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CHAPTER 2

Wood Pellet Grill Simulator for Temperature Control Through a Fuel Delay

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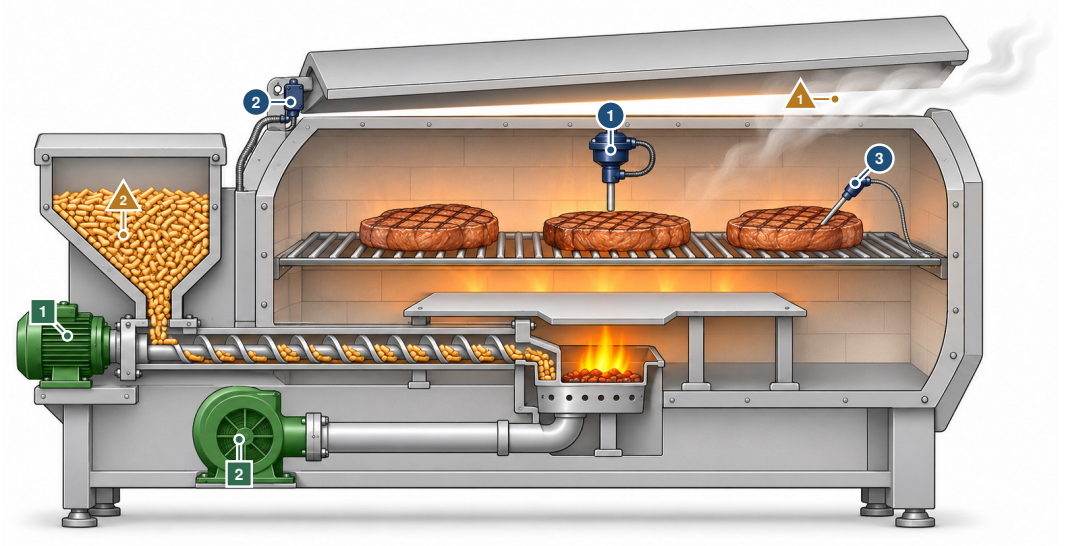
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CHAPTER 2

Wood Pellet Grill Simulator for Temperature Control Through a Fuel Delay



The controller moves 1 motor that turns the auger feeding the pot 2 fan blowing air into the pot

The controller reads 1 grate temperature probe 2 lid switch 3 meat probe

What disturbs it 1 lid opened by the cook 2 damp pellets that burn weaker

Pellets ride the auger from the hopper to the pot and turn to heat only after they catch and burn, while an open lid lets the hot air out within seconds. Illustration, not to scale.

IN PLAIN TERMS Steering a Fire That Is Fed Ahead

The main idea. A pellet grill holds its heat steady by feeding small wood pellets into a fire. A screw carries each pellet to the fire, and the pellet gives off its heat only minutes later, once it has caught and burned.

The challenge. When the lid opens, the hot air escapes within seconds, and a simple controller answers by feeding more pellets. Those pellets burn after the lid is shut, so the grill runs too hot.

Why it matters. A controller that keeps track of the fuel already on its way, and expects the lid to shut soon, holds the grill close to its setting.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the 1957 history segment (Section 2.1), the 1986 history segment and the fact on pellet stoves (Section 2.2), the try-it on the lid (Section 2.4), the try-it on damp pellets and the fact on the pulsed auger (Section 2.6) and the key points
Study	65 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	20 min	the first page, the eighteen measured cases (Section 2.6), Technology in Practice (Section 2.7), Limitations (Section 2.8) and the two open challenges

The control question A pellet grill's controller commands fuel, not heat, and each pellet becomes heat only after it has ridden an auger, caught on the bed and burned. When the lid opens and the chamber air is replaced within seconds, what should the loop do with the feed, and what must it know about the fuel already on its way?

Learning objectives

- Compute the fuel in transit, the heat it carries and the steady heat release from the pellet ledger.
- Fit a first-order plus dead-time model to a feed step, and explain why the dead time depends on the fitting method.
- Derive a SIMC tuning from the fit, and predict the feed a PI loop adds while the lid is up.
- Compare a standard PID, a lid-aware hold and an MPC on the same lid opening, and explain the role of the MPC's assumption about the lid.
- Interpret the heat bias that the extended Kalman filter estimates under damp pellets.

Prerequisites Sections 1.4.2 (dead time and a fitted model), 1.4.3 (prediction through a delay), 1.4.7 (Kalman filter with a disturbance state) and 1.6.2 (velocity-form PID). Section 1.6.3 is needed for the predictive controller.

The app https://apmonitor.com/apps/pellet-grill/

See also Volume 2, Chapter 4 for step tests and tuning on a temperature loop, and Volume 2, Chapter 3 for back-calculation at an actuator limit. Volume 9, Chapter 2 for a burner behind a dead time under predictive control, and Volume 9, Chapter 4 for an offset-free MPC whose estimator carries a fuel bias.

2.1 Introduction

A pellet grill is a thermostat on a wood fire, and its controller cannot add heat to the chamber, only fuel. The fuel it adds becomes heat in three stages. A pellet leaves the hopper on an auger and reaches the burn pot 20 s later, waits about 45 s for the hot bed to light it, and then burns over about 90 s. In the simulator of this chapter, a grill holding 225 °F over a stalled pork shoulder carries 19.6 g of pellets on their way to the fire at any moment, worth 267 kJ of heat, while its steel holds 2137 kJ above the outdoor air. A 5 % step in the feed moves the grate probe by 3.5 °F in five minutes and by 10.5 °F in ten.

The same shape recurs wherever a solid fuel is dosed into a fire. Internal model control of a biomass grate boiler has to be designed around a time-delayed input [1], a small wood pellet boiler emits less carbon monoxide under a nonlinear predictive controller of its airflow [2], and the compensators that handle a known delay, descended from the Smith predictor, work by predicting the delayed effect of the controller's own past moves [3], as Section 1.4.3 shows on the running example of Chapter 1. Model predictive control (MPC) carries that prediction as a model of the plant and is established practice across the process industries [4], and an estimated disturbance added to its model removes the offset that model error would otherwise leave [5].

> ⧖ HISTORY **1957: Predicting Past the Dead Time**
>
> Dead time hurts a feedback loop in two ways: the effects of the controller's moves and of the load take time to reach the measurement, and the controller acts now on an error that began earlier. A loop tuned for speed then oscillates, and one tuned against oscillation turns sluggish. In 1957 Smith proposed a model-based answer, now the best known and most widely used dead-time compensator in academia and industry. It splits the process model into a delay-free part and the delay, and gives the primary controller a virtual signal that anticipates the process output, which removes the delay from the closed-loop characteristic equation when the model is exact [6]. The MPC of this chapter carries the same idea in its own record of the feed in the auger.

The simulator is built against a common belief, that a large disturbance calls for the strongest response a loop can make. The lid of a grill is a disturbance that no controller can counteract while it

lasts, because the chamber air is replaced within seconds, and every pellet fed against it burns after the lid is shut. With the lid up for three minutes, a proportional-integral (PI) loop tuned from a step test raises the feed from 23 to 41.8 % and brings the grate 9.7 °F above its setpoint thirteen minutes after the lid shuts, and brisker tunings overshoot by 28.9 °F and 60.8 °F. A lid-aware variant that holds the feed while the lid switch reads open overshoots by 3.0 °F and settles 8.7 min after the lid shuts, and an MPC that estimates the fuel on the way and assumes the lid will shut at any moment overshoots by 2.7 °F and settles in 0.7 min. Told instead that the lid will stay open, the same MPC feeds 100 % and overshoots by 92.8 °F, so the controller's assumption about the disturbance matters as much as its algorithm.

However, the literatures that touch this plant treat its parts apart. Cooking is modeled as coupled heat and mass transfer inside the meat [7, 8], pellet appliances are characterized by their emissions in steady tests [9], and boiler control is developed for large grates without a lid, a stall or a recipe [1]. This chapter works on a browser simulator of a barrel pellet grill and measures how the fuel in transit, the lid, the meat and the recipe act on one controller, with the true state of the grill logged beside what its instruments report. The lumped model, the velocity-form PID, the offset-free MPC and the recipe logic are established methods. The results are measurements, eight answers to one lid opening and a dozen other cases on a single plant and seed.

Section 2.2 follows the domain from smokehouses and pellet furnaces to the pellet grill. Section 2.3 sets out a 41-state model whose pellet, energy and water ledgers close to integration error, with every equation and constant needed to build the simulator again. Section 2.4 measures the fuel on its way, a feed step with its two dead times, and the lid. Section 2.5 describes the supervisor that lights the fire and declares a flame loss, the standard and lid-aware PID loops, and a predictive controller that carries its own feed history through the auger's delay. Section 2.6 measures eighteen cases, from a feed step and a lid opening through six recipe programs to a set of faults and a graded challenge. Section 2.7 sets the loop on real hardware, and Sections 2.8 and 2.9 state the limits of the model and what the cases show.

2.2 Smokehouses, Pellet Fuel and the Pellet Grill

Cooking meat in smoke under a controlled temperature and humidity began as an industrial process. The smokehouse cooking of meat emulsions was studied in 1987 by raising the smokehouse temperature at 0.42 to 0.98 °C/min and its relative humidity at 1.1 to 3.9 %/min, and by measuring the product's water holding capacity, emulsion stability, shrinkage and texture, with the least shrinkage obtained under a rapid rise of both [10]. The quantities that study controlled, a chamber temperature program and the humidity around a wet product, are the ones a recipe controls in the grill simulated here. Smoke itself comes from incomplete combustion, and the polycyclic aromatic hydrocarbons it deposits on smoked meats are a food-safety concern of their own [11].

Wood pellets made the fire meterable. Densified biomass fuels were characterized with regard to their combustion behavior, with their moisture, ash and heating value among the properties that set how they burn [12]. Pellet stoves for homes were measured across fuel loads, pellet diameters and chimney drafts, and lowering the fuel load raised the emissions of carbon monoxide and hydrocarbons dramatically in a stove that burned continuously, while in a stove that ran intermittently at low load the carbon monoxide rose sharply because of high emissions at the end of each combustion cycle [9]. A grill smoking at 225 °F burns about 0.46 kg of pellets an hour, 23 % of its auger's capacity, so its fire spends most of a cook at low load, and its auger can be driven continuously or in pulses.

The pellet grill came out of the same fuel, through the two patents of the history segment below. The hopper, auger, pot, fan and baffle of the grill simulated here keep that arrangement, and its controller closes a temperature loop on a grate probe, with the recipe programs and lid logic that the problem invites.

> ⧖ HISTORY **1986: A Furnace That Feeds Itself, Then a Barbecue**
>
> A furnace patented in 1986 was meant to run for an extended period without supervision and give a controlled heat output from particulate solid fuel, such as wood waste pellets. A metering wheel took fuel from a hopper, an auger turned by an electric motor moved it along a feed tube to a fire pot, and the fuel was fed intermittently: with no demand for heat, an operating cycle could run, for example, for three minutes every half hour to keep the fire burning [13]. The pellet-fired barbecue patented in 1989 set the pot below the barbecue pan, fed it by an auger under the pan, forced combustion air through it, and covered it with a metallic baffle plate that radiated heat to the food and caught the drippings. With the feed stopped, its burner put itself out relatively rapidly [14].

Larger biomass plants met the same control problem earlier and at a larger scale. Internal model control of a 180 kW biomass grate boiler, designed around a time-delayed input, held high efficiency and low emissions across all load ranges in experiments [1], and a nonlinear predictive controller of the airflow, built on a Hammerstein model of a small wood pellet boiler, cut its carbon monoxide emissions by 35 to 50 % [2]. The grill adds three things those plants do not have, a lid that an operator opens at will, a load that stalls on its own evaporation, and a setpoint that a recipe moves on the clock and on the meat.

> ✦ DID YOU KNOW? **A Pellet Stove's Efficiency**
>
> Certified pellet stoves usually run at 70 to 83 % efficiency, with heating capacities from 8000 to 90 000 Btu per hour [15]. The grill of this chapter burns its pellets at an efficiency of 0.80 and releases 7.6 kW at full feed.

2.3 The Grill as Modeled

The simulated grill is a generic barrel pellet grill rather than a commercial model. A 9 kg hopper feeds an auger rated at 2 kg/h into a burn pot under a baffle and a sloped diffuser and drip tray, a combustion fan delivers up to 36 kg/h of air, a 250 W igniter rod lights the pot, and a single-wall barrel of 0.12 m^3 holds up to two cuts of meat and a water pan, as shown in Figure 2.1. The grate probe TT-101 supplies the controlled variable and the auger feed u_a is the manipulated variable, while the fan follows a schedule on the feed unless it is set by hand or given to the MPC.

