INTERACTIVE CONTROL ENGINEERING
Principles and Applications

Volume 4 Override, Split-Range, and Constraint Control

CHAPTER 8

Growth Control Against a Cash Floor in a Simulated Product Startup

Pages 188–214

Edited by John D. Hedengren

How to cite this chapter. "Growth Control Against a Cash Floor in a Simulated Product Startup," in *Override, Split-Range, and Constraint Control*, John D. Hedengren, Ed., vol. 4 of *Interactive Control Engineering: Principles and Applications*. Advanced Process Solutions, LLC, 2026, ch. 8, pp. 188–214.

The page numbers in this file are those of the printed volume.

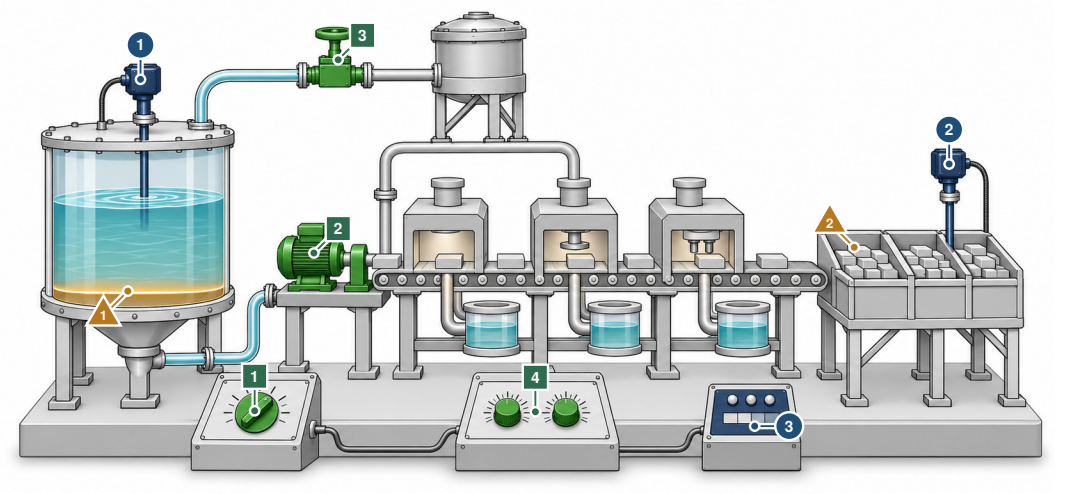
The app for this chapter
https://apmonitor.com/apps/startup-growth/

The volume online: contents, other chapters and citation records
https://apmonitor.com/apps/books/v04-override-constraint/#startup-growth

Copyright © 2026 Advanced Process Solutions, LLC. All rights reserved. Produced by Advanced Process Solutions, LLC.

CHAPTER 8

Growth Control Against a Cash Floor in a Simulated Product Startup



The controller moves 1 credit line draw 2 production rate 3 capital spending on the factory 4 research and marketing budgets

The controller reads 1 cash in the bank 2 weeks of stock on hand 3 the books at each quarterly close

What disturbs it 1 customers paying late 2 a recession or a rival's launch

The cash tank pays for the factory, the stock and the budgets before any sale pays it back, and the amber band near its bottom is the floor that the runway override defends. Illustration, not to scale.

IN PLAIN TERMS **Growing a Company Without Running Out of Cash**

The main idea. A young company pays for stock, a bigger factory and research long before its customers pay it back. So it can show a profit every quarter and still run out of money.

The challenge. Spending more makes the company grow faster, but every bit of growth is paid for first and earned back later. When the cash runs low, something has to give way, and the order in which things give way matters.

Why it matters. A rule written in advance, which cuts marketing and building first, borrows next and cuts research last, lets the company live through late payers and bad years.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the fact on funded startups (Section 8.1), the history segment (Section 8.2), the two try-it segments and the fact on the runway target (Section 8.6) and the key points
Study	60 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the measured cases (Section 8.6), Technology in Practice (Section 8.7), Limitations (Section 8.8) and the open challenge

The control question A product startup can book a profit every quarter and still run out of money, because its stock, its factory and its unpaid invoices are paid for before customers pay. How fast can loops, rules and an economic planner grow such a company before the floor on its cash stops them, and what should give way first when the cash runs short?

Learning objectives

- Account for where the cash of a profitable company goes, using its balance sheet and the delays between a decision and its return.
- Derive Higgins's sustainable growth rate on trailing books and the capital spending that grows a factory at that rate through its build.
- Explain how a runway override split over three bands gives up marketing and investment, then credit, then research.
- Measure what the season, a capacity held back, a late payer and a competitor launch cost the company in the app.
- Judge an economic MPC's financing verdict against its own modeling choices.

Prerequisites Sections 1.4.3 (selectors), 1.4.4 (external reset), 1.4.6 (split range), 1.4.9 (actuators that deplete) and 1.6.2 (velocity form). Section 1.6.3 is needed only for the planner.

The app https://apmonitor.com/apps/startup-growth/

See also Volume 9, Chapter 8 for setpoints chosen on a cost whose optimum sits on a moving constraint and Volume 9, Chapter 6 for a predictive controller that prices a shared resource. Volume 3, Chapter 6 for feedforward with a slow ratio trim and Volume 3, Chapter 4 for selectors that protect a ratio.



8.1 Introduction

Cash is the one stock of a company that has a floor, and every decision that grows the company draws on it before it pays anything back. In the simulator of this chapter a board that spends 15 % of revenue on research and development (R&D) and 15 % on marketing, asks its factory for 40 thousand units a year and adds \$1.5M of factory a year is insolvent after 1.58 yr. At that instant its trailing twelve-month revenue is \$8.37M, up from \$7.26M at the start, its trailing earnings before interest, taxes, depreciation and amortization (EBITDA) are \$1.11M and its net income \$0.51M. A month earlier, at 1.5 yr, its money sits in 26.1 weeks of finished stock carried at \$1.46M, a factory of 42.4 thousand units a year carried at \$4.24M and \$0.92M of receivables. The income statement shows a healthy company and the cash account shows a dead one.

Management science has written the firm as a feedback system for seventy years. Production and work-force scheduling was posed in 1955 with a quadratic approximation to the cost criterion, whose optimum is a linear decision rule [1]. Sales response to advertising was modeled two years later with a decay constant, a saturation level and a response constant [2], and advertising policy was treated as an optimization under dynamic conditions in 1962 [3]. Forrester's market growth model, a system dynamics model of a growing company, has been read as a portrait of bounded rationality, built from rules of thumb that use limited information [4]. Corporate finance supplied the bound. The growth a firm can sustain without new equity at its chosen payout and capital structure is a rate [5], and a profitable company that tries to grow faster than its cash cycle allows can run out of cash [6]. Economic model predictive control (MPC), whose stage cost is a cost in money rather than a tracking error [7], completes the vocabulary. The control frame suits this plant because that literature already writes each of its decisions as a loop, a rule or an optimization.

The structure is general. Any operation that grows by investing from a stock with a floor, and earns its return only after a delay, has this shape, as a hospital adding beds from its operating surplus, a utility building plant from its rate base and a university growing enrollment against its endowment draw all do. Its control problem combines integrating stocks, delays of a year, ratio control of spending to a measured base and an override that protects a hard constraint by taking the actuators away from the loops that normally drive them, all standard in process control [8]. Section 1.4.9 treats such a

stock as a depleting actuator whose runway is itself a limit. The belief the simulator corrects is that profit measures solvency and that sales measure the market. In its fourth case a factory held at $0.4M of investment a year sells $8.97M in year 8 while its customers ask for 2.65 times what it sells, and the growth loop, which sees only sales, raises marketing from 7 % to 20 % of revenue.

> **DID YOU KNOW? Most Funded Startups Still Shut Down**
>
> Census Bureau records show that of new U.S. firms that received venture capital from 1986 to 1997, 75 % had shut down by 2007, against 66 % of matched firms without it. The funded survivors employed 364 % of their group's starting employment, the matched survivors 67 % [9].

However, the finance literature states its bound for a company already growing steadily at constant ratios [5, 6], the system dynamics literature describes the feedback of the firm in decision rules and delays rather than in the setpoints, overrides and constraint handling of a control course [4, 10], and economic MPC is developed for process plants [7], so a student seldom has one company on which the bound, the loops and the planner can be measured together while the calendar, the rule and the financing change one at a time. This chapter quantifies how far feedback loops, two practitioner rules and an economic planner can grow a simulated product startup, whose every decision drains the same cash tank, before its cash floor stops them. The market model, the sustainable growth rate, the production rule, the override structure and economic MPC all come from the literature cited. The measurements are the chapter's own, among them an exact form of Higgins's rate for books kept on trailing twelve months, a sweep that bounds growth for every budget split that earns money, and the sensitivity of the planner's equity round to its own modeling choices. The company has 40 states, and its balance sheet, cash-flow statement and retained earnings reconcile at every step. Each of nine cases, a challenge and a reference isolates one mechanism.

Section 8.2 traces the history, from the learning curve and the linear decision rule to the venture round. Section 8.3 writes the company as stocks and flows, with each equation, constant, lag and bound it uses. Section 8.4 explains why its growth costs cash, through Higgins's rate on trailing books, a sweep of budget splits and the three delays. Section 8.5 gives the board's calendar, its loops, the production rule, the runway override, the rules that set the growth target and the economic MPC. Section 8.6 reports its eleven runs, the cases, the challenge and the reference. Section 8.7 sets the override beside a company's real accounts, budgets and credit line. Section 8.8 states the limitations and Section 8.9 summarizes. The exercises in Section 8.10 run the cases in the app.

8.2 From Learning Curves and Market Growth to the Venture Round

The cost side of the company has the oldest record. The factors that set the cost of building airplanes were analyzed in 1936 [11], the article the application cites for the learning curve on which its unit cost falls with cumulative production. Nineteen years later a research team working with a manufacturing company formalized the setting of a factory's aggregate production rate and work force with a quadratic approximation to the criterion function, and computed an optimal decision rule that is linear in the state of the factory, a rule that, unlike a rule of thumb, can claim to be the best possible in a stated sense [1]. A quadratic cost with a linear optimal rule has the form of the linear quadratic regulator of later control texts, here applied to inventories and payrolls.

Demand acquired dynamics in the same decade. Studies for major industrial concerns described the interaction of advertising and sales with three parameters, a sales decay constant, a saturation level and a response constant, consistent with controlled experiments on a large number of products and several media [2], and an optimal advertising policy under dynamic conditions followed in 1962 [3]. The diffusion of a new product was written in 1969 as a model for the timing of a consumer's first purchase, related to the number of previous buyers through innovative and imitative behavior, and tested against eleven consumer durables with a long-range forecast for color television sets [12]. A generalization added price and advertising as decision variables, reduced to the original model

as a special case, and explained why the original fits so well without them [13]. The market of the simulator is that model with marketing on the innovation coefficient and a logit share against one competitor.

The firm as a whole became a feedback system in the work of Forrester and his students. A 1983 reading of Forrester's market growth model argued that the structure of system dynamics models assumes bounded rationality, with decision functions made of simple rules of thumb and limited information content [4]. The growth trap of the fourth case, a company whose underinvestment produces the evidence for underinvestment, is the behavior the application's briefing attributes to that model. In 2002 a field study of process improvement found that delays between investing in improvement and recognizing its rewards, together with managers' attributions about the cause of poor performance, can trap an organization in a cycle that prevents useful change [10], the mechanism of the fifth case, where an R&D cut pays at once and costs share eighteen months later.

Corporate finance asked the question of this plant directly in 1977, and the runway override of Section 8.5 gives up its actuators in the order of the pecking order theory that the segment below follows.

> HISTORY **1977 to 2001: How Fast a Company Can Grow on Its Own Cash**
>
> In 1977 corporate finance defined a sustainable growth rate, the yearly growth in sales a company can reach while it keeps its target payout ratio and capital structure and issues no new equity. Faster growth forces something in its financial objectives to give, usually to the detriment of financial soundness, and slower growth lets it raise dividends, reduce its borrowing or build up liquid assets [5]. A cash conversion cycle approach to liquidity analysis followed in 1980, because inattention to liquidity can cause severe difficulties even for a firm with favorable long-run prospects [14]. The pecking order theory of 1984 held that firms prefer internal to external funds, and debt to equity when they need outside money [15]. In 2001 a formula written for managers joined the operating cash cycle, the cash needed to finance each dollar of sales and the cash each dollar of sales generates into a self-financeable growth rate [6].

