INTERACTIVE CONTROL ENGINEERING
Principles and Applications

Volume 5 Delays, Slow Measurements, and State Estimation

CHAPTER 13

Navigation Drift as Unimaged Seabed in an AUV Survey Simulator

Pages 319–343

Edited by John D. Hedengren

How to cite this chapter. "Navigation Drift as Unimaged Seabed in an AUV Survey Simulator," in *Delays, Slow Measurements, and State Estimation*, John D. Hedengren, Ed., vol. 5 of *Interactive Control Engineering: Principles and Applications*. Advanced Process Solutions, LLC, 2026, ch. 13, pp. 319–343.

The page numbers in this file are those of the printed volume.

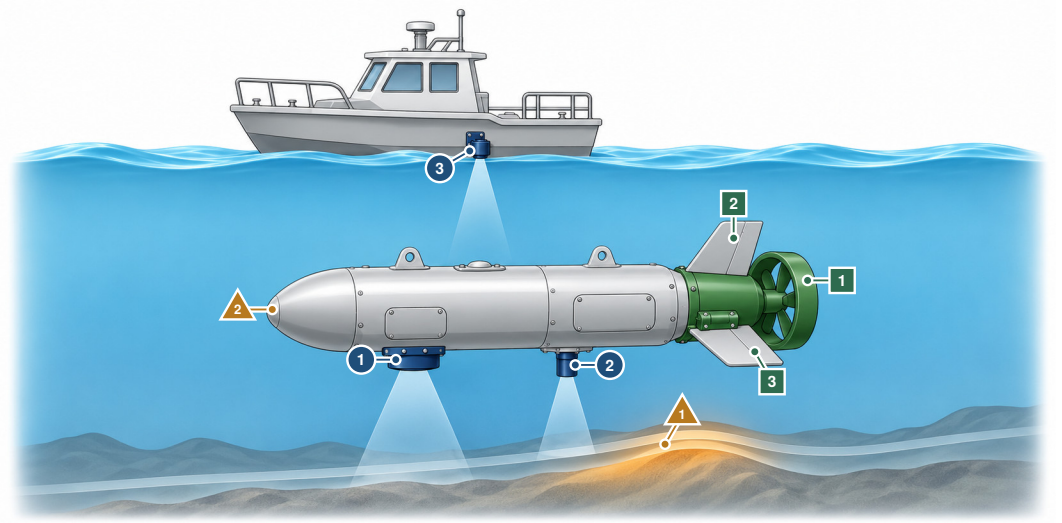
The app for this chapter
https://apmonitor.com/apps/auv-navigation/

The volume online: contents, other chapters and citation records
https://apmonitor.com/apps/books/v05-delays-estimation/#auv-navigation

Copyright © 2026 Advanced Process Solutions, LLC. All rights reserved. Produced by Advanced Process Solutions, LLC.

CHAPTER 13

Navigation Drift as Unimaged Seabed in an AUV Survey Simulator



The controller moves 1 thruster that turns the propeller 2 rudder, moved by a servo 3 stern plane, moved by a servo

The controller reads 1 Doppler log: speed over the seabed or through the water 2 altimeter: height above the seabed 3 ship's acoustic transceiver: position fixes that arrive late

What disturbs it 1 ridge the vehicle must climb 2 water current that carries the vehicle off its line

The vehicle steers on what its Doppler log, its compass and the ship's late fixes tell it, so a current or a compass error can carry it off its lines while every reading looks normal. Illustration, not to scale.

IN PLAIN TERMS A Robot That Maps the Seabed Blind

The main idea. A robot submarine flies back and forth along planned lines and maps the seabed with sound. Under water no radio or satellite signal reaches it, so it works out where it is from its speed and heading, with a sound signal from a ship now and then.

The challenge. Small errors in that reckoning add up, and the robot steers where it believes it is, not where it really is. A strip of seabed can be missed while every reading on board looks normal.

Why it matters. The map is only as complete as the robot's knowledge of where it went.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the fact on the deep surveys (Section 13.1), the history in Section 13.2, the try-it on the cross current and the try-it on the canyon, the fact on the late fixes (Section 13.5) and the key points
Study	50 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	20 min	the first page, the results (Section 13.5), Technology in Practice (Section 13.6), Limitations (Section 13.7) and the two open challenges

The control question A survey vehicle under water steers its own estimate of where it is along the planned lines, while the seabed its sonar images is set by where it actually went. Which errors open unimaged strips that nothing aboard can see, and which measurement, guidance or tuning removes each one?

Learning objectives

- Predict the dead-reckoning error of a constant compass bias from the distance to the last fix, and explain why a lawnmower pattern cancels most of it.
- Compute the swath, the overlap that lines spaced 30 m apart leave, and the uncertainty at which the supervisor stops vouching for coverage.
- Compute the steady offset that line-of-sight guidance leaves in a cross current, and show how crab compensation removes it.
- Explain why a drift on water track is unobservable without a fix or a measured current, and why no tuning recovers the strip it leaves.
- Judge when a filter that applies late fixes behind a chi-square gate will refuse honest fixes, at arrival or after a wild first fix.

Prerequisites Sections 1.4.5, 1.4.6, 1.4.7, 1.6.2 and 1.6.4.

The app https://apmonitor.com/apps/auv-navigation/

See also Volume 20, Chapter 6 for a vessel whose observer carries the slowly varying wind and current as a bias state, Volume 20, Chapter 10 for late camera reports moved on by the belt travel since their capture, and Volume 20, Chapter 2 for a path-following loop on a geometric cross-track error.

13.1 Introduction

Autonomous underwater vehicles map the seafloor at a resolution that surface vessels cannot reach, in deep water especially, and they have become routine instruments of marine geoscience, habitat mapping and seabed survey [1]. They do this without being able to measure where they are. Satellite and radio signals do not reach them, so a submerged vehicle integrates its velocity and heading and corrects the result with acoustic positioning, installed beacons or trips to the surface [2]. The control problem that follows has an unusual shape. The autopilots steer the vehicle's estimate of its position along the planned lines, while the product the survey exists for, the seabed its sonar images, is set by where the vehicle actually went.

+ DID YOU KNOW? 191 Surveys at Mid-Ocean Ridges

Over one decade a single deep-sea survey vehicle conducted 191 surveys of the seafloor at mid-ocean ridges, at an average depth of 2000 m and with its navigation precise to a few meters. Its bathymetric and magnetic maps, photographs and maps of hydrothermal plumes all rested on that navigation [3].

The simulator is aimed at one misconception, that a vehicle whose estimate holds its lines has imaged the seabed between them. In the canyon case of Section 13.5.5, 420 s into the run, the estimate is 3.3 m from line 7 while the vehicle itself is 24.6 m from it, and a strip 25 m wide and 260 m long along the edge of the southern block has been left unimaged. Every trend of the vehicle's own signals is unremarkable at that moment. The estimated heading is 3.1° from its line-of-sight setpoint and the speed through the water 0.01 m/s from its own. A block diagram cannot show the difference, because its feedback path carries the measured output, and the output of a survey is a function of a state that no instrument on the vehicle measures. The simulator therefore keeps three things apart, the true vehicle, what its instruments report and what its filter believes, and paints the survey product from the first.

The class of problem is wider than the ocean. Any machine that covers an area it cannot observe, while steering on its own estimate, belongs to it, and coverage path planning is treated as a problem

of robotics in general [4]. Its navigation half is dead reckoning corrected by absolute fixes that are intermittent and late, which a discrete-time Kalman filter can absorb in several ways at different computational costs [5], and occasionally wild enough that a validation gate must refuse them [6]. Its guidance half is path following by an underactuated craft in a current, where a line-of-sight law without integral action or a drift estimate settles off its path [7, 8]. A process loop that holds an inferential estimate on its setpoint while the unmeasured product drifts has the same structure, the soft sensor of Section 1.4.8.

However, the literature takes the two halves apart. Navigation is reviewed as localization [2] and guidance is analyzed as path following [7, 8], and neither measure is the one a survey is judged by, the seabed actually imaged, which depends on both and on the altitude besides. This chapter scores that dependence on a survey simulator that keeps the truth, the instruments and the estimate apart, and grades every run on the seabed imaged. The maneuvering model, the extended Kalman filter, line-of-sight guidance and the chi-square gate are all standard. The simulator combines them into a 14-state vehicle on Prestero's coefficients, instruments that form the only path from the truth to the controller, a filter that applies each delayed fix at the instant it describes by rewinding a 24 s history, a coverage grid painted from the truth under an explicit holiday rule, and a supervisor that reports from its own covariance when coverage can no longer be vouched for. The chapter sets these down in enough detail to be rebuilt. Eight cases and a graded survey sort the errors by their owners, the hull, the guidance, the heading reference, the current, the fixes and the vertical loops.

Section 13.2 recounts how the domain arrived at aided dead reckoning. Section 13.3 builds the vehicle, the seabed, the survey product and the instruments, and Section 13.4 the navigation filter, the guidance, the four autopilots and the mission supervisor. Section 13.5 reports each case and the graded survey. Section 13.6 describes the real instruments and computers, Section 13.7 states the limitations and Section 13.8 draws the conclusions.

13.2 From Transponder Nets to Aided Dead Reckoning

Underwater vehicles first navigated by listening to sound sources placed around them. A pulse-Doppler system tested at sea in 1976 tracked underwater and surface platforms continuously inside a fixed 50 km^2 net of three reference buoys moored 20 m above the bottom, each carrying a transponder for travel times and a continuous-tone beacon for the Doppler shift, and it placed a platform to within 2 to 3 m of the net [9]. The accuracy came at the cost of the net, which had to be laid before any work began. By 1998 commercial acoustic positioning systems offered full-ocean-depth positioning to 5 m absolute and better than 2 m relative accuracy, and an overview of how they were configured still concluded that the vehicle community needed further development [10].

Dead reckoning moved to the center once Doppler sonar became affordable. Low-cost commercial bottom-lock Doppler sonars were reported in 1999 to augment or replace the acoustic time-of-flight systems then common, and the same work analyzed the effect of heading-sensor errors on bottom-track precision, comparing a low-precision magnetic heading sensor with a ring-laser gyroscope on a survey at 2000 m depth [11]. Small survey vehicles were built around this instrument suite. REMUS, described in 1997, was 19 cm in diameter, 134 cm long and 31 kg, had a range of 25 nautical miles at 3 knots on 400 Wh of lead-acid batteries, and navigated by long and ultra-short baseline acoustics, bottom-lock Doppler and GPS [12]. A six-degree-of-freedom model of that vehicle followed in 2001, its coefficients derived from hydrostatics, lift, drag, added mass and the propeller and fin inputs, and its output verified against motion data collected at sea [13]. Its REMUS coefficients are the declared starting set of this simulator.

The answer in current use is an integrated filter, and the history segment after this paragraph describes an early one. Model-aided inertial navigation later brought a kinetic vehicle model and a real-time sea current estimate into a filter that already took USBL positions, pressure and bottom-track velocity, and was evaluated with the USBL and the Doppler log removed or dropped out [14]. A

Table 13.1 Signals of the Survey by Role, With Their Reference Values

Symbol	Role	Reference	Range
$\hat{\psi}$	CV of HDG, estimated heading	ψ_d from LOS	0–360°
u_m	CV of SPD, DVL water track	1.5 m/s SP	0.5–2.3
a_m, z_m	CV of DEP, altimeter or depth	10 m, 38 m SP	4–25, 5–58
θ_m	CV of PIT, measured pitch	θ_{SP}	±20°
$\hat{e}_\perp$	CV of LOS, estimated cross-track	0	—
δ_r, δ_s	MVs, rudder, stern plane	0	±15°
T	MV, thrust	3.645 N trim	0–10 N
θ_{SP}	MV of DEP, pitch setpoint	—	±20°
V_c, set	DV, current and its direction	0, toward 180°	0–0.6 m/s
b	DV, compass bias, unmeasured	0	±3°
ε_D	DV, DVL scale error, unmeasured	0	±2 %
—	DV, USBL lost pings, outliers	10 %, 0	0–80, 0–30 %
k_σ	DV, USBL noise multiple	1	0.2–5
—	DV, USBL outage, ship station	off, (225, 130) m	event, sliders
u, v, r, q, θ, ψ	states, motion	trim	—
n, e, z	states, position and depth	start pose	—
δ_r, δ_s, T	states, actuators	0, 0, trim	as MVs
E, ℓ	states, energy used, distance	0	—
$\hat{n}, \hat{e}, \hat{b}, \hat{V}_{cn}, \hat{V}_{ce}$	filter states	start pose, 0	—
C, holidays	product, from the truth	—	—

review in 2014 still found every alternative fundamentally limited, whether expensive inertial sensors, installed beacons or periodic surfacing [2].

