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CHAPTER 5

Shear Heating, Overrides and Delayed Density Feedback in a Cereal Extrusion Simulator

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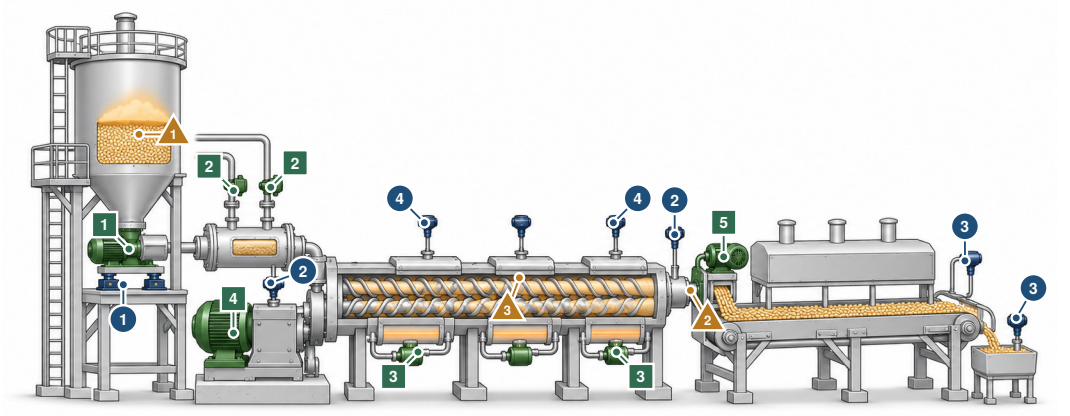
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CHAPTER 5

Shear Heating, Overrides and Delayed Density Feedback in a Cereal Extrusion Simulator



The controller moves 1 flour feeder 2 water and steam valves 3 barrel heating and cooling 4 screw speed 5 cutter knife speed

The controller reads 1 flour weight in the feeder 2 drive torque and die pressure 3 final moisture and bulk density 4 barrel temperatures

What disturbs it 1 a new flour lot 2 deposits in the die holes 3 worn screws

A twin-screw cereal extruder with its preconditioner, die, cutter and belt dryer, in which the torque and the die pressure that the overrides guard are read on the machine while the density that decides the product is known only after the die. Illustration, not to scale.

IN PLAIN TERMS **Cooking Cereal With the Push of Two Screws**

The main idea. An extruder pushes damp flour along two turning screws and out through small holes. The rubbing of the screws, not the heaters, heats the dough, and the water in it turns to steam at the holes and puffs each piece.

The challenge. A wetter or stiffer flour changes the push on the screws, the pressure at the holes and the puff of the pieces all at once. Guard controllers keep the screws and the holes inside safe limits, but the weight of a cup of pieces, the real test, comes back only minutes later.

Why it matters. A line pushed for more output can protect its machine and still fill the reject bin.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the first history segment (Section 5.1), the second history segment (Section 5.2), the fact on maintenance (Section 5.3), the two try-it segments and the fact on the challenge (Section 5.6) and the key points
Study	75 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	25 min	the first page, the results (Section 5.6), Technology in Practice (Section 5.7), Limitations (Section 5.8) and the open challenge

The control question The screws of a cereal extruder heat the melt they carry, so a wetter flour or a stiffer starch moves torque, die pressure and expansion together, while the product's density is known only minutes later. How do overrides realized as clamps, a sampled density trim and a throughput supervisor keep the line inside its limits?

Learning objectives

- Trace the melt's heat to viscous dissipation, not the barrel zones.
- Explain how two overrides realize a maximum and a minimum selector as output limits.
- Compute the error a wrong lot certificate puts into a ratio station.
- Predict where a throughput supervisor stops from its predicted constraints.
- Show that the predictive controller's gain on a wet lot comes from its estimator.

Prerequisites Sections 1.4.3, 1.4.4, 1.4.2, 1.4.6, 1.6.2 and 1.6.4, and Section 1.6.3 for the predictive controller.

The app https://apmonitor.com/apps/food-extrusion/

See also Volume 5, Chapter 10 for a soft sensor corrected by a delayed laboratory, Volume 5, Chapter 3 for control through a slow analyzer and Volume 5, Chapter 4 for estimation ahead of a late analyzer. In this volume, Chapter 4 for throughput pushed against overrides.

5.1 Introduction

Extrusion cooking makes breakfast cereals, expanded snacks and many other foods in one continuous thermal and mechanical process [1, 2]. Flour and water are metered into a preconditioner, sheared, cooked and pressurized by the screws, and forced through a die, where the water superheated in the melt flashes to steam and blows it into a foam [3]. How long this takes depends on the throughput, the screw speed and the screw profile [4]. In the simulator of this chapter the dough spends 31.6 s in the barrel at a recipe of 300 kg/h of flour and 400 rpm, and the pieces then dry on a belt for 12 min before they are sold by their bulk density and their final moisture.

> ⧖ HISTORY **From Ram to Twin Screw, 1930s to 1980s**
>
> The first extruders used for food were pistons and rams that pushed meats and sausages. In the 1930s a single screw replaced the piston and turned a batch process into a continuous one, and single screws were put to work making pasta. Twin-screw extruders were established in the 1960s, and their wider scope led in the 1980s to a large variety of new snack and cereal products. An extension overview of 2017 describes the modern machine as one that mixes, kneads, cooks, forms and cuts in a single piece of equipment, with twin-screw systems the more widely used because of their flexibility [5].

