 °C with the shipped 2 K. On five other seeds the cases are in step after 31.9 to 94.0 min, 2.8 to 8.1 of their cycles, a wider spread than any of these settings gives, and on an infinite suction side of 1×10^{12} L, a pressure no case can move that only the application's checks reach because the parameter stops at 5000 L, none of the five ever synchronizes. Two cases alone, the dairy and produce cases with the other two thermostats in Manual and their valves shut, close a gap of 0.39 of a cycle to 0.10 by their fifth cut-in on the shared side and stay within 0.10 after the first hour, while on the infinite side their gap stays between 0.40 and 0.50 for four hours.

12.4.3 What the Condenser Can Carry

The four cases take 19.64 kW from a 21 °C store. Rejecting that load and the work needed to pump it into air at 38 °C needs a condensing temperature of 48.7 °C with all three fans, 52.5 °C with one out and 62.6 °C with two out, against the 56 °C cut-out, so with two fans out no controller holds both the goods and the rack. The cost of head pressure is steep as well. C1 alone at −10 °C suction, with 6 K of superheat at the coil outlet and 3 K of subcooling, has a cycle coefficient of performance of 4.31 at 25 °C condensing, 2.70 at 40 °C and 1.80 at 55 °C, which is what floating the head pressure on the

Table 12.4 Loops and Supervisory Settings With Their Defaults

Loop	CV → MV	Setting
TC-j	$\hat{T}_{a,j} \to v_j$	on above 3 °C, off below 1 °C (SP ± band/2)
SHC-j	$\widetilde{SH}_j \to u_j$	PI, $K_c = 1.2\,\%/\mathrm{K}$, $\tau_I = 60\,\mathrm{s}$, 5–100 %, $a = -1$
	protection	below 1 K, closes at 30 %/min
PIC-1	$\hat{T}_s \to n_3$	PI, $K_c = 25\,\%/\mathrm{K}$, $\tau_I = 30\,\mathrm{s}$, 30–100 %, $a = -1$
	staging	dead band 2 K, requests held 15 s, 0.5 min between changes
	timers	C1, C2 run 2 min, rest 5 min; C3 run and rest 1 min
PIC-2	$\hat{T}_c \to n_{fan}$	PI, $K_c = 3\,\%/\mathrm{K}$, $\tau_I = 60\,\mathrm{s}$, 0–100 %, $a = -1$
FY-1	floating suction	off; +0.05, −0.25 K/min, duty limit 85 %, −14 to −4 °C
	floating head	off; $\max(25\,^\circ\mathrm{C}, \hat{T}_{amb} + 8\,\mathrm{K})$
KY-1	desynchronization	none; offsets 1.0, 0.8, 1.25, 0.65; scheduler gap 2.5 min
	defrost	staggered, every 6 h from 02:00, 20 min or 10 °C, drip 3 min

ambient buys back. In Volume 9, Chapter 8 the cheapest feasible operating point of a cooling plant likewise moves with the weather.

12.5 Thermostats, Valves, Rack and Fans as Programmed

The controller scans every 2 s and reads only the instruments of Section 12.3. Its loops and their settings are listed in Table 12.4, and every loop has its own Manual and Auto switch, while the header's switch throws all of them at once.

12.5.1 The Shared Controller

The superheat, suction and condenser loops run the shared velocity-form law of Section 1.6.2 with no derivative term and no output bias, so Equations (1.14) to (1.17) reduce to Equation (12.8).

$$
\begin{aligned}
&e_k = a(\mathrm{SP}_k - \mathrm{PV}_k), \qquad P_k = K_c e_k \\
&S_k = S_{k-1} + P_k - P_{k-1} \\
&v_k = S_k + I_{k-1}, \qquad u_k = \min[u_{max}, \max(u_{min}, v_k)] \\
&I_k = I_{k-1} + \left[\frac{K_c}{\tau_I} e_k + \frac{u_k - v_k}{T_t}\right]\Delta t_c
\end{aligned}
\tag{12.8}
$$

where $a = \pm 1$ is the action, $\Delta t_c = 2\,\mathrm{s}$, the integral I carries the output's starting value and the tracking time constant T_t equals τ_I. In Manual the integral is solved each scan as $I_k = u_{op} - S_k$, so the return to Auto is bumpless. No loop carries a rate limit. All three loops have $a = -1$, so a measurement above its setpoint raises the output, which the application's direction checks confirm. A superheat above its setpoint opens the valve, a suction above its setpoint speeds C3 up and a condensing pressure above its setpoint speeds the fans, which is direct action in the ISA sense although comments in the app's source call the loops reverse acting.

12.5.2 Cases and Valves

Each thermostat is the switch with hysteresis of Section 1.4.7. It opens its valve call when its measured air passes the cut-in, the setpoint plus half the band, and closes it below the cut-out, half a band under the setpoint, and a defrost holds it closed. While a case calls, its superheat loop sets the opening, and with the superheat loops in Manual every calling valve takes the operator's opening, which the arrow keys move in quarter percents. Between calls the valve is shut and the loop tracks the opening it closed at, so each restart begins where the last call ended. Below a measured superheat of 1 K a low-superheat protection overrides the loop and closes the valve at 30 %/min, with the loop tracking it. A stuck valve stays where it stuck whatever it is told.

12.5.3 Suction and Staging

C3 trims the suction on its speed, and C1 and C2, two integer actuators in the sense of Section 1.4.8, are staged by the framework's stage sequencer, shared/src/framework/core/sequencer.js. A request for

one more stage is made when the measured suction has been above the top of the 2 K dead band for 15 s with C3 off or at full speed, and a request for one fewer when it has been below the bottom for 15 s with C3 off, fixed or at its least speed. The sequencer starts the free stage of lowest priority, C3 first and then whichever of C1 and C2 has been off longest, stops the running stage of highest priority that has run its minimum, holds 0.5 min between changes unless the suction is a full dead band low, and counts every start inside a trailing hour. The minimum run and rest are 2 and 5 min for C1 and C2 and 1 min each for C3, and C3's loop restarts at its least speed each time C3 starts or stops. In Manual the operator sets C3's speed, which stops it at zero, and the number of fixed compressors. With head-pressure unloading on, the rack adds no stage and caps C3's speed in proportion once the condensing pressure is within 1.5 bar of the cut-out, and sheds a stage within 0.5 bar of it. The cap is applied after the output of Equation (12.8), outside the loop's own limits, so the loop's integral is not told of it and can wind up toward 100 % speed while the cap holds.

> ✦ DID YOU KNOW? **Timers That Move the Cycling**
>
> From the simulator: minimum run and rest times on C1 and C2 cut the rack's starts in three hours of case 3 from 97 to 59, but the variable-speed C3 then makes 20 of them, the suction swings from −15.0 to −1.7 °C instead of −11.9 to −7.7 °C, and R averages 0.37 (Table 12.6).

12.5.4 Condenser and Supervisor

The fans hold the condensing temperature at 40 °C, or with floating head pressure at the measured ambient plus 8 K, never under 25 °C, and in Manual the operator sets their speed. Floating suction averages each case's valve-open share over 30 min as a first-order lag, raises the suction setpoint 0.05 K/min while every share is under 85 %, and lowers it 0.25 K/min as soon as one passes 90 % or a case's air is a kelvin past its cut-in, between −14 and −4 °C, so the case with the least room sets the pressure for all four. Desynchronization by offsets scales the four bands by 1.0, 0.8, 1.25 and 0.65, so the thermostats cycle at four periods. The scheduler holds a case that reaches its cut-in until the last opening is 2.5 min old or the case is 0.6 K past its cut-in, and releases the warmest idle case early once its air is 0.2 K above its setpoint, nobody is waiting and the last opening is 3.75 min old. Defrosts start on a clock, staggered in four slots of an interval or all at once, heat the coil at 2 kW until it reaches 10 °C or 20 min have passed, and drip for 3 min, while a defrost whose termination fails runs to a 60 min failsafe.