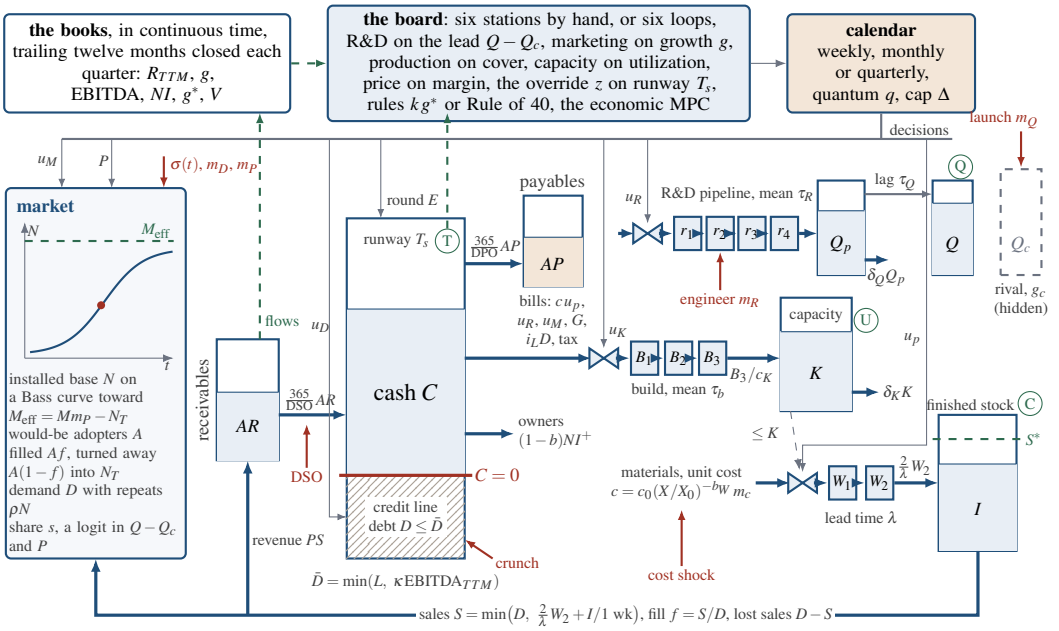
The equity round is the last decision the simulator adds. Entrepreneurship has been described as experimentation whose probabilities of success are low, extremely skewed and unknowable until an investment is made, with a few investors deciding which experiments continue [9]. A survey of 885 institutional venture capitalists at 681 firms found that they weigh the management team above the product or technology and rate deal selection as the most important source of the value they create [16]. On the control side, economic MPC was shown to have an average performance never worse than the best steady state [7], which is the property an exit objective gives up, because an acquisition at a fixed date is a terminal reward and not a steady state. The simulator's planner is a local, successively linearized version of that idea with the round as one more decision.

8.3 The Company as Stocks and Flows

The simulated company makes one hardware product and sells it to consumers against one competitor, drawn in Figure 8.1 as the tanks, pipelines and valves that it is. Money is in millions of dollars ($M), volume in thousand units (k units), time in years, and each flow is a yearly rate. Table 8.1 classifies the signals. All coefficients are the teaching values of Table 8.2, chosen for an imaginary hardware startup and not estimated for any firm. Every stock except the market's is paid for out of the cash tank, and every decision of Section 8.5 opens or closes one of the valves that fill or drain it.

8.3.1 Market

Would-be adopters arrive by Bass diffusion of the company's own installed base N toward an effective potential, and choose this product with a logit share in the quality gap and the price gap, as shown in



Measured live: T runway, U utilization, C cover, Q lead. The spend shares u_R and u_M are paid as shares of R_{TTM} at the last close; every station, by hand, by a loop or by the MPC, moves only on the calendar and on its grid.

Figure 8.1 The startup as stocks and flows, not to scale. Collections from the receivables AR fill the cash tank C, whose floor is the thick red line marked $C = 0$ and below which the hatched credit line holds the debt D within its ceiling $\bar{D}$. Payments of the payables AP, capital spending u_K and dividends drain it. Capital passes a three-stage build chain B_j into capacity K, which bounds the production u_p that passes the work in process W_1, W_2 into finished stock I, sold as S against demand D and booked as revenue PS. R&D spending passes a four-stage pipeline r_j into the quality potential Q_p and the product quality Q, raced against the competitor's hidden Q_c, drawn as a dashed box. Thick blue arrows carry money and goods, red arrows that start at a labeled cause are disturbances, thin gray arrows from the decision bus are the board's decisions on its calendar, and dashed green lines are the measurements and the books, with the circled letters keyed in the line under the drawing.

Table 8.1 Signals of the Company by Role

Symbol	Role	Unit
T_s, g, $I/(D/52)$	CVs, stressed runway, growth at the close, stock cover	mo, –, wk
$\hat{u}_p/K$, ℓ, m_g	CVs, utilization asked, quality lead, gross margin	–, points, –
u_R, u_M	MVs, R&D and marketing as shares of revenue, 0–0.3	–
u_p, u_K	MVs, production, 0–K, and capital spending, 0–20	k/yr, $M/yr
P, u_D	MVs, price, 210–390, and credit draw, −4–4	$/unit, $M/yr
b, E	MVs, retention and an equity round	–, $M
σ, m_D, m_P	DVs, season, demand and potential multipliers	–
m_Q, m_c, m_R	DVs, launch, supplier cost and engineer multipliers	–
DSO, m_L, i_L	DVs, collection days, credit line multiplier, rate	d, –, /yr
C, AR, AP, D	states, cash, receivables, payables, debt	$M
I, W_1, W_2, X	states, stock, work in process, cumulative output	k units
$I^\$$, $W^\$$, F	states, stock and work in process at book, fixed assets	$M
E_P, E_R	states, paid-in and retained equity	$M
K, B_1–B_3	states, capacity and the build chain	k/yr, $M/yr
Q, Q_p, r_1–r_4	states, quality, its potential, the R&D pipeline	–, $M/yr
N, N_T	states, installed base and customers turned away	k units
Σ_x	states, 15 cumulative flows the books close on	$M, k
S, D	measured sales, hidden demand	k/yr
x_{TTM}, V, ω	outputs, trailing books, valuation, founders' share	$M, –

Equation (8.1).

$$\begin{aligned} M_{\text{eff}} &= \max(1, Mm_P - N_T), \quad p = p_0 + \frac{p_M e_M}{M_s + e_M} \\ s &= \left[1 + e^{-\beta_Q(Q-Q_c)+\beta_P(P-P_c)/P_c}\right]^{-1} \\ A &= \left[(p + qN/M_{\text{eff}})(M_{\text{eff}} - N)\right]_+ s\,\sigma\, m_D \\ D &= A + \rho N s\,\sigma\, m_D \\ \dot{N} &= Af, \qquad \dot{N}_T = A(1-f) \end{aligned} \tag{8.1}$$

where $\sigma = \sigma(t)$ is the season, e_M the marketing spend in \$M/yr, $f = S/D$ the share of orders filled, N is held within $[0, M_{\text{eff}}]$ and the exponent of the logit within ± 30. Marketing raises the innovation coefficient by at most p_M, half of it at $e_M = M_s$. A would-be adopter turned away by an empty shelf buys the competitor's product and leaves the potential for good, while a repeat buyer turned away does not defect. The season is a four-term fit through mid-quarter values of 0.7, 0.85, 0.85 and 1.6, as shown in Equation (8.2).

$$\begin{aligned} \sigma(t) &= 1 + a_\sigma\big(0.375\cos\theta - 0.075\sin\theta + 0.225\cos 2\theta\big) \\ \theta &= 2\pi(t - 0.875) \end{aligned} \tag{8.2}$$

At $a_\sigma = 1$ it peaks at 1.602 at $t = 0.866$ of the year, in mid-November, falls to 0.628 at 0.187, in early March, and has quarter means of 0.789, 0.806, 0.924 and 1.481, with an annual mean of exactly one.

8.3.2 Product

R&D money passes a four-stage pipeline with a mean delay τ_R, raises a quality potential with diminishing returns, and the product on sale follows the potential through a lag, as shown in Equation (8.3).

$$\begin{aligned} \dot{r}_1 &= \tfrac{4}{\tau_R}(e_R m_R - r_1) \\ \dot{r}_j &= \tfrac{4}{\tau_R}(r_{j-1} - r_j), \qquad j = 2,3,4 \\ \dot{Q}_p &= \eta_R \frac{(1 + 1/R_s)\, r_4}{1 + r_4/R_s} - \delta_Q Q_p \\ \tau_Q \dot{Q} &= Q_p - Q, \qquad Q_c = (1 + g_c t)\, m_Q \end{aligned} \tag{8.3}$$

where e_R is the R&D spend in \$M/yr and $m_Q = 1.15$ after a competitor launch. At \$1M a year the yield is $\eta_R = 0.1$ quality point a year, at $R_s = \$3M$ a year it is 0.2, half of the 0.4 that no spend can exceed. Near parity a lead of 0.1 point moves the share by 0.124.

8.3.3 Factory and Production Line

Production starts into a two-stage work-in-process line and finishes into stock, a sale ships from stock within a week, and capital spending becomes capacity through a three-stage build, as shown in Equation (8.4).

$$\begin{aligned} \dot{W}_1 &= u_p - \tfrac{2}{\lambda} W_1, \qquad \dot{W}_2 = \tfrac{2}{\lambda}(W_1 - W_2) \\ \dot{I} &= \tfrac{2}{\lambda} W_2 - S, \qquad S = \min\big(D,\ \tfrac{2}{\lambda} W_2 + I/\tau_w\big) \\ \dot{K} &= B_3/c_K - \delta_K K, \qquad 0 \le u_p \le K \\ \dot{B}_1 &= \tfrac{3}{\tau_b}(u_K - B_1), \qquad \dot{B}_j = \tfrac{3}{\tau_b}(B_{j-1} - B_j) \\ c &= c_0\big(\max(X, X_0)/X_0\big)^{-b_W} m_c, \qquad \dot{X} = u_p \end{aligned} \tag{8.4}$$

where $\tau_w = 1/52$ year and $j = 2,3$ in the build chain. A sale the stock cannot make is lost, $D - S$. The exponent $b_W = 0.2345$ makes unit cost fall 15 % with each doubling of cumulative production, $2^{-b_W} = 0.850$. Production is booked into work in process at the unit cost when it starts, $\dot{W}^{\$} =$

$cu_p/10^3 - \frac{2}{\lambda}W_2 w$ with $w = W^{\$}/(W_1 + W_2)$, moves into stock at that average, $\dot{I}^{\$} = \frac{2}{\lambda}W_2 w - S\iota$, and is sold at the book cost of the units available over the shipment window,

$$\iota = \frac{I^{\$}/\tau_w + (2/\lambda)W_2 w}{I/\tau_w + (2/\lambda)W_2}, \qquad \text{COGS} = S\iota.$$

When the denominator is zero, sales and COGS are zero. Since sales do not exceed the available unit flow, COGS cannot exceed $I^{\$}/\tau_w + (2/\lambda)W_2 w$. Thus a zero book inventory has a nonnegative derivative, without a write-off or an artificial clipping flow. This law replaced a small-stock blending law on 2026-11-15.

8.3.4 Statements and Financing

Revenue is booked when the product ships and collected after DSO days, every cost is booked when it is incurred and paid after DPO days, and the cash tank takes what is left, as shown in Equations (8.5) and (8.6).

$$\begin{aligned} \text{Rev} &= PS/10^3, \qquad NI = \Pi - \tau_x[\Pi]_+ \\ \text{EBITDA} &= \text{Rev} - \text{COGS} - (e_R + e_M + G) \\ \Pi &= \text{EBITDA} - \delta_K F - i_L D \end{aligned} \tag{8.5}$$

$$\begin{aligned} \dot{AR} &= \text{Rev} - \tfrac{365}{\text{DSO}}AR \\ \dot{AP} &= cu_p/10^3 + e_R + e_M + G + i_L D + \tau_x[\Pi]_+ \\ &\quad - \tfrac{365}{\text{DPO}}AP \\ \dot{C} &= \tfrac{365}{\text{DSO}}AR - \tfrac{365}{\text{DPO}}AP - u_K + u'_D - (1-b)[NI]_+ \\ \dot{F} &= u_K - \delta_K F, \qquad \dot{D} = u'_D \\ \dot{E}_R &= NI - (1-b)[NI]_+ \end{aligned} \tag{8.6}$$

where Π is profit before tax, taxed at τ_x with no loss carried forward. The credit line holds debt within a ceiling that moves with the company's own earnings, and debt above it is called over a quarter, as shown in Equation (8.7).

$$\begin{aligned} \bar{D} &= \min\big(Lm_L,\ \kappa[\text{EBITDA}_{TTM}]_+\big) \\ u'_D &= \max\big\{\min(u_D, [\bar{D} - D]_+/\Delta t) \\ &\qquad - [D - \bar{D}]_+/0.25,\ -D/\Delta t\big\} \end{aligned} \tag{8.7}$$

where $\Delta t = 1/52$ year and a repayment ($u_D < 0$) passes without the first minimum. An equity round of E adds E to cash and to paid-in equity E_P and multiplies the founders' share ω, 80 % after the seed round, by $V_e/(V_e + E)$, with the pre-money $V_e = \max(1, V + C - D)$. A term loan draws up to the room the ceiling leaves at once. Both are raised after the step in which they are asked for. The balance sheet $C + AR + I^{\$} + W^{\$} + F - AP - D = E_P + E_R$ holds at every step, because each flow enters one asset or liability and its counterpart. Cash may fall below zero. The run then records INSOLVENT and pauses once, and Run carries on the company a bank would have closed, so that it can be watched.

