INTERACTIVE CONTROL ENGINEERING
Principles and Applications

Volume 4 Override, Split-Range, and Constraint Control

CHAPTER 12

Allocating Tires, Suction Fans and Cold-Gas Thrusters on One Nitrogen Tank

Pages 297–319

Edited by John D. Hedengren

How to cite this chapter. "Allocating Tires, Suction Fans and Cold-Gas Thrusters on One Nitrogen Tank," in *Override, Split-Range, and Constraint Control*, John D. Hedengren, Ed., vol. 4 of *Interactive Control Engineering: Principles and Applications*. Advanced Process Solutions, LLC, 2026, ch. 12, pp. 297–319.

The page numbers in this file are those of the printed volume.

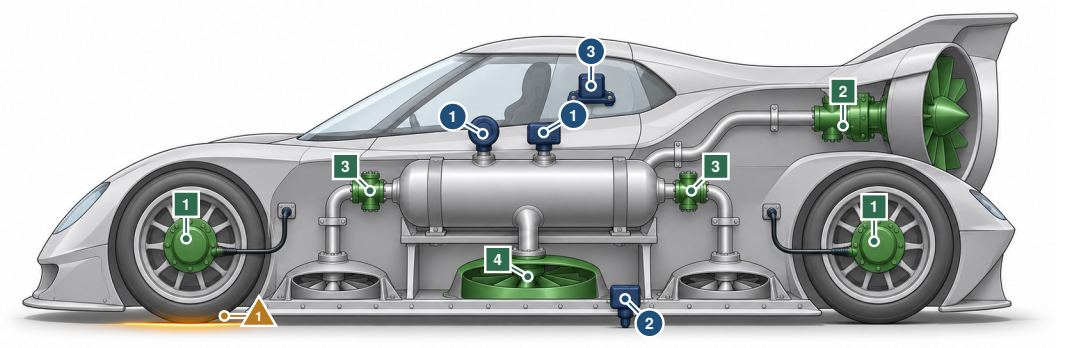
The app for this chapter
https://apmonitor.com/apps/performance-auto/

The volume online: contents, other chapters and citation records
https://apmonitor.com/apps/books/v04-override-constraint/#performance-auto

Copyright © 2026 Advanced Process Solutions, LLC. All rights reserved. Produced by Advanced Process Solutions, LLC.

CHAPTER 12

Allocating Tires, Suction Fans and Cold-Gas Thrusters on One Nitrogen Tank



The controller moves 1 motors that drive the wheels 2 valves of the rear thrusters 3 valves of the corner lift thrusters 4 fan that sucks the floor down

The controller reads 1 tank pressure and temperature 2 height of the floor 3 tilt and turn of the body

What disturbs it 1 a wet patch on the strip

Notice that one tank feeds the rear and the corner thrusters, so the gas a fast launch spends is gone for the flight. Illustration, not to scale.

IN PLAIN TERMS **Three Ways to Push, One Tank of Gas**

The main idea. A test car pushes itself along with its tires, with fans that suck it onto the road for more grip, and with cold gas jets from one tank. The controller uses the free tires first and spends gas only on what they cannot give.

The challenge. The tank cannot be refilled, so gas spent on a fast start is gone for the flight. Escaping gas cools, its pressure falls faster than its amount, and the jets weaken while the gauge still shows gas left.

Why it matters. A machine that draws several actions from one store must keep its reserve in what runs out first.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the fact in the Introduction (Section 12.1), the history segment (Section 12.2), the try-it segment on the inverted ramp (Section 12.4), the try-it segment and the fact in the results (Section 12.6) and the key points
Study	50 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 12.6), Technology in Practice (Section 12.7), Limitations (Section 12.8) and the open challenge

The control question A car is pressed onto the road by suction fans and lifted by cold-gas thrusters, so free tires, cheap fans and costly nitrogen share one demand. What does each allocation choice buy and cost from a tank that cannot be refilled, and why does that tank run out by pressure before it runs out of mass?

Learning objectives

- Compute the suction of a sealed skirt and the tire force it makes available, and explain why the seal is a positive feedback.
- Allocate a launch to the free actuator first and the costly one on the remainder, and price a g target in nitrogen.
- Compute the gas cost of a hover and the runway of a tank, and explain why a runway taken from the gauge's mass is optimistic.
- Compare an integral alone with a mass feedforward on a vehicle that grows lighter as it flies.
- Recognize a demand the tank cannot meet, and set the reserve on the quantity that runs out first.

Prerequisites Sections 1.4.6 (split range), 1.4.8 (allocation), 1.4.9 (depleting actuators), 1.4.5 (an actuator that loses its gain) and 1.6.2 (velocity-form PID).

The app https://apmonitor.com/apps/performance-auto/

See also Volume 20, Chapter 4 for allocation among rotors and a wing whose authority moves with airspeed, Volume 20, Chapter 3 for a load feedforward on a multirotor, and Volume 20, Chapter 7 for actuators that store what they apply and must be emptied.

12.1 Introduction

The app of this chapter, Performance Auto, takes its premise from a 2026 public demonstration of a production-car prototype, a road car with ducted fans that hold it down and compressed nitrogen that lifts it. The generic car built here weighs 1723 kg with 423.0 kg of nitrogen in an 850 L tank at 700 bar. On its tires and fans it reaches 60 mph in 1.47 s. With both rear thrusters wide open it gets there in 0.60 s, at a peak of 4.83 g and for 40.5 kg of nitrogen, while thrust metered to 3.4 g takes 0.83 s and 29.1 kg. A 5 s hover on a full tank costs 114.1 kg, more than a quarter of the fill.

Problems in which several actuators of different speed, limit and cost share one demand are called control allocation, and Section 1.4.8 sets out the general problem. They began with aircraft that carry more control surfaces than axes to control [1] and now cover ships, road vehicles and spacecraft, with costs, limits and failures built into the allocation law [2]. A finite onboard consumable adds a budget to the allocation, the depleting actuator of Section 1.4.9. Cold-gas thrusters, the simplest rocket, are still chosen for small satellites because they are clean and reliable [3, 4], and every newton-second they deliver is drawn from a pressure vessel that empties as it cools. A process engineer meets the same shape in split-range control of a cheap and an expensive utility [5] (Section 1.4.6), and in a plant whose inventory must last a campaign.

✦ DID YOU KNOW? Cold-Gas Thrusters Start at Micronewtons

A 2025 survey of small-spacecraft propulsion lists cold-gas thrusters from 10 µN to 3.6 N, with specific impulses of 40 to 110 s [6]. Each rear thruster of this chapter's car is rated at 25 kN, at an assumed 73 s.

The car is built to correct two beliefs. The first is that the strongest actuator should be used whenever it helps. Opening the thrusters wide saves 0.23 s on the metered launch, spends 11.4 kg more nitrogen and passes the 3.5 g teaching limit. The second is that a tank gauge reports what is left to use. The gas cools as it expands, its pressure falls faster than its mass, and the thrusters are rated at the regulator's pressure, so a 12 s hover ends in a landing at 2.83 m/s with 22.6 % of the nitrogen still in the tank. A transfer function cannot show either mistake, because one is a choice among actuators and the other is a state of the actuator's supply.

Teaching material usually treats these pieces apart. Tire slip is taught on a braking corner with a fixed normal force [7], thruster modulation on a spacecraft in free space [8], and descent guidance on a vehicle whose propellant is not scored [9]. This chapter works with one simulated car that is a ground vehicle and an aircraft in the same run, and it quantifies what each allocation choice buys and what it costs from a single tank. The tire curve, the velocity-form proportional-integral-derivative (PID) loop, the pulse-width modulation of the valves and the descent schedule are established methods. The results are measurements on a single declared car, among them the findings that the tank runs out by pressure before mass, that a bang-bang roll entered at 5.1 m/s of climb ends 1.77 m above its entry rather than level in height, and that metering the launch thrust saves 28 % of its gas at a cost of 0.23 s. The model has 24 states and a tank balance that closes to 4×10^{-12} kg, its gas law reproduces the reference density at the fill to 0.12 %, and a strict split separates what the laws may read from what only the scorecard sees.

Section 12.2 traces the domain: downforce, cold-gas propulsion, allocation and the few control studies of vehicles that both drive and fly. Section 12.3 gives the car, from the tire and the skirt to the tank and the instruments. Section 12.4 draws from the model the three prices and the one budget that shape every choice, and Section 12.5 describes the laws as implemented. Section 12.6 measures ten cases, from a hand-flown launch to a scored one-tank flight. Section 12.7 sets the car's devices beside real hardware, Section 12.8 states the limitations and Section 12.9 concludes. The exercises at the end of the chapter run the cases on the app.

12.2 Suction, Cold Gas and the Allocation of Thrust

Downforce entered racing as a way to add normal force without adding mass. Inverted wings came first, then underbody shapes that accelerate the air between the floor and the track, and in some cars fans that evacuate a sealed space under the body so that the grip no longer depends on speed [10]. Fan suction raced briefly in the 1970s before the rules removed it, while underbody downforce remains the main source of grip in open-wheel racing [10]. The physics of the seal is older and comes from the air-cushion vehicle, whose fans pressurize a skirted cushion and whose fan flow is set by the leakage under the skirt hem [11]. The fan car runs that theory in reverse, with the cushion below ambient and the same leakage, so a hem that lifts a few centimeters lets the suction go.

Cold-gas propulsion is the simplest form of reaction control. A regulated gas is expanded through a nozzle without combustion, and nitrogen gives a specific impulse of the order of 70 s, low against chemical rockets but clean, cheap and safe to handle [12]. Small satellites still carry such systems for attitude control and fine maneuvers [3, 4]. Their tanks are blowdown vessels, and a vessel emptied in seconds behaves almost adiabatically, so the gas left inside cools by tens of kelvins and its pressure falls faster than its mass [13]. At 700 bar nitrogen is far from ideal, and the tables that a simulator can be checked against come from a reference equation of state [14]. On-off thruster valves cannot open for less than a minimum time, and pulse-width pulse-frequency modulation was developed to deliver a commanded average impulse from such valves [8]. The same quantization was flown with a pilot aboard, as the history segment after this paragraph tells.

> ⧖ HISTORY **1967: Rocket Pulses That Hunted in Flight**
>
> On-off thrusters were flown with a pilot aboard long before this car. A free-flight lunar-landing research vehicle carried a vertically mounted jet engine to create a pseudolunar gravity field, and its attitude rockets were fired by on-off solenoid valves. A switching amplifier opened them when the error between the pilot's rate command and the measured rate crossed a threshold. A describing-function analysis mapped the rate dead bands and control authorities that would give a limit cycle, and flight and ground tests in general agreed with it. Pilots found the cycling did not spoil control, but the noise of the rockets firing at about 2 Hz distracted them. In a fixed-base simulator the attitude propellant was least at one middle dead band and rose at smaller ones, partly through limit-cycle operation [15].

