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CHAPTER 13

Stability Lobes and Adaptive Feed Control in a CNC Milling Simulator

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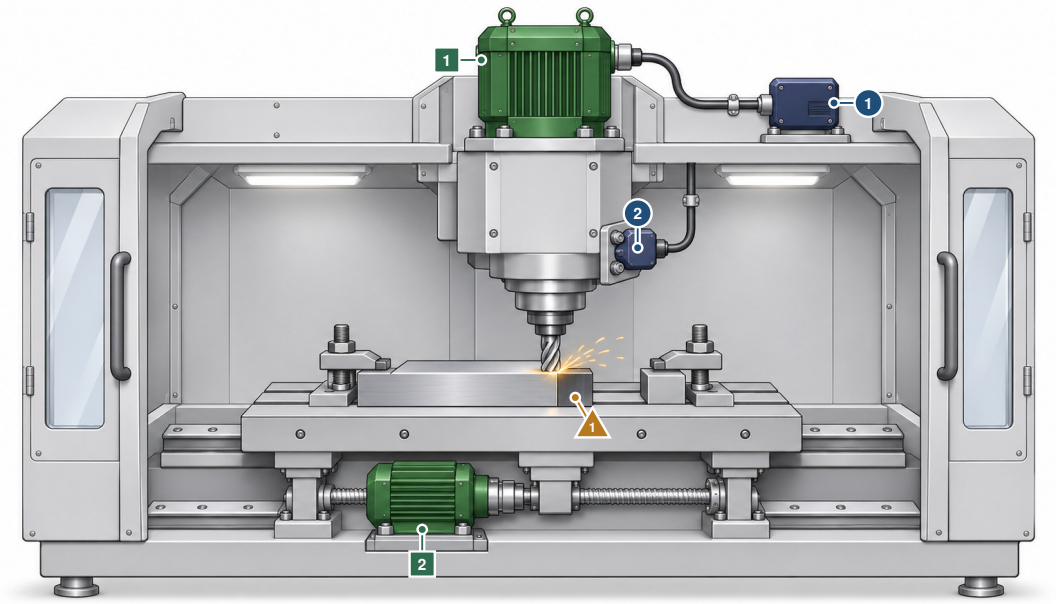
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CHAPTER 13

Stability Lobes and Adaptive Feed Control in a CNC Milling Simulator



The controller moves 1 the spindle drive, which sets the tool's speed
2 the feed drive, which sets how fast the tool moves through the metal

The controller reads 1 the spindle load, read from the drive 2 the vibration sensor on the spindle housing

What disturbs it 1 a harder patch in the metal, or a worn tool

Notice that the feed drive and the spindle drive act on the same cut from different places, one easing the load and the other moving the tool's speed out of chatter. Illustration, not to scale.

IN PLAIN TERMS Two Limits on One Cutting Tool

The main idea. A spinning cutter that clears a pocket in a block of metal can be stopped by two different limits. Its motor can be overloaded, mostly in the corners, and the tool can start to shake and leave a wavy, marked surface.

The challenge. Moving the tool more slowly through the metal eases the motor, but it does not stop the shaking. Only a change of the spinning speed ends it, so the controller must tell the two limits apart while it cuts.

Why it matters. A shop that slows down to cure the shaking loses time and still gets a surface that must be reworked.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the fact in the Introduction (Section 13.1), the two history segments in Section 13.2, the try-its and the fact in Section 13.5, and the key points
Study	45 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 13.5), Technology in Practice (Section 13.6), Limitations (Section 13.7) and the open challenge

The control question An end mill clearing a pocket overloads its spindle in the corners and chatters above a critical depth, and both look like a cut that labors. Which limit does the feed override remove, which one only the spindle speed removes, and how does a controller tell them apart on line?

Learning objectives

- Derive the stability limit of a regenerative cut from its delay equation, and explain why the feed does not appear in it.
- Compute the spindle load from the removal rate, and tell when the spindle power rather than chatter limits a cut.
- Apply the whole-wave speed rule to a measured chatter frequency, including the bounded step.
- Compare, on the app, what an adaptive feed and a speed change each do about a corner, a hard patch, a worn tool and a late load signal.
- Judge a proposed remedy for chatter against the stability lobes.

Prerequisites Sections 1.4.9 (a delay that makes a cycle, bounded by a nonlinearity), 1.4.8 (a lightly damped mode) and 1.4.3 (a gain that moves with the operating point), with Sections 1.6.2 and 1.6.1 for the feed loop and the engine.

The app https://apmonitor.com/apps/cnc-milling/

See also Volume 5, Chapter 8 for a delay that makes a loop of passive parts chatter, Volume 5, Chapter 3 for a tuning that fails once its measurement arrives late, and Volume 5, Chapter 2 for control through a transport delay.

13.1 Introduction

An end mill clearing a pocket is stopped by two limits that an operator meets in the same way, as a spindle that labors and a cut that sounds wrong. The first is the load. The mean cutting power is the specific cutting force times the volumetric removal rate, and into every inside corner the arc of contact grows until the tool is cutting a full slot, so a feed chosen for the straights overloads the tool in the corners. The second is regenerative chatter. Each tooth cuts the surface the previous tooth left one tooth period earlier, a vibration written into that surface returns as a varying chip thickness, and above a critical depth of cut the returning wave grows on every pass [1, 2]. Chatter is among the principal limits on the productivity of machining, because it leaves a surface that must be reworked and it damages the cutting edges and the spindle [3, 4].

+ DID YOU KNOW? Most of the Slab Leaves as Chips

High-speed, high-power spindles were developed to remove more than 95 % of the material from aluminum slabs, leaving monolithic, lightweight parts for the aerospace industry [4]. The aluminum pocket of this chapter is one small step of that kind of work, and chatter is what limits how fast it can be taken.

The two limits call for different actuators, and a block diagram of either one hides that fact. The load is a level, proportional to the feed, and a feed override answers it. The critical depth is the stability boundary of a delay differential equation, set by the frequency response of the tool, the cutting stiffness and the tooth period, and the feed does not enter it at all [2, 5]. Slowing the feed thins the chip and so shrinks a chatter vibration, whose amplitude is held finite by the tool leaving the cut [6], but it does not end the chatter. The simulator is designed to overturn a belief common on the shop floor, that backing off the feed cures any cut that misbehaves. In the application, an adaptive feed loop that holds the load of a worn tool within 1.6 % of its target still leaves 72 mm of chatter marks on a 92 mm pass, while a spindle speed 5.4 % lower ends the chatter of a cut 11 % over its limit within 0.21 s of the command.

The chip regeneration belongs to a general class, the repetitive or multipass process, in which each pass of a tool responds to the profile the previous pass left, and whose classic examples are metal rolling and long-wall coal cutting [7]. A loop of that kind carries a delay equal to the time between

passes, its characteristic equation has infinitely many roots, and its stability can change with the delay while every gain stays fixed [8]. The feed loop carries a second general lesson. Machining control is usually organized in three levels, the servo level that moves the axes, the process level that regulates a cutting quantity such as force or power, and a supervisory level above both [9]. At the process level the load per unit of feed doubles between a straight and a corner, so a fixed-gain force loop tuned for one is mistuned for the other, which is why force control in milling became a problem in adaptive control [10]. The same shape appears wherever a process gain moves with an operating condition that the controller does not measure, as in the neutralizer of Volume 2, Chapter 6, whose gain changes by orders of magnitude across one valve stroke.

The two subjects are normally taught apart. Chatter appears as the stability analysis of a delay differential equation whose result is a lobe diagram, and machining force control as a regulation problem in which the cut is a static gain [4, 9]. However, neither presentation supplies an artifact in which one pocket is cut with the feed loop and the speed decision working together, so that the feed loop can be watched holding the load at a corner while the lobes decide the finish.

This chapter works with a CNC milling simulator in which the load limit and the stability limit act on one pocket, each with its own controller, so that the question of which actuator answers which limit is settled by measurement. The average-tooth stability analysis, the contact-loss nonlinearity, the adaptive force loop, the energy-ratio chatter indicator and the spindle speed rule are established methods, all from the literature reviewed in Section 13.2. The simulator advances a regenerative delay model with contact loss whose simulated stability boundary lies within 0.1 % of the analytic lobes, along a toolpath whose engagement is measured by removing material from a 0.2 mm grid, and both controllers are specified completely enough to rebuild. Eight cases each move a single mechanism, and a graded challenge can be passed only with both loops working.

Section 13.2 traces chatter theory, spindle speed selection and adaptive machining. Section 13.3 builds the tool, chip and machine model, derives the stability lobes and describes the numerical solution and its check against the lobes. Section 13.4 gives the spindle drive, the adaptive feed loop, the chatter detector and the speed rule. Section 13.5 reports the eight cases and the graded challenge, Section 13.6 sets the two loops on machine-tool hardware, Section 13.7 states the limitations and Section 13.8 draws the results together. The exercises in Section 13.9 work through the model and the cases.

13.2 Chatter Theory, Speed Selection and Adaptive Machining

The first stability laws, Tlusty's account of the theory of chatter and the stability analysis of machine tools among them [1], are traced in the segment that follows this paragraph. The regenerative loop is linear only while the tool stays in the material. Once the vibration exceeds the chip thickness the tool leaves the cut for part of each cycle, and that basic nonlinearity is what holds chatter at a finite amplitude [6].

> ⧖ HISTORY **From a Machinist's Puzzle to a Stability Law, 1906 to 1965**
>
> An article of 1906 called chatter the "most obscure and delicate of all problems facing the machinist" [4]. The first scientific stability laws came independently in the 1950s. One gave a critical depth of cut set by the machine's dynamic stiffness and the material's cutting force coefficient, whatever the spindle speed. The other added the regenerative delay of the spindle speed and found the stable pockets, or lobes [4]. Merritt then cast the regeneration as a closed loop with delayed feedback [4] and plotted the borderline of stability as a stability chart. His simpler criterion keeps the directional cutting stiffness below one half of the minimum directional dynamic stiffness of the structure, which assures chatter-free cutting at all spindle speeds. Machine testing with continual improvement of the structure was, he wrote, the best way to combat chatter [2].