8.3.5 Books, Runway and Valuation

The books are kept as practitioners read them. Every flow has a cumulative state Σ_x, and a ring of those states one step apart gives any trailing twelve-month total as $x_{TTM}(t) = \Sigma_x(t) - \Sigma_x(t-1)$, so the season cancels exactly. Before $t = 0$ the ring holds a flat pre-history at the opening rates with the

Table 8.2 Teaching Values by Location: M Listed in Show Model, C on a Controller Card or in the Disturbances, S Fixed in the Source. No Constant Is Hidden From the Student

Symbol	Meaning	Value	Range	
Market, product, disturbances and clocks				
M	potential adopters	2000 k	1000–3500	M
p_0, q	Bass innovation, imitation	0.01, 0.4 yr	0.002–0.03, 0.1–0.8	M
ρ	repeat purchases	0.2 yr	0.05–0.4	M
p_M, M_s	most marketing adds, spend for half	0.02/yr, \$1.5M/yr	0–0.04, 0.2–4	M
β_Q, β_P	share sensitivity to quality, price	5, 3	2–8, 1–6	M
g_c, P_c	rival's quality growth, price	0.08/yr, \$300	0–0.15, 200–400	M
η_R	quality a year from \$1M/yr	0.1	0–0.3	M
R_s	R&D for half the yield	\$3M/yr	0.5–20	M
τ_R, τ_Q	R&D pipeline mean, product lag	1, 0.5 yr	0.5–2, 0.2–1.5	M
δ_Q	obsolescence	0.02/yr	0–0.1	M
P_0	launch price	\$300	200–400	M
a_σ, m_D	seasonal amplitude, demand multiplier	1, 1	0–1.5, 0.5–1.5	C
—	recession: m_D, m_P, length	0.7, 0.8, 1.5 yr	—	S
—	credit crunch m_L, late DSO	0.5, 75 d	—	S
—	rate rise, replayed spread	+0.03, +0.05/yr	—	S
Δt	step, control period, log	1/52 yr each	—	S
Factory, finance and fixed values				
c_0	unit cost at $X_0 = 20$ k	\$150	80–200	M
b_W	learning exponent	0.2345	0–0.5	M
c_K	capital per k/yr of capacity	\$0.1M	0.05–0.2	M
τ_b, δ_K	build mean, depreciation	0.75 yr, 0.1/yr	0.25–1.5, 0.03–0.2	M
λ	production lead time	6 wk	2–16	M
DSO, DPO	collection, payment days	45, 30 d	15–100, 10–90	M
G	fixed G&A	\$1.5M/yr	0–3	M
τ_x, i_L	tax, credit-line rate	0.25, 0.08/yr	0–0.4, 0–0.2	M
L, κ	credit line, covenant multiple	\$1M, 3	0–4, 1–6	M
C_0, K_0, N_0	opening cash (\$M), capacity (k/yr), customers (k)	3, 25, 20	0.5–6, 10–80, 5–400	M
r	owners' discount rate	0.12/yr	0.05–0.2	M
—	opening stock, Q, X	6 wk, 1, 20k	—	S
—	launch m_Q, supplier m_c for 1 yr	1.15, 1.3	—	S
—	viral m_D, engineer m_R	2 for 0.25 yr, 0.6 for 1 yr	—	S
—	seed share, founders	20 %, 80 %	—	S
—	covenant call period	0.25 yr	—	S

season at its mean. The books then give the quantities of Equation (8.8).

$$
\begin{aligned}
g &= \frac{\mathrm{Rev}_{TTM}(t)}{\mathrm{Rev}_{TTM}(t-1)} - 1, \qquad m = \frac{\mathrm{EBITDA}_{TTM}}{\mathrm{Rev}_{TTM}} \\
g^* &= \frac{x}{1-x}, \qquad x = \mathrm{clip}_{[-0.9,0.9]}\left(\frac{b\,NI_{TTM}}{E_P + E_R}\right) \\
V &= [\mathrm{Rev}_{TTM}]_+ \,\mu(g,m) \\
\mu &= \mathrm{clip}_{[1.5,13]}\left(2 + 10\,\mathrm{clip}_{[0,1]}(g) + 2\,\mathrm{clip}_{[0,0.3]}(m)\right)
\end{aligned}
\tag{8.8}
$$

where g^* is Higgins's sustainable growth rate with the return on equity taken at the end of the window, and V is a teaching valuation at the multiple μ, two times revenue at no growth and twelve at 100 % a year, with the shape of the Rule of 40. The founders' value is $\omega[V + C - D]_+$. At every quarter end a close publishes these figures, and the growth and Higgins rules read the close. Runway is $T_r = 12C^+/B$ months at the trailing year's net burn $B = -\mathrm{FCF}_{TTM}$ plus any debt being called, and the override reads a stressed runway T_s whose burn is never less than a quarter of the operating outflows, $\max\big(B,\ 0.25(\mathrm{COGS}_{TTM} + e_R + e_M + G)\big)$, both held within 0 to 120 months.

8.3.6 Start and Integration

The opening company is lined out at its opening decisions. Its sales are the demand those decisions raise at the mean season, or the capacity if that is less, found by a fixed-point iteration because the spends are shares of the revenue they produce. Stock is six weeks of those sales, work in process a lead time of them, receivables and payables their DSO and DPO shares of the flows, every pipeline stage at its opening input, fixed assets $c_K K_0$, and paid-in equity whatever balances the sheet. At the default decisions the company opens with \$6.57M of trailing revenue, 21.9 thousand units a year at \$300, a trailing EBITDA of \$0.14M, a net income of $-\$0.11M$, \$0.81M of receivables, equity of \$6.54M and a credit ceiling of \$0.42M. The engine of Section 1.6.1 integrates the 40 states with classical fourth-order Runge-Kutta on a step of a week. The board's loops execute once a week ahead of each integration step, and the historian records a sample each week. A seasonal pattern and every event enter through time inside the right-hand side, so nothing is random and a case replays exactly. The default seed, 20261004, is not used by any random draw. Over six cases of six years the balance sheet closes to 1.4×10^{-14} \$M, the cash-flow statement to 7.2×10^{-14} \$M and retained earnings to zero, and the installed base follows the closed-form Bass curve to a relative 2.2×10^{-12} when every order is filled. On the balanced reference every derivative of the balance sheet is below 4.5×10^{-16}, and at half the step the challenge's growth changes from 20.7 % to 20.8 %. The application passes its 22 own checks.

8.3.7 Assumptions

One product, one competitor. The rival's quality rises on a fixed trend and never answers a price cut or a launch, there is one market and one factory, and there is no hiring, no wage bill apart from the spends and the overhead, no bad debt and no obsolete stock.

Delays as distributed lags. The R&D pipeline, the work in process and the build chain are chains of first-order stages, which reproduce a mean delay and a spread, not a delivery calendar.

Simplified finance. Tax has no loss carry-forward, the covenant is the only clause of the loan, and rounds are priced at the equity value with no preferences, options or convertible notes.

Teaching values. Every parameter, threshold, multiple and rule is a declared teaching value. Real diffusion coefficients come from fitted diffusion studies [12], real multiples from comparable transactions, real covenants from the loan agreement, and none of the numbers here is an estimate for any company.

8.4 Why Growth Costs Cash

8.4.1 Higgins's Rate on Trailing Books

Higgins's rate is the growth at which equity, raised only by retained earnings, keeps pace with the assets that the sales require. On books kept in continuous time and closed on trailing twelve months the rate is exact. A company growing at the continuous rate γ with constant ratios has $E(t) = E(0)e^{\gamma t}$ and retains $bNI = \dot{E} = \gamma E$, so its trailing net income is a year of that flow, as shown in Equation (8.9).

$$\begin{aligned} \frac{bNI_{TTM}}{E(t)} &= \frac{1}{E(t)} \int_{t-1}^{t} \gamma E(\tau)\,\mathrm{d}\tau = 1 - e^{-\gamma} \\ g^{*} &= \frac{x}{1-x} = e^{\gamma} - 1 \end{aligned} \tag{8.9}$$

The trailing revenue of the same company grows year over year by $e^{\gamma} - 1$, so $g^{*} = g$ holds with no approximation when the ratios hold. Case 2 builds a company in which they nearly do, with no learning curve, no obsolescence, no competitor race, \$0.5M of overhead, no credit line, \$1.5M of cash, and a market of 3.5 million that would buy far more than the factory makes. Growing at $k = 1$ times g^{*}, it settles onto the rate, 21.9 % against 22.2 % at year 4 and 22.8 % against 23.8 % at year 9, and from year 5 to 9.5 growth stays within 1.6 points of g^{*} and cash between 38.6 % and 42.5 % of equity, as shown in Figure 8.2(a).

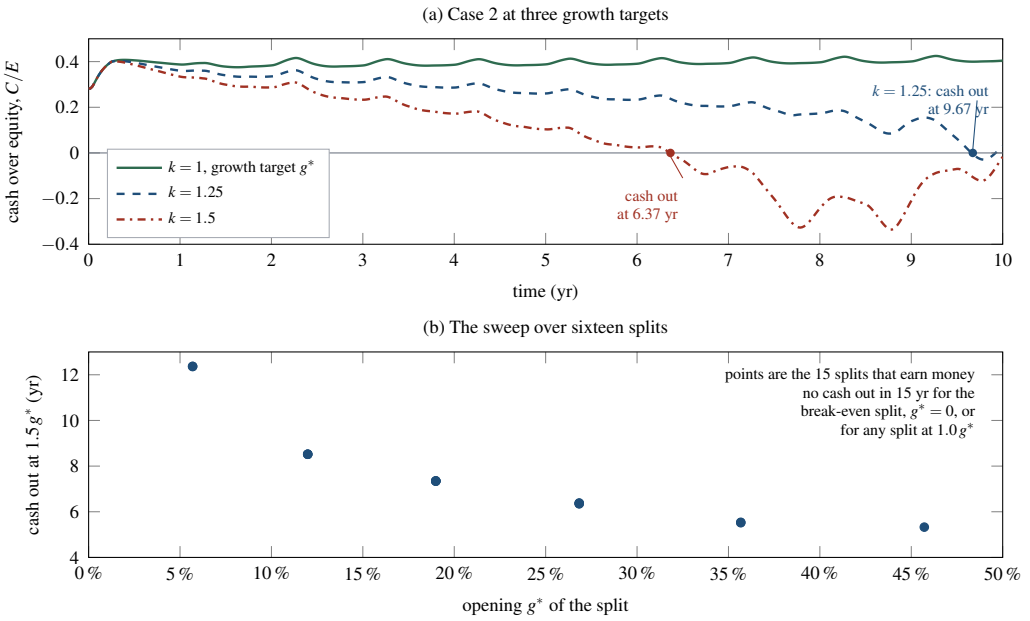


Figure 8.2 Higgins's bound measured on the application's engine. (a) Cash as a share of equity in case 2 with the growth target at 1 (solid), 1.25 (dashed) and 1.5 (dash-dotted) times g^*. (b) The year the cash runs out at $1.5\,g^*$ for each split of R&D and marketing at 5, 10, 15 and 20 % of revenue, against the split's opening g^*. Points of one g^* share a total spend.