The heat is what separates an extruder from a heated vessel. The power the drive puts into the screws is dissipated in the melt, so the specific mechanical energy (SME), the shaft power per unit of melt flow, measures the effort of the drive and the self-heating of the melt at once. At the recipe the screws deliver 14.8 kW, 43.0 kWh/t of melt, and carry the dough from 88.0 °C at the preconditioner to 158.0 °C at the die exit, while the heater bands and cooling water of the five barrel zones exchange a net −0.06 kW with it. That heat depends on a viscosity that falls with temperature and moisture [6], and the expansion depends on the same moisture and melt temperature [7], so moisture alone moves every output. One point of process moisture lowers the torque by 5.0 % of its rating, the die pressure by 0.51 MPa and the melt temperature by 2.3 °C, and it raises the bulk density by 11.5 g/L.

The simulator addresses the belief that an extruder whose barrel zones hold their setpoints is still making the product it made an hour ago. When a flour lot arrives three points wetter than its certificate, every heated zone reads within 0.91 °C of its setpoint over the first 20 min while the density at the cutter climbs from 135 to 164.6 g/L, out of its 120 to 150 g/L window. The coupling also runs the other way, since a falling torque is the first sign of a worn screw. The class of problem is general. In polymer extrusion the rheology of the material decides both the energy the screw consumes and

the thermal quality of the melt it delivers [8], and any process whose quality is set by a nonlinear, self-heating step and measured downstream has the same structure. Inferential control of product quality attributes that are not measured on line has been applied to food cooking extrusion [9].

Control studies of cooking extruders identify dynamic models of the process and design predictive and inferential controllers on them [10, 11]. However, a model identified at one recipe carries its gains as numbers, and a student who tunes a loop against it cannot see that the gains are the heat balance of a melt heating itself, or why they move with every lot of flour.

This chapter accounts for the heat, the pressure and the product of a twin-screw cereal extruder in a simulator whose loops, overrides and quality trim can each be switched, retuned and measured. Extrusion cooking, ratio control, override selection, sampled trims and cascade control are all standard practice [12, 13]. The simulator rests on a 105-state model in which the shaft power equals the heat released in the melt by construction, the dry-solids, water and energy ledgers close to round-off, and every parcel of product keeps its true density and moisture apart from what the instruments report. The model is set down in full so that it can be rebuilt. Its cook, expansion and density correlations are calibrated teaching proxies, and its thermal treatment is not a validated food-safety lethality model. The optional model predictive controller (MPC) treats the overrides as its constraints, as industrial practice does [14]. Three of its elements are specific to this line. The at-line density, reported 90 s after its cup is filled, corrects the prediction made for the moment the cup was taken. The soft sensor's error, shared by every controller that trusts the certificate, is estimated as an unknown moisture entering with the water, so that a torque change the plan did not cause is read as moisture. The torque and the die pressure pay for an excess at every step of the plan's first minute, which is what kept the plan from crossing the 90 % torque limit. Nine cases and an hour-long challenge set a wrong certificate, a stiffer starch, a push for throughput and a worn screw against the same coupling, each from its own side.

Section 5.2 traces how extrusion cooking came to be modeled, measured and controlled. Section 5.3 sets out the model of the line and Section 5.4 its heat balance and steady-state gains. Section 5.5 describes the implemented loops, the two overrides, the density trim, the throughput supervisor and, in Section 5.5.9, the predictive controller. Section 5.6 gives the cases as measured, and Section 5.6.8 compares the predictive controller with the loops. Section 5.7 sets the loops on real line hardware, Section 5.8 states the limitations, Section 5.9 concludes, and Section 5.10 holds the exercises.

5.2 Extrusion Cooking, Its Measurement and Its Control

A review of food extrusion ran to 61 pages by 1979 [1], and the processing studies that followed took the machine apart one mechanism at a time. The mean residence time measured on a co-rotating twin-screw cooking extruder in 1986 depended strongly on the throughput, the screw speed and the screw profile and not on the water content, the die size or the temperature, and a combination of plug flow and perfect mixing described the dimensionless distributions [4]. The rheology of low-moisture molten maize starch was measured the following year [6].

Expansion, the property that decides the product, was modeled next, and the instrument that reads the melt was found wanting, as the history segment below tells. The mechanism of expansion was later summarized for cereal foods. Moisture turns the glassy starch matrix rubbery at process temperatures, superheated steam bubbles form at nuclei and grow, and the biaxial extensional viscosity of the matrix governs how far they expand [3]. Expansion and bulk density can be read only from product that has already left the die.

Control followed the modeling. The modeling and process control of twin-screw cooking extruders were reviewed in 1994 [10]. By 2000 on-line system identification was used to design model predictive and inferential controllers for an extrusion cooking process [11], and in 2001 inferential control of product quality attributes was applied to food cooking extrusion [9], the year a chapter on effective process control appeared in a book on extrusion cooking [12]. The sequence runs from modeling the

process toward inferring what cannot be measured on it, which is also the order of the simulator's layers.

> ⧖ HISTORY **Modeling the Puff, Misreading the Melt, 1988 to 1997**
>
> A 1988 study of corn extrudates from a single-screw extruder put expansion into one model. Lengthwise and volumetric expansion indices were defined beside the sectional one, an inverse relation between radial and lengthwise expansion was confirmed, and one model in the dough moisture, the melt temperature and the die and screw shear strains described all three [7]. Measuring those inputs proved harder. In 1997 infrared and extended melt probes agreed on the bulk temperature of the extrudate, while the standard flush-mounted probe differed substantially from both and may not read it accurately. The heat lost from the surfaces of a twin-screw barrel varied between 2 and 19 % of the total heat input [15].

An extruder also does things that no loop is asked to control. Extrusion cooking destroys antinutritional factors, gelatinizes starch and raises the soluble dietary fiber, while Maillard reactions lower the value of the protein by amounts that depend on the raw material and the conditions, and heat-labile vitamins may be lost to varying extents [2]. Where an extruder is the kill step of a process, its lethality is established by experiment, for example for *Enterococcus faecium* in a balanced carbohydrate-protein meal over a response surface design [16]. The simulator models neither, and its cook index describes starch conversion only.