Milling was harder to analyze, because the number and the direction of the cutting teeth rotate with the spindle and the delay equation acquires periodic coefficients. Averaging the directional factors over a tooth period gave an analytical prediction of the stability lobes of milling [5], the approach this application follows. The average fails in highly interrupted cutting at small radial immersion, where a theory supported by simulation and experiment predicts a doubling in the number of optimally stable speeds [11]. The extra pockets of intermittent cutting and the process damping contributed by the flank face of the tool are part of the modern account, reviewed in [4].

The high-speed machining of the early 1990s, with spindles fast enough to run near the tooth-passing frequencies at which the lobes open into stable pockets, made the stability chart a production tool rather than a laboratory result [12]. Finding the pocket automatically followed. Smith and Tlusty regulated the spindle speed from the measured chatter frequency, so that successive teeth meet the surface wave in phase [13], and a microphone was shown to be a competitive and often superior sensor for detecting chatter, against dynamometers, displacement probes and accelerometers [14]. Later detectors remove the forced vibration at the tooth-passing frequency and its harmonics and declare chatter when the energy of a remaining component grows relative to the forced part [15]. Speed selection is now one of many suppression techniques, passive and active, collected in [3].

Adaptive control of metal cutting dates from the same years as the stability chart, as the segment after this paragraph tells. Force regulation needs adaptation because its process gain moves with the cut. The feed loop of this application is a deliberately simple member of that family, a proportional-integral controller whose gain is divided by an identified load per unit of feed, and its filtered and optionally delayed load signal reproduces the trade-off that such a filter imposes.

> ⧖ HISTORY **Adaptive Control Reaches the Cut, 1964 to 1993**
>
> In 1964 numerical control closed its feedback loops only around the positions of the tool and the workpiece. An adaptive controller reported that year extended the measurement to the interface of tool and work and adjusted the process inputs to optimize a criterion such as cost, under constraints on the surface finish and the machine's horsepower [16]. A model reference adaptive force controller for milling later performed more satisfactorily than fixed-gain controllers but was difficult to implement and tune. Runout of the cutter put noise on the force signal, and the filter that removed it added dynamics that reduced the performance [10]. A review of 1993 ordered the research by the servo, process and supervisory levels of a control hierarchy and highlighted adaptive and optimal control [9].

13.3 Tool, Chip and Machine Model

The simulator clears a pocket 120 mm by 60 mm in 7075-T6 aluminum with a 12 mm three-flute carbide end mill. Seven states are integrated, the tool deflection y and its rate $\dot{y}$ along one bending mode, the spindle speed Ω, the position s along the toolpath, the feed velocity v_f, the tooth phase θ and the flank wear VB. The machine and its two loops, the engagement in plan view and the regenerative chip are shown in Figure 13.1, and the variables are classified in Table 13.1.

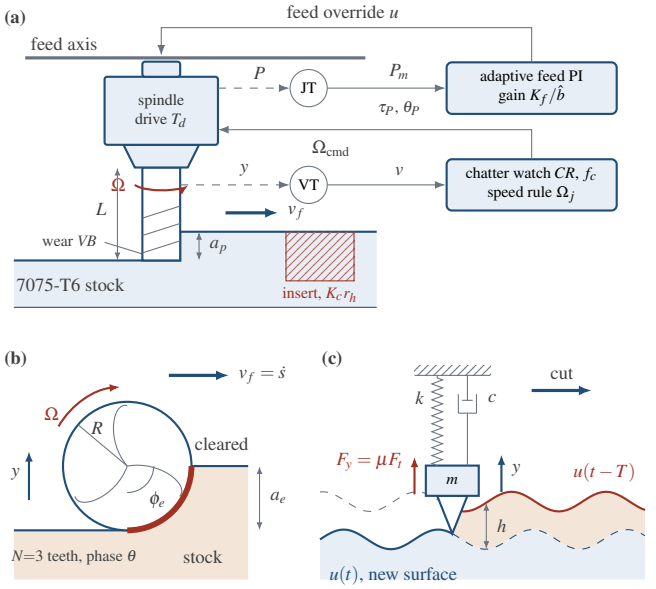


Figure 13.1 End milling with two loops. (a) The tool of overhang L cuts to depth a_p while the feed axis moves it at v_f, the spindle drive turns it at Ω with torque T_d, and flank wear VB grows on its edges. The load transmitter JT passes the cutting power P as P_m, after the load-signal filter τ_P and delay θ_P, to the adaptive feed loop that sets the override u, and the vibration transmitter VT passes the deflection y as v to the chatter watch that sets the speed command Ω_{cmd}. The hatched insert raises the specific cutting force K_c by the ratio r_h. (b) In plan view the tool of radius R and $N = 3$ teeth at phase θ removes a band of radial engagement a_e over the engagement angle ϕ_e, climb milling, and deflects by y away from the stock. (c) The tool mode m, k, c is driven by $F_y = \mu F_t$, and the chip h lies between the surface $u(t-T)$ the previous tooth left and the path the present tooth cuts.

Table 13.1 Variables and Their Roles in the Default Uniform Cut

Symbol	Role	Nominal	Range
$\bar{P}$	CV, spindle load	0.64 kW	SP 0.2–8 kW
CR	CV, chatter indicator	3 %	SP 0
u	MV, feed override	100 %	10–150 %
Ω_{cmd}	MV, speed command	20 000 rpm	3000–24 000
y, $\dot{y}$	states, deflection and rate	2.5 µm, 0	—
Ω	state, spindle speed	20 000 rpm	—
s, v_f	states, path and feed	760 mm, 80 mm/s	0–1044 mm
θ	state, tooth phase	0	0–2π
VB	state, flank wear	50 µm	start 0–400 µm
a_p	setting, axial depth	1.5 mm	0.5–12 mm
F	setting, programmed feed	4800 mm/min	600–9000
a_e	DV, engagement on the path	6 mm	0–12 mm
r_h	DV, insert force ratio	1	1–2.5
L	DV, tool overhang	40 mm	30–60 mm
θ_P	DV, load-signal delay	0 ms	0–200 ms
—	DV, tap on the tool	60 mm/s	event
P_m	measured load	0.64 kW	noise 20 W
v	measured vibration	2.5 µm	noise 0.2 µm

13.3.1 Pocket, Toolpath and Engagement

The pocket is cleared in one layer by a contour-parallel outward spiral, an entry slot 60 mm long along the middle followed by four rectangular loops at a 6 mm stepover, half the tool diameter. Each

loop starts and ends at the middle of its right side, the toolpath is 1044 mm long, and the loops run counterclockwise, which with a clockwise spindle is climb milling. The radial engagement $a_e(s)$ is measured once, before any run, by moving the tool disk along the path over a 0.2 mm material grid, counting the cells removed in each 0.2 mm of travel and smoothing the result with a 1 mm moving average. On a straight the tool cuts a 6 mm band, $a_e = R$, and over the last ten millimeters into each inside corner the arc of contact grows until it cuts a full slot, $a_e = 2R$. The engagement angle, the average number of teeth in the cut and the mean chip are then computed from it by Equation (13.1).

$$\phi_e = \arccos\!\left(1 - \frac{a_e}{R}\right), \quad N^* = \frac{N\phi_e}{2\pi}, \quad \bar{h} = f_z \frac{a_e}{R\,\phi_e} \tag{13.1}$$

for a tool of $N = 3$ teeth and radius $R = 6$ mm, with $f_z = 2\pi v_f/(N\Omega)$ the feed per tooth and $\bar{h}$ the chip thickness averaged over the engaged arc. A straight has $\phi_e = 90^\circ$ and $N^* = 0.75$, a slot $\phi_e = 180^\circ$ and $N^* = 1.5$, and both have $\bar{h} = 0.637 f_z$, so a corner doubles the teeth in the cut at the same chip. A cast-iron insert 20 mm by 18 mm set into the bottom middle of the pocket is crossed by the bottom passes of loops 2, 3 and 4 when its force ratio r_h exceeds 1, and the fraction χ of the removed cells that lie inside it scales the specific cutting force.

13.3.2 Tool Mode

The tool bends along one mode normal to the machined surface, with the deflection y positive away from the material, as shown in Equation (13.2).

$$m\ddot{y} + c\dot{y} + ky = F_y, \quad k = k_0\left(\frac{L_0}{L}\right)^3, \quad m = m_0\,\frac{L}{L_0} \tag{13.2}$$

where L is the overhang, $L_0 = 40$ mm the reference overhang, $k_0 = 12\,\mathrm{N/\mu m}$ the tip stiffness at L_0, $m_0 = k_0/(2\pi f_{n0})^2 = 0.475$ kg with $f_{n0} = 800$ Hz, and $c = 2\zeta\sqrt{km}$ with $\zeta = 0.03$. The stiffness falls with the cube of the overhang and, because the modal mass grows in proportion to it, the natural frequency falls with its square. At 50 mm the tool has 6.14 N/µm and 512 Hz.

13.3.3 Regenerative Chip and Contact Loss

With $u(t)$ the surface left at time t, measured in the same coordinate as y, the chip the tool takes is given by Equation (13.3).

$$\begin{aligned} h(t) &= h_s(t) + u(t-T) - y(t), \qquad T = \frac{2\pi}{N\Omega} \\ h_s(t) &= \bar{h}\,(1 + \beta_1\cos\theta + \beta_2\cos 2\theta) \end{aligned} \tag{13.3}$$

where T is the tooth period, the tooth phase obeys $\dot{\theta} = N\Omega$, and $\beta_1 = 0.15$ and $\beta_2 = 0.05$ give a two-harmonic ripple at the tooth-passing frequency that stands for the teeth entering and leaving the cut. The ripple places forced lines in the spectrum for a detector to reject and leaves the regenerative term untouched. The new surface follows Equation (13.4).

$$u(t) = \min\big(u(t-T) + h_s(t),\; y(t)\big) \tag{13.4}$$

While $h > 0$ the tooth cuts and leaves its own path as the new surface. When the vibration throws the tool clear, $h \le 0$, the tooth cuts nothing and the old surface is carried forward, which is the contact-loss nonlinearity that bounds chatter to a limit cycle [6], the mechanism of Section 1.4.9. In the uniform cut at 3 mm and 20 000 rpm that limit cycle is 60 µm rms, 1.2 times the 51 µm mean chip.