8.4.2 The Honest Bound

Growth above g^* must be paid for by debt, new equity or the cash in the tank. At $k = 1.5$ the factory and the working capital outgrow the equity, the cash share falls and first crosses zero at 6.37 yr. At $k = 1.25$ it first crosses zero at 9.67 yr; the later positive balance is a counterfactual recovery after insolvency, not a solvent ten-year run. The application's checks run the same company for all sixteen splits of R&D and marketing at 5, 10, 15 and 20 % of revenue. At $1.5\,g^*$ every one of the fifteen splits that earns money runs out of cash, between 5.33 yr for 5 % and 5 % and 12.37 yr for 15 % and 20 %, as shown in Figure 8.2(b), and at $1.0\,g^*$ none runs out in fifteen years. The sixteenth split, 20 % and 20 %, opens at break-even, so its g^* is zero and it has no growth to finance. The more profitable a split, the higher its g^*, the faster it is asked to grow and the sooner its cash runs out, from an opening g^* of 45.7 % for the leanest split to 5.7 % for the two splits that spend 35 % of revenue.

8.4.3 Lags, Working Capital and the Season

Three delays separate a decision from its return. A step in R&D on the balanced reference reaches half its final rate of quality gain after 1.385 yr, against the 1.5 yr of pipeline and lag declared in Table 8.2, because the four-stage chain spreads the arrival around its mean. Capacity arrives through a build with a mean of nine months, and an order shipped today is cash 45 days later. Receivables are about DSO/365 of a year's revenue, so on the balanced reference a stretch from 45 to 75 days moves \$0.308M from cash into receivables one for one by year 2.5 while EBITDA is unchanged to the last digit. The season multiplies demand by 1.6 in mid-November, and a production rule that reads last quarter's sales meets each peak a season late.

Table 8.3 Board Settings, Defaults and Ranges. A Marks an Advanced Control

Symbol	Setting	Default	Range
Board			
—	decision policy	six loops	4 policies
—	calendar	quarterly	weekly, monthly
q, Δ	quantum, largest move (A)	1, 100 % of range	0–5, 5–100
Loops: setpoint / K_c / τ_I			
—	R&D on lead ℓ	0 / 1.0 / 4 yr	±0.3 / 0–2 / 0.25–20
—	marketing on growth g	0.25 / 0.2 / 3 yr	0–0.8 / 0–2 / 0.25–10
—	capacity on utilization	0.8 / 5 / 1 yr	0.4–1 / 0–20 / 0.25–10
—	price on gross margin (MANUAL)	0.5 / 100 / 1 yr	0.2–0.7 / 0–500 / 0.25–10
—	override on runway T_s	12 / 0.15 / 0.5 yr	3–30 / 0–1 / 0.1–5
c^*, a_S, β	production: cover, adjustment, line weight	6 wk, 0.5/mo, 1	1–16, 0.05–1, 0–1
θ, ff	forecast smoothing (A), feedforward	0.3/mo, on	0.05–1, off
—	count the R&D pipeline (A)	on	off
Rules and the economic MPC			
k	Higgins multiple of g^*	1	0.5–2
T, H	exit year, cash-flow horizon	6, 5 yr	3–10, 2–15
C_{min}	cash buffer, months of opex	6	0–12
w, E	departure weight, round size (A)	1, \$3M	0.1–10, 0.5–10
—	MPC may raise one round	on	off
b	earnings retained	1	0–1

K_c in share per point of lead or growth, \$M/yr per unit of utilization, \$ per unit of margin and levels per month. Opening stations: u_R 0.15, u_M 0.10, u_p 20 k/yr, u_K \$0.5M/yr, P \$300, u_D 0.

8.5 The Board, Its Loops and Its Planner

The board has six stations, R&D and marketing as shares u_R and u_M of trailing revenue, production u_p, capital spending u_K, price P and the credit draw u_D, together with retention b, which is always the learner's. Each station has its own loop with a MANUAL and AUTO switch, a setpoint and two tuning constants, and a master switch throws all six. The controller executes every week. Table 8.3 lists the settings.

8.5.1 Calendar, Quantum and Cap

Every decision, by the learner's hand, a loop or the planner, passes the calendar and shaping functions of `core/cadence.js` in the collection's framework. The same shaping governs the central bank's rate in Volume 2, Chapter 11 and the intervention desk of Chapter 14, and Section 1.4.5 sets it beside the rate limits of other chapters. Weekly makes every execution a decision, and monthly and quarterly are regular intervals of $1/12$ and $1/4$ year, quarterly as shipped. A quarter is 13 weekly executions, while a month is not a whole number of them, so a monthly decision is taken at the first execution at or after each twelfth of a year, 5, 4 and 4 weeks apart in turn. The decision at $t = 0$ is taken when the case is loaded. A setting changed before Run sets the run up again, so it acts from that decision as it does after Reset, while a setting changed between meetings reaches the stations at the next decision. At a decision each station's request u^w is shaped as shown in Equation (8.10).

$$\begin{aligned} \tilde{u} &= u_{lo} + q_u \,\mathrm{round}\big((u^w - u_{lo})/q_u\big) \\ \Delta_q &= q_u \max\big(1, \lfloor \Delta_u / q_u \rfloor\big) \\ u_k &= \mathrm{clip}_{[u_{lo},u_{hi}]}\big(u_{k-1} + \mathrm{sat}_{\Delta_q}(\tilde{u} - u_{k-1})\big) \end{aligned} \tag{8.10}$$

where ties round away from zero and the bounds snap to the grid, and the station holds until the next decision. At $q = 1$ the quantum q_u is one point of revenue for the two spends, \$3 for the price, 0.5 thousand units a year for production and \$0.05M a year for capital spending and the draw, whose grid is based at zero. The cap Δ_u is a share of the station's range, none as shipped, and production is bounded by the capacity at the decision. The spend shares are paid as dollars at the trailing revenue of

the last close, $e_R = u_R \mathrm{Rev}_{TTM}$ and $e_M = u_M \mathrm{Rev}_{TTM}$, fixed until the next decision. The application's check finds all 72 decisions of three years of case 4 on the quarterly calendar and on their grids.

8.5.2 Four Loops of the Framework's PID

The R&D, marketing, capacity and price loops, and the override's level, run the velocity-form PID of Section 1.6.2 every week with the derivative time zero, as shown in Equation (8.11).

$$\begin{aligned} e_k &= a(r_k - y_k), \qquad P_k = K_c e_k \\ S_k &= S_{k-1} + P_k - P_{k-1} \\ \tilde{u}_k &= u_{b,k} + S_k + I_{k-1}, \qquad u_k = \mathrm{clip}_{[u_{min}, u_{max}]}(\tilde{u}_k) \\ I_k &= I_{k-1} + \left[K_c e_k / \tau_I + (u_k - \tilde{u}_k)/\tau_I\right]\Delta t \end{aligned} \tag{8.11}$$

where $a = \pm 1$ is the action, $u_{b,k}$ a feedforward bias, zero for R&D and marketing, and $\tilde{u}_k$ the output before limiting, v_k in Equation (1.14), with the tracking time $T_t = \tau_I$ of Equation (1.15). The clip is the loop's own bound, the Higgins cap included, while the override, the quantum, the move cap and the calendar act on the request after it, so the back-calculation does not see them, and while the override holds marketing down the growth loop's integral keeps running on its error. Section 1.4.4 contrasts this with an override fed back by external reset. Transfer between MANUAL and AUTO is bumpless as in Section 1.6.2, and the first execution in AUTO starts from the decision in force. Between decisions the request moves every week, and the calendar samples it.

The R&D loop holds the quality lead at zero with its request within 0 and 0.3 of revenue. With its pipeline counted, as shipped, it measures the lead the R&D already spent will deliver, $\ell = Q_p + \eta'_R(\tau_R/4)\sum_j r_j - Q_c - (\tau_R + \tau_Q) g_c m_Q$, with η'_R the marginal yield at the pipeline's mean rate, which is supply-line accounting applied to quality. The marketing loop holds year-over-year growth at the last close at 25 %. The capacity loop acts in reverse on the utilization the production plan asks for, a yearly average of the production request over capacity, with the bias $u_b = \min\left(\bar{u}_K,\ c_K K(\delta_K + g_{ff}\phi)\right)$ that replaces depreciation and grows the factory at the last close's growth g_{ff}, scaled by $\phi = \mathrm{clip}_{[0,1.5]}(2y/r - 1)$ so that an idle factory does not keep growing on last year's growth. The price loop is ratio control, with $u_b = c/(1 - m^*)$ held within \$150 to \$450 and a trim on the gross margin of the units sold, smoothed over about a quarter. It is in MANUAL at \$300 as shipped, so price moves only when the learner moves it.

8.5.3 The Production Rule

Production runs the ordering rule of the milk distribution game of Volume 6, Chapter 11, anchoring and adjustment with supply-line accounting, which the application's source names the Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS), as shown in Equation (8.12).

$$\begin{aligned} O = \max\big\{0,\ F/12 + g(a_S, \delta)(S^* - I) \\ + g(\beta a_S, \delta)(\lambda F - W) + b_P\big\} \\ g(a, \delta) = \left[1 - (1-a)^{\delta}\right]/\delta, \qquad u_p^w = 12\,O \end{aligned} \tag{8.12}$$

where O is a monthly rate, δ the months between decisions (three quarterly), $W = W_1 + W_2$, $S^* = (c^*/52)F$ the stock wanted for a cover of c^* weeks, and b_P a transfer bias that starts at the decision in force and decays at the rule's own rate. The forecast F smooths the sales the company sees, $L \leftarrow L + \gamma(S - L)$ with $\gamma = 1 - (1-\theta)^{12/52}$ each week, so lost orders are invisible to it. With the seasonal feedforward on, as shipped, the level is smoothed with the season divided out, carried forward at the last close's growth over the smoothing's lag and the lead time, and multiplied by the season over the quarter the output will sell, and S^* adds the pre-build that the factory cannot make inside the coming peak. The rule is proportional on the stock and on its own pipeline, so the quantum and the capacity bound are its only anti-saturation.

8.5.4 The Runway Override

The cash loop is a PI controller on the stressed runway against its target $T^* = 12$ months, with the measurement held below 36, whose output is an override level $z \in [0,3]$ split in range over the other loops' requests, as shown in Equation (8.13).

$$\begin{aligned} u_M &\leftarrow u_M^w\,(1-\mathrm{sat}_{[0,1]}z) \\ u_K &\leftarrow u_K^w\,(1-\mathrm{sat}_{[0,1]}z) \\ u_D^w &= (\min(z,2)-1)\,[\bar{D}-D]_+/0.25, \qquad z>1 \\ u_R &\leftarrow u_R^w\,\big(1-0.7\,\mathrm{sat}_{[0,1]}(z-2)\big) \end{aligned} \tag{8.13}$$

From 0 to 1 the level scales marketing and capital spending to nothing, from 1 to 2 it draws the room the ceiling leaves within a quarter, and from 2 to 3 it cuts R&D to 30 % of its request. The three bands are a split range in the sense of Section 1.4.6, ordered by preference rather than by gain. With z at zero and the plain runway above $1.5\,T^*$ it repays any debt over six months. The order follows the pecking order of corporate finance, internal funds before debt and debt before equity [15], and the structure is that of a pressure override in a process plant [8], which acts on the request whoever made it. In MANUAL the station is the learner's draw and no override acts.

8.5.5 Rules That Set the Growth Target

Two practitioner rules replace the growth setpoint with one computed from the plant's own state at the last close. Higgins's rule sets it to $k\,g^*$ and also caps capital spending at $c_K K\big(\delta_K + \ln(1+kg^*)\big)(1+\ln(1+kg^*)\tau_b/3)^3$, the rate that grows the factory at $k\,g^*$ through its three-stage build, so that the company finances its own growth. The Rule of 40 sets it to 0.4 less the trailing free-cash-flow margin. On case 4 with the capacity loop in AUTO, the Rule of 40 ends year 10 at 12.3 % growth with a score of 56.3 %, identically when set before Run or carried through Reset.