Each variable in Table 2.1 carries its role in the loop, controlled (CV), manipulated (MV), disturbance (DV) or state, and the state vector has 41 entries. Ten describe the grill (the hopper, the two pot inventories, the bed, diffuser, shell and gas temperatures, the humidity ratio, the smoke index and the fan speed), seven describe each of two cuts (surface and core temperatures, accessible water, the probe's lag, and bark, smoke and tenderness indices), seven more hold the pan's water and enthalpy, a condensate film on the shell, the steam kit's boiler and reservoir and the lags of the grate and dew-point probes, and ten are ledgers of the pellets fed and burned, the heat released, the energy crossing the boundary and the water entering and leaving. The auger's transport is a delay line of 40 cells outside the integrated state. Units are seconds, kilograms, watts and °C, with kelvin inside the radiation terms, and the interface shows °F by default.

2.3.1 The Pellets' Journey

The hopper empties at the auger's rate, the pellets reach the pot a transport time later, and in the pot they wait as unignited fuel before they burn as the active bed, as shown in Equation (2.1).

$$\begin{aligned} \frac{\mathrm{d}M_h}{\mathrm{d}t} &= -q_{feed}, \qquad q_{feed} = q_{max}\, u_a\, \mathrm{S}\left(\frac{M_h}{0.05\,\mathrm{kg}}\right) \\ q_{arr}(t) &= q_{feed}(t-\theta_t) \\ \frac{\mathrm{d}M_u}{\mathrm{d}t} &= q_{arr} - r_{act}, \qquad \frac{\mathrm{d}M_b}{\mathrm{d}t} = r_{act} - r_{burn} \end{aligned} \tag{2.1}$$

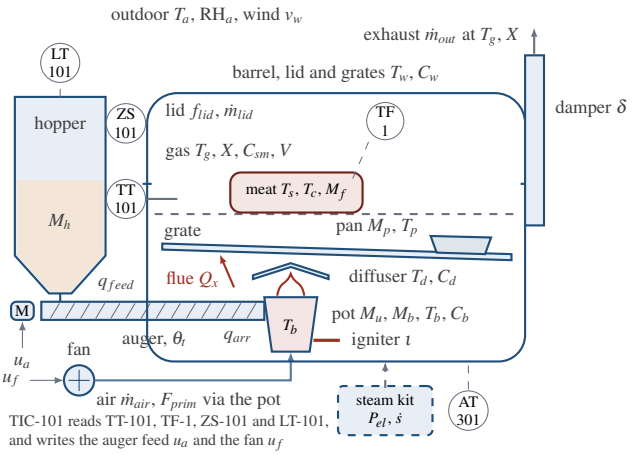


Figure 2.1 Wood pellet grill as modeled. The hopper holds M_h and the auger meters q_{feed}, which reaches the pot as q_{arr} after the transport time θ_t. The pot holds unignited pellets M_u and a burning bed M_b at T_b, lit by the igniter ι and burned in the fan's air $\dot{m}_{air}$, of which the share F_{prim} passes through the pot. The flue gas gives Q_x to the diffuser at T_d, and the chamber gas at T_g with humidity ratio X, smoke C_{sm} and volume V exchanges heat with the barrel, lid and grates at T_w, the meat (surface T_s, core T_c, water M_f) and the pan (M_p, T_p). The lid opens by f_{lid} and exchanges $\dot{m}_{lid}$, the exhaust leaves through the damper δ, and the steam kit and AT-301 are the experimental option. Not to scale.

Table 2.1 Variables of the Grill by Role, With Their Values at the Steady Operating Point of Case 1

Variable	Role	Case 1	Range
TT-101	CV, grate probe	225 °F	SP 150–550 °F
u_a	MV, auger feed	22.9 %	0–100 %, floor 8
u_f	MV, fan (scheduled)	31.5 %	0–100 %
f_{lid}	DV, lid, ZS-101	0	0–1
T_a, RH$_a$	DV, outdoor air	68 °F, 40 %	0–110 °F, 5–100
v_w	DV, mean wind, gusts	2.2 mph	0–30, 0–15 mph
b	DV, air lost at intake	0	0–80 %
d_p	DV, damp pellets	0	0–100 %
b_{TT}	DV, probe bias	0 °F	±20 °F
M_h	state, LT-101	9.0 kg at start	0–9
M_u, M_b	states, pot	5.7 g, 11.4 g	≥ 0
T_b, T_d, T_w	states, steel	643 °C, 376 °F, 147 °F	—
T_g, X, C_{sm}	states, gas	225 °F, dew 105 °F	RH < 100 %
T_s, T_c, M_f	states, per cut	158 °F, 152 °F, 60 %	—
M_p, H_p	states, pan	2 kg	0–4
10 ledgers	states	0	—

where $\mathrm{S}(x) = 3\bar{x}^2 - 2\bar{x}^3$ with $\bar{x}$ the value of x clipped to $[0, 1]$, so the auger starves smoothly as the hopper runs out, and $q_{max} = 2\,\mathrm{kg/h}$. The delay is a line of $\theta_t/\Delta t = 40$ cells at the 0.5 s physics step, filled with the mass fed in each step. In the alternative pulsed drive the motor runs at full speed for $u_a t_{cyc}$ of each cycle of $t_{cyc} = 15\,\mathrm{s}$, and the tube advances one cell only in a step in which the motor turns, so at a duty of 23 % a pellet crosses it in about $\theta_t/u_a = 87\,\mathrm{s}$. Fresh pellets catch only on a hot bed or at the igniter, and the bed burns no faster than its temperature, its inventory and its oxygen

allow, as shown in Equation (2.2).

$$
\begin{aligned}
r_{act} &= \frac{M_u}{\tau_{act}(1+2d_p)}\,\max\big(f_{ign},\,0.35\,\iota\big)\\
r_{burn} &= \operatorname{smin}\big(r_{kin},\,r_{O_2}\big), \qquad r_{O_2} = \frac{0.85\cdot 0.232\,\dot m_{air}}{1.286}\\
r_{kin} &= \frac{M_b}{\tau_{burn}}\,f_T\left(\frac{\dot m_{air}}{\dot m_{ref}}\right)^{0.4}\\
f_{ign} &= \mathrm{S}\left(\frac{T_b - T_{ign} + 50}{100}\right)\\
f_T &= \max\left(\mathrm{S}\left(\frac{T_b - T_{burn} + 50}{100}\right),\,0.6\,\iota\right)
\end{aligned}
\tag{2.2}
$$

where temperatures are in °C, $\iota \in \{0,1\}$ is the igniter, $d_p \in [0,1]$ is the damp-pellet disturbance, 0.232 is the mass fraction of oxygen in air, 1.286 kg of oxygen burn a kilogram of pellets, and 0.85 of the fan's oxygen is usable. The smooth minimum $\operatorname{smin}(a,b) = m\,(1+(m/n)^{16})^{-1/16}$ with $m = \min(a,b)$ and $n = \max(a,b)$ is exactly zero when either rate is, lies within 4.3 % of the true minimum where the two cross, and is exact far from it. With $T_{ign} = 230\,^\circ\mathrm{C}$ and $T_{burn} = 170\,^\circ\mathrm{C}$, no fresh pellet catches on a bed below 180 °C and the bed goes out below 120 °C, so a fire starved of fuel cools, stops lighting new pellets and dies, and a cold pile of pellets stays a pile. The heat released is $Q_{comb} = \eta\,\mathrm{LHV}(1-0.25d_p)\,r_{burn}$ with $\eta = 0.80$ and $\mathrm{LHV} = 17\,\mathrm{MJ/kg}$, 7.6 kW at full feed, and the excess-air ratio of what burns is $\lambda = 0.232\,\dot m_{air}/(1.286\,r_{burn})$.

2.3.2 Air, Flue Gas and the Fire Bed

The fan's air follows its command through a lag, adds to a natural draft through the pot, and loses what a blocked intake or a closed damper takes, as shown in Equation (2.3).

$$
\begin{aligned}
\tau_{fan}\frac{\mathrm{d}f}{\mathrm{d}t} &= u_f - f\\
\dot m_{air} &= \left[\dot m_{air,max}\,f\,\frac{\rho_a}{1.204}\,(1-b) + \dot m_{dr}\right](0.6+0.4\,\delta)\\
\dot m_{in} &= s_k\big[\dot m_{leak}\,\delta\,(1+0.25\,v_w)\\
&\qquad + \dot m_{lid}\,f_{lid}^{1.5}\,(1+0.2\,v_w)\big]
\end{aligned}
\tag{2.3}
$$

where ρ_a is the outdoor air density in $\mathrm{kg/m^3}$, $\dot m_{dr} = 3\times 10^{-4}\sqrt{\max(0,T_b-T_a)/300}$ in kg/s is the draft, $\delta \in [0.2,1]$ is the damper opening, $\dot m_{in}$ is the air entering by leakage and through the lid, and $s_k = \sqrt{\max(0.02,\,(T_g-T_a)/(T_g+273.15))/0.23}$ is a stack factor equal to one at 225 °F in 68 °F air. The flue gas carries the dry air, the dry products and the water of combustion, 0.572 kg per kilogram burned, with 0.007 kg of ash left behind, at a dry flow $\dot m_{fg}$, humidity X_{fg} and heat capacity rate $\dot C_{fg}$. The primary air, $F_{prim} = 0.4$ of the fan's plus the draft, passes up through the bed with the products at the rate $\dot C_{prim}$, and the secondary air mixes in above it at ambient. The bed balance, the mixed flue temperature and the heat the flue gives the diffuser's underside are Equation (2.4).

$$
\begin{aligned}
C_b\frac{\mathrm{d}T_b}{\mathrm{d}t} &= Q_{comb} + P_{ign}\,\iota - \dot C_{prim}(T_b - T_a)\\
&\qquad - Q_{rb} - K_{pot}(T_b - T_a)\\
T_{mix} &= T_a + \frac{\dot C_{prim}}{\dot C_{fg}}(T_b - T_a)\\
Q_x &= \left(1 - \mathrm{e}^{-UA_x/\dot C_{fg}}\right)\dot C_{fg}\,(T_{mix} - T_d)\\
T_{in} &= T_{mix} - Q_x/\dot C_{fg}
\end{aligned}
\tag{2.4}
$$

where $Q_{rb} = 0.9\,\sigma\,(0.012\,\mathrm{m}^2)(T_b^4 - T_d^4)$ is the pot mouth radiating to the diffuser, $K_{pot} = 0.3\,\mathrm{W/K}$ is the pot housing's loss, and T_{in} is the temperature at which the flue gas enters the chamber. In case 1 the bed runs at 643 °C and the fire releases 1733 W at $\lambda = 5.1$.

2.3.3 The Steel and the Chamber Gas

The diffuser, heated from below by radiation and flue gas, loses heat to the gas, the shell, the meat and the pan, and the barrel with its lid and grates loses heat outside, as shown in Equation (2.5).

$$\begin{aligned} C_d \frac{\mathrm{d}T_d}{\mathrm{d}t} &= Q_{rb} + Q_x - hA_d(T_d - T_g) \\ &\quad - Q_{dw} - Q_{df} - K_{dp}(T_d - T_p) \\ C'_w \frac{\mathrm{d}T_w}{\mathrm{d}t} &= Q_{dw} + hA_w(T_g - T_w) + Q_{cw} \\ &\quad - Q_{wf} - Q_{wp} - Q_{wa} - Q_{wo} \end{aligned} \tag{2.5}$$

where $C'_w = C_w + c_w M_{wc}$ includes the condensate film, $h = h_{in}(1 + 1.2 f_{lid})$ because an open lid stirs the chamber, $Q_{dw} = 0.9\sigma A_d(0.55)(T_d^4 - T_w^4)$, Q_{df}, Q_{wf} and Q_{wp} are the heat given to the meat and the pan, Q_{cw} is the latent heat of the film, and the losses outside are $Q_{wa} = (h_{out} + 2.5 v_w^{0.8})A_o(T_w - T_a) + \varepsilon_{out}\sigma A_o(T_w^4 - T_a^4)$ and, with the lid up, $Q_{wo} = f_{lid}\left[(6 + 3v_w)(0.3)(T_w - T_a) + 0.9\sigma A_{open}(T_w^4 - T_a^4)\right]$, with v_w in m/s and the areas $A_w = 1.6$, $A_o = 1.7$, $A_d = 0.30$ and $A_{open} = 0.35\,\mathrm{m}^2$. The chamber gas is well mixed and leaves at T_g, so every stream that enters is written at its own temperature against T_g, as shown in Equation (2.6).

$$\begin{aligned} C_g \frac{\mathrm{d}T_g}{\mathrm{d}t} &= \dot{C}_{fg}(T_{in} - T_g) + \textstyle\sum_i r_i c_{pv}(T_i - T_g) \\ &\quad + \dot{m}_{in,d}(c_{pa} + X_a c_{pv})(T_a - T_g) \\ &\quad + hA_d(T_d - T_g) - hA_w(T_g - T_w) \\ &\quad - \textstyle\sum_f Q_{gf} - hA_p(T_g - T_p) \end{aligned} \tag{2.6}$$

where $C_g = M_d(c_{pa} + X c_{pv})$, $M_d = PV/\big(R_d(T_g + 273.15)(1 + X/0.622)\big)$ is the dry gas the chamber holds, $\dot{m}_{in,d}$ is the dry part of $\dot{m}_{in}$, the sum over i carries the sensible heat of the vapor arriving from each source at its own temperature T_i, and Q_{gf} is the convection to cut f. At 225 °F the gas holds about 110 J/K against 21.2 kJ/K of steel, so it follows the steel and the fire within seconds.