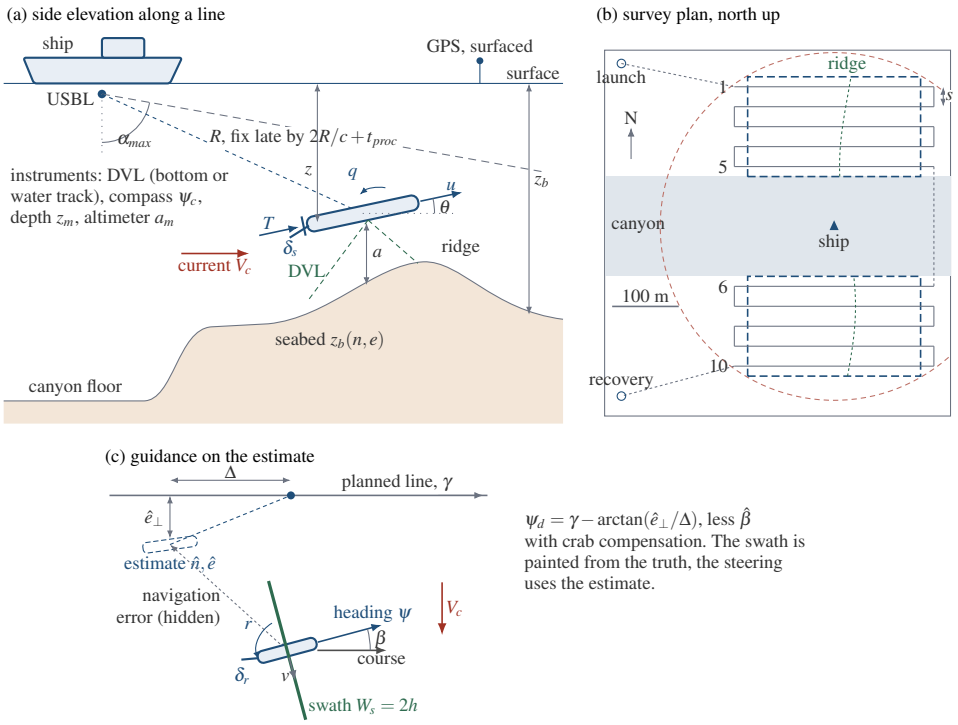
> ⧖ HISTORY **2004: A Lawnmower Pattern Cancels the Drift**
>
> In 2004 a navy's new mine countermeasures vehicle was reported to navigate on an inertial system aided by a Doppler velocity log. An extended Kalman filter, built on a model of the errors, combined the inertial solution with the Doppler log, a pressure sensor and an optional compass, and took GPS fixes at the surface, ranges to transponders, terrain navigation or acoustic fixes from its mother ship when they were available. The growth of its position error depended on the inertial and Doppler errors and on the mission pattern. Low-frequency Doppler errors caused an approximately linear drift on a straight line and tended to cancel on a lawnmower pattern, and the Doppler log's scale error became observable to the filter when position fixes were available or the vehicle turned [15].

Guidance developed beside navigation. An autopilot for the combined speed, steering and diving of a slow vehicle, designed by multivariable sliding modes, was acquiring waypoints by line of sight in 1993 [16]. Line of sight remained the practical law, and because it is susceptible to environmental disturbances, integral action was added to hold a straight path in a constant current [7], as was an adaptive estimate of the sideslip that currents, wind and waves produce [8]. Each of these systems is judged in the end by the chart its vehicle brings back, and the simulator scores its vehicle the same way, from positions the vehicle never sees.

13.3 The Vehicle, the Seabed and the Instruments

The simulated vehicle is a REMUS-class torpedo 0.19 m across and of 30.48 kg, cruising at 1.5 m/s and 10 m above the seabed. It surveys two blocks 260 m wide and 150 m deep, one on each side of a canyon, on ten east–west lines 30 m apart, under a ship that holds station over the canyon, as shown in Figure 13.1. Coordinates are north n, east e and depth z in meters from the south-west corner of the survey, with headings measured clockwise from north.

Table 13.1 sorts the signals by the part each plays. The plant's 14 states are the surge, sway and yaw rate relative to the water, the north and east position, the heading, the pitch rate and pitch, the depth, the rudder and stern-plane angles behind their servos, the thrust behind its motor lag, the energy used and the distance flown.



$\psi_d = \gamma - \arctan(\hat{e}_\perp/\Delta)$, less $\hat{\beta}$ with crab compensation. The swath is painted from the truth, the steering uses the estimate.

Figure 13.1 Survey vehicle, seabed and plan as modeled. (a) Side elevation along a line, schematic: the vehicle at depth z and pitch θ, with surge u, pitch rate q, thrust T and stern plane δ_s, at altitude $a = z_b - z$ above the seabed $z_b(n,e)$, its Doppler log (DVL) beams, and the ship's USBL transceiver at slant range R inside a cone of half-angle α_{max} from the vertical. (b) The survey plan to scale: two blocks, ten lines $s = 30\,\text{m}$ apart with 20 m run-ins, the dotted dive, canyon and return legs, the launch and recovery points, the ridge crest, the canyon with the ship over it, and the USBL footprint at 50 m depth. (c) Guidance on the estimate: a line of direction γ, the estimated cross-track error $\hat{e}_\perp$ and lookahead Δ, the hidden navigation error to the true vehicle, whose heading ψ, rudder δ_r, sway v and yaw rate r leave its course an angle β off the heading in the current V_c, and the swath $W_s = 2h$ painted from the truth.

13.3.1 Maneuvering in the Horizontal Plane

Surge, sway and yaw follow Fossen's form [17] with the REMUS coefficients [13], as shown in Equation (13.1).

$$\begin{aligned} m_{11}\dot{u} &= T + X_{u|u|}u|u| + m_{22}vr \\ m_{22}\dot{v} - Y_{\dot{r}}\dot{r} &= Y_{uv}uv + Y_{v|v|}v|v| + (Y_{ur} - m)ur \\ &\quad + Y_{r|r|}r|r| - Y_\delta\, u|u|\,\delta_r \\ -N_{\dot{v}}\dot{v} + m_{33}\dot{r} &= N_{uv}uv + N_{v|v|}v|v| + N_{ur}ur \\ &\quad + N_{r|r|}r|r| + N_\delta\, u|u|\,\delta_r \end{aligned} \tag{13.1}$$

where u, v and r are relative to the water, $m_{11} = m - X_{\dot{u}} = 31.41\,\text{kg}$, $m_{22} = m - Y_{\dot{v}} = 65.98\,\text{kg}$ and $m_{33} = I_z - N_{\dot{r}} = 8.33\,\text{kg}\,\text{m}^2$. The cross terms Y_{ur} and N_{ur} carry the added-mass contributions, and $N_{uv} = -24.0\,\text{kg}$ carries the Munk moment, the pressure moment that turns a slender body broadside to the flow. Y_δ and N_δ are magnitudes, and the signs make $\delta_r > 0$ swing the bow to starboard, the opposite of Prestero's convention. The signed square $u|u|$ keeps the fins' sense in reverse.

Linearized about the straight line at 1.5 m/s, the sway and yaw pair has eigenvalues of $+0.995$ and $-2.27\,\mathrm{s}^{-1}$, because the Munk moment outweighs the fins' restoring moment. The straight line is therefore an unstable equilibrium, and a disturbance doubles every 0.70 s until crossflow drag balances it in a steady turn. This is the distinctive fact of the hull, and it makes the heading autopilot part of the vehicle rather than an option.

13.3.2 Pitch, Depth, Actuators and Energy

The vertical plane keeps pitch and neglects heave, as shown in Equation (13.2).

$$\begin{aligned} m_{55}\dot{q} &= -M_\delta\, u|u|\,\delta_s + M_{uq}uq + M_{q|q|}q|q| - \overline{BG}\,W\sin\theta \\ \dot{\theta} &= q, \qquad \dot{z} = -u\sin\theta \end{aligned} \tag{13.2}$$

where $m_{55} = I_y - M_{\dot{q}} = 8.33\,\mathrm{kg\,m}^2$, $\delta_s > 0$ is trailing edge down (nose down, a dive) as in Prestero, and the center of gravity $\overline{BG} = 19.6\,\mathrm{mm}$ below the center of buoyancy gives a righting moment of 5.86 N m per radian and a natural period of 7.5 s. The vehicle flies along its pitch attitude. At 1.5 m/s a stern plane of $+5°$ holds $\theta = -11.9°$ and a dive of 0.31 m/s, and a pitch of 20° needs 8.3° of plane, while at 1.0 m/s it needs 18.7°, more than the 15° the planes can move.

The vehicle moves over the ground with the water, as shown in Equation (13.3).

$$\begin{aligned} \dot{n} &= u\cos\theta\cos\psi - v\sin\psi + V_{cn} \\ \dot{e} &= u\cos\theta\sin\psi + v\cos\psi + V_{ce}, \qquad \dot{\psi} = r \end{aligned} \tag{13.3}$$

where the uniform current (V_{cn}, V_{ce}) follows its setting through a 30 s first-order lag, and $\dot{\ell} = (\dot{n}^2 + \dot{e}^2)^{1/2}$ accumulates the distance flown. The actuators and the battery are given by Equation (13.4).

$$\begin{aligned} \tau_f\dot{\delta}_j &= \mathrm{sat}_{\delta_{max}}(\delta_{j,cmd}) - \delta_j, \quad j \in \{r,s\} \\ \tau_T\dot{T} &= \mathrm{sat}_{[0,T_{max}]}(T_{cmd}) - T \\ \dot{E} &= P_{hotel} + \max(T,0)\max(u,0)/\eta_p \end{aligned} \tag{13.4}$$

with $\tau_f = 0.1\,\mathrm{s}$, $\delta_{max} = 15°$, $\tau_T = 0.4\,\mathrm{s}$, $T_{max} = 10\,\mathrm{N}$, $P_{hotel} = 30\,\mathrm{W}$ and $\eta_p = 0.35$. At 1.5 m/s the trim thrust $-X_{u|u|}u^2$ is 3.645 N, the propulsion 15.6 W and the total draw 45.6 W, so the 1000 Wh battery would last 21.9 h, with E kept in watt-hours.

13.3.3 The Seabed and the Plan

The seabed is an analytic height field, as shown in Equation (13.5).

$$\begin{aligned} z_b &= 60 + \xi - h_r\, g\,(1-c) + 95\,c, \quad g = \mathrm{e}^{-(e-e_r)^2/2\sigma_r^2} \\ e_r &= 150 + 12\sin(2\pi n/420) \\ c &= S\!\left(\tfrac{n-150}{35}\right) S\!\left(\tfrac{300-n}{35}\right) \\ \xi &= 0.9\sin(\tfrac{2\pi e}{83} + 0.6)\sin(\tfrac{2\pi n}{127} + 1.1) \\ &\quad + 0.6\sin(\tfrac{2\pi(e+0.6n)}{59} + 2.0) \end{aligned} \tag{13.5}$$

in meters, where $S(x) = x^2(3-2x)$ on $[0,1]$ and is 0 below and 1 above, $h_r = 16\,\mathrm{m}$ and $\sigma_r = 24\,\mathrm{m}$. A 60 m shelf carries small undulations ξ, a ridge crosses both blocks from north to south with its crest 43 to 45 m deep, and a canyon 95 m deeper than the shelf runs east to west between the blocks, its walls falling over 35 m, far steeper than a vehicle held to 20° of pitch can follow. On the planned lines, the faces that the vehicle climbs in each line's own direction rise at 17.9 to 24.2°. The plan starts at a launch point north-west of the north block, dives to line 1, flies lines 1 to 5 of the north block east and west in turn with 30 m turn legs, crosses the canyon on its own leg at the depth the leg began at, flies lines 6 to 10 of the south block, and returns to a recovery point south-west of it. Each line runs 20 m past both sides of its block.