Allocation as a control problem dates from aircraft with redundant control surfaces, where the attainable moment set and its saturation limits were analyzed explicitly [1]. Later work added actuator rates, costs, priorities and failures, and today's allocation laws in marine, automotive and aerospace practice share that formulation [2]. The descent to a landing has its own lineage. The Apollo lunar module flew a guidance law that shaped its altitude and vertical speed toward a gentle touchdown [16], and modern powered descent poses the same problem as a convex trajectory optimization with propellant as the cost [9]. On the ground the friction available to a driven wheel peaks at a small slip and falls past it, the shape that traction control, launch control and anti-lock braking all regulate against [7], and the slip builds over a rolling distance, the tire's relaxation length [17]. Every limit the simulator scores is a teaching value. Real acceleration limits come from human-tolerance data [18], and real touchdown limits from the sink rate a landing gear is designed to absorb [19].

Vehicles that both drive and fly have a control literature of their own, though a short one. A winged flying-car prototype has flown under a single nonlinear position and attitude law whose forces and moments a real-time allocation spreads over distributed propellers and control surfaces [20]. A fly-drive vehicle has been modeled with its rotors, tires, chassis and suspension in one set of equations, and in hardware-in-the-loop simulation its rotors steady the suspension after touchdown through a predictive controller over an allocation layer [21]. A land-air simulation shows the wheel loads falling to zero through the switch from driving to flight and an oscillation once the car is clear [22], and a hybrid vehicle has tracked ground and air trajectories under one adaptive attitude law in experiments [23]. Where the allocation carries a price, a flight-tested law for an over-actuated transition aircraft minimizes total power in place of a norm of the commands [24], and Volume 20, Chapter 4 allocates among the rotors, wing and propeller of a lift-plus-cruise aircraft whose effectors gain and lose authority with the airspeed. In process systems engineering the subject is thin. A search of seven of its journals returned no paper on flying cars, roadable aircraft or fly-drive vehicles, and the nearest works are of two kinds. One is nonlinear model predictive control of a solar aircraft that stores energy in batteries and in altitude [25]. The other is split-range control, which extends the range of one controlled variable by moving through several inputs one at a time [5]. The car sits between them. Its launch is a split range in the sense of Section 1.4.6, tires before thrust, in which the costly input draws on a stock that nothing in the run can replenish, so every allocation choice is also an inventory decision.

12.3 The Car as Modeled

The simulated car is generic and every number is a declared teaching value, as shown in Figure 12.1. A coupe of 1300 kg dry mass rides on four corner springs at $\pm a$ along and $\pm t$ across. One lumped driven wheel of radius R stands for all four. A skirt seals a floor of area A_{floor} to the road, and fans evacuate it. Four corner lift thrusters of 7 kN and two rear thrusters of 25 kN each are fed from one nitrogen tank through a regulator and valves. The car's computer reads only its instruments: wheel speed, a ground-speed radar, a seat accelerometer, lasers for the height, an inertial unit, an optical position fix, the tank's pressure and temperature, and a fan tachometer.

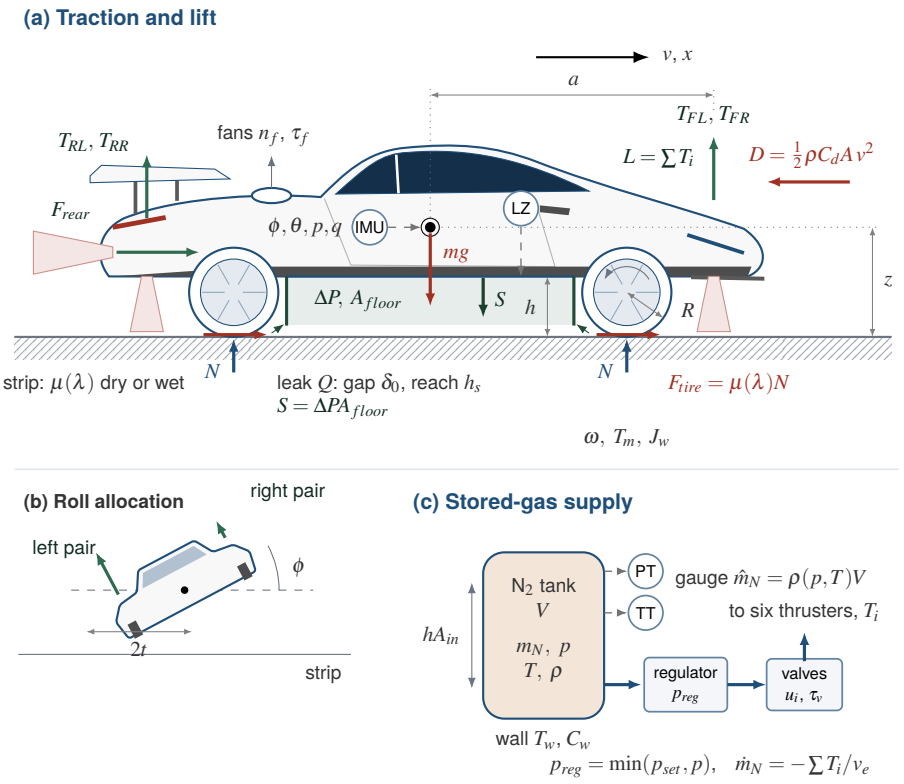


Figure 12.1 Fan car as modeled, in side elevation at the metered launch, with the nitrogen supply and a rear view for the roll. The weight mg, the suction S of the sealed floor and the corner lift L set the road's normal force N, which limits the tire force F_{tire}. The rear thrust F_{rear} and the drag D act along the strip, and z measures the center of mass above the strip. The tank holds m_N at p and T inside a wall at T_w, and the regulator feeds the valves at p_{reg}. LZ is the height laser, IMU the inertial unit, and PT and TT the tank transmitters that drive the gauge. Not to scale.

The variables are classified in Table 12.1. Twenty-four states are integrated: the position and velocity of the center of mass in three axes, the roll ϕ and pitch θ with their rates p and q, the wheel speed ω, the tire slip λ, the motor torque T_m, the fan speed n_f, five valve positions, the nitrogen mass m_N, the gas temperature T, the wall temperature T_w, and two ledgers that integrate the impulse delivered and any leak, so that the tank balance can be checked exactly. Units are seconds, kilograms, meters, newtons, bar absolute and kelvins, with forces quoted in kilonewtons.

12.3.1 The Wheel, the Tire and the Springs

The motors drive the lumped wheel through a first-order torque lag, and the tire pushes with the friction its slip allows on the normal force the springs carry, as shown in Equation (12.1).

$$\begin{aligned} J_w\dot{\omega} &= T_m - RF_x, \qquad F_x = \mu(\lambda)N \\ \sigma\dot{\lambda} &= (R\omega - v) - \max(|v|, v_{min})\,\lambda \\ \tau_m\dot{T}_m &= T_{cmd} - T_m \\ |T_{cmd}| &\le \min\big(F_{mot}R,\ P_{max}/\max(1,|\omega|)\big) \\ \mu(\lambda) &= c_1\big(1 - \mathrm{e}^{-c_2|\lambda|}\big) - c_3|\lambda|, \quad |\lambda| \le 1 \end{aligned} \tag{12.1}$$

Table 12.1 Variables by Role, With Their Values in the Metered Launch of Case 4 at $t = 1.4\,\text{s}$

Variable	Role	At 1.4 s	Range
λ	CV, tire slip	0.047	SP 0.02–0.5
h	CV, wheel height	—	SP 0.5–4 m
ϕ	CV, roll (a profile)	0°	—
T_{cmd}	MV, motor torque	100 %	± motor limit
u_{rear}	MV, rear thrust	25.5 kN	0–50 kN
u_i	MV, corner valves	0	0–1 each
n_{cmd}	MV, fan speed	1	0–1
surface	DV, wet patch	dry	10 m, peak 0.80
F_g	DV, side gust	0	0–6 kN
b, stuck, leak	DV, bump, valve, regulator	none	see text
v	state, speed	27.6 mph	—
N, S	output, road, suction	37.5, 21.1 kN	—
F_{tire}	output, tire force	31.6 kN	$\le \mu(\lambda)N$
m_N	state, nitrogen	409.4 kg	0–423.0 kg
p, T	output, state, tank	621 bar, 11 °C	to 5 bar
a_x	output, seat g (8 Hz)	3.38 g	limit 3.5 g

where $\sigma = 0.25\,\text{m}$ is the relaxation length and $v_{min} = 1\,\text{m/s}$ keeps the slip equation damped at a standstill. The Burckhardt coefficients are those of the anti-lock braking app of Volume 7, Chapter 4, $(c_1, c_2, c_3) = (1.2801, 23.99, 0.52)$ on dry asphalt and $(0.857, 33.822, 0.347)$ on wet, and both apps import the same functions from the framework's shared tire module. The dry peak is 1.170 at $\lambda = 0.170$ and the locked-wheel value 0.760. The slip is the traction form with the car's speed in the denominator, so μ is held at its value for $\lambda = 1$ beyond it, and λ is clipped to ± 1.5.

The four corner springs carry the weight, the suction and whatever the lift takes off, as shown in Equation (12.2).

$$\begin{aligned} \delta_i &= z_{free} - (z + x_i \sin\theta + y_i \sin\phi), \qquad N = \textstyle\sum_i F_i \\ F_i &= \max\!\left(0,\; k\delta_i + c\dot{\delta}_i\right) + 20k \max(0, \delta_i - 0.12) \end{aligned} \tag{12.2}$$

where $(x_i, y_i) = (\pm a, \pm t)$ and the last term is a bump stop 12 cm into the travel. A spring pushes only while compressed and only while the car is roughly upright ($\cos\phi > 0.5$ and $\cos\theta > 0.5$). At rest with a full tank the springs carry 16.90 kN and the center of mass sits at $z = 0.455\,\text{m}$. The tire's force acts at the road, so it pitches the body by $F_x z$, the squat of a launch, and a stiff saturating friction $-0.9N\,\text{sat}(v_y/0.05)$ holds the car laterally while it is in contact.

12.3.2 Suction Under the Skirt

The fans pull air from under the floor, and the outside air returns through the gap under the skirt. Equating the fan curve $\Delta P = \Delta P_{max} n_f^2 - k_Q Q^2$ to the orifice leak $Q = A_{gap}\sqrt{2\Delta P/\rho}$ gives the closed form of Equation (12.3).

$$\begin{aligned} \Delta P &= \frac{\Delta P_{max}\, n_f^2}{1 + 2k_Q A_{gap}^2/\rho}, \qquad S = \Delta P A_{floor} \\ A_{gap} &= L_s\left(\delta_0 + \max(0,\; h - h_s) + b\right) \\ \tau_f \dot{n}_f &= n_{cmd} - n_f \end{aligned} \tag{12.3}$$

where $h = z - d_{floor}$ is the floor's clearance, $h_s = 15\,\text{cm}$ the skirt's reach, $\delta_0 = 5\,\text{mm}$ the sealed leak around $L_s = 12\,\text{m}$ of skirt, and b the extra gap of a bump, 7 mm for 0.3 s. Sealed at full speed the pressure is 6.00 kPa and the suction 23.99 kN, 1.42 times the full-tank weight, for about 60 kW of fan power at an assumed efficiency of 0.6. The seal is positive feedback. A floor 2 cm above the skirt's reach keeps 42.8 % of the sealed suction and one at 5 cm keeps 13.0 %, so a car that starts to lift loses the force that held it. The suction acts at the floor's center and makes no moment.