5.3 Model of the Line

At its recipe the simulated line turns 300 kg/h of flour at 12 % moisture into an expanded cereal. Moisture is a wet-basis mass fraction throughout, except for the dry-basis ratio X on the dryer belt. A loss-in-weight feeder meters the flour into a steam preconditioner, a ratio station adds the water that makes the dough 23.3 % water, and two co-rotating screws carry it through five barrel zones of 220 mm, a feed zone, a conveying zone, a kneading block, a second conveying zone and a metering zone. A die of 20 holes of 2.8 mm diameter sets the pressure, a four-knife cutter sets the piece length, and a belt carries the pieces through the dryer for 12 min. Every actuator, instrument and equation symbol has its place on the drawing of the line in Figure 5.1. A feeder of this kind reads its flow as the falling weight of its hopper, and Volume 12, Chapter 12 shows what such a weighing loop can and cannot see of the product.

> ✦ DID YOU KNOW? **Two Screws Cost More to Keep**
>
> A twin-screw extruder costs 1.5 to 2.0 times as much to maintain as a single-screw one, an extension overview of 2017 notes, yet twin screws are the more common in the food industry for their wide range of operating conditions and products [5].

The variables are sorted by role in Table 5.1, and 105 states are integrated. Each of the two preconditioner cells and ten barrel cells carries six, dry solids S, water W, enthalpy H, cooked solids G, amylose-weighted solids A and a dye R, and there are five barrel-metal temperatures, the filled length, four drives, the dryer air, the die deposit and 21 cumulative ledgers of what has entered and left. The product on the belt is carried as discrete parcels, updated after each integration step.

5.3.1 Feed, Preconditioner and the Chain of Cells

The feeder, the two valves and the main drive follow their commands through first-order lags, as shown in Equation (5.1).

$$\begin{aligned} \tau_F \dot{F} &= u_F F_{max}(1+\varepsilon_F) - F, \qquad \tau_N \dot{N} = N_{cmd} - N \\ \tau_W \dot{W} &= u_W W_{max} - W, \qquad \tau_S \dot{S} = u_S S_{max} - S \end{aligned} \tag{5.1}$$

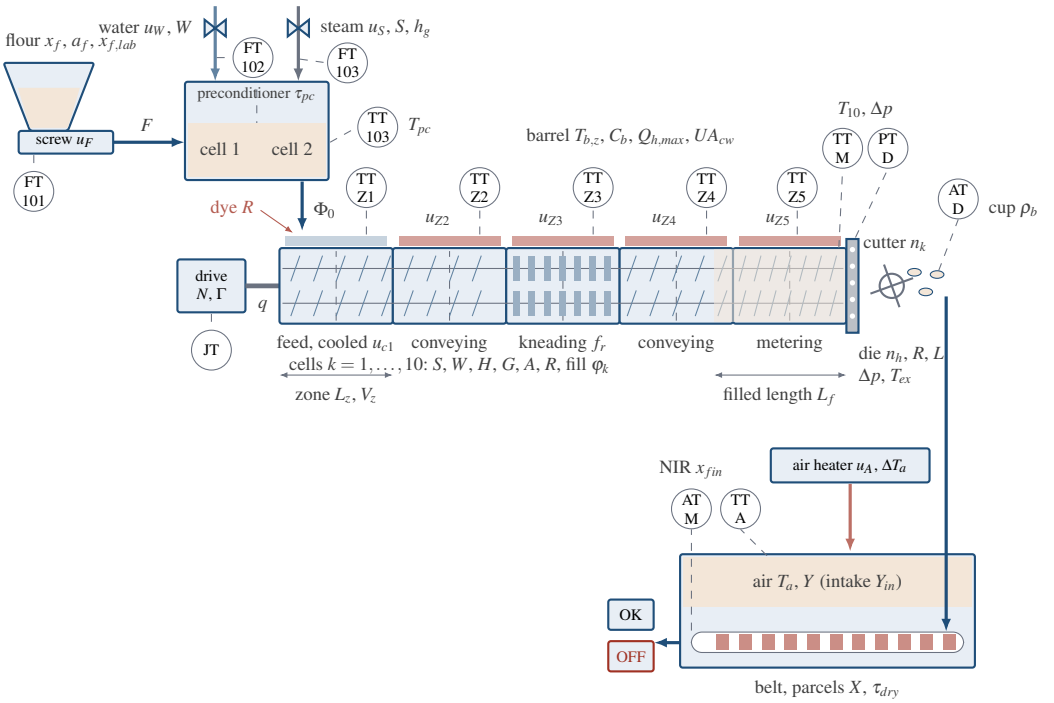


Figure 5.1 Cereal extrusion line as modeled. The loss-in-weight feeder meters flour F of true moisture x_f and amylose a_f, whose certificate states $x_{f,lab}$, and valves add water W and steam S to a preconditioner of two mixed cells. The two screws, turned at speed N against torque Γ, carry the dough through ten mixed cells in five zones, each zone with one split-range output u_Z to its heater bands or cooling water, and the metering cells in front of the die run full over the filled length L_f. The die of n_h holes sets the pressure Δp, the cutter at n_k sets the piece length, and the belt of the dryer, drawn below the barrel and running from right to left, carries 10 s parcels through air at T_a and humidity Y. AT-D weighs a cup at the cutter every minute and AT-M reads the moisture at the belt exit.

where u_F, u_W and u_S are fractions of full output, N is in rpm, and ε_F is a pulsation of the feeder's real flow, colored noise of 0.6 % standard deviation and 2 s correlation time. The two preconditioner cells and the ten barrel cells form one chain. For each cell variable $Z \in \{S,W,H,G,A,R\}$ the balance is Equation (5.2).

$$\frac{\mathrm{d}Z_j}{\mathrm{d}t} = r_{j-1}Z_{j-1} - r_j Z_j + \sigma_j^Z, \qquad T_j = \frac{H_j}{c_s S_j + c_w W_j} \tag{5.2}$$