13.3.4 The Survey Product

The sonar's swath follows the altitude, as shown in Equation (13.6).

$$h(a) = \min\left(a\tan\theta_s,\ \sqrt{R_s^2 - a^2}\right), \qquad W_s = 2h \tag{13.6}$$

for $a_{min} \le a < R_s$ and $h = 0$ outside that range, with $\tan\theta_s = 2$, $R_s = 50\,\text{m}$ and $a_{min} = 2\,\text{m}$. The swath is 40 m at 10 m of altitude, widest at 89.4 m at 22.4 m, and nothing at 50 m. After every step it is painted from the true position, the true heading and the true altitude onto a grid of 1 m cells covering the chart, along a segment perpendicular to the heading sampled every half cell, while the sonar logs on a survey line. Each cell counts the distinct lines that imaged it. The imaged share C is the fraction of the two blocks' 78 000 cells imaged at least once. A holiday is a run of unimaged cells across the lines, within one column of a block, more than 2 m wide, bounded on both sides by imaged seabed or by the block's edge, inside the part of the block already flown, and at least 3 m long once neighboring columns are merged. A column is examined 8 s after the swath last marked it, so a gap still being crossed is not counted. At 10 m of altitude lines 30 m apart overlap by 10 m, 5 m on each side, and that margin has to absorb the cross-track control error, the current's offset, the sag over the ridge and the navigation error together.

13.3.5 Instruments

The instruments are the only path from the truth into the controller, and every random draw is made in them. The Doppler log reports its velocity in the heading frame, over the ground on bottom track and through the water on water track, as shown in Equation (13.7).

$$\begin{aligned} \begin{bmatrix} f_{bt} \\ s_{bt} \end{bmatrix} &= k \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{n} \\ \dot{e} \end{bmatrix} + w_{bt} \\ \begin{bmatrix} f_{wt} \\ s_{wt} \end{bmatrix} &= k \begin{bmatrix} u\cos\theta \\ v \end{bmatrix} + w_{wt}, \qquad \psi_c = \psi + b + \nu_\psi \end{aligned} \tag{13.7}$$

where $k = 1 + \varepsilon_D$ is a common scale error, w_{bt} and w_{wt} are white noise of 0.01 m/s and 0.03 m/s, and ψ_c is the compass, with a constant bias b and colored noise ν_ψ of 0.1° and 2 s. Bottom track exists while $0.3 < a \le 30\,\text{m}$, and neither exists shallower than 1 m, in the surface bubbles. Pitch is measured to 0.05° (colored, 1 s), depth to 0.02 m, and altitude to 0.05 m (colored, 1 s) within 50 m. GPS reports the position to 1.5 m per axis once a second after 20 s with the antenna out of the water ($z < 0.5\,\text{m}$).

The ship interrogates the vehicle's transponder every 15 s. A fix describes the instant t_f of the reply and reaches the vehicle after the reply has traveled to the ship, been processed there and been sent back to the vehicle, as shown in Equation (13.8).

$$\begin{aligned} z_{fix} &= \begin{bmatrix} n \\ e \end{bmatrix}_{t_f} + \sigma_R\, w + o, \qquad \sigma_R = \max(0.2,\ 0.005\,R)\, k_\sigma \\ t_{arr} &= t_f + 2R/c + t_{proc} \end{aligned} \tag{13.8}$$

where R is the slant range from a transducer 4 m below the surface, $c = 1500\,\text{m/s}$, $t_{proc} = 1.0\,\text{s}$, w is a unit normal pair, k_σ a noise multiple of 1, and o an outlier offset, 15 to 40 m in a random direction at a set rate or 50 m on demand. No fix returns from outside the cone of $\alpha_{max} = 80°$ from the vertical, from above 2 m, during an outage, or on one ping in ten. A fix is therefore a sampled measurement with a reporting delay, in the terms of Section 1.4.5, taken every 15 s and describing an instant 1.2 to 1.4 s before it arrives.

Table 13.2 Parameters. The Class Marks Sliders Under Show Model (Adjustable), Values Kept Out of It (Hidden), Constants Set in the Source (Fixed) and Panel Controls

Symbol	Quantity	Value	Class
m, W	mass, weight	30.48 kg, 299 N	adj., hid.
I_y, I_z	pitch, yaw inertia	3.45 kg m^2	hidden
$X_{\dot{u}}$, $Y_{\dot{v}}$	added mass	−0.93 kg, −35.5 kg	hidden
$N_{\dot{r}}$, $M_{\dot{q}}$	added inertia	−4.88 kg m^2/rad	hidden
$Y_{\dot{r}}$, $N_{\dot{v}}$	cross terms	1.93 kg m/rad, 1.93 kg m	hidden
$X_{u\|u\|}$	axial drag	−1.62 kg/m	adjustable
Y_{uv}, $Y_{v\|v\|}$	lift, crossflow drag	−28.6, −131 kg/m	hidden
Y_{ur}, $Y_{r\|r\|}$	sway from yaw	5.22 kg/rad, 0.632 kg m/rad^2	hidden
Y_δ	rudder lift	9.64 kg/(m rad)	hidden
N_{uv}, N_{ur}	Munk, yaw damping	−24.0 kg, −2.0 kg m/rad	adjustable
$N_{v\|v\|}$, $N_{r\|r\|}$	yaw crossflow	−3.18 kg, −94 kg m^2/rad^2	hidden
N_δ, M_δ	rudder, plane moment	6.15 kg/rad	adjustable
M_{uq}, $M_{q\|q\|}$	pitch damping	−2.0 kg m/rad, −188 kg m^2/rad^2	hidden
$\overline{BG}$	CG below CB	19.6 mm	adjustable
δ_{max}, T_{max}	fin, thrust limits	15°, 10 N	hidden
τ_f, τ_T	fin servo, motor lags	0.1, 0.4 s	adjustable
η_p, P_{hotel}	propulsion, hotel load	0.35, 30 W	adjustable
Q, Q_0	battery, charge at launch	1000 Wh, 100 %	adjustable
—	reserve that aborts	5 %	adjustable
$\tan\theta_s$, R_s	swath, slant range	2, 50 m	adjustable
a_{min}, cell	imaging floor, grid	2, 1 m	hidden
—	holiday width, altitude floor	2, 3 m	adjustable
α_{max}, t_{proc}	USBL cone, processing	80°, 1.0 s	adjustable
—	DVL lock range, noise	30 m, 0.01, 0.03 m/s	fixed
—	compass, pitch noise	0.1° (2 s), 0.05° (1 s)	fixed
—	depth, altimeter noise	0.02, 0.05 m (1 s)	fixed
—	GPS fix, noise	after 20 s, 1.5 m	fixed
—	USBL noise, lost pings	0.005R, 10 %	fixed, DV
q_p, q_b, q_V	filter process noise	5×10^{-4}, 1×10^{-9}, 4×10^{-6}	fixed
σ_b, σ_V	filter priors	2°, 0.1 m/s	control
Δt, Δt_c	step (RK4), controller	0.05, 0.1 s	fixed
—	historian, seed	2 Hz, 20261103	fixed

q_p in m^2/m, q_b in rad^2/s, q_V in m^2/s^3. The vehicle rows are Prestero's REMUS set.

13.3.6 Initial State and Numerics

Table 13.2 collects the parameters. Each case opens trimmed, either on a leg at a stated along-track distance and altitude or depth, or floating at the launch point, with the surge at its setpoint, the thrust equal to the drag, the fins at zero, the current already at its setting and the filter on the true position with a standard deviation of 1.5 m for a start at the surface or 1.0 m for a start underway. The engine of Section 1.6.1 advances the plant by classical fourth-order Runge-Kutta at $\Delta t = 0.05$ s, runs the instruments, filter, supervisor and loops every $\Delta t_c = 0.1$ s, keeps two rows a second in the historian and draws every random number from one generator seeded with 20261103. After each step the vehicle is held at the surface once it reaches it, stopped if its hull touches the seabed and left without its motor once the battery is empty, and the swath is painted. Halving the step moves the lowest altitude of case 8 by 0.013 m and the largest navigation error of case 4 by 0.01 %, and cells of 0.5 m move case 4's imaged share by 0.058 percentage points and its widest holiday from 7 to 7.5 m. Headless, a step takes about 3.5 µs, so the 43 min survey runs in under 0.2 s, and at 60 times real time it plays in 43 s.

13.3.7 Assumptions

Prestero's constant coefficients. They are taken as the declared set for a REMUS-class hull, constant with speed and with linear fin lift up to the 15° limit. The yaw instability they produce is kept as declared, and the heading loop is tuned for this model rather than for a particular vehicle. Fin stall, a

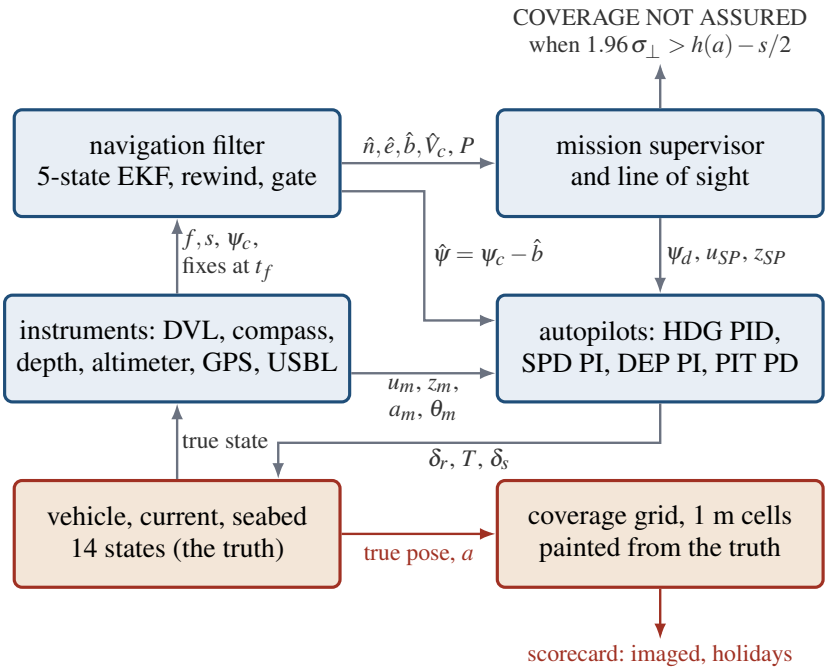


Figure 13.2 Control structure as implemented. The instruments are the only readers of the true state. The filter's estimate feeds the supervisor and, through the bias-corrected heading, the heading loop, and the coverage grid is painted from the truth and read by nothing that steers.

speed-dependent Munk moment and propeller torque are absent, so the model is least trustworthy near the fin limit and far from the cruise speed.

No heave and no roll. The vehicle flies along its pitch attitude with no angle of attack, so depth changes only through pitch, and a turn does not bank the swath. Both fail in steep, fast maneuvers.

A uniform current. The relative-velocity form of Equation (13.3) is exact only for a current constant in space and slow in time. Water track measures exactly the velocity through the water, so the current that bottom track less water track reveals is the current at the vehicle. With shear it would be the current at the tracking layer.

A smooth, charted seabed and a nadir swath. The supervisor knows the seabed of Equation (13.5) exactly. The swath is computed from the altitude straight below, as if the seabed under it were flat, and the sonar's nadir gap, resolution, image quality and sensitivity to pitch and roll are not modeled. Coverage is a yes-or-no grid, and on a steep or rough seabed the nadir altitude misstates the swath.