12.5.5 Monitors and the Models Kept Apart

Two monitors read the controller's own signals. The hunting detector records the measured superheat's turning points with 1 K of hysteresis and flags a coil that holds, in the last 12 min, at least five swings of 2 K or more and three troughs at least 0.5 K under the line, ignoring the first 3 min after each opening. The synchrony monitor gives each case the phase $\varphi_j = 2\pi(t - t_{in,j})/P_j$ since its last cut-in over its last period and computes the order parameter of Equation (12.9).

$$R = \frac{1}{N}\left|\sum_{j=1}^{N} \mathrm{e}^{i\varphi_j}\right| \tag{12.9}$$

where R is 1 when the cases cut in together and near 0 when they are spread, and the status reads SYNCHRONIZED once R has stayed above 0.9 for 20 min. Equations (12.1) to (12.7) are the simulation model. The identified model of the superheat at 6 K is the first-order lag with dead time of Section 12.4, and the controller holds no model of the plant, since the floating suction reads duty as a proxy for room and the scheduler needs only the openings.

12.6 Measured Cases, From a Hand on the Valve to a Hot Day

Table 12.5 lists the cases and Table 12.6 their measured results. Each starts from the reference settings of Tables 12.1 and 12.4 and changes only what its row lists, and among the original cases only case 6

Table 12.5 The Cases, Each Changing the Reference Settings as Listed

#	Change from the reference, duration	Teaching point
1	cases held calling, valves by hand from 32 % up 0.5 % every 6 min, suction −6 °C, 48 min	the line by hand
2	as 1 with the superheat loops on, SP 6, 2 and 4 K, 50 min	hunting below the line
3	staging alone, C3 fixed, no timers, scheduler, 3 h	starts against timers
4	free-running thermostats from seeded phases, 3 h	synchrony
5	started in step, offset differentials, 3 h	desynchronization
6	2 kg of frost, all four defrosted at 01:00, scheduler, 4 h	a scheduled peak
7	floating suction from −12 °C, meat case at 1 °C, scheduler, 6 h	the case that binds
8	13:00 start, two fans out, heat wave at 14:00, floating head, unloading, scheduler, 6 h	an infeasible afternoon
9	meat case restocked at 15 min, scheduler, 5 h	goods the air hides
10	predictive case admission, 3 h	restricted hybrid scheduling
11	a summer day, see Table 12.7	graded challenge

Table 12.6 Measured Results of the Cases and Their Variants

Case and variant	Result
1, 32 to 33.5 %	(6.82 ± 0.11) K to (4.43 ± 0.16) K, coils 77 to 89 % wet
1, 34, 34.5, 35 %	(2.66 ± 0.64) K, (0.34 ± 0.35) K, coil full and 0.18 kg of liquid by 48 min
2, SP 6, 2, 4 K	(5.98 ± 0.16) K; 0.67 to 7.84 K, hunting from 18.9 min; (3.98 ± 0.25) K
3, no timers	97 starts, suction −11.9 to −7.7 °C
3, C1, C2 timed 2 and 5 min	59 starts (C3 20), −15.0 to −1.7 °C, *R* 0.37 mean
3, timed, C3 variable	no start, −10.7 to −9.4 °C
4, free running	in step at 65.2 min, 14 starts in the first hour, 30 in the next two
4, infinite suction side	never in step, *R* averaging 0.45 from hour 1 to 4, no restaging
5, offsets, scheduler, none	24, 8 and 45 starts; *R* 0.33, 0.07 and 1.00 at 3 h
6, all at once, staggered	56.2 kW and 46.4 kW rejected at peak, all four cooling 17.5 and 2.8 min after 01:00
7, floating, fixed	43.21 and 45.42 kWh, 27 and 58 starts, setpoint −9.39 °C mean
8, unloading on, off	no trip, goods out 205 to 208 min; 26 trips, goods to 14.87 °C
9, restocked	goods 5.22 °C, out of band 147 min, air at most 3.58 °C

and the summer day defrost. No interface takes part in the results below, which run the engine from its default seed over each case's own duration unless another span is named. A setting that a variant changes is made at the case's start, before the run, as a learner makes it, and the application gives the same run whether it is made before the first Run or carried through a Reset (Section 1.6.1). A start-up parameter such as the suction side volume, changed during a run, waits for the next Reset. Where the noise matters, the spread over five other seeds is given beside the default seed's value.

12.6.1 The Line by Hand

Case 1 holds all four cases calling with their thermostats in Manual and opens the four valves by half a percent every six minutes from 32 %, with the suction held at −6 °C. From 32 to 33.5 % the superheat falls from 6.82 to 4.43 K and stays within ±0.11 to ±0.19 K, while the coils go from 77 to 89 % wet. The next half percent takes it under the line, to a mean of 2.66 K that wanders between 1.52 and 4.00 K, at 34.5 % it sits at 0.34 K with the coils 98 % wet, and at 35 % they are full and liquid starts to reach the accumulator, 0.18 kg of it by 48 min. The case has no loop, so it floods rather than hunts. The measured points of Figure 12.2(a) follow the steady curve for the air of each step until the line. At 34 % the mean stays above the curve's 0.44 K because the coil is still filling when the step ends, and with the wander and the sensor noise switched off its true superheat is still falling, from 4.32 K at 24 min to 0.83 K at 30 min.

12.6.2 The Loop That Hunts

Case 2 puts the same coils on their superheat loops at 6 K, where the superheat holds (5.98 ± 0.16) K with the valves between 31.7 and 32.6 %. At 10 min the setpoint drops to 2 K, a kelvin under the line. The superheat never settles. It swings between 0.67 and 7.84 K, averaging (3.27 ± 1.98) K, the valves travel from 17.5 to 38.0 %, the low-superheat protection acts for 6 % of the time from 12 to 30 min, and the loop is flagged hunting from 18.9 min. Raised to 4 K at 30 min, a kelvin above the line, the

superheat holds (3.98 ± 0.25) K, as shown in Figure 12.2(b). The cure is margin above the line, not a smaller gain.

Pause. Try Exercise 12.2 before reading on.

12.6.3 Starts Against Timers

Case 3 holds the suction at −10 °C by staging alone, with C3 at fixed speed, a 2 K dead band, no timers and the scheduler keeping the cases apart. The rack makes 97 starts in three hours, 48 of C1 and 49 of C2, and after the first half hour the suction stays between −11.9 and −7.7 °C at its 1st and 99th percentiles. Minimum run and rest times of 2 and 5 min on C1 and C2 cut the starts to 59, but C3 takes up the cycling with 20 of them, the suction swings from −15.0 to −1.7 °C and lies outside its dead band 49 % of the time, and the swings begin to pull the thermostats toward step, R averaging 0.37 after the first half hour against 0.09 untimed. Five other seeds give 97 to 104 starts untimed and 54 to 67 timed. Giving C3 its speed back removes every start on the default seed, leaves 2 to 7 on the other five, and holds the suction between −10.7 and −9.4 °C.