12.3.3 Flight in Five Degrees of Freedom

Pitch is applied first and roll second, with no yaw, so the body's longitudinal and vertical axes in the world are $\mathbf{e}_x = (\cos\theta, 0, \sin\theta)$ and $\mathbf{e}_z = (-\sin\theta\cos\phi, -\sin\phi, \cos\theta\cos\phi)$. The translation and the two rotations are given by Equation (12.4).

$$\begin{aligned} m\ddot{\mathbf{r}} &= L\,\mathbf{e}_z + F_{rear}\,\mathbf{e}_x + (F_x - F_r)\mathbf{e}_X + F_y\mathbf{e}_Y \\ &\quad + (N - S)\,\mathbf{e}_Z - mg\,\mathbf{e}_Z - \mathbf{D} + F_g\,\mathbf{e}_Y \\ I_{xx}\ddot{\phi} &= t\,(T_{FL} + T_{RL} - T_{FR} - T_{RR}) + \textstyle\sum_i y_i F_i - 0.25F_g \\ I_{yy}\ddot{\theta} &= a\,(T_{FL} + T_{FR} - T_{RL} - T_{RR}) + \textstyle\sum_i x_i F_i + F_x z \end{aligned} \tag{12.4}$$

where $L = \sum T_i$ over the corners, $F_r = f_r N$ is the rolling resistance, and the drag components are $\frac{1}{2}\rho A_j v_j |v_j|$ with $A_x = C_d A = 0.6\,\mathrm{m}^2$, $A_y = 3.6\,\mathrm{m}^2$ and $A_z = 8\,\mathrm{m}^2$. The gust F_g is a 2 s half sine that starts 1 s into the hover and acts 0.25 m above the center of mass. Roll and pitch are integrated as independent axes, which is exact for a roll with the pitch held at zero. The mass and inertias follow the tank, $m = m_{dry} + m_N$ and $I = I_{full}\, m/m_{full}$.

12.3.4 Thrusters, Valves and the Tank

Each thruster delivers its rating times its valve position, scaled by the regulator, and every newton costs mass at the exhaust speed, as shown in Equation (12.5).

$$\begin{aligned} T_i &= T_{rated}\,u_i\,\frac{p_{reg}}{p_{set}}, \qquad p_{reg} = \min(p_{set},\, p) \\ \tau_v \dot{u}_i &= u_{i,cmd} - u_i \\ \dot{m}_N &= -\frac{\sum T_i}{v_e} - \dot{m}_{leak}, \qquad v_e = I_{sp}\,g_0 \end{aligned} \tag{12.5}$$

where $p_{set} = 60\,\mathrm{bar}$ is the pressure at which the thrusters are rated, $v_e = 716.1\,\mathrm{m/s}$ for $I_{sp} = 73\,\mathrm{s}$, and τ_v is 20 ms for proportional valves and 5 ms for on-off valves. Below 5 bar the valves pass nothing and TANK EMPTY shuts every thruster. Holding the full-tank weight of 16.90 kN costs 23.6 kg/s, and 50 kN of rear thrust costs 69.8 kg/s. A yaw pair of up to 2 kN is assumed to hold the heading, and its gas is counted while a gust acts.

The tank's pressure comes from a real-gas law and its temperature from the energy balance of the gas left behind, as shown in Equation (12.6).

$$\begin{aligned} p &= \rho R_{N_2} T\, Z(\rho, T), \qquad \rho = m_N/V \\ Z &= 1 + \textstyle\sum_{k=1}^{5} r^k \left(a_{k0} + a_{k1}\tau + a_{k2}\tau^2\right) \\ m_N c_v \dot{T} &= hA_{in}(T_w - T) - \dot{m}_{out}\,\frac{T}{\rho}\left(\frac{\partial p}{\partial T}\right)_\rho \\ C_w \dot{T}_w &= hA_{in}(T - T_w) + hA_{out}(T_{air} - T_w) \end{aligned} \tag{12.6}$$

where $r = \rho/(1000\,\mathrm{kg/m^3})$, $\tau = 100\,\mathrm{K}/T$, and $c_v = e_0 + e_1 r + e_2 r\tau + e_3 r^2 + e_4 r^2\tau + e_5 r^3$ in J/(kg K). The fifteen a_{kj} and six e_j are listed in Table 12.2. They are least-squares fits to the reference tables for nitrogen [14] between 140 and 320 K and 5 and 705 bar, with worst errors of 0.9 % in pressure and 0.5 % in density. At 700 bar and 20 °C the fit gives 497.6 kg/m^3 against the tables' 498.2, so the tank holds 423.0 kg, of which 4.9 kg stays behind at 5 bar. For an ideal gas the last term of the energy balance reduces to $\dot{m}_{out} R_{N_2} T$, the familiar adiabatic cooling. With the wall switched off the blowdown follows the reference isentrope from the fill within 0.7 K down to 200 kg/m^3. The gauge inverts the same law at the measured pressure and temperature, $\hat{m}_N = \rho(\hat{p}, \hat{T})V$, by twelve fixed Newton iterations.

Table 12.2 Coefficients of the Nitrogen Fit of Equation (12.6)

k	a_{k0}	a_{k1}	a_{k2}
1	1.244544798	−2.619325373	−4.784349976
2	5.289904277	−20.10382874	27.53234519
3	−8.408539214	50.59369007	−58.79664928
4	10.45578251	−50.99223434	37.20049222
5	−0.337057974	23.03951548	−0.8679504764
c_v	e_0, e_1	e_2, e_3	e_4, e_5
	727.51089, 0.16531432	1698.7723, −930.60672	−2469.4212, 1991.6526

Table 12.3 Model Parameters. H Is Hidden and Fixed, A Is Adjustable in the Advanced Parameters Panel

Symbol	Meaning	Value	Unit	
m_{dry}	car, driver, empty tank	1300	kg	A
I_{xx}, I_{yy}	inertias, full tank	550, 2200	kg m^2	A
a, t	half wheelbase, half track	1.35, 0.90	m	H
z_{free}	CG height, springs unloaded	0.49	m	H
k, c	corner spring, damper	120, 9	kN/m, kN s/m	A
C_dA	drag area	0.6	m^2	A
ΔP_{max}, k_Q	fan rise, curve droop	6.35, 9.83	kPa, Pa s^2/m^6	A, H
A_{floor}, L_s	floor area, skirt perimeter	4.0, 12	m^2, m	H
δ_0, h_s	sealed gap, skirt reach	5, 150	mm	A, H
d_{floor}	floor below the CG	0.35	m	H
τ_f	fan spool lag	0.5	s	A
R, J_w	wheel radius, lumped inertia	0.34, 8	m, kg m^2	H, A
F_{mot}, P_{max}	tractive force, power limit	34, 1.0	kN, MW	A
τ_m, σ	motor lag, relaxation length	20, 0.25	ms, m	A
f_r	rolling resistance	0.012	—	A
T_c	corner thruster, each of four	7	kN	A
T_{rear}, T_{yaw}	rear pair, yaw pair	50, 2	kN	A
τ_v	valve lag, proportional, on-off	20, 5	ms	A
I_{sp}	specific impulse	73	s	A
V, p_{fill}	tank volume, fill pressure	850, 700	L, bar	A
T_{fill}	fill temperature	293.15	K	H
p_{set}, p_{empty}	regulator, empty	60, 5	bar	A, H
C_w, hA_{in}	wall capacity, gas to wall	60, 800	kJ/K, W/K	A
hA_{out}	wall to air	50	W/K	H
v_{hard}, v_{stop}	hard landing, stop	1.5, 3.0	m/s	A
a_{lim}	driver g limit (warning)	3.5	g	A
g, ρ, T_{air}	gravity, air, ambient	9.81, 1.2, 293.15	SI	H

12.3.5 Events, Instruments and Numerics

Hard events act between integration steps. The takeoff is the instant every corner unloads. The touchdown is located inside its step by interpolating to the first corner contact, with the vertical speed carried from the last free step's acceleration, and a touchdown faster than 1.5 m/s latches HARD LANDING, faster than 3 m/s stops the run. A touchdown faster than 1.5 m/s rolled past 25° or pitched past 15°, or any body corner reaching the road while the car is not wheels-down, latches CRASHED. On the inverted ramp the road is a ceiling, gravity's component along the road normal changes sign, and the car has fallen once it hangs 0.3 m below its unloaded ride height. Every instrument is the truth plus seeded colored noise, with standard deviations of 0.05 rad/s on the wheel speed, 0.03 m/s on the ground speed, 4 mm on the height, 0.02 m/s on the vertical speed, 2 cm on the position, 0.1° on the attitude, 0.3 bar and 0.3 K on the tank, and 0.5 % on the fan speed. The seat's acceleration is filtered with a 20 ms lag, close to 8 Hz, and that filtered value is the g the limit and the scorecard use. The engine of Section 1.6.1 integrates with fourth-order Runge-Kutta steps of $\Delta t = 1$ ms, the computer runs every 10 ms and the historian logs at 100 Hz. Halving the step leaves the metered launch, the hover, the roll and case 9 unchanged to the printed digit, and the four measured touchdowns agree to 0.001 m/s.

12.3.6 Assumptions

The parameters of Table 12.3 describe no real vehicle. One lumped wheel replaces four, so load transfer between axles never changes the grip, and the tire curve was fitted to braking slip and is read here in traction. The skirt is one gap around a flat floor, which is why the suction makes no pitch or roll moment, and the inverted ramp is a flat ceiling the car is already on. Roll and pitch are independent axes, which breaks down in a tumble. The specific impulse is held at 73 s, as if the gas were warmed to ambient between the regulator and the nozzles, while the tank itself cools. A nozzle fed at −150 °C would deliver nearer 47 s. Below 140 K the gas law is extrapolated, and at the end of case 9 (about 125 K and 27 bar) its density is within 1.1 % of the tables while its c_v is about 12 % low, so the last seconds' cooling is slightly overstated. Condensation is not modeled. The g limit, the landing speeds and the reserve are teaching values.

12.4 Three Prices and One Budget

12.4.1 A Normal Force on Command

With the fans off the springs carry the weight alone, and the dry tire can push with at most $1.17 \times 16.9 = 19.8$ kN, 58 % of the motors' 34 kN. Fully spooled the fans add 24.0 kN to the normal force, and the tire's limit rises to 47.8 kN, past the motors. The fans make the tire's limit a manipulated variable, as actively pitched wings do for a race car's tires [26]. They are slow, though, because the suction grows with the square of a speed that lags its command by 0.5 s, so 0.3 s after a start the fans turn at 45 % and give 20 % of their suction. On the ground the allocation is therefore a priority with a time dimension: spool the fans first, push through the tires, and add thrust only for the remainder. Chapter 1 calls this a split range in time (Section 1.4.6).