Generic instruments. The noise and bias models stand for no particular product. The compass bias is constant and independent of heading, and the DVL scale error is not among the filter's states. A heading-dependent compass error, or a scale error, would leave a residual the filter cannot learn.

Teaching thresholds. The line spacing, the 3 m altitude floor, the 2 m holiday, the battery reserve and every supervisor threshold are teaching values. A real survey takes its coverage and gap criteria from a standard such as IHO S-44, and the battery is treated as a store of energy drawn by a constant hotel load and one propulsive efficiency.

13.4 Navigation, Guidance and the Autopilots

Every 0.1 s the controller reads the instruments, advances and corrects the navigation filter, lets the supervisor choose the leg and the setpoints, and runs four loops, as shown in Figure 13.2. Nothing in it reads the true position, heading or altitude.

13.4.1 Dead Reckoning and the Filter

One extended Kalman filter, the filter of Equation (1.11) linearized at every step, carries $X = [\hat{n}, \hat{e}, \hat{b}, \hat{V}_{cn}, \hat{V}_{ce}]^\top$, the position, the compass bias and the current, in every navigation mode. It predicts on the Doppler log and the compass, as shown in Equation (13.9).

$$\begin{aligned} \hat{\psi} &= \psi_c - \hat{b}, \qquad \begin{bmatrix} d_n \\ d_e \end{bmatrix} = \begin{bmatrix} \cos\hat{\psi} & -\sin\hat{\psi} \\ \sin\hat{\psi} & \cos\hat{\psi} \end{bmatrix} \begin{bmatrix} f \\ s \end{bmatrix} \\ \hat{n}^+ &= \hat{n} + (d_n + \omega\hat{V}_{cn})\Delta t_c, \quad \hat{e}^+ = \hat{e} + (d_e + \omega\hat{V}_{ce})\Delta t_c \\ P^+ &= FPF^\top + \operatorname{diag}(q_p\delta\ell, q_p\delta\ell, q_b\Delta t_c, q_V\Delta t_c, q_V\Delta t_c) \end{aligned} \tag{13.9}$$

where (f, s) is the velocity the Doppler log reports, $\omega = 0$ on bottom track, which already contains the current, and $\omega = 1$ on water track, which does not, F is the identity plus $d_e\Delta t_c$ and $-d_n\Delta t_c$ in the position rows of the bias column and $\omega\Delta t_c$ in those of the current columns, and $\delta\ell$ is the distance moved in the step (plus $10^{-8}\,\mathrm{m}^2$ per step). With no Doppler velocity in the surface bubbles, f is the propeller speed $(T_{cmd}/-X_{u|u|})^{1/2}$, $s = 0$ and $\omega = 1$. The bias and the current start at zero with standard deviations $\sigma_b = 2^\circ$ and $\sigma_V = 0.1\,\mathrm{m/s}$. Dead reckoning is this prediction alone. On bottom track with a constant bias, the estimated velocity is the true one rotated by b, so the estimated track is the true one rotated by b about the last fix, and the error follows Equation (13.10).

$$|\hat{x} - x| = 2\sin(b/2)\, d \tag{13.10}$$

Here d is measured from the last fix rather than along the track flown, and $2\sin(0.5^\circ) = 1.745\,\%$. On water track with $\hat{V}_c = 0$ the error grows instead at the current's own speed.

The aided mode applies each USBL fix at the instant it describes. The filter keeps 240 steps (24 s) of its states, covariances and inputs, finds the newest stored step k_f at or before t_f, and updates there by Equation (13.11),

$$\begin{aligned} \nu &= z_{fix} - HX_{k_f}, \qquad S = HP_{k_f}H^\top + \sigma_R^2 I \\ d^2 &= \nu^\top S^{-1}\nu \le \gamma_g, \qquad K = P_{k_f}H^\top S^{-1} \\ X_{k_f} &\leftarrow X_{k_f} + K\nu \\ P_{k_f} &\leftarrow (I - KH)P_{k_f}(I - KH)^\top + \sigma_R^2 KK^\top \end{aligned} \tag{13.11}$$

in which H picks out the position and $\gamma_g = 9.21$ is the 99 % point of the chi-square law for two degrees of freedom. The filter then predicts again through every stored step since k_f with its stored inputs, so a late fix also corrects the bias and the current that those steps used. Of the three ways to fuse a delayed measurement set out in Section 1.4.7, which trade optimality against computation [5], this is the exact and costly one, affordable for five states. Volume 20, Chapter 10 takes a cheaper route for late camera reports and moves each one on by the belt travel since its capture. A fix with $d^2 > \gamma_g$ is refused. In the aided mode the Doppler log's own current, bottom track less water track rotated by $\hat{\psi}$ and averaged over ten samples, is applied once a second as an ungated measurement of $(\hat{V}_{cn}, \hat{V}_{ce})$ with a standard deviation of 0.02 m/s, and through the filter's correlations it also corrects the position that drifted while the current was unknown. The current is thus a slowly varying environmental state carried in the filter, as the bias state of the vessel observer in Volume 20, Chapter 6 carries wind and current. A GPS fix at the surface, in the GPS and aided modes, resets the position to the fix with a variance of $(1.5\,\mathrm{m})^2$, removes its correlations with the bias and the current, which keep what they learned, and clears the history, since no older fix can be applied after it. An advanced option applies each fix as if it described its arrival.

Table 13.3 Loops as Shipped, All at $\Delta t_c = 0.1\,\text{s}$

Loop	PV	MV	a	K_c	τ_I, τ_D, τ_{df}	output
HDG	$\hat{\psi}$	$\delta_{r,cmd}$	+1	1.2	60, 1.2, 0.2 s	±15°
SPD	u_m (lag 1 s)	T_{cmd}	+1	10	5 s, —, —	0–10 N
DEP	z_m	θ_{SP}	−1	8.6	40 s, —, —	±20°
PIT	θ_m	$\delta_{s,cmd}$	−1	3	∞, 1.0, 0.3 s	±15°

K_c in ° per °, N per m/s, ° of pitch per m, ° per °.

13.4.2 Line of Sight and the Current

The supervisor steers the heading toward a point a lookahead Δ ahead on the current leg, and it measures everything on the estimate, by Equation (13.12),

$$\begin{aligned} \psi_d &= \gamma - \arctan(\hat{e}_\perp/\Delta) - c_\beta\,\hat{\beta} \\ \hat{e}_\perp &= -(\hat{n} - n_A)\sin\gamma + (\hat{e} - e_A)\cos\gamma \end{aligned} \tag{13.12}$$

in which (n_A, e_A) is the start of the leg, γ its direction, $\Delta = 20\,\text{m}$ by default, and $\hat{e}_\perp$ is positive to starboard. The cross-track error is a geometric quantity, as in the lane keeping of Volume 20, Chapter 2, but here it is measured on the estimate. With crab compensation $c_\beta = 1$ and $\hat{\beta}$ is the drift angle of the velocity over the ground in the heading frame, $\text{atan2}(s_G, f_G)$ lagged by 2 s, taken from bottom track or from water track plus the current estimate and set to zero below $f_G = 0.2\,\text{m/s}$. Without compensation $c_\beta = 0$, and a steady cross current that needs a crab angle β, with $\sin\beta = V_\perp/U$, can only be met by sitting off the line until $\arctan(e_\perp/\Delta) = \beta$, a steady offset of $\Delta\tan\beta$ downstream. A leg ends when the estimate passes the perpendicular through its end.

13.4.3 The Four Loops

The four loops use the velocity-form PID of Section 1.6.2, Equations (1.15) to (1.18), at Δt_c and with no output bias. In each loop $a = \pm 1$ is the loop's direction and PV^f is the measurement through the exact first-order lag of time constant τ_{df}, so the derivative acts on the measurement. Back-calculation, with the tracking time equal to the integral time, runs only in the heading, speed and depth loops, which have an integral. The clamp acts on the emitted output, and no loop has a rate limit. In MANUAL each loop solves for $I_k = u_{op} - S_k$, which makes it emit exactly the operator's setting, while the pitch setpoint follows the measured pitch, so the return to AUTO is bumpless. The speed loop's integral starts at the trim thrust.

The pairings and tunings are listed in Table 13.3. The heading setpoint is unwrapped next to the estimated heading so that the error is taken the short way round. The speed loop acts on the water-track velocity lagged by 1 s. The depth loop acts in both vertical modes. At constant depth its setpoint is z_{SP}, and in terrain following it is the depth that puts the altitude setpoint over the seabed the altimeter sees, $z_{SP} = \overline{z_m + a_m} - a_{SP}$, the seabed depth lagged by 1 s and held when the altimeter has no reading. Its output, the pitch setpoint, is limited to $\pm 20°$, and because the pitch loop is proportional and derivative only, it holds about 17.7° against the righting moment when that setpoint is pinned at 20°.

13.4.4 The Mission Supervisor

The supervisor walks the plan on the estimated position. It dives from the launch point in depth mode toward the charted seabed at the start of line 1 less the altitude setpoint until the altimeter reads within 3 m of it, flies each line with the sonar logging, holds the depth the canyon leg began at across the canyon, and starts the final ascent where a climb at 17° reaches the surface at the recovery point. In the GPS mode it surfaces at the end of the first line to finish after the interval since the last fix, climbing in a 20 m loiter circle centered 25 m beyond the line's end, waits at the surface for the fix, descends to the altitude setpoint and rejoins the next line 30 m before its start. A battery at its

Table 13.4 Scenarios of the Survey Simulator and What Each Teaches

#	Scenario and settings	Teaching point
1	Manual, free flight at 32 m from line 1	a hull that will not fly straight
2	Lines 1 and 2 in still water	the loops, the turn, the ridge
3	0.4 m/s current toward 180°, lines 1 to 3	crab angle, LOS offset
4	DR, bias 1°, north block from launch	the pattern rotates
5	DR, 0.3 m/s toward 180°, line 5 to line 7	bottom lock lost
6	DR with GPS every 15 min, bias 1°	a reset and its cost
7	USBL, bias 1°, 50 m outlier at 400 s	delay, noise, gate
8	Lines 1 and 2 at 8 m altitude	the pitch limit
9	Full survey, DR, bias 1°, 0.25 m/s, 20 % battery	graded challenge

5 % reserve aborts the mission to the surface. On every line it also judges, from its own covariance, whether the overlap can still be trusted, as shown in Equation (13.13).

$$\begin{aligned} 1.96\,\sigma_{\perp} &> h(a_{plan}) - s/2 \\ \sigma_{\perp}^{2} &= \mathbf{g}^{\top} P_{pos}\,\mathbf{g}, \qquad \mathbf{g} = [-\sin\gamma,\ \cos\gamma]^{\top} \end{aligned} \tag{13.13}$$

where P_{pos} is the position block of the covariance and a_{plan} is the altitude setpoint, or the charted altitude at constant depth. At 10 m the right side is 5 m. When the test holds on a line inside a block, the supervisor records COVERAGE NOT ASSURED for it. The vehicle cannot see a holiday, but it can know when it may be leaving one.

13.4.5 Plant, Filter and Supervisor Models

Equations (13.1) to (13.8) are the simulation model, 14 states with the current's lag and the coverage grid beside them. The controller's internal models are the kinematics of Equation (13.9) with random-walk bias and current, the geometry of Equation (13.12), the drag law behind the propeller speed, the swath law and the charted seabed that the supervisor uses. None of them contains the hydrodynamics of Equation (13.1) or the Doppler scale error. No control model is identified. The tunings of Table 13.3 were set on the simulation model, and every result below is measured on it.

13.5 Measured Behavior of the Cases

Table 13.4 summarizes the eight cases and the graded survey. All of them begin from the configuration of Table 13.3, aided by the USBL with the gate on, in terrain following at 10 m and 1.5 m/s, and each row names what its case changes. The numbers below were measured headlessly at the default seed, stepping the engine as the browser does, and where one differs from the application's instructions guide the engine's value is the one printed. Each variant is set on the case as loaded, before Run, and the application gives the same run when the setting is carried through a Reset (Section 1.6.4). The application's own 41 checks, 24 of them specific to it, and its ten scenario-progress checks pass. Table 13.5 gathers the scorecard of every case.