12.6.4 Synchrony and Its Cures

Case 4 runs the four thermostats free from the seeded phases 0.59, 0.67, 0.38 and 0.08 of a cycle. R climbs from 0.59 at 20 min to 1.00 at an hour, and the cases are in step from 65.2 min, 5.7 cycles of 11.4 min, as the screenshot of Figure 12.3 shows at two hours. In step, the rack makes 14 starts in the first hour and 30 in the next two, and the suction swings from −12.4 to −3.0 °C in the last hour against −12.2 to −7.5 °C in the first half hour. On the infinite suction side R averages 0.45 from the first hour to the fourth and the rack never restages.

> **TRY IT IN THE APP** **4 · Four Thermostats Fall Into Step**
>
> Load the scenario and press Run. Watch the four hands of the thermostat dial on the Store view draw together. R climbs from 0.59 at 20 min to 1.00 at an hour, the cases are in step from 65.2 min, and the status line then reads SYNCHRONIZED. On the trends the four case air temperatures merge into one trace.

Case 5 starts the cases a hundredth of a cycle apart. With offset differentials R falls from 1 to 0.30 in 30 min and averages 0.40 from the first hour to the third, and the rack makes 24 starts against 45 free running, but the four periods let the phases slide back through one another and R is back to 0.33 at three hours. The scheduler spreads them and keeps them spread, R 0.42 at 30 min, 0.16 at an hour, 0.12 at two and 0.07 at three, with 8 starts in three hours and 3 and 2 early releases of the meat and produce cases. Five other seeds give 22 to 28, 6 to 9 and 45 starts with offsets, the scheduler and neither. The scheduler costs the warmest air 0.47 K, a peak of 4.09 °C against 3.61 °C free running.

> **TRY IT IN THE APP** **5 · Desynchronization: Offsets and a Scheduler**
>
> Load the scenario and press Run. With Desynchronization on Offsets, R falls from 1 to 0.30 in 30 min, the rack makes 24 starts in three hours, and R is back to 0.33 at the end. Press Reset, set Desynchronization to Scheduler on the KY-1 card before pressing Run, and run again: 8 starts, and R 0.07 at three hours.

12.6.5 A Defrost Is a Scheduled Load

Case 6 gives each display case 2 kg of frost, keeps the cases apart with the scheduler and defrosts all four at 01:00. Each heater melts its frost, 670 kJ of latent heat, and warms the coil to 10 °C in 7.2 to 9.0 min, ending by temperature, and a 3 min drip follows. From 01:00 to the end of the four hours all four cool at once for 17.5 min, against 2.8 min with the defrosts staggered 90 minutes apart, and five other seeds give 16.0 to 20.0 against 1.1 to 4.0 min. Both runs bring in all three compressors, but all at once the condenser rejects 56.2 kW at its peak against 46.4 kW, the electric power peaks at 15.4 kW against 13.5 kW, and the condensing temperature reaches 49.8 °C against 44.3 °C. The

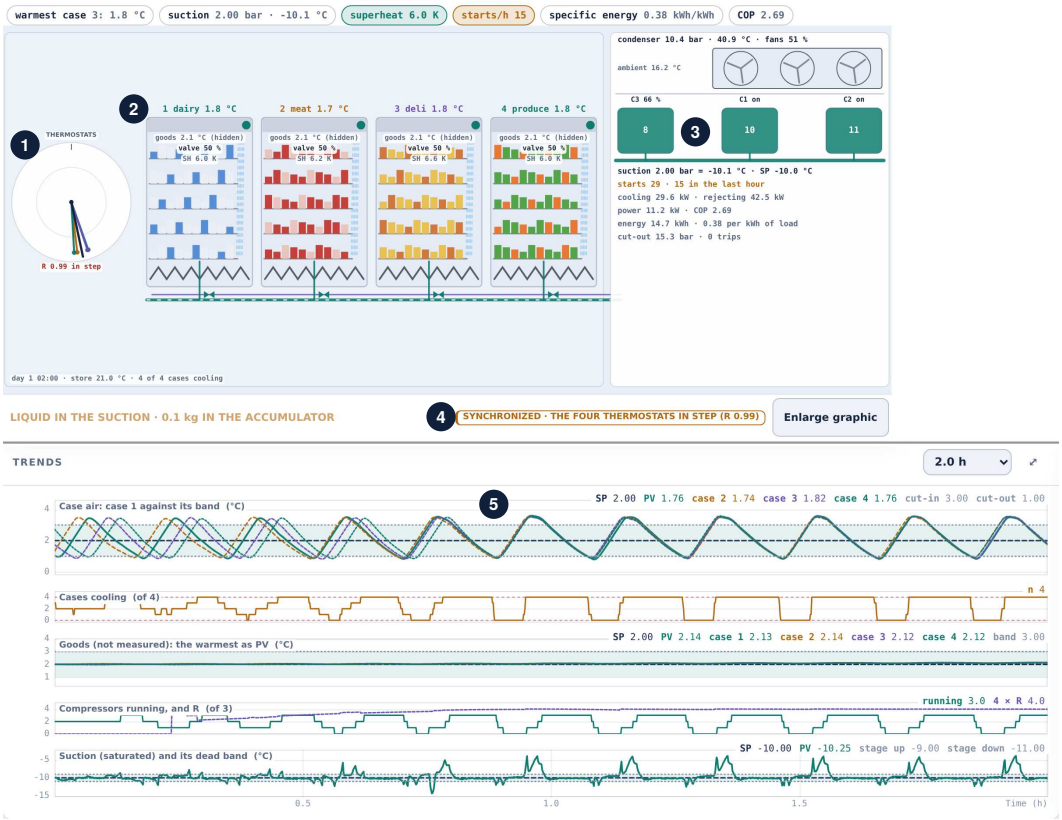


Figure 12.3 Case 4, Four thermostats fall into step, at $t = 2.00$ h in the Store view with the 2 h trend window and no control changed, a composite of the app's plant view with its status line and its trend strips; the header, the control panel and the Learn panel are cut. (4) The status line reads SYNCHRONIZED with the four thermostats in step at R 0.99, and (1) the thermostat dial draws the four hands together. (2) All four cases cool, their air at 1.8, 1.7, 1.8 and 1.8 °C, every valve at 50 % and measured superheats of 6.0 to 6.6 K, with the hidden goods at 2.1 °C. (3) The rack runs C3 at 66 % with C1 and C2, after 8, 10 and 11 starts, 29 in all and 15 in the last hour, holding the suction at 2.00 bar, −10.1 °C, and the condenser at 10.4 bar, 40.9 °C, with the fans at 51 % against a 16.2 °C ambient. The coils take 29.6 kW, the condenser rejects 42.5 kW and the pack draws 11.2 kW of electricity, the COP over the compressors' power is 2.69, and the run so far has used 0.38 kWh per kWh of case load. (5) In the trends the four case air temperatures cycle apart in the first half hour and as one trace by the second hour, the cases cooling step between none and four, the compressors running follow them as four times R climbs to 4.0, and the suction swings each time the rack restages.

defrost brings the cases back in step, and the scheduler spreads them again, R averaging 0.22 from 01:40 to 02:40 against 0.15 staggered.

12.6.6 The Case That Binds a Floating Suction

Case 7 sets the meat case at 1 °C and the other three at 2 °C on a cautious fixed −12 °C, with floating suction on. The meat case binds from the first hour to the last, reaching a duty of 78 % at six hours while the others sit at 66 to 72 %, and the setpoint floats between −10.81 and −8.12 °C, averaging −9.39 °C from the first hour to the sixth. The six hours cost 43.21 kWh against 45.42 kWh fixed, 4.9 % less, the compressors' share 4.9 % less for the 2.6 K gained, and the compressors start 27 times

instead of 58. Five other seeds save 5.4 to 6.3 % with 19 to 25 starts against 66 to 72. The coldest case taxes every case on the line.

