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CHAPTER 11

Supply-Line Feedback and the Bullwhip Effect in a Perishable Dairy Chain

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CHAPTER 11

Supply-Line Feedback and the Bullwhip Effect in a Perishable Dairy Chain



The controller moves 1 the plant's bottling plan 2 the depot's order to the plant 3 the supermarket's order to the depot

The controller reads 1 the stock record at the plant and the depot 2 the tills' sales record

What disturbs it 1 shoppers' demand, which a promotion doubles

Notice that each stage sees only its own stock and the orders or sales that reach it, while every order waits days for its truck. Illustration, not to scale.

IN PLAIN TERMS **Milk Orders That Grow on the Way Up**

The main idea. A supermarket, a depot and a dairy each order milk from the one above, and every order takes days to arrive. A manager who forgets the milk already on its way orders the same milk twice.

The challenge. A one-week sale at the store then reaches the dairy as a swing many times larger. Milk goes past its date on the shelf, and the dairy pours away fresh milk from the farms when the orders stop.

Why it matters. Counting the milk on its way, and letting every stage see what shoppers buy, keeps the chain steady and saves the milk.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history in Section 11.2, the fact in Section 11.3, the two try-its and the fact in Section 11.6, and the key points at the end of Section 11.9
Study	50 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 11.6), Technology in Practice (Section 11.7), Limitations (Section 11.8) and the two open challenges

The control question Three sensible managers each order milk from the stage above through a delay, and a one-week promotion at the till reaches the plant as a swing many times larger. How far does the classical analysis of the ordering rule predict what a perishable, capacity-limited chain does, and which cure removes the amplification: a different rule, shared information or one planner for the whole chain?

Learning objectives

- Explain why a stage that ignores the milk already on order orders the same milk twice, and name the term of the rule that fixes it.
- Compute the frequency response $|\Gamma(e^{j\omega})|$ of an ordering rule at chosen frequencies and judge whether a chain of such stages is string stable.
- Compute the variance floor of order-up-to under local and shared information and compare it with a measured ratio.
- Trace where the farms' milk goes when the orders collapse, and price the waste, the drain and the dryer.
- Compare the shipped rule, the supply-line-aware rule, shared sales and the centralized MPC on the 26-week challenge.

Prerequisites Sections 1.4.8 (disturbance propagation and string stability), 1.6.1 (engine, seed and Reset), 1.6.3 (MPC option) and 1.6.4 (scenarios, scorecard and export).

The app https://apmonitor.com/apps/milk-distribution-game/

See also Volume 1, Chapter 10 for a stock whose level is set by adapting inflows decided on a calendar with a quantum and a cap, and Volume 1, Chapter 9 for an inventory that is a pure integrator. Volume 7, Chapter 14 for string stability of a line of cars, the same criterion applied to accelerations.

11.1 Introduction

An inventory is an integrating state, and an order is an input that reaches it after a delay. A shortage observed today says nothing about how much milk is already on its way, so a manager who orders again for the same gap orders the same milk twice. When every stage of a distribution chain corrects its own stock from the orders it receives, the corrections travel upstream as apparent demand, and the variance of the orders grows from stage to stage, the bullwhip effect [1]. Treating this as a control problem is older than the name, because production control was written as a servomechanism problem in 1952 [2] and periodic replenishment as a discrete servo problem in 1955 [3]. The inventory and order based production control system of the production-control literature is a feedback loop with a demand-averaging time, a feedback gain and a production delay [4], the bullwhip itself has been measured and avoided with control theory [5], and model predictive control (MPC) has been applied to supply chain optimization and to demand networks [6, 7]. The plant here is a chain of stocks and pipelines, the actuator is an order rate and the controller is a decision rule, so the tools of feedback control apply to it directly.

A common belief holds that sensible managers at every stage cannot together produce a crisis, and the simulator puts that belief to a test. In its second case a one-week promotion doubles what shoppers buy, from 20 to 40 kL/d, and every manager runs the same anchoring-and-adjustment rule. The depot then orders 65.5 kL/d and ten days later nothing, the plant's daily orders vary 12.70 times as much as the shoppers' demand, 32.5 kL of finished milk ages past its code date on the supermarket's shelf and 30.5 kL of raw milk goes down the drain when the dryer is full. Counting every liter already on order pours none. With order-up-to at every stage and every assumption of the analytical theory in force, the plant's variance ratio stays above its floor for every forecast window measured, 11.36 against 8.58 at a week, and sharing the supermarket's sales takes it to 2.73. A block diagram of three loops shows none of these consequences, because the cost of a policy lies in the ages of the cartons, the censoring of demand by an empty shelf and the farm milk that the trough in orders leaves unbottled.

The structure is general. A cascade in which each element answers its neighbor with the same local feedback passes a wave on multiplied by the element's gain at that frequency, and the requirement that no frequency grows along the cascade is string stability, a property defined for infinite strings of

interconnected nonlinear subsystems [8]. Section 1.4.8 applies the same criterion to vessels whose level loops pass flow variation downstream. A line of cars under adaptive cruise control is one such cascade, studied in Volume 7, Chapter 14, and a distribution chain is another. Perishable stocks add a deadline to every unit, and ordering policies for stock with a fixed life were reviewed with blood banks as an application [9]. Case studies of fresh chains for milk, fish and poultry found that their performance improves with more efficient sharing of demand and shelf-life data [10], and dairy planning joins an uncertain raw-milk supply with products as different as cheese, yogurt, powdered milk and ultra-high-temperature milk [11].

However, the record of the problem comes from board games and behavioral experiments [12–14] and from linear analysis [5, 15], so a student seldom has a chain on which the frequency-domain criterion, the analytical variance floor and the waste of a perishable product can be measured together while the rule, the information or the calendar changes one at a time. This chapter measures, on a perishable dairy chain where the order a manager controls can empty a shelf or pour milk away days later, how far the classical analysis of the ordering rule describes what the nonlinear chain actually does. The ordering rule, the order-up-to policy, the variance floor and the centralized MPC are established results. The analysis is a set of measurements on one simulator, including a sweep of the forecast window that places order-up-to at or above its floor every time. The simulator is an age-resolved hourly chain, with three decision calendars and a daily constrained predictive controller, and every equation, constant, delay and limit is stated in the sections that follow. Nine cases, a constant-flow reference and a 26-week challenge report service, waste, cost and order variance together.

Section 11.2 follows the problem from servomechanism theory to the beer game and to fresh food chains. Section 11.3 sets out the chain, its ages, the plant's balance and the cost ledgers, and Section 11.4 the ordering rules, the decision calendar and the centralized planner. Section 11.5 derives the rule's frequency response and the variance floor of order-up-to and measures the floor where its assumptions hold. Section 11.6 reports the nine cases, the challenge and the constant-flow reference, Section 11.7 describes the real stock records, orders and refrigerated trucks, Section 11.8 states the model's limits and Section 11.9 summarizes. The cases and the challenge carry completion rules, and the exercises of Section 11.10 are built on them.

11.2 From Servomechanisms to the Beer Game and Fresh Chains

Inventory control met feedback theory early. The problem of controlling the production rate of a single product was stated in 1952 in terms of servomechanism theory, with Laplace transforms introduced to study the stability and steady-state behavior of the system and a cost criterion constructed for evaluating alternative decision rules [2]. Three years later the simplest periodic replenishment case was analyzed with discrete variable servo theory, and a reorder rule was derived that minimizes the variance of the inventory balance, with that variance written explicitly in terms of the sales forecast error and the system constants [3]. The rules of this simulator are periodic replenishment rules of the same kind, decided once a day.

The game the simulator descends from began with appliances, not beer, as the box below tells. Forrester's article of 1958 described the structure and behavior of a three-stage system, with the oscillation, amplification and phase lag that the game still produces [12, 16]. The Chain view of the simulator is drawn as the game's board, with crates on docks, trucks on a road and order slips moving the other way.

The game became a research instrument in 1989. Subjects managing a simulated distribution system with multiple actors, feedbacks, nonlinearities and delays produced aggregate behavior that diverged systematically from the optimum, and an anchoring-and-adjustment heuristic for stock management, tested econometrically, explained their orders well. The estimates identified misperceptions of feedback, among them a failure to account for the supply line [12, 13]. Production engineers wrote the same loop as an inventory and order based production control system and

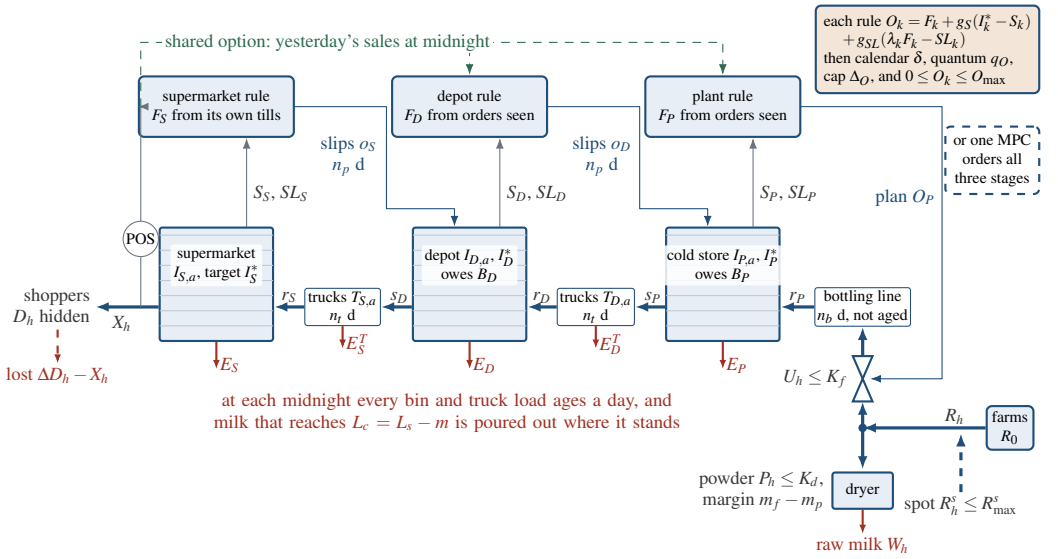


Figure 11.1 The dairy chain as stocks, delays and valves, not to scale. Each stock is a stack of daily age bins $I_{k,a}$ with its target I_k^*, and the depot and the plant carry what they owe downstream, B_D and B_P. Trucks $T_{S,a}$ and $T_{D,a}$ spend n_t days on the road, order slips o_S and o_D spend n_p days in processing, and bottled milk U_h spends n_b days on the line before it enters the cold store. The farms' milk R_h, with optional spot milk R_h^s, is split by the bottling valve, the dryer and the raw-milk drain. Shoppers take the oldest carton first except for a share f_L who take the freshest, and the point-of-sale (POS) record X_h is a censored measurement of the hidden demand D_h. Expired milk E_k and E_k^T leaves every stock and truck. Each rule sees its own stock, its supply line and its forecast input, with the shared option sending yesterday's sales to the depot and the plant at midnight.

showed that its feedback path is essential, that feedforward alone cannot give adequate performance, and that one good compromise makes the time to adjust inventory, the demand averaging time and the production delay comparable [4]. Later experiments found the bullwhip effect persisting after batching, price variation and demand estimation had been removed, with decision makers consistently underweighting the supply line, and found that sharing inventory information helped only somewhat [14].