13.3.4 Cutting Force and Spindle Power

The tangential force follows the chip and the force along the mode is a fixed fraction of it, Equation (13.5).

$$F_t = \begin{cases} K a_p N^* h + K_{\text{rub}} VB\, a_p N^*, & h > 0 \\ 0, & h \le 0 \end{cases} \qquad K = K_c\,[1 + (r_h - 1)\chi]\,(1 + k_{VB} VB), \qquad F_y = \mu F_t \tag{13.5}$$

where a_p is the axial depth of cut, $K_c = 800\,\text{N/mm}^2$ the specific cutting force of the sharp tool in 7075-T6, $k_{VB} = 1.2\,\text{mm}^{-1}$ the rise of the cutting force with flank wear, $K_{\text{rub}} = 40\,\text{N/mm}^2$ a rubbing force per unit area of wear land and $\mu = 0.6$ the directional orientation factor of the average-tooth model [5]. The spindle load is the cutting power $P = F_t R\,\Omega$. Averaged over a tooth period on a steady cut, Equation (13.1) gives $N^* \bar{h} = N f_z a_e / (2\pi R)$, so the load is the specific cutting force times the removal rate, $\bar{P} = K a_p a_e v_f$, plus the rubbing term. On the 1.5 mm straight at 4800 mm/min the removal rate is 43.2 cm^3/min, a sharp tool would draw 0.58 kW, and the default 50 µm of flank wear raises the load to the 0.64 kW the application reports.

13.3.5 Stability Lobes

While the tool stays in the cut and the ripple is set aside, $u(t - T) = y(t - T)$, and Equations (13.2) to (13.5) reduce to a linear delay differential equation with the characteristic equation shown in Equation (13.6).

$$1 + \mu K a_p N^* \left(1 - \mathrm{e}^{-sT}\right) G(s) = 0 \tag{13.6}$$

where $G(s) = 1/(ms^2 + cs + k)$ is the receptance of the tool mode. The exponential gives the equation infinitely many roots [8]. Setting $s = j\omega_c$ yields the depth at which a root crosses the imaginary axis and the chatter frequency there, Equation (13.7).

$$a_{\text{lim}} = \frac{-1}{2\mu K N^* \operatorname{Re} G(j\omega_c)}, \qquad \omega_c T = 3\pi + 2\psi + 2\pi k \tag{13.7}$$

where $\psi \in (-\pi, 0)$ is the phase of $G(j\omega_c)$ and $k = 0, 1, 2, \ldots$ numbers the lobe. For each lobe the phase condition has one root above the natural frequency, which the application finds by bisection, and the boundary is the lowest lobe, the one-mode form of the average-tooth result [5]. The feed does not appear. The lowest depth over all speeds is $a_{\text{crit}} = 2k\zeta(1 + \zeta)/(\mu K N^*)$, which is Merritt's criterion of half the minimum dynamic stiffness [2], and it is 1.94 mm on the straight. The boundary for the straight at two overhangs is shown in Figure 13.2. At 20 000 rpm the limit is 2.10 mm with chatter predicted at 816 Hz, and at 16 300 rpm, where the 815 Hz tooth-passing frequency sits just above the natural frequency, it is 17.9 mm. The 50 mm tool moves the pockets to lower speeds, and its limit at 16 300 rpm is 1.19 mm.

Pause. Try Exercise 13.1 before reading on.

13.3.6 Spindle, Feed Axis and Wear

The spindle obeys Equation (13.8).

$$J\dot{\Omega} = T_d - F_t R - c_\Omega \Omega, \qquad |T_d| \le \min\left(T_{\max}, \frac{P_{\max}}{\Omega}\right) \tag{13.8}$$

where $J = 0.004\,\text{kg m}^2$, $T_{\max} = 10\,\text{N m}$ and $P_{\max} = 5.5\,\text{kW}$, so above the base speed of 5252 rpm the drive is power limited, and $c_\Omega = 2 \times 10^{-5}\,\text{N m s}$ is the bearing drag. The feed velocity follows its reference through a servo lag, $\tau_F \dot{v}_f = v_{\text{ref}} - v_f$ with $\tau_F = 5\,\text{ms}$, and $\dot{s} = v_f$. Flank wear grows while the tool cuts, as given by Equation (13.9).

$$\frac{\mathrm{d}VB}{\mathrm{d}t} = w_0 \left(\frac{\Omega R}{v_0}\right)^2 \min\left(1, \frac{N^*}{0.75}\right) \tag{13.9}$$

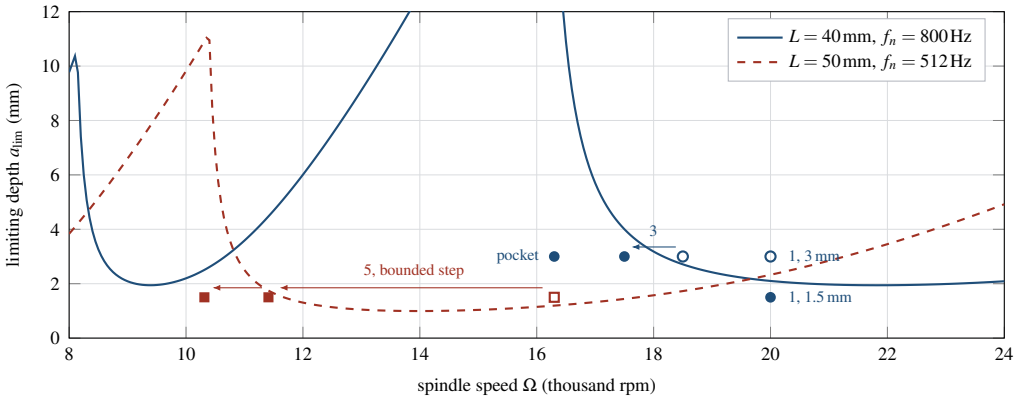


Figure 13.2 Stability lobes of a 50 % straight, $a_e = 6$ mm, from Equation (13.7) with the default 50 µm of flank wear. The solid line is the boundary of the 40 mm tool and the dashed line that of the 50 mm tool. Circles are operating points of the 40 mm tool and squares of the 50 mm tool, filled where the cut is stable and open where it chatters. Case 1 is stable at 1.5 mm and chatters at 3 mm, case 3 steps from 18 500 to 17 500 rpm at 3 mm, and case 5 takes a bounded step from 16 300 to 11 410 rpm and then moves to 10 314 rpm. The pocket near 16 300 rpm rises above the frame to 17.9 mm.

with $w_0 = 2\,\mu\text{m/s}$ at the reference surface speed $v_0 = 750\,\text{m/min}$, a rate the application accelerates about a thousandfold over a real tool so that wear shows within one pocket.

13.3.7 Measurements

Neither controller reads the state. The spindle load $\bar{P}$ is the cutting power through the drive's 10 ms average, and it is what the load trend and the scorecard report. The feed loop reads P_m, that average passed through a first-order load-signal filter of $\tau_P = 20$ ms, delayed by θ_P and corrupted by white noise of 20 W. The vibration sensor reads $v = y + n_v$ at every physics step, with n_v white noise of 0.2 µm, and the encoder gives the spindle speed exactly.

13.3.8 Numerical Solution

The engine's fourth-order Runge-Kutta integrator (Section 1.6.1) advances the equations in steps of 20 µs, 50 steps per tooth period at 20 000 rpm. The surface u is a ring buffer of 2048 entries written once per completed step, and every Runge-Kutta stage reads $u(t - T)$ by linear interpolation between the two entries that bracket it, with T computed from the spindle speed of that stage. At the top speed of 24 000 rpm a tooth period of the three-tooth tool spans more than 41 steps, so every stage query falls inside the stored history and nothing is extrapolated. The engagement, the cutting coefficient and the mean-chip factor are refreshed once per step and held through its four stages, because the tool moves about 2 µm in a step while the engagement changes over millimeters. The surface update (13.4), the wrap of the tooth phase and the vibration sensor run after each step, and every random draw is made outside the derivative function, so the right-hand side stays deterministic within a step. The controllers execute every 1 ms, the historian logs at 100 Hz, and the raw vibration is kept in its own 4096-sample buffer for the scope and the spectrum. A flying start places the tool at the case's starting point on the path at the commanded feed and speed, with the static deflection $\mu F_t / k$ of the steady cut, the surface history filled with that deflection and $\theta = 0$, and it seeds the drive's load averages, the load signal and $\hat{b}$ at their steady values. A start at the entry begins at rest with $v_f = 0$ and $y = 0$. Settings changed before the first step are part of the program and re-seed this initial state as Reset does (Section 1.6.1), so a setting made before the run and the same setting carried through Reset give one run.