2.3.4 Water in the Chamber

The humidity ratio is fed by the air, the water of combustion, the meat, the pan and the steam kit, and carried out with the exhaust, as shown in Equation (2.7).

$$\begin{aligned} M_d \frac{\mathrm{d}X}{\mathrm{d}t} &= \dot{m}_{fg}(X_{fg} - X) + \dot{m}_{in,d}(X_a - X) \\ &\quad + \textstyle\sum_f r_f + r_p + \dot{s} - r_{wc} \\ p_v &= \frac{PX}{0.622 + X}, \qquad T_{dew} = T_{sat}(p_v) \\ \mathrm{RH} &= \frac{p_v}{p_{sat}(T_g)} \le \frac{P}{p_{sat}(T_g)} \end{aligned} \tag{2.7}$$

where p_{sat} and its inverse T_{sat} are the region-4 equations of IAPWS-IF97 [16], r_f and r_p are the evaporation of the meat and the pan, $\dot{s}$ is the steam, and r_{wc} is condensation on the shell. Above the boiling point the saturation pressure exceeds the chamber pressure, so the relative humidity cannot approach 100 %. In case 1 it is 5.8 % and could not pass 77.6 % however much water went in, while the dew point is 105 °F. A smoke index follows $M_d \dot{C}_{sm} = 1000\,y_s r_{burn} - \dot{m}_{out,d} C_{sm} - 2 \times 10^{-4} M_d C_{sm}$, with a yield $y_s = 1 + 6\,\mathrm{S}((1.4 - \lambda)/0.6) + 3(1 - f_T)$ that rises when the fire is short of oxygen or smoldering. Its chemistry is not modeled.

Table 2.2 The Cuts. Mass m, Area A, Surface-to-Core Conductance K_{sc}, Surface Share ϕ_s, Water Share, Critical Share and Skin Coefficient k_m

Cut	m (kg)	A (m^2)	K_{sc} (W/K)	ϕ_s	water	crit.	k_m (kg/(s Pa m^2))
pork shoulder	4.0	0.155	1.8	0.30	0.34	0.22	0.9×10^{-8}
beef brisket	5.5	0.25	2.2	0.30	0.32	0.22	0.9×10^{-8}
spare ribs	1.6	0.15	4.0	0.35	0.28	0.32	0.9×10^{-8}
whole chicken[a]	1.8	0.085	1.6	0.30	0.30	0.22	2.0×10^{-8}
thick ribeye	0.6	0.045	1.4	0.35	0.25	0.40	0.9×10^{-8}
second butt	3.0	0.13	1.5	0.30	0.34	0.22	0.9×10^{-8}

[a]Water activity $a_w = 0.98$, all others 0.97. The water share sets $M_{f,0} = \text{water}\cdot m$, and M_{crit} is the critical share of $M_{f,0}$.

2.3.5 The Meat and the Stall

Each cut is an outer layer that exchanges heat and evaporates and a core that the probe reads, with the accessible water held in the core's heat capacity and migrating out to the surface, as shown in Equation (2.8).

$$\begin{aligned} C_s\frac{\mathrm{d}T_s}{\mathrm{d}t} &= Q_g + Q_d + Q_w - K_{sc}(T_s - T_c) - Q_o \\ &\quad - L_v(T_s)\,r + c_w\,r\,(T_c - T_s) \\ C_c'\frac{\mathrm{d}T_c}{\mathrm{d}t} &= K_{sc}(T_s - T_c), \qquad \frac{\mathrm{d}M_f}{\mathrm{d}t} = -r \\ r &= \omega\,\mathrm{smin}\big(k_m A\,[w^2 a_w\,p_{sat}(T_s) - p_v]^+,\; R_{mig}Aw\big) \end{aligned} \tag{2.8}$$

where $C_c' = C_{c,dry} + c_w M_f$, $w = \min(1, M_f/M_{crit})$, $[x]^+ = \max(0,x)$, $R_{mig} = 1.8\,\mathrm{kg/(h\,m^2)}$ caps how fast water reaches a wet surface, $\omega = 0.05$ for a wrapped cut and 1 otherwise, and $L_v(T) = L_0 - (c_w - c_{pv})T$ with $L_0 = 2.501\,\mathrm{MJ/kg}$. The heat arriving is $Q_g = \rho_h h A(T_g - T_s)$, $Q_d = \rho_r(0.85)\sigma A\beta(T_d^4 - T_s^4)$ from the diffuser below and $Q_w = \rho_r(0.85)\sigma A(0.9-\beta)(T_w^4 - T_s^4) + 0.3(A/0.155\,\mathrm{m^2})(T_w - T_s)$ from the shell and the grate, with $\beta = 0.2$ facing the diffuser, and a wrap sets $\rho_h = 0.9$ and $\rho_r = 0.5$ (both 1 otherwise). With the lid up the surface also radiates $Q_o = 0.3 f_{lid}\rho_r(0.9)\sigma A(T_s^4 - T_a^4)$ to the sky. When the gas is above the surface's dew point the surface condenses at $r = 9\times10^{-8}A(p_{sat}(T_s) - p_v) < 0$ and the condensate soaks into the core, and a cut resting on the counter loses $Q_o = 6A(T_s - T_a) + 0.9\sigma A(0.7)(T_s^4 - T_a^4)$ to still air. The cuts are listed in Table 2.2, with $C_s = \phi_s m c_m$, $C_{c,dry} = (1-\phi_s)mc_m - c_w M_{f,0}$ and $c_m = 3300\,\mathrm{J/(kg\,K)}$. No plateau is written anywhere in Equation (2.8). A wet surface cannot heat past the temperature at which its latent loss balances the heat it receives, and the core cannot pass the surface, so the stall emerges from the evaporation and ends as w falls, as in the coupled heat and mass transfer models of convection cooking [7]. The migration cap stands for the water that a shrinking protein network expels toward the surface [8].

Three qualitative indices, from 0 to 100, are labeled as such in the interface. Bark grows as $\dot{B} = \mathrm{S}((T_s - 60)/25)(0.25 + 0.75(1-w))(1 + 200\,\mathrm{S}((T_s - 105)/25))(100 - B)/9000\,\mathrm{s}$ on an unwrapped cut, smoke deposits while the surface is cool at $\dot{S}_m = C_{sm}(0.35 + 0.65\,\mathrm{S}((75 - T_s)/25))(100 - S_m)/(100\cdot 5400\,\mathrm{s})$, and tenderness follows an Arrhenius rate of 110 kJ/mol referenced to 85 °C above a 55 °C onset with a 5760 s time constant. None is a food-safety or doneness prediction.

2.3.6 The Water Pan, the Film and the Steam Kit

The pan is carried as an enthalpy, so that it boils at the pressure's boiling point instead of heating past it, as shown in Equation (2.9).

$$\begin{aligned}
\frac{\mathrm{d}M_p}{\mathrm{d}t} &= -r_p \\
\frac{\mathrm{d}H_p}{\mathrm{d}t} &= Q_{gp} + K_{dp}(T_d - T_p) + Q_{wp} - h_v(T_p)\, r_p \\
T_p &= \frac{H_p}{C'_p} - \left[\frac{H_p}{C'_p} - T_{boil}\right]^+ \frac{M_p}{M_p + 0.005\,\mathrm{kg}} \\
r_p &= 9\times 10^{-8} A_p \left(p_{sat}(T_p) - p_v\right)\psi_p \\
&\quad + \left[\frac{H_p - C'_p T_{boil}}{5\,\mathrm{s}\cdot L_v(T_{boil})}\right]^+ \psi_p
\end{aligned} \tag{2.9}$$

where $C'_p = 400\,\mathrm{J/K} + c_w M_p$, $\psi_p = M_p/(M_p + 0.01\,\mathrm{kg})$ applies to evaporation only, $A_p = 0.06\,\mathrm{m}^2$, $h_v(T) = L_0 + c_{pv}T$ and $T_{boil} = T_{sat}(P)$. An empty pan is steel alone and heats up. The condensate film on the shell grows at $9\times 10^{-8} A_w(p_v - p_{sat}(T_w))$ while the gas is above the shell's dew point and dries back at the same coefficient scaled by $M_{wc}/(M_{wc} + 0.002\,\mathrm{kg})$. The experimental steam kit is a boiler of $C_{sg} = 1.5c_w + 800\,\mathrm{J/K}$ whose 1 kW heater P_{el} tapers off between T_{boil} and $T_{boil} + 2\,^\circ\mathrm{C}$ and whose valve passes $\dot{s} = u_s \dot{s}_{max}\,\mathrm{S}((T_{sg} - T_{boil} + 4)/4)$ up to $\dot{s}_{max} = 1.1\,\mathrm{kg/h}$, both scaled by $M_{res}/(M_{res} + 0.02\,\mathrm{kg})$, with feedwater at 15 °C and a loss of 1.5 W/K.

2.3.7 Instruments and the Lid

Every probe lags its quantity to first order, as shown in Equation (2.10).

$$\begin{aligned}
\tau_{TT}\frac{\mathrm{d}T_{pr}}{\mathrm{d}t} &= T_g - T_{pr}, \qquad \tau_{food}\frac{\mathrm{d}T_{fp}}{\mathrm{d}t} = T_c - T_{fp} \\
\tau'_{dew}\frac{\mathrm{d}T_{dp}}{\mathrm{d}t} &= T_{dew} - T_{dp}
\end{aligned} \tag{2.10}$$

with $\tau_{TT} = 8\,\mathrm{s}$, $\tau_{food} = 15\,\mathrm{s}$ and $\tau'_{dew} = 20\,\mathrm{s}\,(1 + 5\varphi)$ lengthened by the fouling φ. TT-101 reads $T_{pr} + b_{TT}$ plus first-order colored noise of 0.3 °C standard deviation and 2 s correlation, rounded to 0.1 °C. The meat probes add 0.1 °C of noise and read to 0.1 °C, AT-301 reads $T_{dp} - 4\varphi$ with 0.2 °C of noise and is flagged invalid above its 180 °C head rating, the lid switch ZS-101 reads open whenever $f_{lid} > 0.05$, and the level LT-101 reads the hopper to 0.1 kg. The lid moves at $0.5\,\mathrm{s}^{-1}$ toward its target, fully open or shut in 2 s, and a timed opening shuts itself.

2.3.8 Parameters, Initialization and Numerics

The adjustable and hidden parameters are listed in Table 2.3, with the constants $c_{pa} = 1010$, $c_{pv} = 1900$ and $c_w = 4186\,\mathrm{J/(kg\,K)}$ and $R_d = 287.05\,\mathrm{J/(kg\,K)}$. Every constant is an illustrative teaching value, calibrated so that 225 °F with a stalled shoulder and a pan burns about 1 lb of pellets an hour and full feed takes the empty grill to about 590 °F. It is not a measurement of a commercial grill.