> ⧖ HISTORY **1956 to 1984: From an Appliance Factory to the Beer Game**
>
> In 1956 and 1957 managers of a household appliance factory in Kentucky were troubled by production that swung from three shifts, seven days a week, to laying off half the people two or three years later. A hand simulation with columns for employees, production rate, backlog and inventory, one line per week, showed that the system created the instability even with a constant demand from the market. That notebook exercise became the production-distribution game, used in a university summer session from 1958 with retail, distributor and factory stages. Beer first appeared as its context in 1973, and the name Beer Distribution Game was first used in 1984. The session that one school plays each year with its incoming business class often has more than 400 participants [12].

The name and the information view arrived in 1997. Orders were shown to be a distorted signal of sales, with the variance of orders larger than that of sales and growing upstream, from four sources, which are demand signal processing, the rationing game, order batching and price variation [1]. Three years later the effect of forecasting and lead time was quantified for a retailer and a manufacturer, extended to chains of several stages with and without centralized customer demand information, and

Table 11.1 Signals of the Chain by Role

Symbol	Role	Unit
I_S, S_D, S_P	CVs, shelf stock and the net stocks $I_k - B_k$	kL
I_S^*, I_D^*, I_P^*	SPs, 3, 2 and 2 days of D_0	kL
O_S, O_D, O_P	MVs, two order rates and the bottling plan, 0 to 80	kL/d
D_h, R_h	DVs, shoppers' demand and the farms' milk	kL/d
—	DVs, promotion, heat, holiday, road and dryer events	events
$I_{k,a}$, $T_{k,a}$	states, age bins on docks and trucks	kL
B_k, SL_k	states, backlogs and supply lines	kL
X_h	measured, the tills' sales	kL
o_k^{in}	measured, orders arriving upstream	kL
F_k	estimates, each stage's forecast	kL/d
U_h, P_h, W_h	outputs, bottled, powder and raw drain	kL/d
C, ρ_k	outputs, cost ledgers and variance ratios	k$, —

shown to be reducible but not eliminable by centralizing that information [15]. Control theory then supplied a way to measure the effect and to avoid it [5], and the frequency response of Section 11.5 is drawn in that spirit.

Perishable stock has a theory of its own. Ordering policies for inventory with a fixed life and for inventory subject to exponential decay were reviewed in 1982, for deterministic and stochastic demand, with a brief review of their application to blood bank management [9]. Case studies of fresh food chains in the Nordic countries, milk among them, found that their performance improved with more efficient sharing of demand and shelf-life data, together with changes in the chain's structure that sped up deliveries [10]. Planning a real dairy chain now treats raw-milk supply and product demand as uncertain together, and in one two-stage stochastic formulation the value of the stochastic solution rose to 21.1 % above the deterministic one [11]. On the control side, chemical engineers posed supply chain optimization as model predictive control [6] and a framework of the same kind managed multi-product, multi-echelon demand networks [7]. The simulator stops short of an industrial plan. It keeps the board game's structure and gives every order a perishable product and a supply that cannot be turned away.

11.3 The Chain and Its Ledgers

The simulated chain is one product, three stocks and the farms that supply them, drawn in Figure 11.1 as the application draws it, milk moving from the farms at the right to the shoppers at the left and orders moving the other way. The supermarket (S) sells to shoppers and orders from the depot (D), the depot orders from the plant (P), and the plant sets a bottling plan and takes the farms' milk, which it cannot refuse. Table 11.1 classifies the signals. Volumes are thousand liters (kL), flows kL/d, money thousand dollars, and a kiloliter at a dollar a liter is $1k. The engine's clock runs in weeks on a one-hour step, so a day is exactly 24 steps.

11.3.1 One Hour in the Chain

Each hour the chain advances in a fixed order. Trucks and the bottling line deliver whatever is due, and orders that have finished processing reach the stage upstream. The shoppers then buy, the depot and the plant ship what they owe oldest first, the orders decided for this hour enter processing, and the plant splits the farms' milk. Every queue entry carries the step at which it arrives, so a road closure delays a truck without losing its milk. With $\Delta = 1/24$ day, the conservation laws on the aggregate

ledgers are Equations (11.1) to (11.4).

$$I_{S,h+1} = I_{S,h} + r_{S,h} - X_h - E_{S,h} \tag{11.1}$$
$$I_{k,h+1} = I_{k,h} + r_{k,h} - s_{k,h} - E_{k,h}, \quad k \in \{D,P\} \tag{11.2}$$
$$B_{k,h+1} = B_{k,h} + o^{\text{in}}_{k,h} - s_{k,h}, \quad k \in \{D,P\} \tag{11.3}$$
$$SL_{k,h+1} = SL_{k,h} + o_{k,h} - r_{k,h} - E^T_{k,h} \tag{11.4}$$

where $I_k = \sum_a I_{k,a}$ is milk on the dock, r_k the receipts, s_k the shipments, E_k and E^T_k the milk that expires on the dock and on the trucks bound for stage k, and o^{in}_D and o^{in}_P the supermarket's and the depot's orders placed n_p days earlier. The volumes are per step, not rates. Orders placed are $o_{k,h} = \Delta \max(0, O_k)$ for $k \in \{S,D\}$ and the volume actually bottled, $o_{P,h} = \Delta U_h$, at the plant. The supply line of the supermarket is its orders in processing, the depot's backlog B_D and the trucks on the road, that of the depot is the same one stage up, and that of the plant is the bottling line. The flows are

$$\begin{aligned} X_h &= \min(\Delta D_h,\ I_{S,h} + r_{S,h}), \\ s_{k,h} &= \min(B_{k,h} + o^{\text{in}}_{k,h},\ I_{k,h} + r_{k,h}), \end{aligned} \tag{11.5}$$

where D_h is the shoppers' demand rate and a shipment rides n_t days to the stage below. Shipments leave oldest first, and the shoppers' purchase takes a share f_L from the youngest bins and the rest from the oldest. A sale the shelf cannot make is lost, $\Delta D_h - X_h$, while an upstream order that cannot be filled stays in B_k until milk arrives.

11.3.2 Ages and the Code Date

An age bin is a state, not a label attached after the run. Receipts enter the bin of the age they carry, and at every midnight each dock and each truck load shifts by one day, as shown in Equation (11.6).

$$\begin{aligned} I^+_{k,a} &= I_{k,a-1}, \qquad & I^+_{k,0} &= 0 \\ E_k &= \sum_{a \ge L_c} I^+_{k,a}, \qquad & L_c &= L_s - m \end{aligned} \tag{11.6}$$

where L_s is the shelf life, m the sell-by margin and L_c the code date, 11 days by default, and the milk at or past it is poured out where it stands, on a dock or on a road. Milk that spoils on a truck also leaves the receiving stage's supply line in Equation (11.4). The bottling line holds milk that has not yet aged, so a liter enters the cold store at age 0 one day after bottling and is pulled at the code date counted from that arrival. Twenty-four bins hold ages 0 to 23 days.

11.3.3 The Plant's Balance

The farms' milk R_h arrives every hour whatever the plant plans. With bottling plan O_P, line capacity K_f, dryer capacity K_d (zero during a dryer outage) and spot milk up to $R^s_{\max}$ when the option is on, the plant splits its milk as shown in Equations (11.7) to (11.10).

$$R^s_h = \min\left(R^s_{\max},\ [\min(O_P, K_f) - R_h]_+\right) \tag{11.7}$$
$$U_h = \min(O_P,\ K_f,\ R_h + R^s_h) \tag{11.8}$$
$$P_h = \min(K_d,\ R_h + R^s_h - U_h) \tag{11.9}$$
$$W_h = R_h + R^s_h - U_h - P_h \tag{11.10}$$

where $R^s_h = 0$ with spot milk off, U_h is bottled, P_h dried into powder and W_h poured away raw, all in kL/d. Powder is a product, not waste, so only its lower margin is charged. The identity $R_h + R^s_h = U_h + P_h + W_h$ holds at every step to 2.2×10^{-16} by the application's own check.

🕮 **Pause.** Try Exercise 11.3 before reading on.

11.3.4 Shoppers, Herds and Events

Demand and the farms' milk follow a calendar. At day d the calendar week is $y_d = w_0 + (d + \frac{1}{2})/7$, a seasonal bump is $G_c(y) = \exp\{-\frac{1}{2}[\mathrm{wrap}_{52}(y-c)/5]^2\}$ with the distance wrapped around a 52-week year, and the rates are Equations (11.11) and (11.12).

$$D_h = D_0\,\omega_d\,[1-\delta_s G_{31}(y_d)]\,e_h\,\max(0,1+\sigma\varepsilon_d) \tag{11.11}$$

$$R_h = R_0\,[1+\phi_f G_{19}(y_d)-\phi_t G_{43}(y_d)]\,e_h^R \tag{11.12}$$

where $\omega_d = 7/(5+2w_e)$ on Monday to Friday and $7w_e/(5+2w_e)$ on Saturday and Sunday keeps the weekly mean at one, e_h is 2 during a seven-day promotion, 1.2 during a 14-day heat wave and 0.85 during a 14-day school holiday, $e_h^R = 0.9$ during the heat wave and 1 otherwise, σ is the demand-noise slider divided by 100 and ε_d a standard normal deviate drawn once a day from the scenario's seeded generator. Two further sliders multiply D_h and R_h by $1+s/100$ with s from -30 to 30, a road closure holds every truck bound for the supermarket for a day, and a dryer outage sets $K_d = 0$ for two days. An event begins at the first step at or after its scheduled time, and the run starts on a Monday.

