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CHAPTER 5

# Forecasting Frozen-In Stress in a Float Glass Line Simulator

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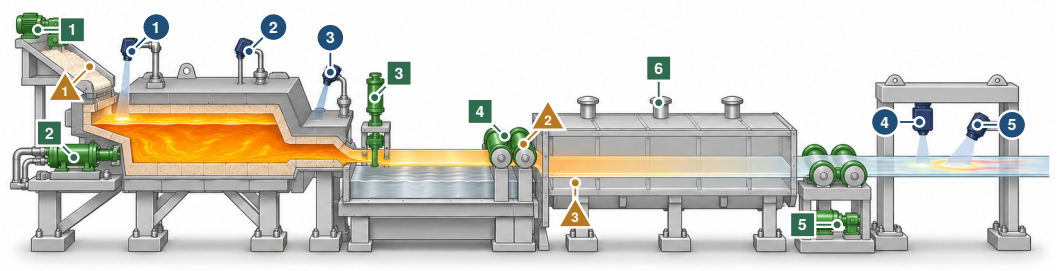
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CHAPTER 5

# Forecasting Frozen-In Stress in a Float Glass Line Simulator



**The controller moves** 1 batch charger 2 furnace firing 3 tweel, the gate that meters the glass 4 top rolls that set the stretch 5 lehr drive that sets the ribbon speed 6 heating and cooling of the six lehr zones

**The controller reads** 1 melt level 2 crown temperature 3 canal temperature 4 thickness gauge 5 polarimeter that reads the stress kept

**What disturbs it** 1 more cullet in the charge 2 top rolls losing their grip 3 a lehr zone failing

*Notice how far the two cold-end gauges stand from the top rolls and the first lehr zones, where the thickness and the stress are fixed. Illustration, not to scale.*

### IN PLAIN TERMS Glass Whose Quality Is Set Before It Is Measured

**The main idea.** A float line pours melted glass onto a bath of liquid tin and pulls it into a long, flat ribbon. Rollers stretch the ribbon to the right thickness, and a long oven then cools it slowly so that little strain stays locked inside.

**The challenge.** The gauges that check thickness and strain stand at the cold end, reading glass shaped about a quarter of an hour before. The controller must act on what it predicts, and say plainly when no oven setting can save the glass.

**Why it matters.** Without a forecast, the line learns of a fault only after many meters of glass are already spoiled.

### THREE WAYS TO READ THIS CHAPTER

| | | |
|---|---|---|
| **Explore** | 11 min | the first page, the history segment (Section 5.2), the fact on furnace size (Section 5.3), the try-its on the thickness gauge and the failed lehr zone and the fact on the failed zone (Section 5.6) and the key points |
| **Study** | 65 min | the whole chapter with its three pause points and the twelve exercises at its end |
| **Practice** | 20 min | the first page, the results (Section 5.6), Technology in Practice (Section 5.7), Limitations (Section 5.8) and the open challenge |

**The control question** A float line settles the thickness of its glass on the tin and the stress the glass keeps in the first meters of the lehr, minutes before the gauge and the polarimeter at the cold end can read either. How can the line control qualities that are fixed before they can be measured, and when must its controller report that no setting of the lehr can meet the limit?

**Learning objectives**

- Compute the delays of the line as distances over the lehr speed, and show how they move with the product.
- Derive the stretch an order needs from the forming relation, and explain why top rolls that lose their grip thicken the glass.
- Locate where along the lehr the stress is kept, and explain why the supervisor searches a grid of offsets instead of closing a loop on its forecast.
- Compare a loop on the delayed gauge, the same loop as a Smith predictor and a loop on the mass-balance thickness by the glass each rejects.
- Assess a predictive controller that plans the annealing zones from the job card against the forecast supervisor.

**Prerequisites** Sections 1.4.4 (delay that moves with flow), 1.4.3 (Smith predictor), 1.4.8 (soft sensor and bias update) and 1.6.2 (velocity-form PID), and Chapter 3 for a quality read late. Sections 1.4.7 and 1.6.3 are needed only for the predictive controller.

**The app** https://apmonitor.com/apps/float-glass/

**See also** Volume 9, Chapter 5 for releases planned from a published schedule through a transport delay, and Volume 9, Chapter 4 for a predictive controller made offset free by a disturbance estimate.

## 5.1 Introduction

A product that carries its own history can be judged only once that history is complete, and float glass shows this plainly. Its thickness is controlled by stretching the ribbon while it is still viscous on molten tin [1]. Its residual stress comes from the temperature differences through its thickness while it cools [2, 3], and it affects how well the glass cuts [3]. The stress is read photoelastically on glass that has cooled [4], and in-line systems have been built to read the thickness in the hot zone of the lehr rather than at its end [5].

The float process pours molten glass from a furnace onto a bath of molten tin, where it spreads and is drawn off as a continuous ribbon [6]. In the line of this chapter a furnace holding 1.27 kt of glass, 2.1 days of its 600 t/day pull, meters the glass under a refractory gate called the tweel. The lehr draws the ribbon 1.611 times as fast as the top rolls release it, which thins it from the 6.0 mm it would settle to on the tin to 4.00 mm, and the ribbon crosses a 130 m lehr at 11.26 m/min. The thickness gauge, 171 m past the top rolls, reads glass formed 15.2 min earlier. The ribbon passes its annealing point 18 m into the lehr and its strain point 21 m further on, and the polarimeter stands another 100 m downstream, 8.9 min of travel.

The plant differs from a loop with a fixed dead time in three ways. Its delays are distances divided by the lehr speed, so they move with the product, from 15.2 min for 4 mm glass to 22.8 min for 6 mm glass at the same pull, a transport delay defined in distance as Section 1.4.4 defines it in volume. Its forming is viscous, which makes width and thickness one mass balance set by two speeds, so top rolls that slip and narrow the ribbon also thicken it. Its stress is a record of the thermal path each piece of glass has followed, fixed over the 21 m in which the glass stops flowing, so a lehr trimmed on the polarimeter acts on glass that was annealed nine minutes before. Upstream, the furnace answers its firing and its pull over hours, its melt with a time constant of 96 min.

A reading at the cold end is easily taken for a report on the glass being made. A gauge at 4.00 mm and a polarimeter at 1.0 MPa describe glass formed a quarter of an hour ago and annealed nine minutes ago, and neither says anything about the glass in the lehr now. The class of problem is broad.

Wherever a quality is measured downstream of the place where it is made, a dead-time compensator of the Smith predictor family can take a known delay out of the loop [7], and an inferential controller can act on an estimate built from faster secondary measurements [8], the two remedies of Sections 1.4.3 and 1.4.8. Steel strip in a continuous annealing furnace likewise takes its temperature along a path whose speed the line sets (Volume 16, Chapter 9). The furnace upstream is the slow half of the same line, and model-based control has been applied to glass melting as well [9]. However, a transfer function with a constant dead time represents neither a delay that changes with the product nor a quality set by a thermal path. It cannot show a stress forecast running ahead of the polarimeter, or a supervisor reporting that no setting of the lehr can anneal the ribbon it has.

This chapter charts where the thickness and the stress of float glass are made and where they are measured. The simulator follows every 3 m of ribbon from the tin to the cut with its own temperature and stress through its thickness, infers the thickness at the width camera from the mass relation, and anneals the ribbon from a forecast made on an estimator's copy of every parcel. Section 5.2 traces the float process, its annealing and its control. Section 5.3 sets out the model of the whole line, from a lumped furnace through viscous forming to a ribbon whose stress follows a Maxwell element at every depth, specified fully enough to rebuild, with the truth, the measurements and the estimates logged apart. Section 5.4 finds where along the lehr the stress is kept and how it depends on the annealing offset. Section 5.5 describes the controller: the PI loops and their feedforwards, the Smith predictor and the inferred thickness, the thermal-history estimator, the supervisor and a model predictive controller built on the estimator. Section 5.6 measures nine cases and a graded product change, which separate what the cold end can report from what the glass in the lehr will keep, and in Section 5.6.7 it sets the predictive controller against the supervisor on a scheduled job change. Section 5.7 sets the loops on plant hardware, and Sections 5.8 and 5.9 state the limitations and the conclusions.

## 5.2 Float Glass, Its Annealing and Its Control

The float process was described by its inventor in a 1969 review lecture, which set it against the methods used to make flat glass before and at the time of its invention, traced its development and treated three of the main problems solved along the way [6]. Before it, flat glass was drawn from the melt as sheet or cast and then ground and polished as plate [6]. Glass poured on tin spreads toward an equilibrium thickness, and thinner glass has to be stretched [1, 6]. The control of thickness by stretching the ribbon was analyzed in 1977 as a one-dimensional problem for a free ribbon without edge constraints. Hot ribbons were simulated to show their width and thickness attenuating at different rates, and the calculated width was compared with in-plant measurements [1]. Top rolls that hold the edges, as on the simulated line, decide how each stretch is shared between the two.

> ⧖ HISTORY **1950s: Glass Floated on Tin**
>
> Nearly all flat glass in the United States is made by the float process, developed in England in the 1950s and licensed in the United States in 1962. It replaced the plate glass process, whose grinding and polishing could take as much as 3 million Btu per ton of glass. Float glass leaves the bath fire-polished, close to the quality of ground and polished plate, and larger, more efficient furnaces lowered its energy further. The molten glass flows under a tweel and over a lip onto a pool of molten tin about 160 to 190 feet long, kept under nitrogen so that the tin does not oxidize, and takes on the flatness of the tin. The process is so dominant that the terms float glass and flat glass are now used interchangeably [10].

Annealing is older than the float process. How the stress in glass relaxes at the temperatures of its annealing range, and how a cooling schedule follows from the rate of that relaxation, was set out in 1920 [11]. The three-constant viscosity equation that the simulator uses was published in 1925 [12]. Viscoelastic analysis came later. A theory built on measured relaxation data computed the stress history of a glass plate cooled symmetrically from both surfaces in 1965 [2]. Compared with

measured stress distributions in 1969, it agreed for glass cooled from well above the strain point once its numerical formulation was changed, but not for glass quenched from lower temperatures [13]. A model of structural relaxation that carries the memory of the thermal history followed in 1971 [14].

The lehr became a control problem of its own on float lines. An analytical model of a float glass annealing lehr was published in 1977, with radiation between the moving ribbon and its enclosure and gas convection in a flue system. It illustrated how modeling could improve the design and operation of the lehr, and it proposed a linear algorithm for on-line temperature control, checked by computer experiments [15]. The stress a lehr leaves can be measured photoelastically, because glass is birefringent under stress [3, 4], and the residual stress of commercial float glass of several thicknesses has been quantified this way because it affects cutting quality [3]. At the hot end, simplified first-principles models of the combustion chambers of glass furnaces have been developed [16], and model-based control of a glass melting furnace has been reported [9]. Energy is part of the motivation, since the options for decarbonizing this energy-intensive industry have been reviewed [17].

## 5.3 The Line From Charger to Cut

At its design point the line makes 600 t/day of 4.00 mm glass 3.70 m wide, which leaves 3.40 m of net ribbon once 0.15 m is trimmed from each edge. Positions are measured along the ribbon from the spout. The furnace, the working end and canal, the tweel, the float bath with its top rolls, the six lehr zones and the cold end are shown in Figure 5.1 with every symbol of the equations.

> **DID YOU KNOW? Furnaces Sized in Hundreds of Tons a Day**
>
> A 2002 profile of the United States glass industry found that every float line melts its glass in a side-port regenerative furnace, typically of 500 to 1000 tons per day, large because both the volume and the quality of flat glass demand it [10]. The simulated line pulls 600 t/day.

Table 5.1 classifies the variables. The 33 lumped states are the batch blanket and the glass inventory, the crown, melt and working-end temperatures, four actuator lags, the lehr and top-roll speeds, the flow and thickness leaving the top-roll zone, six zone temperatures and fourteen energy and mass ledgers. About 59 parcels of ribbon, held in 72 slots, carry the rest of the state, each with its position, mass, thickness, width, the order in force when it was formed and ten node values.

### 5.3.1 Furnace and Tweel

The charger lays a blanket of batch and cullet on the melt, and the blanket melts with a time constant that shortens with temperature and with cullet, as shown in Equation (5.1).

$$\begin{aligned} \frac{\mathrm{d}M_b}{\mathrm{d}t} &= F_{ch} - F_m, \qquad F_m = \frac{M_b}{\tau_b}, \qquad \frac{\mathrm{d}M_g}{\mathrm{d}t} = Y F_m - G \\ \tau_b &= \tau_B \, \mathrm{e}^{-(T_m - T_B)/\theta_b} \, \max(0.2,\ 1 - k_{cul} f_c) \\ Y &= f_c + (1 - f_c) Y_b, \qquad L = \frac{M_g - \rho_m A_f h_0}{\rho_m A_f} \end{aligned} \tag{5.1}$$

Here $F_{ch} = x_{ch} F_{ch,max}$ is the charge, $T_B = 1450\,^{\circ}\mathrm{C}$, and $Y$ is the glass per kilogram of mix, less than one because raw batch loses 17 % of its mass as carbonate gas while cullet loses nothing. The level $L$ is the height of the glass above the depth $h_0$ in a melter of area $A_f$. At the design point the charger lays 694 t/day of mix for 600 t/day of glass, and the 17.4 t blanket melts with $\tau_b = 36$ min at the 1446 °C melt. A millimeter of level is 1.06 t of glass. The level enters the tweel flow through the head alone, and the pull loop removes even that, so under automatic control the level integrates the difference between melting and pull behind a melting lag.