Table 13.2 Model Parameters. Class Marks a Value the Application Lets the User Adjust (adj.), One It Keeps Hidden, and One Fixed in Its Code

Symbol	Meaning	Value	Class
N, R	teeth, tool radius	3, 6 mm	adj., fixed
L	tool overhang	40 mm	adj.
f_{n0}, k_0	mode at $L_0 = 40$ mm	800 Hz, 12 N/µm	adj.
ζ	damping ratio	0.03	adj.
K_c	specific cutting force, sharp	800 N/mm^2	adj.
μ	directional factor	0.6	hidden
β_1, β_2	tooth-pass ripple	0.15, 0.05	hidden
VB_0	flank wear at the start	0.05 mm	adj.
k_{VB}	force rise with wear	1.2 mm^{-1}	adj.
K_{rub}	rubbing force per wear area	40 N/mm^2	adj.
w_0	wear rate at 750 m/min	2 µm/s	adj.
J	spindle inertia	0.004 kg m^2	hidden
$P_{\max}$, $T_{\max}$	spindle ratings	5.5 kW, 10 N m	adj.
$\alpha_{\max}$	spindle acceleration	8000 rpm/s	adj.
K_ω	drive speed bandwidth	100 rad/s	hidden
c_Ω	bearing drag	2×10^{-5} N m s	hidden
$a_{\max}$	feed axis acceleration	3 m/s^2	adj.
τ_F	feed servo lag	5 ms	hidden
τ_P, θ_P	load-signal filter, delay	20 ms, 0 ms	adj.
σ_P, σ_v	load, vibration noise	20 W, 0.2 µm	adj.
—	load cap (challenge)	5.5 kW (2.2 kW)	adj.
—	waviness mark threshold	5 µm	adj.
—	drive load average	10 ms	fixed
—	drive torque estimate	2 ms	fixed
Δt, Δt_c	physics step, control period	20 µs, 1 ms	fixed

The delay treatment was checked against Equation (13.7). A tap on the tool on a clean straight, with the ripple, the noise and the wear removed, gives a free vibration whose logarithmic envelope grows or decays, and bisection on the depth places the change of sign at 2.103 mm against the analytic 2.101 mm at 20 000 rpm, 0.095 % apart, and at 6.011 mm against 6.007 mm on the $k = 1$ lobe at 12 000 rpm, 0.064 % apart. Halving the step to 10 µs changes the growth rate at $1.1\,a_{\text{lim}}$ by 0.79 % and the amplitude of a fully developed chatter limit cycle by 0.071 %. The parameters are listed in Table 13.2.

13.3.9 Assumptions

The force is the average-tooth, or zeroth-order, force. Real milling forces vary with the angle of every tooth, and below about a quarter of the diameter of radial engagement that variation adds period-doubling lobes and shifts the boundary [11]. The pocket is cut at half and full engagement, where the average is closer, although the thinning engagement at the end of each loop lies outside that range.

One bending mode along the chip-thickness direction represents the tool, with a fixed directional factor of 0.6. A real tool has two orthogonal modes, the holder and the spindle add their own, and the effective direction turns with the cut [4].

The cutting force is linear in the chip, with no size effect and no process damping. Process damping from the flank rubbing on the surface waves raises the limit sharply at low surface speed [4], so the lobes below about 5000 rpm are pessimistic.

The engagement is measured once from the planned toolpath. The vibration does not change what the next pass sees in plan view, and the axes take every corner at the commanded feed with no corner slowdown.

Wear is accelerated about a thousandfold and acts linearly, raising K by $k_{VB}VB$ and adding a rubbing force proportional to the wear land.

The load signal is the drive's estimate of the cutting power, and the vibration sensor reads the tool deflection with white noise and no dynamics of its own. A real accelerometer or microphone sees the tool through the spindle housing, and the workpiece and the machine frame are rigid here.

13.4 Two Controllers on One Machine

Three models should be kept apart. The simulation model is the delay equation with contact loss of Section 13.3. The feed loop's internal model is a static gain, the load per unit override, identified on line. The chatter watch has no model of the tool at all, and uses only the measured chatter frequency. The lobe diagram the application draws is computed from Equation (13.7) for display and is read by neither controller.

13.4.1 Spindle Drive and Feed Axis

The actuators are machine hardware and are not tuned by the user. The speed command is clamped between 3000 and 24 000 rpm, its reference ω_{ref} slews at $\alpha_{\max} = 8000\,\mathrm{rpm/s}$, and the drive torque is $T_d = J[K_\omega(\omega_{\mathrm{ref}} - \Omega) + \dot{\omega}_{\mathrm{ref}}] + \hat{T}_L + c_\Omega\Omega$, held inside the limit of Equation (13.8), where $K_\omega = 100\,\mathrm{rad/s}$ and $\hat{T}_L$ is a 2 ms average of the cutting torque. The feed command is $v_{\mathrm{cmd}} = \min(uv_{\mathrm{prog}}, \sqrt{2a_{\max}(s_{\mathrm{end}} - s)})$, where v_{prog} is the programmed feed F, so the axis stops exactly at the end of the path, and its reference slews at $a_{\max} = 3\,\mathrm{m/s^2}$. If the speed falls below half its reference the spindle is declared stalled and the feed stops.

13.4.2 Adaptive Feed on the Measured Load

To first order the feed loop sees a static gain from override to load, $b = Ka_p a_e v_{\mathrm{prog}}$, behind the feed-axis lag, the drive's 10 ms average, the 20 ms load-signal filter and any delay θ_P. On the default straight $b = 0.64\,\mathrm{kW}$ per unit override, and it doubles in a slot or a corner. Each execution, every $\Delta t_c = 1\,\mathrm{ms}$, first updates the identified gain, Equation (13.10).

$$\hat{b}_k = \mathrm{clamp}\Big(\hat{b}_{k-1} + \frac{\Delta t_c}{\tau_b}\Big(\frac{P_{m,k}}{\tilde{u}_k} - \hat{b}_{k-1}\Big),\ b_{\mathrm{lo}},\ b_{\mathrm{hi}}\Big) \tag{13.10}$$

where $\tilde{u}$ is the applied override passed through the same filter τ_P and the same delay θ_P as the load signal, though not through the drive's 10 ms average, $\tau_b = 0.1\,\mathrm{s}$, $b_{\mathrm{lo}} = 0.2\,\mathrm{kW}$ and $b_{\mathrm{hi}} = 20\,\mathrm{kW}$, and the update runs only while $P_m > 100\,\mathrm{W}$ and $\tilde{u} > 0.05$. The proportional gain is then $K_{P,k} = K_f/\hat{b}_k$, so the loop gain, the product of $K_{P,k}$ and the true b, stays near the dimensionless K_f on a straight, in a slot and at a corner. With adaptation switched off, $\hat{b}$ is replaced by the sharp-tool gain of a 50 % straight, $K_c a_p R\, v_{\mathrm{prog}}$.

The loop runs the velocity-form PID algorithm of Section 1.6.2 with a zero derivative time, written for this loop in Equations (13.11) to (13.14), where $\hat{u}$ is the override before the clamp and p the proportional term.

$$e_k = P_{\mathrm{SP}} - P_{m,k}, \qquad p_k = K_{P,k}\, e_k \tag{13.11}$$

$$S_k = S_{k-1} + (p_k - p_{k-1}) \tag{13.12}$$

$$\hat{u}_k = S_k + I_{k-1}, \qquad u_k = \mathrm{clamp}(\hat{u}_k, 0.1, 1.5) \tag{13.13}$$

$$I_k = I_{k-1} + \Big[\frac{K_{P,k}}{\tau_I}\, e_k + \frac{u_k - \hat{u}_k}{T_t}\Big]\Delta t_c \tag{13.14}$$

These are Equations (1.12) to (1.15) with S_k never clamped, so S_k equals p_k at the gain in force and a change of $\hat{b}$ reaches the whole proportional term at once. The back-calculation uses the tracking time $T_t = \tau_I$, and with no bias the integral carries the initial override from the reset. No slew limit is placed on the override, because the feed axis ramps at its own acceleration limit. The error is $P_{\mathrm{SP}} - P_m$, so a load above target lowers the override, the reverse-acting sense that a positive process gain requires. The defaults are $K_f = 0.6$ and $\tau_I = 50\,\mathrm{ms}$ with adaptation on.

Three conditions replace the update by tracking, in which the integral is set so that $S_k + I_k$ equals the held override exactly. Below $0.2\,P_{SP}$ the tool is taken to be cutting air, the override is held at $\min(u_{k-1}, 1)$, and the loop leaves the air cut without the jump to 150 % that an unattended integrator would bring to the next engagement. On the stopping ramp at the end of the path the axis, not the override, sets the feed, and the override is frozen. With the feed control set to Constant, or in MANUAL, the override is the operator's value. Switching to Adaptive is therefore bumpless. In AUTO each manual station follows its controller's output (Section 1.6.2), and the header MANUAL and AUTO switch moves both loops at once, so a loop handed to the operator starts from the value it last applied. Reset keeps the selected modes. Returning to AUTO restores the feed and spindle modes selected before the handover, including after Reset, while a freshly loaded case supplies its own modes. AUTO does not enable Adaptive feed or Chatter control unless that mode was selected.

13.4.3 Chatter Detection

The chatter watch runs at the physics rate. Forced vibration repeats every tooth and chatter does not, so the watch subtracts a synchronous average of the vibration kept in 32 bins of tooth phase and compares the energy of the remainder with the energy of the whole, as shown in Equation (13.15).

$$\begin{aligned} r_n &= v_n - \bar{v}_{b(n)}, \qquad b(n) = \lfloor 32\,\theta_n / 2\pi \rfloor \\ CR &= \min\left(1,\ \frac{\overline{r^2}}{\overline{(v-\bar{v})^2} + E_0}\right) \end{aligned} \tag{13.15}$$

where each bin averages its first 20 visits equally and then updates with a weight of 0.05, about 13 tooth periods of memory at 20 000 rpm, the overbars are first-order averages of 20 ms, $\bar{v}$ is a 50 ms running mean, and $E_0 = (1\,\mu\text{m})^2$ keeps a quiet cut from reading as chatter. The chatter frequency f_c is the count of upward zero crossings of r in the last 40 ms, less one, over the time between the first and the last, with a hysteresis of $\max(3\sigma_v, 0.3\,r_{rms})$ and at least six crossings. Chatter is declared when CR stays above the threshold $CR_{th} = 40\,\%$ for a dwell of 40 ms. The dwell may run only after 50 ms of learning, and a change of the speed command by more than 1 % clears the bins so that the new tooth-pass pattern is learned again. This is the energy-ratio idea of [15], with a synchronous average in place of a Kalman filter.