12.6.7 An Afternoon No Setting Saves

Case 8 starts at 13:00 with two of the three fans out of service, and at 14:00 a heat wave lifts the ambient 10 K over an hour, to 38 °C at 15:00, holding until 18:00. Section 12.4 showed that the load needs 62.6 °C of condensing temperature with two fans out against the 56 °C cut-out. With head-pressure unloading the rack never trips. It holds the condensing temperature to 54.7 °C by refusing capacity for 292 min, and from 15:32 every case's goods leave their band, for 205 to 208 min, reaching 5.13 °C. Without unloading the rack trips 26 times, the first at 14:24 when 4 K of the heat wave has arrived, and the goods reach 14.87 °C. Five other seeds trip 31 to 33 times, the first between 13:01 and 13:16 before the heat wave begins, so the time of the first trip depends on the noise while the outcome does not. With one fan repaired at the start unloading holds every case in its band, and with both repaired it is never needed. Unloading chooses warm goods over a stopped rack, and only a repaired fan avoids both.

12.6.8 Goods the Thermostat Cannot See

Case 9 replaces two fifths of the meat case's goods with goods at 10 °C at 15 min. The goods reach 5.22 °C and take 147 min to return under 3 °C, 140 to 145 min on five other seeds, while the air never leaves its ordinary cycling, reaching at most 3.58 °C after the restock, and the thermostat's only sign is a half-hour duty that rises from 59 % just before the restock to 74 % 50 minutes after it. A thermostat on the air holds the air, and no tuning of it holds a goods band.

🕮 **Pause.** Try Exercise 12.3 before reading on.

12.6.9 The Summer Day

The challenge runs a day on fixed suction (−10 °C) and condensing (40 °C) setpoints with free-running thermostats, a 2 K superheat setpoint and all four cases defrosted at once at 04:00, 10:00, 16:00 and 22:00. A heat wave arrives at 14:00 and the meat case is restocked at 16:00. The target asks for no high-pressure trip, at most 150 compressor starts, every case's goods in their band for all but 240 min, at most 20 min of hunting and 90 min with liquid in the accumulator, and at most 0.41 kWh per kWh of case load. The day is judged as given, so a changed model parameter or disturbance setting, or an extra disturbance button, voids the attempt. Table 12.7 compares the configurations, and Figure 12.4 draws three of its columns against the limits. As shipped the rack trips 9 times, the first at 16:11, just after the 16:00 defrost and two hours into the heat wave, makes 383 starts with the cases synchronized for 848 min, leaves the worst goods out of their band for 473 min at up to 9.82 °C, and floods the accumulator for 877 min. The suggested configuration, floating suction and head pressure, the scheduler, a staggered defrost and a 6 K superheat setpoint, passes with no trip, 108 starts and 188 min out of band, all of them in the restocked case. On five other seeds the shipped day also trips 9 times, with 385 to 391 starts, and the suggested one passes with 100 to 113 starts. Undoing the scheduler, the staggered defrost or the 6 K setpoint fails the target, and so does fixing both pressures, while either float alone is enough. With the defrost all at once the rack trips 11 times, with the thermostats free running it makes 178 starts, at 2 K the loops hunt for 340 min, and with both pressures fixed the day costs 0.417 kWh per kWh, against 0.405 with only the suction fixed and 0.407 with only the head pressure fixed, and offset differentials in place of the scheduler also pass, with 139 starts. Removing the trip alone does not rescue the shipped day, which still makes 354 starts staggered at 6 K, while head-pressure unloading can stand in for the staggering.

12.6.10 An Eight-Minute Hybrid Scheduler

The Desynchronization control also offers an MPC choice, which addresses the shared load by choosing case calls while the rack still enforces its compressor timers. Every minute a mixed-integer linear program selects one eight-minute profile per case from 00000000, 11111111, 11001100 and

Table 12.7 The Summer Day in Twelve Configurations. Limits: No Trip, 150 Starts, 240 Goods Minutes, 20 Hunting, 90 Liquid, 0.41 kWh per kWh

Configuration	Trips	Starts	Goods	Hunt	Liquid	kWh/kWh
as shipped	9	383	473	17	877	0.443
suggested	0	108	188	0	19	0.392
suggested, unloading	0	115	191	3	26	0.392
suggested, defrost at once	11	151	471	0	218	0.389
and unloading	0	117	164	0	29	0.392
suggested, 2 K superheat	0	128	177	340	192	0.394
suggested, free running	0	178	188	0	40	0.393
suggested, offsets	0	139	200	2	29	0.392
suggested, fixed suction	0	95	197	7	2	0.405
suggested, fixed head	0	91	189	6	7	0.407
suggested, fixed pressures	0	66	205	1	5	0.417
shipped, staggered, 6 K	0	354	219	8	179	0.439

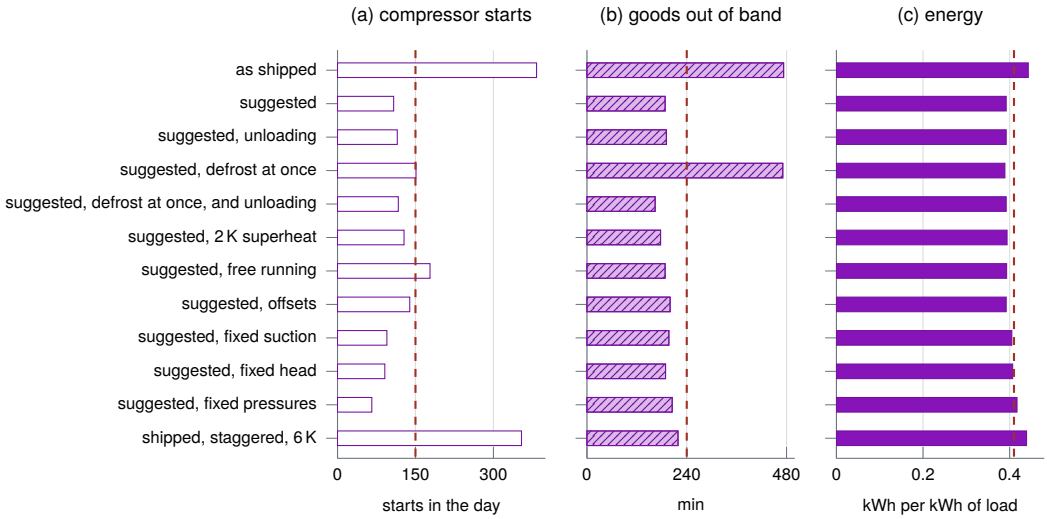


Figure 12.4 The summer day of Table 12.7 in its twelve configurations: (a) compressor starts, (b) the worst case's goods minutes out of band and (c) the specific energy, with the challenge limits dashed. The table prints the values. Notice that fixing both pressures gives the fewest starts and fails on energy alone, while the free-running thermostats fail on starts alone.

00110011. Only the first call is applied before replanning. This is a restricted profile library, not unrestricted compressor commitment or an electrical energy optimum. The mixed-integer formulation makes a small receding decision problem explicit alongside the usual MPC architecture [21], whose form in the apps is set out in Section 1.6.3.