Here r_j is the discharge rate of cell j, $x_j = W_j/(S_j+W_j)$ its moisture, $c_j = G_j/S_j$ its cook index and $a_j = A_j/S_j$ its amylose share. The first cell receives dry solids $F(1-x_f)$, water Fx_f+W+S, enthalpy $Fh_{fl}+Wc_wT_w+Sh_g$ and amylose $a_fF(1-x_f)$, where $h_{fl} = [(1-x_f)c_s + x_f c_w]T_{fl}$, so the steam condenses completely. The dye enters with the dough at the first barrel cell. In the preconditioner $r_j = o/(\tau_{pc}/2)$, with $o = \mathrm{clamp}(N/N_{min},0,1)$ so that a stopped screw also stops the discharge, the enthalpy source is the heat loss $-(UA_{pc}/2)(T_j - T_{amb})$, and the cook source is thermal only.

5.3.2 The Starve-Fed Barrel

The feeder, not the screws, sets the throughput. A barrel cell passes on what it holds at the drag capacity of the screws, except the fraction f_k of its volume that is completely full, which passes on

Table 5.1 Roles and Recipe Values of the Line Variables

Variable	Role	Recipe value	Range
ρ_b (AT-D)	CV, QIC, sampled	135 g/L SP	120–150
x_{fin} (AT-M)	CV, MIC, delayed	3.00 % SP	2–4
$\hat{x}$	estimate, ratio	23.3 % SP	19–28
F (FT-101)	CV, FIC-101	300 kg/h SP	150–540
W (FT-102)	CV, FFC-102	22.5 kg/h	0–100
T_{pc} (TT-103)	CV, TIC-103	88 °C SP	60–98
$T_{b,2..5}$ (TT-Z)	CVs, split range	115/140/140/140 °C	60–180
Γ (JT)	constraint, TQC	65.4 %	85, trip 100
Δp (PT-D)	constraint, PIC	6.77 MPa	9, trip 12
T_{ex}	constraint	158.0 °C	up to 170
T_a (TT-A)	inner CV, TIC-A	133.7 °C	90–149
u_F, u_W, u_S	MVs	50.0/22.5/54.3 %	0–100
$u_{Z2..Z5}$	MVs, heat or cool	44/40/14/−19 %	±100
N	MV of TQC	400 rpm	150–450
F_{SP}	MV of PIC	300 kg/h	60–540
Δx	MV of QIC	0	±4 points
u_A	MV of TIC-A	72.4 %	0–100
n_k	knife, ratio	755 rpm	100–3000
x_f, a_f	DV, unmeasured	12, 25 %	8–18, 15–40
$x_{f,lab}$	certificate	12 %	8–18
w, h_3	DV, wear, heaters	0, 100 %	0–80, 0–100
r_δ, Y_{in}	DV, deposit, air	0, 12 g/kg	0–2, 3–60
S, W, H, G, A, R	states, 12 cells	see text	—
$T_{b,z}$, L_f, T_a, δ	states	74.6–140, 163 mm	—

only what arrives, as shown in Equation (5.3).

$$\begin{aligned} r_k^{-1} &= \frac{V_c(1-f_k)}{qN/60} + \frac{\rho V_c f_k}{\Phi_{k-1}} \\ f_k &= \max\Big[f_{r,z},\ \mathrm{clamp}\Big(\frac{L_f-(10-k)L_c}{L_c},0,1\Big)\Big] \end{aligned} \tag{5.3}$$

where $k = 1,\ldots,10$ counts the barrel cells, $V_c = V_z/2$ and $L_c = L_z/2$ are the volume and length of a cell, $qN/60$ is the volumetric drag capacity, Φ_{k-1} is the mass flow into the cell, $f_{r,z}$ is the fraction a restrictive element holds full (0.35 in the kneading block, zero elsewhere) and L_f is the filled length in front of the die. At steady state the fill $\varphi_k = (S_k + W_k)/(\rho V_c)$ is $\varphi_{st} + (1-\varphi_{st})f_k$ with $\varphi_{st} = \Phi/(\rho qN/60)$. At the recipe the drag capacity is 1092 kg/h against a flow of 344.2 kg/h, so the conveying cells run 31.5 % full, the kneading block 55.5 % and the last cell full. A faster screw holds less dough, which is how screw speed sets the residence without changing the throughput.

5.3.3 Viscosity, Dissipation and Torque

The melt is a power-law fluid of index n whose consistency is the product of four factors, as shown in Equation (5.4).

$$\begin{aligned} K = K_{ref}\,\mathrm{e}^{E_\eta\left(\frac{1}{T^*}-\frac{1}{T^*_{ref}}\right)}\,\mathrm{e}^{-\beta_X(x-x_{ref})} \\ \times\left[1-\kappa_c(1-c)\right]\mathrm{e}^{\beta_A(a-a_{ref})} \end{aligned} \tag{5.4}$$

with T^* the absolute temperature, floored at 30 °C. The consistency falls with temperature and moisture and rises with the cook and with the amylose share of the starch. Each barrel cell dissipates

Table 5.2 Screw Profile, Fixed in the Source

Zone	Element	C_γ	χ	f_r
1	feed, cooled at u_{c1}	20	0.10	0
2	conveying	20	0.30	0
3	kneading block	40	0.55	0.35
4	conveying	25	0.40	0
5	metering	30	0.60	0

the power of Equation (5.5).

$$P_k = \chi_z K_k \dot{\gamma}_z^{n+1} V_c \varphi_k, \qquad \dot{\gamma}_z = C_{\gamma,z} m_z \frac{N}{60} \tag{5.5}$$

where χ_z and $C_{\gamma,z}$ are the dissipation factor and the shear rate per revolution per second of the screw elements in zone z, listed in Table 5.2, and $m_z = \max(0.1, 1 - w_s w)$ in zones 3 to 5 and 1 elsewhere, with w the wear fraction. The same P_k heats the cell and loads the shaft, as shown in Equation (5.6).

$$P_{sh} = \sum_{k=1}^{10} P_k + \Delta p\, Q_v, \qquad \Gamma = \frac{P_{sh}}{2\pi N/60}, \qquad \mathrm{SME} = \frac{P_{sh}}{\Phi_{10}} \tag{5.6}$$

with Δp the die pressure, Q_v the volumetric flow through the die and Φ_{10} the melt flow. Shear heating and mechanical work therefore cannot disagree. The application checks $2\pi N\Gamma/60$ against the sum to 10^{-12} and the integral of $\mathrm{SME}\,\Phi_{10}$ against the energy ledger to 1.8×10^{-5} over 600 s.