11.3.5 Ledgers and Scores

The cost ledgers integrate, per step, the increments of Equation (11.13).

$$\begin{aligned}
\Delta C_h &= c_h \Delta \textstyle\sum_k I_k, \qquad & \Delta C_l &= c_l \Delta (B_D + B_P)\\
\Delta C_b &= c_b(\Delta D_h - X_h), & \Delta C_s &= c_s \Delta R_h^s\\
\Delta C_w &= c_w\big(\Delta W_h + \textstyle\sum_k E_k + \sum_k E_k^T\big)\\
\Delta C_{\mathrm{div}} &= (m_f - m_p)\Delta P_h
\end{aligned} \tag{11.13}$$

where the stocks are those on the three docks after the hour's receipts, sales and shipments, and expired milk is charged at the midnight it is poured. The chain's operating cost is $C_h + C_l + C_b + C_w + C_s$, and the total cost adds the margin C_{div} lost to the dryer. Service is cumulative sales over cumulative demand, waste is the finished milk poured out over the milk bottled, and the longest stockout counts consecutive hours in which a sale was lost. The amplification of stage k is a ratio of variances of daily totals, Equation (11.14).

$$\rho_k = \frac{\mathrm{Var}_{d \ge 7}\left(\sum_{h \in d} o_{k,h}\right)}{\mathrm{Var}_{d \ge 7}\left(\sum_{h \in d} \Delta D_h\right)} \tag{11.14}$$

where the first week is skipped, the plant's daily order is its bottling plan ΔO_P summed over the day rather than what the farms let it bottle, and the denominator is the true demand, lost sales included.

> ✦ DID YOU KNOW? **Milk Lost Before It Is Drunk**
>
> In 2010 the United States had an estimated 53.8 billion pounds of fluid milk at the retail level. About 12 percent of it was lost in stores and 20 percent by consumers. Overstocking, because the number of customers is hard to predict, is one listed cause of loss in stores [17].

11.3.6 Start, Integration and Checks

Every case starts lined out. Each stock is set to its target at age 0, every queue is filled at a common steady rate, the chain runs 21 days at that rate with every order equal to it so that the age profiles become periodic, and then the clock and the cumulative ledgers return to zero. At 20 kL/d the supply lines hold 40 kL, 40 kL and 20 kL, the stocks 60 kL, 40 kL and 40 kL, and the oldest cartons are 9, 5 and 2 days old. The engine advances the chain one hour and then integrates the 26 aggregate states (the stocks, backlogs, supply lines, cumulative flows and six cost ledgers) by an Euler step over the same flows, which is exact because every flow is constant over its hour. The controller runs before each step and the historian logs every hour. Over every shipped case, initial milk plus farm intake and

Table 11.2 Teaching Values and Where Each Is Set: M in the Show Model Dialog, C in a Control Card or the Disturbance Panel, A Among the Advanced Controls, S in the Source. No Constant Is Hidden From the Reader

Symbol	Meaning	Default	Range		Symbol	Meaning	Default	Range	
D_0	shoppers' mean demand	20 kL/d	5–60	M	a_S	stock adjustment	0.26/d	0.02–1	C
w_e	weekend lift	1.3	1–1.8	M	β	supply-line weight, a_{SL}/a_S	0.34	0–1	C
δ_s	summer dip at week 31	0.10	0–0.3	M	θ	forecast smoothing	0.36/d	0.02–1	C
w_0	calendar week at the start	9	0–51	M	p	moving-average window	7 d	3–28	A
R_0	farms' mean milk	30 kL/d	10–60	M	δ	decision interval	1 d	hour, day, week	C
ϕ_f, ϕ_t	flush at week 19, trough at 43	0.15, 0.10	0–0.4, 0–0.3	M	q_O	order quantum	0.5 kL/d	0–5	A
$R^s_{\max}$	spot milk available	10 kL/d	0–30	M	Δ_O	largest move per decision	40 kL/d	1–80	A
m_f, m_p	fluid and powder margins	0.40, 0.15 \$/L	0.1–1, 0–0.5	M	I^*_S, I^*_D, I^*_P	stock wanted, days of D_0	3, 2, 2	0.5–6	C
c_h	holding	0.01 \$/L/d	0–0.05	M	K_f	bottling capacity	40 kL/d	10–60	A
c_l	late order, upstream backlog	0.02 \$/L/d	0–0.2	M	K_d	dryer capacity	20 kL/d	0–40	A
c_b	lost sale	0.40 \$/L	0–2	M	n_p, n_t	order processing, transit	1, 1 d	1–3	A
c_w	milk poured out	0.60 \$/L	0–2	M	n_b	bottling to the cold store	1 d	1–2	A
c_s	spot premium	0.20 \$/L	0–1	M	L_s, m	shelf life, sell-by margin	14, 3 d	7–21, 0–5	A
σ	demand noise, day to day	0 %	0–30	C	f_L	shoppers taking the freshest	0 %	0–100	A
q, w	MPC stock and move weights	2, 2	0.1–50, 0–50	C	N	MPC horizon	14 d	7–28	A
$O_{\max}$	station limit on every order	80 kL/d	—	S	—	step, age bins, warm-up	1 h, 24, 7 d	—	S
—	line-out, seasonal width	21 d, 5 wk	—	S	—	default seed, cases 4, 6 and challenge	20261004, 11, 7	—	S

spot milk equals milk held on docks, trucks and the line plus sales, powder, raw discards and expired milk to 7.0×10^{-11} kL, and the age bins sum to the aggregate stocks and the supply lines to their recount from the queues within the same tolerance. Seeding, speed and Reset are the framework's of Section 1.6.1. At half the step the challenge's cost under the suggested tuning changes by a relative 5.9×10^{-6}. The application passes its 42 checks.

11.3.7 Assumptions

One product and three stocks. One supermarket, one depot and one plant stand for an aggregated chain. There is no network of stores, no substitution between products or stores and no returns, and a lost sale is lost for good.

A fixed code date for quality. Milk is pulled at the same age everywhere, trucks included. Temperature, handling and microbial growth, which decide real shelf life, are absent, so this is a logistics

model and not a food-safety model.

> ⚠ **OPEN CHALLENGE** **A Shelf Life That Updates Itself**
>
> Most shelf-life models for fresh dairy products estimate the shelf life once, from the product's condition at the start, while the product meets changing temperatures in distribution, at retail and at home. Industry would gain from systems that update the remaining shelf life as sensor readings, measurements or temperature records arrive. Such a system needs a validated model of deterioration, an estimate of the product's present state and its storage history, and uncertainty is seldom treated. Progress would be multi-batch data, realistic time and temperature histories and validation where the prediction is used [18]. The chapter's fixed code date is the case with none of these.

A smooth, compulsory supply. The farms' milk follows a seasonal curve with no day-to-day variation and must be taken in full, and every liter becomes bottled milk, powder or a raw discard. Cream separation, packaging losses and quality-dependent yield are absent. The milk evaporator of Chapter 4 and the milk powder spray dryer of Volume 5, Chapter 10 control unit operations that this plant reduces to a capacity and a margin.

Daily aging on an hourly clock. Flows move every hour, but ages move at midnight and the bottling day is not aged, so the effective selling window is a whole number of days measured from arrival in the cold store.

Teaching values. Every price, capacity, shelf life, gain and limit, the challenge's among them, is chosen to expose a mechanism. Real values come from a dairy's own costing, its cold-chain and food-safety rules and its contracts with retailers, as in planning studies of real dairy chains [11], and the cost ledger excludes capital, energy and the revenue from fluid sales, so no total here is a profit estimate.

11.4 Ordering Rules on a Decision Calendar

Each stage decides its order on the chain's calendar, through the same quantum and the same cap whether a rule or a person decides. The controller executes every hour, and an execution is a decision only when the calendar says so.

11.4.1 Anchoring and Adjustment With the Supply Line

Every automatic stage runs one rule by name, anchoring and adjustment with supply-line accounting, as shown in Equations (11.15) and (11.16).

$$g(a,\delta)=\frac{1-(1-a)^{\delta}}{\delta},\qquad a_{SL}=\beta a_S \tag{11.15}$$

$$O_k^{\mathrm{req}}=\Big[F_k+g(a_S,\delta)(I_k^*-S_k)+g(a_{SL},\delta)(\lambda_k F_k-SL_k)+b_k\Big]_+ \tag{11.16}$$

where F_k is the stage's forecast, S_k its stock net of what it owes ($S_S=I_S$), $\lambda_k F_k$ the supply line it wants, with $\lambda_S=\lambda_D=n_p+n_t=2\,\mathrm{d}$ and $\lambda_P=n_b=1\,\mathrm{d}$, δ the days between decisions and b_k a transfer bias that is zero in steady use. The fractions are per day, so a weekly or hourly calendar closes the same share of each gap per day, compounded, and $g(a,1)=a$. The application's source names the structure APIOBPCS after the production-control literature, and Volume 4, Chapter 8 copies the same function for its production loop. In control terms the stock term is a proportional controller on an integrating plant with a dead time, and the supply-line term is a second proportional term on the controller's own pipeline. With β small, the loop orders again for a gap already on its way. The shipped human rule has $a_S=0.26/\mathrm{d}$, $\beta=0.34$ and $\theta=0.36/\mathrm{d}$, the shape of the rule fitted to beer-game players [13] with teaching values applied per day, not the published weekly estimates. The supply-line-aware rule

sets $\beta = 1$, and order-up-to sets $a_S = a_{SL} = 1$ on a p-day moving average. An advanced option runs the chosen rule at one stage only and the human rule at the other two.

The forecasts are updated as their data arrive, and a decision samples them. The supermarket observes its own sales every hour, which undercount demand while the shelf is empty, and the depot and the plant observe the orders that reach them. With shared information the two upstream stages forecast instead from the supermarket's sales of the day just ended, published at midnight. Exponential smoothing applies $F_k \leftarrow F_k + \gamma(x - F_k)$, with x the observation as a daily rate and $\gamma = 1 - (1-\theta)^{1/24}$ for an hourly observation or $\gamma = \theta$ for a published day, and order-up-to takes the mean of the last $24p$ hourly observations or the last p published days.

11.4.2 Shaping a Decision

The calendar and the shaping come from the framework's `shared/src/framework/core/cadence.js`, which also shapes the quota decisions of Volume 1, Chapter 10. A calendar decides at every execution (hourly), at every midnight (daily, as shipped) or at every seventh midnight from the start, a Monday (weekly), so every instant falls on the hourly step and the realized intervals are exactly 1, 24 and 168 steps. At a decision the request becomes the order in force as shown in Equation (11.17).

$$\begin{aligned} \tilde{O} &= q_O \,\mathrm{round}\left(O_k^{\mathrm{req}}/q_O\right) \\ O_k &= \Pi_{[0,O_{\max}]}\left(O_k^- + \mathrm{sat}_{\Delta_O}(\tilde{O} - O_k^-)\right) \end{aligned} \tag{11.17}$$

where O_k^- is the order in force before the decision, ties round away from zero, sat caps the move at the largest whole number of quanta not above Δ_O, and Π clips to the station's limits pulled onto the quantum grid. Between decisions the order holds. With $q_O = 0.5\,\mathrm{kL/d}$, $\Delta_O = 40\,\mathrm{kL/d}$ and $O_{\max} = 80\,\mathrm{kL/d}$ as shipped, the quantum and the cap are the only anti-saturation the rules need, because Equation (11.16) has no integral or derivative term to wind up or to filter. The plant's plan is further limited by Equation (11.8), and a request above the station limit or the line's capacity is flagged as saturated.