12.4.2 Positive Feedback at the Seal

The skirt seals until the floor is 15 cm up, 1 cm above the height at which the wheels unload. Past that the leak grows with the clearance and the suction collapses, as Equation (12.3) gives. On a ceiling the consequence is a fall. At 89 % fan speed the suction is 1.12 times the weight, and a bump that opens the skirt 7 mm for 0.3 s cuts it to 0.79 of sealed, 0.89 of the weight. The wheels unload, the floor moves off the ceiling, and the seal never recovers. At takeoff the same feedback works for the car, through the fall of the wheel loads to zero that other flying-car models also show [22]. Fully spooled the fans add 24.0 kN to the 16.9 kN weight, against 28 kN of lift, so the car cannot leave with them at full speed, and the sequencer spins them down first.

> **TRY IT IN THE APP** **5 · The Inverted Ramp**
>
> Load the scenario and press Run. The car drives on a ceiling held by its suction alone with the fans at 89 %, and the bump at 2 s drops it at 2.42 s. Press Reset, set Fan speed to 100 % and run again. The tires now keep at least 0.86 kN on the ceiling through the bump.

12.4.3 A Hover on a Lightening Car

A hover costs mg/v_e, 23.6 kg/s on a full tank, and the car loses a fifteenth of its all-up mass in a 5 s hover, 114.1 kg. An integral alone must find the weight and then chase it down, while a feedforward from the gauge carries it directly. The hover demand is 60 % of each corner thruster, which on on-off valves with a 10 ms minimum on and off time in a 20 ms period lies between the 50 and 100 % that a pulse can give. The minimum impulse bit is 70 N s per thruster.

12.4.4 The Tank Runs Out by Pressure

A blowdown of seconds is nearly adiabatic, so the gas cools and its pressure falls faster than its mass, as shown in Figure 12.2b for case 9. By the takeoff at 3.48 s the metered launch and the rollout have spent 7.1 % of the nitrogen, while the pressure has fallen 23.1 % and the gas has cooled to 1 °C. At

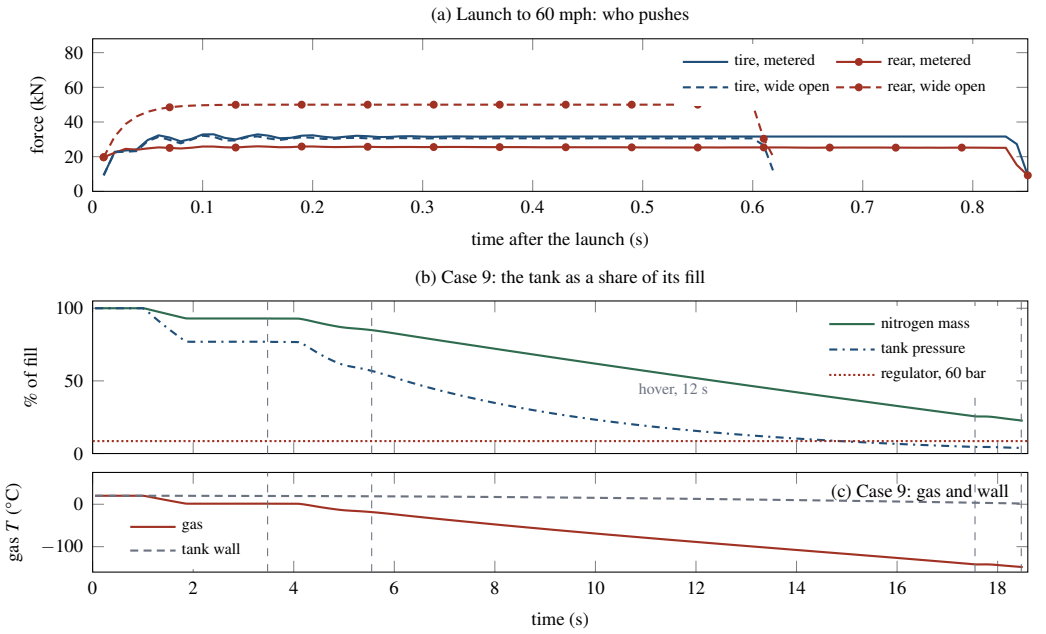


Figure 12.2 The launch allocation and the tank's two budgets, from the engine at the shipped settings. (a) Tire force and rear thrust after the launch, metered to 3.4 g (solid, case 4) and wide open (dashed, case 3), the rear thrust marked with dots. (b) Case 9 nitrogen mass (solid) and tank pressure (dash-dotted) as shares of their fill, with the regulator's 60 bar line (dotted), and thin dashed lines at the takeoff, the start and end of the 12 s hover and the touchdown. (c) The gas (solid) and wall (dashed) temperatures.

the start of the hover the tank holds 84.9 % of its mass at 56.9 % of its pressure, and by the end of a 12 s hover 25.6 % of the mass remains at 31.7 bar and −142 °C. The pressure met the 60 bar regulator at 14.86 s with 38.1 % of the mass left. From there the thrust falls with the pressure, and at the end of the hover full lift is 14.8 kN against a weight of 13.8 kN, 1.07 times it. The valves can still open fully, so the thrusters are the actuator of Section 1.4.5 whose gain falls without reaching its clamp. A runway computed from the gauge's mass with Equation (1.11) is therefore optimistic, since the thrust gives out before the mass does. The wall warms the gas by only about 2 to 6 K over a run, because the time constant $m_N c_v / h A_{in}$ is several minutes.

🕮 **Pause.** Try Exercise 12.1 before reading on.

12.4.5 The Price of a Roll

One corner pair gives 12.6 kN m about the roll axis. The roll profile asks for 90 % of it, $\alpha = 0.9 \cdot 2T_c t / \hat{I}_{xx}$, and a bang-bang turn of 2π takes $T_{roll} = 2\sqrt{2\pi/\alpha}$, 1.04 s in the challenge and 1.08 s in case 7, where the car is heavier. While the car is on its side its lift points sideways, and inverted it points at the road, so a roll from a hover at 2 m reaches the ground. The turn lingers near level at its start and end, where the firing pair still lifts, so the mean lift is not zero. Entered at 5.1 m/s of climb the roll ends 1.77 m above its entry in case 7, and the entry rate that ends the roll at its entry height is about 3.6 m/s.

12.5 The Controllers as Implemented

Every law runs every 10 ms on a measurement packet that holds only the instrument readings of Section 12.3 and the gauge, never the true mass, suction, tire force, surface, gust, stuck valve or

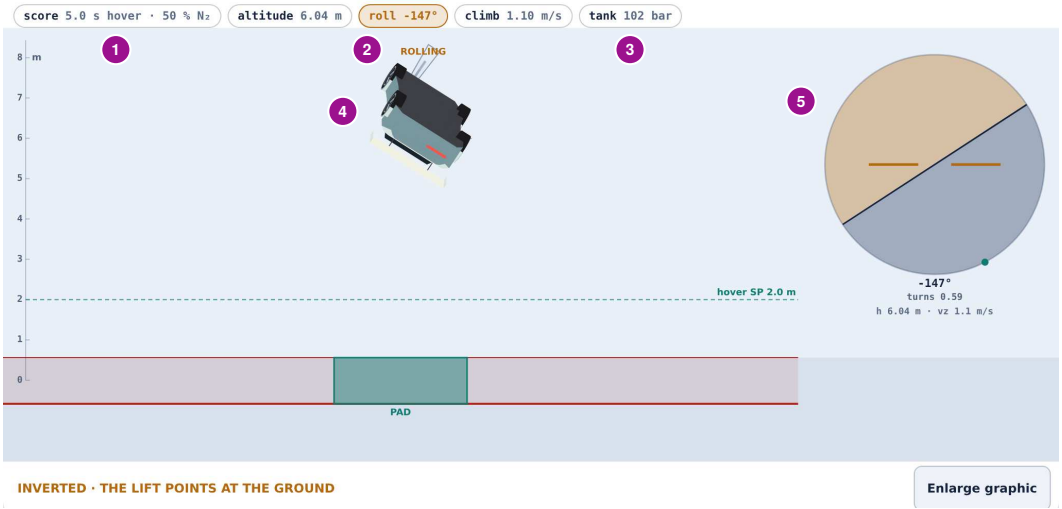


Figure 12.3 The one-tank challenge on its suggested settings, cropped to the Roll view and captured at $t = 11.70$ s, 0.57 s into the 1.04 s roll. The five badges above the scene give the score (1, 5.0 s of hover in band and 50 % of the nitrogen left), the altitude, the roll (2), the climb rate and the tank pressure (3, 102 bar). The car (4) is past inverted at −147°, which the dial (5) counts as 0.59 of a turn, 6.04 m up and still climbing at 1.10 m/s. At that instant the right pair was braking the turn, the collective stood at one pair's 14.0 kN against a weight of 14.8 kN, and the tank held 212 kg, 50 % of its fill.

leak. Truth feeds only the scorecard, the budget shading and the drawing. A stage sequencer from the shared framework carries the run as one active phase at a time: ARMED, SPOOL, LAUNCH, ROLLOUT, TAKEOFF, CLIMB, HOVER, LOB, ROLL, ARREST, LAND and DONE. SPOOL lasts the pre-spool lead, LAUNCH ends at 60 mph (26.82 m/s), ROLLOUT brakes to a stop, TAKEOFF ends at 5 cm of height, CLIMB ends within 0.15 m and 0.3 m/s of the hover, HOVER lasts its set time, LOB ends at the entry climb rate, ROLL lasts $T_{roll} + 0.05$ s, ARREST ends when the descent falls below 0.2 m/s, and LAND ends on the ground. The fans spin down at takeoff and play no part in flight.

12.5.1 The Slip Loop

The motor torque comes from the shared velocity-form PID of Section 1.6.2, Equations (1.12) to (1.15), with the error $e_k = \lambda_{SP} - \hat{\kappa}_k$ and the derivative term on the measured slip. Here $\hat{\kappa} = (R\hat{\omega} - \hat{v})/\max(|\hat{v}|, 3\,\mathrm{m/s})$ is the measured slip with its denominator held at 3 m/s at low speed. The running sum is never clamped, and the integral is corrected by back-calculation from the clamped output, with the tracking time constant equal to τ_I. The defaults are $\lambda_{SP} = 0.17$, $K_c = 40$ kN m per unit slip, $\tau_I = 50$ ms and no derivative. The output range is 0 to the motor limit in the launch and $-T_{lim}$ to 0 in the rollout, where the same loop brakes at $\lambda_{SP} = -0.10$ and a hold torque of 3 kN m opposes any motion slower than 0.3 m/s. At the launch the integral is preset to $0.9\,\mu_{peak}\hat{N}R$, the torque the tire can take on the normal force the gauge and the fan law predict. In Manual the throttle sets the torque and the loop follows it, which makes the switch back to Auto free of bumps.