13.4.4 Speed Selection

When chatter is declared with f_c between 150 and 4000 Hz, the spindle is moved to a speed at which a whole number j of chatter waves fits between two teeth, Equation (13.16).

$$\Omega_j = \frac{60 f_c}{N j}, \qquad j \in \{\lfloor q \rfloor - 1,\ \lfloor q \rfloor,\ \lceil q \rceil,\ \lceil q \rceil + 1\} \tag{13.16}$$

with speeds in rpm, $j \geq 1$ and $q = 60 f_c / (N\Omega)$ the ratio of the chatter frequency to the tooth-passing frequency. At Ω_j the tooth period holds exactly j periods of the measured chatter, so the wave a tooth leaves is met in phase by the next and the chip variation it regenerates vanishes, the principle of spindle speed regulation [13]. Because f_c lies just above the natural frequency, Ω_j falls near the top of the stable pocket between lobes $j-1$ and j. Among the candidates inside the speed range the one nearest the present speed is chosen, and if it is more than the largest step, 30 % by default, the spindle takes the bounded step toward it instead. The supervisor then waits until the speed is within 1 % of its target, waits a further 0.15 s, resets the dwell and watches again.

🕮 **Pause.** Try Exercise 13.2 before reading on.

13.4.5 Finish and Scoring

Once per tooth pass the true deflection is recorded, and the waviness is the rms residual of the last 16 records about their least-squares line, so a ramp in the deflection where the engagement changes is

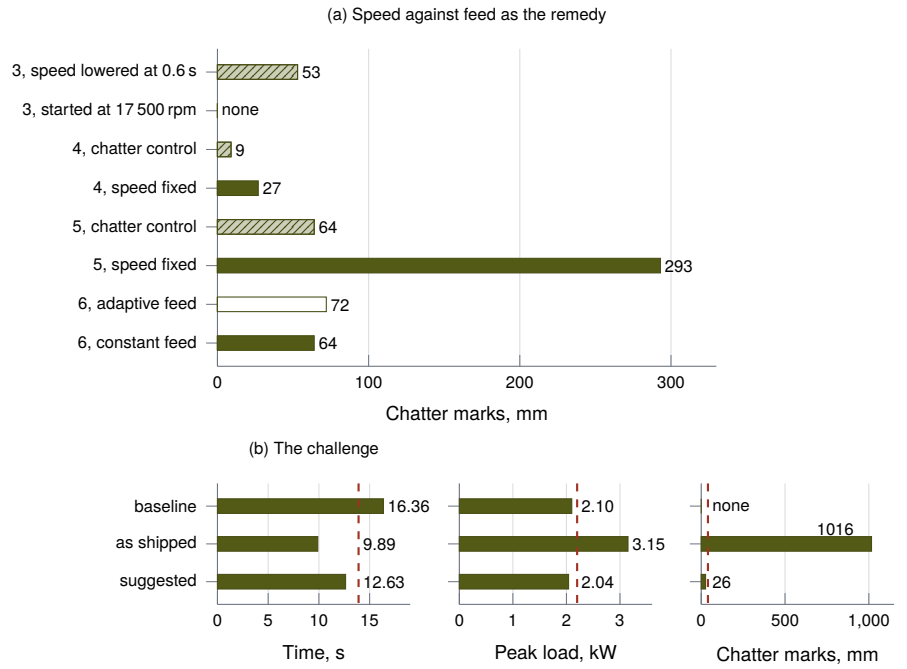


Figure 13.3 Rows of Table 13.4. (a) Chatter marks on cases 3 to 6: hatched bars where the spindle speed was changed or chosen, solid bars where nothing acted on the chatter, and the open bar where the adaptive feed slowed the cut. (b) The challenge, with its targets of 13.91 s, 2.2 kW and 40 mm as dashed lines. Notice that in cases 4 and 5 the speed change leaves far fewer marks than a fixed speed, while the slower feed of case 6 leaves more marks than a constant feed, and that only the suggested configuration meets all three targets.

not counted as waviness. A millimeter of path is marked as chattered when the waviness stays above 5 µm for 24 consecutive tooth passes, which the free ring-down after a corner cannot sustain, and only where material is being removed. The scorecard reports the machining time, the peak of the 10 ms load, the length of chatter marks, the removal rate, the peak rms vibration, the mean waviness, the flank wear and the number of speed changes. Beside them the framework's scorecard (Section 1.6.4) scores the load loop, and the chatter indicator as a loop with setpoint zero.

13.5 Results on the Pocket

Table 13.3 lists the cases. Each one cuts a single stretch of the pocket from a flying start, except case 8 and the challenge, which begin at the entry hole. The values in Table 13.4 were measured on the running engine at the default seed (Section 1.6.4), with each case as shipped unless the condition names a change made before the run. Figure 13.3 draws the chatter marks of cases 3 to 6 and the three challenge rows of the table.

13.5.1 The Load Scales and the Stability Does Not

Case 1 runs the long top side of the outer loop, a 92 mm straight at 50 % engagement, at 1.5 mm, 20 000 rpm and 4800 mm/min, a feed of 0.08 mm per tooth. The load is 0.64 kW, and the only vibration is forced, 0.5 µm rms at the 1000 Hz tooth-passing frequency, with the chatter indicator at 3 % and the waviness at most 0.33 µm. Raised to 3 mm the load doubles to about 1.3 kW, but the depth is now 43 % over the 2.10 mm limit. The waves grow until the tool loses contact for part of each cycle, and the cut settles into chatter at 823 Hz, just above the 800 Hz mode and off the tooth line,

Table 13.3 The Eight Cases and the Challenge

#	Stretch and settings	Teaching point
1	loop 4 top, 1.5 mm, 20 000 rpm, constant feed	load against limit
2	loop 3, 1.5 mm, adaptive feed at 0.65 kW	the corner overshoot
3	loop 4 top, 3 mm, 18 500 to 17 500 rpm at 0.6 s	the lobes
4	loop 4 bottom, insert $r_h = 1.6$, 3.5 mm, both loops	feed for load, speed for chatter
5	loop 4, $L = 50$ mm, 16 300 rpm, chatter control	lobes belong to the tool
6	loop 4 top, $VB_0 = 0.25$ mm, 1.8 mm, adaptive at 0.77 kW	load restored, finish not
7	loop 3, $\theta_P = 80$ ms, 3 mm, 16 300 rpm, $K_f = 1.2$, $\tau_I = 30$ ms	dead time in the feed loop
8	slot and loop 1, 8 mm, 16 300 rpm, constant feed	the power limit
C	whole pocket, 3 mm, insert, 2.2 kW cap, 20 000 rpm, adaptive at the cap	both loops together

Table 13.4 Results Measured on the Running Engine

Case and condition	Result
1, 1.5 mm	0.64 kW, 0.5 µm rms at 1000 Hz, *CR* 3 %, limit 2.10 mm
1, 3 mm	chatter 823 Hz, 60 µm waviness, 88 mm of 92 mm marked
1, 3 mm at 16 300 rpm	8.8 µm rms at 815 Hz, waviness 0.7 µm, no marks
2, constant / adaptive	corner peak 1.27 / 0.87 kW, time 3.38 / 3.65 s
2, $K_f = 1.2$, $\tau_I = 30$ ms	corner peaks 0.74–0.78 kW, run peak 0.84 kW at the step-out
3, as shipped	chatter 812 Hz, 69 µm, 53 mm marked, below 2 µm 0.21 s after the change
3, start at 17 500 rpm	at most 2.1 µm, no marks
4, as shipped	override 63 %, peak 1.97 kW, 811 Hz to 16 212 rpm, 9 mm marked
4, speed fixed / feed also constant	27 mm, 114 µm / peak 2.50 kW
5, as shipped	546 Hz to 11 410 rpm, 516 Hz to 10 314 rpm, 64 mm marked
5, speed fixed	293 mm marked
6, adaptive / constant	71 %, load within 1.6 %, 32 µm, 72 mm, 1.65 s / 1.09 kW, 42 µm, 64 mm, 1.19 s
7, θ_P 80 ms / detuned / 0 ms	peak 3.65 / 3.19 / 2.08 kW
8, constant / adaptive	6.78 kW, 14 675 rpm / 6.29 kW, 16 227 rpm at the first corner
C, baseline	16.36 s, 2.10 kW, no marks
C, as shipped	9.89 s, 3.15 kW, 1016 mm marked
C, suggested	12.63 s, 2.04 kW, 26 mm marked

with 60 µm of waviness and 88 mm of the pass marked. At 16 300 rpm the same 3 mm cut leaves no marks, because the limit there is 17.9 mm. The tool shakes harder, 8.8 µm rms, because the teeth now pass at 815 Hz, close to its natural frequency, but every tooth shakes it the same way, the indicator reads 0.4 %, and the waviness stays at 0.7 µm. The load scaled with the depth and the stability did not.

The deepest cut that leaves no marks can also be found by hand. Mapped in case 1 at a series of fixed speeds, it traces the lobe of Figure 13.2 to within 2 % between 17 000 and 22 000 rpm. From 14 000 to 16 000 rpm the pocket lies above the 12 mm depth the control allows, so no depth on the slider chatters there.

13.5.2 A Corner Arrives Before the Loop Sees It

Case 2 runs once around the third loop at 1.5 mm and ends with the 6 mm step across into the band of the outer loop. At constant feed the corners reach 1.27 kW against 0.64 kW on the straights. With the adaptive feed holding 0.65 kW, the override falls to between 56 and 58 % into each corner and the load is held, but the loop learns of a corner only through the drive's average and the load-signal filter, after the engagement has begun to grow, so each corner still peaks at 0.86 to 0.87 kW, 33 to 34 % over target. A gain of 1.2 with a 30 ms integral time pulls the override to 43 to 45 % at the corners and holds their peaks to 0.74 to 0.78 kW, 15 to 20 % over. Where the loop closes on its own start the engagement thins to air, and the override runs to 150 % until the air-cut hold returns it to about

100 %. The step across that follows is nearly a full slot met at that held override. It sets the lowest override of the run, 44 % with the stock tuning and 26 % with the faster one, and for the faster tuning also the highest load, 0.84 kW. At this target the adaptive run is slower, 3.65 s against 3.38 s, because it slows for every corner while the straights need only 100 %. The time is won with a higher target, as the challenge shows.