A warm case starts from the steady state the grill reaches at its setpoint with the meat and the pan where the case puts them. The plant itself is run for three hours in 2 s fourth-order Runge-Kutta steps (shorter when the fan or probe lag is set faster than 2 s) with the meat, the pan and the ledgers frozen, under a PI loop of $K_c = 1.5\,\%$ per °C and $\tau_I = 600\,\mathrm{s}$ on the true gas temperature with the fan on its schedule, and the auger is then filled at the steady feed. A cold case starts with everything at the outdoor temperature and the grill OFF. The engine of Section 1.6.1 integrates the plant by the fourth-order Runge-Kutta method with a fixed step $\Delta t = 0.5\,\mathrm{s}$, and the historian logs every 5 s. Over an hour of case 4 with the pan and the steam kit running, the pellets balance to $1.2\times 10^{-14}\,\mathrm{kg}$, the energy ledger closes to 1.8 J of 6.64 MJ released and the water ledger to $4.8\times 10^{-7}\,\mathrm{kg}$ of 0.79 kg.

Table 2.3 Model Parameters. Hidden Constants Are Shown but Cannot Be Edited in the Application

Symbol	Meaning	Value	Unit	Visibility
q_{max}	auger feed at 100 %	2.0	kg/h	adjustable
LHV	heating value as fed	17	MJ/kg	adjustable
η	combustion efficiency	0.80	—	adjustable
θ_t	auger transport time	20	s	adjustable
τ_{act}, τ_{burn}	ignition lag, burn time	45, 90	s	adjustable
$M_{h,max}$	hopper capacity	9	kg	adjustable
P_{ign}	igniter power	250	W	adjustable
T_{ign}, T_{burn}	ignition, burning thresholds	230, 170	°C	hidden
$\dot{m}_{air,max}$	fan air at 100 %	36	kg/h	adjustable
$\dot{m}_{ref}$	air at which τ_{burn} applies	12.6	kg/h	hidden
τ_{fan}	fan lag	2	s	adjustable
$\dot{m}_{leak}$, $\dot{m}_{lid}$	leakage, open-lid exchange	2.16, 216	kg/h	adjustable
C_b, C_d, C_w	bed, diffuser, shell	1.2, 4, 16	kJ/K	adjustable
UA_x	flue gas to diffuser	4	W/K	adjustable
h_{in}, h_{out}	convection in, out (calm)	8, 3	W/(m^2 K)	adjustable
ε_{out}	outside emissivity	0.80	—	adjustable
V	chamber volume	0.12	m^3	hidden
$M_{p,0}$, K_{dp}	pan water, tray contact	2, 1.5	kg, W/K	adjustable
P_{el}, $\dot{s}_{max}$	steam heater, valve	1000, 1.1	W, kg/h	adjustable
$M_{res,0}$	steam reservoir	5	kg	adjustable
τ_{TT}, τ_{food}, τ_{dew}	probe lags	8, 15, 20	s	adjustable
$T_{dp,max}$	dew-point probe rating	180	°C	adjustable
P	atmospheric pressure	101.3	kPa	adjustable

Halving the step from 0.5 to 0.25 s reduces the energy residual over the first 25 min of case 4 from 1.30 to 0.028 J, so the residual is integration error and not a bookkeeping term. The noise is seeded, seed 20260928, so every case replays identically.

2.3.9 Assumptions

One well-mixed chamber, one bed, one diffuser, one shell. A real barrel has a hot zone over the pot, a cooler end by the chimney and stratified air. The lumping keeps the fact the lesson needs, that the steel stores far more heat than the air the probe reads, and it fails wherever the position of the meat or the probe in the barrel matters.

Fuel as mass, not particles. The pellets move through a delay and two inventories, and ignition and burning are smooth thresholds of one bed temperature. Pellet size, fines, ash accumulation and the bed's geometry are outside the model, and the heating value, efficiency, time constants and the primary-air share are declared values rather than measurements of a pellet or a pot [12].

Constant transfer coefficients. Convection uses fixed coefficients with a wind term outside, and radiation fixed view factors between gray bodies. The calibration against a pound of pellets an hour makes the totals plausible, not the split between the paths.

A two-node meat. The stall comes from surface evaporation limited by a critical water content and a migration rate, and the constants are chosen so that the default cuts cook in about the times pitmasters quote. Conduction inside the meat, fat rendering, drip loss, collagen chemistry and the smoke ring are not modeled, where a transport model of cooking would resolve them [7, 8].

A single smoke index and a single humidity ratio. Smoke is transported and deposited but has no chemistry, so its yield under a starved fire is qualitative, while real smoke composition sets flavor and the polycyclic aromatic hydrocarbons deposited on the meat [11].

Repeatable numerics. Because the step is fixed and the random sequences seeded, every machine produces the same cook, a repeatability that no grill's microcontroller reading a thermocouple can offer.

2.4 Dynamic Character

2.4.1 Fuel on Its Way and Heat in the Steel

Two inventories set how the grill answers its controller. At 30 minutes into case 1 the tube holds 2.5 g of pellets, the pot 5.7 g that are catching and 11.4 g that are burning, 19.6 g in all and 267 kJ at η LHV, while the steel holds 2137 kJ above the outdoor air. At the 1733 W the fire releases, the pellets already delivered are about 2.6 minutes of fire that the controller cannot take back, and with the auger stopped the fire still releases 80 % of its heat 60 s later, 39 % after 150 s and 11 % after five minutes. The steel's store is eight times larger, which is why the chamber recovers by itself once a lid is shut. The shell passes 841 W to the outdoor air and the flue stream carries 334 W of sensible heat up the chimney, and the rest goes into the meat and the pan as latent heat.

2.4.2 A Feed Step and Two Dead Times

Case 3 holds an empty grill at 225 °F by hand, the auger at 18.2 % and the fan fixed at 27 %, and steps the feed up by 5 % five minutes in. The extra pellets reach the pot exactly $\theta_t = 20$ s after the step, and by 60 s the heat release has moved 12 % of the way to its new value, 91 % by five minutes. Because the manual feed and fan are rounded from the steady values of the warm start, the grill drifts up 0.65 °F over four hours without any step, so the response is taken as the stepped run minus an otherwise identical run with the scripted step removed, which also cancels the probe noise. The grate is then 3.5 °F up five minutes after the step, 10.5 °F at ten and 36.5 °F after four hours. A first-order plus dead-time (FOPDT) model, Equation (2.11), fits the response.

$$\frac{T_{TT}(s)}{U_a(s)} = \frac{K\,\mathrm{e}^{-\theta s}}{\tau s + 1} \tag{2.11}$$

The gain is $K = 4.05$ °C per percent (7.29 °F per percent) and the time constant $\tau = 15.5$ min, while the dead time is 4.7 min by Smith's two-point method and 2.8 min by the tangent at the steepest point. The two fits disagree because the delay is a transport time in series with the ignition lag, the burn time and the lag of the steel, and no single dead time represents that chain. Section 1.4.2 sets this fit beside the other chapters of this volume, among them Chapter 3, which fits a dead time to a column that has no delay at all. The SIMC rule, an internal model control (IMC) derivation, with a closed-loop time constant $\tau_c = \tau/2$ gives $K_c = \tau/(K(\tau_c + \theta)) = 0.31$ % per °C and $\tau_I = \tau = 15.5$ min on Smith's dead time [17], which the application ships as 0.3 % per °C and 15.5 min. The tuning is conservative for a setpoint change, and the lid measures what that costs.

🕮 **Pause.** Try Exercise 2.2 before reading on.

2.4.3 The Lid

With the lid up in the default 2.2 mph breeze, the exchange of Equation (2.3) rises to 0.072 kg/s, about 73 W/K of fresh air against a gas that holds 110 J/K, so the chamber air is replaced within seconds. In case 4 the air falls 95.7 °F in three minutes while the shell loses 16.0 °F, and TT-101 bottoms at 128.7 °F. A PI loop's answer to the opening can be written down. The error reaches 53 °C as the lid shuts, so the proportional term raises the feed by $K_c e \approx 16$ %, and the error averages 50 °C over the three minutes, so the integral adds $(K_c/\tau_I)\,\bar{e}\,t_{lid} \approx 2.9$ %, which accounts for the measured rise from 23 to 41.8 %. The proportional part disappears within a minute of the lid shutting because the air recovers on the steel's heat, but the pellets fed meanwhile are already on their way and the integral keeps its 2.9 %. Over the 20 minutes after the lid opens the standard PID feeds 0.180 kg of pellets against 0.161 kg for the lid-aware PID and 0.166 kg for the MPC, and the extra 14 to 19 g, 190 to 250 kJ at η LHV, is what burns into the closed chamber.

🕮 **Pause.** Try Exercise 2.1 before reading on.

Table 2.4 Supervisor States and Transitions. TT_0 Is the Grate Reading at Start

State	Actuators	Leaves when
OFF	feed 0, fan 0	Start is pressed
IGNITING	igniter on, feed 50 then 15 %, fan 40 %	TT $\geq$ $TT_0 + 14$ °C, or rising 3 °C in 2 min and TT $\geq$ $TT_0 + 8$ °C (RUNNING), or 900 s pass (FAULT)
RUNNING	the loop in charge, feed floor 8 %	probe failed, TT $>$ 316 °C (600 °F), auger jammed 60 s, or flame loss (FAULT), Shut down (SHUTDOWN)
SHUTDOWN	feed 0, fan 60 %	TT $<$ 60 °C after 300 s (OFF)
FAULT	feed 0, fan 50 % for 10 min	Start is pressed

TRY IT IN THE APP 4 · A Three-Minute Lid Opening

Load the scenario and press Run with the Standard PID as shipped. Watch the feed strip while the lid is up from 20 to 23 minutes: the feed climbs to 41.8 %. Then watch the grate after the lid shuts. It recovers on the steel's heat, keeps rising, and peaks 9.7 °F over the setpoint thirteen minutes later.

2.5 Implemented Control System

The controller scans every $\Delta t_c = 0.5$ s, one physics step. The supervisor, the recipe, the auger drive and the fan act every scan, the PID and the estimator every second, and the MPC every 10 s. Each physics step runs, in order, the controller, the integrator, the auger's transport with the lid, and finally the outputs and the log. Only the MPC's labeled perfect-state teaching mode ever reads a true temperature.

2.5.1 Supervisor

The supervisor of Table 2.4 decides whether there is a fire to control. Start lights the igniter and primes the pot at 50 % feed for 60 s, then trickles at 15 % with the fan at 40 %, and hands over to the loop once TT-101 has risen 14 °C (25 °F) above its value at Start, or has risen 3 °C over two minutes of samples taken every 10 s while standing at least 8 °C above it. From cold, case 2 hands over 5.0 minutes after Start with the grate at 83 °F. A flame loss is declared from a five-minute memory of the grate and the feed, when the lid has been shut for five minutes, the grate has fallen more than 8 °C, the mean feed is above 30 % and the grate is more than 15 °C below the setpoint, because a loop cannot tell a dead fire from a heavy load. A fault stops the feed and runs the fan at 50 % for 10 min, and nothing returns to RUNNING without an explicit Start.

2.5.2 TIC-101 and the Lid-Aware Hold

The standard loop is the velocity-form PID of Section 1.6.2, run every second on TT-101 with the derivative on the filtered measurement, as shown in Equation (2.12).

$$
\begin{aligned}
e_k &= T_{SP,k} - T_{TT,k}, \qquad P_k = K_c e_k \\
D_k &= -K_c \tau_D \dot{\tilde{T}}_{TT,k} \\
s_k &= s_{k-1} + (P_k - P_{k-1}) + (D_k - D_{k-1}) \\
\hat{u}_k &= s_k + I_{k-1}, \qquad \bar{u}_k = \min\big(100,\ \max(u_{fl},\ \hat{u}_k)\big) \\
I_k &= I_{k-1} + \frac{\Delta t_{PID}}{\tau_I}\big[K_c e_k + (\bar{u}_k - \hat{u}_k)\big]
\end{aligned}
\tag{2.12}
$$

where $\tilde{T}_{TT}$ is TT-101 through a first-order filter of $\max(1\,\text{s}, \tau_D/8)$, $u_{fl} = 8\,\%$ is the feed floor while RUNNING, and $\Delta t_{PID} = 1$ s. A colder grate raises the feed, so the loop is reverse acting. The symbols map onto Equations (1.15) to (1.18) of Chapter 1 as $s_k = S_k$, $\hat{u}_k = v_k$ and $\bar{u}_k = u_k$, with no output bias, a tracking time $T_t = \tau_I$ and no rate limit, so anti-windup acts on the integral alone [18]. In MANUAL, and whenever another controller or the supervisor has the auger, the integral is reset every second to $I_k = u - s_k$ for the feed u actually applied, so every transfer is bumpless. The defaults are $K_c = 0.3\,\%$

per °C, $\tau_I = 930\,\text{s}$ and $\tau_D = 0$, from sliders of 0.05 to 5 % per °C, 1 to 60 min and 0 to 240 s. The MPC's horizon runs from 5 to 40 min, its move suppression from 0 to 20 and its model delay from 0 to 120 s.