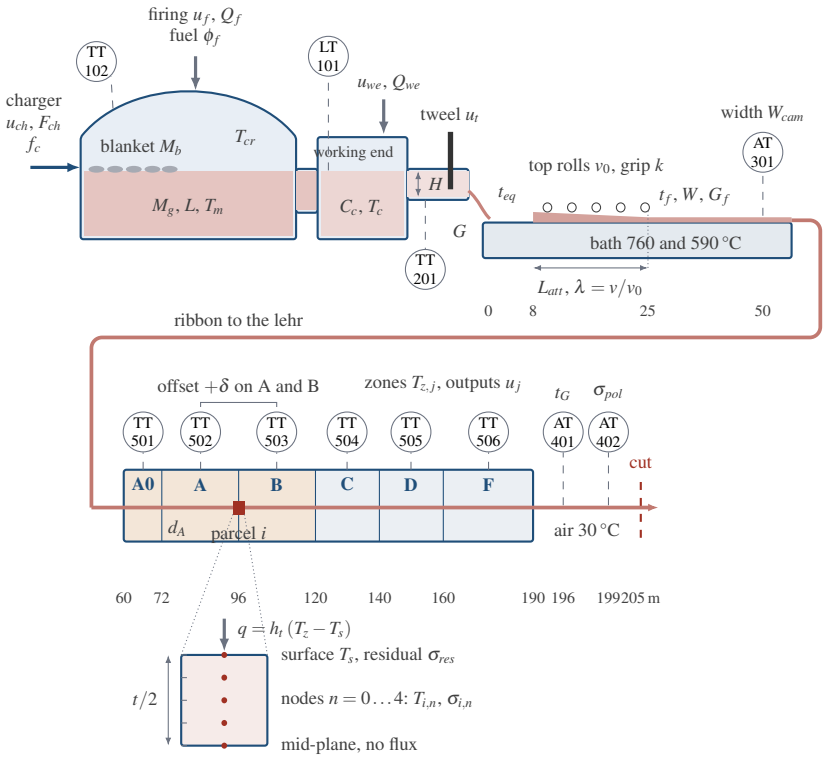



**Figure 5.1** Float line as modeled, drawn out of scale with positions in meters from the spout, the furnace and the float bath above and the lehr and the cold end below. The charger lays batch and cullet (fraction $f_c$) on the melt as a blanket $M_b$ that melts into the glass inventory $M_g$, whose level $L$ LT-101 reads. The firing $u_f$ heats the crown space ($T_{cr}$, TT-102), the working end ($C_c$, $T_c$) conditions the glass on its way to the canal (TT-201), and the tweel at opening $u_t$ passes the pull $G$ under the head $H$. On the tin the glass spreads toward $t_{eq}$ and is stretched over $L_{att}$ between the top rolls ($v_0$, grip $k$) and the lehr ($v$), leaving the forming zone at 25 m with thickness $t_f$, width $W$ and flow $G_f$. The width camera AT-301 stands at 50 m, the six lehr zones A0 to F hold $T_{z,j}$ with outputs $u_j$, zones A and B carry the supervisor's offset $\delta$, and zone A can fail with its damper stuck at $d_A$. The gauge AT-401, the polarimeter AT-402 and the cut follow. Each 3 m parcel $i$ carries five nodes of temperature and stress over its half thickness $t/2$, heated or cooled at its surface by the flux $q$.

Three heat stores, the combustion space under the crown, the melt and the working end, are coupled as shown in Equation (5.2).

$$\begin{aligned} C_{cr}\frac{\mathrm{d}T_{cr}}{\mathrm{d}t} &= \eta Q_f - UA_{cm}(T_{cr}-T_m) - UA_{cr}(T_{cr}-T_a) \\ C_m\frac{\mathrm{d}T_m}{\mathrm{d}t} &= UA_{cm}(T_{cr}-T_m) - UA_m(T_m-T_a) \\ &\quad - F_m\left[Yc_p(T_m-T_a) + (1-f_c)\,q_r\right] \\ C_c\frac{\mathrm{d}T_c}{\mathrm{d}t} &= Gc_p(T_m-T_c) - UA_c(T_c-T_a) + Q_{we} \end{aligned} \tag{5.2}$$

Here $Q_f = Q_{f,max}\min(x_f, \phi_f)$ is the firing capped by the fuel available $\phi_f$ and $\eta$ the share of the firing that stays in the furnace. The term $q_r$ is the reaction and gas heat of raw batch, and $T_a = 25\,°\mathrm{C}$. The working end gives $Q_{we} = x_{we}Q_{wh}$ with its burners for $x_{we} \geq 0$ and $x_{we}Q_{wc}$ with its cooling otherwise. The four actuator positions $x$ follow their commands through first-order lags of 1 min for the charger,

**Table 5.1** Roles, Design Values and Ranges of the Line's Variables

| Symbol | Role | Design value | Range |
|---|---|---|---|
| $L$ | CV, melt level (LIC-101) | 0 mm SP | SP ±3 |
| $T_{cr}$ | CV, crown (TIC-102) | 1580 °C | SP 1500–1640 |
| $T_c$ | CV, canal (TIC-201) | 1100 °C | SP 1060–1140 |
| $\hat{G}$ | CV, pull estimate (FIC-301) | 600 t/day | SP 400–850 |
| $t$ | CV, thickness (AIC-401) | 4.00 mm | SP 3–8 |
| $W_{cam}$ | CV, width (AIC-301) | 3.70 m | SP 3.4–4.0 |
| $T_{z,j}$ | CVs, six zones (TIC-501 to 506) | Table 5.2 | SP 20–620 |
| $\hat{\sigma}$ | CV, stress forecast (supervisor) | 1.00 MPa | SP 0.4–2 |
| $u_{ch}$, $u_f$ | MVs, charger, firing | 57.4, 74.4 % | 0–100 |
| $u_{we}$ | MV, working end | −46.5 % | ±100 |
| $u_t$ | MV, tweel opening | 50.0 % | 0–100 |
| $\lambda$ | MV, attenuation $v/v_0$ | 1.611 | 0.8–3.0 |
| $v$ | MV, lehr speed | 11.26 m/min | 3–25 |
| $u_j$ | MVs, zone heat and cooling | Table 5.2 | ±100 |
| $\delta$ | MV, annealing offset on A, B | 6.7 K | 0–40 |
| $\delta_j$ | MVs, MPC offsets on A0, A, B | — | 0–$\delta_{max}$ |
| $f_c$ | DV, cullet in the charge | 20 % | 0–60 |
| $\phi_f$ | DV, fuel available | 100 % | 40–100 |
| $k$ | DV, top-roll grip (nominal 0.70) | 100 % | 30–100 |
| $w$, $s$ | DVs, lip wear, $t_{eq}$ shift | 0 % | −5–8, ±5 |
| $d_A$ | DV, zone A damper stuck | 0 % | 0–100 |
| $b_A$, $e_h$ | DVs, TC bias, estimator error | 0 | ±20, ±30 |
| $M_b$, $M_g$ | states, blanket, glass | 17.4 t, 1.27 kt | — |
| $T_m$ | state, melt temperature | 1446 °C | — |
| $v_0$, $G_f$, $t_f$ | states, forming | 6.99 m/min, — | — |
| parcels | $x$, $m$, $t$, $W$, $t_{SP}$, $T_n$, $\sigma_n$ | about 59 | 72 slots |

the fuel valves and the working end and 0.5 min for the tweel. At 600 t/day with 20 % cullet the firing is 44.6 MW, 6.43 GJ per metric ton of glass. Of the 26.8 MW that stays in the furnace, 20.8 MW crosses from the crown to the melt and 6.0 MW leaves through the crown and walls. The melt spends 11.8 MW heating the new glass, 4.0 MW on reactions and gas and 5.0 MW on floor and side losses. The glass gives up 2.9 MW between the 1446 °C melt and the 1100 °C canal, 1.7 MW as losses and 1.2 MW to the working-end cooling. With their neighbors held, the melt, the crown space and the working end settle with time constants of 96, 20 and 40 min.

Glass flows under the tweel in proportion to its opening and to the head over the lip, and inversely to its viscosity at the canal, as shown in Equation (5.3).

$$\begin{aligned} G &= G_{ref}\,\frac{x_t}{u_{ref}}\,\frac{H}{H_{ref}}\,10^{\ell(T_{c,ref})-\ell(T_c)}\,(1+w) \\ \ell(T) &= A+\frac{B}{T-T_0} \end{aligned} \tag{5.3}$$

with $H = H_{ref} + L$, $\ell = \log_{10}\eta$ for $\eta$ in Pa s and $w$ the lip wear the model does not know. The Vogel-Fulcher-Tammann (VFT) form [12] takes $A = -3.13$, $B = 5030$ K and $T_0 = 211$ °C, fitted to typical soda-lime float fixed points. They are 10 Pa s at 1450 °C, $10^3$ at 1020 °C, $10^{6.6}$ at the 725 °C softening point, $10^{12}$ at the 548 °C annealing point and $10^{13.5}$ at the 511 °C strain point. The fit meets them within 0.14 in $\log_{10}\eta$ apart from the annealing point, which it misses by 0.20. At the 1100 °C canal a degree of cooling cuts the flow by 1.46 %, so the conditioning temperature is a flow control, and one percent of opening at the 50 % reference passes 12 t/day.

### 5.3.2 Viscous Forming on Tin

On the tin the glass would settle to its equilibrium thickness $t_{eq}$. The lehr draws the ribbon away at $v$, faster than the top rolls, which grip its edges and turn at $v_0$, and the grip $k$ decides how much of the stretch $\lambda = v/v_0$ goes into thickness rather than width, as shown in Equation (5.4).

$$\begin{aligned} t(\lambda) &= t_{eq}(1+s)\,\lambda^{-(1+k)/2}, \qquad W = \frac{G_f}{\rho\, t_f\, v} \\ \tau_F \frac{\mathrm{d}t_f}{\mathrm{d}t} &= t(\lambda) - t_f, \qquad \tau_F \frac{\mathrm{d}G_f}{\mathrm{d}t} = G - G_f \\ \tau_F &= L_{att}/v \end{aligned} \tag{5.4}$$

Here $s$ is the fractional shift of the equilibrium thickness with the bath atmosphere, and $k$ runs from 0 for a free sheet that narrows and thins alike to 1 for edges held fast. The thickness and the flow reaching the end of the 17 m attenuation zone lag their inputs by the zone's length over the speed, 1.51 min at 11.26 m/min. With $k = 0.70$ the thickness goes as $\lambda^{-0.85}$, and at a given flow and top-roll speed the width goes as $\lambda^{-0.15}$, so the glass spreads to 3.97 m before it is stretched to 3.70 m and 4.00 mm at $\lambda = 1.611$ with the top rolls at 6.99 m/min. The lehr and top-roll drives follow their commands with a 6 s lag. The glass leaves the top rolls at 860 °C with a viscosity of $4.2 \times 10^4$ Pa s, so this is the forming of a liquid, in which the stretch sets the thickness and the mass relation $G_f = \rho W t v$ sets the width.

🕮 **Pause.** Try Exercise 5.3 before reading on.

### 5.3.3 The Ribbon as Parcels

Past the forming zone the ribbon is carried as parcels $\Delta S = 3$ m long that move at the lehr speed, the tracking of material of Section 1.4.4. The mass crossing the end of the forming zone accumulates until the ribbon has advanced 3 m, and a parcel is then born with that mass $m_i$, the thickness $t_f$ being formed and the width $W_i = m_i/(\rho\, t_f\, \Delta S)$, so the mass relation holds parcel by parcel. It starts at a uniform 860 °C with no stress. Each parcel also keeps the order $t_{SP}$ in force when it was formed, against which it is later judged. Along its path it sees the bath at 760 °C from 20 to 40 m and at 590 °C from 40 to 58 m, both with a convective coefficient of 15 W/(m$^2$ K), and a lift-out at 570 °C and 10 W/(m$^2$ K) up to 60 m. The six lehr zones follow to 190 m, then cold-end air at 30 °C and 12 W/(m$^2$ K), and a parcel that straddles two of these surroundings sees each in proportion to its overlap.

### 5.3.4 Heat and Stress Through the Thickness

Both faces of the ribbon are cooled alike, so half of the thickness is modeled, on five nodes from the mid-plane to the surface, as shown in Equation (5.5).

$$\rho c_p \frac{\partial T}{\partial t} = k_g \frac{\partial^2 T}{\partial z^2}, \qquad -k_g \left.\frac{\partial T}{\partial z}\right|_s = h_t\,(T_s - T_z) \tag{5.5}$$

There is no flux at the mid-plane, and $h_t = h_c + \varepsilon\sigma_{SB}(T_z^2 + T_s^2)(T_z + T_s)$, in kelvin, linearizes the radiation about the surface temperature of the previous step. The grid is node-centered with half cells at both ends, and each 1 s step is backward Euler with one tridiagonal solve, which is stable at any step. The heat a parcel gives up is handed to the zones it overlaps on the next step, so the energy ledger of the lehr closes.

The glass is a plate free to expand in its own plane, with a Maxwell element at every depth, as shown in Equation (5.6).

$$\begin{aligned} \frac{\mathrm{d}\sigma_n}{\mathrm{d}t} &= E'\alpha\left(\left\langle\frac{\mathrm{d}T}{\mathrm{d}t}\right\rangle - \frac{\mathrm{d}T_n}{\mathrm{d}t}\right) + \left\langle\frac{\sigma}{\tau}\right\rangle - \frac{\sigma_n}{\tau(T_n)} \\ \tau(T) &= \frac{\eta(T)}{G_r} \end{aligned} \tag{5.6}$$

**Table 5.2** The Six Lehr Zones. Output Is the Case 1 Mean From 60 to 120 min, Negative for Cooling

| Zone | Span (m) | SP (°C) | $Q_H$ (kW) | $Q_C$ (kW) | $h_c$ (W/(m$^2$ K)) | $C_z$ (MJ/K) | Output (%) |
|---|---|---|---|---|---|---|---|
| A0 | 60–72 | 550 | 300 | 500 | 10 | 4 | −52 |
| A | 72–96 | $505+\delta$ | 300 | 800 | 10 | 4 | −45 |
| B | 96–120 | $455+\delta$ | 250 | 1000 | 10 | 4 | −35 |
| C | 120–140 | 360 | 150 | 1300 | 12 | 4 | −44 |
| D | 140–160 | 230 | 0 | 1800 | 25 | 5 | −46 |
| F | 160–190 | 45 | 0 | 2800 | 60 | 6 | −63 |

Here $E' = E/(1-\nu)$, $\alpha$ is the expansion of the solid glass, $G_r$ the relaxation modulus and $\langle\cdot\rangle$ the mean over the thickness, and the mean terms keep $\langle\sigma\rangle = 0$, the force balance of a free plate [2]. Each step applies the elastic change, then the exact relaxation $\sigma_n \mathrm{e}^{-\Delta t/\tau}$ at the node's mean temperature over the step, complete above 700 °C and absent below 400 °C, and then removes the mean. The stress that would remain if the temperature equalized now is the residual of Equation (5.7).

$$\sigma_{res,n} = \sigma_n + E'\alpha\,(T_n - \langle T\rangle) \tag{5.7}$$

The rest of $\sigma_n$ is temporary and vanishes with the gradient. Tension is positive and $E'\alpha = 0.831\,\mathrm{MPa/K}$, so each kelvin between the surface and the mean that is frozen in leaves 0.83 MPa. The application reports $-\sigma_{res}$ at the surface as the compression kept.