13.5.1 A Hull That Will Not Fly Straight

Case 1 starts in MANUAL at 32 m depth, 28.8 m above the seabed, heading east at 1.5 m/s with the fins at zero and the thrust at its 3.645 N trim. Undisturbed, it flies straight for the 400 s tested, because nothing breaks the symmetry. Two degrees of rudder for 5 s build 12.1 °/s of yaw rate, and when the rudder is centered the turn does not stop. It settles at 9.85 °/s on a 7.1 m radius, with 5.0° of sideslip and the speed down to 1.22 m/s at the unchanged thrust, which is the unstable pole of Section 13.3 ending where crossflow drag balances the Munk moment. The stern plane behaves differently. A plane of $+5°$ holds the pitch at $-11.9°$ and the depth then grows at 0.31 m/s, and $+10°$ set at the start reaches the seabed 49 s later. Switched to AUTO, with setpoint tracking handing the autopilot the heading and depth the vehicle has, the heading loop stops the turn within 2.8 s, overshooting the captured heading by about 17°.

Table 13.5 Scorecard at the Default Seed. Navigation Error Is the Hidden Truth on the Survey Lines, and a Survey Ends When the Vehicle Surfaces

Case	Imaged (%)	Holidays, widest (m)	Lowest alt. (m)	Nav. error RMS, max (m)	End (s)	Miss (m)
2	22.01	0	6.41	0.56, 1.74	533	—
3	33.11	2, 6	5.95	0.52, 1.73	768	—
4	49.41	2, 7	5.44	6.25, 8.77	1329	—
5	25.69	1, 25	5.55	20.23, 21.56	698	—
6 on DR only	98.80	4, 7	5.44	7.75, 11.35	2574	9.7
6, GPS surfacing	99.61	3, 5	5.34	5.40, 8.75	3075	8.8
6 on USBL	99.96	0	5.58	0.57, 3.65	2574	0.7
7	99.95	0	5.61	0.56, 3.65	2575	0.9
7, gate off	99.94	0	5.61	1.51, 7.05	2577	0.5
7, fixes at arrival	99.90	1, 3	5.65	1.92, 3.65	2584	1.5
8	20.47	2, 5	4.40	0.54, 1.74	536	—
8, stronger depth loop	20.72	0	5.12	0.54, 1.74	536	—
8 at 1.0 m/s[a]	20.41	2, 9	3.77	0.47, 1.19	830	—

[a] Set before Run, so the case starts trimmed at 1.0 m/s.

13.5.2 The Overlap Budget: Autopilots and the Ridge

Case 2 flies lines 1 and 2 in still water on the USBL. The true cross-track error inside the block is 0.49 m RMS on line 1 and 0.46 m on line 2, the navigation error 0.56 m RMS, and 25 fixes are used while one honest fix is refused by the gate. The turn onto line 2 swings 6.0 m off the line inside its 20 m run-in and is 1.8 m off at the block's edge. Over the ridge the depth loop can climb only at its pitch limit, so the vehicle falls to 6.41 m above the seabed on line 2 and its swath narrows to 25.6 m. The neighboring line crosses the same place on the far side of its own climb, higher, so the two still overlap, but by 3 m at the crest instead of 10 m, and no holiday opens. The ridge alone spends 7 m of the overlap before any navigation error. The lookahead barely matters in still water on this agile hull, 0.62 m RMS on line 1 at 5 m and 0.40 m at 40 m.

Case 8 flies the same lines at 8 m, where the swath is 32 m and each side has only 1 m to spare, so the supervisor records COVERAGE NOT ASSURED on both lines before anything has gone wrong. The face that line 2 climbs rises at up to 23.1° along the planned line, while the pitch loop, its setpoint pinned at 20° for 21 s of that climb, holds a median of 17.7° with the planes near −6.7°. The seabed gains on the vehicle, which falls to 4.40 m above it near the crest with a swath of 17.6 m, and two holidays open over the ridge, the widest 5 m. A stronger depth loop, 14.3° of pitch per meter with a 30 s integral time, starts the climb sooner and holds 5.12 m with no holiday, and flying at 10 m holds 6.41 m. Slowing to 1.0 m/s makes it worse, 3.77 m and a 9 m holiday, because the planes run near −13.5° of their 15° and hold a median of only 15.5° of pitch, while the climb is limited by its angle and not by its speed. No gain can climb faster than the pitch limit allows. What the loop decides is how early the climb starts.

13.5.3 A Current Across the Lines

In case 3 a 0.4 m/s current flows south across the east–west lines. The vehicle crabs 15.5° into it on every line, the arcsine of 0.4 over 1.5, and its navigation stays at 0.52 m RMS because bottom track measures the velocity over the ground. Pure line of sight commands the heading, so it leads the line by the crab angle only by sitting downstream of it. The mean offsets inside the block are 5.4, 5.0 and 5.1 m on the three lines at $\Delta = 20\,\text{m}$, against $20\tan 15.5^\circ = 5.5\,\text{m}$, always to the south whichever way the line is flown. The whole pattern slides south together, so the gap opens at the block's upstream edge, two holidays along the north edge, the widest 6 m. A 10 m lookahead halves the offsets to 2.8, 2.5 and 2.6 m with no holiday, and 40 m doubles them to about 10 m and opens a 12 m holiday. Crab compensation, steering the course over the ground that the Doppler log measures, brings them to 1.6, 0.4 and 0.2 m with no holiday, the last two slightly upstream. Line 1 keeps 1.6 m because the setting is in force when the heading loop is primed, so the loop's integral starts by cancelling the crab

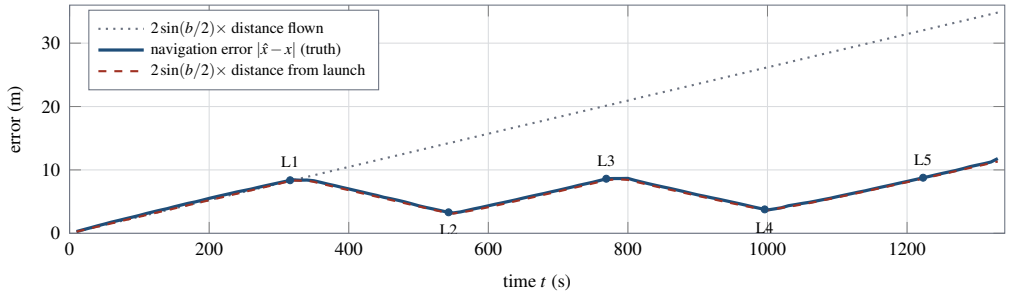


Figure 13.3 Dead-reckoning error in case 4 with a 1° compass bias. The engine's error (thick solid) follows $2\sin(b/2)$ times the distance from the launch point (dashed) and not the distance flown (dotted), and the dots mark the ends of lines 1 to 5.

command and unwinds it over about a minute. Crab compensation is the same remedy that integral and sideslip-compensating line-of-sight laws provide in theory [7, 8].

> **TRY IT IN THE APP** **3 · A Cross Current**
>
> Load the scenario, press Run and watch the trend across the line. The hidden truth sits about 5 m south of every line while the estimate holds it, and two holidays open along the north edge, the widest 6 m. Then set Crab compensation to On, press Reset and run again. The offsets fall to 1.6, 0.4 and 0.2 m with no holiday.

13.5.4 A Compass Bias Rotates the Survey

Case 4 dives from the launch point and flies the north block on dead reckoning with the compass 1° high. Equation (13.10) predicts the error from the distance to the launch point, and the engine follows it to within 0.32 m until the vehicle surfaces at the end, 0.18 m on average, as shown in Figure 13.3. The error is 8.37 m at the east end of line 1, 471 m from the launch point (1.78 %), 3.31 m at the west end of line 2, 182 m away after 814 m flown (1.82 %), 8.61 m at the east end of line 3 and 3.77 m at the west end of line 4 after 1495 m flown. The naive 1.7 % of the distance flown would have given 26 m there, seven times the error measured. A lawnmower pattern is known to cancel low-frequency Doppler errors that drift linearly on a straight line [15], and a constant heading bias cancels the same way here, each line undoing much of the one before.

Pause. Try Exercise 13.1 before reading on.

A rotation keeps the lines parallel and evenly spaced, so no gap opens between them. What it moves is the block's edge. The lines sit about 6 m north of their plan in the middle of the block and 8 m at its east end, and the south edge is left unimaged in two holidays, 7 m wide where the ridge also narrows line 5's swath and 5 m wide at the east end. The supervisor records COVERAGE NOT ASSURED 129 s into the dive, 193 m from the launch point, when its own 95 % uncertainty across the line passes the 5 m of spare overlap, and it records the same flag at the same instant with no bias at all, when dead reckoning stays within 0.24 m RMS, because the filter allows 2° of bias that nothing aboard can rule out. The flag reports what can be vouched for rather than what went wrong. At 2° of bias the error is 12.3 m RMS with a 12 m holiday, and on the USBL it is 0.73 m with none.

13.5.5 The Canyon: A Drift Nothing Aboard Can See

Case 5 finishes line 5 on dead reckoning and crosses the canyon in a 0.3 m/s current flowing south. The supervisor holds 49.8 m across the canyon, the altitude reaches 105.7 m, beyond the altimeter's 50 m as well as the Doppler log's 30 m, and bottom lock is lost for 71.8 s. On water track the current is not in the reckoning, and the error grows from 0.05 to 21.46 m, at 0.298 m/s, and Figure 13.4 shows the run on line 7 at 420 s. Every line of the southern block is then flown about 21 m south of

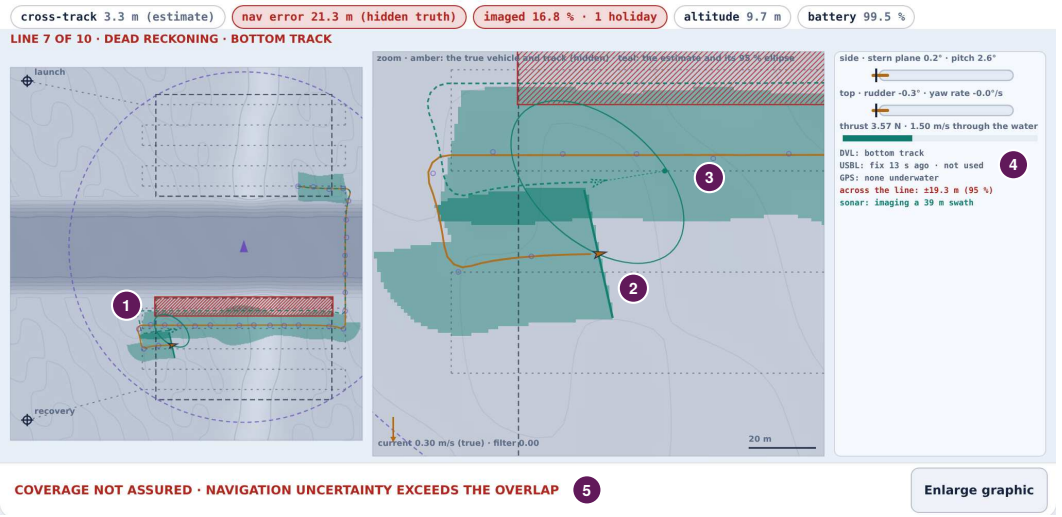


Figure 13.4 Case 5, The canyon, at $t = 7.00$ min (420 s) with no control changed, 34 s after the vehicle turned onto line 7 on dead reckoning, cropped to the Chart view (the application header, the trends, the control panel and the Learn panel are cut). The header line reads line 7 of 10, dead reckoning, bottom track. The survey chart shows the launch and recovery points, the two dashed blocks with their dotted lines, the canyon band with the ship over it inside the dashed USBL circle at the vehicle's depth, the seabed imaged at the east end of line 5, along line 6 and at the start of line 7, and a holiday (1) hatched along the whole north edge of the southern block. The zoom, centered on the true vehicle (2, amber, the hidden truth), shows it south of line 7 while the estimate (3, teal) lies near the line inside its 95 % ellipse, with the line-of-sight point ahead, the 39 m swath across the true vehicle and the unused USBL fixes as open circles. The badges give the cross-track error of the estimate (3.3 m), a navigation error of 21.3 m (hidden truth), 16.8 % imaged with one holiday, an altitude of 9.7 m and 99.5 % of the battery, the inset (4) reports bottom track, a USBL fix 13 s old and not used, and ±19.3 m across the line at 95 %, and the status line (5) reads COVERAGE NOT ASSURED.

where the vehicle believes it is, and the block's north edge is left unimaged in one holiday 25 m wide and 260 m long. The filter's own uncertainty grows honestly across the crossing, and the supervisor records COVERAGE NOT ASSURED on lines 6 and 7.