12.5.2 Metered Rear Thrust

The rear thrusters are off, wide open, or metered to a g target a_t by supplying only what the tires do not, as shown in Equation (12.7).

$$\begin{aligned} F_{rear} &= \mathrm{clip}\big(\hat{m}a_t + \hat{D} - \hat{F}_{tire},\ 0,\ T_{rear}f\big) \\ \hat{F}_{tire} &= (T_{cmd} - J_w\hat{\dot{\omega}})/R, \qquad \hat{D} = \tfrac{1}{2}\rho C_d A\hat{v}^2 \end{aligned} \tag{12.7}$$

where $\hat{m} = m_{dry} + \hat{m}_N$, $f = p_{reg}/p_{set}$ from the measured pressure, and $\hat{\dot{\omega}}$ is the wheel's measured acceleration through a 10 ms lag. The motor command rather than the lagged torque is used because the rear valves lag the same 20 ms as the motors, so the two arrive together. The default target is 3.4 g.

Pause. Try Exercise 12.2 before reading on.

12.5.3 Altitude, Attitude and the Mixer

The altitude loop is a cascade. A ramped height setpoint h_{SP}, moving at 2.5 m/s, asks for a climb rate in proportion to the height error, and a PI on the climb-rate error adds lift to the mass feedforward, as shown in Equation (12.8).

$$\begin{aligned} v_{z,SP} &= \mathrm{clip}\big(K_h(h_{SP} - \hat{h}),\ \pm 2.5\,\mathrm{m/s}\big) \\ L &= \frac{\hat{m}g}{\cos\hat{\phi}\cos\hat{\theta}} + \mathrm{PID}\big(v_{z,SP} - \hat{v}_z\big) \end{aligned} \tag{12.8}$$

with $K_h = 2.5\,\mathrm{s}^{-1}$, a climb-rate gain of 10 kN s/m, $\tau_I = 2$ s and the cosine floored at 0.5. Without the feedforward the integral carries the whole weight. The rocket of Volume 3, Chapter 10 also carries a gravity feedforward while it burns its own propellant. In the lob and the arrest the collective is full lift, in the roll it is one pair's worth, and the loop tracks meanwhile so it takes over on the feedforward. The landing schedule is $v_{z,SP} = -\min\big(4,\ \max(0.5,\ 2\sqrt{\hat{h}})\big)$ m/s, a constant deceleration of $2\,\mathrm{m/s}^2$ that ends at a 0.5 m/s touchdown rate.

Roll and pitch are held by PD laws whose gains follow a natural frequency $\omega_a = 8\,\mathrm{rad/s}$ and damping $\zeta_a = 0.8$ on the gauge's inertias, with the profile's acceleration as feedforward in the roll, as shown in Equation (12.9).

$$\begin{aligned} \tau_\phi &= \hat{I}_{xx}\big(\alpha_{ff} + \omega_a^2 e_\phi + 2\zeta_a\omega_a(p_{ref} - \hat{p})\big) \\ \tau_\theta &= \hat{I}_{yy}\big(\omega_a^2(\theta_{SP} - \hat{\theta}) - 2\zeta_a\omega_a\hat{q}\big) \\ T_i &= \frac{L}{4} + s_{\phi,i}\frac{\tau_\phi}{4t} + s_{\theta,i}\frac{\tau_\theta}{4a}, \qquad 0 \le T_i \le T_c f \end{aligned} \tag{12.9}$$

where $s_{\phi,i}$ and $s_{\theta,i}$ are ± 1 by corner. The mixer gives attitude priority. If the torques alone need more spread than one thruster's range they are scaled down together, and the collective is then clipped to what they leave. Position loops lean the car, $\phi_{SP} = -\mathrm{clip}\big((0.8e_y - 1.4\hat{v}_y)/g,\ \pm 12^\circ\big)$ and the same in pitch within $\pm 8^\circ$, except in the lob, the roll and the arrest. The roll reference is the bang-bang profile, $\phi_{ref} = \frac{1}{2}\alpha s^2$ for the first half of the turn and $2\pi - \frac{1}{2}\alpha(T_{roll} - s)^2$ for the second, with its acceleration led by the valve lag.

12.5.4 Gas-Aware Abort and Pulsed Valves

The sequencer lands at once, never in the middle of a roll, when the gauge reads less than 8 % of the fill or, if the option is on, when the measured tank pressure reaches the regulator setpoint. On-off valves are pulsed every 20 ms and can neither open nor close for less than 10 ms, so a pulse is 0, 10 or 20 ms wide. The nearest-width law, which the app's Pulse law option labels Nearest width, opens each valve for the allowed width nearest its demand. The pulse-width law keeps an impulse account per valve, fires the widest allowed pulse the account holds, and banks at most one period, so the average delivered equals the average asked.

Pause. Try Exercise 12.3 before reading on.

Table 12.4 The Ten Cases and Their Lessons

#	Case as shipped	Teaching point
1	Manual, fans off, throttle 100 %	wheelspin past the friction peak
2	Fans and slip loop, 0.3 s pre-spool lead	suction raises the tire's limit, a slow actuator needs a lead
3	Rear thrusters wide open	fast, over the g limit, expensive
4	Rear thrusters metered to 3.4 g	the free actuator first, the costly one on the remainder
5	Inverted ramp at 12 m/s, fans 89 %, bump at 2 s	a seal's margin must cover a disturbance
6	On the pad, 5 s hover at 2 m, integral alone	the unstick, feedforward against integral
7	Hovering at 2 m, roll from the hover	a roll needs climb rate in hand
8	Hovering on on-off valves, nearest-width pulses	a minimum impulse bit sets a limit cycle
9	Metered launch, 12 s hover, abort off	a demand the tank cannot meet
10	Challenge: wide open, integral alone, on-off valves, roll from the hover, wet patch, 3 kN gust	the whole allocation on one tank



Figure 12.4 Two scene views at their telling instants. Left: the metered launch of case 4 at $t = 1.4$ s on the Forces view, 0.39 s after the launch at 27.6 mph. The road's 37.5 kN carries the 16.8 kN weight and 21.1 kN of suction, the tire gives 31.6 kN of its 43.9 kN limit at a slip of 0.05 with the motors saturated, and the rear thrusters add 25.5 kN toward the 57.1 kN the 3.4 g target asks. Right: case 9 as shipped at $t = 18.5$ s on the Budget view, just after the hard landing, with 96 kg of 423 kg left at 27 bar, −148 °C and 44 % thrust, the pressure under the regulator's line while the mass line still stands well above the reserve.

12.5.5 Three Models Kept Apart

The plant is everything in Section 12.3. The controllers' model is smaller: the declared gas law in the gauge, the fan law in the launch preset, the declared inertias in the roll profile and the PD gains, and the drag estimate in the metering law. No identified model is fitted inside the application, and every controller reads the plant only through its instruments.

12.6 Ten Cases, Measured

The ten cases are listed in Table 12.4. They share one reference configuration, the challenge's suggested settings less the choices each case makes for itself, and depart from it as their rows say. Each value below comes from a headless run at the default seed, advanced in the same fixed steps the browser takes (Section 1.6.1), with the change a row names applied before Run. Each case was also loaded three ways (first, from the list after another case, and by Reset), and the three runs were identical.

12.6.1 Launching on Three Actuators

Floored by hand with the fans off, the slip runs to 1.5, the wheels spin for 3.64 s and the car takes 3.68 s to 60 mph at a peak of 0.96 g. At 60 % throttle the slip peaks at 0.14, below the curve's peak,

Table 12.5 Launch to 60 mph on the Declared Car, Against the Design-Stage Estimate

Launch (fans on unless stated)	0–60 (s)	Peak (g)	N_2 (kg)	Estimate (s, g, kg)
Manual, floored, fans off (case 1)	3.68	0.96	0	—
Manual, 60 %, fans off (case 1)	2.43	1.15	0	—
Slip loop, fans off	2.49	1.15	0	2.3, 1.2, 0
Slip loop, 0.3 s lead (case 2)	1.76	1.93	0	—
Slip loop, 1 s lead	1.47	1.93	0	1.26, 2.2, 0
Rear wide open (case 3)	0.60	4.83	40.5	0.51, 5.5, 35
Metered to 3.0 g	0.94	3.00	24.3	—
Metered to 3.4 g (case 4)	0.83	3.39	29.1	0.81, 3.4, 21
Metered to 3.8 g	0.75	3.79	33.2	—
Metered, wet patch	0.83	3.41	29.4	—

and the time falls to 2.43 s at 1.15 g. At 61 % the wheels break loose and the time is back to 3.48 s, because past the peak the force falls as the slip grows. The launch results are collected in Table 12.5. With the fans spooled the motors rather than the tire set the pace, and a lead of 0.3 s costs 0.29 s against a lead of a second.

> **TRY IT IN THE APP** **1 · By Hand: Fans Off, Foot to the Floor**
>
> Load the scenario and press Run in Manual. Watch the slip badge and the WHEELSPIN warning: the slip runs to 1.5, the wheels spin for 3.64 s, and the car needs 3.68 s to 60 mph. Press Reset, set Throttle (motor torque) to 60 % and run again. The slip now peaks at 0.14 and the time falls to 2.43 s.

The metered launch is the allocation drawn in Figure 12.2a and on the Forces view of Figure 12.4. At the slowest playback speed, 0.1 ×, it lasts 8.3 s of wall time, slow enough to watch the allocation on the traction bar. Between 0.2 and 0.6 s after the launch the tire gives an average of 31.6 kN on 37.5 kN of normal force and the rear thrusters add 25.5 kN, so the seat reads 3.37 g. Wide open the tire gives about the same, 30.6 kN, and the thrusters 50.0 kN, which buys the last 0.23 s for 11.4 kg and 1.4 g over the target. On the wet patch the tire gives less and the metering law makes it up, for 0.3 kg more. The design-stage estimates were made for a lighter car, 1595 kg, with the motors' full force from the first instant. The engine's car weighs 1723 kg, its wheels and rotors add $J_w/R^2 = 69\,\text{kg}$ of effective mass, and the motors lag 20 ms, which together account for the differences. Figure 12.5 sets the ten launches of Table 12.5 side by side.

12.6.2 Suction, Takeoff and Hover

On the inverted ramp at 89 % the bump drops the car at 2.42 s, the suction collapsing to 5 % of sealed as the floor moves off the ceiling. At 94 % the car still falls, at 2.61 s, at 95 % it only just holds with the seal at 0.79, and at 100 % the tires keep at least 0.86 kN on the ceiling through the bump. The flight results are collected in Table 12.6. In case 6 the integral alone leaves the pad 0.97 s after Run, climbs at most at 1.58 m/s, hovers between 1.914 and 1.998 m and lands at 10.21 s. With the feedforward the car leaves at 0.75 s, hovers within 1.985 to 2.003 m and is down at 8.20 s. The 5 s hover costs about the same either way, and the 39.5 kg difference comes from the integral's slow climb and descent, which keep the car in the air 2.0 s longer. Lifting at once with the feedforward, while the fans are still spooling down, the car leaves at 0.27 s with 8.1 kN of suction still under the floor, which collapses as the skirt lifts.