### 5.3.5 Lehr Zones and the Cut

Each lehr zone is one heat store, heated electrically in A0 to C and cooled with air in all six, as shown in Equation (5.8).

$$\begin{aligned} C_{z,j}\frac{\mathrm{d}T_{z,j}}{\mathrm{d}t} &= Q_{H,j}\,u_j^{+} - Q_{C,j}\,c_j \\ &\quad - UA_z(T_{z,j} - T_a) + Q_{g,j} \end{aligned} \tag{5.8}$$

with $u_j^{+} = \max(0, u_j)$ and $c_j = \max(0, -u_j)$, while $Q_{g,j}$ counts the heat the parcels gave zone $j$ over the previous step. When zone A fails its heaters trip and its damper sticks at least $d_A$ open, so its heat is zero and $c = \max(c, d_A)$. The zones are listed in Table 5.2. At 205 m each parcel is cut, and the scorecard tallies its net area $\Delta S(W_i - 2e)$ as accepted or rejected on the inspection record. The inspection record holds each instrument's reading of the parcel itself, its own width at the camera, thickness at the gauge and stress at the polarimeter plus that instrument's noise, and the verdict, written at the polarimeter, rejects a thickness reading more than 0.09 mm from the order, a net width reading under 3.31 m or a stress reading over 1.45 MPa. The scorecard checks the truth separately, against a thickness within 0.10 mm of the parcel's order, a net width of at least 3.30 m and a residual surface compression of at most 1.5 MPa.

### 5.3.6 Positions, Initial State and Numerics

Table 5.3 lists the parameters, and the guard bands are 0.01 mm, 0.01 m and 0.05 MPa. The attenuation zone runs from 8 to 25 m, the width camera stands at 50 m, the bath ends at 58 m, the lehr runs from 60 to 190 m, and the gauge, the polarimeter and the cut stand at 196, 199 and 205 m. Every case starts from the furnace at rest for its pull, crown and canal setpoints and cullet, and from the forming at rest, with the lehr speed from the mass relation and $v_0 = v/\lambda$. The zones start at their setpoints plus the supervisor's first offset, and the ribbon is filled every 3 m from 28 to 202 m with the steady path of one parcel. The zone outputs start where they balance the heat the ribbon gives them, and every loop is seeded by tracking. The lumped states are integrated by classical fourth-order Runge-Kutta with a 1 s step, and the parcels are stepped after each one. Halving the step moves the residual of the stable ribbon by 0.03 %, nine nodes instead of five move it by 1.19 %, and 1.5 m parcels instead of 3 m by 0.28 %, all within the application's own checks.

**Table 5.3** Parameters of the Line. The Class Says Whether the Parameter Panel Offers the Value (vis.), Keeps It Out of Sight (hid.) or Cannot Change It (fixed)

| Symbol | Meaning | Value | Class |
|---|---|---|---|
| $A_f$, $h_0$ | melt area, depth at zero level | 450 m$^2$, 1.2 m | hid. |
| $\rho_m$ | molten glass density | 2350 kg/m$^3$ | hid. |
| $C_m$, $C_{cr}$, $C_c$ | heat stores | 960, 190, 24 MJ/K | vis. |
| $UA_{cm}$ | crown to melt | 155 kW/K | vis. |
| $UA_{cr}$, $UA_m$, $UA_c$ | losses | 3.85, 3.50, 1.60 kW/K | hid. |
| $\eta$, $Q_{f,max}$ | heat kept, firing capacity | 0.60, 60 MW | vis. |
| $q_r$, $Y_b$ | batch heat, glass per kg | 0.62 MJ/kg, 0.83 | vis. |
| $\tau_B$ | melting time at $T_B$, no cullet | 40 min | vis. |
| $\theta_b$, $k_{cul}$ | melting scale, cullet effect | 60 K, 0.8 | hid. |
| $F_{ch,max}$ | charger capacity | 1210 t/day | vis. |
| $\tau_{ch}$, $\tau_f$ | charger, fuel valve lags | 1, 1 min | hid. |
| $Q_{wh}$, $Q_{wc}$ | working-end heat, cooling | 1.5, 2.5 MW | vis. |
| $\tau_{we}$, $\tau_t$ | working-end, tweel lags | 1, 0.5 min | hid. |
| $G_{ref}$ | flow at the reference | 600 t/day | vis. |
| $u_{ref}$, $T_{c,ref}$ | reference opening, canal | 50 %, 1100 °C | hid. |
| $H_{ref}$ | head over the lip | 0.20 m | vis. |
| $\rho$ | glass density | 2500 kg/m$^3$ | vis. |
| $t_{eq}$, $k$ | equilibrium thickness, grip | 6.0 mm, 0.70 | vis. |
| $L_{att}$, $T_{form}$ | attenuation zone, exit | 17 m, 860 °C | hid. |
| $\tau_d$, $e$ | drive lag, edge trim | 6 s, 0.15 m | hid. |
| $c_p$ | glass heat capacity | 1200 J/(kg K) | vis. |
| $k_g$, $\varepsilon$ | conductivity, emissivity | 1.5 W/(m K), 0.70 | vis. |
| $E$, $\nu$ | modulus, Poisson ratio | 72 GPa, 0.22 | hid. |
| $\alpha$, $G_r$ | expansion, relaxation | 9 ppm/K, 30 GPa | vis. |
| $A$, $B$, $T_0$ | VFT constants | −3.13, 5030 K, 211 °C | vis. |
| $C_z$ | zone store, A0 to C | 4 MJ/K | vis. |
| $UA_z$ | zone shell loss | 100 W/K | hid. |
| $\tau_L$, $\tau_{TC}$ | level, furnace TC lags | 1, 1 min | hid. |
| $\tau_Z$, $\tau_{cam}$, $\tau_{ga}$ | zone TC, camera, gauge | 18, 12, 12 s | hid. |
| $\Delta t$, $W_{min}$ | tolerance, net width | 0.10 mm, 3.30 m | vis. |
| $\sigma_{lim}$ | residual stress limit | 1.5 MPa | vis. |
| — | guard bands $t$, $W$, $\sigma$ | 0.01, 0.01, 0.05 | hid. |
| $\sigma_{tmp}$ | temporary tension alarm | 10 MPa | hid. |
| $T_a$, $\Delta S$ | ambient, parcel length | 25 °C, 3 m | fixed |
| — | nodes, parcel slots | 5, 72 | fixed |

### 5.3.7 Assumptions

The property values, capacities, limits and correlations are illustrative calibrations chosen for teaching, not those of a particular plant or glass. The furnace is three lumped heat stores with a batch blanket and one glass inventory, and the melt's heat capacity is an active 800 t. A real melting tank has convection currents and a temperature field in three dimensions, and fining, seeds, stones and cord are absent, so a cold melt shows only as a growing blanket of unmelted batch.

Forming is quasi-steady and reduced to an equilibrium thickness and one grip, with a first-order lag over the attenuation zone. There is no thickness profile across the ribbon, no edge bulb and no asymmetry between the tin side and the top side. Glass thicker than $t_{eq}$ would need $\lambda < 1$, which a real line does not use.

Heat moves only through the thickness, with both faces alike, constant properties, gray radiation and a conductivity that includes internal radiation. The ribbon is uniform across its width, so there is no edge cooling and no edge stress. The zones change temperature in steps at their boundaries, smoothed only by the parcel length, which the residual hardly notices while the peak temporary tension rises from 4.79 to 5.05 MPa with 1.5 m parcels.

The stress is one Maxwell element per depth with a single relaxation time. Real glass relaxes over a spectrum of times, and its structure relaxes as well [14], so the residual is a proxy for the permanent through-thickness stress, and the 1.5 MPa limit and 1.0 MPa target are teaching values rather than a

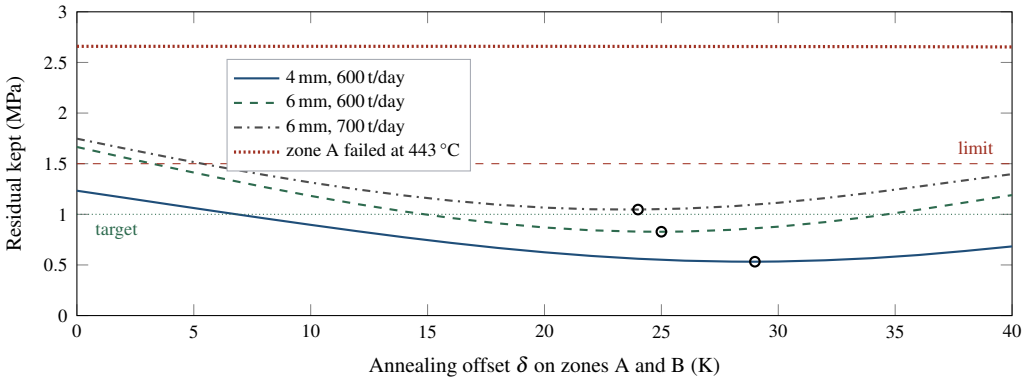



**Figure 5.2** Residual surface compression kept by a steady ribbon against the annealing offset added to zones A and B. Each point is one parcel carried from the forming zone to the cut at 1 s steps through fixed zone temperatures, and the circles mark the minima. With zone A failed at 443 °C the offset acts on zone B alone.

standard.

The instruments are lags and filters with colored noise and no failure modes other than the scenario events. The polarimeter is taken to read the permanent stress, and each parcel is judged against its own order with guard bands the size of the instruments' noise. Rejected glass is not returned to the furnace as cullet.

## 5.4 Where the Ribbon Keeps Its Stress

In case 1 the ribbon enters the lehr at 607 °C with 3.1 K between its mid-plane and its surfaces. Averaged from 60 to 120 min, it passes the annealing point at 78.0 m, in zone A, and the strain point at 98.7 m, just inside zone B. Across those 21 m, 1.8 min of travel, the relaxation time $\eta / G_r$ grows from 21 s to 24 min. The residual of Equation (5.7), the stress the glass would keep if its temperature equalized at that point, is 1.69 MPa at the lehr entrance and 1.00 MPa at 96 m, where the ribbon is at 519 °C. Over the remaining 94 m of lehr the ribbon cools to 89 °C and its residual changes by less than 0.01 MPa, so it reaches the cut with 1.01 MPa. The stress is settled within the first 36 m of the lehr, and the other 94 m only cool the glass.

🕮 **Pause.** Try Exercise 5.1 before reading on.

The forecast the supervisor makes is this residual carried forward, and its dependence on the annealing offset is the characteristic that shapes the control design, as shown in Figure 5.2 for steady ribbons annealed with each offset from the start. For 4 mm glass at 600 t/day the stress falls from 1.23 MPa on the recipe to the 1.0 MPa target at 6.8 K, close to the 6.7 K the supervisor holds on average from 60 to 120 min in case 1. It reaches a minimum of 0.532 MPa at 29 K and rises again to 0.683 MPa at 40 K. Zone A does nearly all the work, since raising zone A alone by 30 K takes the stress to 0.563 MPa, while raising zone B alone takes it only to 1.213 MPa. Past the minimum the ribbon leaves zone A still at the top of its annealing range, at 547 °C with 0.27 MPa kept at 40 K of offset, and finishes freezing in zone B, which the recipe holds 50 K below zone A, where it gains another 0.41 MPa.

Thicker glass keeps more for the same schedule, because at a given cooling rate the temperature difference through a plate grows with the square of its thickness. For 6 mm glass at 600 t/day the target needs 14.8 K, close to the 14.6 K the supervisor reaches in case 8. At 700 t/day the 6 mm ribbon cannot reach the target at all, with a minimum of 1.047 MPa at 24 K, although 5.4 K takes it under the limit. A 4 mm ribbon at 850 t/day, 42 % faster, needs 15.1 K, so even the largest pull the application allows leaves the lehr feasible. With zone A failed and held at 443 °C the curve is flat, 2.659 MPa

with no offset and 2.653 MPa with 40 K on zone B, because the ribbon passes both fixed points inside zone A.

Two consequences follow for the controller. The sensitivity of the stress to the offset changes sign at the minimum, so an integrating loop closed on the forecast would push the offset the wrong way on the far side of it, and the supervisor searches a grid of offsets instead. And the exit temperature of the lehr says nothing about the stress. On the recipe, cooling zone F from 45 to 20 °C takes the ribbon out of the lehr 19 K colder and to the cut 15 K colder with its residual unchanged, and it only raises the peak temporary tension from 4.56 to 5.05 MPa.

**Pause.** Try Exercise 5.2 before reading on.

## 5.5 Implemented Controller

The loops, the estimator and the supervisor run every 5 s on a clock kept in minutes. The controller executes on every fifth 1 s physics step, and each step then integrates the lumped states and moves, heats and stresses the ribbon. The historian logs every 20 s, and the application runs from a quarter of a minute to 20 min of plant per second, 2 min by default. The three clocks are those of Section 1.6.1.

### 5.5.1 Instruments and the Pull Estimate

No loop reads the truth. Each measurement is the true value passed through a first-order lag or filter, plus colored noise, and each ribbon instrument reads the parcels passing its position. The level has a 1 min lag and 0.02 mm of noise, and the crown and canal thermocouples have 1 min lags with 1 and 0.1 °C. The zone thermocouples have 18 s and 0.3 °C, and the width camera, the gauge and the polarimeter have 12 s filters with 2 mm, 2 µm and 0.03 MPa. The noise is correlated over 12 to 60 s, and the lehr speed is read with 0.02 % of noise. The filters feed the loops and the trends. The inspection record of a parcel takes its own value plus the noise at that scan, because a filtered reading mixes adjacent 3 m parcels, which differ by up to 0.20 mm when the thickness hunts.