11.4.3 Manual Stations and Transfer

Each stage has its own MANUAL and AUTO switch, and a master switch throws all three. A station in MANUAL holds the learner's value, decided on the same calendar and shaped the same way. Switching to MANUAL puts the order in force on the station, so nothing moves. On the way back to AUTO the rule sets $b_k = O_k^- - O_k^{\mathrm{raw}}$, with O_k^{raw} the value of Equation (11.16) before the floor at zero, so its first request is the order in force, and later decisions multiply b_k by $(1 - \max(a_S, 0.05))^{\delta}$, the manual reset of a proportional controller decaying at the rule's own rate. The PID loops of Section 1.6.2 reach the same end through their running sum, which this rule does not have. While a stage is automatic its manual station follows the rule's request before the quantum, the cap and the limits, so that a station never ratchets on a shaped value. In the application's check a transfer from AUTO to MANUAL and back moves no order of any of the four rules.

11.4.4 Seasonal Feedforward

An advanced option, off as shipped, gives the anchor a calendar. A stage that forecasts from the shoppers' sales smooths their level with the known weekly and summer pattern divided out and multiplies it by the pattern on the day its milk will be sold, 2, 5 and 7 days ahead at the supermarket, the depot and the plant. A stage that forecasts from the orders it receives knows no calendar for them and is unchanged. On the constant-flow reference the option moves no order at all.

11.4.5 One Planner for the Chain

The centralized controller replaces the three rules by one quadratic program (QP) at each decision, built with the framework's condensed linear MPC, `shared/src/framework/core/linmpc.js`, of

Section 1.6.3. Its internal model steps one day at a time, as shown in Equation (11.18) for the default one-day delays.

$$\begin{aligned} I_S^{j+1} &= I_S^j + z_{S2}^j - \hat{D}_j, \quad I_D^{j+1} = I_D^j + z_{D2}^j - z_{S1}^j \\ I_P^{j+1} &= I_P^j + z_B^j - z_{D1}^j, \quad z_{S1}^{j+1} = u_S^j,\ z_{S2}^{j+1} = z_{S1}^j \\ z_{D1}^{j+1} &= u_D^j,\ z_{D2}^{j+1} = z_{D1}^j, \quad z_B^{j+1} = u_P^j \end{aligned} \tag{11.18}$$

where u are the three daily orders, z_{S1} and z_{D1} hold an order in processing, z_{S2} and z_{D2} the same order on its truck, z_B the bottling line, and the stocks are net, so a negative value is a backlog. At each solve the eight states are read from the chain itself, every queued order, backlog, truck and bottle put in the slot for the day it will move. The demand forecast $\hat{D}_j$ is a level smoothed at θ from the published sales divided by the known weekly and seasonal pattern, times that pattern on day j, so the planner reads the shared sales whatever the information setting. The farms' milk $\hat{R}_j$ is today's intake carried along its known season. No promotion or heat wave is previewed. The horizon of $N = 14$ days is split into eight move blocks, four of one day, two of two days and two of three, or into weeks on the weekly calendar, and a stage in MANUAL is fixed at its station. The QP is Equation (11.19), the objective of Equation (1.16) with a terminal term and linear prices on the slacks.

$$\begin{aligned} \min\ & \sum_{j=1}^{N} \sum_k q'(I_k^j - I_k^*)^2 + 5q' \sum_k (I_k^N - I_k^*)^2 \\ & + w' \sum_j \sum_k (u_k^j - u_k^{j-1})^2 + \sum_i (\varepsilon s_i^2 + \sigma_i s_i) \\ \text{s.t. } & 0 \le u_S, u_D \le O_{\max}, \qquad 0 \le u_P \le K_f \\ & |u_k^j - u_k^{j-1}| \le \Delta_O, \qquad u_P \le \min(K_f, \bar{R} + R_{\max}^s) \\ & I_k^j \le \bar{F}\, c_k + s_k^f, \qquad I_k^j \ge -s_k^0 \\ & z_B^j \ge \hat{R}_j - K_d - s^d, \qquad s \ge 0 \end{aligned} \tag{11.19}$$

where $q' = 10^{-3} q = 0.002$ and $w' = 10^{-3} w = 0.002$ convert the sliders to thousand dollars, the terminal day carries six times the stage weight in all, u^j are the inputs of move block j with u^0 the orders in force, $\bar{R}$ the forecast intake averaged over a block, with $R_{\max}^s$ only when spot milk is on, and $\bar{F}$ the mean forecast over the horizon. The freshness ceilings c_k are the code date less the days of target stock and transit that the milk spends at the other stages, at least one day, 5, 4 and 4 days of forecast at the default targets and transits, a linear stand-in for the expected discards of the age bins. One slack per soft band carries a linear price σ_i, c_w for the three ceilings and the dryer floor, c_b for the supermarket's floor at zero and $c_b/2$ for the two upstream floors, and a small quadratic $\varepsilon = 10^{-3}$ keeps the program strictly convex. A band-price setting beside the planner's weights multiplies every σ_i and is one as shipped. The program has 31 variables (24 block inputs and 7 slacks) and 209 constraint rows, and the dual active-set method of Goldfarb and Idnani [19] solves it, as in Section 1.6.3. Only the first day's orders are applied, shaped by Equation (11.17). A failed solve holds the orders in force and counts the failure, and the first automatic decision after a manual transfer holds the order in force. The planner tracks stocks with economic penalties and is not a profit optimum.

11.4.6 Plant, Analysis and Planner Models

The simulation model is Equations (11.1) to (11.14), with every carton aged, every sale censored and every limit hard. The ordering rules hold no model at all beyond the lead times in λ_k. The analysis model of Section 11.5 is the rule linearized, without the floor at zero, the quantum, the cap or any expiration, and it is used only to draw $|\Gamma(e^{j\omega})|$ and to compute the variance floor. The MPC's internal model is Equation (11.18), which knows the delays and the limits but neither the ages of the cartons

nor the censoring of sales, and whose demand forecast knows the calendar but no event. No model is identified from data.

11.5 The Rule's Frequency Response and the Floor

A chain of stages that each answer the stage below is a platoon of inventories, and the application draws its criterion as Volume 7, Chapter 14 draws the cars' and as Section 1.4.8 draws a level loop's. Linearized with daily decisions and no floor at zero, Equation (11.16) with net stock $S_t = S_{t-1} + O_{t-1-\lambda} - d_{t-1}$ and supply line $SL_t = O_{t-1} + \cdots + O_{t-\lambda}$ gives the ratio of a stage's orders to the orders it receives, Equation (11.20).

$$\Gamma(z) = \frac{(1 + a_{SL}\lambda)Q(z)H_F(z) + a_S z^{-1}}{Q(z) + a_S z^{-(\lambda+1)} + a_{SL}Q(z)\sum_{i=1}^{\lambda} z^{-i}} \tag{11.20}$$

where $Q = 1 - z^{-1}$, $z = e^{j\omega}$ with ω in radians per day, and H_F is the forecast filter, $\theta z^{-1}/[1-(1-\theta)z^{-1}]$ for exponential smoothing or $p^{-1}\sum_{i=1}^{p} z^{-i}$ for a moving average. $\Gamma(1) = 1$ for every rule, because a stage must in the end order what it sells, so a lasting change passes on whole and a chain is string stable only if $|\Gamma(e^{j\omega})| \le 1$ at every frequency [8]. For order-up-to the denominator is exactly one and $\Gamma = z^{-1} + (1+\lambda)QH_F$.

For white demand the variance ratio of stage k is the mean of $|T_k|^2$ over frequency, Equation (11.21), by Parseval's theorem.

$$\frac{\mathrm{Var}(O_k)}{\mathrm{Var}(D)} = \frac{1}{\pi}\int_0^{\pi} |T_k(e^{j\omega})|^2 \,\mathrm{d}\omega, \qquad T_k = z^{-1}\Gamma_k T_{k-1} \tag{11.21}$$

where T_k is stage k's orders over the shoppers' demand, $T_1 = \Gamma_1$ at the supermarket, and the factor z^{-1} is the day by which each upstream stage sees the orders below it late. With shared sales the forecast of each upstream stage is driven by demand instead of by the orders it receives. For order-up-to on a p-day moving average, with $L = \lambda + 1$ the lead time plus the review day, three days at the supermarket and the depot and two at the plant, the classical expressions are Equation (11.22).

$$\begin{aligned} \rho_k^{\mathrm{loc}} &\ge \prod_{i \le k}\left(1 + \frac{2L_i}{p} + \frac{2L_i^2}{p^2}\right) \\ \rho_k^{\mathrm{sh}} &= 1 + \frac{2L_1}{p} + \frac{2\sum_{i \le k} L_i^2}{p^2} \end{aligned} \tag{11.22}$$

where the first line is the floor under local information of Chen and coauthors [15], exact for one stage, and the second is what this chain gives with shared sales. The application's checks reproduce the one-stage factor from Equation (11.21) within 1.8×10^{-15} at five windows from 1 to 28 days, and the shared expression from the linear chain within a relative 3.1×10^{-15} at the seven windows of the sweep below. The shared expression is not the centralized value $1 + 2\sum L/p + 2(\sum L)^2/p^2$ of the same reference, 5.90 at the plant for $p = 7$, because here the orders reach each stage a day late while the sales reach every stage at the same midnight. Each stage then adds its own L^2 once instead of multiplying the stage below, so sharing does not remove the squared lead-time term.

The three rules have three different peaks, as shown in Figure 11.2(a). The shipped human rule peaks at 1.89 at 0.395 rad/d, a wave of 15.9 days, so any swing near that period leaves each stage larger than it arrived. Counting the supply line lowers the peak to 1.56 at 0.324 rad/d, a 19.4-day wave, and the challenge's suggested $a_S = 0.3$ with $\beta = 1$ peaks at 1.60. Order-up-to on a 7-day average peaks at $13/7 = 1.857$ at the two-day wave, $\omega = \pi$, with ripples that touch one at multiples of $2\pi/7$. None of the three is string stable, which is one reason the cures that work in practice change the information rather than the rule alone. The curve is not drawn for the MPC or for hourly and weekly calendars, where the linearization would misdescribe the controller.