8.5.6 The Economic MPC and the Round

The economic MPC replaces every AUTO loop's request at a board decision and plans at most once a quarter. At each plan it simulates its model of the company, the right-hand side of Equations (8.1) to (8.7) with every parameter at its declared default and no shock, forward a week at a time with explicit Euler steps over N quarters, holding the loops' requests $\bar{u}$ and producing to keep six weeks of demand, to give a nominal path $\bar{x}_k$. It linearizes that right-hand side by central differences about its estimate of the present state, with the sales minimum smoothed by a sixth-power norm, and discretizes it exactly for a zero-order hold over a quarter with the framework's `core/linmpc.js`. The prediction is $x_{k+1} = \bar{x}_{k+1} + A_d(x_k - \bar{x}_k) + B_d(u_k - \bar{u})$, and the quadratic program (QP) of Equation (8.14) chooses the decisions of the stations in AUTO in move blocks of 1, 1, 2, 2, 4, 6 and 10 quarters, the last taking what is left. The planner reads the company's books, the credit and the rate the bank reports, its own unit cost, its sales, the quality lead and the competitor's launch, and the seasonal calendar. The states the books do not show, the installed base, the customers turned away, the quality potential and the R&D and construction pipelines, come from the declared model run beside the company on its own decisions, with the quality potential corrected through the declared lag by the measured quality. One demand factor, updated whenever the shelf did not limit sales and held over the plan, scales the model's demand to the sales seen. On the declared company, unshocked, the estimate matches the simulated state bit for bit, so in case 8 the planner starts from the company's true state. The QP keeps the move suppression and priced slacks of Equation (1.17) and replaces its tracking term by

the founders' discounted value at the exit.

$$\begin{aligned}\min_{u,\varepsilon}\ & -\omega\phi e^{-r(T-t)}\big[V_T'(R_N-R_{N-4})\\ &\quad +V_P'(R_{N-4}-R_{N-8})+C_N-D_N\big]\\ &\quad +\textstyle\sum_j\|\Delta u_j\|_S^2+\sum_k\|u_k-\bar{u}\|_W^2\\ &\quad +\textstyle\sum_i(\rho_i\varepsilon_i^2+\varsigma_i\varepsilon_i)\\ \text{s.t. } & C_k\ge C_{buf}-\varepsilon_1,\qquad D_k\le\bar{D}+\varepsilon_2\\ & 2\bar{d}-\varepsilon_3\le I_k\le 12\bar{d}+\varepsilon_3,\qquad \varepsilon\ge 0\\ & u_{min}\le u_k\le u_{max},\qquad |\Delta u_j|\le\Delta u_{max}\\ & u_{p,k}\le K_k\end{aligned}\tag{8.14}$$

where R_k is the predicted cumulative revenue, so that R_N-R_{N-4} is the trailing revenue at the exit, $N=$ round$(4(T-t))$ held within 4 to 60, ϕ the dilution of a round, C_{buf} six months of operating expense, $\bar{d}$ a week of trailing demand and $\bar{D}$ the ceiling now. The terminal gradients are $V_T'=\mu_0+10(1+g_0)$ and $V_P'=-10(1+g_0)^2$, the derivatives of Equation (8.8) at the last close's growth g_0 and multiple μ_0 when $0<g_0<1$, and μ_0 and zero otherwise. Decisions are in model units, R&D and marketing in \$M/yr with bounds of 0 and 0.3 times revenue, and a station in MANUAL is held at its value by setting both of its bounds to it. Production $u_{p,k}$ is held within K_k, the capacity predicted for the start of quarter k, by one hard row per quarter, since capacity reaches the linearized flows only through the clip on production. The move weights S, departure weights W (scaled by w) and quarterly move caps Δu_{max} are, for (e_R,e_M,u_p,u_K,P,u_D), $(0.2/\text{Rev},\ 0.2/\text{Rev},\ 2\times10^{-4},\ 0.02,\ 2\times10^{-5},\ 0.02)$, $w(1/\text{Rev},\ 1/\text{Rev},\ 10^{-3},\ 0.1,\ 10^{-4},\ 0.1)$ and $(0.05\text{Rev},\ 0.05\text{Rev},\ 10,\ 1,\ 15,\ 4)$, with Rev the trailing revenue at the close and the draw bounded between repaying all debt and filling the ceiling within a quarter. The slacks carry quadratic prices of 10^3, 10^3 and 10^2 and linear prices of 10^2, 10^2 and 10, so the bands bind as hard constraints whenever they can be met. The cash-flow objective replaces the bracket by the discounted free cash flow over a horizon H plus the discounted value at its end. With the exit objective and no round raised, the QP is solved for seven candidates, no round, or E or $2E$ now, in a year or in two, each round adding its cash at its quarter with $\phi=V_e/(V_e+E)$ at the equity value grown at g_0, and the best objective is kept. Only a round chosen for now is raised, and a later one is re-decided each quarter. The first solve of case 8 has 45 variables (42 inputs and 3 slacks) and 286 rows. The Goldfarb–Idnani dual active-set solver [17] handles the 59 QPs of the case in a mean of 91.3 iterations and at most 169, with no failure. Over its first five years the plan meets its cash floor and its ceiling in its own prediction at all 19 plans. The plan's first block replaces each AUTO loop's request, its last plan stands in for the requests between plans, and the override still acts on it.

8.5.7 Company, Rules and Planner

Equations (8.1) to (8.8), with the disturbances of Table 8.2, are the simulated company. The loops hold almost no model, only the production lead time, the R&D pipeline's delay and the build chain's lag inside their feedforward terms, each at its declared default. The planner's model is the same right-hand side with every parameter at its declared default, linearized at its estimate of the present state, with sales smoothed, production bounded by the capacity it predicts, the covenant ceiling frozen at its present value, a round priced at the present equity value grown at g_0, the valuation linearized about the present growth, and no foresight of scheduled shocks. The models they stand for, Bass diffusion, Higgins's identity, the learning curve and the anchoring-and-adjustment rule, were fitted or derived in the works cited in Section 8.2. The application fits nothing to data of its own.

8.6 Measured Cases

All measurements in this section were regenerated on 2026-11-15 using the shipment-window accounting law of Section 8.3. The sensitivity counterfactuals use the same revised company.

Table 8.4 The Cases, the Challenge and the Reference

#	Change from the defaults	Teaching point
1	all MANUAL: u_R, u_M 0.15, u_p 40, u_K 1.5	insolvent while profitable
2	constant-ratio company, Higgins's rule, $k = 1$	the sustainable rate
3	$K_0 = 45$, only production in AUTO, feedforward off	the season as feedforward
4	capacity in MANUAL at \$0.4M/yr	the growth trap
5	R&D in MANUAL, cut from 0.15 to 0.05 at year 1	a pipeline hides a cost
6	customers pay in 75 days from year 1	profit without cash
7	rival launch $\times 1.15$ at year 2	two levers, fast and slow
8	economic MPC to an exit at year 6	dilution against speed
9	$L = 3$, $C_0 = 2$, \$2.5M loan at 1.5, recession at 2, runway MANUAL	a covenant that moves
ch	12 yr, recessions at 3 and 8, launch at 4, all MANUAL	everything at once
ref	125k repeat buyers, no season, nothing moves	the identities

Table 8.4 lists the cases. Each starts from the defaults of Tables 8.2 and 8.3, selected from the dropdown, with the change its row names. Each independent variant starts from a fresh scenario with the stated settings. A setting chosen before Run is applied to the opening board decision, with the same trajectory when it is carried through Reset. A loop switched to MANUAL holds the decision then in force, while an explicitly edited station keeps the learner's value. The comparisons distinguish those two manual plans. A run that becomes insolvent pauses there, and its later values are read after Run is pressed again. Table 8.5 collects the outcomes over each run's window, all measured on the engine headless. The scorecard also shows integrated errors, which grow with the length of the run and are therefore left out here. The exported file of Section 1.6.4 holds one row a week, with columns for each loop's setpoint, measurement, station, request and saturation, the override level, the statements, the valuation, the true demand beside the sales and the forecast, and the MPC's solves and failures. Playback runs the company at up to two years a second.

8.6.1 Insolvent While Profitable

Case 1 hands the board every station. The company is insolvent at 1.58 yr and its cash bottoms at $-\$0.38M$ at 1.79 yr, while its trailing revenue climbs to \$15.38M by year 4, growing 25.7 % a year, and its cash recovers to \$4.44M only because the model lets the company run on below zero. Reloaded and set before Run to 22 thousand units a year and \$0.5M of investment, the same company never falls below \$2.20M, with the same result through Reset, but sells \$6.60M in year 4, less than the \$7.26M it opened with, and fills 69.7 % of orders. At the first insolvency sample of the shipped plan, $t = 1.577$ yr, trailing revenue is \$8.37M, EBITDA \$1.11M and net income \$0.51M. The danger is visible only in the cash, because every outflow in Figure 8.1 buys an asset that the balance sheet counts and that a credit line sized on trailing EBITDA does not lend against.

> **TRY IT IN THE APP** **1 · The Board Game by Hand**
>
> Load the scenario and press Run on the Company view. Watch the cash tank empty until the status strip reads INSOLVENT, at about 1.58 yr, while trailing revenue is still rising. Then open the Statements view and find where the cash went: stock, factory and receivables.

Pause. Try Exercise 8.1 before reading on.

8.6.2 The Season and the Trap

Case 3 runs only the production rule on a factory of 45 thousand units a year that never limits it. With the feedforward off the shelf is short for 29 weeks of three years, 91.6 % of orders are filled and 6.98 thousand units are lost, and the first quarter after each peak is overstocked, 17.1 weeks of cover at 0.27 yr against the 6 wanted. With the feedforward on every order is filled and the cover

Table 8.5 Measured Cases Over Each Run's Window: Lowest Cash, First Insolvency, Trailing Revenue, Orders Filled, Valuation and Founders' Value at the End

Case and configuration	yr	C_{min} ($M)	insolv. (yr)	Rev_{TTM} ($M)	filled (%)	V ($M)	founders ($M)
1, as shipped	4	−0.38	1.58	15.38	98.8	79.5	67.1
1, u_p 22, u_K 0.5	4	2.20	—	6.60	69.7	14.8	14.4
2, $k = 1$	10	1.54	—	40.31	53.2	196.3	170.7
2, $k = 1.5$	10	−17.12	6.37	77.19	79.3	312.2	248.8
3, feedforward off	3	2.37	—	8.97	91.6	40.9	36.0
3, feedforward on	3	2.17	—	10.45	100.0	51.9	45.1
4, u_K 0.4 by hand	8	2.53	—	8.97	59.4	24.1	26.7
4, capacity AUTO	8	0.90	—	36.11	99.9	197.7	175.6
5, R&D cut to 0.05	5	0.96	—	9.84	99.7	25.6	26.3
5, R&D held at 0.15	5	1.07	—	15.36	99.8	76.8	64.0
5, R&D loop AUTO	5	0.99	—	16.01	99.8	86.7	72.0
6, override	4	0.79	—	11.57	99.7	56.6	47.1
6, override MANUAL	4	−2.03	1.88	13.14	99.7	69.3	55.1
6, on time, override	4	0.99	—	12.49	99.7	77.5	64.0
6, on time, MANUAL	4	−1.25	2.37	13.14	99.7	69.3	56.2
7, as shipped	8	0.82	—	11.85	98.5	51.8	43.7
7, $255 at the launch	8	0.73	—	14.40	91.5	62.6	51.6
7, price loop at 40 %	8	0.40	—	7.54	74.0	19.5	16.1
7, no launch	8	0.99	—	36.32	99.9	194.4	174.1
8, MPC, one round	6	1.88	—	29.41	100.0	177.3	140.0
8, MPC, no round	6	1.47	—	22.92	100.0	128.8	108.6
8, six loops	6	0.99	—	21.69	99.8	133.4	112.0
8, loops, $3M at yr 2	6	0.99	—	22.61	99.9	132.6	104.2
8, explicit manual plan	6	2.34	—	6.00	65.2	14.1	14.7
8, hold decisions in MANUAL	6	−1.35	4.02	3.45	54.4	6.9	4.4
9, runway MANUAL	5	−0.01	2.79	13.81	97.9	94.5	76.1
9, override, T^* 24	5	1.50	—	8.80	90.7	49.9	42.3
9, override, T^* 12	5	0.62	—	12.39	97.1	90.3	74.0
ref	8	3.00	—	3.75	100.0	7.8	8.6

Founders' value is $\omega[V + C - D]_+$, and so exceeds V where cash is large.

On time, held and no launch are case 8 under the six loops, with the runway or R&D loop in MANUAL where named, which has no scripted event. The $3M round is the round button pressed at year 2.

stays between 0.84 and 8.6 weeks. Feedback alone corrects a seasonal error one season late, which is out of phase.

Case 4 holds capital spending at $0.4M a year by hand. Revenue creeps from $7.62M at year 2 to $8.97M at year 8 while demand runs 1.32 times sales at year 2 and 2.65 times at year 8, and 59.4 % of orders are filled. By year 8, 112.7 thousand would-be customers have bought elsewhere and left the potential, against 191.2 thousand served. The marketing loop, measuring stalled sales, raises marketing from 7 % to 20 % of revenue and buys demand the factory cannot serve. With the capacity loop in AUTO the company reaches $36.11M at year 8 and 99.9 % of orders filled, and its valuation is $197.7M instead of $24.1M. A loop on sales cannot see the demand it turns away, and a utilization target of 100 % never gives the capacity loop a reason to build.

🕮 **Pause.** Try Exercise 8.2 before reading on.