13.5.3 Five Percent of Spindle Speed

Case 3 cuts the top side at 3 mm and lowers the speed from 18 500 to 17 500 rpm at 0.6 s, a change the spindle makes in 0.125 s at its acceleration limit. The limit is 2.71 mm at the first speed, with chatter predicted at 810 Hz, and 4.00 mm at the second, as shown in Figure 13.2. At 18 500 rpm the cut is 11 % over its limit, the waviness passes 5 µm at 0.11 s, the first mark is recorded at 0.14 s once it has stayed above that level for 24 consecutive teeth, and chatter at 812 Hz grows to 69 µm of waviness. After the change the cut is 25 % under its limit, and the waviness is below 2 µm at 0.81 s, 0.21 s after the command, with 53 mm of the pass marked. Started at 17 500 rpm, the same cut leaves at most 2.1 µm and no marks. The load is 1.3 kW both ways.

> **TRY IT IN THE APP 3 · Stable and Unstable Speeds**
>
> Load the scenario, press Run and watch the spectrum and the vibration scope as the program lowers the speed from 18 500 to 17 500 rpm at 0.6 s. Chatter at 812 Hz grows to 69 µm of waviness, and 0.21 s after the change the waviness is below 2 µm. The load is 1.3 kW both ways.

13.5.4 Two Faults at the Insert

Case 4 crosses the cast-iron insert, 1.6 times harder, at 3.5 mm and 17 500 rpm with both loops on. The limit is 4.00 mm in the aluminum and 2.50 mm in the iron. At the insert the load jumps by 60 %, the feed loop pulls the override to 63 %, and the load peaks at 1.97 kW against the 1.5 kW target. The cut is then 40 % over its new limit. Eleven millimeters after the tool meets the iron, the chatter watch declares chatter at 811 Hz and moves the spindle to 16 212 rpm, where one chatter wave fits between two teeth, and the pass ends with 9 mm of marks. With the speed fixed the insert chatters to its end, with 27 mm of marks and a worst waviness of 114 µm, and with the feed constant as well the load reaches 2.50 kW. Each loop answers the fault it can see.

> **TRY IT IN THE APP 4 · A Harder Patch**
>
> Press Run and watch the load, the feed override and the chatter indicator as the tool meets the iron. The override falls to 63 % and the load peaks at 1.97 kW, then the chatter watch moves the spindle to 16 212 rpm and the pass ends with 9 mm of marks. Press Reset, set Spindle speed to Fixed and run again: the marks grow to 27 mm.

13.5.5 A Longer Tool and a Worn One

Case 5 fits a tool with 50 mm of overhang, 6.14 N/µm and 512 Hz, and starts it at 16 300 rpm, the best speed of the short tool, where the long tool's limit is 1.19 mm. At 1.5 mm it chatters at once, and at 0.09 s the watch reads 546 Hz, near its own mode. The nearest whole-wave speed, 10 925 rpm, is 33 % away, so the first move is the bounded step to 11 410 rpm. On the straight that speed is linearly stable, with a limit of 1.75 mm, and a run started there leaves no marks. The chatter grown during the ramp through unstable speeds is still decaying when the 0.15 s settling time ends, and the indicator still reads above 90 %, so at 0.88 s the watch reads 516 Hz and moves on to 10 314 rpm, where the limit is 10.9 mm. The loop ends with 64 mm of marks, against 293 mm at the fixed speed. The energy ratio cannot tell a decaying chatter from a growing one, which is why the second move happens.

Pause. Try Exercise 13.3 before reading on.

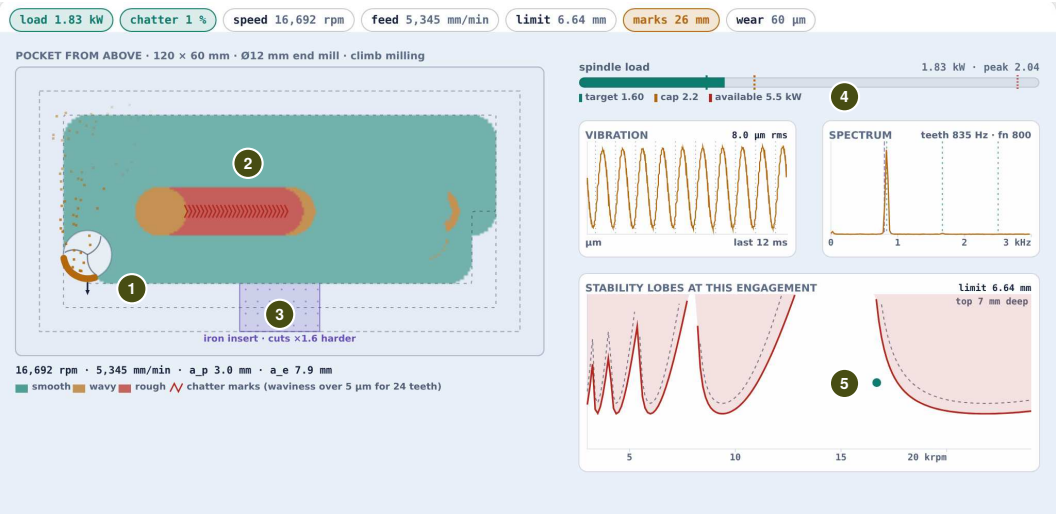


Figure 13.4 The CNC milling application at $t = 7.50\,\mathrm{s}$ of the challenge with the suggested configuration, a 1.6 kW load target, $K_f = 1.5$, $\tau_I = 30\,\mathrm{ms}$ and chatter control, cropped to the plant view. The pocket seen from above shows the start of the entry slot along the middle painted rough and crossed with zigzag chatter marks (2), the first two loops and most of the third cleared smooth around it, the tool on the left side of the third loop heading into its lower corner with its arc of contact drawn (1), and the cast-iron insert below (3). Beside it the load bar (4) reads 1.83 kW against the 1.60 kW target and the 2.2 kW cap with a 2.04 kW peak, the scope and spectrum show 8.0 µm rms of forced vibration at the 835 Hz tooth-passing frequency beside the 800 Hz natural frequency, and the lobes for the present 7.9 mm engagement place the operating point (5) at 16 692 rpm and 3 mm in a stable pocket below its 6.64 mm limit.

Case 6 runs a tool with 0.25 mm of flank wear on the top side at 1.8 mm and 20 000 rpm. The wear raises K by 30 % over a sharp edge, the load at constant feed peaks at 1.09 kW against the 0.77 kW of the default tool, and the limit falls from 2.10 to 1.71 mm. The adaptive feed settles near 71 % and holds the load within 1.6 % of 0.77 kW, and the cut chatters anyway, with 32 µm of waviness and 72 mm of marks, against 42 µm and 64 mm at constant feed. The slower feed made the waves about a quarter shallower, since the limit cycle scales with the chip, and made the pass 39 % longer, 1.65 s against 1.19 s, while the limit did not move.

> **DID YOU KNOW? A Steady Load Over a Chattering Cut**
>
> From the simulator: with a worn tool the adaptive feed holds the load within 1.6 % of its target, yet the pass leaves 72 mm of chatter marks against 64 mm at constant feed (Table 13.4). A load held on target says nothing about the finish.

13.5.6 Dead Time and the Power Limit

Case 7 delays the load signal by 80 ms on the third loop at 3 mm and 16 300 rpm, where the cut is stable, with a tuning that holds a 1.7 kW target to a 2.08 kW corner peak when the signal is prompt. With the delay the override swings between its 10 % and 150 % limits and the load peaks at 3.65 kW in the first corner. Detuned to a gain of 0.3 and an integral time of 200 ms, the loop stops swinging and the override stays above 87 %, but it is then too slow for the corners and they peak at 3.19 kW. Neither tuning comes near the 2.08 kW of the prompt signal, because dead time takes phase without taking gain. Volume 5, Chapter 3 measures the same loss in a composition loop, where a tuning made for a prompt measurement fails once the analyzer is slow.

Case 8 spends the depth that the pocket at 16 300 rpm allows. At 8 mm, below the slot's limit of 8.9 mm, the entry slot needs 6.78 kW from a 5.5 kW spindle. The drive holds its torque limit for 1.64 s, the rest comes out of the rotor, and the speed sags to 14 675 rpm, 10 % down, with every tooth then taking a thicker chip. With the adaptive feed at a 4.5 kW target the override falls to 51 % at the entry and holds near 65 % along the slot, the entry peaks at 5.41 kW, and through the slot the speed stays within 20 rpm of its command. The overload moves to the first inside corner of loop 1, which the loop meets at the override of about 134 % that the half-width straight needs, so the load reaches 6.29 kW there and the speed dips to 16 227 rpm. Once chatter is out of the way, power sets the removal rate and a load loop is still needed. The feed rate of a combine harvester in Volume 4, Chapter 4 meets the same kind of limit, an engine that slows everything it drives when the crop asks for more power than it has.

13.5.7 The Graded Challenge

The challenge clears the whole pocket at 3 mm with the insert under the bottom passes. The fixed-feed baseline, 80 % override at 16 300 rpm, finishes in 16.36 s with a 2.10 kW peak, no marks and a removal rate of 78 cm^3/min. The targets are a time of at most 13.91 s, 85 % of that baseline, a peak load of at most 2.2 kW and at most 40 mm of marks, each scored as the worst case of the whole run. As shipped, with the adaptive target at the cap and the spindle at 20 000 rpm, the pocket takes 9.89 s, but the corners reach 3.15 kW and chatter marks 1016 mm of the 1044 mm path, and the load spends 1.99 s above the cap.