The lid-aware variant runs the same loop until ZS-101 reads open. It then holds the feed at $u_{hold} = \min(\bar{u}_{120}, u_{lid})$, where $\bar{u}_{120}$ is the feed averaged with a 120 s first-order filter that is frozen while the lid is up and $u_{lid} = 30\,\%$ is a cap, keeps holding for 45 s after the lid shuts while the air recovers, and does not call the PID meanwhile, so the integral resumes where it was before the lid opened. The derivative filter is reset to the current reading on release, so the step the air made during the hold produces no derivative kick.

2.5.3 Fan and Auger Drive

By default the fan follows a schedule on the feed that the fire has been getting, $u_f = \min(1,\ (12 + 0.85\,\bar{u}_{90})/100)$ with $\bar{u}_{90}$ the feed through a 90 s first-order lag, so that the air follows the fuel rather than the command and the fan's own effect on the grate does not mask the fuel delay during a step test. Its gain on the grate can take either sign. With the fire short of oxygen more air releases more heat, and with the fire short of fuel more air carries more heat out of the chimney. The continuous auger meters the feed as a mean rate, and the pulsed drive of Section 2.3 turns it into an on-time in every 15 s cycle.

2.5.4 Model Predictive Control

The MPC runs every $\Delta t_m = 10\,\text{s}$ over a horizon of 20 min, $N_p = 120$ steps. Section 1.6.3 describes the option in general. Here the controller's model is the plant's own Equations (2.1) to (2.10) with the declared parameters, the defaults of Table 2.3 fixed when the application loads, the cuts on the grate, the mean weather and no damp pellets or blocked intake, run on the estimate rather than on the truth. The estimated state is $\hat{x} = [M_u, M_b, T_b, T_d, T_w, T_g, T_{s1}, T_{c1}, T_{s2}, T_{c2}, T_{pr}, d]$, where d is a heat bias in watts added to the gas balance of Equation (2.6). The meat's water, the pan, the humidity and the film are carried open loop by the same model, and the hopper comes from LT-101. The pellets in the auger are known without measurement, because the controller keeps its own record of every feed it commanded and passes it through its model's delay, a shift register of $n_d = \theta_m/\Delta t_m$ cells with $\theta_m = 20\,\text{s}$ by default, which may be set away from the plant's θ_t.

At each execution the model is linearized at the estimate by finite differences and discretized exactly for a zero-order hold of Δt_m with the matrix exponential of the augmented system $[A\ B\ c;\ 0\ 0\ 0]\Delta t_m$, with the lid's effect taken as the secant between shut and open. The prediction of TT-101 over the horizon is $\hat{y} = y_{free} + Sz$, where z holds twelve blocked moves of 1, 1, 1, 1, 2, 2, 3, 4, 6, 9, 15 and 75 steps and two slacks, and the quadratic program is Equation (2.13).

$$
\begin{aligned}
\min_{z}\ J = {} & \tfrac{1}{2}\textstyle\sum_{k=1}^{N_p} w_k\,(\hat{y}_k - r_k)^2/(5\,\text{K})^2 \\
& + \tfrac{1}{2} w_{\Delta u} \textstyle\sum_j (u_j - u_{j-1})^2 + w_f\, q_{max} \textstyle\sum_j \tau_j u_j \\
& + \tfrac{1}{2}\left(50\, s_{hi}^2 + 10^4 s_{lo}^2\right) \\
\text{s.t.}\ \ & u_{fl} \le u_j \le 1, \qquad |u_j - u_{j-1}| \le 0.25 \\
& q_{max} \textstyle\sum_j \tau_j u_j \le \hat{M}_h - 0.02\,\text{kg} \\
& \hat{y}_k \le r_k + 15\,\text{K} + s_{hi}, \qquad s_{hi},\ s_{lo} \ge 0
\end{aligned}
\tag{2.13}
$$

where u_j is the feed as a fraction in block j of τ_j seconds with u_0 the feed applied now, r_k is the setpoint, previewed from the recipe when it runs on the clock, $w_{\Delta u} = 10$, $w_f = 0$ by default, and the ceiling is imposed on every other step. The tracking weight is $w_k = 0.3$ for the first three steps and 1 after them, and zero while an announced or programmed lid opening has the lid up and for 30 s after it. An optional stage-hold policy adds the soft bounds $T_{SP} - 3\,^\circ\text{C} - s_{lo} \le \hat{y}_k \le T_{SP} + 3\,^\circ\text{C} + s_{lo}$ until the current stage ends. The fan may be a second decision variable, pulled toward its schedule

and held inside the envelope $u_f \geq 0.554\,u_a$, and the steam valve a third, with the dew point a second output. The dense dual active-set method of Goldfarb and Idnani [19] solves the program, and the first move is applied. If an execution fails, the last plan is followed, and on the third failure in a row the lid-aware PID takes over from the MPC's last output. On case 4 the program has 14 variables and 111 constraint rows, and over 361 executions it needs 0.07 active-set steps on average and 7 at most, with none failing and a median of about 0.5 ms of wall-clock time for a whole execution in the V8 JavaScript engine of the Chromium browsers.

What the MPC assumes about an open lid is a setting. By default it assumes the lid will shut at any moment, so that it predicts the chamber's recovery on the steel's heat and feeds only what the steel and the meat lost. It can instead assume that the lid stays open for the whole horizon, or that an opening lasts as long as a cook announced it would.

The estimator is an extended Kalman filter run every second, the filter of Equation (1.11) linearized afresh at each step, as shown in Equation (2.14).

$$\begin{aligned} \hat{x}^- &= \hat{x} + f(\hat{x}, v)\,\Delta t, \qquad F = I + \frac{\partial f}{\partial x}\Delta t \\ P^- &= FPF^\top + Q\,\Delta t \\ K &= P^- H^\top (HP^- H^\top + R)^{-1} \\ \hat{x} &= \hat{x}^- + K(y - H\hat{x}^-) \end{aligned} \tag{2.14}$$

where v holds the pellets arriving through the model's delay, the fan, the steam valve, the lid switch and the igniter, y holds TT-101 and the meat probes, applied as scalar updates in turn, and the Jacobian is taken by forward differences. The process noise per second is $4\,\mathrm{K}^2$ on T_b, 0.2 on T_g, 0.05 on T_d, 0.01 on T_w, T_s and T_{pr}, 1×10^{-4} on T_c, 1×10^{-9} on the fuel, and $4\,\mathrm{W}^2$ on the bias d, a random walk that makes the controller offset-free [5], the disturbance state of Section 1.4.7. Volume 9, Chapter 4 carries a fuel bias in its estimator for the same reason. The probe is trusted to $R = 1\,\mathrm{K}^2$, above its 0.3 °C noise, because a tighter filter passes the noise into the feed, and the meat probes to $R = 0.0225\,\mathrm{K}^2$. A perfect-state teaching mode replaces the physical estimates with the truth and leaves only the bias to be estimated. The filter starts from the warm or cold start of Section 2.3 computed with the declared parameters, which is the plant's own start when the parameters are at their defaults.

2.5.5 Dew-Point Loop and the Three Models

With the steam kit fitted, AIC-301 is a PI loop from AT-301 to the steam valve, $K_c = 8\,\%$ per °C and $\tau_I = 180\,\mathrm{s}$, which tracks with the valve shut whenever AT-301 is invalid. The application keeps three models apart. The plant is everything in Section 2.3. The identified model is the FOPDT fit of Equation (2.11), with its two dead times, which the PID tuning uses and nothing else does. The controller's internal model is the MPC's copy of the plant's equations, which knows neither the damp pellets, the blocked intake, the gusts, the probe's bias nor a parameter changed from its default, and whose auger delay may differ from the plant's, and whose heat bias absorbs what it gets wrong.

2.6 Eighteen Cases, Measured

The eighteen cases appear in Table 2.5. All of them share one reference configuration (the standard PID at its IMC-based tuning, the scheduled fan, the continuous auger and the MPC's defaults) and depart from it only as their rows say. The figures quoted below all come from the engine run headless on its default seed, stepped exactly as the browser steps it, with each setting made before Run, which gives the same run as a setting carried through Reset (Section 1.6.1). The comparisons quoted were repeated over three to five noise seeds. At the fastest of the app's speeds a sixteen-hour cook plays in 96 s, and the exported file keeps each probe beside the true quantity it measures, the pellets in the tube, in the pot and burning, and the MPC's heat bias. The scorecard counts the share of the scored time with TT-101 within 5 °F of the setpoint, the worst excursion above the setpoint once the loop

Table 2.5 The Eighteen Cases

#	Case and event	Teaching point
1	Steady at 225 °F, stalled shoulder	fuel on its way
2	Cold start, Start at 30 s	ignition, first overshoot
3	Manual, feed +5 % at 5 min	FOPDT, two dead times
4	Lid up 20 to 23 min	feeding a disturbance
5	Overnight shoulder, 3 stages	recipe, the stall
6	Brisket, wrap at 165 °F core	a load change with a lid
7	Ribs, 3-2-1	clock stages, unwrap
8	Smoke, then roast at 350 °F	recipe preview
9	Reverse sear to 450 °F, MPC	carry-over, browning
10	Cold 3 kg butt at 30 min	a lid and a load
11	9 mph gusting wind, 60 to 35 °F	unmeasured losses
12	Damp pellets, MPC	model error, heat bias
13	0.5 kg in the hopper	flame loss, restart
14	0.3 kg pan at 250 °F	a vanishing load
15	Steam kit, dew point 113 °F	dew point, not RH
16	300 °F for 1 h, then 180 °F, MPC	pre-cooling, stage hold
17	Probe +10 °F, intake 30 % blocked	sensor and airway faults
18	Challenge, brisk PID, two lids and a cold load	the whole problem

has reached it, the worst excursion in the 20 minutes after a lid shuts, and the recovery, the time from the lid shutting until TT-101 is back within 5 °F and stays there for five minutes.

2.6.1 The Steady Point and a Cold Start

Case 1 holds 225 °F over the stalled shoulder. Over its scored 80 minutes TT-101 stays within 5 °F the whole time, with an RMS error of 0.55 °F under the standard PID and 0.57 °F under the MPC, at a mean feed of 22.9 %, and the grill burns 0.69 kg (1.52 lb) of pellets in 90 minutes. A steady operating point hides the delay, because nothing asks the fire to change. Case 2 does ask. From cold the supervisor hands over to the loop 5.0 minutes after Start, with the grate at 83 °F, and the standard PID first comes within 5 °F of 225 °F 20.2 minutes after Start and then overshoots by 21.4 °F, because its integral has grown through a climb of more than 100 °F and the pot still holds fuel fed for a colder grill. The MPC is in the band 9.8 minutes after Start and overshoots by 1.9 °F, because it predicts the arrival of what it has already fed.

2.6.2 The Lid and Four Answers

Case 4 opens the lid for three minutes at 20 minutes. Table 2.6 gives eight configurations on the identical opening and seed, and Figure 2.2 traces four of them, and Figure 2.3 draws the overshoot and the time in the band of all eight. The standard PID feeds up to 41.8 % while the lid is up, peaks at 234.7 °F, 9.7 °F over the setpoint, thirteen minutes after the lid shuts, and has not settled back inside the band for good by the end of the hour. Brisker tunings recover sooner by overshooting more, 28.9 °F with a 22.1 min recovery at 0.6 % per °C and 10 min, and 60.8 °F at 1.0 % per °C and 6.5 min. The lid-aware PID holds the feed at 23 % while the lid is up and for 45 s after, feeds no more than 26 % once it resumes, overshoots by 3.0 °F and recovers in 8.7 min. The MPC first cuts the feed to its floor as the probe collapses, then feeds up to 39.1 % to restore the heat that the steel and the meat are losing, and 51 % for a moment as the lid shuts, overshoots by 2.7 °F and is back in the band for good 0.7 min after the lid shuts, 93.9 % of the hour in the band against 53.4 % for the standard PID. Over five noise seeds the overshoot spans 9.0 to 9.7 °F for the standard PID, 2.7 to 3.4 °F for the lid-aware PID and 2.7 to 3.2 °F for the MPC, so those two differ in how fast they settle rather than in how far they overshoot.