8.6.3 Pipelines and Receivables

Case 5 cuts R&D from 15 % to 5 % of revenue at year 1. At 1.5 yr the cash is $2.20M against $1.95M with R&D kept, and the share is 0.449 in both runs. The market notices from year 2, 0.422 against 0.442 at 2.5 yr and 0.282 against 0.475 at year 5, when the quality lead is −0.19 against −0.02. The cut company holds more cash at year 5, $7.33M against $3.21M, but sells $9.84M a year instead of $15.36M and is valued at $25.6M instead of $76.8M. The company with R&D kept is case 8 under the six loops with the R&D loop in MANUAL at 15 %, which differs from case 5 only by the cut. The R&D loop in AUTO, which counts its own pipeline, reaches $16.01M and $86.7M.

Case 6 stretches collections from 45 to 75 days at year 1. At year 2 receivables stand at \$1.96M against \$1.30M on time, in case 8 under the six loops, the same company without the late payments, while trailing EBITDA is \$1.348M against \$1.406M. The override cuts marketing to nothing, its level passes 1 at the execution of 1.69 yr, the board draws the line from its meeting at 1.75 yr, at most \$0.31M, and the level peaks at 1.31, short of the band that cuts R&D, so R&D stays at 15 % of revenue and the lowest cash is \$0.79M against \$0.99M on time. With the override in MANUAL the company is insolvent at 1.88 yr, with a trailing EBITDA at year 2 of \$1.560M, exactly what it earns with customers on time, and cash of $-\$0.29M$ against \$0.52M. Without the override and with customers on time it is insolvent at 2.37 yr, because every loop in it spends before it earns and only the override sees them all.

Pause. Try Exercise 8.3 before reading on.

> **TRY IT IN THE APP** **6 · Customers Pay Late**
>
> Run the scenario for three years and watch the Cash card: the override level passes 1, the credit line is drawn and R&D stays at 15 % of revenue. Set the Runway loop and override to Manual, press Reset and run again. The company is insolvent at 1.88 yr.

8.6.4 Two Levers on Share

Case 7 has the rival launch a product 15 % better at year 2. As shipped, share falls from 0.446 to 0.246 by 2.85 yr, the R&D loop goes to its cap of 30 % of revenue and share recovers to 0.37 by year 8, with a valuation of \$51.8M against \$194.4M without the launch, case 8 under the six loops. A price cut to \$255 decided at the meeting of year 2, the launch's quarter, which offsets about 0.09 of quality deficit at $\beta_P/\beta_Q = 0.6$, holds share at 0.338 or better and reaches 0.569 by year 8 and a valuation of \$62.6M, with the lowest cash \$0.73M. Set once the launch is announced, the cut waits for the meeting of 2.25 yr, share first falls to 0.249 and the valuation reaches \$59.9M. Letting the price loop chase a 40 % margin drives the price to its \$210 floor at the meeting of 2.50 yr, and the lower price raises demand while reducing margin. With the setting chosen before Run, the company stays solvent over eight years, with minimum cash of \$0.40M, trailing revenue of \$7.54M, 74.0 % of orders filled and a valuation of \$19.5M. Solvency alone does not make that price policy a better outcome than the unchanged \$300 price. Two such launches at once, the launch button pressed at year 2 beside the scripted one, a factor of 1.32, take share below 0.04, and all five configurations measured, among them a price of \$240 or \$210 and the R&D loop at its largest gain of 2 with $\tau_I = 2$ yr, are insolvent between 2.79 yr and 4.27 yr. The design of the application first proposed that launch, and the case uses 15 % instead.

8.6.5 An Acquisition Planned by the MPC

Case 8 hands every station in AUTO to the economic MPC with an exit at year 6, while price stays in MANUAL at \$300. The plan at 1.5 yr chooses \$3M now, raised at 1.52 yr at a \$35.24M pre-money, reducing the founders from 80 % to 73.7 %. At year 6 trailing revenue is \$29.41M, growing 34.3 %, valuation is \$177.29M and cash is \$12.56M, giving founders' value of \$139.96M (Table 8.5), and Figure 8.3 shows the cash, the growth, the valuation and the founders' value as the app rounds them. Without the round the planner reaches \$108.63M, below the six loops' \$111.95M; the loops with a \$3M round forced at year 2 reach \$104.23M. The explicit manual plan reaches \$14.68M: R&D at 15 %, marketing at 10 %, production at 20 thousand units a year, investment at \$0.5M/yr, price at \$300 and no credit draw. Switching all loops to MANUAL without editing the stations instead holds the opening production decision of 11.5 thousand units a year and is insolvent at 4.02 yr, both before Run and through Reset. The MPC's round adds \$31.33M for the founders on this model. It buys both scale and faster growth: the bootstrapped planner grows 30.2 % on \$22.92M of revenue.

The funded planner builds 126.8 thousand units a year of capacity by year 6, and the bootstrapped planner 97.3 thousand, against the loops' 110.6 thousand. The bootstrapped local plan trails the

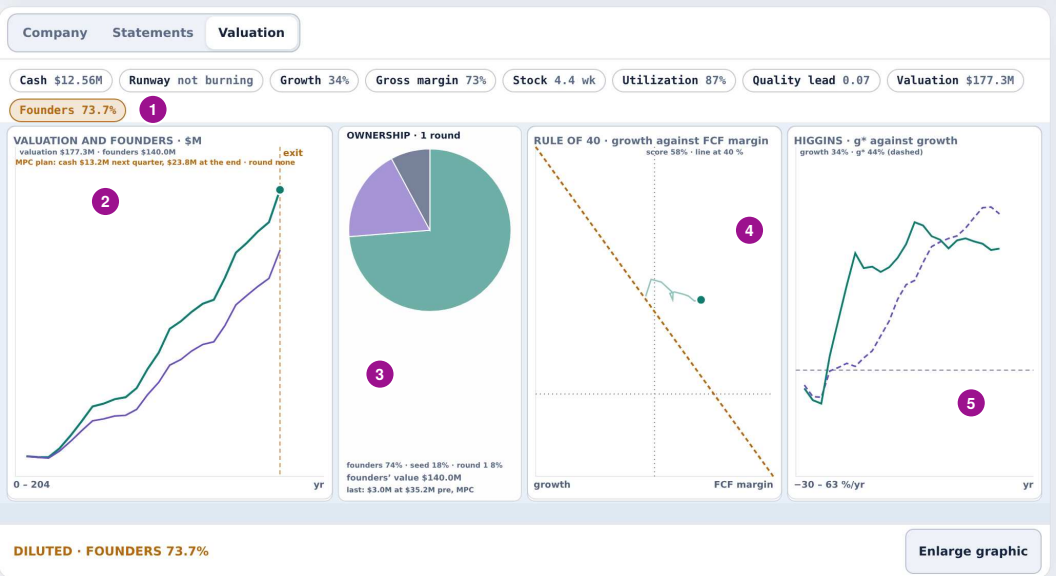


Figure 8.3 Case 8 at $t = 6.00$ yr in the Valuation view, left in AUTO as shipped, cropped to the view: the header, the control panel, the trends and the learning panel are cut. (1) The badges show cash \$12.56M, growth 34 %, gross margin 73 %, stock 4.4 weeks, utilization 87 %, quality lead 0.07, valuation \$177.3M and founders 73.7 %. (2) The valuation panel rounds founders' value to \$140.0M; its full value is \$139.96M. The last plan, made at 5.75 yr, expects cash of \$13.2M next quarter and \$23.8M at its end. (3) The ownership panel shows the round of \$3.0M at a \$35.2M pre-money. (4) Rule of 40 reads 58 % and (5) Higgins's g^* 44 %.

Table 8.6 Case 8's Founders' Value at Year 6 Under Changes to the Planner

Planner	one round ($M)	no round ($M)	round chosen
Current application			
$w = 1$, as shipped	140.0	108.6	\$3M at 1.52 yr
$w = 0.3$	168.2	134.2	\$3M at 1.52 yr
$w = 3$	148.8	124.5	\$3M at 1.52 yr
$w = 10$	70.8	94.9	\$3M at 1.77 yr
price loop in AUTO	96.6	93.9	\$3M at 3.52 yr
Counterfactuals, same revised company			
omit both fixes	153.8	165.0	\$3M at 1.52 yr
fixed production ceiling	63.6	62.3	\$6M at 1.52 yr

The six loops reach \$112.0M, or \$44.4M with the price loop in AUTO.
Each independent setting is chosen before Run; carrying it through Reset gives the same result.

loops in founders' value; predictive control does not guarantee dominance on the realized nonlinear company.

The counterfactual rows use the same accounting law and declared-model estimator. Omitting both the terminal-gradient factor $(1 + g_0)$ and the MANUAL-price hold yields \$153.8M with a round and \$165.0M without. Keeping the current formulation but bounding every future production decision by today's factory instead of its predicted capacity yields \$63.6M with a round and \$62.3M without; the funded factory reaches only 69.6 thousand units a year. The latter constraint makes an investment look like cash spent without the production it would enable.

The verdict on the round also moves with the departure weight w. It adds \$31.33M at $w = 1$,

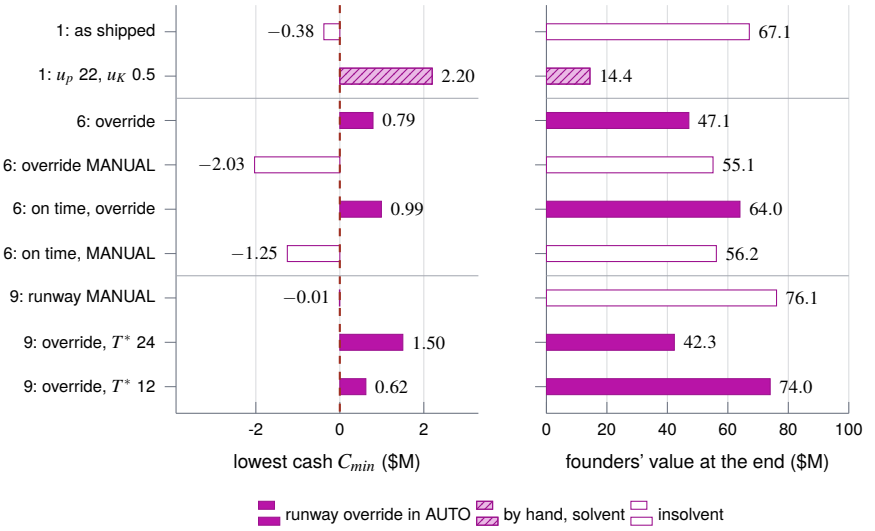


Figure 8.4 Lowest cash and founders' value of cases 1, 6 and 9 from Table 8.5, grouped by case (thin rules). The dashed line is the cash floor. Dark bars ran with the runway override in AUTO, the hatched bar by hand without insolvency, and white bars became insolvent, whose later values are counterfactual because the model runs on below zero. Notice that every run without the override in cases 6 and 9 falls below the floor, and that in case 9 the 24-month target keeps the most cash and the least founders' value.

$34.03M at 0.3 and $24.27M at 3, but loses $24.06M at 10, where both plans end below the six loops. With price in AUTO the planner reaches $96.61M with a round and $93.94M without, while the six loops reach $44.35M. An economic optimum on a linearized model belongs to that model and to the weights around it.

8.6.6 A Covenant That Moves

Case 9 opens with $2M of cash, borrows $2.5M on a $3M line at 1.5 yr and meets a recession at year 2 that takes 30 % of demand and 20 % of the potential for eighteen months. As trailing EBITDA falls the ceiling drops to $1.16M, the bank calls $0.87M from 2.75 yr, and with the runway loop in MANUAL the company is insolvent at 2.79 yr. With the override in AUTO and a 24-month target it cuts marketing and investment from its first year and repays the debt down to $0.83M by year 2, so only $0.17M is called and the Lowest cash tile rounds to $1.50M, while a 12-month target also holds with a lowest cash of $0.62M. The buffer costs growth, a founders' value at year 5 of $42.3M at 24 months and $74.0M at 12. With the credit line priced at the inflation-targeting application's replayed policy rate plus 5 points, 10.125 % at year 5, the unprotected company is insolvent at 2.73 yr. A covenant is a constraint whose bound moves with the plant's own output. Figure 8.4 sets the lowest cash and the founders' value of cases 1, 6 and 9 side by side.