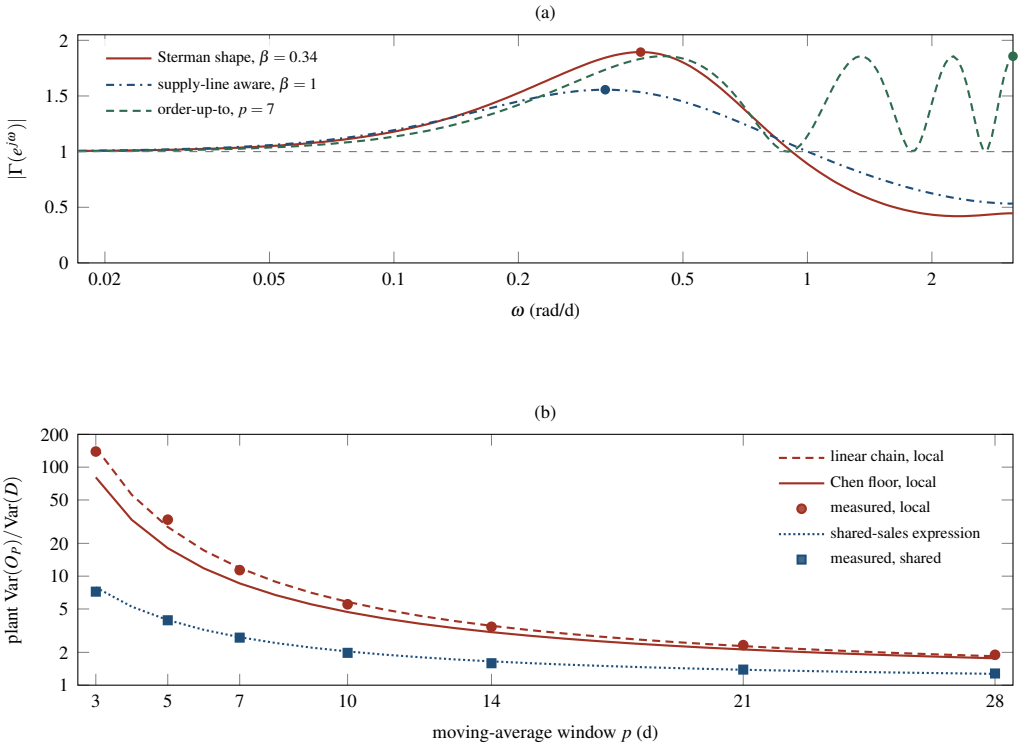


Figure 11.2 The rule and the floor, from the application's own functions. (a) $|\Gamma(e^{j\omega})|$ of the depot's rule, $\lambda = 2$ d, for the shipped human rule (solid), the same rule with $\beta = 1$ (dash-dotted) and order-up-to on a 7-day average (dashed), peaks marked by dots and unity by a thin dashed line. (b) The plant's variance ratio against the forecast window under the assumptions of case 4, with the Chen floor of Equation (11.22) (solid), the linear chain's own local ratio from Equation (11.21) (dashed), the shared-sales expression (dotted), and the ratios measured over 104 weeks at seed 11 (dots local, squares shared).

The floor holds where its assumptions hold. Case 4 runs Chen's chain, with order-up-to at every stage, no quantum and a cap of 80 kL/d, a 60 kL/d bottling line and a 40 kL/d dryer, a 21-day shelf life pulled at its date, spot milk on so that no plan is short of milk, and 2 % white daily noise on a flat calendar. Swept over $p = 3$ to 28 days for 104 weeks at seed 11, the plant sits at or above its floor every time, from 139.1 against 80.6 at $p = 3$ through 11.36 against 8.58 at $p = 7$ to 1.90 against 1.77 at $p = 28$, as shown in Figure 11.2(b). The floor is a bound, not the expected value. The linear chain's own ratio at $p = 7$ is 11.89, because the orders that reach the depot carry the supermarket's forecast and are correlated from day to day. Over seeds 1 to 8, which only a headless run can choose because the case fixes its seed, 104 weeks give a mean of 11.80 and a minimum of 11.22. At 104 weeks all eight seeds sit above the floor at every window, while at 52 weeks single seeds dip by up to 1.5 % below it at $p = 21$ and 28, 2.12 against 2.13 and 1.74 against 1.77, where the expected value exceeds the floor by only 4 to 7 %. With shared sales the same sweep falls below the local floor at every window and within 9 % of the shared expression, 7.21 against 7.89 at $p = 3$, 2.73 against 2.76 at $p = 7$ and 1.28 against 1.27 at $p = 28$. In the perishable, capacity-limited chain of the other cases orders are clipped, sales censored and milk poured out, and the floor there is a reference, not a bound.

One plausible expectation fails by measurement. With $a_S = 1$ and $\beta = 1$ the plant's order might be expected not to overshoot at a step, but a lasting 10 % demand step on the constant-flow reference lifts it to 34.9 kL/d before it settles at 22, because the supply line wanted, $\lambda_k F_k$, rises with the forecast

Table 11.3 The Cases and the Challenge

#	Change from the defaults	Teaching point
1	supermarket in MANUAL, flat calendar, promotion at week 2	the lead time by hand
2	flat calendar, promotion at week 2	the bullwhip and its milk
3	case 2 with $\beta = 1$	count the supply line
4	order-up-to under Chen's assumptions, 2 % noise, seed 11	a floor no tuning beats
5	$\beta = 1$, shared sales, seasons from week 9	the push and the dryer
6	case 4 with shared sales	information moves the floor
7	case 2 with $L_s = 10$ d, $f_L = 50\,\%$	perishability multiplies waste
8	MPC, shared sales, $K_f = 30$, $K_d = 15$ kL/d, seasons, promotion at week 2	limits planned for
9	$\beta = 1$, shared, $a_S = 0.3$, $R_0 = 22$, weekend lift, heat at week 2	who carries the shortage
ch	seasons, 5 % noise, seed 7, promotion at 10, heat at 20	several limits at once
ref	flat calendar, nothing moves	the equilibrium check

Table 11.4 Measured Cases. Total Cost Is the Operating Cost Plus the Margin Lost to the Dryer

Case and setting	wk	ρ_P	poured (kL)	raw (kL)	lost (kL)	outage (h)	total (k$)
1, order held at 20	8	0.00	0.0	0.0	80.0	96	226.1
2, as shipped	8	12.70	32.5	30.5	31.0	41	254.7
3, $\beta = 1$	8	7.61	0.0	0.0	27.0	44	205.8
4, $p = 7$	26	11.19	0.0	0.0	0.0	0	708.2
5, as shipped	16	0.21	0.0	0.0	0.0	0	528.8
5, $K_d = 12$	16	0.21	0.0	167.0	0.0	0	587.2
6, shared, $p = 7$	26	2.52	0.0	0.0	0.0	0	708.1
7, as shipped	8	8.95	261.4	0.0	24.8	43	298.7
8, MPC	8	0.35	0.0	0.0	0.0	0	206.7
8, $\beta = 1$ rule	8	2.11	13.1	8.4	1.5	4	227.7
9, spot off	8	6.15	0.0	0.0	0.0	0	78.8
9, spot on	8	0.74	2.4	0.0	0.0	0	118.2
ref	8	—	0.0	0.0	0.0	0	218.4

ρ_P the plant's ratio, Equation (11.14), poured the finished milk past its code date.

and must be filled once. What $\beta = 1$ removes is the oscillation. The plant's net stock crosses its target twice and overshoots it by 2.73 kL, and its order never falls below 21.3 kL/d after the peak, while at $\beta = 0.34$ with the same a_S the stock crosses its target 19 times and the order swings between 0 and the 80 kL/d limit.

11.6 Nine Cases on One Chain

Table 11.3 lists the cases. Each starts from the defaults of Table 11.2, the shipped human rule at every stage, local information and daily decisions, and changes only what its row lists. The figures below come from the engine run headless on each case's seed, over the window named, and Table 11.4 collects them. A variant that a case's briefing reaches by changing a setting and pressing Reset is measured that way, and the same change made before the first Run gives the same run. Integrated errors appear on the scorecard but are not quoted, because they keep accruing with the window.

11.6.1 The Lead Time by Hand

In case 1 the learner sets the supermarket's order while the depot and the plant run the human rule, and at the midnight of day 14 a promotion doubles demand to 40 kL/d for seven days. An order raised at that midnight is processed on day 15 and its first milk reaches the shelf in the first hour of day 16, while one raised at noon on day 14 waits for the next midnight and arrives on day 17. Left at 20, the 60 kL shelf empties early on day 17 and 80 kL of sales are lost, with the shelf empty for 96 hours in a row. Raised to 40 at the promotion's midnight and set back to 20 on day 19, it loses 6.5 kL, because

the depot behind it cannot ship the whole jump at once, and two days at 60 followed by three at 40 lose nothing. Because every order of the supermarket stays at 20 in the first run, the stages above see no variation at all and their ratios are zero. A turn-based option pauses the run at every midnight, and the application's check confirms 84 pauses in 84 days over four runs on three calendars, so a class can play it as the board game.

> **TRY IT IN THE APP** **1 · Play the Supermarket**
>
> Set Turn-based in Advanced so the run pauses at every midnight. At the midnight that opens the promotion on day 14, raise Supermarket order to 40, and set it back to 20 on day 19. The first extra milk reaches the shelf on day 16, and the shelf loses 6.5 kL of sales against 80 kL with the order left at 20.

11.6.2 The Bullwhip and Its Milk

Case 2 gives every stage the human rule. The supermarket's order peaks at 49.5 kL/d on day 17, the depot's at 65.5 kL/d on day 21 before falling to zero on day 31, and the plant's plan reaches the station's 80 kL/d limit on day 23 while the farms' 30 kL/d is all it can bottle. By day 40 it bottles 1.0 kL/d. Over eight weeks the ratios are 1.90, 6.36 and 12.70 at the supermarket, the depot and the plant. The overshoot ages on the supermarket's shelf until 32.5 kL, 2.54 % of what was bottled, is poured out at the four midnights that open days 44 to 47, and in the trough the dryer runs full for 144 hours and 30.5 kL of raw milk goes down the drain. The shelf is empty for 65 hours in all and 41 in the longest stretch, and the depot and the plant owe the stage below them for 270 and 430 hours. Figure 11.3 shows the trough at 5.75 weeks. A sweep in the application's checks makes the price explicit, because over 26 weeks of this case the plant's ratio and the milk poured out rise together as a_S goes from 0.1 to 1, from 2.5 and 18.5 kL to 72.1 and 141.5 kL, and as β falls from 1 to 0, from 7.9 and nothing to 29.8 and 183.0 kL.