No meter measures molten glass, so the pull is estimated from Equation (5.3) with the measured opening, level and canal temperature, multiplied by $1+\beta$ and smoothed over 1 min before the loops use it. The correction $\beta$ is learned from the cold end. Each parcel reaching the gauge carries its camera width $W_{cam}$, its gauge thickness $t_G$ and the lehr speed $v_B$ at its birth, and so its own pull $G_{ce} = \rho W_{cam} t_G v_B$. This is compared with the model pull tagged to it when it was formed, and $\beta$ moves 5 % of the way toward the ratio's excess over one, within $\pm 8$ %. Three percent of lip wear is learned as 2.93 % within 90 min, and the true pull returns to 600.4 t/day.

### 5.5.2 PI Loops and Their Feedforwards

Every loop is a PI controller written in the framework's velocity form of Section 1.6.2, Equation (5.9).

$$
\begin{aligned}
e_k &= a\,(\mathrm{SP}_k - \mathrm{PV}_k), \qquad P_k = K_c e_k \\
S_k &= S_{k-1} + (P_k - P_{k-1}), \qquad c_k = b_k + S_k + I_{k-1} \\
u_k &= \mathrm{slew}\big(\mathrm{clamp}(c_k, u_{min}, u_{max}),\ u_{k-1}\big) \\
I_k &= I_{k-1} + \Big[\frac{K_c}{\tau_I}\, e_k + \frac{u_k - c_k}{\tau_I}\Big]\Delta t_c
\end{aligned}
\tag{5.9}
$$

Here the sign $a = \pm 1$ fixes the direction, $b_k$ carries the loop's feedforward and $\Delta t_c = 5$ s. These are Equations (1.15) to (1.18) with $c_k = v_k$, the feedforward $b_k$ in place of the bias $u_b$, no derivative term and the tracking time $T_t = \tau_I$, so the clamp and the slew limit act only on the emitted output and back-calculation on the integral alone. Every loop here has integral action, and the derivative action is switched off in every loop. In manual the integral tracks as in Section 1.6.2, with $I_k = u_{op} - b_k - S_k$, and switching a feedforward moves the integral by the difference between the old and the new feedforward, so that the output does not jump.

**Table 5.4** Loops as Shipped. All Are PI in the Velocity Form With Integral Times in Minutes

| Tag | CV to MV | $K_c$ | $\tau_I$ | Output | $a$, $b_k$ |
|---|---|---|---|---|---|
| LIC-101 | level, charger | 30 %/mm | 200 | 0–100 % | +1, charge |
| TIC-102 | crown, firing | 1.0 %/K | 20 | 0–$\phi_f$ | +1 |
| TIC-201 | canal, working end | 5 %/K | 15 | ±100 % | +1 |
| FIC-301 | $\hat{G}$, tweel | 0.035 %/(t/day) | 2 | 0–100 % | +1 |
| AIC-401 | thickness, $\lambda$ | 0.20 per mm | 4 | 0.8–3.0 | −1, $\lambda_{ff}$ |
| AIC-301 | width, $v$ | 1.0 min$^{-1}$ | 5 | 3–25 m/min | −1, $v_{ff}$ |
| TIC-50$j$ | zone $j$, $u_j$ | 3 %/K | 5 | ±100 % | +1 |

The lehr speed slews at most 3 m/min per minute, zones D and F cannot heat.

The loops are listed in Table 5.4. Their directions follow from the process gains. A low level charges more, a cold crown fires more, a cold canal takes the working end toward heat, a low pull estimate opens the tweel and a cold zone heats, all with $a = +1$. Glass that is too thick must be stretched more and a ribbon that is too wide must run faster, so the two forming loops run with $a = -1$. The charger feedforward is the mix that melts into the estimated pull at the recipe's cullet, $100\,\hat{G}/(YF_{ch,max})$ in percent, and the pull setpoint is ramped at 60 t/day per hour.

### 5.5.3 Forming Model, Smith Predictor and Inferred Thickness

With the forming model on, the thickness loop takes its bias from the equilibrium thickness, $\lambda_{ff} = (t_{eq}/t_{SP})^{2/(1+k)}$ with the nominal grip, which is 1.611 for 4 mm and 1.000 for 6 mm. The width loop takes the lehr speed from the mass relation, $v_{ff} = \hat{G}/(\rho W_{SP}\hat{t})$, where $\hat{t}$ is the thickness the model says is being formed, so that a change of thickness does not spill into the width. The same model turns the thickness loop into a Smith predictor [7], as shown in Equation (5.10), the predictor of Equation (1.4) with the delay tagged to each parcel instead of fixed in time.

$$\mathrm{PV}_t = t_m + \tilde{t}, \qquad \tau_s \frac{\mathrm{d}\tilde{t}}{\mathrm{d}t} = \left(t_{meas} - t_{pred}\right) - \tilde{t} \tag{5.10}$$

Here $t_m$ is the model's thickness forming now, computed from the measured speed ratio with the nominal grip, no bath drift and the forming lag, $t_{pred}$ is the model thickness tagged at its birth to the glass now under the instrument, and $\tau_s = 15$ s. A transition that the model predicts is then not an error, and a constant mismatch is removed without waiting out the delay on every scan. The instrument is either the gauge or the mass-balance thickness at the camera, an inferential measurement [8] built from the model pull $\hat{G}_B$ and the lehr speed $v_B$ tagged to the glass under the camera, $t_{cam} = \hat{G}_B(1+\beta)/(\rho W_{cam} v_B)$. The camera sees glass 2.2 min after it is formed and the gauge 15.2 min after, and the mass balance is exactly as good as the pull estimate, which is why the pull is learned from the cold end. A soft sensor corrected by a late, exact measurement is also the structure of Chapter 10, where a laboratory corrects a psychrometric estimate of powder moisture. With the forming model off both feedforwards are zero and the loop acts on the raw measurement.

### 5.5.4 Thermal-History Estimator

The estimator keeps its own copy of every parcel's temperature and stress, stepped at the 5 s control period with the kernel of Equations (5.5) to (5.7), the measured zone temperatures, the tracked positions, each parcel's tagged thickness $\hat{t}$ and an optional heat-transfer error $e_h$. When a parcel reaches the polarimeter the estimator's residual for it is compared with the reading, and a bias $b$ moves a tenth of the way toward the difference, within ±0.5 MPa. This is the bias update of Section 1.4.8 with a weight of 0.1, which this chapter does not call $\lambda$, since $\lambda$ is the attenuation here. It is aligned by construction, since each reading is compared with the forecast for the parcel it was taken on. In case 1 at 90 min the corrected forecast for the glass entering the lehr and the polarimeter differ by 0.057 MPa. With the supervisor off, the offset held at 6.5 K and a 10 °C bias on zone A's thermocouple, the true

zone runs 10.00 °C below its setpoint, and at 65 min the polarimeter reads 1.35 MPa, 0.35 MPa more than the uncorrected forecast, while $b$ has grown to 0.36 MPa.

### 5.5.5 Supervisor

Once a minute the supervisor forecasts the residual of the parcel nearest the lehr entrance from the estimator's state, at the measured lehr speed, to the lehr exit with a 4 s step, for offsets on a 2.5 K grid from 0 to $\delta_{max} = 30$ K on zones A and B. A zone whose loop is in automatic and unsaturated is assumed to hold its setpoint, while a zone whose loop is saturated, or any zone in manual, is assumed to stay at its reading and takes no offset. The target offset and the verdict follow Equation (5.11).

$$\begin{aligned} \delta^{*} &= \min\{\delta : \hat{\sigma}(\delta) + b \le \sigma_{SP}\} \\ \text{infeasible} &\iff \min_{\delta} \hat{\sigma}(\delta) + b > \sigma_{lim} \end{aligned} \tag{5.11}$$

The smallest offset is interpolated between grid points and sought only below the grid's minimum, and when no offset meets the target the supervisor takes the offset of least forecast stress. The offset moves toward $\delta^{*}$ at 2 K/min. When the lehr is infeasible the supervisor also forecasts the best offset at 95 down to 60 % of the present speed, in steps of 5 %, and names the first pull that would meet the limit. The 4 s step alone makes the forecast from the lehr entrance read about 1.9 % low, which the bias absorbs. A search is used instead of a loop on the forecast because of the sign change in Figure 5.2. Once a minute the estimator also forecasts, for every parcel between 48 and 190 m, the residual it will keep under the present setpoints, and the scene draws these forecasts as ticks on the stress bars.

### 5.5.6 Annealing Model Predictive Controller

The lehr structure has a third setting beside the manual offset and the supervisor, a model predictive controller (MPC) [18, 19] of the kind Section 1.6.3 describes, that sets zones A0, A and B through three separate offsets $\delta_j$ above their recipe setpoints. Zones C, D and F keep their recipe, because a sustained kelvin on any of them moves the residual by less than 0.0001 MPa at the four steady products measured (4 mm at 600 and 680 t/day, 6 mm at 600 and 700 t/day), where zone A moves it by $-0.030$ to –0.033 MPa/K (and –0.0002 MPa/K for the 6 mm ribbon at 700 t/day, near its minimum in Figure 5.2), zone B by $-0.0005$ to $-0.0031$ and zone A0 by $+0.0020$ to $+0.0067$. A warmer A0 sends the glass into zone A hotter, so it then crosses its strain point faster and keeps slightly more. Every zone loop of Table 5.4 acts beneath the plan, the manual offset is unused as under the supervisor, and the supervisor remains the default structure of the application and of every case except case 9.

The controller's model is the estimator's own, which makes it wrong only where the estimator is. At each decision the glass between the lehr entrance at 60 m and the end of zone B at 120 m is divided into one-minute cohorts at the measured lehr speed, $N_a = \lceil 60\,\text{m}/(v \cdot 1\,\text{min}) \rceil$ of them, 6 at 11.26 m/min and at most 20 at the 3 m/min floor. One parcel per cohort is flown by the kernel of Equations (5.5) to (5.7), at the supervisor's 4 s step and at 1 s from 157 to 167 m, from its estimated temperature and stress to the cold end, under the scheduled speed and with the zones at their applied setpoints, or at their readings when saturated or in manual. Glass not yet formed starts at $T_{form}$ with the thickness of its order, or with the largest thickness the top rolls can form at $\lambda = 0.8$ when the order asks for more. Each flight gives the residual $x_a$ the cohort will keep and the largest surface tension $z_a$ it meets in the lehr, which for every product from 3 to 10 mm at 4 to 20 m/min falls at zone F's entrance, 161 to 165 m. A second flight with 1 K more on zone $j$ while the cohort is $a$ minutes past the entrance gives the sensitivities $s_{a,j}$ and $s'_{a,j}$ at the measured speed and the thickness entering

now, and the cohorts are chained as shown in Equation (5.12).

$$\begin{aligned} x_{a+1}[k+1] &= x_a[k] + \textstyle\sum_j s_{a,j}\,\Delta\bar{T}_j[k], \quad & y_k &= x_{N_a}[k] + b \\ z_{a+1}[k+1] &= z_a[k] + \textstyle\sum_j s'_{a,j}\,\Delta\bar{T}_j[k], \quad & y'_k &= z_{N_a}[k] - b \\ \frac{\Delta T_j(s)}{\Delta\delta_j(s)} &= \frac{1+\tau_{n,j}s}{a_{2,j}s^2 + a_{1,j}s + 1}, & & j \in \{\mathrm{A0}, \mathrm{A}, \mathrm{B}\} \end{aligned} \tag{5.12}$$

Here $k$ counts 1 min steps, $x_0[k]$ and $z_0[k]$ are the free forecasts of the cohort entering the lehr during step $k$, $\Delta\bar{T}_j$ is the mean over the step of zone $j$'s deviation from its reference, $y_k$ is the residual of the cohort leaving zone B and $y'_k$ its peak surface tension. The bias $b$ of the estimator enters the tension with a minus sign, because the surface tension is the temporary stress less the compression the surface keeps. On the steady ribbon the chain's summed sensitivity to zone A agrees with the kernel's steady derivative within 0.7 %.

The last line of Equation (5.12) is each zone's closed loop, identified from ±5 K setpoint steps on the engine at 4 mm and 600 t/day, each averaged about an unstepped run on the same seed so that the noise cancels. The fits give $a_{1,j} = 6.34$, 6.08 and 5.63 min, $a_{2,j} = 20.98$, 13.33 and 10.12 min$^2$ and $\tau_{n,j} = 4.90$, 4.96 and 4.88 min for A0, A and B, with residuals of 0.03 to 0.07 % of the step. The zero sits at the loops' 5 min integral time, the fitted $a_{2,j}$ lie within 10 % of the 22.2, 13.9 and 11.1 min$^2$ that the zone store, the loop gain and the cooling capacity give by hand, and at 6 mm no coefficient moves by more than 6.1 %. The fits are sampled exactly at 1 min, and a steady-state Kalman filter (Section 1.4.7) built on them and on the 18 s thermocouple lag estimates each zone's two states from its thermocouple at every 5 s scan, with a measurement variance of 0.09 K$^2$. The bias $b$ is the output disturbance that makes the plan offset free [20, 21], as the fuel bias in the estimator of Volume 9, Chapter 4 does there. It is the estimator's own polarimeter correction, so each laboratory reading is aligned with the parcel it was taken on, and a reading that arrives between two decisions moves $b$ at once and enters the next plan. In a test, one parcel made to read 0.40 MPa more and reaching the polarimeter 20 s after the 40 min decision raises the A and B offsets of the 41 min decision against the same run without it.