Each case has nominal air, goods and coil estimates. With minutes as time, the predictor uses

$$320\dot{\hat{T}}_a = 60[.24(21-\hat{T}_a)+.35-U_g(\hat{T}_a-\hat{T}_g) \\ -1.4(\hat{T}_a-\hat{T}_w)]+320b, \tag{12.10}$$

$$1400\dot{\hat{T}}_g = 60U_g(\hat{T}_a-\hat{T}_g), \tag{12.11}$$

$$25\dot{\hat{T}}_w = 60[1.4(\hat{T}_a-\hat{T}_w)-.78v(\hat{T}_w-T_s)], \tag{12.12}$$

where $U_g = 1400/7200$ kW/K, v is the binary call and T_s is the current measured suction temperature, held throughout the forecast. The factor $.78 = 1.2(.65)$ fixes the wetted fraction at .65. Exact zero-order holds propagate each on/off interval. These coefficients stay at declared defaults when the

Table 12.8 Matched Three-Hour Profile Comparison

Method	Energy (kWh)	Rack starts	Air IAE (K min)
None	22.0537	45	557.51
Offsets	21.9225	24	481.92
Scheduler	21.5574	8	540.19
MPC	21.5184	24	384.14

learner changes the plant. Goods and coil estimates start at measured air temperature, then follow the nominal balances and the controller's own calls. Air is assimilated each two-second control scan. A rate-residual filter with gain .15 and bounds ± 1 K/min supplies b. It is a bounded disturbance correction, motivated by offset compensation [22], rather than a proof of zero offset for this profile library.

For each candidate, the cost sums absolute air error, twenty times air-band exceedance, .15 per on-minute and .4 per transition. Two continuous variables represent the peak number of simultaneous calls and of simultaneous openings. Their default prices are four and two. Four one-hot rows and sixteen coupling inequalities constrain sixteen binary and two continuous variables. Current manual calls are fixed, and an active defrost fixes its case off. Bands are soft. The defrost, superheat loops, rack staging, run/rest timers, pressure switches and manual stations keep authority. Measured air more than .6 K beyond either band edge overrides the call. The two-minute episodes inside the library therefore do not establish a dwell time on the executed sequence.

Only an optimal, finite, integral solution satisfying every bound and row, with a consistent objective and gap, is accepted. A rejected solve holds the current call vector on its first scan, then thermostat hysteresis takes over until a valid plan recovers. Historian marks and notes identify both transitions. The PIs continue their existing operation and tracking because the scheduler changes call permissions rather than their continuous commands. The first optimization occurs at initialization. Eight appended CSV fields include solver iterations, nodes, band exceedance, status, planning time and the largest predicted air temperature at the horizon. The scene badge displays that endpoint, which is not a prediction of the warmest goods.

An independent enumeration of all 256 combinations agrees with the MILP objective on twelve varied measurement packets. A separate RK4 integration agrees with the nominal exact hold within 1.5×10^{-14} K. These algebra checks do not validate the reduction against refrigerant transients. Removing boiling immediately when a call ends ignores liquid remaining in the coil. At $\hat{T}_w = -2$ and $T_s = -10$ degrees C with a .65 wetted fraction, that omission changes the coil rate by 14.976 K/min. Frost, superheat heat removal, future defrost, doors, weather and restocking are also absent. Goods remain unmeasured.

Table 12.8 and Figure 12.5 use case 10's nearly synchronous start, the default seed and three hours under each method. The MPC reduces measured air IAE summed across the cases from 557.51 to 384.14 K min and lowers energy and starts against free running. The gap scheduler makes fewer starts. A simple change of its gap to 1.5 min gives five starts at 21.8414 kWh, so low start counts do not require prediction. MPC makes 106 case cut-ins against 66 with the default gap scheduler. Pricing peak calls does not minimize motor starts.

A wider comparison of 44 runs covers defrost, floating suction, restocking, heat-wave operation, a half-size physics step, four scheduler gaps and two alternative concurrency prices. At 180 min in the heat wave, MPC gives 31.95 min of goods outside band against 27.88 with the gap scheduler and 16.14 with None. Halving air-to-coil conductance raises MPC air IAE to 232.78 K min against 163.19 for None. On the suggested day, MPC uses 175.3384 kWh and 103 starts against 183.5111 kWh and 108 for the suggested scheduler, but goods outside band rise from 188.22 to 215.02 min. Both pass. On the as-loaded day both fail with nine trips, despite MPC reducing starts from 383 to 184. Half-step runs preserve the rounded three-hour results. Six Chrome captures time sixty decisions

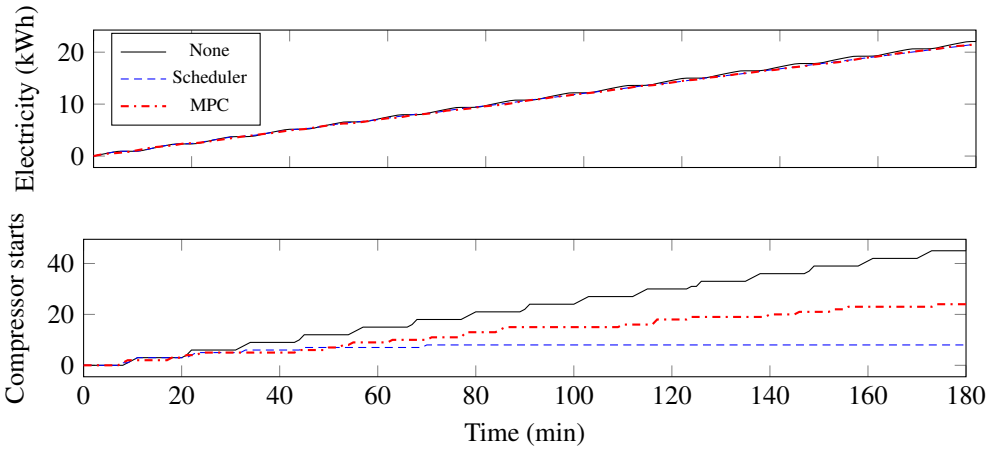


Figure 12.5 Case 10 uses the same instruments and initial phases under each choice: None is thin solid, the scheduler densely dashed and MPC thick dash-dotted. A smaller cumulative energy demand accompanies more compressor starts for MPC than for the gap scheduler.

each at .332–.360 ms per complete plan, .0332–.0360 percent of one core at one simulated minute per wall second. At the 900-fold top speed the same work costs 29.9–32.4 percent, excluding observation, physics and drawing. Prediction buys one trade, rather than replacing equipment capacity or the simpler scheduler.

12.7 Technology in Practice

What Measures It

In the basic control structure of a refrigeration system the suction pressure is measured at the compressor inlet, and with no pressure drop in the suction line it equals the evaporating pressure [16]. This chapter's model makes that assumption, one suction pressure for four cases. Each display case carries an air temperature sensor next to the goods, used in its control loop as an indirect measure of the goods temperature [16]. Case 9 shows what that indirection hides. The superheat of each coil needs a second pair of readings, the outlet temperature and the evaporation pressure.

> **TECHNOLOGY** **Sensor: Suction Pressure and a Clamped Thermometer**
>
> Superheat is not measured directly. It is the evaporator outlet temperature minus the evaporation temperature, and the evaporation temperature is normally found from a measurement of the evaporation pressure [14]. On one laboratory refrigeration system the pressure transmitters gave a voltage in proportion to the pressure, and platinum resistance thermometers were mounted on the copper tubes under insulation, so they read the tube rather than the refrigerant and added a small time constant [16]. Close to a flooded coil the outlet temperature grows restless: on two test systems its variance rose considerably at low superheat and fell again once the evaporator flooded [14]. The 6 s lag and 0.08 K noise of $\widehat{SH}_j$ in Table 12.1 stand for that clamped sensor.