> **TRY IT IN THE APP** **2 · The Bullwhip**
>
> Load the scenario, press Run and watch the depot's order on the trends through week 8. The one-week promotion at the till becomes a depot order of 65.5 kL/d on day 21 and nothing at all on day 31. Then open the Bullwhip view, where the plant's bar reads 12.70 against the shoppers' 1.

Case 3 changes only β, to 1. No milk is poured out, finished or raw. The depot's orders stay between 7.0 and 57.0 kL/d, the plant never bottles less than 14.5 kL/d, and the plant's ratio by week 8 is 7.61. The shelf still empties, losing 27.0 kL of sales against 31.0, and the plan still reaches 80 kL/d, because the backlog behind the promotion is real and the farms let the plant bottle only 30 kL/d. Counting the pipeline removes the double ordering, not the lead time or the farms' ceiling. The calendar matters as much as the gains. Deciding every hour instead of every midnight cuts case 2's lost sales from 31.0 to 8.4 kL and its longest stockout from 41 to 28 hours, but raises the plant's ratio from 12.70 to 13.89, while deciding weekly on Mondays orders the promotion week at the old rate and loses 80.0 kL, the board game's slowness.

Pause. Try Exercise 11.1 before reading on.

11.6.3 The Floor and Shared Sales

Over 26 weeks case 4 measures 2.13, 5.31 and 11.19 against floors of 2.22, 4.95 and 8.58. The supermarket lands on its value within the scatter of a 26-week estimate, because for one stage the expression is exact, and the depot and the plant land above theirs. At $p = 14$ the plant measures 3.34 against 3.07 and at $p = 28$ 1.90 against 1.77. Nothing is poured away and no sale is lost, because the shelf life is long and the capacities ample. This is the honestly infeasible case. With local information no forecast window gets under the floor, and the floor approaches one only as the window grows without end. Case 6 changes only the information, every stage forecasting from the published sales.

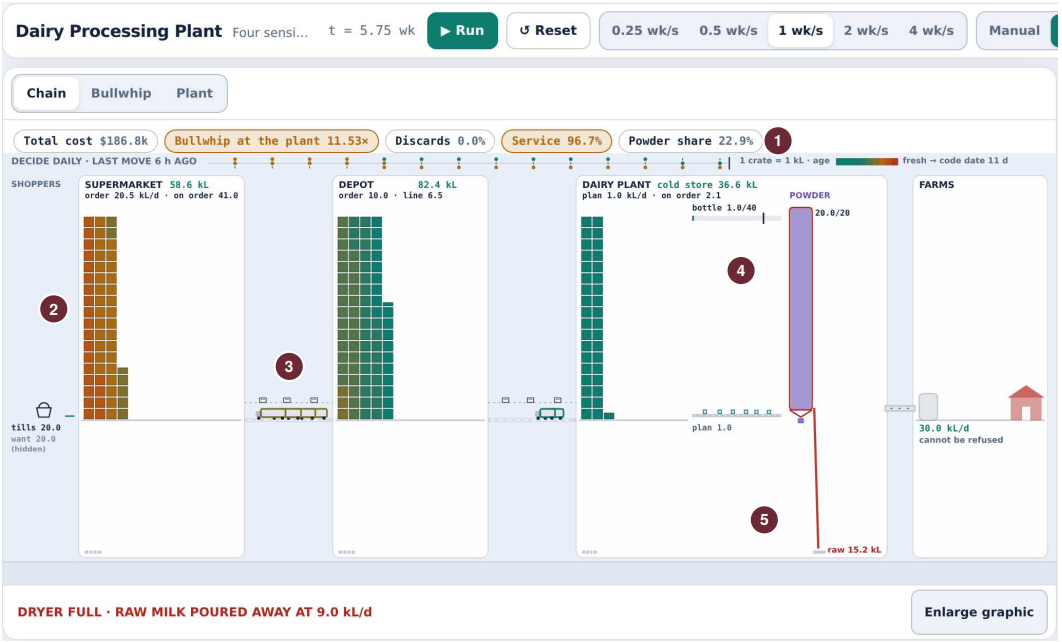


Figure 11.3 Case 2, the bullwhip, at $t = 5.75$ wk (day 40, 6 a.m.) in the Chain view, run in AUTO with nothing changed and an 8-week trend window, cropped to the header and the Chain view. The badges (1) read total cost $186.8k, bullwhip at the plant 11.53×, discards 0.0 %, service 96.7 % and powder share 22.9 %, and the status line reads DRYER FULL, raw milk poured away at 9.0 kL/d. The supermarket (2) holds 58.6 kL with 41.0 kL on order, every carton 7 to 9 days old against an 11-day code date, order slips and trucks (3) ride the lanes between the stages, the depot holds 82.4 kL and orders 10.0 kL/d, and the plant's cold store holds 36.6 kL while its line (4) bottles 1.0 kL/d of its 40, so its dryer takes 20 of its 20 and the drain (5) has taken 15.2 kL of raw milk. The decision strip reads daily, last move 6 h ago. In the app's trends, cut from this picture, the promotion at week 2 appears in the shoppers' demand, the depot's order climbs to its peak and falls to zero, and the plant's net stock falls far below its target while its bottling plan rests at the station limit. The app's scorecard, also cut, gives an operating cost of $117.6k, a powder diversion loss of $69.2k, a longest supermarket stockout of 41 h and 15.2 kL of raw milk discarded.

The ratios become 2.13, 2.51 and 2.52, the supermarket unchanged because it always forecast from its own tills, and the plant far under its local floor and near the shared value of 2.76. The total cost barely moves ($708.2k against $708.1k), because with a 21-day shelf life the extra variance costs only holding.

🕮 **Pause.** Try Exercise 11.2 before reading on.

11.6.4 Farms That Push

Case 5 starts in calendar week 9 with the weekend lift, the summer dip and the herds' season on, every stage counting the supply line and reading the shared sales. Over 16 weeks the farms send 3700 kL, peaking at 34.5 kL/d on day 69, while bottling stays between 18.0 and 22.0 kL/d. All of the difference goes to powder, 1486 kL or 40.2 % of the intake, a margin of $371.6k lost to the dryer. Without the flush the same run dries 1146 kL for $286.5k, so the flush itself costs $85.1k. With the dryer cut to 12 kL/d it is full for 2112 of the 2688 hours, 167.0 kL of raw milk is poured away, and the operating cost rises from $157.2k to $257.4k. The plant then behaves as a ratio controller on a flow it cannot change, splitting it between two outlets with limits, and the dryer is its surge capacity,

sized by the flush rather than by the average. The seasonal feedforward makes the supermarket chase the weekly pattern, raising its ratio from 0.94 to 2.65 and lowering the total by \$0.9k, because the shelf absorbs a weekly pattern almost free.

11.6.5 A Shorter Shelf Life

Case 7 is case 2 with a shelf life of 10 days instead of 14, pulled three days before its date, and half the shoppers taking the freshest carton. Over eight weeks 261.4 kL is poured out, 17.0 % of what was bottled, against case 2's 32.5 kL. The two causes compound. At 10 days with every shopper taking the oldest first the chain pours out 151.5 kL (10.5 %), and at 14 days with half of them taking the freshest it pours out 71.6 kL (5.3 %). Service barely moves, 98.0 % against 97.5 %, because the waste comes from the overshoot aging on the shelf. At a 7-day shelf life the milk passes its 4-day code date before it reaches the shelf and 88.4 % of what is bottled is poured out, which teaches nothing about the rule, so the case uses 10 days.

11.6.6 Planning Around the Limits

Case 8 hands every order to the MPC, with the line cut to 30 kL/d and the dryer to 15 kL/d so that both limits bind through the promotion, and the seasons on. It solves all 56 midnights of eight weeks without a failure, in a mean of 9.6 active-set iterations and at most 27. It ramps the supermarket's order to 47.5 kL/d on day 17, holds the plant at its 30 kL/d line for 312 hours, keeps the dryer at no more than 14.6 of its 15 kL/d, and serves every shopper with nothing poured away, at an operating cost of \$68.7k. The supply-line-aware rule on the same chain and the same shared sales asks for up to 55.5 kL/d against the line, overflows the dryer for 192 hours, empties the shelf for 4 hours (1.5 kL of lost sales) and pours away 13.1 kL of finished and 8.4 kL of raw milk, at \$92.9k. The planner meets the limits in its prediction, while the rule meets them as backlog and overflow.

11.6.7 Heat and Spot Milk

Case 9 is a plant that sells nearly all its milk fluid, the farms sending 22 kL/d to shoppers who buy 20 on average with a weekend lift, and a two-week heat wave at week 2 that raises demand 20 % and cuts the herds' milk 10 %. The stocks carry the shelf through, and no sale is lost either way. Without spot milk the shortage lands upstream, the plant owing the depot for 423 hours, up to 32.8 kL. With spot milk on the plant buys 69.8 kL and owes nothing, but the run costs \$118.2k against \$78.8k, the premium plus farm milk displaced into powder afterward (99 kL dried against 35). The override decides who carries the shortage, and here the safety stock carried it for free.