Every 12 control scans, on a clock rounded to the minute and first at $t = 0$, the controller solves the quadratic program of Equation (5.13) over $N = 40$ steps, twice the longest chain, with the offsets held over ten move blocks, two of 1 min, two of 2 min, one each of 3, 4, 5 and 6 min and two of 8 min.

$$\begin{aligned} \min\ & \textstyle\sum_{k=1}^{N} Q\,(y_k - \sigma_{SP})^2 + \sum_{k\in\mathscr{K}} \rho_T\,\varepsilon_k^2 \\ & + \textstyle\sum_{c=L,C} (\rho_c\,\varepsilon_c^2 + \sigma_c\,\varepsilon_c) \\ & + \textstyle\sum_{i=1}^{10} \left[ S\,\|\delta_i - \delta_{i-1}\|^2 + n_i R\,\|\delta_i - \delta_0\|^2 \right] \\ \text{s.t. } & y_k \le \sigma_{SP} + \varepsilon_k, \quad y_k \le \sigma_{lim} - g_\sigma + \varepsilon_L \\ & y'_k \le \sigma_{tmp} + \varepsilon_C, \quad \varepsilon \ge 0, \quad k \in \mathscr{K} \\ & 0 \le \delta_{j,i} \le \delta_{max}, \qquad |\delta_{j,i} - \delta_{j,i-1}| \le r_\delta \cdot 1\,\mathrm{min} \end{aligned} \tag{5.13}$$

Here $\delta_i$ holds the three offsets of block $i$, which lasts $n_i$ steps, $\delta_0$ the offsets in force, $\mathscr{K}$ the steps 1, 3, …, 39 at which every band is judged, $g_\sigma = 0.05$ MPa the guard band, $\sigma_{tmp} = 10$ MPa the temporary tension of Table 5.3 that risks a check of the lehr, and $\delta_{max}$ and $r_\delta$ the supervisor's largest offset and offset rate, 30 K and 2 K/min. The weights are $Q = 200$, $\rho_T = 10^3$, $\rho_L = 10^5$, $\sigma_L = 10^3$, $\rho_C = 10^6$, $\sigma_C = 10^4$, with stresses in megapascals, and $S = 0.1$, $R = 0.002$ per square kelvin. The target term holds the cohort on $\sigma_{SP}$, the band above it, priced again with one slack per judged step, makes glass above the target costlier than glass below it, and the two horizon-wide slacks soften the reject level and the check level, so the program is never infeasible. The offsets' box is hard, as is the rate, applied at the first move and at every later block boundary. Thirty offsets and 22 slacks make 52 decision variables, and the QP's regularization is fixed at $10^{-9}(1 + 2\rho_L)$ so that the large check-level price

does not pull the offsets toward zero. Between decisions the applied setpoints slew toward the first move at $r_\delta$, as the supervisor's offset does.

The preview is the job card, the order, pull and width changes scheduled in the case ahead of now. The forecasts convert them into the lehr speed through the mass relation's ratio to the present speed, with the pull ramp and the width loop's 3 m/min per minute slew, and give the glass formed after a change the thickness of its new order. With the preview Off the plan assumes the present order, pull ramp and speed relation for all glass to come, and a change made by the operator is unannounced in both settings. The controller accepts a plan with an optimal status whose moves, predictions, cost and slacks are all finite, whose arrays have their shapes, whose planned offsets lie in their box and whose first entries equal the move. Otherwise the historian records the reason, the supervisor takes over from where the zones stand, zone A's offset becoming its $\delta$ while A0 and B ramp back within the rate, and the next valid plan resumes control. Twelve injected solver faults are all rejected, a forced fallback and resumption each move a setpoint by at most the rate over one scan, and no fallback occurs in any case measured below. In manual the MPC idles.

The controller reads only a packet of the estimator's parcels, their tracked positions and tagged thicknesses, the measured speed and zone temperatures, the zones' saturation flags, the applied offsets, the recipe, the target, the limit, the guard band, the check level, the forming range, the offset bounds and the schedule. A source check and a replay test enforce the separation, and they catch 21 of 21 copies of the controller doctored to read a plant or parameter field and 4 of 4 behavioral mutants. In Chrome a decision takes 2.22 to 2.82 ms on average on four cases (4.2 ms at the 95th percentile, 8.0 ms at most), 0.28 % of a core at the default rate of a minute of plant per second and 5.6 % at the top rate of twenty, and a frame guard drops a decision rather than a frame at high speed.

### 5.5.7 Plant, Control and Estimator Models

Section 5.3 is the simulation model. The control model is the forming model of Equation (5.4) with the nominal grip and equilibrium thickness, the mass relation, and the tweel equation with its learned correction. The estimator's model is the thermal kernel run on measured inputs, the same in structure as the plant and wrong only through the instruments, the tagged thickness and the error $e_h$. No model is identified inside the application, and the shipped tunings are not derived from one. The MPC holds the estimator's kernel linearized into Equation (5.12) at each decision and the zone loops identified offline at the design point, so a change of the glass properties reaches the estimator and the plan alike, while a change of the zone stores, the grip or the lip wear is model error for both.

## 5.6 Cases and Their Measured Outcomes

The cases and the challenge appear in Table 5.5 in the order the application lists them. Each has a completion rule, and the challenge is judged on the case as given, so a changed model parameter or disturbance voids the attempt (Section 1.6.4). Each case begins from one reference configuration, with the forming model, the mass-balance thickness, the cold-end learning, the charger feedforward and the supervisor on (the MPC in case 9), and every event comes at 20 min unless stated. Case 1 was taken up in Section 5.4. Every value quoted below was measured on the engine, run headless at its default seed. Signals that move with the noise are quoted as extremes of the true value or as averages over the stated window, and the application's 52 checks pass.

### 5.6.1 Two Clocks: Pull, Fuel and Cullet

Raising the pull target from 600 to 680 t/day at 60 t/day per hour opens the tweel from 50.0 to 56.8 % and speeds the lehr to 12.78 m/min, and the thickness and width never leave their tolerance, with 10 077 $m^2$ accepted and none rejected in 240 min. With the crown held by its loop, more heat can reach the melt only if the melt cools, and it settles 12.6 °C colder at steady state, reaching 1434.9 °C at 240 min and still falling, while the blanket grows from 17.4 to 23.6 t. The charger feedforward adds the new charge at once, but the batch takes 36 min or more to melt, and the true level falls 0.24 mm, lowest at 69 min, or 0.57 mm at 49 min with a ramp of 400 t/day per hour. Downstream the ribbon

**Table 5.5** Case Sequence and What Each Case Teaches

| # | Scenario and event | Teaching point |
|---|---|---|
| 1 | Stable ribbon | where the stress is decided |
| 2 | Pull 600 to 680 t/day, ramped | two clocks |
| 3 | Fuel capped at 70 % | the working end absorbs it |
| 4 | Cullet 20 to 40 % | a saving after an upset |
| 5 | Bath drift +4 %, gauge | a late measurement |
| 6 | Top-roll grip to 60 % | necking thickens |
| 7 | Zone A fails, repaired at 100 min | an honest infeasibility |
| 8 | Order 4 to 6 mm | a coordinated move |
| 9 | Job change, MPC | a plan over the transit |
| 10 | 300 min product change | graded challenge |

**Table 5.6** Headline Numbers of the Cases

| Case | Measured outcome |
|---|---|
| 1, lehr | 548 °C at 78.0 m, 511 °C at 98.7 m |
| 1, stress | 1.00 MPa kept by 96 m, 1.01 MPa at the cut |
| 1, furnace | 44.6 MW, 6.43 GJ/t, blanket 17.4 t |
| 2, level | true dip 0.24 mm at 69 min |
| 2, furnace | melt 12.6 °C colder at steady state |
| 2, lehr | exit 89 to 101 °C, offset 6.5 to 8.8 K |
| 3, working end | cooling 46.5 to 40.8 %, canal 1100.1 °C |
| 3, loop frozen | canal −11.3 °C, tweel 50.0 to 59.1 % |
| 4, level | +1.02 mm at 37 min, −0.65 mm at 75 min |
| 4, steady | 6.43 to 6.18 GJ/t, −3.85 % |
| 5, first reading | gauge 36.1 min, mass balance 22.8 min |
| 5, rejected | 734 gauge, 212 mass, 1827 $m^2$ no model |
| 6, width | camera 3.512 m at 24.5 min, $\lambda$ to 1.768 |
| 6, rejected | 579 $m^2$, 1131 $m^2$ with the gauge |
| 7, forecast | infeasible at 27.0 min, best 1.96 MPa |
| 7, polarimeter | over 1.5 MPa at 33.5 min, peak 2.72 MPa |
| 7, cut | 3121 $m^2$ rejected, worst accepted 1.46 MPa |
| 8, forming | $\lambda$ at 0.80 for 0.8 min, $t_f$ peak 6.111 mm |
| 8, rejected | 118 $m^2$, 1037 $m^2$ without the model |

spends 12 % less time in every zone and leaves the lehr at 101 °C instead of 89, and the supervisor raises its offset from 6.5 K at the change to 8.8 K at 240 min to keep 1.0 MPa. The furnace answers in hours and the lehr in minutes.

Capping the firing at 70 % while the furnace needs 74.4 % cools the crown by 23.6 °C and the melt by 15.0 °C by 240 min, and grows the blanket to 22.3 t. The working end absorbs all of it. It cuts its cooling from 46.5 to 40.8 %, holds the canal at 1100.1 °C, and nothing on the ribbon moves. With the conditioning loop's gain set to zero, so that the working end holds its output, the canal falls 11.3 °C, the glass at the tweel is 18 % more viscous, and the pull loop opens the tweel from 50.0 to 59.1 % to keep 599 t/day. The fault still shows where it began, in a firing pinned at its limit and a blanket 4.9 t heavier.

Raising the cullet from 20 to 40 % of the charge makes the charger feedforward cut the mix by 3.8 % at once, since cullet loses no gas. The blanket already on the melt then melts in 29 min instead of 36, and in 26 min once the melt has warmed, so it releases glass that nothing asked for. The true level rises 1.02 mm by 37 min and falls 0.65 mm below its setpoint at 75 min before it settles. At steady state the firing falls from 44.6 to 42.9 MW, 3.85 %, or from 6.43 to 6.18 GJ/t, the blanket shrinks from 17.4 to 12.1 t, and by 240 min the melt runs 6.3 °C warmer. The saving arrives only after the inventories in front of it have moved.

### 5.6.2 A Thickness Measured Late

In case 5 the bath atmosphere changes so that the glass settles 4 % thicker on the tin, and the loop measures thickness with the gauge. The formed thickness passes 4.05 mm at 20.6 min and the gauge reads it at 36.1 min, 15.5 min later, of which 15.2 min is travel and the rest the gauge's filter. The glass formed from 21.3 to 41.2 min lies more than the 0.09 mm the inspection accepts from its order, and 734 $m^2$ is rejected. The mass-balance thickness passes 4.05 mm at 22.8 min, the glass out of tolerance is formed only from 21.3 to 26.9 min, and the rejected area falls to 212 $m^2$. The gauge loop works at all because the forming model makes it a Smith predictor, whose robustness to a delay the model gets right is that of Section 1.4.3. With the model off the same tuning hunts on the delayed gauge. After its first correction the formed thickness swings between 3.85 and 4.13 mm up to 120 min with a period of about 54 min, each swing about three quarters of the one before, and 1827 $m^2$ is rejected in 120 min.

> **TRY IT IN THE APP** **5 · Thickness-Gauge Delay**
>
> Press Run and watch the thick glass travel toward the gauge, which reads it at 36.1 min, and 734 $m^2$ is rejected. Press Reset, set Thickness measured by to Mass balance before 20 min and run two hours. The mass-balance thickness passes 4.05 mm at 22.8 min, and the rejected area falls to 212 $m^2$.

### 5.6.3 Necking

In case 6 the top rolls lose 40 % of their grip. At the same stretch a grip of 0.42 would form 4.28 mm glass, and the formed thickness reaches 4.235 mm at 23.4 min as the ribbon narrows, because the same pull at the same speed is spread over less width. The camera reads 3.512 m at 24.5 min, and the width loop slows the lehr from 11.26 to 10.53 m/min and has the camera above 3.68 m again by 26.7 min. The thickness loop raises the stretch from 1.611 to 1.768, against the 1.770 that Equation (5.4) gives at $k = 0.42$, since with less grip more of each stretch narrows the ribbon instead of thinning it. The cut rejects 579 $m^2$, all of it for thickness and 97 $m^2$ of it narrow as well, and 1131 $m^2$ when the loop measures with the gauge.

### 5.6.4 A Lehr Zone That Cannot Be Made Up For

Case 7 fails zone A. Its heaters trip and its damper sticks wide open, and it falls from 511 to 443 °C by the repair at 100 min, with its loop at full heat from 26.3 min. The ribbon then crosses from its annealing point to its strain point in about 6.5 m, from 75 to 81 m at 70 min, instead of 21 m, and both points now lie inside zone A. The forecast does not see the failure at first, because it assumes that a zone whose loop is unsaturated will reach its setpoint, and it stays near 1.0 MPa until zone A's loop saturates. The next forecast, at 27.0 min, finds 1.96 MPa at best against the 1.5 MPa limit. The supervisor declares the lehr infeasible and ramps the offset to its 30 K maximum, reached at 38.4 min. For its first 3 min of infeasibility it names 360 t/day, 60 % of the speed, as a pull that would meet the limit, and from 30.0 min, with zone A at 472 °C and falling, it names none. The polarimeter crosses the limit at 33.5 min, 6.5 min after the forecast and 13.5 min after the failure, and peaks at 2.72 MPa at 105.4 min. The state at 60 min is shown in Figure 5.3.

> **TRY IT IN THE APP** **7 · Lehr-Zone Failure**
>
> Load the case and press Run on the Lehr view. Watch zone A fall below its setpoint, then the supervisor card. It declares the lehr infeasible at 27.0 min, with 1.96 MPa at best, while the polarimeter on the stress trend crosses the 1.5 MPa limit only at 33.5 min.

The cold end rejects 3121 $m^2$ in all, every square meter for stress, and it accepts no parcel over the limit, the worst accepted keeping 1.46 MPa. The supervisor finds the lehr feasible again at 105.0 min, and good glass is cut from 115.3 min. Within its offsets and its pulls down to 60 % of the speed,




**Figure 5.3** The float glass application in case 7, the lehr-zone failure, at $t = 60.00$ min, 40 min after zone A lost its heaters and its damper stuck open, cropped to the plant panel of the Lehr view. The header, the control panel with the supervisor card and the trends are cut. (1) The zone row shows zone A hatched red at 445.4 °C against its setpoint of 505 plus the 30.0 K offset with its loop at 100 % heat, and zone B heating at 54 % at 484.3 °C, while the badges read 4.00 mm, 2.60 MPa, 1285 $m^2$ accepted, 3.70 m and 601 t/day. (2) The ribbon's mean temperature falls through the annealing and strain points inside zone A and then rides zone B. (3) Beneath it the stress bars are red above the dashed limit wherever the glass has frozen and gray, with red forecast ticks, where it is still above 480 °C. (4) The forecast against the offset is flat near 2.6 MPa, above the limit and marked infeasible, and the status line says the lehr cannot anneal this ribbon, with a forecast of 2.58 MPa. (5) The parcel at 77 m and 535 °C would keep the solid parabola if its temperature equalized now, 2.66 MPa of compression at its surfaces and 1.44 MPa of tension in its middle by the application's own measurement, which the panel draws without printing.

nothing could save the glass annealed while zone A was down, and the forecast said so 6.5 min before the polarimeter could.