Table 8.7 The Twelve-Year Challenge in Seven Configurations

Configuration	C_{min} ($M)	growth (%/yr)	filled (%)	V_{12} ($M)	Target
shipped, all MANUAL	−28.64	−23.0	100.0	0.6	not met
suggested, quarterly	0.85	20.7	97.7	272.5	met
suggested, monthly	1.00	20.6	98.8	284.5	met
suggested, weekly	1.08	20.6	99.1	286.6	met
suggested, cap 10 %	0.15	20.2	89.3	339.0	not met
Higgins's rule	2.13	17.7	79.5	306.8	not met
MPC, cash flow, $H = 5$	0.84	−5.7	99.5	6.5	not met

Insolvent at 4.40 yr as shipped, read at year 12 after Run past the pause.
Changes carried through Reset. Growth compounded from Rev_{TTM} before $t = 0$.

✦ DID YOU KNOW? A Longer Runway Target Costs the Founders

From the simulator: in case 9 the company with its runway loop in MANUAL is insolvent at 2.79 yr. The override with a 24-month target keeps its lowest cash at $1.50M but ends year 5 with a founders' value of $42.3M, against $0.62M and $74.0M with a 12-month target (Table 8.5).

8.6.7 Twelve Years Without Outside Money

The challenge runs twelve years on the quarterly calendar through recessions at years 3 and 8, a launch at year 4 and the season, and asks, judged at year 12, for no insolvency, no equity round, trailing revenue compounding at least 15 % a year from the $6.57M of the year before $t = 0$, at least 95 % of orders filled and a valuation of at least $150M. Table 8.7 compares seven configurations. The shipped board holds R&D and marketing at 10 %, production at 30 thousand units a year and investment at $1M a year, and is insolvent at 4.40 yr in the first recession, as production and investment run on while demand falls. The suggested settings put every loop except price in AUTO, start R&D at 15 % with a lead setpoint of 0.1 and a gain of 2, hold twelve weeks of stock and 70 % utilization with a capacity gain of 10, and pass with a lowest cash of $0.85M, 20.7 % growth, 97.7 % of orders filled and a valuation of $272.5M, identically before Run and through Reset. Its margins over the targets are 5.7 points of growth, 2.7 points of orders filled and $122.5M of valuation.

Deciding more often helps a little. Monthly, with 144 decisions, fills 98.8 % of orders with a lowest cash of $1.00M, and weekly, with 624, 99.1 % with $1.08M, while growth stays at 20.6 %, likely because production and capacity follow the season sooner. A move cap of 10 % of each station's range per decision fills only 89.3 % and fails, while a quantum of zero or three changes the fill by at most two tenths of a point. Higgins's rule on the same loops grows 17.7 % a year with $2.13M at the lowest and fills only 79.5 % of orders, which makes it sustainable by turning customers away. The economic MPC with the cash-flow objective over five years harvests the company and shrinks it 5.7 % a year. All these independent configurations give the same result before Run and through Reset. An earlier version of the challenge asked instead for stock never below two weeks, a floor that a policy can meet by rationing sales, and it rewarded such a policy.

The balanced reference stands still. Cash holds at exactly $3.00M for eight years and equity at $5.10M, with EBITDA of $0.15M replacing the factory as it wears out and net income zero, and customers paying late there move cash into receivables one for one.

8.7 Technology in Practice

A real company reads the same stocks and moves the same handles as the board of Section 8.5, but its instruments are accounts, statements and contracts. Figure 8.5 draws the runway override with those instruments in place of the schematic's symbols.

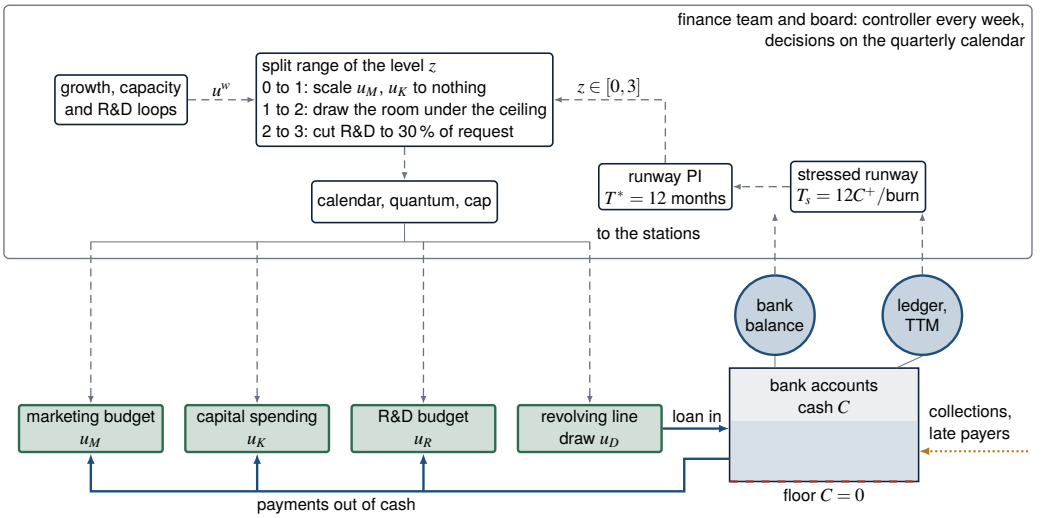


Figure 8.5 The runway override of Section 8.5 with a company's real instruments. Green boxes are the final elements, three budget lines and the draw on a revolving line of credit, blue circles the measurements, the bank balance and the ledger kept on trailing twelve months (TTM). Solid arrows carry money, dashed arrows signals, and the dotted arrow the collections that a late payer delays. Notice that one level z acts on four stations in a fixed order, and that its request waits for the board's calendar before any station moves.

What Measures It

The controlled variable of the override is a runway, cash divided by a burn, and both come from the books. Most firms use accrual accounting, and the statement of cash flows most often follows the indirect method, which starts from net income and adjusts it for noncash expenses and for changes in receivables, inventory and payables. Income statements of publicly traded companies are available quarterly [18]. Receivables are watched through an aging schedule, which sorts unpaid invoices by the days since the sale and shows which customers pay late [18]. The trailing twelve-month ring of Section 8.3 stands for that bookkeeping, kept weekly and exactly.

> **TECHNOLOGY Sensor: The Cash Balance and the Cash Budget**
>
> A company's cash holdings are mostly checks waiting to be deposited and the balances of its checking accounts. Profit is a different reading: under accrual accounting a sale is recorded when it occurs, not when cash moves, so a firm can be profitable and still lack an adequate flow of cash, and the statement of cash flows reconciles the two [18]. A ratio taken at one date says little about future liquidity [18], so the cash budget projects the balance forward, divided into the shortest period possible [19]. The model's sensor is its own books on trailing twelve months, from which the override reads a stressed runway. A late payer shows there first as receivables that rise while EBITDA holds.

What Moves It

The board's stations are budget lines and a draw on a bank line. The master budget has an operating part, which projects revenue and expenses, and a financial part, which projects assets, liabilities and cash, and its most common period is one year [19]. Credit terms are the disturbance side of the same ledger: net terms usually range between 30 and 90 days, depending on the industry [18], and case 6 moves the model's collections from 45 to 75 days inside that range.

> **TECHNOLOGY** **Final Element: Budgets and a Revolving Credit Line**
>
> Companies often arrange a line of credit as an emergency source of cash, much like overdraft protection on a checking account [19]. Under one federal loan guarantee program an asset-based revolving line lets a business draw continually against its existing assets and repay as its cash cycle dictates, and a revolving loan to a manufacturer is reviewed each year: an unsatisfactory review converts it to a term loan with no further draws [20]. The line may also be out of reach. Of small employer firms that applied for financing in the year before the 2025 survey, 22 % received none [21]. The model's line has a ceiling of three times trailing EBITDA, so it moves with the company's own earnings.

Where the Controller Runs

A company's bylaws determine how often its board meets [18]. The cash budget is commonly geared toward monthly or quarterly figures, and a rolling budget is adjusted each month, with a new month added as each one passes [19]. The application's controller executes every week, and its decisions pass the quarterly calendar of `core/cadence.js` as shipped, so the override's request reaches the stations at the next meeting. The structure lives in the cash budget's minimum balance, and the split range of Equation (8.13) writes down in advance what gives way first.

What Is Modernizing It

Three developments bear on this loop. Lending has moved online: among small employer firms that applied for loans, lines of credit or cash advances, the share that sought financing at online lenders rose from 17 % in the 2020 survey to 29 % in the 2025 survey, and 60 % of those that borrowed there reported costs higher than expected [21]. Artificial intelligence has reached planning: 46 % of firms in the same survey use it, 51 % of users for planning or analysis, and accuracy was the challenge users named most [21]. Faster measurement shortens the loop: electronic invoicing most likely cuts the average collection period by removing mail delays [18]. The economic MPC of Section 8.5 adds a planner whose stage cost is money [7], and case 8 shows how far its verdict depends on its own weights.

> **IN PRACTICE** **A Minimum Cash Balance in the Cash Budget**
>
> A cash budget adds the period's receipts to the beginning balance and subtracts its payments to give a potential ending balance. Organizations typically require a minimum cash balance: if the potential ending balance falls short, management plans financing to reach it, and an excess may repay loans and interest [19]. A bank may itself require a minimum balance in the company's checking account [18]. Forecasts are redone for scenarios such as a recession, and sensitivity analysis changes one input at a time, sales 10 % lower, for example [18]. The runway override turns that review into a standing rule.

8.8 Limitations

The company is one product against one rival that never responds, its delays are distributed lags, and its finance omits loss carry-forward, preferences and the clauses of a real loan. Every coefficient, multiple, threshold and rule, the challenge's 15 %, 95 % and $150M included, is a teaching value for an imaginary hardware startup, and nothing here is investment or business advice. Higgins's g^{*} is a bound only where its ratios hold, as in case 2, and in the default company the learning curve, the overhead and the rival move the ratios. The valuation multiple, the Rule of 40 and the runway are teaching rules with a practitioner's shape, not appraisals. Cash may fall below zero and the company runs on, so any recovery after INSOLVENT is a counterfactual. The economic MPC is local and successively linearized, holds the declared parameters, so that a changed parameter or a shock reaches it only through the books and the sales, and does not preview scheduled shocks, and its result in case 8 moves between $70.8M and $168.2M across the departure weights of Table 8.6.

OPEN CHALLENGE **Financing That May Not Be There**

Startup outcomes are hard to predict even for experienced investors: at one venture capital firm the scores that partners gave at the first investment correlated with the return multiple at only 0.1. Investors also speak of financing risk, the chance that an otherwise sound project cannot raise capital for its next stage. Taking larger sums less often protects against it but reduces the value of the option to abandon, and it bites hardest on experimental new technologies that must return to the market many times [9]. In case 8 a round the planner chooses is always raised. A planner that keeps its cash floor when the round may not come would be progress.

8.9 Conclusion

The simulator turns a startup into four coupled inventories, cash, stock, capacity and quality, with one floor, and measures what each decision costs in the tank. A board that grows by hand is insolvent at 1.58 yr with trailing EBITDA of $1.11M. On books kept on trailing twelve months Higgins's rate equals year-over-year growth exactly, a company held to it keeps its cash near 40 % of equity, and at 1.5 times the rate every split that earns money runs out of cash between 5.33 yr and 12.37 yr. The season's feedforward fills every order where feedback fills 91.6 %, a factory held at replacement fills 59.4 %, and the runway override survives late payments by drawing $0.31M without cutting R&D. The economic MPC reaches $139.96M at an acquisition with a round and $108.63M without; the latter trails the six loops' $111.95M, and the verdict on the round changes sign with the departure weight. In the challenge the suggested loops pass at 20.7 % growth and 97.7 % of orders filled where the board's fixed split fails at 4.40 yr. The model is one product, its finance is simplified and its values are for teaching. A fuller model would add a rival that answers, loss carry-forward and loan clauses, an estimator for the hidden demand, and a planner that previews scheduled shocks. The application makes the cost of growth a measurement in the one stock that ends the run.

KEY POINTS

- A company can book a profit and still run out of cash, because stock, factory and unpaid invoices are paid for before customers pay.
- At 1.5 times Higgins's rate every budget split that earns money runs out of cash, between 5.33 yr and 12.37 yr.
- The runway override scales marketing and capital spending to nothing, then draws the credit line, then cuts R&D.
- With customers paying late the override survives by drawing $0.31M without cutting R&D. Without it the company is insolvent at 1.88 yr.
- The verdict of the economic MPC on the equity round changes sign with the departure weight.

Where else this applies. Chapter 14 defends a reserve with a floor of its own, and Volume 6, Chapter 11 runs the same anchoring-and-adjustment ordering rule on a supply chain.