What Moves It

The rack's compressors are switched on and off to hold the suction pressure, and one of them may run on a frequency converter for continuous action. The condenser pressure is held the same way, by switching fans or by changing their speed on a frequency converter [16]. The pack of this chapter has both kinds: C1 and C2 are staged while C3 trims on its speed, and PIC-2 sets the speed of three fans. Figure 12.6 draws the loops on that hardware.

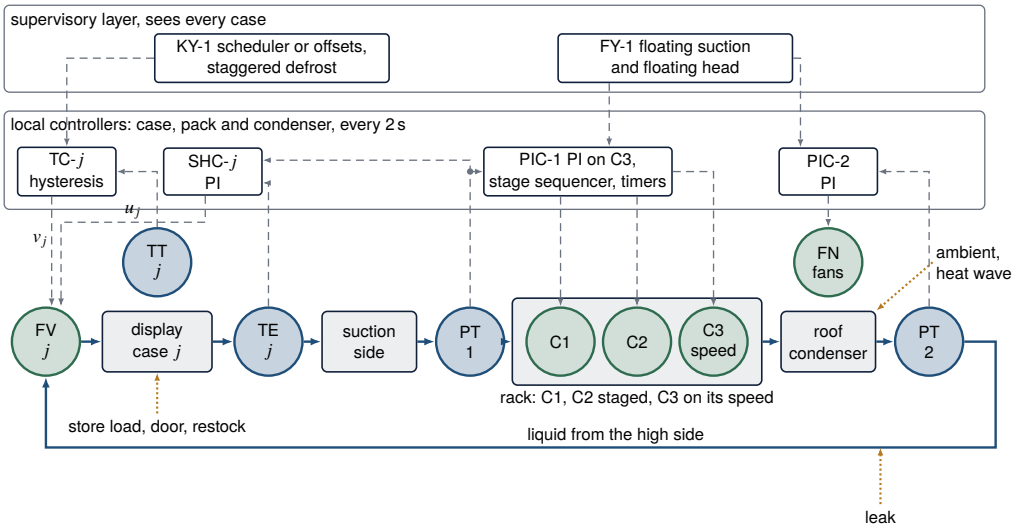


Figure 12.6 The chapter's loops on store hardware. In each case controller the thermostat TC-j reads TT-j and gives the inlet valve FV-j its call v_j, and SHC-j sets its opening u_j from the outlet temperature TE-j and the suction pressure PT-1. The pack controller holds the suction with C3's speed and stages C1 and C2 under their timers, and the condenser controller sets the fan speed from PT-2. The supervisory layer spreads the cases' calls and defrosts and floats both pressure setpoints. Dotted arrows mark disturbances.

> **TECHNOLOGY** **Final Element: An Inlet Valve That Opens and Pulses**
>
> In a traditional display case an electronic on/off inlet valve, opened and closed by the case's temperature controller, sits in series with a thermostatic expansion valve that holds the superheat while the inlet valve is open. In some versions the expansion valve is left out, and the inlet valve regulates the superheat itself by pulse width modulation [16]. A thermostatic valve meters the flow on the temperature difference across the evaporator, keeps liquid from returning to the compressor, and is stable enough that it typically needs no adjustment [23]. The response from opening to superheat is in general very nonlinear, which makes an electronic valve's loop hard to tune [14]. This chapter's valve carries both roles, the thermostat's call v_j and the superheat loop's opening u_j.

Where the Controller Runs

The control of a supermarket pack is distributed. The rack has its suction pressure controller, the condenser its pressure controller, and each display case a superheat and a temperature controller. A monitoring center receives data from the local controllers through a gateway and can change their setpoints and parameters, but the infrastructure allows only limited sharing of information directly between the local loops [16]. A building automation system of the same kind records historical data and alerts staff to a device failure or an alarm, and in such operational technology the supervisory and lower loops run with cycle times from milliseconds to minutes [24]. The chapter's controller scans every 2 s. Read as store hardware, the thermostats and superheat loops belong in the case controllers, PIC-1 and the stage sequencer in the pack controller, and the floating pressures, the scheduler and the staggered defrost in a supervisory layer that must see every case.

IN PRACTICE **Head Pressure, Superheat and Charge**

A federal retrofit guide for grocery stores sets the routine. Head pressure is set from the maker's chart for the refrigerant and the desired temperature, and lowered only in small increments while product temperatures are watched, because not every system meets its load at a lower head pressure. A check of the superheat confirms that the expansion valve works, and the suction pressure then falls in line. The charge is checked and the system inspected for leaks at least yearly. An under-charged system runs its compressor continuously at low suction and head pressures and misses its setpoints, and an over-charged one raises head pressure [23]. The leak of Table 12.1 starves this chapter's valves once the high side runs low.

What Is Modernizing It

Three developments touch this pack. Defrost cycles typically run on a time schedule whether a defrost is needed or not, and adaptive defrost controllers learn a baseline after installation and skip the unneeded cycles [23]. Head pressure reset combined with electronic expansion valves can lower the minimum condensing temperature to 50 to 60 °F, with a variable-speed drive on the condenser fan [23], which is the floating head pressure of Section 12.5. New systems have moved from conventional centralized direct expansion toward distributed, secondary loop, cascade and transcritical carbon dioxide systems, which typically leak less or use refrigerants with better environmental profiles [19]. Model-based setpoint optimization drove a test refrigeration system close to its optimum and cut its power consumption by up to 20 % [16], and the optimizers of Section 12.2 carry the same idea to the switching of whole packs.

12.8 Limitations

The cases are lumped, with no air curtain, shelf profile or product core, and frost carries no latent heat. The stable line is three declared factors on one exponential, a teaching reduction of a boundary that real evaporators show by experiment [13], and its numbers describe no particular coil. The suction side is one pressure with saturated vapor density and no line pressure drop, the compressors are ideal displacements that start at once, and the condenser is one temperature. The refrigerant comes from fits rather than from the standard formulation [20], and the R404A and R290 alternatives are approximate.

OPEN CHALLENGE **Controlling a Temperature Field, Not a Point**

Hysteresis control of display cases is kept mainly because it gives a uniform temperature through the case and a less harmful frost on the evaporator. If the frost and the temperature distribution in a case could be modeled, continuous control that does not synchronize could be developed. Such a model would probably be a set of partial differential equations, and the task would become control of a temperature distribution rather than of one point [16]. Case 9 shows the gap: the goods leave their band for 147 min while the air thermostat sees only its ordinary cycling (Table 12.6).

Every threshold is a teaching value. The 3 K line, the 56 °C cut-out, the −25 °C low-pressure stop, the goods band of 1 to 3 °C and the challenge's limits were set on this model, and real cut-outs follow the system's pressure rating, the compressor maker and the standard the application names. The weak dependence of the lock on the suction volume and the rack's tuning is measured on this model and its rack logic only. The thermostats' supervisor is rule-based. The profile MILP of Section 12.6.10 omits suction inventory and cannot reproduce the full hybrid optimizers in the benchmark literature [7, 9]. Its band penalties and warm-air overrides provide no goods-temperature guarantee. The numbers describe this model and no particular store.

OPEN CHALLENGE Keeping Many Cases Apart in Real Time

A desynchronizing predictive controller has held two display cases apart in simulation, but its complexity, and with it the computation time, grows with the size of the system, so a larger pack needs simplifications to make it practical [16]. A nonlinear scheme that tuned the cases' own controllers from above worked on a system of moderate size but not on larger ones, because its search space grew combinatorially, and it handled sudden large changes of the disturbances poorly [17]. Progress would be a coordinator that splits the problem into parallel subproblems of lower complexity, as the thesis proposed [16], and still keeps every case's goods in their band.