5.3.4 Barrel Metal and the Split-Range Zones

Each zone's barrel section is one heat capacity, heated by its bands or cooled by its water through a single output $u_z \in [-1, 1]$, a split range at zero output in the sense of Section 1.4.6, as shown in Equation (5.7).

$$\begin{aligned} C_b \frac{\mathrm{d}T_{b,z}}{\mathrm{d}t} &= u_z^+ h_z Q_{h,max} - u_z^- U A_{cw} (T_{b,z} - T_{cw}) \\ &\quad - \sum_{k \in z} q_k - U A_{amb} (T_{b,z} - T_{amb}) \\ q_k &= \tfrac{1}{2} U A_b \left[0.3 + 0.7 \min(1, \varphi_k)\right] (T_{b,z} - T_k) \end{aligned} \tag{5.7}$$

where $u^+ = \max(0, u)$, $u^- = \max(0, -u)$, h_z is the share of heater capacity still working (1 except in zone 3, where it is a disturbance) and q_k is the heat passed to the dough in cell k, scaled by how much of the wall the dough wets. The enthalpy source of a barrel cell is $q_k + P_k$. The feed zone has no loop, and its cooling valve is held at $u_1 = -u_{c1}$.

5.3.5 Die and Filled Length

The die is n_h capillaries of radius $R = R_h(1 - \delta)$, where the deposit δ grows at the disturbance rate r_δ in percent of the radius per minute and is capped at 0.6. The pressure drop of a power-law melt with the end correction of [17] is Equation (5.8).

$$\begin{aligned} \Delta p &= \frac{2\tau_w (L + eR)}{R}, \qquad \tau_w = K_{10} \dot{\gamma}_w^n \\ \dot{\gamma}_w &= \frac{3n+1}{n} \frac{Q_v}{n_h \pi R^3} \end{aligned} \tag{5.8}$$

where $Q_v = \Phi_{10}/\rho_m$ and e is the entrance correction in radii. At the recipe the wall shear rate is 2259/s, the apparent viscosity 58.9 Pa s and the pressure 6.77 MPa. The screws must run completely

full over the length that builds this pressure, which follows its target through a one-second lag, as shown in Equation (5.9).

$$\begin{aligned} \tau_L \dot{L}_f &= \mathrm{clamp}(\Delta p / G_p,\, 0,\, 3L_z) - L_f \\ G_p &= \frac{c_G K_{10} \dot{\gamma}_5^{n}}{h} (1-\varphi_{st})(1-w) \end{aligned} \tag{5.9}$$

with h the channel depth, $\varphi_{st} \le 0.95$ and both bracketed factors floored at 0.05. A screw turning below $N_{min}/2$ holds its filled length. The die adds the heat of its own pressure work, so the melt leaves at $T_{ex} = T_{10} + \Delta p/(\rho_m c_p)$ with $c_p = (1-x)c_s + x c_w$, 2.28 °C above the melt in front of the die at the recipe. A filled length beyond $2L_z$ reaches the kneading block and raises an alarm that clears below $1.8L_z$.

5.3.6 Cook

The cook index is first order in the uncooked starch, as shown in Equation (5.10).

$$\begin{aligned} \sigma_j^{G} &= \nu_j \,(S_j - G_j) \\ \nu_j &= k_c \,\mathrm{e}^{-E_c\left(\frac{1}{T^*} - \frac{1}{T^*_{c,ref}}\right)} \mathrm{e}^{\beta_c (x - x_{ref})} + k_M \frac{P_j}{S_j + W_j} \end{aligned} \tag{5.10}$$

Its thermal part, with the temperature floored at 20 °C, rises steeply with temperature and with moisture, and its mechanical part exists only in the barrel, so a screw that shears less also cooks less. Because it is evaluated in every mixed cell, the die sees the cook averaged over the whole residence-time distribution, 0.945 at the recipe, with 0.196 already done in the preconditioner.

5.3.7 Flash, Expansion and Bulk Density

This part of the model is a calibrated teaching proxy, chosen to give the directions reported for expanded cereals [3, 7] and labeled illustrative in the application. It is not a validated texture or density model for any flour or extruder. At the die exit the melt flashes adiabatically to 100 °C, losing the mass fraction and keeping the moisture of Equation (5.11).

$$e = \min\!\left[\frac{c_p (T_{ex} - 100)}{\lambda},\; x - 0.03\right], \qquad x_{ext} = \frac{x - e}{1 - e} \tag{5.11}$$

The sectional, lengthwise and volumetric expansion indices and the bulk density of the dried pieces follow from Equation (5.12).