### 5.6.5 Changing the Order

Case 8 changes the order from 4 to 6 mm at 600 t/day. The attenuation feedforward falls to 1.000, since 6 mm is the equilibrium thickness, but the proportional term acts on the same step in the setpoint, and the attenuation sits at its 0.80 limit from 20.0 to 20.8 min. The lehr slows toward 7.50 m/min at its limit of 3 m/min per minute and undershoots to 6.81 m/min at 24.3 min. The formed thickness overshoots to 6.111 mm at 23.8 min, and the camera width swings between 3.34 and 3.99 m, staying outside $3.70 \pm 0.05$ m until 39.0 min. The feedforwards still move both speeds together, and 118 $m^2$ is rejected, 89 $m^2$ of it for thickness, 87 $m^2$ for width and 20 $m^2$ for stress, the three overlapping. With the forming model off the formed net width stays below 3.30 m from 20.3 to 58.7 min and 1037 $m^2$ is rejected. The gauge now reads glass formed 22.8 min earlier and the lehr takes 17.3 min, and the supervisor raises its offset from 6.5 to 14.6 K to keep 1.0 MPa, where an offset held at 6.3 K leaves the 6 mm glass with 1.36 MPa.

### 5.6.6 The Production Transition

The challenge runs 300 min. At 20 min the order changes from 4 mm at 600 t/day to 6 mm at 700 t/day, and the line starts with the forming model off, the gauge measuring thickness and the supervisor off

**Table 5.7** Product-Change Challenge Over 300 min. Worst Is the Highest True Residual Among Accepted Parcels, and Off-Spec Counts Accepted Parcels Outside a Limit

| Configuration | Acc. ($m^2$) | Rej. ($m^2$) | Worst (MPa) | Off spec | Pass |
|---|---|---|---|---|---|
| as shipped | 1367 | 8820 | 1.245 | 0 | no |
| suggested[a] | 8745 | 118 | 1.361 | 0 | yes |
| model, mass balance, $\delta = 0$ | 1377 | 7486 | 1.501 | 1 | no |
| same, $\delta = 5$ K | 2553 | 6309 | 1.521 | 12 | no |
| same, $\delta = 7.5$ K | 8275 | 587 | 1.422 | 0 | yes |
| same, $\delta = 10$ K | 8755 | 108 | 1.325 | 0 | yes |
| same, $\delta = 20$ K | 8755 | 108 | 1.074 | 0 | yes |
| model, gauge, supervisor | 8641 | 222 | 1.358 | 0 | yes |
| no model, mass, supervisor | 7921 | 968 | 1.061 | 0 | no |
| no model, gauge, supervisor | 1468 | 8718 | 1.207 | 0 | no |

[a] forming model, mass-balance thickness and supervisor on

with no offset. The target is 8000 $m^2$ accepted with no accepted parcel outside $\pm 0.10$ mm, 3.30 m or 1.5 MPa. The configurations are compared in Table 5.7 and drawn in Figure 5.4. As shipped the thickness loop hunts on the delayed gauge and 1367 $m^2$ is accepted against 8820 $m^2$ rejected, all of it glass formed before the order change. The forming model and the mass balance alone accept 1377 $m^2$, because 6 mm glass at 700 t/day keeps 1.75 MPa on the recipe. The suggested configuration, the forming model, the mass-balance thickness and the supervisor, accepts 8745 $m^2$ and rejects 118 $m^2$, its worst accepted parcels at 1.36 MPa, 0.091 mm from the order and 3.319 m wide, for 837 GJ of fuel. The shipped run falls 6633 $m^2$ short of the target and fails, while the suggested one clears it by 745 $m^2$ with no accepted parcel off specification and passes. The supervisor ends at 22.5 K, near the minimum of Figure 5.2, since the target cannot be met at this pull.

A fixed offset also works, and the smallest on the supervisor's grid that passes is 7.5 K, with 8275 $m^2$. At 10, 12.5, 15 and 20 K it accepts 8755 $m^2$, one parcel more than the supervisor, while 5 K passes 12 parcels whose true stress reaches 1.52 MPa, and no offset passes one at 1.501 MPa, because the 0.05 MPa guard band is less than twice the polarimeter's 0.03 MPa noise. With the forming model off and the supervisor on, the mass balance accepts 7921 $m^2$, short of the target, and the gauge 1468 $m^2$. With the thickness hunting on the gauge, adjacent parcels differ by up to 0.20 mm, yet no parcel outside the tolerance passes, because each parcel's record differs from its own thickness by the gauge noise alone, at most 0.006 mm in this run. The application replays one noise sequence. Run with 20 other sequences the suggested configuration accepts 8695 to 8755 $m^2$ and passes in every one.

### 5.6.7 Predictive Control of the Annealing Zones

Case 9 announces on the job card that at 30 min the order changes from 4 mm at 600 t/day to 6 mm at 650 t/day, and it runs the lehr on the MPC of Section 5.5.6 with the preview On. The change reaches the lehr twice. The whole ribbon slows at once, so the thin glass already in the lehr cools more slowly and keeps less, and the thick glass, which needs a much warmer zone A, first enters the lehr at 35.4 min. The structures are compared in Table 5.8 on case 9, case 8, the challenge with its suggested settings, the pull ramp of case 2 and the stable ribbon. Each row has the same seed, duration (360 min, the challenge 300 min), instruments and noise, the MPC keeps the settings of case 9 throughout, and each 3 m parcel is judged by its true residual at the cut. The MPC solved 300 to 360 programs per case and rejected none.

On case 9 the MPC leaves 6 m of ribbon outside the band, all of it above, with a worst cut of 1.114 MPa and 165 K of travel, against the supervisor's 36 m, 1.470 MPa and 241 K. The plan warms zone A0 to 15.2 K at 29 min, before the line slows, because the job card says the thin glass then in

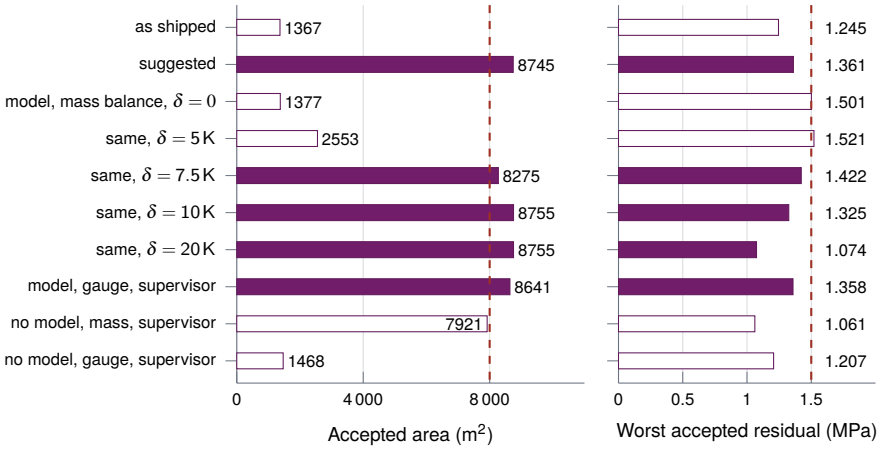



**Figure 5.4** The product-change challenge by configuration, from Table 5.7: the accepted area over 300 min (left, dashed line at the 8000 $m^2$ target) and the highest true residual among accepted parcels (right, dashed line at the 1.5 MPa limit). Dark bars pass the challenge and white bars fail it. Notice that the forming model with the mass-balance thickness is not enough without annealing (third row), and that the supervisor and a fixed offset of 7.5 K or more both pass.

**Table 5.8** Annealing Structures Against the Job Changes. Ribbon Outside the Target ±0.1 MPa as Meters Over/Under, the Worst Residual Cut and the Setpoint Travel of Zones A0, A and B Over the Whole Run

| Structure | Case 9, 4 to 6 mm, 650 t/day | | | Case 8, 4 to 6 mm | | |
|---|---|---|---|---|---|---|
| | Out (m) over/under | Worst (MPa) | Travel (K) | Out (m) over/under | Worst (MPa) | Travel (K) |
| offset fixed at 6.5 K | 2481/21 | 1.409 | 0 | 2370/21 | 1.357 | 0 |
| supervisor, shipped | 30/6 | 1.470 | 241 | 33/0 | 1.456 | 158 |
| supervisor, retuned[a] | 12/6 | 1.175 | 61 | 12/36 | 1.172 | 124 |
| supervisor, job-card FF[b] | 0/90 | 1.042 | 230 | 0/96 | 1.047 | 140 |
| MPC, preview On | 6/0 | 1.114 | 165 | 6/9 | 1.110 | 173 |
| MPC, preview Off | 6/45 | 1.125 | 143 | 9/45 | 1.118 | 154 |
| MPC, change 5 min late | 9/0 | 1.119 | 168 | 9/15 | 1.121 | 179 |
| MPC, original rule[c] | 0/324 | 1.018 | 22 | 0/240 | 1.018 | 23 |

| Structure | Challenge, suggested | | | Case 2, pull | | Case 1, stable | |
|---|---|---|---|---|---|---|---|
| | Out (m) over/under | Worst (MPa) | Travel (K) | Worst (MPa) | Travel (K) | Worst (MPa) | Travel (K) |
| offset fixed at 6.5 K | 2199/21 | 1.464 | 0 | 1.075 | 0 | 1.024 | 0 |
| supervisor, shipped | 33/0 | 1.461 | 100 | 1.035 | 155 | 1.035 | 130 |
| supervisor, retuned[a] | 1686/12 | 1.175 | 54 | 1.035 | 155 | 1.035 | 130 |
| supervisor, job-card FF[b] | 0/99 | 1.058 | 92 | 1.037 | 161 | — | — |
| MPC, preview On | 3/0 | 1.102 | 198 | 1.038 | 143 | 1.038 | 154 |
| MPC, preview Off | 6/45 | 1.114 | 167 | 1.038 | 143 | 1.038 | 154 |
| MPC, change 5 min late | 6/3 | 1.112 | 200 | 1.038 | 140 | — | — |
| MPC, original rule[c] | 0/240 | 1.043 | 47 | — | — | — | — |

No row has ribbon outside the band in cases 2 and 1, a meter over the 1.5 MPa limit or an accepted parcel off specification. [a] Offset rate 10 K/min, largest offset 15 K. [b] From the change on, the larger of the supervisor's offset and the offset it forecasts for the scheduled product. [c] $Q = 3$, $S = 2$, rate capped between block centers.

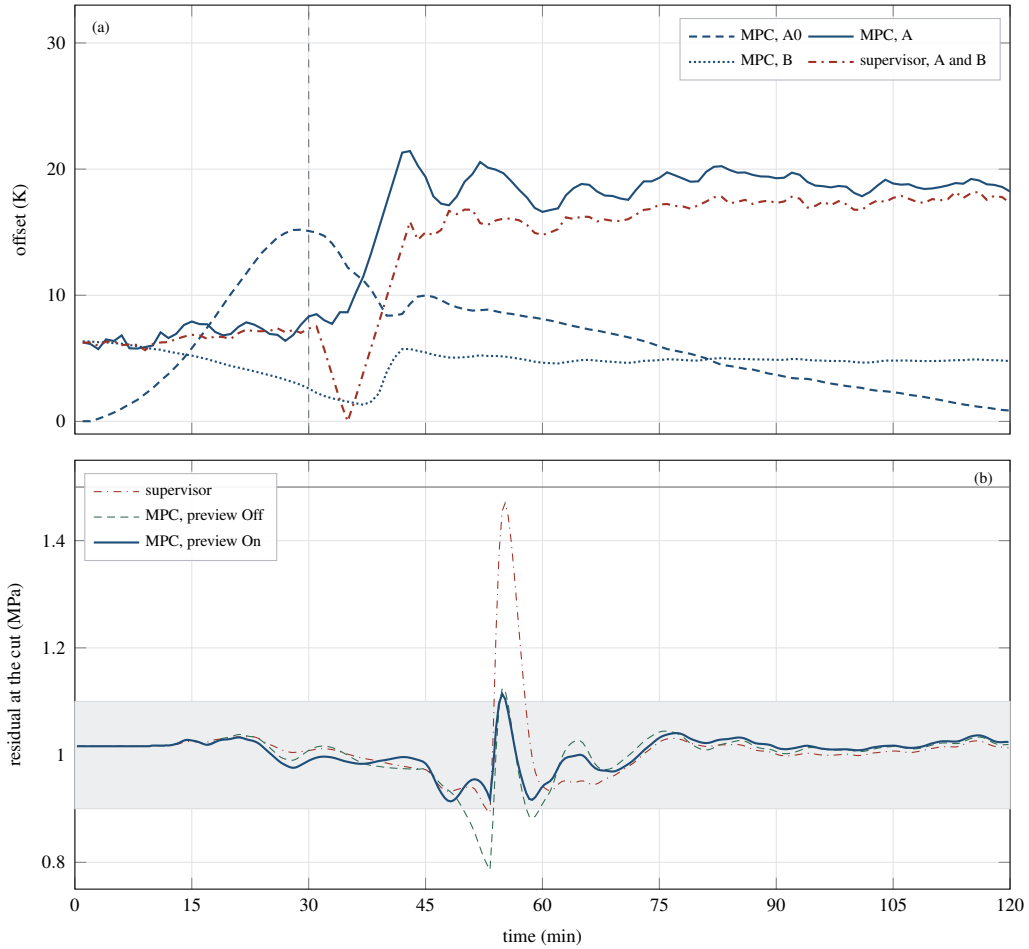



**Figure 5.5** Case 9 from 0 to 120 min, the job change at 30 min dashed. (a) Offsets above the recipe applied to zones A0 (dashed), A (solid) and B (dotted) by the MPC with the preview On, all thick, and to zones A and B by the supervisor (dash-dotted). (b) True residual of each 3 m parcel at the cut under the supervisor (thin dash-dotted), the MPC with the preview Off (thin dashed) and On (thick solid), with the 1.0 MPa $\pm$ 0.1 band shaded and the 1.5 MPa limit a thin gray line.

the lehr is about to over-anneal and a warmer A0 makes it keep more. It then raises zone A at the 2 K/min rate to 21.4 K by 43 min, so the first 6 mm glass, cut near 54 min, already finds a warm zone, as shown in Figure 5.5. The supervisor forecasts only the glass entering the lehr. It drops zones A and B to the recipe by 35 min for the slowed thin glass and climbs back after the thick glass has arrived, 15.8 K at 43 min, and its first 6 mm parcels keep up to 1.470 MPa at 55.2 min, 0.03 MPa under the limit. The plan pays for its lead with zone A0, which keeps a mean of 8.9 K from 40 to 60 min while zone A works against it, an offset the supervisor never uses. Case 8, the same order change at 600 t/day and outside the tuning, gives 15 against 33 m, and the challenge, with the MPC in place of the supervisor in the suggested configuration, gives 3 against 33 m while accepting 8755 $m^2$ against 8745 $m^2$.