12.9 Conclusion

The simulated pack shows two failures of loops that behave well alone. A superheat loop that holds (5.98 ± 0.16) K at 6 K swings between 0.67 and 7.84 K at 2 K with the same gains, because below the 3 K line the coil's response steepens more than tenfold, slows threefold and wanders, and the opening at which a valve meets that line drifts 1.76 points in 24 minutes. Four independent thermostats lock in 65.2 min through the one suction pressure, and a larger suction volume does not prevent it (52.6 minutes at 5000 L), nor does the rack's tuning, but the lock never forms on a pressure no case can move. The cures are measured beside the faults. The scheduler cuts three hours' starts from 45 to 8, staggering cuts a defrost's peak rejection from 56.2 to 46.4 kW, floating suction saves 4.9 % of six hours' energy, and together with a 6 K superheat and floating head pressure they turn a day of 9 trips and 383 starts into one of none and 108. The model is lumped, its stable line and thresholds are teaching values, and its supervisor is rule-based. The hybrid scheduler lowers energy against free running in the matched case, while the gap scheduler makes fewer starts and model mismatch can warm goods. A fuller model would add humidity, refrigerant inventory and the latent load of frost to the predictor, line pressure drops, and frozen cases on a second suction group. The pack lets a reader measure the coupling that a shared utility puts between loops before meeting it in a plant.

KEY POINTS

- Four thermostats that never exchange a signal still pull each other into step, because every case boils at the one suction pressure the rack holds.
- A larger suction volume does not prevent the lock (52.6 minutes at 5000 L), but a pressure no case can move does.
- A superheat loop that holds (5.98 ± 0.16) K at 6 K hunts at 2 K with the same gains, and the cure is margin above the line, not a smaller gain.
- The scheduler cuts three hours' starts from 45 to 8, and staggering cuts a defrost's peak rejection from 56.2 to 46.4 kW.

Where else this applies. The rack's minimum run and rest times (Table 12.4) are the minimum dwell that the cooldown of Chapter 14 applies to a fleet of servers.

References

[1] S. A. Tassou, Y. Ge, A. Hadawey, and D. Marriott, "Energy consumption and conservation in food retailing," *Applied Thermal Engineering*, vol. 31, no. 2-3, pp. 147–156, 2011.

[2] L. F. S. Larsen, C. Thybo, R. Wisniewski, and R. Izadi-Zamanabadi, "Synchronization and desynchronizing control schemes for supermarket refrigeration systems," in *2007 IEEE International Conference on Control Applications*, 2007, pp. 1414–1419.

[3] D. Docimo and H. K. Fathy, "Demand response using heterogeneous thermostatically controlled loads: Characterization of aggregate power dynamics," *Journal of Dynamic Systems, Measurement, and Control*, vol. 139, no. 10, p. 101009, 2017.

[4] S. H. Strogatz, "From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators," *Physica D: Nonlinear Phenomena*, vol. 143, no. 1-4, pp. 1–20, 2000.

[5] X.-D. He, S. Liu, and H. H. Asada, "Modeling of vapor compression cycles for multivariable feedback control of HVAC systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 119, no. 2, pp. 183–191, 1997.

[6] L. F. S. Larsen, R. Izadi-Zamanabadi, and R. Wisniewski, "Supermarket refrigeration system - benchmark for hybrid system control," in *2007 European Control Conference (ECC)*, 2007, pp. 113–120.

[7] D. Sarabia and C. de Prada, "Hybrid predictive control of supermarket refrigeration systems: an optimal time switching strategy," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 10 916–10 922, 2008.

[8] N. L. Ricker, "Predictive hybrid control of the supermarket refrigeration benchmark process," *Control Engineering Practice*, vol. 18, no. 6, pp. 608–617, 2010.

[9] T. G. Hovgaard, L. F. S. Larsen, K. Edlund, and J. B. Jørgensen, "Model predictive control technologies for efficient and flexible power consumption in refrigeration systems," *Energy*, vol. 44, no. 1, pp. 105–116, 2012.

[10] W.-D. Gruhle and R. Isermann, "Modeling and control of a refrigerant evaporator," *Journal of Dynamic Systems, Measurement, and Control*, vol. 107, no. 4, pp. 235–240, 1985.

[11] K. A. James and R. W. James, "Transient analysis of thermostatic expansion valves for refrigeration system evaporators using mathematical models," *Transactions of the Institute of Measurement and Control*, vol. 9, no. 4, pp. 198–205, 1987.

[12] A. Outtagarts, P. Haberschill, and M. Lallemand, "The transient response of an evaporator fed through an electronic expansion valve," *International Journal of Energy Research*, vol. 21, no. 9, pp. 793–807, 1997.

[13] W. Chen, Z. Chen, R. Zhu, and Y. Wu, "Experimental investigation of a minimum stable superheat control system of an evaporator," *International Journal of Refrigeration*, vol. 25, no. 8, pp. 1137–1142, 2002.

[14] K. Vinther, C. H. Lyhne, E. B. Sorensen, and H. Rasmussen, "Evaporator superheat control with one temperature sensor using qualitative system knowledge," in *2012 American Control Conference (ACC)*, 2012, pp. 374–379.

[15] B. P. Rasmussen and A. G. Alleyne, "Control-oriented modeling of transcritical vapor compression systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 126, no. 1, pp. 54–64, 2004.

[16] L. F. S. Larsen, "Model based control of refrigeration systems," Ph.D. dissertation, Department of Control Engineering, Aalborg University, Aalborg, Denmark, 2006, iSBN 87-90664-29-9. [Online]. Available: https://vbn.aau.dk/ws/files/549479206/thesis.pdf

[17] C. Sonntag, A. Devanathan, and S. Engell, "Hybrid NMPC of a supermarket refrigeration system using sequential optimization," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 13 901–13 906, 2008.

[18] L. S. Larsen and C. Thybo, "Potential energy savings in refrigeration systems using optimal set-points," in *Proceedings of the 2004 IEEE International Conference on Control Applications*, 2004, pp. 701–704.

[19] U.S. Environmental Protection Agency, "GreenChill: Keeping cool for fifteen years (2007–2022)," U.S. Environmental Protection Agency, Report, 2022. [Online]. Available: https://www.epa.gov/system/files/documents/2022-09/GreenChill-Keeping-Cool-for-15-Years-2022_1.pdf

[20] R. Tillner-Roth and H. D. Baehr, "An international standard formulation for the thermodynamic properties of 1,1,1,2-tetrafluoroethane (HFC-134a) for temperatures from 170 K to 455 K and pressures up to 70 MPa," *Journal of Physical and Chemical Reference Data*, vol. 23, no. 5, pp. 657–729, 1994.

[21] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.

[22] G. Pannocchia and J. B. Rawlings, "Disturbance models for offset-free model-predictive control," *AIChE Journal*, vol. 49, no. 2, pp. 426–437, 2003.

[23] National Renewable Energy Laboratory, "Advanced energy retrofit guide: Practical ways to improve energy performance; grocery stores," U.S. Department of Energy, Building Technologies Program, Report DOE/GO-102012-3655, revised, 2013. [Online]. Available: https://www.osti.gov/servlets/purl/1045045

[24] K. Stouffer, M. Pease, C. Tang, T. Zimmerman, V. Pillitteri, S. Lightman, A. Hahn, S. Saravia, A. Sherule, and M. Thompson, "Guide to operational technology (OT) security," National Institute of Standards and Technology, Gaithersburg, MD, Special Publication 800-82r3, 2023.

12.10 Exercises

12.1 Concept check · 2 min

In Case 4 the four thermostats run free from seeded phases and share nothing but the suction line. Why are they in step after about an hour?