11.6.8 Twenty-Six Weeks Under Three Limits

The challenge runs 26 weeks from calendar week 9 through the spring flush, with a promotion at week 10, a heat wave at week 20 and 5 % daily noise at seed 7. Its target asks, at week 26, for a total cost of at most \$810k, finished milk poured out at most 3 % of bottling and no supermarket stockout longer than 24 hours. Table 11.5 compares seven configurations. The shipped rule fails all three clauses, at \$898.2k, 3.72 % and 44 hours, missing the limits by \$88.2k, 0.72 points and 20 hours. Each cure alone fails. Counting the supply line costs \$817.8k and still leaves a 44-hour stockout, and sharing the sales alone costs \$820.8k. Both together with $a_S = 0.3$, the suggested tuning, cost \$791.3k, pour out 1.28 % and leave a longest stockout of 6 hours, inside the limits by \$18.7k, 1.72 points and 18 hours, and at a half-hour step the result is the same to \$0.1k. The MPC with shared sales does better still, \$774.4k, 0.47 % and no stockout, over 182 solves without a failure. The diversion is \$468k to \$517k of every total, because the farms send half again what the shoppers buy. Weekly decisions break the suggested tuning, at \$916.6k, 4.82 % and 56 hours, while hourly decisions pass at \$789.2k with no stockout. The limit is \$810k rather than the \$820k first calibrated, because at \$820k sharing the sales alone missed by only \$0.8k. The verdicts do not hinge on seed 7. Over seeds 1 to 10, run headless because the case fixes its seed, the same four configurations fail and the same three pass on every seed, and the MPC and the hourly calendar cost less than the suggested tuning on every one,

Table 11.5 The 26-Week Challenge in Seven Configurations

Setting	chain (k$)	div. (k$)	total (k$)	waste (%)	outage (h)	ρ_S, ρ_D, ρ_P	target
shipped	430.1	468.1	898.2	3.72	44	1.43, 4.07, 9.51	not met
$\beta = 1$	307.9	509.9	817.8	1.49	44	1.01, 1.81, 4.92	not met
shared	325.1	495.7	820.8	2.29	20	1.72, 1.82, 1.78	not met
suggested	280.1	511.2	791.3	1.28	6	1.53, 1.25, 1.50	met
MPC, shared	257.5	517.0	774.4	0.47	0	1.00, 0.82, 0.67	met
hourly	277.0	512.1	789.2	1.14	0	1.30, 1.42, 2.29	met
weekly	423.2	493.4	916.6	4.82	56	0.70, 0.30, 0.21	not met

Suggested: $\beta = 1$, shared sales, $a_S = 0.3$, also run hourly and weekly.
Service 98.97 % shipped and 99.94 % suggested.

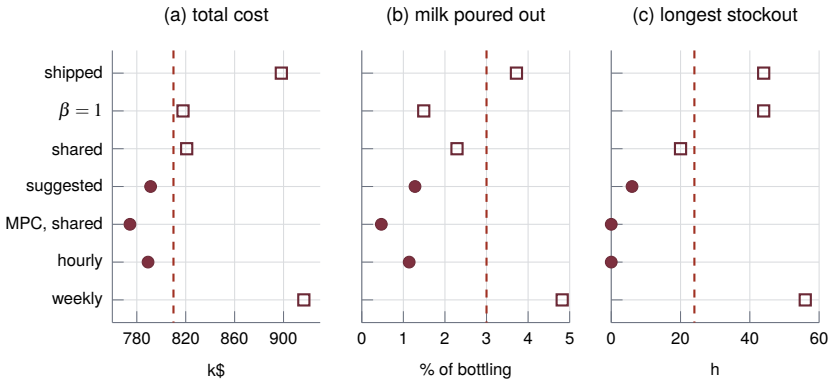


Figure 11.4 The 26-week challenge of Table 11.5 in its seven configurations against the three limits of the target (dashed): (a) a total cost of $810k, (b) 3 % of bottling poured out and (c) a 24-hour stockout. Filled circles mark the settings that meet the target and open squares those that do not. Notice that counting the supply line alone passes only (b), sharing the sales alone passes (b) and (c) but fails (a), and only their combination, the planner and the hourly calendar pass all three.

although the MPC's longest stockout reaches 12 hours on seed 4. The challenge is judged on the case as given, so a changed model parameter or disturbance setting, or an extra disturbance button, voids the attempt. Figure 11.4 draws the seven configurations against the challenge's three limits.

> ✦ **DID YOU KNOW? The Smallest Bullwhip, the Worst Result**
>
> From the simulator (Table 11.5). Deciding weekly gives the challenge its smallest variance ratios, 0.70, 0.30 and 0.21 from the supermarket to the plant. It is also the costliest of the seven settings, at $916.6k with 4.82 % poured out and a 56-hour stockout. A small ratio alone does not make a good chain.

In the constant-flow reference every order stays at exactly 20 kL/d for eight weeks and every stock returns to the same value at every midnight, while the plant dries a third of its milk for $140.0k of margin.

11.7 Technology in Practice

What Measures It

Each rule of this chapter reads three things: its stage's stock, the milk it has on order, and a forecast input, either the orders that reach it or the supermarket's published sales. In a real store these come from one stock record that software keeps from the deliveries received and the sales scanned at the tills, and the same software forecasts demand and triggers replenishment [20]. The model's

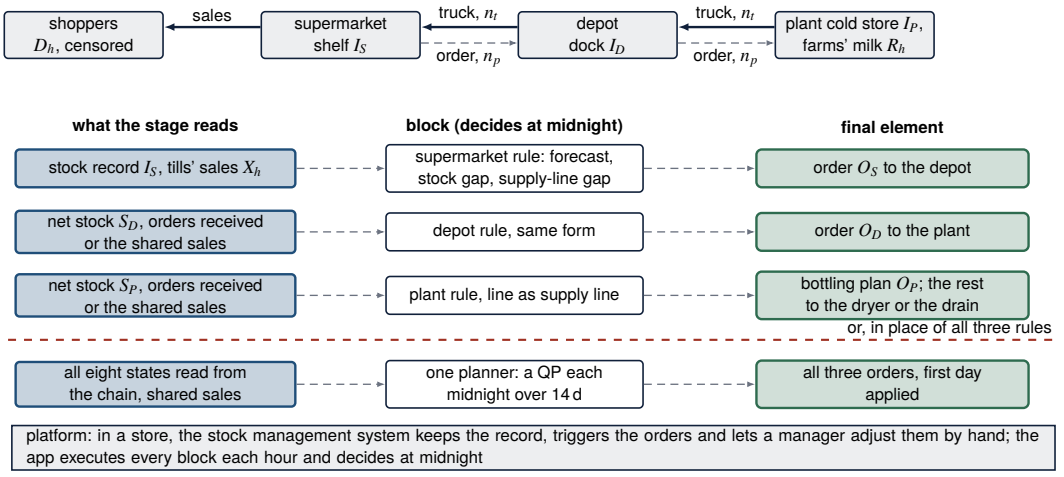


Figure 11.5 The ordering loops of Section 11.4 on real hardware, from what each stage reads to the order it sets, under the chain whose milk and orders move in opposite directions. Notice that the three decentralized rules each see one stock and one input, while the planner below the dashed line reads the whole chain and sets every order at once.

supermarket reads its shelf exactly, and its tills undercount demand only while the shelf is empty, as Section 11.4 states. A real record also drifts away from the shelf, and a store corrects it only by counting [20].

> **TECHNOLOGY** **Sensor: The Stock Record and the Tills**
>
> A store's stock is not read by a gauge on the shelf. Software keeps a stock record from the deliveries received and the sales scanned at the tills, and the tools that forecast demand and trigger replenishment read that record and the sales. A gap between the recorded and the actual stock is called inventory record inaccuracy, and earlier studies found 20 to 70 percent of records wrong. When the record shows more than the shelf holds, the system orders too late, and the shelf can empty without the system knowing. When it shows less, the system orders milk that is not needed. In a study of grocery stores, perishable items had the larger inaccuracy, and the stores corrected their records with physical counts up to four times a year [20].

What Moves It

The final element of every loop is an order: the supermarket's and the depot's order rates and the plant's bottling plan, each rounded to a quantum and capped, as Figure 11.5 draws them. An order becomes milk on refrigerated trucks, which the model reduces to a fixed transit of n_t days with every carton aging by the calendar. The Grade "A" Pasteurized Milk Ordinance governs that cold chain, from cooling the milk before it is filled to the temperature on the delivery vehicles [21].

TECHNOLOGY Final Element: The Order and the Refrigerated Truck

The same stock management system releases the order: it triggers orders automatically, and a store manager may adjust an order or trigger one by hand [20]. The milk the order calls for moves in refrigerated vehicles. The Grade "A" Pasteurized Milk Ordinance requires every vehicle that carries pasteurized milk to be built and operated so that the milk stays at 7 °C (45 °F) or less, and every refrigerated room that stores it to have a thermometer in its warmest zone [21]. A truck that runs warm does not spill the milk. Higher storage temperatures speed microbial growth in pasteurized milk and shorten the time it takes to spoil [18]. In this chapter's model no truck runs warm.

Where the Controller Runs

In a store the ordering rule runs inside the stock management system, which triggers orders automatically while the managers keep the option to adjust an order or to trigger one by hand [20], the role of the MANUAL stations of Section 11.4. No source read for this section prints the cycle of such a system, so none is given here. The app executes every rule each hour and lets a decision through only at midnight on the daily calendar, as shipped, or every hour or every Monday on the other two. The chapter's structure decides where the rules sit: three decentralized rules, one per stage, each with its own stock, supply line and forecast, or one planner that reads all eight states and the shared sales at each midnight and sets the three orders together.

What Is Modernizing It

Two developments bear on this chapter. Sensors and models aim to predict the shelf life left in fresh dairy products: spectroscopic, imaging and sensor-based methods give analytical fingerprints of quality, and chemometric and machine-learning methods relate those signals to quality and to microbial behavior [18]. The model instead pulls every carton at one code date. Radio-frequency identification, used during stock counts or to track stock movements, has lowered record inaccuracy substantially but not completely, and a grocery study found that counts correcting records that overstate the shelf raise sales, so the tags serve as a complement to targeted counts rather than a substitute [20]. Both feed the rules of this chapter better measurements: a stock that is really on the shelf and an age that the milk really has.

IN PRACTICE Date Codes Set by Practice, Not One Rule

In the United States, federal law generally does not require or regulate date labels, infant formula excepted. A review of 2013 found that most states that require dates apply them to specific foods such as milk and dairy, very few define what the words mean, and virtually none set how the date is determined. A sell-by date serves the retailer's stock control [22]. On the shelf, perishable goods add first-in, first-out rotation, date checks, markdowns and the removal of expired goods to a store's routines, and each step is a further chance for a record error [20].

11.8 Limitations

The chain is one product, one store, one depot and one plant, and its managers run one rule with teaching gains, while real people vary from one another and from round to round. Ages move once a day, the bottling day is not aged, and quality is a single code date rather than a function of temperature and handling. The farms' milk is smooth and compulsory, the spot market unlimited up to a fixed amount at a fixed premium, and the cost ledger leaves out capital, energy and fluid revenue. The MPC's daily prediction is linear, knows no carton ages and replaces expiry by freshness ceilings, so its comparison with the rules is a seeded teaching result, not evidence of general optimality. The variance floor holds only under the assumptions of case 4, and a single run's ratio is an estimate that scatters around its expected value, by up to 1.5 % below the floor at 52 weeks. Every threshold, the challenge's three among them, is a teaching value.

OPEN CHALLENGE **Stock Records Nobody Has Counted**

Every rule and the planner of this chapter read their stocks exactly. A real one reads a record that drifts from the shelf, and the drift cannot be seen when it occurs: it is measured only afterward, by a deliberate count. In one grocery study a store-wide count lifted sales by about 11 percent, all of it on items whose record had shown more stock than the shelf held, and more on perishable items. Radio-frequency tags improve visibility but do not remove the errors, so the study expects most from selective technology combined with focused counts and disciplined routines [20].