The preview serves the thin glass, not the thick glass. With the preview Off the worst cut is still 1.125 MPa, because the new order is known once it is in force, 5.4 min before the first thick glass reaches the lehr, but 45 m of the slowed 4 mm glass falls under the band, and zone A0 never rises

**Table 5.9** Peak Surface Tension in the Lehr on the Two Extreme Job Changes, Before and After the Check-Level Constraint: Peak, Seconds Above the 10 MPa Check Level and Meters Cut Over the Stress Limit

| | 6 to 3 mm, 600 t/day | | | 4 to 8 mm, 850 t/day | | |
|---|---|---|---|---|---|---|
| Structure | MPa | s | m | MPa | s | m |
| offset fixed at 6.5 K | 9.90 | 0 | 51 | 9.98 | 0 | 2367 |
| supervisor, shipped | 10.32 | 84 | 39 | 10.27 | 3355 | 1293 |
| supervisor, retuned | 10.32 | 81 | 39 | 10.23 | 2761 | 1425 |
| MPC On, without | 10.64 | 160 | 33 | 10.43 | 5045 | 1104 |
| MPC On, with | 9.84 | 0 | 75 | 10.01 | 16 | 1383 |
| MPC Off, without | 10.32 | 90 | 36 | 10.43 | 5054 | 1107 |
| MPC Off, with | 10.28 | 74 | 39 | 10.01 | 16 | 1374 |
| MPC On, 5 min late, with | 10.26 | 55 | 66 | 10.03 | 49 | 1356 |

above 3.7 K before 40 min. A change made 5 min later than the job card announces costs 9 instead of 6 m on case 9 and 24 instead of 15 m on case 8. A schedule known ahead pays on other transport systems too, as Volume 9, Chapter 5 shows for releases made a travel time ahead of published demand. Two classical structures given more than the shipped supervisor were measured beside it. The supervisor retuned once by the rule that chose the MPC's weights, over offset rates from 0.5 to 10 K/min and largest offsets from 15 to 40 K, takes 10 K/min and 15 K and halves the shipped 36 m to 18 m on case 9, but its 15 K cannot anneal 6 mm glass at 700 t/day, and it leaves 1698 m of the challenge outside the band, none of it over the limit. The supervisor given the job card as a feedforward cuts the lowest peaks of all, 1.042 to 1.058 MPa, at the price of 90 to 99 m over-annealed, and of 24 pairs of lead from −4 to 8 min and gain from 0.25 to 1 none leaves less than 45 m on case 9. The pull ramp and the stable ribbon are ties, with no meter outside the band in any row, and on the stable ribbon the MPC moves the zones more, 154 against 130 K.

The weights were chosen on case 9 alone by a rule fixed before the first grid, no meter over the limit, then the fewest meters off target, then the least travel within 3 m of the best. The rule as first written counted only glass above the target, and the winner of its first grid of 108 settings parks the zones hot from the start, with 0 over and 324 m under on case 9 (the last row of Table 5.8). The second criterion was amended after that grid, openly, to count glass off target on both sides, as the target and the supervisor do, and the weights in use come from 150 further settings. Under either rule the MPC leaves less glass over the band on case 9 than the shipped supervisor, 0 or 6 m against 30 m, and both rules retune the supervisor to the same setting.

The surface tension of the ribbon in the lehr was not in the plan as first built, and the measurement exposed it. On the two most extreme changes, 6 to 3 mm at 600 t/day, where the line nearly doubles its speed with 6 mm glass in the lehr, and 4 to 8 mm at 850 t/day, an order the top rolls can form only as 7.25 mm, the plan without the tension chain raised the peak to 10.64 and 10.43 MPa, above the supervisor's 10.32 and 10.27 MPa and the 10 MPa check level, as shown in Table 5.9. The cause was none of the plan's particular choices. The rate cap between block centers gives 10.60 and 10.43 MPa, no band above the target 10.64 and 10.42 MPa, and offsets weighted toward the recipe 10.64 and 10.43 MPa. The peak lies at zone F's entrance, where the surface tension is the temporary stress less the compression the surface keeps, so a kelvin that takes compression off the glass puts tension on its surface. On 6 mm glass at 15.9 m/min a kelvin on zone B lowers the compression kept by 0.024 MPa and raises the peak by 0.030 MPa. The plan was annealing glass that would be rejected anyway, its worst cuts at 2.30 and 1.69 MPa against the supervisor's 2.54 and 1.79 MPa, and the surfaces of that glass met zone F in more tension. The second chain of Equation (5.12) with its soft ceiling at $\sigma_{tmp}$ was the fix, its price set at ten times the reject level's before any run, because a check risks the ribbon while a reject loses only glass. The one-step forecast of each cohort's peak runs a median of 0.030 to 0.045 MPa above the peak it then meets on four cases, with fifth percentiles of −0.004 to

**Table 5.10** Worst Cases Under Every Structure: Meters Cut Over the 1.5 MPa Limit and the Worst Residual Cut (MPa), the Fixed Offset Being 6.5 K

| Case | Fixed | Supervisor | Retuned | MPC On | MPC Off |
|---|---|---|---|---|---|
| 7, zone A failed | 918/2.673 | 915/2.670 | 915/2.673 | 915/2.670 | 915/2.670 |
| damper 50 % | 0/1.197 | 0/1.193 | 0/1.193 | 0/1.173 | 0/1.173 |
| 4 to 8, 600 t/day | 1818/1.544 | 0/1.317 | 0/1.165 | 0/1.101 | 0/1.113 |
| 4 to 8, 850 t/day | 2367/1.976 | 1293/1.787 | 1425/1.798 | 1383/2.004 | 1374/2.004 |
| 6 to 3 mm | 51/2.506 | 39/2.537 | 39/2.571 | 75/2.644 | 39/2.710 |
| pull to 850 t/day | 0/1.196 | 0/1.036 | 0/1.036 | 0/1.038 | 0/1.038 |
| TC +20 K | 3669/1.714 | 138/1.808 | 126/1.739 | 135/1.726 | 135/1.726 |
| TC −20 K | 0/1.022 | 0/1.035 | 0/1.035 | 0/1.038 | 0/1.038 |
| $e_h = -30\,\%$ | 0/1.357 | 0/1.396 | 0/1.415 | 0/1.261 | 0/1.260 |
| $e_h = +30\,\%$ | 0/1.357 | 0/1.463 | 0/1.201 | 0/1.087 | 0/1.097 |

Damper stuck from 20 to 100 min, pull ramped at 400 t/day per hour, thermocouple (TC) bias on zone A at 20 min, $e_h$ on case 8.

–0.051 MPa. With the constraint the peaks fall to 9.84 and 10.01 MPa, under the supervisor's in every preview setting, and the price is paid in glass, the meters cut over the stress limit rising from 33 to 75 and from 1104 to 1383, more than the supervisor's 39 and 1293. Equal pricing gives 10.32 and 10.14 MPa with 42 and 1311 m, which only ties the supervisor's peak, and it was not kept. Case 9, case 8, the challenge and cases 1, 2 and 7 are unchanged in every measured column, since there the ceiling never binds.

The worst cases of Table 5.10 take the extreme job changes, the fastest pull ramp, the zone faults and the estimator's heat-transfer error under every structure. No structure accepts a parcel off specification in any of them, and the failed zone A of case 7 defeats every structure alike, with 915 to 918 m over the limit. Against the supervisor the MPC cuts less over the limit with zone A's thermocouple reading 20 K high (135 against 138 m, worst 1.726 MPa against 1.808 MPa) and a lower worst residual under either heat-transfer error (1.261 and 1.087 MPa against 1.396 and 1.463 MPa), and it loses the two extreme changes as described above, with higher worst cuts there as well. On 4 to 8 mm at 600 t/day no structure but the fixed offset cuts glass over the limit, but the preview costs ribbon off target. The plan as first built forecast an order of 8 mm at 5.6 m/min where the line was making 7.25 mm at 6.2 m/min, and once the forecasts were clipped to the thickness the top rolls can form, its 153 m outside the band fell to 81 m, and its travel from 509 to 235 K. The 81 m still exceed the preview Off's 51 m and the supervisor's 48 m, because the rate cap applied at every block boundary makes the plan climb zone A early, while the slowed thin glass is still in it. A cap between block centers leaves 21 and 42 m with the preview On and Off, but 36 m on case 9 instead of 6, because it lets the plan believe that a coarse later block can jump several kelvin and postpone its climb at every decision. The cap at every boundary was kept for case 9, and this loss is its measured cost.

> ✦ **DID YOU KNOW? No Structure Rescues a Failed Zone**
>
> From the simulator: with zone A failed as in case 7, the fixed offset, the shipped and the retuned supervisor and the predictive controller with or without its preview all cut 915 to 918 m of ribbon over the stress limit (Table 5.10). The glass freezes inside the failed zone, where no other zone can reach it.

## 5.7 Technology in Practice

### What Measures It

A float line measures its two qualities where the glass is cold and hard. The gauge stands 171 m past the top rolls and the polarimeter just beyond it, so both read glass shaped and annealed minutes earlier. A 2002 roadmap for the industry found that current sensors and process controls cannot measure and control melting, refining and forming in real time [22]. The chapter's answer is to infer the thickness

at the width camera from the mass relation, and to forecast the stress from the estimator's copy of every parcel, with the cold-end readings kept to correct the inference and the forecast.

> **TECHNOLOGY Sensor: The Polarimeter That Reads Stress**
>
> Glass under stress is birefringent. Polarized light entering it splits into two waves that travel at different speeds, and by the stress-optic law the retardation between them grows with the stress, the thickness and the stress-optical constant of the glass. A polariscope reads that retardation. Newer instruments image it pixel by pixel through a polarization camera, or at several wavelengths at once, the method the latest inline quality systems use. Readings taken by hand at a few points are complex, prone to error and dependent on the operator, and their results vary with who takes them [23]. The chapter's polarimeter is a 12 s filter with 0.03 MPa of noise that reads only the permanent stress.

### What Moves It

Four final elements decide the two qualities. The top rolls and the lehr drive set the stretch and the speed, and so the thickness and the width through one mass balance. The heaters and cooling air of the six lehr zones set the thermal path that freezes the stress. Where rolls are driven by alternating-current motors, their speed is set by variable frequency drives, which vary the frequency of the power fed to the motor and are important final control devices in industrial systems [24]. The chapter's drives follow their commands through a 6 s lag, and each zone output acts on its heat and cooling capacity directly, as in Equation (5.8). Figure 5.6 draws the loops of Section 5.5 on these devices.

> **TECHNOLOGY Final Element: Heaters and Cooling Air of the Lehr Zones**
>
> Modern lehrs are modular or zonal, with heat sources placed where they are needed, and many recirculate hot air within each zone and recover waste heat. In 2002 over 90 % of lehrs were fired with natural gas, while electric lehrs, cleaner and cheaper to build but costlier to run, annealed storefront and safety glass [10]. Cooling air is throttled by dampers, multi-vane devices for large flows of air at low pressure [24]. Zones A0 to C of this chapter heat electrically and all six cool with air. A damper stuck open shows on a trend as a zone falling below its setpoint while its loop sits at full heat, as zone A does in case 7.

### Where the Controller Runs

Process plants run such loops in a distributed control system, whose supervisory level oversees field controllers that hold their variables at setpoints, with cycle times from milliseconds to minutes [25]. The application executes its loops and its estimator every 5 s on a 1 s physics step, the supervisor once a minute and the predictive controller every 12 scans. The velocity-form PI loops of Table 5.4, with their forming-model feedforwards and the Smith predictor, are the base layer. The estimator and the supervisor, or the MPC in its place, sit above them and write the offsets of the annealing zones.

### What Is Modernizing It

Three developments bear on this line. The first is model-based control. The 2002 roadmap planned intelligent process control in stages, from models of process parameters, through stand-alone advisory systems for operators, to neural network systems that adjust the process automatically, and it asked for real-time dynamic models for process control [22]. The estimator and the predictive controller of this chapter are such models run in closed loop. The second is the smart sensor, which reports when it is drifting or failing and calibrates itself [22]. The bias update from the polarimeter does part of that job here, by model rather than by instrument. The third is inline stress measurement. Multi-wavelength photoelasticity is used in the latest inline systems for quality control, where manual point readings are time-consuming under production conditions [23].

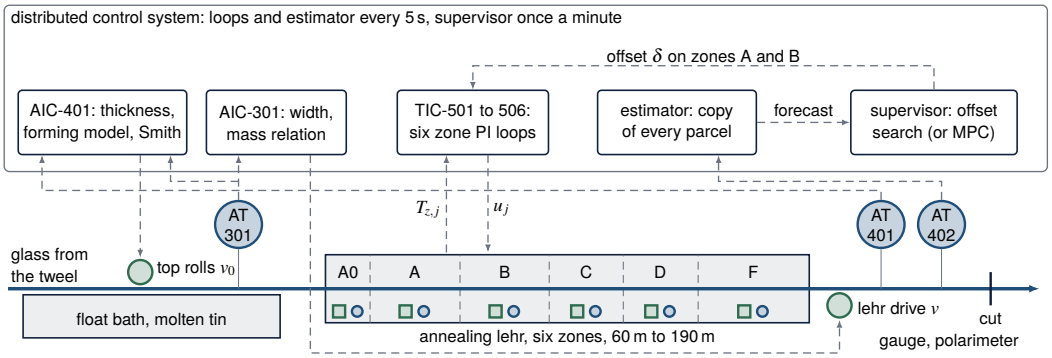



**Figure 5.6** The forming and lehr loops of the float line on plant hardware. The thickness loop (AIC-401) moves the stretch through the top rolls, the width loop (AIC-301) moves the lehr drive, and six zone loops (TIC-501 to 506) move the heaters and cooling air of the lehr. The width camera (AT-301) feeds the mass-balance thickness, the gauge (AT-401) the Smith predictor, and the polarimeter (AT-402) the estimator's bias, while the supervisor writes the offset of zones A and B. Notice that both quality instruments stand past the end of the lehr, far downstream of the top rolls and of zones A and B, where thickness and stress are made.