The last three rows separate what the MPC knows from what it assumes. With the true state in place of the estimate it overshoots by 1.9 °F instead of 2.7 °F (1.8 to 2.1 °F against 2.7 to 3.2 °F over five seeds), so the estimator costs little. Told that the lid will stay open for the whole horizon, the same controller predicts a chamber it cannot heat, feeds 100 % and overshoots by 92.8 °F, peaking at

Table 2.6 Case 4, the Three-Minute Lid Opening, on One Seed. Feed Is the Largest While the Lid Is Up, and Over Is Measured in the 20 Minutes After It Shuts

Configuration	Feed (%)	Low (°F)	Over (°F)	Recovery (min)	In band (%)
standard PID	41.8	128.7	9.7	—	53.4
PID, 0.6 %/°C, 10 min	61.5	131.0	28.9	22.1	61.4
PID, 1.0 %/°C, 6.5 min	89.1	132.3	60.8	14.7	72.9
PID, pulsed auger	41.4	129.6	12.7	—	45.7
lid-aware PID	23.0	123.6	3.0	8.7	82.5
MPC, lid shuts	39.1	125.1	2.7	0.7	93.9
MPC, perfect state	33.7	124.7	1.9	0.8	93.8
MPC, lid stays open	100	133.5	92.8	7.4	83.5

A dash means the grate had not settled for good by the end of the hour.

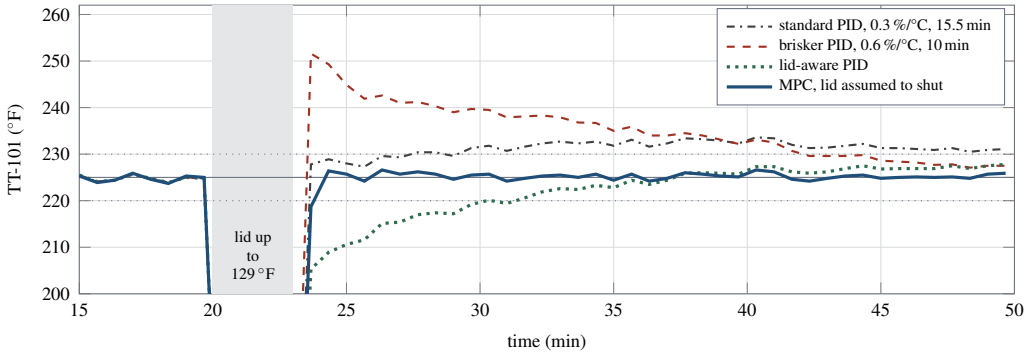


Figure 2.2 Grate probe TT-101 through the three-minute lid opening of case 4, recorded from the running application, with the lid up in the shaded band and the 225 °F setpoint and its ±5 °F band as thin solid and dotted gray lines. The standard PID (dash-dotted) peaks 9.7 °F over thirteen minutes after the lid shuts, the brisker PID (dashed) 28.9 °F within a minute, and the lid-aware PID (thick dotted) and the MPC (thick solid) stay within about 3 °F.

317.8 °F. The opening is the same in both runs and only the assumption differs, which is the point the case is built to make. Pulsing the auger at the same mean feed makes the standard PID worse, 12.7 °F over and 45.7 % in the band, because a pulsed tube advances only while the motor turns and its transport time grows as the feed falls.

🕮 **Pause.** Try Exercise 2.3 before reading on.

✦ **DID YOU KNOW? Pulsing the Auger Makes the Same Loop Worse**

From the simulator: on the lid opening of case 4 the standard PID overshoots by 9.7 °F with a continuous auger and by 12.7 °F with a pulsed one at the same mean feed, and its time in the band falls from 53.4 to 45.7 % (Table 2.6). The pulsed delay is longest when the feed is lowest.

2.6.3 Recipes, the Stall and Wrapping

Case 5 puts a 4 kg shoulder on at 9 PM straight from the refrigerator under a program of 225 °F for six hours, 250 °F for three, then 275 °F until the probe reads 200 °F, and a 165 °F hold, while the night cools from 68 °F to 50 °F. With the standard PID the second stage begins at 3:00 AM and the third at 6:00 AM, and the core passes 150 °F at 4.41 h and does not reach 165 °F until 7.59 h. That stall of 3.2 h happens with the grate on target, because the surface is evaporating, and a tighter grate band cannot shorten it. The probe reads 200 °F at 7:41 AM, and the grill burns 6.87 kg (15.1 lb) in

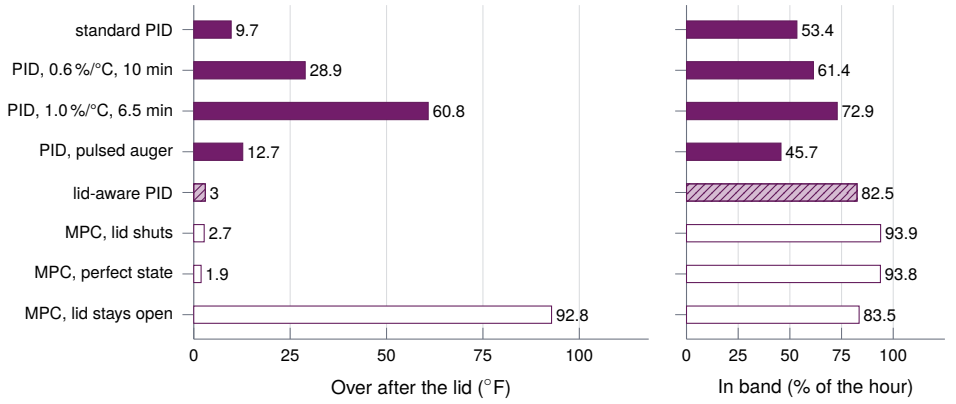


Figure 2.3 The eight answers to the three-minute lid opening of case 4 from Table 2.6: the overshoot in the 20 minutes after the lid shuts (left) and the share of the hour within ±5 °F (right). Dark bars are standard PID loops, the hatched bar the lid-aware PID and white bars the MPC. Notice that brisker PID tunings buy time in the band with larger overshoots, while the MPC told that the lid stays open overshoots most of all.

sixteen hours. The MPC keeps 97.1 % of the night in the band against 87.4 % and finishes at 7:37 AM, although its worst excursion is larger, 14.9 °F against 7.7 °F, because it pre-heats ahead of each programmed step. At 1 PM the core has cooled to 158.6 °F in the 165 °F hold, since a hold holds the chamber and the meat keeps evaporating.

Wrapping acts on the evaporation itself. Case 6 smokes a 5.5 kg brisket from 10 PM at 225 °F until the probe reads 165 °F, prompts a wrap with the lid up three minutes, and runs 250 °F to a 200 °F core. The probe reads 165 °F at 6:38 AM, 8.63 h in, and the wrapped brisket reaches 200 °F at 8:01 AM against 8:16 AM unwrapped. At that hold it keeps 14.3 % of its water against 6.8 % and has a bark index of 28 against 52.5. After the wrap's lid the standard PID recovers in 7.3 min and the MPC in 0.8 min. Case 7 runs spare ribs on the 3-2-1 schedule. The core is 161.6 °F at 3 h, reaches 204.2 °F in the foil by 5 h, falls to 186.3 °F half an hour after the unwrap as the wet surface evaporates, and is back at 195.3 °F at 6 h. After the wrap the standard PID takes 45.4 min to settle, because the wrapped rack suddenly draws less heat and the conservative loop trims the feed slowly, while the MPC settles in 0.7 min, and the grate is in the band for 82.4 % of the six hours against 96.0 %.

2.6.4 Preview and the Stage-Hold Policy

A recipe on the clock is information that feedback cannot use and prediction can, and it can be used too early. Volume 9, Chapter 5 uses published orders the same way, releasing water one travel time ahead of the demand. Case 8 smokes a chicken at 225 °F for 45 minutes and then roasts it at 350 °F to a 165 °F core. The standard PID reaches 345 °F 54.1 min after the step and the chicken is done at 6:07 PM. The MPC sees the step 20 minutes ahead, the grate is already at 256.8 °F when the roast begins, it reaches 345 °F 2.5 min later, and the chicken is done at 5:57 PM. With the stage-hold policy on, the grate is held to 234.9 °F at the step and still reaches 345 °F in 3.1 min. Case 16 holds 300 °F for an hour and then asks for a 180 °F hold. The MPC cuts the feed to its floor well before the transition and the grate is at 218.5 °F when the program changes, 81.5 °F under the stage it was meant to hold, and reaches 185 °F 10.5 min later. With the stage-hold policy it holds 292.8 °F to the end of the stage and takes 23.5 min, while the standard PID holds 300.4 °F, takes 42.4 min, and cuts the feed from 32.5 to no lower than 12.1 %. Using the preview early is optimal for the objective of Equation (2.13) and wrong for a recipe that wanted every minute of each stage, which is what the policy is for. Case 9 shows a limit of the grill itself. A thick ribeye cooked to a 115 °F core and then seared at 450 °F passes its 130 °F pull temperature 8.2 min into the sear, less than a minute after the grate reaches

450 °F, and its core peaks at 135.4 °F on the counter while its surface never passes 169.9 °F, because it is still evaporating, so a convection grill at 450 °F cannot brown a wet steak before its core is done.

2.6.5 Loads the Loop Has to Find

Case 10 puts a cold 3 kg butt on at 30 minutes with the lid up for a minute. The standard PID keeps 96.5 % of the three hours in the band, overshoots by 1.0 °F after the lid and recovers in 7.3 min, while its feed settles from 22.9 to 24.2 % for the new load. Case 11 blows a 9 mph gusting wind across the barrel as the night falls from 60 °F to 35 °F. The standard PID keeps 89.5 % of the four hours in the band and the MPC 93.6 %, the feed rises from 27.6 to 30.4 % in ten-minute means, and the grill burns 2.31 kg against 1.83 kg on a calm 68 °F night. Over five seeds of the gusts the standard PID keeps 82 to 90 % in the band and the MPC 88 to 94 %, so the MPC's advantage holds while its size varies. Case 12 gives the MPC damp pellets that catch more slowly and carry less heat than its model believes. The heat bias reaches −232 W by two hours, a phantom heat loss that stands in for the weaker fuel, the feed climbs from 22.9 to 27.8 %, and the MPC keeps 97.0 % of the two hours in the band, the same as with dry pellets, and overshoots the lid at 70 minutes by 1.9 °F, against 62.8 % for the standard PID. With the model's auger delay set to 0, 60 and 120 s instead of the plant's 20 s, the lid overshoot is 2.1, 1.9 and 3.9 °F. The first two lie within the 1.9 to 2.3 °F that five noise seeds give, and only the 100 s error doubles the overshoot, so the bias absorbs most of a wrong delay, and the lags in the pot rather than the tube dominate the fire's response.

> **TRY IT IN THE APP** **12 · Damp Pellets From a New Bag**
>
> Press Run and watch the heat bias on the MPC card after the damp pellets reach the pot at 30 minutes. It falls to −232 W by two hours, a phantom heat loss that stands in for the weaker fuel, while the feed climbs from 22.9 to 27.8 % and the grate stays in the band for 97.0 % of the two hours.

2.6.6 Faults and the Chamber's Water

Case 13 starts with 0.5 kg in the hopper. The auger runs dry at 1.18 h, the fire burns down what the pot held while the PID raises the feed from 23 to 33 %, and at 75.3 min, with the grate at 181.0 °F, the supervisor declares FLAME OUT and stops the auger. Left alone, the meat's core falls to 114.4 °F by three hours, and refilled and restarted at 90 minutes the grill is RUNNING again 5.7 min after Start. Case 14 cooks at 250 °F over a pan of 0.3 kg. The pan is down to its last 10 g at 47.7 min, the grate, relieved of the pan's latent load, peaks at 259.9 °F, and the dew point falls from 103.7 to 92.1 °F. In case 17 the grate probe begins to read 10 °F high at 20 minutes, and at 50 minutes TT-101 reads 225.9 °F on target while the air around the meat is 216.5 °F. The MPC, fed the same probe, is fooled the same way, since feedback cannot correct its own sensor. As Section 1.4.6 shows for an analyzer, a constant bias seen through the one measurement it corrupts cannot be estimated without a second, unbiased reading. At 80 minutes ash blocks 30 % of the intake, and by 133 minutes the excess-air ratio has fallen from 5.25 to 3.88 while the fire still releases 1617 W. The fire has five times the air it needs, which is also why a small pellet boiler's emissions respond to accurate airflow control [2].