The suggested configuration lowers the target to 1.6 kW, raises the gain to 1.5 with a 30 ms integral time and turns the chatter control on. The chatter watch reads 835 Hz in the slot at 0.09 s, the learning time plus the dwell, and moves the spindle to 16 692 rpm. The 26 mm of marks all lie in the first 28 mm of the slot, and nothing else on the pocket chatters. At each of the three steps from an air cut into a new loop the indicator crosses its threshold for only 13 to 15 ms, inside the dwell. The highest load, 2.04 kW, comes at the first of those steps, where the tool leaves the air cut that closes loop 1 for the band of loop 2, while the insert peaks at 2.00 kW and the corners at 1.92 kW. The pocket is finished in 12.63 s (Table 13.4), 23 % faster than the baseline at 101 cm^3/min, and Figure 13.4 shows the run at 7.50 s. A 1.5 kW target peaks at 1.96 kW but takes 13.50 s, and 1.7 kW finishes in 11.88 s but peaks at 2.14 kW in the insert, 2.8 % under the cap. The target trades time against the cap, and the speed is a separate decision.

Judged by the challenge's own rule, the shipped configuration fails two of the three targets, with a peak 43 % over the cap and 1016 mm marked, while the suggested one passes with 1.27 s, 0.16 kW and 14 mm to spare. Changing a model parameter or the insert voids an attempt.

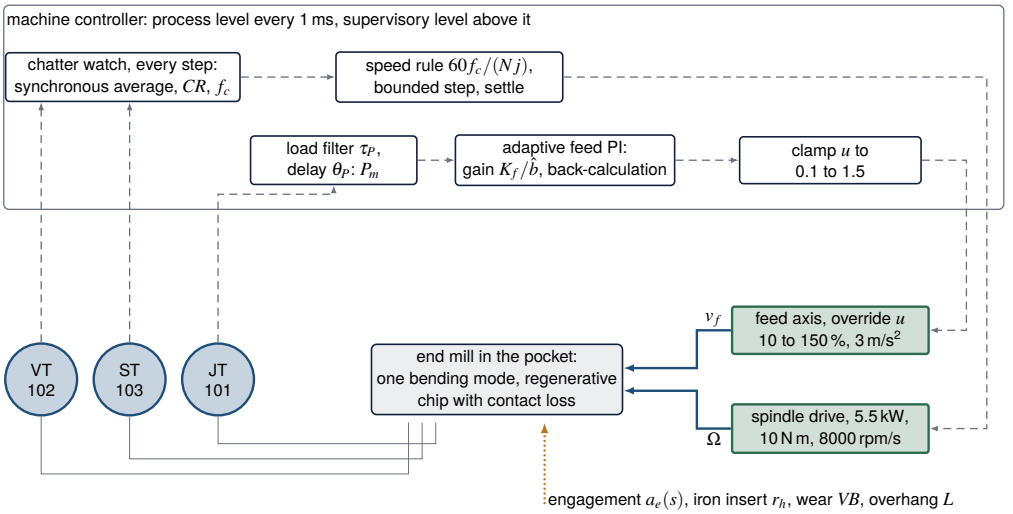


Figure 13.5 The two loops on machine-tool hardware. The load JT-101 passes through the load-signal filter and delay to the adaptive feed loop, whose clamped override drives the feed axis. The vibration VT-102 and the spindle speed from ST-103 feed the chatter watch, and the speed rule sends a new command to the spindle drive. Notice that the two loops share the cut but no signal: the feed loop never reads the vibration, and the speed rule never reads the load.

13.6 Technology in Practice

What Measures It

The two controllers of this chapter read two signals, a load and a vibration. In service the load usually comes from the spindle drive as motor current or power, and the vibration from a sensor on the machine or in the tool holder, or from a microphone [14, 17]. The chapter's load signal is the cutting power through the drive's 10 ms average, a 20 ms filter, an optional delay and noise, and its vibration signal is the tool deflection with white noise and no sensor dynamics.

> **TECHNOLOGY** **Sensor: Load and Vibration on the Machine**
>
> Reviews of machining monitoring rank cutting force and torque, motor current and power, vibration, temperature and acoustic emission as the most significant signals. Their sensors sit in the spindle and axis drives, on the frame, the table or the fixture, or in the tool holder, and they detect small changes better the closer they are to the cut [17]. The microphone named in Section 13.2 hears the whole room, and isolation, collection or intensity methods are needed to reject the background noise [14]. On a rotating tool holder the sensor's own rotation disturbed the acceleration signals, though their rms still marked chatter clearly [17]. The chapter's load signal stands for the drive's power estimate and its vibration signal for a sensor at the tool.

What Moves It

Two final elements share the cut. The feed override scales the programmed feed, and the feed axis follows it through its servo. The spindle drive turns the tool, and the chapter's drive is torque limited below its base speed and power limited above it. A speed change is made by the controller that sits above both and that, in a conventional numerical control, runs the program open loop with no process feedback [18].

Where the Controller Runs

Machining control is organized in servo, process and supervisory levels [9]. In an open machine controller architecture, the trajectory generator sends position and velocity setpoints at the interpolation interval to a servo module that computes the actuator commands at a higher rate. Constant-force machining is named there as a case in which the trajectory itself needs sensor feedback, and chatter control is one of the feedback functions such a controller was meant to add [18]. The chapter places its feed loop at the process level, running every 1 ms, and its chatter watch and speed rule at the supervisory level, with the watch reading the vibration at the 20 µs physics rate. Figure 13.5 draws both loops on that hardware.

> **TECHNOLOGY** **Final Element: The Spindle and the Feed Override**
>
> In a numerical controller the spindle normally runs independently of the axes, held at a constant velocity for machining and changed infrequently. The axes follow setpoints that a trajectory generator picks off the programmed path at each interpolation interval [18]. A change of spindle speed is therefore a supervisory decision, as it is in this chapter. Chatter accelerates the wear of the tools and of machine parts such as the spindle bearings [17], so a cut left chattering wears the final element itself. The chapter's spindle is power limited above its base speed, and its feed override scales the programmed feed between 10 and 150 %.

What Is Modernizing It

Three developments are changing the hardware around these loops. Tool holders now carry strain gauges, an accelerometer and a temperature sensor, powered by a generator that draws on the tool's own rotation and sending their data wirelessly. In one such holder the bending moment gave a reliable chatter feature [17]. Active chatter control applies a computed force in real time through actuators in the spindle, in the tool holder or on the workpiece, such as piezoelectric stacks and magnetic bearings, where passive tools with variable pitch or helix break up the regeneration by their geometry [19]. And the field's own review asks for physics-based stability prediction to be joined with on-line adaptive learning and tuning, so that detection and avoidance stay correct as the machine and the tool change [4]. The chapter's speed rule uses only the measured chatter frequency, the simplest member of that family.

> **IN PRACTICE** **Matching the Teeth to the Tool's Mode**
>
> Machine tools are tested dynamically, and their designers identify and stiffen the mode shapes that limit stability. A rule from high-speed milling matches the tooth-passing frequency, the spindle speed times the number of teeth, to the first bending mode of the spindle, so that the machine runs in the highest-speed lobe and removes metal fastest. The tool-point response is measured by modal testing, which needs equipment and expertise that a production shop may not have [4]. Case 1 of this chapter shows the rule: at 16 300 rpm the teeth pass at 815 Hz, beside the 800 Hz mode, and a 3 mm cut leaves no marks.

13.7 Limitations

The average-tooth force misses the extra lobes of light radial immersion [11], and the thin engagement at the end of each loop falls in that range.

One mode along a fixed direction stands in for the tool, the holder and the spindle, whose several modes shift and split the lobes of a real assembly [4].

There is no process damping and no size effect, so the lobes at low speed are pessimistic [4].

The engagement comes from the planned toolpath, the axes take corners with no slowdown, and wear is accelerated about a thousandfold.

The sensors are idealized. The load signal is the cutting power itself through a filter, a delay and noise, the vibration signal is the deflection with white noise, and there is no sound, so the microphone route to detection [14] is not represented.

The chatter watch has no growth test, so it reacts to the ring-down of chatter that its previous move has already cured, as in case 5. The speed rule uses only the measured frequency, and no supervisor chooses speeds from the lobes.

OPEN CHALLENGE **Knowing the Tool's Dynamics in Production**

A stability chart is only as good as the tool-point response, the cutting force coefficients and the process damping behind it. The response can change with the spindle speed, the cutting load and the position of the machine, a thin-walled part changes as metal is removed, and wear changes the rubbing that gives process damping. A review of the field calls for physics-based chatter prediction made off line to be combined with on-line adaptive learning and tuning [4]. Progress would be lobes that stay correct along the toolpath while the tool wears. The lobes of this chapter come from one fixed mode and are read by neither controller.

13.8 Conclusion

A pocket cut by a slender end mill is limited by a load that the feed can hold and a stability boundary that the feed cannot move, and the simulator of this chapter puts both on one toolpath with a controller for each. The regenerative chip is advanced as a delay differential equation with contact loss, and its simulated boundary lies 0.095 % from the analytic lobe at 20 000 rpm and 0.064 % at 12 000 rpm. In the uniform cut 1.5 mm draws 0.64 kW with 0.5 µm of forced vibration, while 3 mm doubles the load and chatters at 823 Hz with 60 µm of waviness against a 2.10 mm limit. The adaptive feed holds a corner to 34 % over its target where a constant feed reaches 95 %, and holds a worn tool's load within 1.6 % while the cut leaves 72 mm of marks. The chatter watch and the speed rule end chatter by moving the spindle to $60 f_c/(N j)$, and with both loops the pocket is cleared in 12.63 s against a 16.36 s baseline, under the 2.2 kW cap with 26 mm of marks. The model carries one tool mode, an average-tooth force and no process damping, so its lobes are pessimistic at low speed and incomplete at light immersion. A fuller model would add a second orthogonal mode, time-periodic force coefficients, process damping, a growth test in the chatter watch and a feedforward from the planned engagement. The simulator puts the two limits of a milled pocket, the one a feed can remove and the one only the spindle speed can, side by side where both can be measured.

KEY POINTS

- Slowing the feed lowers the load on the spindle, but it does not move the depth at which the cut starts to chatter.
- The critical depth is set by the tool's frequency response, the cutting stiffness and the tooth period, and the feed does not enter it.
- An adaptive feed loop that holds a worn tool's load within 1.6 % of its target still leaves 72 mm of chatter marks.
- A spindle speed 5.4 % lower ends the chatter of a cut 11 % over its limit within 0.21 s.