$$\begin{aligned} \mathrm{SEI} &= 1 + (\mathrm{SEI}_{max} - 1)\, g_T\, \mathrm{e}^{-\beta_E (x - x_{ref})} \left(\frac{\Delta p}{p_{ref}}\right)^{p_E} c^3 \\ g_T &= \left(\frac{T_{ex} - 100}{\theta_T}\right)^{m_T} \left[1 + \mathrm{e}^{(T_{ex} - T_{col})/w_{col}}\right]^{-1} \\ \mathrm{LEI} &= \mathrm{LEI}_{ref}\, (\mathrm{SEI}/4.5)^{-\lambda_L} \\ \rho_b &= \frac{k_{pack}\, \rho_m (1-x)}{\mathrm{SEI} \cdot \mathrm{LEI}\, (1 - x_{fin})} \end{aligned} \tag{5.12}$$

where Δp is floored at 0.2 MPa, $g_T = 0$ below 100 °C and x_{fin} is the final moisture. The expansion rises with the superheat, the cook and weakly with the die pressure, falls with moisture, collapses above T_{col}, and trades length against section, which is the inverse relation of [7]. At the recipe the melt flashes 5.66 % of its mass, leaves at 18.70 % moisture with an SEI of 4.53 and an LEI of 1.047, and makes 135.0 g/L at the 3.0 % target moisture. The pieces are $D = 2R\sqrt{\mathrm{SEI}}$ across, 5.96 mm, and $\ell = v\,\mathrm{LEI}/f_{cut}$ long, with v the velocity in a hole and $f_{cut} = n_{knife} n_k / 60$ the cutting frequency, 12.0 mm at a knife speed of 755 rpm.

5.3.8 Belt Dryer and the Product Record

The cutter forms a parcel every 10 s from the dry solids and the water that left the die, less the flashed vapor, with flow-weighted means of the expansion, density, cook and piece size. Each parcel rides the belt for τ_{dry} and dries first-order toward the equilibrium of the air it meets, which is integrated exactly across each step, as Equation (5.13) states.

$$\begin{aligned} \dot{X} &= -k(X - X_{eq}), \qquad X_{eq} = a_E \left(\frac{RH}{1 - RH} \right)^{b_E} \\ k &= k_d \, \mathrm{e}^{(T_a - T_{d,ref})/\theta_D} \, \frac{\max(0, T_a - T_{wb})}{T_{d,ref} - 42} \end{aligned} \tag{5.13}$$

where X is the dry-basis moisture, X_{eq} follows the isotherm of [18], $RH = p_v / p_{sat}(T_a)$ is capped at 0.95, $p_v = 101.325\,Y/(0.622 + Y)$ kPa, p_{sat} comes from the Antoine equation, and the wet-bulb temperature is a linear fit to the psychrometric chart between 100 and 160 °C, $T_{wb} = 42 + 0.12(T_a - 130) + 250(Y - 0.020)$. The air follows its heater through $\tau_a \dot{T}_a = T_{in} + u_A \, \Delta T_a - T_a$, and it carries the intake humidity plus the water just evaporated, $Y = Y_{in} + E/\dot{m}_a$, with E the evaporation rate through a 30 s lag. A parcel leaving the belt is judged on its true properties: dense above 150 g/L, light below 120 g/L, wet above 4.0 % and dry below 2.0 %. The controllers never read these records.

5.3.9 Numbers, Initialization and Numerics

Every parameter is listed in Table 5.3. Fifty are shown in the parameter panel and may be moved while the line runs, and 35 are hidden. Each scenario opens on the steady state of the line at its recipe. When a setting chosen before Run would put the torque or the die pressure past an override's limit, the start is the steady state with that override holding it, the screws faster or the feed lower, found by false position. The solver sets the preconditioner analytically, with the water from the certificate and the steam that holds its setpoint, solves each barrel cell's energy balance by bisection, which is monotone because a hotter melt dissipates less, iterates the cook and the dissipation together, and closes the filled length by substitution damped by 0.6. At the recipe it returns 22.48 kg/h of water, 21.71 kg/h of steam, a filled length of 163.2 mm, a torque of 353.1 N m (65.38 %), the feed-zone metal at 74.6 °C and the zone outputs at 44.0, 39.9, 14.2 and −19.1 %. The dryer air, 133.7 °C, is found by bisection for the 3.0 % target, and the belt is filled with 72 parcels at the steady drying profile. The largest relative derivative at the stored state is 1.7×10^{-16}.

On the three clocks of Section 1.6.1, the integrator is fourth-order Runge-Kutta at a fixed 0.1 s step, the controllers execute every 0.5 s, and the historian logs once per simulated second. Halving the step moves the cutter density at 900 s of the wetter-flour case by 2.6×10^{-12} g/L, because the controllers then execute at the same instants, and parcels of 5 s instead of 10 s move the final moisture of the humid-air case at 1200 s by 1.4×10^{-3} points. Over 30 min of the wetter-flour case with a forced trip and a restart the dry-solids and water ledgers close to 1.9×10^{-15} of their throughput, and over 20 min of the heater case the energy ledger closes to 1.2×10^{-15}. These are among the application's 55 passing checks. The random streams start from the seed 20261204, the default seed, so a run replays to the digit.

5.3.10 Assumptions

The barrel is ten well-mixed cells in series with residences from a starve-fed conveying law. The cells are a residence-time model, not a discretization, and they spread a pulse from its first appearance at the die, while a real screw adds a plug-flow delay [4] and narrower tails.

The melt is a power-law fluid with one index everywhere, and its consistency is an exponential product of temperature, moisture, cook and amylose. Real starch melts are viscoelastic, and the expansion mechanism depends on their extensional viscosity [3], which the model does not carry. The correlations are meant for about 19 to 28 % moisture and 100 to 180 °C.