> **TRY IT IN THE APP** **5 · The Canyon**
>
> Load the scenario, press Run and watch the navigation error trend as the vehicle crosses the canyon. Bottom lock is lost for 71.8 s, and the error climbs to about 21 m while the estimated cross-track error stays near zero. A holiday 25 m wide then opens along the north edge of the southern block.

This is the case that no tuning can recover, and the simulator says so. A lookahead of 10 m leaves a 23 m holiday, 40 m leaves 28 m, and crab compensation leaves 21 m, because without a fix or a current measurement the drift is unobservable (Section 1.4.6), and the application's own checks confirm that every one of these variants is flagged. In still water the same crossing costs nothing, and on the USBL the navigation holds 0.41 m RMS through it, leaving only the current's line-of-sight offset as one 4 m holiday. The remedy is a measurement, a fix from outside or the current measured while the seabed was still in view, the combination that model-aided navigation with sea current estimation was built for [14].

Pause. Try Exercise 13.2 before reading on.

13.5.6 Buying a Fix: Surfacing Against Acoustic Aiding

Case 6 flies the whole survey on dead reckoning with the 1° bias and surfaces for GPS at the end of the first line after each 15 min. The first surfacing follows line 4, at 996 s. Line 7 ends 900.15 s after the last fix of that surfacing, so the second follows line 7, at 2027 s, and a line 7 that ended a fraction of a second sooner would have sent the vehicle on to line 8. Each surfacing climbs from about 50 m in 105 and 107 s in its loiter circle, waits 20 s for the fix, descends and rejoins the next line, which takes 256 s and 3.45 Wh from the end of one line to the start of the next the first time and 257 s and 3.42 Wh the second. The fixes leave the estimate 1.65 and 0.76 m from the truth, against 3.77 and 8.45 m at the ends of lines 4 and 7, but the bias survives them and the pattern goes on rotating, now about each surfacing point. A fix taken between two neighboring lines also changes their spacing. Over the ridge, line 7 runs 5.0 m north of its plan and line 8, flown after the fix, 2.3 m south of its own, and a 5 m holiday opens between them. On the same seed, dead reckoning alone images 98.80 % with four holidays and surfaces 9.7 m from the recovery point at 2574 s on 33.67 Wh, the surfacings image 99.61 % with three holidays, an 8.8 m miss, 3075 s and 40.33 Wh, and USBL aiding images 99.96 % with none, a 0.7 m miss, at 2574 s. A surfacing bounds the error at the fix and corrects nothing that caused it, and in a 0.2 m/s current the two surfacings do not reach the canyon's drift, which leaves a 27 m holiday and a 29.6 m miss.

Case 7 flies the same survey aided by the USBL. The fixes arrive 1.2 to 1.4 s after the instant they describe, with standard deviations of 0.48 to 1.32 m, and 38 of the 172 interrogations bring nothing back. Of those, 14 are lost pings and 24 find the vehicle outside the 80° cone, during the dive, during the final ascent, and on the lines farthest from the ship where the vehicle rises over the ridge. Fixes taken on lines flown east and west see the bias from opposite sides, which makes it observable in the sense of Section 1.4.6, and its estimate reaches 1.005° by 300 s. With the bias removed from the heading the survey images 99.95 % with no holiday, a navigation error of 0.56 m RMS, and surfaces 0.9 m from the recovery point. The multipath fix that arrives at 407 s lies 50 m off at a chi-square distance of 2382 against 9.21 and is refused, and with the gate off it is accepted and the error jumps to 7.05 m before the next fixes pull it back. Applying every fix as if it described its arrival compares it with a position about 1.3 s of travel, some 2 m, further along the line, the gate then refuses 47 honest fixes besides the outlier, and the error RMS more than triples, to 1.92 m. The supervisor records COVERAGE NOT ASSURED once even here, on line 1 at 129 s, because the vehicle is outside the transceiver's cone until 116.5 s and the first fix arrives only at 137 s.

> **+ DID YOU KNOW? About a Second Late, Three Times the Error**
>
> From the simulator: in case 7 every acoustic fix describes an instant 1.2 to 1.4 s before it arrives. Applied at that instant, the fixes hold the navigation error to 0.56 m RMS with no holiday. Applied as if they described their arrival, the same fixes give 1.92 m and a 3 m holiday (Table 13.5).

13.5.7 The Graded Survey

The challenge flies the whole survey launched with 20 % of the battery, in a 0.25 m/s current toward 190°, with the compass 1° high and 3 % of the USBL fixes wild. The target is at least 95 % imaged, no holiday wider than 2 m, never below 3 m above the seabed, surfacing within 20 m of the recovery point and at least 15 % of the battery left, judged when the vehicle surfaces, with the model parameters and disturbances as given. A changed parameter or disturbance voids the attempt. It ships on dead reckoning with a 40 m lookahead, no crab compensation and a soft vertical pair, 3.4° of pitch per meter with a 60 s integral time and a pitch gain of 1.5. Table 13.6 compares the configurations, and Figure 13.5 draws three of its columns against their targets.

As shipped the survey images 83.86 % with nine holidays, the widest 42 m, comes within 1.72 m of the seabed over the ridge and surfaces at 2559 s, 61.3 m from the recovery point. It fails four of the five criteria and meets only the battery, with 16.67 % left. The suggested configuration images 99.95 % with no holiday, never comes closer than 5.50 m, and surfaces 1.9 m from the recovery point

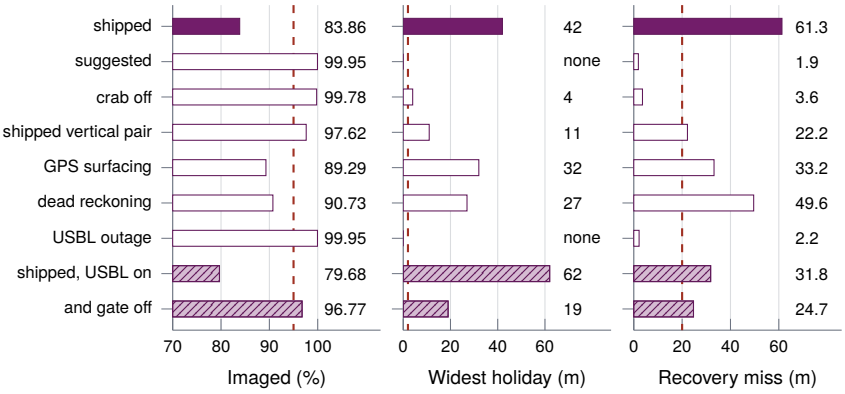


Figure 13.5 The graded survey of Table 13.6: the seabed imaged (left, dashed line at the 95 % target), the widest holiday (middle, dashed line at 2 m) and the miss at the recovery point (right, dashed line at 20 m). The dark bar is the shipped configuration, white bars the suggested one and its single changes, and hatched bars the shipped configuration with the USBL switched on. Notice that switching on the USBL alone images less than dead reckoning did, while the suggested configuration and its USBL outage are the only rows inside all three lines.

Table 13.6 Graded Survey at the Default Seed, With Single Changes From the Shipped and the Suggested Configurations

Configuration	Imaged (%)	Holidays, widest (m)	Lowest (m)	Miss (m)	Battery (%)	Fixes[b]
shipped	83.86	9, 42	1.72	61.3	16.67	—
suggested[a]	99.95	0	5.50	1.9	16.62	125, 7
crab off	99.78	1, 4	5.23	3.6	16.66	123, 3
shipped vertical pair	97.62	12, 11	2.22	22.2	16.61	130, 4
GPS surfacing	89.29	3, 32	5.08	33.2	15.95	—
dead reckoning	90.73	4, 27	4.77	49.6	16.61	—
USBL outage[c]	99.95	0	5.50	2.2	16.62	105, 8
shipped, USBL on	79.68	11, 62	1.11	31.8	16.66	1, 132
and gate off	96.77	13, 19	1.92	24.7	16.68	125, 0

[a] USBL with the gate, $\Delta = 20$ m with crab compensation, 8.6°/m and 40 s, pitch gain 3. [b] Used, refused. [c] From 1150 to 1450 s.

at 2577 s with 16.62 % left. It is inside every limit, the battery by the smallest margin, 1.62 percentage points, and it passes on each of twelve further seeds as well. Each part has an owner. Without crab compensation one 4 m holiday remains, with the shipped vertical pair the vehicle comes to 2.22 m above the ridge and twelve holidays open, GPS surfacing in place of the USBL leaves a 32 m holiday and takes 3078 s, and on dead reckoning the canyon's drift leaves a 27 m holiday whatever else is set, with COVERAGE NOT ASSURED on every line, which is the infeasible variant. The aided filter survives a USBL outage across the whole canyon leg, from 1150 to 1450 s, with no holiday, because it measured the current while it still had bottom lock and had learned the bias from the fixes before.

The most natural first change fails in a way the other cases do not teach. Switching only the navigation of the shipped configuration to the USBL images 79.68 %, less than dead reckoning did, with a 62 m holiday. The first fix to arrive, at 137 s, is one of the wild ones, 24.2 m from the truth, while the filter's own standard deviation across the line has grown to 7.17 m, most of it from the 2° of bias it allows over the dive, so the fix's chi-square distance is 8.04 and the gate accepts it. The filter explains most of it as compass bias, $\hat{b} = -5.41°$ against a true 1°, and its position standard deviation falls to 1.2 m. Every one of the next 132 fixes, 129 of them honest, then lies at a distance of 21 to 479 from a confident and wrong prediction and is refused, and the navigation error reaches 73.4 m.

With the gate off the same configuration uses 125 fixes and images 96.77 %. On twelve other seeds the same change refuses no more than seven fixes and images 96.55 to 96.75 %, still failing for the guidance and the vertical loops. A validation gate is only as tight as the covariance behind it [6], and the first fix of a mission meets it at its widest.

Pause. Try Exercise 13.3 before reading on.

13.6 Technology in Practice

What Measures It

Depth, heading, pitch and roll each have a direct sensor, but no equivalent sensor exists for the two horizontal coordinates [3], so the position is reckoned from a Doppler velocity log and a heading reference and corrected by acoustic fixes, and no instrument aboard measures it. A Doppler log sends four acoustic beams, taken in one analysis as a quarter turn apart around the vertical with the first usually 45° off the forward direction, and turns the Doppler shifts of their echoes into the velocity in the vehicle's frame. Its scale is proportional to the sound speed at the transducer [15]. Bottom-lock logs measure the velocity over the seabed to 0.3 % or better at up to 5 Hz, from altitudes of about 18 to 100 m that vary with the log [3], and a higher frequency buys accuracy at the cost of range [15]. A calibrated magnetic compass gives the heading to the order of 1 to 3° [3], the range of the chapter's compass bias. The chapter's log locks on the bottom within 30 m and falls back to water track above it, which is where the canyon's drift begins.