(a) The suction loop is tuned too aggressively, and its own oscillation entrains the four cases

(b) The four cases carry identical loads, so their periods match and the phase gaps they start with persist

(c) The scheduler releases each case early once the last opening is old enough, which lines them up

(d) A case that opens raises the suction pressure and slows the cases already cooling, so the phases draw together

12.2 Concept check · 2 min

In Case 2 the superheat loop holds 6 K and then hunts after its setpoint is lowered to 2 K, a kelvin under the 3 K stable line. Which change does the chapter show restores a steady superheat?

(a) Raising the setpoint to 4 K, a kelvin above the line, with the same gains

(b) Lowering the loop gain at the 2 K setpoint until the swings die out

(c) Holding the valves in Manual at the opening where the steady curve meets the line

(d) Shortening the lag of the outlet sensor so that the loop sees each trough sooner

12.3 Concept check · 1 min

In Case 9 two fifths of the meat case's goods are replaced by goods at 10 °C, and the goods take about two and a half hours to return under 3 °C. Why does the air thermostat not prevent this?

(a) Its 2 K band is too wide, so the air is allowed to warm along with the goods

(b) It measures the air, which recovers within its usual cycling while the unmeasured goods lag by hours

(c) The scheduler holds back the meat case's cut-ins, so the case cannot cool while the goods are warm

(d) The restock raises the suction pressure, which takes cooling away from the meat case

12.4 Theory and calculation · 5 min

The order parameter of Equation (12.9) measures how close the thermostats are to cutting in together.

(a) Case 4 starts the four thermostats at phases 0.59, 0.67, 0.38 and 0.08 of a cycle, so φ_j is 2π times each. Compute R. *Checkpoint:* $R = 0.30$

(b) Show that for two cases a gap of g cycles gives $R = |\cos(\pi g)|$, and evaluate it for the dairy and produce cases of Section 12.4, which start 0.39 of a cycle apart and close to 0.10. *Checkpoint:* 0.34 and 0.95

(c) The status reads SYNCHRONIZED once R has stayed above 0.9 for 20 min. What is the largest gap between two cases that $R > 0.9$ allows, in cycles and in minutes of an 11.4 min cycle? *Checkpoint:* 0.144 of a cycle, 1.6 min

12.5 Theory and calculation · 7 min

Equation (12.3) joins two branches at the dry length λ_{mss}.

(a) Differentiate both branches with respect to λ at λ_{mss}, and show that the slope below the line is the slope above it times $\rho = G\,SH_{mss} / \left[(\Delta T - SH_{mss})(-\ln(1 - SH_{mss}/\Delta T))\right]$, whatever a is. *Checkpoint:* ρ as given

(b) Evaluate ρ at the start of Case 2: air at 2.14 °C, suction at −6 °C, $G = 8$ and $SH_{mss} = 3$ K. *Checkpoint:* $\rho = 10.2$

(c) The superheat loop has a loop gain of 1.24 at 6 K. Taking the slope above the line as a rough stand-in for the slope at 6 K, estimate the loop gain just below the line, and give the superheat lag there from Equation (12.4) with $\beta = 1$. *Checkpoint:* about 13, 54 s

12.6 Theory and calculation · 6 min

The condenser's conductance at fan speed $n = 1$ with N_{out} of its three fans failed follows the $UA_c(n)$ law of Section 12.3 with $UA_c = 2.7$ kW/K and $UA_{c0} = 0.35$ kW/K.

(a) Compute the conductance with no fan, one fan and two fans out. *Checkpoint:* 2.70, 2.05 and 1.33 kW/K

(b) Section 12.4 finds that the four cases at 2 °C need a condensing temperature of 48.7 °C with all fans into air at 38 °C. How much heat does the condenser reject there? *Checkpoint:* about 28.9 kW

(c) Holding that heat, estimate the condensing temperature with one and with two fans out. Compare with the chapter's 52.5 and 62.6 °C and the 56 °C cut-out, and say what your estimate leaves out. *Checkpoint:* 52.1 °C and 59.8 °C

12.7 App challenge · 8 min

Load Scenario 2, *The superheat loop and the stable line*, at the default seed and change nothing. Run its 50 min at speed 5 (about 10 s) and export the CSV file.

(a) From column `SH1_K`, the dairy coil's measured superheat, give its mean over the first 10 min and its lowest and highest values from 10 to 30 min. *Checkpoint:* 5.98 K; 0.74 and 7.82 K

(b) At what time does column `hunting` first read 1? *Checkpoint:* 18.9 min

(c) Give the mean of `SH1_K` from 36 to 50 min, and explain the difference between the three periods. *Checkpoint:* 3.99 K

12.8 App challenge · 8 min

Load Scenario 4, *Four thermostats fall into step*, at the default seed and change nothing. Run the 3 h at speed 20 (about 9 s) and export the CSV file.

(a) Read `sync_R` at 20 min and the first time at which column `status` reads SYNCHRONIZED. *Checkpoint:* 0.59; 65.2 min

(b) Read `metric_starts` at 60 and at 180 min, and give the starts in the first hour and in the next two. *Checkpoint:* 14 and 30

(c) In the `sync_R` column, find when R last rose above 0.9 before the status changed, and explain the difference from (a). *Checkpoint:* 45.2 min

12.9 App challenge · 7 min

Load Scenario 7, *Floating suction: the energy and the case that sets it*, at the default seed and change nothing. Run the 6 h at speed 60 (about 6 s), and read the Energy and Compressor starts tiles and the case the FY-1 card names as binding. Then reload the scenario, set Floating suction pressure to Off before Run, and run the 6 h again.

(a) Give the energy and the starts of both runs. *Checkpoint:* 43.2 and 45.4 kWh; 27 and 58

(b) What share of the energy does the float save? *Checkpoint:* 4.9 %

(c) Which case binds, and why that one? *Checkpoint:* case 2, the meat case

12.10 Application area · 4 min

Case 6 gives each coil 2 kg of frost and defrosts all four at once.

(a) What share of the clean coil's air-to-coil conductance is left with 2 kg of frost, and after how many hours of frosting at the declared rate does a wet, cold coil carry that much? *Checkpoint:* 71 %; 4 h

(b) How much heat does melting 2 kg take, and how long does a 2 kW heater need for that alone? Compare with the measured defrost length. *Checkpoint:* 668 kJ; 5.6 min

(c) A store engineer asks why the defrosts should be staggered. Answer from the case's results. *Checkpoint:* 56.2 kW against 46.4 kW

12.11 Application area · 4 min

Floating head pressure lets the condensing temperature follow the ambient down to a floor of 25 °C.

(a) With the cycle coefficients of performance of Section 12.4, 4.31, 2.70 and 1.80 at 25, 40 and 55 °C condensing, what compressor power carries the 19.64 kW of case load at each, and what does floating from 40 to 25 °C save? *Checkpoint:* 4.56, 7.27 and 10.91 kW; 37 %

(b) The valve law makes the flow at a given opening proportional to $\sqrt{P_c - P_{suc}}$. With the saturation fit of Table 12.2 and the suction at −10 °C, what share of its capacity at 40 °C condensing does a valve keep at 25 °C, ignoring the change in liquid density? Why is there a floor? *Checkpoint:* about 0.75

12.12 Review the agent · 5 min

An AI assistant reviewing a store's rack writes: "The cases fall into step because each valve opening kicks a small 250 L suction side. Add a 5000 L buffer volume. The suction pressure will then move twenty times less per opening, the coupling will be twenty times weaker, and the cases will stay apart without any scheduler." Find the flaw, and say how you would check the claim in the app.