Where else this applies. Volume 4, Chapter 4 holds a harvester's feed rate against the power of its engine, the load half of this chapter's problem.

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13.9 Exercises

13.1 ⌕ Concept check · 2 min

A straight cut chatters at a fixed spindle speed with the feed override at 100 %. The operator lowers the override to 60 % and changes nothing else. What does the model of this chapter predict?

(a) The chatter goes on with shallower waves, because the feed does not enter the stability limit

(b) The chatter stops, because a thinner chip lowers the cutting stiffness below the stability limit

(c) The chatter stops, because a slower feed lengthens the tooth period and moves the cut into a pocket

(d) The chatter grows, because a slower feed raises the gain the adaptive loop identifies per unit feed

13.2 ⌕ Concept check · 1 min

The chatter watch moves the spindle to $\Omega_j = 60 f_c/(N j)$ for a measured chatter frequency f_c. Why does a speed chosen this way end the chatter?

(a) The tooth-passing frequency moves far from the natural frequency, so the teeth stop exciting the tool

(b) The tooth period holds a whole number of chatter waves, so each tooth meets the wave left before it in phase

(c) The surface speed falls, so the damping from the flank rubbing on the surface waves rises sharply

(d) The chip grows thinner in proportion to the speed, so the tool no longer leaves the cut on each cycle

13.3 Concept check · 2 min

In case 5 the chatter watch first takes the bounded step from 16,300 to 11 410 rpm, where the 1.5 mm cut of the long tool is stable, and then at 0.88 s moves on to 10 314 rpm. Why the second move?

(a) Flank wear grown during the ramp has lowered the limit at the first new speed below the depth

(b) The spindle overshot its new target and came back through a band of unstable speeds

(c) The chatter grown during the ramp is still ringing down, and the energy ratio reads it as chatter

(d) The first new speed put the tooth-passing frequency below the natural frequency, which always forces a move

13.4 Theory and calculation · 6 min

The tool mode of Equation (13.2) has $k_0 = 12\,\mathrm{N/\mu m}$ and $f_{n0} = 800\,\mathrm{Hz}$ at $L_0 = 40\,\mathrm{mm}$, with $\zeta = 0.03$. Take $\mu = 0.6$ and the cutting coefficient of the default tool, $K = K_c(1 + k_{VB}VB)$ with $K_c = 800\,\mathrm{N/mm^2}$, $k_{VB} = 1.2\,\mathrm{mm^{-1}}$ and $VB = 0.05\,\mathrm{mm}$.

(a) Find the modal mass m_0, and the stiffness and natural frequency of the tool at an overhang of 50 mm. *Checkpoint:* 0.475 kg, 6.14 N/μm and 512 Hz.

(b) Find the lowest limiting depth over all speeds, $a_{\mathrm{crit}} = 2k\zeta(1+\zeta)/(\mu K N^*)$, on a straight ($N^* = 0.75$) for both overhangs. *Checkpoint:* 1.94 mm and 0.995 mm.

(c) Say which of these cuts is stable at every spindle speed: case 1 at 1.5 mm with the 40 mm tool, case 5 at 1.5 mm with the 50 mm tool, and the 8 mm entry slot of case 8 ($N^* = 1.5$) with the 40 mm tool. *Checkpoint:* Only case 1. The slot's lowest limit is 0.97 mm.

13.5 Theory and calculation · 5 min

Use the steady load of Section 13.3, $\bar{P} = K a_p a_e v_f$ plus the rubbing term, in which the rubbing force $K_{\mathrm{rub}} VB\, a_p N^*$ with $K_{\mathrm{rub}} = 40\,\mathrm{N/mm^2}$ acts at the surface speed $R\Omega$, $R = 6\,\mathrm{mm}$. The default tool has $K = 848\,\mathrm{N/mm^2}$ and $VB = 0.05\,\mathrm{mm}$, and the feed is 4800 mm/min.

(a) Case 1 cuts a straight, $a_e = 6\,\mathrm{mm}$ and $N^* = 0.75$, at 1.5 mm and 20 000 rpm. Find the removal rate and the load. *Checkpoint:* 43.2 $\mathrm{cm^3}$/min and about 0.64 kW.

(b) Case 8 cuts the entry slot, $a_e = 12\,\mathrm{mm}$ and $N^* = 1.5$, at 8 mm and 16 300 rpm. Find the load and compare it with the 5.5 kW spindle. *Checkpoint:* About 6.76 kW.

(c) With $T_{\max} = 10\,\mathrm{N\,m}$, find the speed above which the drive is limited by power rather than torque. *Checkpoint:* 5252 rpm.

13.6 Theory and calculation · 7 min

Apply the speed rule of Equation (13.16) with $N = 3$, the speed range 3000 to 24 000 rpm and a largest step of 30 %. The candidate nearest the present speed is chosen.

(a) Case 4 declares chatter at 811 Hz with the spindle at 17 500 rpm. Find q, the candidate speeds and the new speed. *Checkpoint:* $q = 0.927$, candidates 16,220 and 8110 rpm, new speed 16 220 rpm.

(b) Case 5 declares chatter at 546 Hz with the long tool at 16 300 rpm. Find the nearest candidate and the speed the spindle actually takes. *Checkpoint:* 10 920 rpm is 33 % away, so the bounded step to 11 410 rpm.

(c) The watch then reads 516 Hz at 11 410 rpm. Find the next speed and the tooth-passing frequency there. *Checkpoint:* 10 320 rpm, with the teeth passing at 516 Hz.

13.7 App challenge · 8 min

Load scenario 1, Uniform cut, with the default seed. Each pass stops itself when it is complete, at 1.19 s, so any playback speed works. For each run read the Peak load, Chatter marks and Vibration (peak rms) tiles of the scorecard, and the limit line of the depth trend.

(a) Run the scenario as shipped, at 1.5 mm. *Checkpoint:* 0.64 kW, no marks, limit 2.10 mm.

(b) Press Reset, set Axial depth of cut to 3 mm before Run, and run the pass. *Checkpoint:* 1.32 kW, 88 mm, 60.1 µm.

(c) Press Reset, keep 3 mm, set Spindle speed to 16 300 rpm before Run, and run the pass. *Checkpoint:* 1.27 kW, no marks, 8.8 µm, limit 17.9 mm.

Accept loads within 0.02 kW, marks within 2 mm and vibration within 0.5 µm. Explain why run (c) shakes the tool far more than run (a) and still leaves no marks.

13.8 App challenge · 8 min

Load scenario 2, Corner engagement, with the default seed. Each run stops itself at the end of the loop. Read the Peak load and Machining time tiles.

(a) Run the scenario as shipped, with the adaptive feed at 0.65 kW, $K_f = 0.6$ and $\tau_I = 50$ ms. *Checkpoint:* 0.87 kW and 3.65 s.

(b) Press Reset, set Feed control to Constant before Run, and run again. *Checkpoint:* 1.27 kW and 3.38 s.

(c) Press the reload button, set Loop gain to 1.2 and Integral time to 30 ms before Run, and run again. Export the CSV file and find the lowest value of `feed_override_pct` with its `path_mm`. *Checkpoint:* 0.84 kW, 3.67 s, lowest override 25.9 % at about 730 mm.

Accept loads within 0.02 kW, times within 0.02 s and the override within 1 %. Say whether the faster tuning's peak and its lowest override are at a corner.

13.9 App challenge · 8 min

Load scenario 7, A late load signal, with the default seed. The load signal is delayed by 80 ms, and the loop runs $K_f = 1.2$ and $\tau_I = 30$ ms with a 1.7 kW target. Each run stops itself at the end of the loop. Read the Peak load tile, and the lowest feed override on the trend or in `feed_override_pct` of the CSV file.

(a) Run the scenario as shipped. *Checkpoint:* 3.65 kW, override down to 10 %.

(b) Press Reset, set Loop gain to 0.3 and Integral time to 200 ms before Run, and run again. *Checkpoint:* 3.19 kW, override never below about 87.6 %.

(c) Press the reload button, open Show Model, set the Load signal delay to 0 ms, and run the original tuning. *Checkpoint:* 2.08 kW.

Accept peaks within 0.02 kW and the override within 1 %. State which remedy recovers the corner peak and why detuning cannot.

13.10 Application area · 4 min

Case 6 runs a tool with 0.25 mm of flank wear on the case 1 straight at 1.8 mm and 20 000 rpm, with the adaptive feed holding the 0.77 kW that the default tool draws there.

(a) Find the factor by which the wear raises K over a sharp edge and over the default tool. With the rubbing term of Equation (13.5) at a surface speed of 12.6 m/s, estimate the load at constant feed. *Checkpoint:* 1.30 and 1.23, and about 1.07 kW.

(b) If the whole load scaled with the feed, what override would hold 0.77 kW? Name the logged signal a tool-condition monitor should watch while an adaptive feed is on. *Checkpoint:* About 72 %. The override, or the load per unit override.

13.11 Application area · 4 min

A shop changes from a 40 mm to a 50 mm tool overhang to reach a deeper pocket, and keeps the 16 300 rpm that worked before.

(a) Using $f_n \propto L^{-2}$, find the natural frequency of the long tool and the speed at which its tooth-passing frequency equals it, for $N = 3$. Compare with the 16 000 rpm of the short tool. *Checkpoint:* 512 Hz and 10 240 rpm.

(b) Say what the shop should measure before trusting a speed, and name two features of a real tool assembly that this model leaves out. *Checkpoint:* A tap test of the tool in its holder. Several modes, two directions, process damping.

13.12 Review the agent · 5 min

An AI assistant reviews case 6, A worn tool, and replies as follows. “The worn tool chatters because it is overloaded: at constant feed it draws 1.09 kW against 0.77 kW for a good tool. Lower the Spindle load target from 0.77 to 0.50 kW. The adaptive feed will cut the chip by a third, the cutting force falls with it, and the 72 mm of chatter marks will disappear.” Find the flaw and say how you would check it on the app, and what would work instead.