12.6.3 The Roll and the Pulsed Valves

Rolled from the hover, the car is falling within a quarter turn and a body corner reaches the ground 0.73 s into the roll, at 75° of roll. With the lob it climbs at full lift to 5.12 m/s, enters the roll at 3.88 m, ends it 0.3° from level at 5.64 m, arrests the fall and lands at 1.23 m/s. A lob to 6 m/s ends the roll at 7.40 m, 2.80 m above its entry. On on-off valves the nearest-width law rounds the 60 % demand to

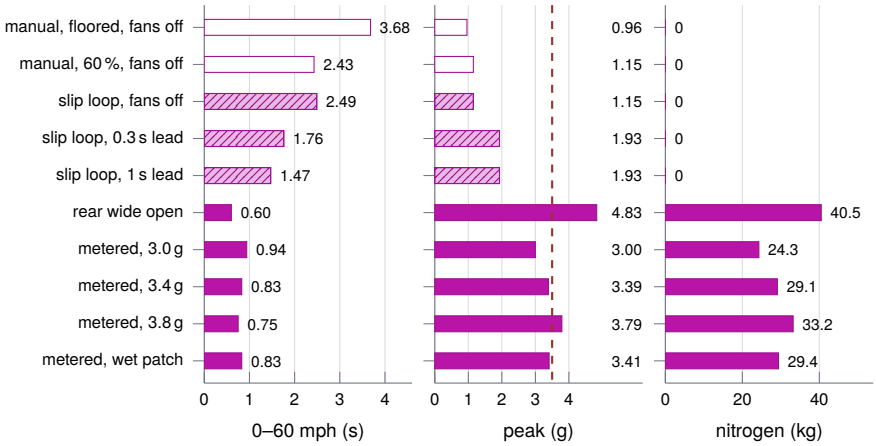


Figure 12.5 Every launch of Table 12.5: time to 60 mph, peak g and the nitrogen spent. Open bars are tires driven by hand, hatched bars the slip loop, and solid bars add the rear thrusters. The dashed line is the 3.5 g teaching limit. Notice that the fans and the slip loop take the launch from 3.68 to 1.47 s for no gas at all, and that past the slip loop each tenth of a second is bought with nitrogen, the last 0.23 s at the price of the g limit.

Table 12.6 Flight Cases. Hover Band After Its First Second, Worst Error Over the Whole Hover, Gas Spent Over the Whole Run

Case	Configuration	Hover (m)	Worst	N_2 (kg)
6	integral alone (shipped)	1.914–1.998	15.4 cm	207.6
	mass feedforward	1.985–2.003	3.5 cm	168.1
8	on-off, nearest width (shipped)	1.902–1.987	9.8 cm	204.2
	on-off, pulse-width law	1.992–2.008	1.0 cm	200.8
	proportional valves	1.995–2.006	0.6 cm	200.9
		Entry	Roll end	Touchdown
7	roll from the hover (shipped)	0.01 m/s, 2.00 m	CRASHED	—
	lob to 5.1 m/s	5.12 m/s, 3.88 m	0.3°, 5.64 m	1.23 m/s
	lob to 3.6 m/s	3.61 m/s, 2.92 m	0.3°, 2.95 m	1.34 m/s
	lob to 3 m/s	3.08 m/s, 2.69 m	0.1°, 2.15 m	1.31 m/s
		Hover	Left	Touchdown
9	12 s, abort off (shipped)	12.00 s	22.6 %	2.83 m/s
	pressure abort on	9.30 s	33.2 %	0.94 m/s
	10.8 s, abort off	10.80 s	—	1.47 m/s
	10.9 s, abort off	10.90 s	—	1.59 m/s

50, and the hover hunts between 1.902 and 1.987 m, the lift on each corner jumping between half and full as the loop's demand crosses the rounding point. The pulse-width law brings the worst error from 9.8 to 1.0 cm and spends 3.4 kg less.

12.6.4 An Honestly Infeasible Hover

Case 9 asks for 12 s of hover after the metered launch with the pressure abort off. The hover is flown, and the landing comes down at 2.83 m/s, a HARD LANDING, with 22.6 % of the nitrogen in the tank at 27 bar and −148 °C, the thrust down to 44 %, as the Budget view of Figure 12.4 shows. The gas has cooled 168 K from its fill. With the abort on, the car starts down when the tank reaches 60 bar after 9.30 s of hover and lands at 0.94 m/s. Without it, 10.8 s is the longest hover that still lands softly. A sweep of 864 combinations of the abort, the feedforward, the valves and pulse law, the climb-rate gain and integral time, the height gain and the reserve found none that hovers 12 s and lands under 1.5 m/s, the softest coming down at 2.38 m/s, and the best of 36 hand-flown descents lands at 2.49 m/s. The

Table 12.7 The One-Tank Challenge. Values at the Default Seed, Nitrogen Left at the End of the Run, Passes Over the Default Seed and Seeds 1–20

Configuration	0–60 (s, g)	Roll end	Touch (m/s)	Left (%)	Seeds passed
shipped	0.60, 4.82	crash	5.81	—	0/21
suggested	0.83, 3.41	0.1°	1.22	37.9	21/21
suggested, integral alone	0.83, 3.41	0.1°	0.50	25.9	21/21
suggested, on-off valves	0.83, 3.41	0.7°	1.21	37.6	21/21
suggested, wide open	0.60, 4.82	0.0°	1.26	35.7	0/21
suggested, roll from hover	0.83, 3.41	crash	—	—	0/21

limit belongs to the tank. Thrusters rated at 40 bar land the same hover at 1.08 m/s, and a gas-to-wall conductance of 10 kW/K lands it at 1.44 m/s.

12.6.5 The One-Tank Challenge

The challenge asks for 0–60 under 0.85 s at no more than 3.5 g, a roll ending within 10° of level, a 5 s hover within 0.2 m of 2 m, a touchdown under 1.5 m/s and at least 5 % of the nitrogen left, with the declared parameters. The run is judged when the car is down and settled, and a change to a model parameter, the hover height or a disturbance voids the attempt. The challenge configurations are compared in Table 12.7. As shipped the run fails twice. The wide-open launch reaches 60 mph in 0.60 s at 4.82 g, and the roll from the hover meets the ground at 5.81 m/s and 60° of roll, CRASHED. The suggested settings (the metered launch, the mass feedforward, proportional valves and the lob) pass at each of 31 seeds, at the native step and with it halved, and the shipped settings fail at all 62 of those runs. They spend 262.7 kg: 29.4 on the launch, 0.7 in the rollout, 33.9 to take off and climb, 110.4 in the hover, 24.7 on the lob, 21.7 on the roll, 5.4 in the arrest and 36.5 to land. The roll ends 0.1° from level and 2.04 m above its 3.60 m entry, and the car lands at 1.22 m/s with 37.9 % left, down and settled 14.3 s after Run. Two of the four changes are not needed on their own, as the table shows, and the scorecard prices them instead. The car rolling on the suggested settings is shown in Figure 12.3.

> **+ DID YOU KNOW? The Softest Landing Is Not the Cheapest**
>
> From the simulator: on the challenge's suggested settings, the integral alone in place of the mass feedforward gives the softest touchdown of Table 12.7, 0.50 m/s against 1.22 m/s, but leaves 25.9 % of the nitrogen against 37.9 %.

12.7 Technology in Practice

A real car of this kind would read and move what the laws of Section 12.5 read and move. Figure 12.6 draws the slip loop, the metered rear thrust and the tank's gauge with field hardware in place of the schematic's symbols.

What Measures It

The slip loop needs two speeds, the wheel's and the car's, and the model reads them from a wheel-speed sensor and a ground-speed radar with noise of 0.05 rad/s and 0.03 m/s. Its error is the measured slip, with the car's speed held at 3 m/s or more in the denominator. What no on-board instrument reads is the friction the road can give, the open challenge at the end of the Limitations. The tank carries a pressure and a temperature transmitter, and the gauge turns their readings into a mass.

TECHNOLOGY Sensor: Pressure and Temperature as a Tank Gauge

A gas tank has no level to read, so what it holds is computed from its pressure, its temperature and its known volume through an equation of state. Spacecraft tanks are gauged from the same three readings by the pressure-volume-temperature method, which rests on conservation of the gas's mass, so no venting or leakage is allowed. In one ground test its errors stayed within 1.5 %. It was most accurate in steady, uniform conditions, had more uncertainty at low tank pressures, and is not considered an accurate real-time technique. A pressure transducer failing slowly showed as readings that grew progressively different from those of a second transducer [27]. The model's gauge inverts the declared gas law from readings with 0.3 bar and 0.3 K of noise.

What Moves It

The gas reaches the valves through a pressure regulator. In a self-operated, spring-loaded regulator a spring pushes the plug off its seat while the downstream pressure acts on a diaphragm to push it back, so the less the downstream pressure, the wider the trim opens, and the spring sets the setpoint [28]. The model's regulator holds 60 bar until the tank falls to it and then passes the tank's pressure, $p_{reg} = \min(p_{set}, p)$ in Equation (12.5), which is why the thrusters lose gain below 60 bar while their valves still open fully. The motors and the fans are the other final elements, each modeled as a first-order lag on its command.

TECHNOLOGY Final Element: Solenoid Thruster Valves

A common on-off valve is the solenoid valve. An energized coil pulls a ferrous armature into its center, and the armature moves a small valve element, most often a poppet that covers a seat. A return spring holds the valve in its normal position, and the process pressure should act to help that spring, since enough pressure against it can keep the valve from returning when the coil is switched off. Fast action demands a coil force that greatly exceeds the spring's [28]. No open source read gives the opening time of a thruster valve. The model's on-off valves lag 5 ms and cannot open or close for less than 10 ms, and a stuck valve is one of its disturbances.

Where the Controller Runs

Control in a car is executed by electronic control units, microprocessors with input and output interfaces. Once computers entered the car they took on antilock braking, electronic braking and stability control, which use accelerometers and gyroscopes to brake the wheels one by one [29]. Every law of this chapter runs each 10 ms on one computer: the slip loop with its clamp on the motor torque, the metering law, the altitude cascade and the mixer that gives attitude priority. The stage sequencer and the gas-aware abort sit above them as a supervisory layer that chooses the active phase.

IN PRACTICE What a Stability Control Must Be Able to Do

In the United States, Federal Motor Vehicle Safety Standard No. 126 (49 CFR 571.126) defines electronic stability control by six attributes. Among them, it brakes the wheels one by one to induce a correcting yaw moment, uses a closed-loop algorithm to limit oversteer and understeer, and has an algorithm and a means to modify engine torque as necessary. It must work in every phase of driving except below 20 km/h, in reverse or during initialization, and stay capable of activation while antilock braking or traction control is active. An off control is allowed, but each new ignition cycle restores the default mode [30]. Torque taken away by a protective loop is the override of this volume.