TECHNOLOGY **Sensor: The Ship's Ultra-Short Baseline Fix**

An ultra-short baseline system needs only a transducer on the hull of the ship. A sonar array in it measures the range and the bearing to the vehicle, and the system needs an alignment calibration with the ship's own positioning system, typically GPS. Every acoustic time-of-flight method needs an accurate sound velocity and is limited by the speed of sound in water, about 1500 m/s, and vehicles deep below the ship are especially challenged, because the sound velocity varies with the temperature and density of the water [3]. The chapter's fix has a noise of 0.5 % of the slant range, reaches the vehicle after the two-way travel and 1.0 s of processing, and fails as a lost ping, outside the 80° cone or as an outlier that the gate must refuse.

What Moves It

The vehicle has three final elements. The rudder and the stern plane each sit behind a servo modeled as a 0.1 s lag and limited to 15°, and the thruster sits behind a 0.4 s motor lag and is limited to 0 to 10 N. The sources opened for this section give no fin or thruster data for a real survey vehicle, so these limits are the model's own.

IN PRACTICE **How a Survey Is Judged on Its Gaps**

A United States hydrographic specification defines a holiday as a gap in gridded data larger than the features the survey claims to detect. On its own surveys it allows no data gap of 3 by 3 grid nodes or more, 3 m by 3 m on a 1 m grid, and the cause of any such gap must be explained in the report of survey. A towed side scan sonar must fly at 8 to 20 % of its range scale above the bottom, and when conditions shorten its effective range the coverage claimed should shrink with it [18]. The international standard asks for 100 % bathymetric coverage in its Special Order and 200 % in its Exclusive Order [19].

Where the Controller Runs

One computer on the vehicle runs the navigation filter, the mission supervisor and the four autopilot loops. The sources opened give no scan period for the vehicle's computer. The chapter's controller reads the instruments, runs the filter and the supervisor and executes the four loops every 0.1 s, and the ship interrogates the transponder every 15 s. Figure 13.6 draws the loops on that hardware.

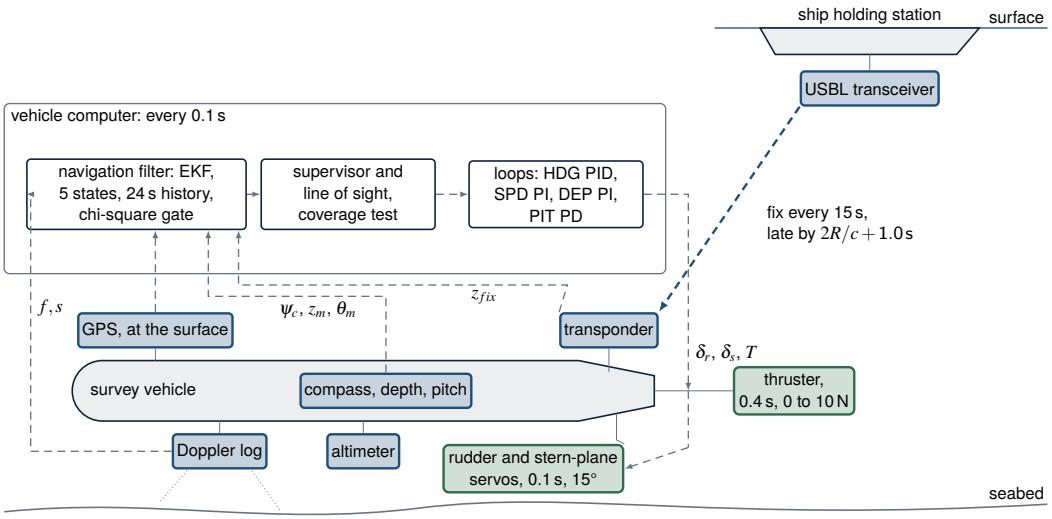


Figure 13.6 The survey loops on vehicle hardware. The Doppler log, the compass and the depth and pitch sensors feed the navigation filter, which also takes GPS at the surface and the ship's USBL fixes. The altimeter feeds the depth loop in terrain following (line not drawn). One computer runs the filter, the supervisor and the four loops every 0.1 s, and the servos and the thruster move the rudder, the stern plane and the propeller. Notice that each USBL fix reaches the vehicle 1.2 to 1.4 s after the instant it describes, and that no line carries the true position: the coverage the survey exists for is never measured aboard.

TECHNOLOGY Platform: The Navigation Filter and Its Clock

On a survey vehicle the inertial navigation system computes position, velocity and attitude from high-rate inertial data, and an extended Kalman filter on an error-state model corrects it with the Doppler log, the pressure sensor, the compass and any position fixes. Every measurement must carry the time it describes. When the filter compares a Doppler velocity with its own, a time-stamp error times the vehicle's acceleration becomes a velocity error, and an analysis of the dynamics during surface fixes asked for Doppler time stamps better than 10 ms, preferably with the log reporting its own latency in each message [15]. The chapter's filter keeps 24 s of history so that each fix is applied at the instant it describes.

What Is Modernizing It

The first development is the heading. Heading accuracy is a principal error source of Doppler navigation, and low-cost north-seeking optical gyrocompasses, with a dynamic heading accuracy of the order of 0.1°, effectively ameliorate it [3], and by Equation (13.10) a smaller bias rotates the pattern less. The second is ranging. Acoustic modems with synchronized clocks allow ranging by one-way travel time, in which a ship broadcasts its GPS position and every vehicle in range computes a running fix, while a long-baseline net of seafloor transponders typically reaches up to 10 km with a precision of 0.1 to 10 m [3]. The third is the seabed itself. Terrain-relative navigation and simultaneous localization and mapping correct the position from the vehicle's own bathymetric sonar or cameras against a map, given in advance or built as the vehicle goes, within a sensor range of the order of 10 to 100 m for a sonar [3]. Over the canyon the vehicle flies 105.7 m above the floor.

13.7 Limitations

The vehicle is Prestero's REMUS set with constant coefficients, no heave and no roll, so it teaches the structure of an underactuated, yaw-unstable hull rather than the handling of a particular vehicle,

and its turns do not bank the swath. The current is uniform and unsheared, so water track measures exactly the velocity through the water and the current it reveals is the current at the vehicle.

The seabed is smooth and known exactly to the supervisor, the swath is taken from the altitude at nadir over a locally flat seabed, and the sonar's image quality, nadir gap and resolution are not modeled. The altitude loop sees only the altimeter straight below, with no preview of the seabed ahead. The instruments are generic, the compass bias is constant, and the Doppler scale error is outside the filter.

> **OPEN CHALLENGE** **Navigating Far From Seabed and Surface**
>
> GPS has solved navigation at the surface, and Doppler sonars and landmark navigation have greatly improved it near the seabed. In the mid-depth zone, far from both, only acoustic time-of-flight systems and inertial units measure the horizontal position, which limits how precisely a vehicle can navigate there. A survey of the field expected the advances to come from model-based state estimators, helped by vehicles working near the surface or the seabed at the same time [3]. The canyon of case 5 is a small mid-depth zone, where the chapter's filter loses the current. Progress would be an estimate of the current or the position that holds between the walls.

The filter's covariance is honest only for the errors it models, and nothing in it detects a run of refused fixes or recovers from one, so the lockout of Section 13.5.7 persists to the end of the survey. At the surface the filter reckons on the propeller speed through the steady drag law, which undercounts while the hull coasts through a change of speed. Every threshold the supervisor and the scorecard use is a teaching value.

> **OPEN CHALLENGE** **Navigation and the Survey as One Problem**
>
> Ocean engineers have traditionally worked on determining the position in the water and flown trajectories defined in advance, which suits bathymetry. A survey of the field described studies that analyze vehicle navigation, trajectory planning and the data of the science sensors together, to learn how each design choice affects the accuracy of the mapped process, and noted that observability, uniqueness and lower bounds for such systems were only beginning to be studied [3]. This chapter scores navigation and guidance on the seabed imaged, but on a fixed plan. Progress would be a vehicle that replans its lines from its own covariance to cover the strips it may have missed.

13.8 Conclusion

This chapter has treated an AUV survey as a control problem whose output is never measured. The simulator keeps the true vehicle, its instruments and its filter apart so that the difference can be scored on the seabed. Prestero's coefficients leave the hull unstable in yaw, with a pole at $0.995\ \mathrm{s}^{-1}$, so the heading loop is part of the vehicle. The ridge alone spends the 10 m of overlap down to 3 m, and a 0.4 m/s current holds pure line of sight 5.4 m downstream until crab compensation removes the offset. A 1° compass bias rotates the dead-reckoned survey about the launch point, 1.78 % of the distance from the fix rather than of the distance flown, and over the canyon a 0.3 m/s current carries the vehicle 21.46 m from its estimate and leaves a 25 m holiday that no tuning recovers. GPS surfacing costs about 256 s and 3.4 Wh a time and leaves the bias in place, while USBL aiding learns it and images 99.95 %. The graded survey rises from 83.86 % imaged with nine holidays to 99.95 % with none on thirteen seeds of thirteen, although switching the shipped navigation to the USBL alone locks the gate out after one wild fix. The model omits heave, roll, current shear and the sonar's image. The natural additions are a lockout monitor in the filter, a forward-looking altimeter, a sheared current and a long-baseline net. The survey simulator is a laboratory for the difference between where a vehicle was and where it believed it was.

KEY POINTS

- A survey vehicle steers its own estimate of where it is, while the seabed its sonar images is set by where it actually went.
- A 1° compass bias rotates the dead-reckoned survey about the launch point, 1.78 % of the distance from the fix rather than of the distance flown.
- Over the canyon a 0.3 m/s current carries the vehicle 21.46 m from its estimate and leaves a 25 m holiday that no tuning recovers.
- USBL aiding learns the bias and images 99.95 %, but one wild first fix can lock the gate out.

Where else this applies. Volume 20, Chapter 6 carries wind and current as a slowly varying bias state of its observer, and Volume 20, Chapter 10 moves each late camera report on by the belt travel since its capture.

References

[1] R. B. Wynn, V. A. I. Huvenne, T. P. Le Bas, B. J. Murton, D. P. Connelly, B. J. Bett, H. A. Ruhl, K. J. Morris, J. Peakall, D. R. Parsons, E. J. Sumner, S. E. Darby, R. M. Dorrell, and J. E. Hunt, "Autonomous underwater vehicles (AUVs): Their past, present and future contributions to the advancement of marine geoscience," *Marine Geology*, vol. 352, pp. 451–468, 2014.

[2] L. Paull, S. Saeedi, M. Seto, and H. Li, "AUV navigation and localization: A review," *IEEE Journal of Oceanic Engineering*, vol. 39, no. 1, pp. 131–149, 2014.

[3] J. C. Kinsey, R. M. Eustice, and L. L. Whitcomb, "A survey of underwater vehicle navigation: Recent advances and new challenges," https://robots.engin.umich.edu/publications/jkinsey-2006a.pdf, 2006.

[4] E. Galceran and M. Carreras, "A survey on coverage path planning for robotics," *Robotics and Autonomous Systems*, vol. 61, no. 12, pp. 1258–1276, 2013.

[5] T. D. Larsen, N. A. Andersen, O. Ravn, and N. K. Poulsen, "Incorporation of time delayed measurements in a discrete-time Kalman filter," in *Proceedings of the 37th IEEE Conference on Decision and Control (Cat. No.98CH36171)*, vol. 4. IEEE, 1998, pp. 3972–3977.

[6] Y. Bar-Shalom, X.-R. Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation*. Wiley, 2002.

[7] E. Borhaug, A. Pavlov, and K. Y. Pettersen, "Integral LOS control for path following of underactuated marine surface vessels in the presence of constant ocean currents," in *2008 47th IEEE Conference on Decision and Control*. IEEE, 2008, pp. 4984–4991.

[8] T. I. Fossen, K. Y. Pettersen, and R. Galeazzi, "Line-of-sight path following for Dubins paths with adaptive sideslip compensation of drift forces," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 2, pp. 820–827, 2015.

[9] R. Spindel, R. Porter, W. Marquet, and J. Durham, "A high-resolution pulse-Doppler underwater acoustic navigation system," *IEEE Journal of Oceanic Engineering*, vol. 1, no. 1, pp. 6–13, 1976.