Table 5.3 Parameters of the Line. A Marks a Value Shown and Adjustable in the Parameter Panel and H a Hidden One

Symbol	Meaning	Value		Symbol	Meaning	Value	
q	drag volume per revolution	35 cm^3	A	τ_{pc}	preconditioner residence	150 s	A
V_z, L_z	free volume, length per zone	1.0 L, 220 mm	H	UA_{pc}	preconditioner loss	25 W/K	H
ρ, ρ_m	dough in barrel, melt at die	1300, 1350 kg/m^3	H	S_{max}, W_{max}	steam, water valve capacity	40, 100 kg/h	A
N_{min}, N_{max}	screw speed range	150, 450 rpm	H, A	h_g	steam enthalpy, 3 bar	2725 kJ/kg	H
Γ_{rated}	rated torque, both shafts	540 N m	A	T_{fl}, T_w, T_{amb}	flour, water, room	20, 20, 25 °C	H
τ_N	drive speed lag	1.5 s	H	τ_W, τ_S, τ_F	water, steam, feeder lags	2, 3, 1 s	H
w_s	shear lost per unit wear	0.6	H	F_{max}	feeder capacity	600 kg/h	A
n	power-law index	0.40	A	σ_F	feeder pulsation, 1σ	0.6 %	A
K_{ref}	consistency at reference	7000 Pa s^n	A	k_c	cook rate at 140, 23.3	0.06/s	A
T_{ref}, x_{ref}, a_{ref}	consistency reference	150 °C, 23.3, 25 %	H	$T_{c,ref}$	cook reference temperature	140 °C	H
E_η	viscosity activation	3500 K	A	E_c, β_c	cook activation, moisture	11 000 K, 10	A
β_X, β_A	moisture, amylose exponents	12, 4	A	k_M	mechanical cook rate	10 kg/MJ	A
κ_c	consistency lost uncooked	0.6	A	SEI$_{max}$	expansion scale	4.1	A
UA_b	barrel to dough, wetted	200 W/K	A	θ_T, m_T	reference superheat, exponent	50 K, 2.0	A
$Q_{h,max}$	heater capacity per zone	6.0 kW	A	T_{col}, w_{col}	bubble collapse, width	185 °C, 6 K	A, H
UA_{cw}	cooling water, valve open	60 W/K	A	β_E, p_E	moisture, pressure exponents	10, 0.10	A
C_b	barrel capacity per zone	20 kJ/K	A	p_{ref}	expansion reference pressure	6 MPa	H
UA_{amb}, T_{cw}	barrel loss, cooling water	8 W/K, 25 °C	H	LEI$_{ref}$, λ_L	lengthwise index, exponent	1.05, 0.4	A, H
u_{c1}	feed-zone cooling valve	25 %	A	k_{pack}	packing fraction in a cup	0.60	A
n_h, R_h, L	die holes, radius, land	20, 1.4, 30 mm	A	τ_{dry}, Δt_p	belt residence, parcel	720, 10 s	A, H
e	entrance correction	4 radii	H	k_d, $T_{d,ref}$	drying rate, reference air	0.004/s, 130 °C	A, H
h, c_G	channel depth, pressure factor	5 mm, 6	H	θ_D	air scale of drying rate	40 K	A
τ_L	filled-length settling	1.0 s	H	a_E, b_E	Oswin coefficient, exponent	0.13, 0.40	A
p_{trip}	die-pressure trip	12 MPa	A	T_{in}, ΔT_a	intake air, heater rise	25 °C, 150 K	H, A
Γ_{trip}, Γ_{max}	torque trip, hard limit	100, 90 %	A	τ_a, $\dot{m}_a$	air lag, dry-air flow	45 s, 1.2 kg/s	H
p_{max}, T_{max}	die, melt hard limits	10 MPa, 170 °C	A	P_{air}, n_{knife}	air heater rating, knives	240 kW, 4	H
$T_{a,max}$	dryer air hard limit	150 °C	A	ρ_{lo}, ρ_{hi}, x_{lo}, x_{hi}	specification	120, 150 g/L, 2, 4 %	A
c_s, c_w	solids, water heat capacity	1600, 4180 J/(kg K)	fixed	λ	latent heat at 100 °C	2257 kJ/kg	fixed

Dissipation uses one characteristic shear rate per zone, proportional to the screw speed. Wear is one number that lowers the shear in zones 3 to 5 and the pressure built per filled meter, while real wear is uneven and opens leakage paths that also change the residence distribution.

Heat transfer between barrel and dough is one conductance per zone scaled by the wetted wall. The heater bands act at once, there is no conduction between zones, and the melt thermocouple reads the melt in front of the die exactly apart from its lag and noise, although a real flush probe may not [15].

The preconditioner is two mixed cells of fixed residence in which the steam condenses completely. Expansion, bulk density and cook are the illustrative proxies of Equations (5.10) to (5.12), and there

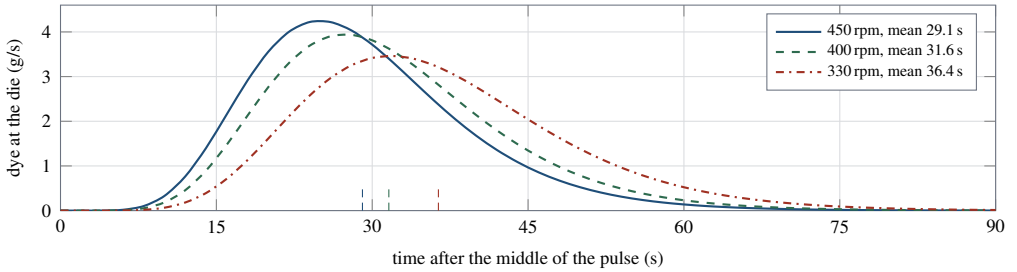


Figure 5.2 Dye leaving the die after a 0.1 kg pulse at three screw speeds, with the feed held at 300 kg/h and the noise switched off: 450 rpm solid, 400 rpm dashed and 330 rpm dash-dotted. The short vertical ticks on the axis, in the order 450, 400 and 330 rpm from left to right, mark the first moment of each curve, which equals the barrel holdup over the throughput.