What Is Modernizing It

Three developments bear on this car. The first is the allocation itself. A flight-tested law for an over-actuated transition aircraft minimizes total power in place of a norm of the commands [24], and a fly-drive vehicle has used a predictive controller over an allocation layer [21], where this chapter

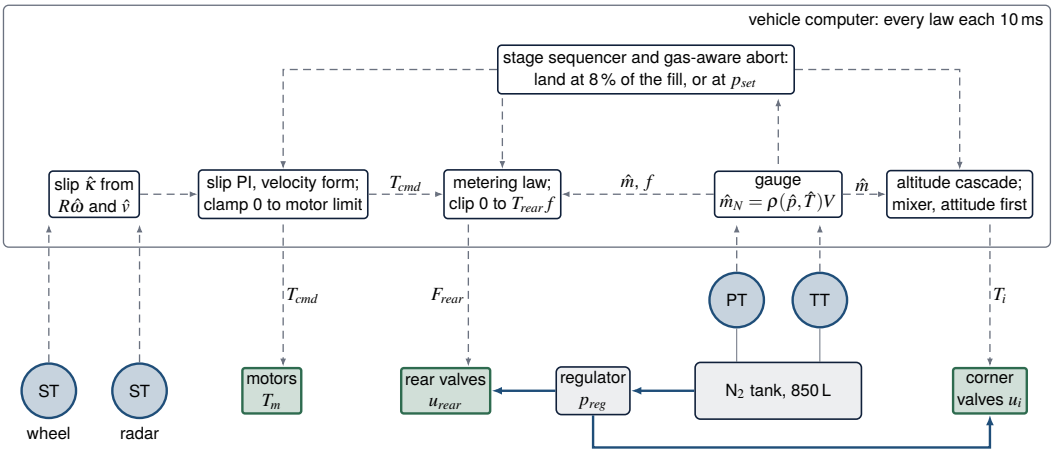


Figure 12.6 The allocation on field hardware. A wheel-speed sensor and a ground radar (ST) give the slip, and its PI loop drives the motors with the output clamped to the motor limit. The metering law asks the rear valves for only the force the tire does not give toward the g target, from the motor command and the gauge's mass. Pressure (PT) and temperature (TT) transmitters on the tank feed the gauge, whose mass also serves the altitude cascade and the gas-aware abort. Every thruster draws through one regulator. Notice that the tank pressure reaches the thrusters twice, as a reading through the gauge and as gas through the regulator, so a falling pressure cuts their thrust while the gauge still reads mass.

keeps a fixed priority. The second is the road. Intelligent tires that carry their own accelerometers are one sensing route to the friction coefficient, and neural networks trained on data are another [31]. The third is security. United Nations Regulation No. 155 makes cybersecurity a condition of a vehicle type's approval. Its list of threats includes sending arbitrary messages on the vehicle's network through an unprotected gateway and manipulating what a sensor collects, and it notes that data manipulation on sensors can be mitigated by correlating data from different sources [32].

12.8 Limitations

The car is generic and its numbers are teaching values, chosen so that the launch, the hover, the roll and the landing fit one tank. The model lumps four wheels into one, ignores load transfer and yaw, reads a braking tire curve in traction, and treats the skirt as one gap around a flat floor. The thrusters hold a constant specific impulse although the gas they draw reaches −148 °C, so the hover's gas cost is understated at the end of a long flight, and the gas law is extrapolated below 140 K. Condensation, nozzle icing and the regulator's own dynamics are outside the model. The tank was enlarged from a first estimate of 600 L to 850 L because on this model the smaller tank either loses thrust before the landing or ends at nitrogen's dew line, and the case 9 infeasibility belongs to that declared tank, not to control.

The controllers know the declared gas law, inertias and fan law, so model error reaches them only through the noise and the disturbances a case introduces. None of them is an optimization, and the allocation is a fixed priority rather than a solved program with costs such as a power-priced allocation [24] or a predictive controller over an allocation layer [21]. Chapter 13 solves a constrained least-squares allocation every period for the feet of a walking robot. The height-neutral entry of about 3.6 m/s is taught in case 7 but not flown in the challenge, which keeps an entry rate of 5.1 m/s, and the challenge's thresholds do not force the mass feedforward or proportional valves, which the scorecard prices instead. The g limit, the landing speeds and the reserve are teaching values, not certified limits.

OPEN CHALLENGE **Knowing the Grip Before the Tire Finds It**

The friction between tire and road sets the best slip of antilock braking and the yaw rate a stability control may ask for, yet no on-board sensor measures it. Estimators built on the vehicle's dynamics resist outside interference but need suitable excitation, added sensors that look at the road need none but are easily disturbed, and data-driven estimators depend on the completeness of their data. A 2022 review names fused estimators, friction shared among connected vehicles and the prediction of friction ahead as the ways forward [31]. The slip loop of this chapter keeps the dry curve's peak of 0.17 as its setpoint on the wet patch too, since the surface is hidden from the controllers.

12.9 Conclusion

A fan car with cold-gas thrusters has three actuators priced very differently, and this chapter measured what each allocation choice buys from one 850 L tank of nitrogen. The fans raise the tire's limit from 19.8 to 47.8 kN at no gas cost, metering the rear thrust to 3.4 g saves 28 % of the launch gas at a cost of 0.23 s, and a mass feedforward from the gauge saves 39.5 kg of a takeoff and hover against an integral alone. A roll on lift thrusters needs climb rate in hand, and a bang-bang turn entered at 5.1 m/s ends 1.77 m high. The tank runs out by pressure, so a 12 s hover lands at 2.83 m/s with 22.6 % of the nitrogen still inside, and no controller among 864 tried lands it softly.

The model is a teaching model with a constant specific impulse and a lumped wheel. A fuller model would add a nozzle temperature dependence, yaw and a lateral tire, a solved allocation with explicit costs [24] or a trajectory optimized against the stored gas as an energy budget is optimized for a solar aircraft [25], and a regulator with its own dynamics. Even as a teaching model, the car shows allocation against a budget that runs out in a quantity the gauge does not show.

KEY POINTS

- A car with tires, fans and gas thrusters has three ways to push, priced very differently, and all of its gas comes from one tank.
- The fans raise the tire's limit from 19.8 to 47.8 kN at no gas cost, and metering the rear thrust to 3.4 g saves 28 % of the launch gas for 0.23 s.
- A mass feedforward from the gauge saves 39.5 kg of a takeoff and hover against an integral alone.
- The tank runs out by pressure, so a 12 s hover lands at 2.83 m/s with 22.6 % of the nitrogen still inside.

Where else this applies. Volume 20, Chapter 4 allocates among rotors and a wing whose authority moves with the airspeed, and Volume 7, Chapter 4 holds the same tire curve near its friction peak while braking.

References

[1] W. C. Durham, "Constrained control allocation," *Journal of Guidance, Control, and Dynamics*, vol. 16, no. 4, pp. 717–725, 1993.

[2] T. A. Johansen and T. I. Fossen, "Control allocation—a survey," *Automatica*, vol. 49, no. 5, pp. 1087–1103, 2013.

[3] A. Anis, "Cold gas propulsion system – an ideal choice for remote sensing small satellites," in *Remote Sensing – Advanced Techniques and Platforms*. InTech, 2012.

[4] K. Lemmer, "Propulsion for CubeSats," *Acta Astronautica*, vol. 134, pp. 231–243, 2017.

[5] A. Reyes-Lúa and S. Skogestad, "Multi-input single-output control for extending the operating range: Generalized split range control using the baton strategy," *Journal of Process Control*, vol. 91, pp. 1–11, 2020.

[6] S. V. Weston, C. D. Burkhard, J. M. Stupl, R. L. Ticknor, B. D. Yost, R. A. Austin, P. Galchenko, L. K. Newman, and L. Santos Soto, "State-of-the-art small spacecraft technology," National Aeronautics and Space Administration, Ames Research Center, Moffett Field, CA, Technical Publication NASA/TP-20250000142, Jan. 2025. [Online]. Available: https://ntrs.nasa.gov/citations/20250000142

[7] U. Kiencke and L. Nielsen, *Automotive Control Systems: For Engine, Driveline, and Vehicle*, 2nd ed. Berlin: Springer Berlin Heidelberg, 2005.

[8] B. Wie, *Space Vehicle Dynamics and Control*, 2nd ed. American Institute of Aeronautics and Astronautics, 2008.

[9] B. Açıkmeşe and S. R. Ploen, "Convex programming approach to powered descent guidance for mars landing," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 5, pp. 1353–1366, 2007.

[10] J. Katz, "Aerodynamics of race cars," *Annual Review of Fluid Mechanics*, vol. 38, no. 1, pp. 27–63, 2006.

[11] L. Yun and A. Bliault, "Air cushion theory," in *Theory and Design of Air Cushion Craft*. Elsevier, 2000, pp. 48–83.

[12] G. P. Sutton and O. Biblarz, *Rocket Propulsion Elements*, 8th ed. Chichester, UK: John Wiley and Sons, 2010.

[13] A. Haque, S. Richardson, G. Saville, and G. Chamberlain, "Rapid depressurization of pressure vessels," *Journal of Loss Prevention in the Process Industries*, vol. 3, no. 1, pp. 4–7, 1990.

[14] R. Span, E. W. Lemmon, R. T. Jacobsen, W. Wagner, and A. Yokozeki, "A reference equation of state for the thermodynamic properties of nitrogen for temperatures from 63.151 to 1000 K and pressures to 2200 MPa," *Journal of Physical and Chemical Reference Data*, vol. 29, no. 6, pp. 1361–1433, 2000.

[15] C. R. Jarvis, "Flight-test evaluation of an on-off rate command attitude control system of a manned lunar-landing research vehicle," National Aeronautics and Space Administration, Washington, DC, NASA Technical Note D-3903, Apr. 1967.

[16] A. R. Klumpp, "Apollo lunar descent guidance," *Automatica*, vol. 10, no. 2, pp. 133–146, 1974.

[17] H. B. Pacejka, "Single-contact-point transient tire models," in *Tire and Vehicle Dynamics*. Elsevier, 2012, pp. 329–354.

[18] A. M. Eiband, "Human tolerance to rapidly applied accelerations: A summary of the literature," National Aeronautics and Space Administration, NASA Memorandum 5-19-59E, Jun. 1959.

[19] N. S. Currey, *Aircraft Landing Gear Design: Principles and Practices*. American Institute of Aeronautics and Astronautics, 1988.

[20] X. Shi, K. Kim, S. Rahili, and S.-J. Chung, "Nonlinear control of autonomous flying cars with wings and distributed electric propulsion," in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 5326–5333.

[21] Q. Tan, X. Zhang, H. Liu, S. Jiao, M. Zhou, and J. Li, "Multimodal dynamics analysis and control for amphibious fly-drive vehicle," *IEEE/ASME Transactions on Mechatronics*, vol. 26, no. 2, pp. 621–632, 2021.

[22] T. Ai, B. Xu, C. Xiang, W. Fan, and Y. Zhang, "Modeling and multimode analysis of electrically driven flying car," in *2020 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, 2020, pp. 1565–1571.

[23] J. Colmenares-Vázquez, P. Castillo, N. Marchand, and D. Huerta-García, "Nonlinear control for ground-air trajectory tracking by a hybrid vehicle: Theory and experiments," *IFAC-PapersOnLine*, vol. 52, no. 8, pp. 19–24, 2019.

[24] O. Pfeifle and W. Fichter, "Minimum power control allocation for incremental control of over-actuated transition aircraft," *Journal of Guidance, Control, and Dynamics*, vol. 46, no. 2, pp. 286–300, 2023.