Case 15 fits the experimental steam kit and a dew-point PI that holds 113 °F while a shoulder cooks at 250 °F with no pan. At 1 h the valve runs about 51 %, the relative humidity is only 4.7 % and could not pass 49.1 % at 250 °F however much steam went in, and with the valve shut and wide open the reachable dew point is 90.3 to 126.4 °F. With the steam the core reaches 165 °F at 3.0 h against 3.58 h without, because the surface evaporates less and runs hotter, but 180 °F at 7.55 h against 7.44 h, because the water it kept makes the evaporating period last longer. The steam costs 4.8 kWh and the bark index ends at 76.9 against 95.2.

2.6.7 The Challenge

The challenge holds 225 °F for two hours through a three-minute opening at 20 minutes, a cold 3 kg butt with the lid up for a minute at 45 minutes and another three-minute opening at 75 minutes. The

Table 2.7 The Challenge, Two Lid Openings and a Cold Load Over Two Hours

Configuration	In band (%)	Over (°F)	Recoveries (min)	Pellets (lb)
standard PID as shipped[a]	70.1	73.2	13.1, 8.0, 12.8	2.33
lid-aware PID, shipped gains	85.9	4.6	3.4, 4.2, 3.3	2.19
lid-aware PID[b]	88.4	3.7	2.4, 2.5, 2.4	2.19
lid-aware PID[c]	90.7	2.5	1.5, 1.5, 1.4	2.19
MPC	92.5	3.4	0.7, 0.8, 0.6	2.19

[a] $K_c = 1.2$ % per °C, $\tau_I = 400$ s. [b] $K_c = 2$ % per °C, $\tau_I = 5$ min.
[c] $K_c = 5$ % per °C, the top of its range, $\tau_I = 5$ min.

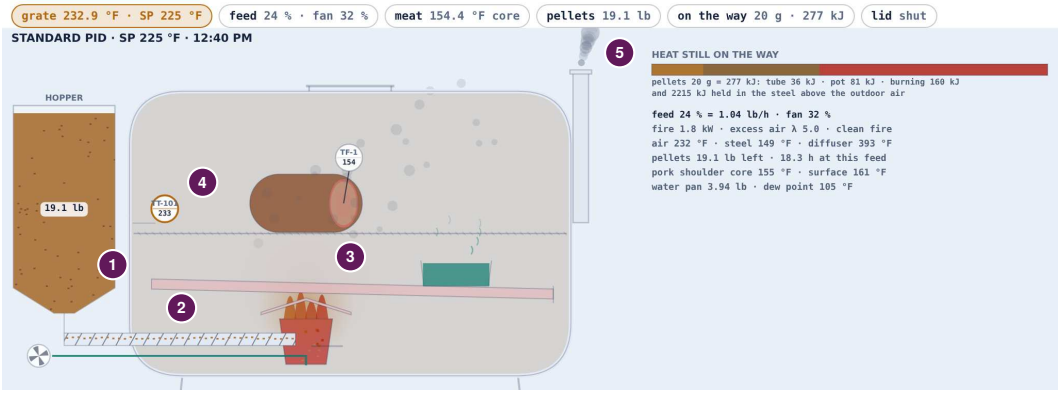


Figure 2.4 The three-minute lid opening of case 4 at $t = 40.0$ min in the Grill view, run with the standard PID and cropped to the plant view (the header, control panel, trends and scorecard are cut). The badge row reads the grate at 232.9 °F, 7.9 °F above the setpoint, and the cutaway shows the shoulder on the grate with its cut end in section, the water pan and the chimney. The callouts mark (1) the hopper at 19.1 lb, (2) the pellets in the auger tube, (3) the fire under its baffle, (4) TT-101 at the grate and (5) the panel that gives 20 g of pellets and 277 kJ still on their way against 2215 kJ in the steel.

target is TT-101 within 5 °F for at least 90 % of the two hours, no more than 10 °F over the setpoint after any of the three openings, and every recovery inside five minutes, judged on the case as given. Table 2.7 gives five configurations. The brisk standard PID it ships with meets no part of the target, and its slowest recovery is 13.1 min. At its shipped gains and at 2 % per °C the lid-aware PID meets the overshoot and recovery parts but not the 90 %, with 85.9 and 88.4 %. With $\tau_I = 5$ min it passes narrowly once K_c reaches 3.3 % per °C, eleven times the step-test gain, and at 5 % per °C it keeps 90.7 % in the band on all five seeds tried, with recoveries of 1.4 to 1.5 min. The MPC meets every part at its defaults with the widest margin, 92.5 % in the band, 3.4 °F over and a slowest recovery of 0.8 min, margins of 2.5 points, 6.6 °F and 4.2 min on the target, and about 6 % of the two hours (7.0 min) is lost to the openings themselves, with the lid up, whatever the controller does. The plant view of the run of case 4 with the standard PID is shown in Figure 2.4, and Table 2.6 gives that run beside the MPC's.

2.7 Technology in Practice

A real grill reads and moves what the loop of Section 2.5 reads and moves. Figure 2.5 draws TIC-101, the lid-aware hold and the MPC with field devices in place of the schematic's symbols.

What Measures It

The controlled variable is the grate temperature that TT-101 reads. Of the contact sensors, thermocouples are by far the most rugged and wide-ranging, while resistance temperature detectors are more fragile but need no reference-junction compensation and are inherently more linear. A thermowell around the element adds a first-order lag through its mass and specific heat, and an air gap between element and well adds a second one [20]. The chapter lumps every such lag into the 8 s of τ_{TT} in Equation (2.10).

> **TECHNOLOGY** **Sensor: The Grate Thermocouple**
>
> A thermocouple is a junction of two dissimilar metals. In industrial use the wires are protected by a stainless steel or ceramic sheath. Its range depends on its type: about 32 °F to 2300 °F for type K and −300 to 700 °F for type T, and narrower where a tighter accuracy is wanted. Its most common failure is to fail open, or burn out. The open input then picks up noise from nearby motors and drives that reads as a wildly varying temperature, so the instrument is given a burnout mode that drives the reading high or low [20]. TT-101 of this chapter is an 8 s lag with colored noise and an optional constant bias, the fault of case 17, and a failed probe sends the supervisor to FAULT.

What Moves It

The manipulated variable is the auger feed u_a, and the combustion fan follows a schedule on that feed. The furnace patented in 1986 already fed its fuel intermittently and could be controlled by a thermostat or other heat sensitive means, and a blower system supplied the combustion air that flowed into the fire pot [13]. The final element of the chapter is the auger alone. Its delay of $\theta_t = 20\,\text{s}$ is a line of 40 cells, and its two drives differ only in how that line advances.

> **TECHNOLOGY** **Final Element: An Auger on a Gear Motor**
>
> In both patents an electric motor turns the auger along its feed tube [13, 14], and in process plants powders are metered the same way, by variable-speed screw conveyors. A direct-current motor's power is commonly set by pulse-width modulation, the fraction of each period its switches conduct, and the same time-proportioned output serves an electric oven whose heater can only be fully on or fully off [20]. Pellet stoves need an electrical supply because motorized components feed the fire and blow the air [15]. The application offers both drives, a continuous auger that meters a mean rate and a pulsed one that runs at full speed for a share of every 15 s cycle.

Where the Controller Runs

A grill's controller is the microcontroller reading a thermocouple that the Assumptions of this chapter mention. Embedded digital controls have replaced analog mechanical ones in rotating machines and engines, and supervisory and lower-level loops run with cycle times from milliseconds to minutes [21]. The application's controller scans every 0.5 s, runs the PID and the estimator every second and the MPC every 10 s, all short beside the 20 s transport time and the 15.5 min time constant of the fit. The velocity-form PID of Equation (2.12) is the block that holds the standard loop, the lid-aware hold is a switch around it, and the MPC replaces it with a plan made from its own record of the feed.

What Is Modernizing It

Three developments bear on this loop. The first is the estimator inside predictive control. A survey of industrial MPC found that most products did not carry a process state and fed back an ad hoc bias, while only two used a Kalman filter, one of them an extended filter for a nonlinear model, and it urged practitioners to recognize the rich framework the filter provides [4]. The MPC of this chapter takes that route, with an extended filter and a heat bias. The second is connectivity. Low-cost Ethernet, Internet Protocol and wireless devices are replacing older proprietary technologies in control systems, which brings remote access and raises the likelihood of cybersecurity vulnerabilities and

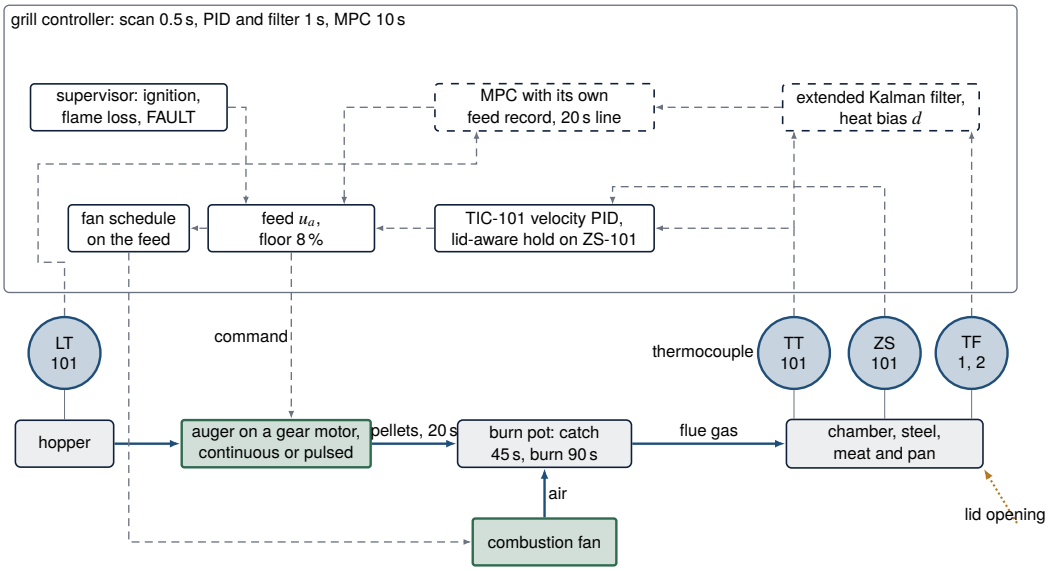


Figure 2.5 TIC-101 and its alternatives on grill hardware. A thermocouple at the grate (TT-101), the lid switch (ZS-101), the meat probes (TF-1, TF-2) and the hopper level (LT-101) feed one embedded controller. The velocity-form PID holds the feed while the lid switch reads open, the dashed MPC and its extended Kalman filter replace the PID when selected, and the supervisor takes the auger for ignition and faults. The fan follows a schedule on the feed. Notice that every path from the controller to the grate passes through the auger's 20 s line and the pot's ignition and burn lags, which only the MPC carries in its model, with its own record of the feed.

incidents [21]. The third is the fuel. Pellets differ in how well they are made and how well they burn, lower quality pellets may give less heat, and third-party certification programs let users identify consistent, reliable pellets [15]. A steadier fuel shrinks the disturbance that case 12 imposes and that the heat bias has to absorb.

> **IN PRACTICE** **Reading a Bump Test**
>
> A process is characterized in the field by an open-loop test. The controller is put in manual, the final element is stepped, and the trend is read for two features: the dead time before the measurement first moves, and the reaction rate, the steepest slope that follows. Dead time tends to stay the same whatever the size of the step, while the reaction rate grows with it, so the step size enters the tuning calculation. Ziegler and Nichols built their second tuning rule on these two readings in their 1942 paper, where they called dead time a lag, a word that now means something else [20]. Case 3 is such a test, run on the auger.

2.8 Limitations

The combustion, heat-transfer and cooking constants are illustrative, chosen so that the totals are plausible, a pound of pellets an hour at 225 °F and about 590 °F at full feed in an empty grill, without being fitted to a grill. The chamber is one well-mixed node and the steel two, so the model cannot say where in the barrel the meat cooks fastest or where a probe should sit. The fire is a mass balance with smooth thresholds, and ash accumulation, pellet quality beyond the damp disturbance, and the chemistry of smoke are outside it. The meat is two nodes, so its stall and its carry-over are qualitatively right while its internal temperature profile is not resolved, and no endpoint in the application is a food-safety assessment.