11.9 Conclusion

The dairy chain gives an ordering mistake a physical destination, a shelf of cartons near their code date or a drain under a full dryer. With the shipped human rule a one-week promotion becomes a depot order of 65.5 kL/d and then nothing, a plant ratio of 12.70, 32.5 kL of milk poured out and 30.5 kL of raw milk drained, and counting the supply line pours none. Order-up-to under the analytical assumptions sits at or above the local floor at every window from 3 to 28 days, 11.36 against 8.58 at a week, while shared sales take it to 2.73. A 10-day shelf life multiplies case 2's waste by eight, and the MPC holds a cut bottling line for 312 hours without a lost sale. In the challenge the shipped rule costs $898.2k with a 44-hour stockout, each cure alone fails, and both together pass at $791.3k and 6 hours. The chain is aggregated and its values are for teaching. A fuller model would add a network of stores, quality driven by temperature, a stochastic farm supply and planners that see the ages of their stock. The simulator makes the bullwhip a measurement of milk, money and hours rather than a ratio alone.

KEY POINTS

- A manager who forgets the milk already on order orders the same milk twice, and the error grows from stage to stage.
- With the shipped human rule a one-week promotion becomes a depot order of 65.5 kL/d and then nothing, a plant ratio of 12.70 and 32.5 kL of milk poured out, and counting the supply line pours none.
- Order-up-to under the analytical assumptions sits at or above the local floor at every window, 11.36 against 8.58 at a week, while shared sales take it to 2.73.
- In the challenge each cure alone fails, and both together pass at $791.3k with a 6-hour stockout.

Where else this applies. The product startup of Volume 4, Chapter 8 sets its production with the same anchoring-and-adjustment rule, its supply-line term counting the work in process, decided on a quarterly calendar as shipped.

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11.10 Exercises

11.1 Concept check · 2 min

In case 2 the supermarket's shelf is still short a day after a large order left the store, although that order is already on a truck. The rule of Equation (11.16) orders more milk again. Which change to the rule stops it ordering the milk on the truck a second time, while it still corrects the stock gap?

(a) Lower the stock adjustment a_S, so that each day closes a smaller share of the stock gap

(b) Raise the supply-line weight β to 1, so that every liter on order is counted

(c) Lower the forecast smoothing θ, so that the forecast follows the orders more slowly

(d) Raise the order quantum q_O, so that small corrections round away to zero

11.2 Concept check · 2 min

In case 4, order-up-to on a 7-day moving average with local information, the plant's variance ratio over 26 weeks is 11.19 against a floor of 8.58 from Equation (11.22). Which single change takes the plant's ratio below the local floor computed for the setting in use?

(a) Raise the moving-average window from 7 to 14 days
(b) Lower every stage's stock target from 2 or 3 days of demand to 1 day
(c) Forecast every stage from the supermarket's shared sales
(d) Count the supply line with $\beta = 1$ at every stage

11.3 Concept check · 2 min

At 5.75 weeks in case 2 the plant bottles 1.0 kL/d while the farms deliver 30 kL/d, the dryer's capacity is 20 kL/d and spot milk is off. Where do the other 29 kL/d go?

(a) Back to the farms, which hold their milk until the bottling plan rises again
(b) Into the cold store, where they wait to be bottled when the orders return
(c) To spot buyers, who take the surplus at the premium c_s
(d) 20 kL/d to powder and 9 kL/d down the raw-milk drain

11.4 Theory and calculation · 5 min

Take the shipped human rule, $a_S = 0.26/\text{d}$ and $\beta = 0.34$, in Equations (11.15) and (11.16).

(a) Compute the gain $g(a_S, \delta)$ for daily, weekly and hourly decisions, $\delta = 1$, 7 and $1/24$ day. *Checkpoint:* 0.260, 0.1255 and 0.2992 per day.

(b) Show that a gap left unchanged for a week is closed by the same share on all three calendars, and give that share. *Checkpoint:* $1-(1-a_S)^7 = 0.878$.

(c) The supermarket forecasts 20 kL/d. Its stock is 10 kL below target, and the order it raised yesterday has put its supply line 10 kL above the $\lambda_S F_S$ it wants. Compute the daily order of the human rule and of the supply-line-aware rule, after the quantum of 0.5 kL/d. *Checkpoint:* 21.5 and 20.0 kL/d.

11.5 Theory and calculation · 6 min

Use Equation (11.20) with daily decisions.

(a) Show that $\Gamma(1) = 1$ for every choice of a_S, a_{SL}, λ and forecast filter, and say what it means for a lasting change in demand. *Checkpoint:* $\Gamma(1) = a_S/a_S = 1$.

(b) For order-up-to, $a_S = a_{SL} = 1$, show that the denominator is one. Then evaluate $|\Gamma|$ at $\omega = \pi$ for the depot, $\lambda = 2$, with a 7-day moving average, and give the period of that wave in days. *Checkpoint:* $13/7 = 1.857$, a 2-day wave.

(c) Evaluate $|\Gamma|$ for the same rule at $\omega = 2\pi/7$. *Checkpoint:* 1.

11.6 Theory and calculation · 6 min

Use Equation (11.22) for case 4: lead times plus the review day $L = 3$, 3 and 2 days at the supermarket, the depot and the plant.

(a) Compute the local floors of the three stages at $p = 7$. *Checkpoint:* 2.22, 4.95 and 8.58.

(b) Compute the plant's shared-sales value and the centralized value $1 + 2\sum L/p + 2(\sum L)^2/p^2$ at $p = 7$, and say why the chain's shared value is the smaller. *Checkpoint:* 2.76 and 5.90.

(c) Case 4's completion rule asks for a run at $p \geq 14$ with the plant's ratio at least 95 % of its floor. Compute the floor at $p = 14$ and check the measured 3.34 against the rule. *Checkpoint:* Floor 3.07; $3.34/3.07 = 1.09$.

11.7 App challenge · 8 min

Scenario 1, Play the supermarket, default seed. The supermarket's orders are in Manual at 20 kL/d, and the promotion starts at the midnight that opens day 14 (week 2.00).

(a) Before running, predict the day on which the first milk of an order raised at that midnight reaches the shelf, and the day for one raised at noon on day 14. *Checkpoint:* Day 16 and day 17.

(b) Press Run and leave the order at 20 for 8 weeks at 1 week per second (8 s). Read the Longest supermarket stockout tile, and the last value of the exported `service_fraction` column. Convert the service into lost sales, given that the shoppers want 1260 kL over the 8 weeks. *Checkpoint:* 96 h; 0.9365, so 80 kL (within 0.5 kL).

(c) Reload the case. In Advanced set Turn-based: pause at every midnight to On. Press Run at each pause. At the pause at day 14 set Supermarket order to 40, and at the pause at day 19 (week 2.71) set it back to 20. Then set Turn-based to Off and run to week 8. Read the same two values. *Checkpoint:* 24 h; 0.9948, so 6.5 kL (within 0.5 kL).

11.8 App challenge · 8 min

Scenario 2, The bullwhip, default seed, at 1 week per second.

(a) Run 8 weeks. Read the three stages' bars on the Bullwhip view (or the last row of `S_bullwhip`, `D_bullwhip` and `P_bullwhip`), the Raw milk discarded tile and the last value of `finished_waste_kL`. *Checkpoint:* 1.90, 6.36, 12.70; 30.5 kL; 32.5 kL (all within 0.1).

(b) In the Ordering policy card set Decide to Hourly, press Reset and run 8 weeks again. Read the plant's bar, the Longest supermarket stockout tile and the last `service_fraction`. *Checkpoint:* 13.89; 28 h; 0.9934.

(c) Explain why deciding every hour cuts the lost sales but raises the plant's ratio, and name the setting that would cut both. *Checkpoint:* The supply-line weight, $\beta = 1$.

11.9 App challenge · 8 min

The challenge, Twenty-six weeks (the tenth case in the list), default seed (7), at 4 weeks per second (6.5 s per run).

(a) Run the shipped managers for 26 weeks. Read the Total cost including diversion tile, the Longest supermarket stockout tile, the Challenge tile and the last `finished_waste_fraction`. *Checkpoint:* \$898.2k, 44 h, not met, 0.0372.

(b) Press Reveal suggested tuning, then Reset, and run 26 weeks. Read the same values. *Checkpoint:* \$791.3k, 6 h, met, 0.0128.

(c) Keep those settings, set Ordering rule to Centralized MPC, press Reset and run 26 weeks. Read the same values and the MPC solves / failures tile. *Checkpoint:* \$774.4k, 0 h, met, 0.0047; 182 / 0.

Give costs within \$0.1k.

11.10 Application area · 4 min

Case 5 runs 16 weeks (112 days) through the spring flush. The farms send 3700 kL in all, peaking at 34.5 kL/d on day 69, while bottling stays between 18.0 and 22.0 kL/d and 1486 kL is dried.

(a) Compute the mean powder rate and the dryer capacity needed at the flush peak, and say why a 12 kL/d dryer must pour raw milk away. *Checkpoint:* 13.3 kL/d; 12.5 to 16.5 kL/d.

(b) With the dryer cut to 12 kL/d, 167.0 kL of raw milk is poured away. Price it at c_w and compare with the rise in operating cost the chapter reports. *Checkpoint:* \$100.2k, the whole rise from \$157.2k to \$257.4k.

11.11 Application area · 4 min

In Table 11.5 the shipped managers lose the least margin to the dryer, \$468.1k against \$511.2k for the suggested tuning. The powder margin loss is $m_f - m_p = 0.25\,\$/\mathrm{L}$ and milk poured out costs $c_w = 0.60\,\$/\mathrm{L}$.

(a) Convert both diversion losses into kiloliters of powder and give the difference. *Checkpoint:* 1872.4 and 2044.8 kL; 172.4 kL.

(b) A plant manager concludes that the shipped managers use the farms' milk best. Explain where the milk the shipped chain did not dry went, and why that is the more expensive outlet. *Checkpoint:* Bottled and later poured out, or drained raw, at 0.60 $/L.

11.12 Review the agent · 5 min

An AI assistant reviews case 2 and writes: "The bullwhip comes from managers who react too slowly. Raise the stock adjustment a_S from 0.26 to 1/d at every stage, so each stage closes its stock gap in one day. The shelf will be empty less often, and the swings will die out sooner." Find the flaw, say how you would check it in the app, and propose a better change.