[10] K. Vickery, "Acoustic positioning systems. A practical overview of current systems," in *Proceedings of the 1998 Workshop on Autonomous Underwater Vehicles (Cat. No.98CH36290)*. IEEE, 1998, pp. 5–17.

[11] L. Whitcomb, D. Yoerger, and H. Singh, "Advances in Doppler-based navigation of underwater robotic vehicles," in *Proceedings 1999 IEEE International Conference on Robotics and Automation (Cat. No.99CH36288C)*, vol. 1. IEEE, 1999, pp. 399–406.

[12] B. Allen, R. Stokey, T. Austin, N. Forrester, R. Goldsborough, M. Purcell, and C. von Alt, "REMUS: a small, low cost AUV; system description, field trials and performance results," in *Oceans '97. MTS/IEEE Conference Proceedings*, vol. 2. IEEE, 1997, pp. 994–1000.

[13] T. Prestero, "Development of a six-degree of freedom simulation model for the REMUS autonomous underwater vehicle," in *MTS/IEEE Oceans 2001. An Ocean Odyssey. Conference Proceedings (IEEE Cat. No.01CH37295)*, vol. 1. IEEE, 2001, pp. 450–455.

[14] Ø. Hegrenaes and O. Hallingstad, "Model-aided INS with sea current estimation for robust underwater navigation," *IEEE Journal of Oceanic Engineering*, vol. 36, no. 2, pp. 316–337, 2011.

[15] B. Jalving, K. Gade, K. Svartveit, A. Willumsen, and R. Sørhagen, "DVL velocity aiding in the HUGIN 1000 integrated inertial navigation system," *Modeling, Identification and Control: A Norwegian Research Bulletin*, vol. 25, no. 4, pp. 223–236, 2004.

[16] A. J. Healey and D. Lienard, "Multivariable sliding mode control for autonomous diving and steering of unmanned underwater vehicles," *IEEE Journal of Oceanic Engineering*, vol. 18, no. 3, pp. 327–339, 1993.

[17] T. I. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control*. Wiley, 2011.

[18] NOAA Office of Coast Survey, *Hydrographic Survey Specifications and Deliverables, Version 2026.0.03*, National Oceanic and Atmospheric Administration, 2026. [Online]. Available: https://nauticalcharts.noaa.gov/publications/documents/HSSD_2026-0-03.pdf

[19] International Hydrographic Organization, *IHO Standards for Hydrographic Surveys, S-44 Edition 6.1.0*, International Hydrographic Organization, Monaco, 2022. [Online]. Available: https://iho.int/uploads/user/pubs/standards/s-44/S-44_Edition_6.1.0.pdf

13.9 Exercises

13.1 Concept check · 2 min

In case 4 the vehicle dead reckons the north block with its compass 1° high. After 1495 m flown, at the west end of line 4, the navigation error is only 3.77 m, not the 26 m that 1.745 % of the distance flown would give. Why?

(a) The bias rotates the estimated track about the last fix, so the error grows with the distance from the fix, not the distance flown

(b) The filter learns the bias from bottom track on each line, so the error it leaves shrinks from one line to the next

(c) The Doppler log's scale error reverses with the direction of each line, so its share of the error cancels in pairs

(d) The supervisor's COVERAGE NOT ASSURED flag resets the estimate at the end of each line to the charted position

13.2 Concept check · 1 min

In case 5 the vehicle crosses the canyon on dead reckoning in a 0.3 m/s current and then flies lines 6 and 7 about 21 m south of where it believes it is. Why does neither a 10 m lookahead nor crab compensation recover the 25 m holiday?

(a) The lookahead and the crab angle act only once the vehicle reaches the south block, too late to correct line 6

(b) The guidance steers an estimate that is itself 21 m wrong, and nothing aboard can observe that drift without a fix

(c) The depth loop holds the vehicle too high over the canyon walls for the sonar to image the block's north edge

(d) The heading loop saturates its rudder in the turn onto line 6 and loses the line for the rest of the case

13.3 Concept check · 2 min

In the graded survey, switching only the shipped navigation to the USBL images 79.68 %, less than dead reckoning. The Fixes tile ends at 1 used and 132 refused. What went wrong?

(a) Each fix arrives about 1.3 s late, so the gate compares it with a position 2 m behind the vehicle and refuses it

(b) The gate's threshold of 9.21 is too tight for honest fixes at the long ranges of the south block

(c) A wild first fix passes the gate while the covariance is wide, and the confident, wrong filter then refuses the honest fixes

(d) The vehicle stays outside the transceiver's 80° cone for most of the survey, so almost no fix returns

13.4 Theory and calculation · 6 min

Use Equations (13.6) and (13.13) with $\tan\theta_s = 2$, $R_s = 50\,\text{m}$ and lines $s = 30\,\text{m}$ apart.

(a) Compute the swath, the overlap between neighboring lines and the spare overlap $h - s/2$ on each side at altitudes of 10 and 8 m. *Checkpoint:* 40 m, 10 m and 5 m at 10 m. 32 m, 2 m and 1 m at 8 m.

(b) Find the altitude at which the swath is widest, and its width. *Checkpoint:* 22.4 m and 89.4 m.

(c) Find the standard deviation across the line $\sigma_\perp$ above which the supervisor records COVERAGE NOT ASSURED at each altitude of part (a), and say why case 8 is flagged before anything has gone wrong. *Checkpoint:* About 2.55 m at 10 m and 0.51 m at 8 m.

13.5 Theory and calculation · 7 min

Use Equation (13.10) and the case 4 and case 5 results of Section 13.5.

(a) Compute $2\sin(b/2)$ for a bias of 1°, and predict the error at the east end of line 1, 471 m from the launch point, and at the west end of line 2, 182 m from it. Compare with the measured 8.37 and 3.31 m. *Checkpoint:* 1.745 %, about 8.2 m and 3.2 m.

(b) Compute the error that 1.745 % of the 1495 m flown to the end of line 4 would predict, and its ratio to the 3.77 m measured. *Checkpoint:* About 26.1 m, 6.9 times the measured error.

(c) In case 5 bottom lock is lost for 71.8 s in a 0.3 m/s current and the filter's current estimate is zero. Predict the error added over the crossing and compare it with the measured growth. *Checkpoint:* About 21.5 m, against 0.05 to 21.46 m.

13.6 Theory and calculation · 6 min

Line-of-sight guidance without crab compensation meets a steady cross current $V_\perp$ only by sitting off the line (Equation (13.12)). The vehicle moves at $U = 1.5\,\text{m/s}$ through the water.

(a) For the 0.4 m/s current of case 3, compute the crab angle and the steady offset at lookaheads of 10, 20 and 40 m. Compare with the measured mean offsets of 2.8, 5.4 and about 10 m on line 1. *Checkpoint:* 15.5°, and 2.8, 5.5 and 11.1 m.

(b) The graded survey runs in 0.25 m/s toward 190° with a 40 m lookahead as shipped. Find the current across the east–west lines, the crab angle and the steady offset, and compare it with the 5 m of spare overlap at 10 m. *Checkpoint:* About 0.246 m/s, 9.4° and 6.7 m, more than the spare.

(c) Explain why in case 3 the gap opens at the north edge of the block, whichever way a line is flown. *Checkpoint:* The offset is always downstream, south, so the whole pattern slides south together.

13.7 App challenge · 8 min

Load scenario 4, Dead reckoning with a compass bias, with the default seed. Run at 20× until the vehicle surfaces and the run pauses, at about 1331 s on the clock, and read the Seabed imaged, Holidays and Navigation error tiles. Reload the scenario before each new configuration and make the change before pressing Run.

(a) Run as loaded. Report the three tiles. *Checkpoint:* 49.4 %, 2 holidays with the widest 7.0 m, and 6.2 m RMS with 8.8 m at most, each within one unit of the last digit.

(b) Set the Compass bias to 2° and run. Report the same tiles. *Checkpoint:* 48.0 %, 2 holidays with the widest 12.0 m, and 12.3 m RMS with 17.3 m at most, each within one unit of the last digit.

(c) Set the Navigation to USBL, with the bias at 1°, and run. Report the same tiles and the Fixes tile, and explain why the error falls by a factor of about nine. *Checkpoint:* 50.0 %, no holiday, 0.7 m RMS with 3.7 m at most, 64 fixes used and none refused, each within one unit of the last digit.

13.8 App challenge · 7 min

Load scenario 5, The canyon, with the default seed. Run at 20× until the vehicle surfaces and the run pauses, at about 700 s on the clock. Read the Holidays, Navigation error and Fixes tiles, and watch the status line while lines 6 and 7 are flown. Reload the scenario before each new configuration and make the change before pressing Run.

(a) Run as loaded, and watch the navigation error strip while the Doppler log is on water track. Report the tiles and what the status line says on lines 6 and 7. *Checkpoint:* 1 holiday, 25.0 m wide, and 20.2 m RMS with 21.6 m at most, each within one unit of the last digit. Coverage not assured.

(b) Set Crab compensation On on the LOS card and run. *Checkpoint:* 1 holiday, 21.0 m, and 20.5 m RMS with 21.7 m at most, each within one unit of the last digit.

(c) Set the Navigation to USBL and run. Report the tiles, and explain what the aided filter measures that dead reckoning does not. *Checkpoint:* 1 holiday, 4.0 m, 0.4 m RMS with 1.7 m at most, 36 fixes used.

13.9 App challenge · 9 min

Load the challenge, The ten-line survey, with the default seed. Each run ends when the vehicle surfaces and the run pauses, about 2560 s on the clock, or 43 s at 60×. Read the Seabed imaged, Holidays, Lowest altitude, Recovery miss, Battery and Fixes tiles. Reload the scenario before each new configuration and make the change before pressing Run.

(a) Run as loaded. Report the tiles and which of the five criteria fail. *Checkpoint:* 83.9 %, 9 holidays with the widest 42.0 m, 1.7 m, 61.3 m, 16.7 %, each within one unit of the last digit. Four criteria fail, only the battery passes.

(b) Set only the Navigation to USBL and run. Report the tiles. *Checkpoint:* 79.7 %, 11 holidays with the widest 62.0 m, 1.1 m, 31.8 m, 1 USBL fix used and 132 rejected, each within one unit of the last digit.

(c) Press Reveal suggested tuning and run. Report the tiles, and name the setting of the suggestion that deals with each failure of part (a). *Checkpoint:* 100.0 % (99.95), no holiday, 5.5 m, 1.9 m, 16.6 %, 125 fixes used and 7 rejected, each within one unit of the last digit.

13.10 Application area · 4 min

A survey company plans a dead-reckoning survey with long lines and wants to choose the heading reference and the line spacing. Use Equations (13.10), (13.6) and (13.13).

(a) Compute the dead-reckoning error 2 km from the last fix with a compass biased by 1° and with a gyrocompass biased by 0.05°. *Checkpoint:* About 34.9 m and 1.75 m.

(b) At 10 m altitude, the navigation is expected to hold a 95 % uncertainty across the line of 8 m. Find the widest line spacing for which the supervisor's test still vouches for coverage, and name one other error the spacing must leave room for. *Checkpoint:* 24 m. The sag over a ridge, or the current's line-of-sight offset.

13.11 Application area · 3 min

In case 6 each GPS surfacing takes about 256 s and 3.45 Wh from the end of one line to the start of the next.

(a) A vehicle on dead reckoning with GPS surfacing every 15 min is to fly 6 h of survey lines. Estimate the extra time and energy the surfacings cost, and the share of the mission spent on them. *Checkpoint:* 24 surfacings, about 1.7 h and 83 Wh, about 22 % of the mission.

(b) Name what the surfacings do not correct, and what USBL aiding needs instead. *Checkpoint:* The compass bias stays, so the pattern keeps rotating. The USBL needs a ship on station with the vehicle inside its cone.

13.12 Review the agent · 5 min

An AI assistant is asked how to cut the navigation filter's memory and computation. It replies: "Drop the 24 s history and apply each USBL fix when it arrives. A fix is only about 1.3 s late, which is 2 m of travel at 1.5 m/s, well inside the 5 m each swath can spare, so coverage will not suffer." Find the flaw, and say how you would check it.