OPEN CHALLENGE **Delays That Move With the Feed**

Dead-time compensation still has open questions that a pellet fire raises. Extending the properties of compensators descended from Smith's predictor to nonlinear input-output models, such as Hammerstein models, is an open research topic. So is a robust stability test for delays that vary in time that is less conservative than those available [6]. The grill has both: the burn rate depends on the bed's temperature and oxygen, and the pulsed auger's transport time grows as the feed falls. A compensator whose guarantees hold while the delay moves with the feed it sets would be progress.

The controller's model is the plant's own structure with declared parameters, so in the cases the MPC meets model error only through the disturbances they apply, and a parameter changed from its default gives it a plant it was not designed for. A real controller would also face a model of a different structure. The MPC is linearized afresh at every execution and solved as a convex program, and it makes no guarantee beyond its horizon. The supervisor's thresholds are educational and not certified appliance logic, the steam kit and the high-temperature dew-point probe are an experimental teaching configuration, and the lid, the loads and the faults arrive as scheduled events rather than as a cook's habits.

OPEN CHALLENGE **A Biased Probe Looks Like a Good One**

In most industrial predictive controllers a sensor fault is recognized only as a complete failure beyond preset limits. A significant bias or drift that stays within normal limits is generally neither detected nor identified [4]. The probe bias of case 17 is such a fault, and it fools the MPC of this chapter as it fools the PID. Fault detection that finds a bias within the normal range, without slowing the loop, would be progress.

2.9 Conclusion

This chapter studied a pellet grill, a plant whose controller commands fuel that becomes heat through a 20 s transport time, an ignition lag and a burn time, on a simulator that logs every inventory of that fuel beside what its instruments report. At 225 °F the fuel on its way is worth 267 kJ against 2137 kJ in the steel, and a 5 % feed step fits a gain of 7.29 °F per percent and a time constant of 15.5 min with a dead time of 4.7 or 2.8 min depending on the method. A three-minute lid opening leads a PI loop tuned from that step to feed 41.8 % and overshoot by 9.7 °F after the lid shuts, while a lid-aware hold overshoots by 3.0 °F and an MPC that assumes the lid will shut overshoots by 2.7 °F and recovers in 0.7 min, and the same MPC told that the lid stays open overshoots by 92.8 °F. Overnight the meat stalls for 3.2 h with the grate on target, and a recipe previewed by the MPC can pre-heat or pre-cool a stage unless a policy forbids it. The model is lumped and its constants illustrative. A fuller model would add a distributed chamber, a pellet quality that the estimator must identify, a controller whose model differs in structure from the plant, and emissions tied to the excess air. The grill shows that knowing what the actuator has already done decides how a loop behaves when its plant is disturbed.

KEY POINTS

- A pellet grill's controller can add fuel but not heat, and each pellet becomes heat only after it has ridden the auger, caught on the bed and burned.
- A 5 % feed step fits a dead time of 4.7 or 2.8 min depending on the method.
- After a three-minute lid opening a PI loop tuned from that step overshoots by 9.7 °F, a lid-aware hold by 3.0 °F, and an MPC that assumes the lid will shut by 2.7 °F.
- Told that the lid stays open, the same MPC overshoots by 92.8 °F, so its assumption about the disturbance matters as much as its algorithm.

Where else this applies. Volume 9, Chapter 2 puts a burner behind a dead time under predictive control, and Volume 9, Chapter 4 carries a fuel bias in its estimator for the same reason as this grill.

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2.10 Exercises

2.1 Concept check · 2 min

In case 4 the standard PID raises the feed to 41.8 % while the lid is up, and thirteen minutes after the lid shuts the grate peaks 9.7 °F over the setpoint. What is the main cause of this overshoot?

(a) Pellets fed while the lid was up are still in the tube and the pot when it shuts, and the integral keeps what it gained

(b) The steel lost most of its stored heat with the lid up, and the loop overcorrects as it replaces that heat

(c) The 8 s lag of the grate probe hides the recovery of the air from the loop for several minutes

(d) Anti-windup clamps the running sum while the lid is up, and the sum is released in one step as it shuts

2.2 Concept check · 1 min

The feed step of case 3 is fitted with a first-order plus dead-time model. Smith's two-point method gives a dead time of 4.7 min and the tangent at the steepest point gives 2.8 min, although the auger delay is only 20 s. Why?

(a) The probe noise biases the tangent, which is drawn from two samples near the steepest point

(b) The fuel passes through ignition, burning and the steel's heating in series, and no single dead time represents that chain

(c) The auger's transport time grows as the feed rises, so the delay changes during the step

(d) The scheduled fan adds a second delay that the tangent method cannot see

2.3 Concept check · 2 min

On the lid opening of case 4, the MPC that assumes the lid will shut at any moment overshoots by 2.7 °F. Told that the lid will stay open for the whole horizon, the same MPC feeds 100 % and overshoots by 92.8 °F. What does the comparison show?

(a) The extended Kalman filter loses track of the fuel while the lid is up, so the plan starts from a wrong state

(b) The 20 min horizon is too short to see the lid shut, so the plan ignores the recovery

(c) What the controller assumes about the disturbance's future matters as much as its algorithm

(d) The move suppression of 10 is too weak to stop the feed from reaching its limit

2.4 Theory and calculation · 6 min

The auger delivers $q_{max} = 2\,\mathrm{kg/h}$ at full feed, the pellets have a heating value of 17 MJ/kg as fed and the fire burns them at an efficiency $\eta = 0.80$.

(a) Case 1 holds 225 °F at a mean feed of 22.9 %. Find the pellets burned per hour in steady state and the heat the fire releases. *Checkpoint:* 0.458 kg/h (about 1.01 lb/h) and 1.73 kW.

(b) At 30 minutes into case 1, 19.6 g of pellets are in the tube, in the pot and burning. Find the heat they release in the fire, at η, and how many minutes of fire they represent at the 1733 W the fire releases. Compare the heat with the 2137 kJ the steel holds above the outdoor air. *Checkpoint:* 267 kJ, 2.6 min, and the steel holds 8.0 times more.

(c) With the pulsed auger drive, the tube advances only while the motor turns. Estimate the transport time of a pellet at a duty of 23 % with $\theta_t = 20\,\mathrm{s}$. *Checkpoint:* About 87 s.

2.5 Theory and calculation · 8 min

The feed step of case 3 fits Equation (2.11) with $K = 4.05\,^{\circ}\mathrm{C}$ per percent and $\tau = 15.5\,\mathrm{min}$, with a dead time of 4.7 min by Smith's method and 2.8 min by the tangent. The SIMC rule takes $K_c = \tau/\big(K(\tau_c + \theta)\big)$ and $\tau_I = \tau$ with $\tau_c = \tau/2$.

(a) Find K_c and τ_I on Smith's dead time, and give K_c in percent per °F as the app shows it. *Checkpoint:* 0.31 % per °C (0.17 % per °F) and 15.5 min.

(b) Find K_c on the tangent's dead time, and say which fit gives the more cautious loop. *Checkpoint:* 0.36 % per °C. Smith's dead time gives the more cautious loop.

(c) The shipped loop runs $K_c = 0.3$ % per °C and $\tau_I = 930$ s. In case 4 the error is 53 °C as the lid shuts and averages 50 °C over the 180 s it is up. Estimate the feed the proportional and integral terms add to the steady 23 %. *Checkpoint:* About 15.9 % and 2.9 %, a feed of about 41.8 %.

2.6 Theory and calculation · 4 min

The chamber is at the atmospheric pressure $P = 101.3$ kPa. Steam tables give a saturation pressure of 130.5 kPa at 225 °F (107.2 °C) and of 7.61 kPa at 105 °F (40.6 °C).

(a) Find the highest relative humidity the chamber gas can reach at 225 °F, however much water enters. *Checkpoint:* 77.6 %.

(b) In case 1 the dew point is 105 °F. Find the relative humidity, and say why the humidity loop of the steam kit controls the dew point instead. *Checkpoint:* 5.8 %.

2.7 App challenge · 8 min

Load scenario 4, A three-minute lid opening, and keep the Standard PID as loaded. Run to 60 min at a speed of 60 with the default seed, and read the tiles In ±5 °F, Over SP after a lid and Lid recovery. Press Reset, set Automatic controller to Lid-aware PID, and repeat. Press Reset again, set Automatic controller to MPC, and repeat.

(a) Report the three tiles for each controller. *Checkpoint:* Standard PID 53.4 %, 9.7 °F, not yet. Lid-aware PID 82.5 %, 3.0 °F, 8.7 min. MPC 93.9 %, 2.7 °F, 0.7 min. Within 0.3 %, 0.2 °F and 0.2 min.

(b) The lid-aware PID and the MPC overshoot by nearly the same amount. Explain from the feed strip why the MPC recovers more than ten times faster. *Checkpoint:* The MPC feeds to replace the heat the steel and the meat lost and predicts its arrival. The hold feeds no more than before the lid opened.

2.8 App challenge · 9 min

Load scenario 3, A feed step to identify the fire, and change no setting. The grill stays in Manual with the auger at 18.2 % and the fan at 27 %, and the feed steps up by 5 % at 5 min. Run to four hours at a speed of 600 with the default seed and export the CSV file. Use the noise-free true chamber air, column `T_chamber_C`.

(a) Read the column at 300 s and at 14 400 s, and find the rise and the gain of the whole step in °C per percent. *Checkpoint:* A rise of 20.61 °C (37.10 °F) and 4.12 °C per percent, within 0.05 °C.

(b) The grill drifts up 0.65 °F in four hours with no step at all. Correct the gain for the drift and give it in °F per percent. *Checkpoint:* 4.05 °C per percent, 7.29 °F per percent.

(c) Find the times after the step at which the air has covered 28.3 % and 63.2 % of the rise of part (a), and apply Smith's method, $\tau = 1.5(t_{63} - t_{28})$ and $\theta = t_{63} - \tau$. *Checkpoint:* $t_{28} = 9.75$ min and $t_{63} = 20.2$ min, within 0.1 min, so $\tau = 15.6$ min and $\theta = 4.5$ min.

2.9 App challenge · 8 min

Load scenario 12, Damp pellets from a new bag, with the MPC as loaded. Damp pellets reach the pot at 30 min, when the Damp pellets slider moves to 70 %, and the lid is up for three minutes at 70 min. Run to two hours at a speed of 60 with the default seed and export the CSV file.

(a) Read the MPC's heat bias, column `mpc_bias_W`, and the share in the band, column `metric_in_band_pct`, in the row at 7200 s. Explain the sign of the bias. *Checkpoint:* −232 W, within 3 W, and 97.0 %, within 0.2 %.

(b) Reload the scenario, set Auger delay in the MPC's model to 120 s, and run to 95 min. Read Over SP after a lid, and compare it with the 1.9 °F of the correct 20 s delay. *Checkpoint:* 3.9 °F, within 0.2 °F.

2.10 Application area · 3 min

A 9 kg hopper feeds an overnight cook. In case 5 the grill burns 6.87 kg of pellets in sixteen hours. In case 11 it burns 2.31 kg in four hours of a 9 mph gusting wind against 1.83 kg on a calm 68 °F night.

(a) Find the mean burn rate of case 5 and the pellets left at the end. *Checkpoint:* 0.43 kg/h, and 2.13 kg (4.7 lb) left.

(b) Find what a sixteen-hour cook would need at the windy rate, and what a cook should do before an unattended night in wind. *Checkpoint:* About 9.24 kg, more than the hopper holds.

2.11 Application area · 4 min

In case 17 the grate probe begins to read 10 °F high at 20 min. At 50 min TT-101 reads 225.9 °F on target while the air around the meat is 216.5 °F.

(a) How much colder than displayed does the meat cook, and could the MPC correct it? *Checkpoint:* About 9.4 °F. No: it is fed the same probe.

(b) The chapter's model has one well-mixed chamber. Describe how a practicing engineer would detect a probe bias on a real grill. *Checkpoint:* An independent reference reading at the grate, compared with TT-101 at a steady point.

2.12 Review the agent · 5 min

An AI assistant is asked how to pass the challenge (case 18, Two lid openings and a cold load) with a PID. It replies: "The lid-aware PID cannot meet the 90 % target at any tuning. A gain search reaches only 88.4 % at its best point, 1.11 % per °F (2 % per °C) and an integral time of 5 min, because more gain only adds overshoot after each lid. The MPC is the only way to pass." Find the flaw, and say how you would check it in the app.