References

[1] C. C. Holt, F. Modigliani, and H. A. Simon, "A linear decision rule for production and employment scheduling," *Management Science*, vol. 2, no. 1, pp. 1–30, 1955.

[2] M. L. Vidale and H. B. Wolfe, "An operations-research study of sales response to advertising," *Operations Research*, vol. 5, no. 3, pp. 370–381, 1957.

[3] M. Nerlove and K. J. Arrow, "Optimal advertising policy under dynamic conditions," *Economica*, vol. 29, no. 114, 1962.

[4] J. D. W. Morecroft, "System dynamics: Portraying bounded rationality," *Omega*, vol. 11, no. 2, pp. 131–142, 1983.

[5] R. C. Higgins, "How much growth can a firm afford?" *Financial Management*, vol. 6, no. 3, 1977.

[6] N. C. Churchill and J. W. Mullins, "How fast can your company afford to grow?" *Harvard Business Review*, vol. 79, no. 5, 2001, PMID 11345910.

[7] D. Angeli, R. Amrit, and J. B. Rawlings, "On average performance and stability of economic model predictive control," *IEEE Transactions on Automatic Control*, vol. 57, no. 7, pp. 1615–1626, 2012.

[8] D. E. Seborg, T. F. Edgar, D. A. Mellichamp, and F. J. Doyle, III, *Process Dynamics and Control*, 4th ed. Hoboken, NJ: John Wiley & Sons, 2016.

[9] W. R. Kerr, R. Nanda, and M. Rhodes-Kropf, "Entrepreneurship as experimentation," *Journal of Economic Perspectives*, vol. 28, no. 3, pp. 25–48, 2014.

[10] N. P. Repenning and J. D. Sterman, "Capability traps and self-confirming attribution errors in the dynamics of process improvement," *Administrative Science Quarterly*, vol. 47, no. 2, pp. 265–295, 2002.

[11] T. P. Wright, "Factors affecting the cost of airplanes," *Journal of the Aeronautical Sciences*, vol. 3, no. 4, pp. 122–128, 1936.

[12] F. M. Bass, "A new product growth for model consumer durables," *Management Science*, vol. 15, no. 5, pp. 215–227, 1969.

[13] F. M. Bass, T. V. Krishnan, and D. C. Jain, "Why the Bass model fits without decision variables," *Marketing Science*, vol. 13, no. 3, pp. 203–223, 1994.

[14] V. D. Richards and E. J. Laughlin, "A cash conversion cycle approach to liquidity analysis," *Financial Management*, vol. 9, no. 1, 1980.

[15] S. C. Myers, "The capital structure puzzle," *The Journal of Finance*, vol. 39, no. 3, pp. 574–592, 1984.

[16] P. A. Gompers, W. Gornall, S. N. Kaplan, and I. A. Strebulaev, "How do venture capitalists make decisions?" *Journal of Financial Economics*, vol. 135, no. 1, pp. 169–190, 2020.

[17] D. Goldfarb and A. Idnani, "A numerically stable dual method for solving strictly convex quadratic programs," *Mathematical Programming*, vol. 27, no. 1, pp. 1–33, 1983.

[18] J. Dahlquist and R. Knight, *Principles of Finance*. Houston, TX: OpenStax, 2022. [Online]. Available: https://openstax.org/books/principles-finance/pages/1-why-it-matters

[19] M. Franklin, P. Graybeal, and D. Cooper, *Principles of Accounting, Volume 2: Managerial Accounting*. Houston, TX: OpenStax, 2019. [Online]. Available: https://openstax.org/books/principles-managerial-accounting/pages/1-why-it-matters

[20] "SBA lenders: Types of 7(a) loans," U.S. Small Business Administration, 2026, accessed 2026-11-21. [Online]. Available: https://www.sba.gov/sba-lenders/#types-7a-loans

[21] Federal Reserve Banks, "2026 report on employer firms: Findings from the 2025 small business credit survey," Federal Reserve Banks, Small Business Credit Survey, Tech. Rep., 2026.

8.10 Exercises

8.1 Concept check · 2 min

In case 1 the run pauses at INSOLVENT at 1.58 yr. The trailing net income is then \$0.51M, nothing has been borrowed and every dollar of earnings is retained. Where did the opening \$3M of cash go?

(a) Into interest and tax, which are paid before the revenue they follow is collected

(b) Into dividends, since the board pays out the trailing net income every quarter

(c) Into a larger factory and a growing stock of finished units, bought before customers pay

(d) Into operating losses, which the trailing income statement reports as a profit

8.2 Concept check · 1 min

In case 4 the board holds capital spending at \$0.4M a year. Between year 2 and year 8 the growth loop raises marketing from 7 % to 20 % of revenue. Why does the loop keep spending more?

(a) The capacity loop, in AUTO, asks for more demand to fill the factory it builds

(b) The runway override releases marketing as cash piles up in the tank

(c) The R&D loop loses the quality race, and the growth loop makes up the lost share

(d) It measures sales, which the factory caps, so it sees stalled growth and buys demand it cannot serve

8.3 Concept check · 2 min

In case 6 the runway override's level peaks at 1.31. Which describes the stations at that moment?

(a) Marketing and capital spending at zero, the credit line being drawn, R&D at its loop's request
(b) Marketing at a third of its request, capital spending untouched, no draw, R&D at its request
(c) Marketing and capital spending at zero, no draw, R&D cut to a third of its request
(d) Every request divided by 1.31, so marketing, capital spending and R&D fall in step

8.4 Theory and calculation · 6 min

A company grows with constant ratios at the continuous rate γ, so its equity is $E(t) = E(0)e^{\gamma t}$ and it retains $bNI = \dot{E}$.

(a) Show that $bNI_{TTM}/E = 1 - e^{-\gamma}$ on books closed on trailing twelve months, and that Higgins's rate $g^* = x/(1-x)$ then equals the year-over-year growth $e^{\gamma} - 1$. *Checkpoint:* $x = 1 - e^{-\gamma}$, so $x/(1-x) = e^{\gamma} - 1$.

(b) Case 2 reports $g^* = 22.2\,\%$ at year 4. Find γ and the retained return x behind it. *Checkpoint:* $\gamma = 0.2005\,\text{yr}$ and $x = 0.1817$.

(c) At the opening close of case 2, $g^* = 26.8\,\%$ and $K = 25$ thousand units a year. With $c_K = \$0.1M$ per thousand units a year, $\delta_K = 0.1\,\text{yr}$, $\tau_b = 0.75\,\text{yr}$ and $k = 1$, find the cap that Higgins's rule puts on capital spending, and the cap without the factor $(1 + r\tau_b/3)^3$, where $r = \ln(1 + kg^*)$. *Checkpoint:* \$1.00M a year, and \$0.84M a year without the factor.

8.5 Theory and calculation · 5 min

Receivables follow $\dot{AR} = \text{Rev} - (365/\text{DSO})AR$ of Equation (8.6), and the credit ceiling is $\bar{D} = \min(Lm_L,\ \kappa[\text{EBITDA}_{TTM}]_+)$.

(a) At the default decisions the company opens with \$6.57M of trailing revenue, a trailing EBITDA of \$0.14M, DSO of 45 days, $L = \$1M$, $m_L = 1$ and $\kappa = 3$. Find the receivables in steady state and the credit ceiling. *Checkpoint:* \$0.81M and \$0.42M.

(b) The balanced reference sells \$3.75M a year. Customers stretch from 45 to 75 days. Find the cash that moves into receivables, and the time constant of the receivables before and after. *Checkpoint:* \$0.308M, with 0.123 yr (6.4 weeks) before and 0.205 yr (10.7 weeks) after.

(c) Explain why the ceiling does not rise to finance the receivables of part (b).

8.6 Theory and calculation · 6 min

The share is the logit of Equation (8.1) with $\beta_Q = 5$, $\beta_P = 3$ and $P_c = \$300$, and the competitor's quality is $Q_c = (1 + g_c t)\,m_Q$ with $g_c = 0.08\,\text{yr}$.

(a) Show that at a fixed price $\mathrm{d}s/\mathrm{d}(Q - Q_c) = \beta_Q s(1-s)$, and find the change in share for a lead of 0.1 point at $s = 0.449$. *Checkpoint:* 0.124.

(b) In case 7 the launch at year 2 sets $m_Q = 1.15$. Find the jump in Q_c and the share just after it from $s = 0.446$, with the quality and the price unchanged. *Checkpoint:* 0.174 and 0.252.

(c) Find the price that would offset the whole jump, and the quality deficit that the cut to \$255 of case 7 offsets. *Checkpoint:* \$213 and 0.09 point.

8.7 App challenge · 9 min

Load scenario 1, The board game by hand, at the default seed.

(a) Press Run and let the run go on until it pauses at INSOLVENT. Read the clock, and export the CSV file. In its last row read `revenue_ttm_musd`, `ebitda_ttm_musd` and `ni_ttm_musd`. *Checkpoint:* 1.58 yr, then \$8.37M, \$1.11M and \$0.51M, within 0.01.

(b) From the first and last rows of the same file, find the change in `cash_musd`, `fixed_musd`, `stock_book_musd`, `wip_book_musd`, `AR_musd`, `AP_musd` and `equity_musd`, and show that they account for the cash. *Checkpoint:* -3.02, $+1.82$, $+1.17$, $+0.10$, $+0.01$, $+0.09$ and -0.01 \$M.

(c) Press the reload button, set Production rate to 22 and Capacity investment to 0.5 in their cards, and run to year 4. Read the Lowest cash and Orders filled tiles and `revenue_ttm_musd` at year 4. *Checkpoint:* \$2.20M, 69.7 % and \$6.60M, within 0.01.

Is the plan of part (c) a better plan?

8.8 App challenge · 8 min

Load scenario 6, Customers pay late, at the default seed. The runway override is in AUTO with Override gain 0.15.

(a) Run three years. Read the Lowest cash tile, and in the exported CSV file find the largest `override_level` and `debt_musd` and the smallest `rd_share`. *Checkpoint:* \$0.79M, then 1.31, \$0.31M and 0.15, within 0.01.

(b) Set the Runway loop and override to Manual, press Reset and run until the run pauses at INSOLVENT. Read the clock. *Checkpoint:* 1.88 yr, within 0.02.

(c) Press the reload button, set Override gain to 0.50 and run three years. Read the same four values. *Checkpoint:* \$1.09M, then 3.00, \$0.98M and 0.05, within 0.01.

What does the higher gain buy, and what does it give up?

8.9 App challenge · 9 min

Load the challenge, Twelve years without outside money, at the default seed. For each run read the Lowest cash, Orders filled and Challenge tiles, and in the exported CSV file `revenue_ttm_musd` and `valuation_musd` at year 12. Compute the growth as $(\mathrm{Rev}_{TTM}(12)/6.57)^{1/12}-1$.

(a) Run as loaded until the run pauses, and read the clock. *Checkpoint:* INSOLVENT at 4.40 yr.

(b) Press Reveal suggested tuning, press Reset and run twelve years. *Checkpoint:* \$0.85M, 97.7 %, met, \$62.54M and \$272.5M, growth 20.7 %.

(c) Set Decision policy to Higgins, press Reset and run twelve years. *Checkpoint:* \$2.13M, 79.5 %, not met, \$46.44M and \$306.8M, growth 17.7 %.

Which target does run (c) miss, and why does a sustainable rule miss it?

8.10 Application area · 4 min

In case 9 the company borrows \$2.5M on a \$3M line, and the bank calls any debt above three times trailing EBITDA over a quarter.

(a) Find the trailing EBITDA below which the bank starts to call the loan, and the trailing EBITDA at the bottom of the recession, when the ceiling is \$1.16M. *Checkpoint:* \$0.83M and \$0.39M.

(b) Using the chapter's results and limitations, say what the 24-month runway target buys and costs, and what a real loan agreement adds that the model leaves out.

8.11 Application area · 4 min

A seller into a holiday peak runs its factory flat out all year at the mean demand of 21.9 thousand units a year. The season of Equation (8.2) has quarter means of 0.789, 0.806, 0.924 and 1.481.

(a) Find the stock the factory must have built by the start of the fourth quarter to fill every order in it, and its book value at the opening unit cost of \$150. *Checkpoint:* 2.63 thousand units and about \$0.39M.

(b) Name two costs of such a pre-build that the model leaves out, and say what case 3 shows about feedback without the calendar.

8.12 Review the agent · 5 min

An AI assistant reads the results of case 8 in the current application and writes: "The economic MPC raised $3M at 1.52 yr and ended with founders' value of $139.96M, against $108.63M for the same planner without a round. The round is worth $31.33M to the founders, so the company should raise it." Find the flaw and say how you would check it.