> ⚠ **IN PRACTICE** **Annealing in Stages, Checked in Polarized Light**
>
> Flat glass is annealed so that it is virtually strain-free and can be cut easily. A 2002 profile describes the lehr in stages. The glass is held just above its annealing temperature, about 1000 to 1050 °F for soda-lime glass, until its stresses relax, then cooled slowly so that no gradient forms between its inside and its surfaces [10]. ASTM C1279 sets out photoelastic test methods for the edge and surface stress of tempered glass, reading edge stress by hand with a polarimeter at two points [23]. The target and the limit of this chapter are teaching values, not those of a standard.

## 5.8 Limitations

The furnace is lumped, and a cold melt appears only as a growing blanket because glass quality is not modeled. Forming has no cross-ribbon profile, and glass thicker than $t_{eq}$ can be made only with $\lambda < 1$.

The stress model has one relaxation time per depth and no structural relaxation [14], and the ribbon has no edges. The polarimeter is taken to read the permanent stress, but at 199 m the ribbon in case 1 still carries 0.59 K from its middle to its surfaces, a temporary 0.32 MPa of surface tension that a real instrument would see together with the 1.01 MPa kept.

The loops are PI controllers with feedforwards, and the supervisor moves one offset on two zones, while a plant would also schedule the later zones for the product and the pull. Manual and automatic switch every loop together, so a single loop is frozen by setting its gain to zero, as in case 3. The inspection reads each parcel's own value with only the instrument's noise, while a real gauge averages over its spot and its readings must be aligned with the moving ribbon. The forecast treats a zone as healthy until its loop saturates, so the first 6.3 min of the zone A failure pass unseen. Rejected glass is not recycled as cullet.

The MPC's model is the estimator linearized at each decision, at the measured speed and the thickness entering now, so a change of speed or thickness inside the horizon reaches its sensitivities only at the next decision, and its zone loops were identified at 4 mm and 600 t/day and are not refit when an operator retunes them. Its rate cap at every block boundary is pessimistic about the coarse later blocks, which is right on case 9 and costs 81 against 21 m on the 4 to 8 mm change. It leaves zones C, D and F at their recipe because they keep no stress, although on the formable products a kelvin more on zone F lowers the peak surface tension by 0.020 to 0.036 MPa, a lever on the check

level that costs no kept stress and that the plan does not use. Using it would need zone F identified and a limit on the temperature of the glass leaving the lehr, which the model does not constrain and which a kelvin on zone F raises by 0.4 to 0.6 K at the cut.

**OPEN CHALLENGE Measuring a Ribbon Before Its Faults Freeze**

A 2002 roadmap for the glass industry found that current sensors and process controls cannot measure and control melting, refining and forming in real time, and that an upset can affect product quality for several days. It listed poor control of temperature gradients during forming among the barriers [22]. Here the forecast doubts a failing zone only once its loop saturates, so the first minutes of the zone A failure pass unseen. An estimate that caught a falling zone sooner would be progress.

## 5.9 Conclusion

This chapter studied a float line, which fixes its two qualities far upstream of the instruments that measure them. The simulator follows every 3 m of ribbon so that the fixing and the measuring can be compared. In the reference case the stress is settled within the first 36 m of a 130 m lehr and changes by less than 0.01 MPa afterward, while the gauge reads glass formed 15.2 min earlier. The stress against the annealing offset has a minimum near 29 K, which argues for a search rather than a loop on the forecast.

The mass-balance thickness at the camera sees a 4 % bath drift 2.3 min after forming instead of 15.5 min and cuts the rejected glass from 734 to 212 $m^2$. With zone A failed the forecast declares the lehr infeasible at 27.0 min, 6.5 min before the polarimeter, and no glass over the limit is accepted. In the product change the forming model, the inferred thickness and the supervisor raise the accepted area from 1367 to 8745 $m^2$.

An MPC that plans zones A0, A and B from the same estimator and the job card cuts the ribbon off target on a scheduled job change from 36 to 6 m. On changes too extreme to anneal it briefly exceeds the lehr check level, with 10.01 MPa and 16 s above the check, while sending more glass over the limit.

The model is reduced order, with a lumped furnace, one Maxwell element per depth and a polarimeter that reads only the permanent stress. A fuller model would add structural relaxation, a cross-ribbon thickness profile, recycling of rejected glass and a forecast that doubts a falling zone before its loop saturates. Within these limits the line shows what a cold-end reading can and cannot say about the glass being made.

**KEY POINTS**

- A float line makes its thickness on the tin and its stress in the first meters of the lehr, minutes before the cold-end instruments read either.
- The delays are distances over the lehr speed, from 15.2 min for 4 mm glass to 22.8 min for 6 mm glass at the same pull.
- The mass-balance thickness sees a 4 % bath drift 2.3 min after forming instead of 15.5 min and cuts the rejected glass from 734 to 212 $m^2$.
- The stress has a minimum against the annealing offset, so the supervisor searches a grid, and with zone A failed it declares the lehr infeasible 6.5 min before the polarimeter.

**Where else this applies.** Volume 16, Chapter 9 carries steel strip along a temperature path whose speed the line sets, and Chapter 10 corrects a soft sensor by a late, exact measurement in the same way.

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## 5.10 Exercises

**5.1** Concept check · 2 min

In case 1 the ribbon enters the lehr at about 607 °C and leaves it at about 89 °C. Where along the lehr is the residual stress it keeps at the cut decided?

(a) At the lehr exit, where the cold-end air takes over the cooling of the glass
(b) In zone F, where the cooling is strongest and the surface tension reaches its peak
(c) Between the annealing and strain points, within the first 36 m of the lehr
(d) On the tin, where the glass is formed and its thickness is fixed for good

**5.2** Concept check · 2 min

The supervisor forecasts the stress for offsets on a 2.5 K grid once a minute and picks the smallest offset that meets the target. Why does it not close a PI loop from the forecast to the offset instead?

(a) The forecast is too noisy for integral action, and a search averages the noise out
(b) A PI loop cannot respect the 2 K/min rate at which the offset may move
(c) A forecast made once a minute is too slow for the integral action of a PI loop
(d) The stress has a minimum against the offset, so the loop's gain changes sign past it

**5.3** Concept check · 1 min

In case 6 the top rolls lose 40 % of their grip, while the pull and the lehr speed have not yet moved. What happens to the ribbon leaving the forming zone?

(a) It narrows and thickens, because the same mass flow spreads over less width
(b) It narrows and thins, because a sheet held more loosely gives way to the stretch
(c) It thickens at the same width, because the grip acts only on the thickness
(d) It widens and thins, because the edges no longer hold the glass against the stretch

**5.4** Theory and calculation · 5 min

The line pulls 600 t/day of glass of density 2500 kg/m$^3$ as a ribbon 3.70 m wide. The thickness gauge stands 171 m past the end of the forming zone and the width camera 25 m past it.

(a) Find the lehr speed $v = G/(\rho W t)$ for 4 mm and for 6 mm glass. *Checkpoint:* 11.26 m/min and 7.51 m/min.

(b) Find the travel time from the forming zone to the gauge and to the camera for each product. *Checkpoint:* Gauge 15.2 min and 22.8 min. Camera 2.22 min and 3.33 min.

(c) A predictor that holds the gauge delay fixed in time at its 4 mm value is used on the 6 mm product. By how much is its delay wrong? *Checkpoint:* By 7.6 min, a third of the true delay.

**5.5** Theory and calculation · 5 min

The forming relation of Equation (5.4) is $t = t_{eq}\lambda^{-(1+k)/2}$ with $t_{eq} = 6.0$ mm and the nominal grip $k = 0.70$, with no bath drift.

(a) Find the attenuation $\lambda$ that forms 4 mm and 5 mm glass. *Checkpoint:* 1.611 and 1.239.

(b) The grip falls to $k = 0.42$ while the stretch stays at 1.611. Find the thickness formed. *Checkpoint:* 4.28 mm.

(c) Find the stretch that restores 4.00 mm at $k = 0.42$. *Checkpoint:* 1.770.

**5.6** Theory and calculation · 7 min

The viscosity of the glass follows the VFT form $\log_{10}\eta = A + B/(T - T_0)$ with $A = -3.13$, $B = 5030$ K and $T_0 = 211$ °C, $\eta$ in Pa s, and the relaxation time is $\eta/G_r$ with $G_r = 30$ GPa.

(a) Find the relaxation time at the annealing point, 548 °C, and at the strain point, 511 °C. *Checkpoint:* 20.8 s and 24.1 min.

(b) The canal runs at 1100 °C. By what percentage does a degree of cooling cut the flow under the tweel, Equation (5.3)? *Checkpoint:* 1.46 %.

(c) With $E = 72$ GPa, $\nu = 0.22$ and $\alpha = 9$ ppm/K, find $E'\alpha = E\alpha/(1-\nu)$ and the frozen difference between the surface and the mean temperature that leaves the 1.0 MPa target. *Checkpoint:* 0.831 MPa/K and 1.20 K.

**5.7** App challenge · 8 min

Load scenario 5 · Thickness-gauge delay with the default seed. The bath atmosphere changes at 20 min and the glass settles 4 % thicker on the tin. Use a speed of 2 (minutes of plant per second), so each 120 min run takes one minute.

(a) Run to 120 min as loaded. In the exported CSV find the first row at which `thick_gauge_mm` reaches 4.05 mm, and read the Rejected ribbon tile. *Checkpoint:* 36.3 min on the log (the chapter's 36.1 min), within 0.4 min. 734 $m^2$, within 5 $m^2$.

(b) Press Reset, set Thickness measured by to Mass balance, run to 120 min and repeat with `thick_massbal_mm`. *Checkpoint:* 23.0 min on the log (the chapter's 22.8 min), within 0.4 min. 212 $m^2$, within 5 $m^2$.

(c) The formed thickness passed 4.05 mm at 20.6 min. Account for the gauge's lag with the speed of 11.26 m/min and the gauge's 12 s filter, and say why the mass balance is quicker. *Checkpoint:* 15.2 min of travel plus about 0.2 min of filter. The camera sees the glass 2.2 min after forming.

**5.8** App challenge · 8 min

Load scenario 7 · Lehr-zone failure with the default seed. Zone A loses its heaters at 20 min and is repaired at 100 min. Run to 120 min at a speed of 5, which takes 24 s.

(a) In the exported CSV find the first row with `supervisor_infeasible` equal to 1 and the first row with `stress_pol_MPa` above 1.5. How much warning does the forecast give? *Checkpoint:* Rows at 27.3 and 34.0 min (the chapter's 27.0 and 33.5 min), within 0.6 min. About 6.5 min.

(b) Read the Rejected ribbon tile, the share of it rejected for stress in the tile's tooltip, and the Worst accepted stress tile. *Checkpoint:* 3121 $m^2$, all for stress, within 5 $m^2$. 1.46 MPa.

(c) The failure starts at 20 min. Explain why the forecast needs about 7 min to see it, and propose a change to the forecast rule. *Checkpoint:* A zone whose loop is unsaturated is forecast at its setpoint. Zone A's loop saturates at 26.3 min.

**5.9** App challenge · 9 min

Load scenario 9 · Annealing MPC: a job change planned ahead, with the default seed. The job card changes the order from 4 mm at 600 t/day to 6 mm at 650 t/day at 30 min, and the lehr runs on the MPC with the preview On. Run to 60 min at a speed of 2, which takes 30 s.

(a) In the exported CSV read `mpc_off_A0_K` at 29 min and `mpc_off_A_K` at 43 min, and read the Worst accepted stress tile. *Checkpoint:* 15.2 K and 21.4 K, within 0.3 K. 1.05 MPa.

(b) Press Reset, set Lehr supervisor to On, and run to 60 min. Read `anneal_offset_K` at 35 min, the largest value of `stress_cut_true_MPa` in the file and the Worst accepted stress tile. *Checkpoint:* 0 K. 1.47 MPa near 55 min, within 0.01 MPa. 1.40 MPa.

(c) Explain why the MPC warms zone A0 before the job change, and why the supervisor's Worst accepted stress tile reads less than the worst parcel it cut. *Checkpoint:* The plan knows the line will slow and over-anneal the thin glass. The worst parcels were transition glass the inspection rejected.

**5.10** Application area · 4 min

The model assumes the polarimeter reads the permanent stress. At the polarimeter, 199 m along the line, the case 1 ribbon still carries 0.59 K from its mid-plane to its surfaces, and it keeps 1.01 MPa of surface compression.

(a) Take the profile through the thickness as a parabola, so the surface lies two thirds of the mid-to-surface difference below the mean. Estimate the temporary surface tension with $E'\alpha = 0.831\,\mathrm{MPa/K}$. *Checkpoint:* About 0.33 MPa. The engine gives 0.32 MPa.

(b) What would a real instrument at that position read, and how would an engineer make its reading usable for the bias update? *Checkpoint:* About 0.69 MPa of compression. Move it to cooler, more uniform glass or correct the reading with the estimated gradient.

**5.11** Application area · 4 min

The batch plant raises the cullet from 20 to 40 % of the charge at 600 t/day. Raw batch yields $Y_b = 0.83$ of its mass as glass and cullet yields all of it.

(a) The firing settles at 42.9 MW instead of 44.6 MW. Find the fuel saved per day and per metric ton of glass. *Checkpoint:* About 147 GJ per day, 0.25 GJ/t.

(b) Find the cut in the mix charged for the same pull, and explain why the melt level still rises before it settles. *Checkpoint:* 3.8 %. The blanket already on the melt melts faster and releases glass nobody asked for.

**5.12** Review the agent · 5 min

An AI assistant reviewing the challenge writes: *"The supervisor's search is needless. Close a PI loop from the stress forecast to the annealing offset with a setpoint of 1.0 MPa. Integral action removes any steady error, so the 6 mm glass at 700 t/day will be held at exactly 1.0 MPa once the loop settles, with no grid and no infeasibility logic."* Find the flaw, say what the loop would do, and say how you would check it in the app.