[25] R. A. Martin, N. S. Gates, A. Ning, and J. D. Hedengren, "Dynamic optimization of high-altitude solar aircraft trajectories under station-keeping constraints," *Journal of Guidance, Control, and Dynamics*, vol. 42, no. 3, pp. 538–552, 2019, correction: vol. 43, no. 11, p. 2201, 2020, doi:10.2514/1.G003737.c1.

[26] F. Diba, A. Barari, and E. Esmailzadeh, "Handling and safety enhancement of race cars using active aerodynamic systems," *Vehicle System Dynamics*, vol. 52, no. 9, pp. 1171–1190, 2014.

[27] N. T. Van Dresar and G. A. Zimmerli, "Pressure-volume-temperature (PVT) gauging of an isothermal cryogenic propellant tank pressurized with gaseous helium," National Aeronautics and Space Administration, Glenn Research Center, Cleveland, OH, Technical Publication NASA/TP-2014-218083, Feb. 2014. [Online]. Available: https://ntrs.nasa.gov/citations/20140012695

[28] T. R. Kuphaldt, *Lessons in Industrial Instrumentation*, version 2.33 ed. Open textbook, Creative Commons Attribution 4.0 license, ibiblio.org/kuphaldt/socratic/sinst, 2024.

[29] K. J. Åström and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2nd ed. Princeton, NJ: Princeton University Press, 2021.

[30] National Highway Traffic Safety Administration, "Standard no. 126; electronic stability control systems for light vehicles, 49 CFR 571.126," Code of Federal Regulations, Title 49, Chapter V, edition of October 1, 2024, U.S. Government Publishing Office, 2024. [Online]. Available: https://www.govinfo.gov/content/pkg/CFR-2024-title49-vol6/pdf/CFR-2024-title49-vol6-sec571-126.pdf

[31] Y. Wang, J. Hu, F. Wang, H. Dong, Y. Yan, Y. Ren, C. Zhou, and G. Yin, "Tire road friction coefficient estimation: Review and research perspectives," *Chinese Journal of Mechanical Engineering*, vol. 35, no. 1, p. 6, 2022.

[32] United Nations Economic Commission for Europe, "UN regulation no 155: Uniform provisions concerning the approval of vehicles with regards to cybersecurity and cybersecurity management system," Official Journal of the European Union, L 82, 9 March 2021, pp. 30–59, 2021. [Online]. Available: https://eur-lex.europa.eu/eli/reg/2021/387/oj/eng

12.10 Exercises

12.1 Concept check · 2 min

At the end of the 12 s hover of case 9, about a quarter of the nitrogen fill is still in the tank, yet the car comes down at 2.83 m/s, a hard landing. Which explanation is right?

(a) The gauge over-reads at low temperature, so far less gas is left than the tile shows

(b) The climb-rate integral wound up in the hover and overshoots the landing schedule

(c) The valves were pulsed, so the hover burned gas faster than mg/v_e

(d) The gas cooled as it expanded, so its pressure fell below the regulator's and the lift fell with it

12.2 Concept check · 1 min

The metered launch sets the rear thrust to $\mathrm{clip}(\hat{m}a_t + \hat{D} - \hat{F}_{tire}, 0, T_{rear}f)$, as in Equation (12.7). Which allocation does this law carry out?

(a) The tires give what they can for free, and the thrusters add only the remainder toward the g target

(b) The thrusters carry the g target, and the tires add whatever the slip loop allows on top of it

(c) The demand is split between the tires and the thrusters in proportion to their force ratings

(d) The thrusters fire only while the slip is above its setpoint, so that the tire is relieved

12.3 Concept check · 2 min

In case 8 the hover needs about 60 % of each corner thruster, and the on-off valves can stay open for 0, 10 or 20 ms of each 20 ms period. With the nearest-width pulse law (Nearest width in the app), what does the hover do?

(a) It holds 2 m, because the pulses average to the demand over many periods

(b) It hunts below 2 m, because the demand rounds to half and the corner lift jumps between half and full

(c) It climbs above 2 m, because the demand rounds up to a full pulse in every period

(d) It settles a few centimeters low and stays there, because the integral cannot see the rounding

12.4 Theory and calculation · 6 min

The skirt of Equation (12.3) has $\Delta P_{max} = 6.35\,\mathrm{kPa}$, $k_Q = 9.83\,\mathrm{Pa\,s^2/m^6}$, $\rho = 1.2\,\mathrm{kg/m^3}$, $L_s = 12\,\mathrm{m}$, $\delta_0 = 5\,\mathrm{mm}$ and $A_{floor} = 4.0\,\mathrm{m^2}$. The full-tank car weighs 16.90 kN, and the dry tire peaks at $\mu = 1.170$.

(a) Find the sealed suction at full fan speed and its ratio to the full-tank weight. *Checkpoint:* 23.99 kN, 1.42 times the weight.

(b) Find the largest tire force on the dry strip with the fans off and with them fully spooled, and compare both with the motors' 34 kN. *Checkpoint:* 19.8 kN (58 % of the motors) and 47.8 kN.

(c) On the inverted ramp a bump opens the gap by a further 7 mm. Find the share of the sealed suction left during the bump, and the lowest fan speed at which the bumped suction still equals the weight. *Checkpoint:* 0.79 of sealed, and a fan speed of about 94.4 %.

12.5 Theory and calculation · 6 min

The thrusters have $I_{sp} = 73\,\mathrm{s}$ with $g = 9.81\,\mathrm{m/s^2}$. The full-tank car has an all-up mass of 1723 kg, the rear pair is rated at 50 kN together, and a proportional valve follows its command with a first-order lag of $\tau_v = 20\,\mathrm{ms}$.

(a) Find the exhaust speed, the gas rate of a hover on a full tank and the gas rate of the rear pair wide open. *Checkpoint:* 716.1 m/s, 23.6 kg/s and 69.8 kg/s.

(b) Find the gas spent in a 5 s hover from a full tank, allowing for the mass that the hover itself removes. *Checkpoint:* 114.1 kg.

(c) In case 3 both rear valves are commanded wide open at the launch and the car reaches 60 mph 0.60 s later. Estimate the gas spent to that instant, allowing for the valve lag. *Checkpoint:* 40.5 kg.

12.6 Theory and calculation · 6 min

At the start of the hover of case 9 the tank holds 84.9 % of its 423.0 kg fill. The dry mass is 1300 kg, $v_e = 716.1\,\mathrm{m/s}$, $g = 9.81\,\mathrm{m/s^2}$, and the gauge reserve is 8 % of the fill.

(a) Find the gas rate of the hover at that instant. *Checkpoint:* 22.7 kg/s.

(b) Find the runway to the reserve with Equation (1.11) at that rate, and again allowing for the rate falling as the car lightens. *Checkpoint:* 14.3 s and 15.9 s.

(c) With the pressure abort on, the car starts down after 9.30 s of hover. Compare and explain. *Checkpoint:* The mass runway is 1.5 to 1.7 times the pressure runway.

12.7 App challenge · 7 min

Load scenario 4, Metered to 3.4 g: the tires first, at the default seed. Each run ends by itself when the car has braked to a stop, about 3.5 s after Run.

(a) Run it as shipped and read the 0–60 mph, Peak g (launch) and N_2 for the launch tiles. *Checkpoint:* 0.83 s, 3.39 g, 29.1 kg.

(b) Press Reset, set Metered g target to 3.00 g, and run again. *Checkpoint:* 0.94 s, 3.00 g, 24.3 kg.

(c) Load scenario 3, Rear thrusters wide open, and run it as shipped. From the three runs, find the nitrogen spent per second of 0–60 time saved, from 3.0 g to 3.4 g and from 3.4 g to wide open. *Checkpoint:* 0.60 s, 4.83 g, 40.5 kg. About 44 and 50 kg/s.

Read the tiles to their last digit, and accept a price within 3 kg/s.

12.8 App challenge · 9 min

Load scenario 9, A 12 s hover and a soft landing on one tank, at the default seed. Each run ends by itself when the car is down, 16 to 19 s after Run.

(a) Run it as shipped and read the Touchdown and N_2 left tiles. *Checkpoint:* 2.83 m/s (hard), 95 kg and 23 %.

(b) Press Reset, set Land when the tank nears the regulator On, and run again. Read how long the car hovered (Hover in band) and the Touchdown and N_2 left tiles. *Checkpoint:* 9.3 s, 0.94 m/s, 140 kg and 33 %.

(c) Press Reset, set Land when the tank nears the regulator Off and Hover time to 10.5 s, and run. Then press Reset, set Hover time to 11.0 s, and run again. Read the Touchdown tile each time. *Checkpoint:* 1.15 m/s (soft) and 1.72 m/s (hard).

Say what limits the hover and how much gas the soft landing leaves unused.

12.9 App challenge · 8 min

Load the challenge, One tank, at the default seed. Each run ends when the car is down and settled or has crashed, within about 17 s of Run.

(a) Run it as shipped. Read the Peak g (launch), Touchdown and Run tiles, and name the two clauses of the target that fail. *Checkpoint:* 4.82 g, 5.81 m/s, CRASHED.

(b) Press Reset, set Rear thrusters to Metered, Mass feedforward On, Thruster valves to Proportional and Aerial roll to Lob first, and run again. Read Peak g (launch), Roll ended, Touchdown, N_2 left and Challenge target. *Checkpoint:* 3.41 g, 0.1°, 1.22 m/s, 160 kg and 38 %, met.

(c) Press Reset, set Mass feedforward Off, and run again. Read Hover worst error, Touchdown, N_2 left and Challenge target. *Checkpoint:* 15.4 cm, 0.50 m/s, 109 kg and 26 %, met.

Say what the feedforward is worth on this flight.

12.10 Application area · 4 min

The model holds I_{sp} at 73 s, as if the gas were warmed to ambient, 293 K, between the regulator and the nozzles, while the tank gas cools to near −150 °C. For an ideal nozzle the exhaust speed scales with the square root of the absolute temperature of the gas at its inlet.

(a) Estimate I_{sp} for gas fed to the nozzles at −150 °C. *Checkpoint:* About 47.3 s.

(b) At the end of the hover of case 9 the car weighs 13.8 kN. Find the hover gas rate with each specific impulse, and say which results of the chapter would move and in which direction. *Checkpoint:* 19.3 kg/s and 29.7 kg/s.

12.11 Application area · 3 min

In case 9 as shipped the tank pressure meets the regulator's 60 bar after 14.86 s, with 38.1 % of the nitrogen fill still in the tank. The gauge reserve that triggers a landing is 8 % of the fill.

(a) Which limit is reached first in this flight, and what share of the fill would a reserve on mass need to protect full thrust here? *Checkpoint:* The pressure, and at least 38.1 %.

(b) Explain why no fixed share of mass serves as the reserve on every flight, and name the measurement the app's abort uses instead. *Checkpoint:* The tank pressure against the regulator setpoint.

12.12 Review the agent · 5 min

An AI assistant reviews case 6, Takeoff and a 5 s hover, and writes: "The mass feedforward only improves the hover's accuracy. A hover costs mg/v_e whichever loop holds it, so the run spends the same nitrogen with the feedforward on or off. Leave it off to keep the controller simple, since the integral will find the weight." Find the flaw in this claim and say how it could be checked.