Table 5.4 Steady-State Gains at the Recipe, by Central Differences on the Model's Own Steady-State Solver

Step	Γ (%)	Δp (MPa)	T_{10} (°C)	ρ_b (g/L)
process moisture, +1 point	−5.04	−0.505	−2.32	+11.48
amylose, +1 point	+2.11	+0.194	+0.75	−2.75
feed, +10 kg/h	+1.53	+0.103	−0.17	+0.59
screw speed, +10 rpm	−0.64	−0.048	+0.41	−0.97
zone 3 setpoint, +1 °C	−0.21	−0.008	+0.07	−0.22
zone 5 setpoint, +1 °C	−0.15	−0.050	+0.41	−0.91
preconditioner setpoint, +1 °C	−0.10	−0.004	+0.08	−0.47

is no protein texturization, no microbial kinetics and no lethality. The pieces keep the volume they expand to and dry on the belt with no shrinkage, case hardening or temperature history, in one air temperature and humidity. The limits and trips are teaching values for an illustrative line, not ratings of any machine.

5.4 Where the Heat Comes From

The heat budget of the recipe is the first result, averaged over 300 to 600 s of the first case. The steam brings 15.94 kW relative to water at 20 °C, and apart from the 1.62 kW the preconditioner loses, all of it goes into raising the dough from 20 to 88 °C. The shaft then delivers 14.80 kW. Of that, 0.48 kW is the pressure work of the die, and the 14.32 kW dissipated in the barrel splits into 0.49, 1.22, 6.27, 1.17 and 5.16 kW across the five zones, so the kneading block and the metering zone release 80 % of it. The heater bands deliver 5.88 kW to the barrel metal, the cooling water removes 2.06 kW and the room takes 3.88 kW. Zones 2 and 3 pass 1.92 and 1.47 kW to the dough, and zones 1, 4 and 5 take 1.14, 0.07 and 2.24 kW back, a net −0.06 kW. The whole rise from 88.0 °C to 158.0 °C is mechanical work, and the barrel only shapes the temperature profile along the way.

Screw speed does two things at once, as shown in Figure 5.2 for the dye pulse that case 1 adds at 120 s. At the fixed feed of the recipe a faster screw runs emptier, so the mean residence falls from 36.4 s at 330 rpm to 31.6 at 400 rpm and 29.1 s at 450 rpm, with the conveying cells 38.2, 31.5 and 28.0 % full. It also shears harder, so the SME rises from 38.2 to 43.0 and 46.1 kWh/t and the melt from 152.8 to 155.7 and 157.7 °C, at a torque that falls from 70.5 to 65.4 and 62.4 %. The two effects pull the cook in opposite directions, and the heating wins in this model, with the cook index rising from 0.938 to 0.945 and 0.950 while the density falls from 142.3 to 135.0 and 130.4 g/L. Measured from the middle of its 2 s pulse at 400 rpm, the dye reaches 1 % of its peak rate after 7.7 s, peaks at 27.4 s and is 90 % out by 46.2 s. Its first moment, 31.59 s, matches the 3.021 kg holdup over the 344.2 kg/h throughput, both averaged over the two minutes of its passage, to 0.001 %.

Table 5.5 Instruments. Noise Is Colored With the Correlation Time Given, and the Feeder's Is a Fraction of the Reading

Tag	Measures	Lag or delay	Noise 1σ, time
FT-101	feeder rate	2 s	0.5 %, 1 s
FT-102, FT-103	water, steam	1 s	0.3, 0.2 kg/h, 0.5 s
TT-103	preconditioner discharge	5 s	0.1 °C, 2 s
TT-Z1 to Z5	barrel metal	3 s	0.1 °C, 1 s
TT-M	melt in front of the die	2 s	0.2 °C, 1 s
PT-D	die pressure	0.3 s	0.03 MPa, 0.5 s
JT	drive torque	0.5 s	0.4 % rated, 0.5 s
TT-A	dryer air	5 s	0.2 °C, 1 s
AT-M	NIR at the belt exit	30 s + 10 s	0.05 points, 5 s
AT-D	at-line bulk density	60 s, 90 s	1.5 g/L, white

Moisture is the master variable, as the gains of Table 5.4 show. One point of process moisture moves the density by 11.48 g/L, while the metering zone's setpoint needs 12.7 °C, the screw speed 118 rpm and the kneading zone 52 °C to do the same at the local gains. The kneading zone, where most of the cook happens, moves the melt at the die by only 0.07 °C per degree of its barrel. The melt also pulls itself back toward where it was. Raising the moisture from 23.3 to 24.3 % lowers the consistency of Equation (5.4) by $1 - e^{-0.12} = 11.3\,\%$ at a fixed state, but the thinner melt dissipates less and runs 2.25 °C cooler, so the consistency at the die falls by only 7.8 %. A 32 instead of 25 % amylose flour raises the consistency law by 32.3 % at a fixed state, and the realized rise at the die is 21.1 % because the melt runs 5.58 °C hotter. Every gain in Table 5.4 is therefore a property of the heat balance, and it moves with the recipe. Across 21.3 to 25.3 % moisture the density gain runs from 10.6 to 11.8 g/L per point and the torque gain from −5.50 to −4.58 % per point.

5.5 Implemented Controller

All loops and the supervisor execute every 0.5 s, at 0.5, 1.0, 1.5 s and so on after a priming call at $t = 0$, in the order of work of Section 1.6.1, and each call receives $\Delta t_c = 0.5\,\text{s}$. The density trim executes once per at-line result, every 60 s. The speed selector plays the line at 1 to 60 times real time, 10 by default, so a 30 min case takes three minutes to watch.

5.5.1 Instruments and the Soft Sensor

No controller reads a model state directly. Each measurement is the true value filtered by an exact first-order lag, with colored noise added at the levels of Table 5.5, and only the screw speed is read exactly. The NIR analyzer reads the moisture of the parcel leaving the belt 30 s late through a 10 s lag, and the at-line check weighs a cup of pieces at the cutter every minute and reports it 90 s later, referred to the target moisture. The process moisture entering the extruder has no instrument. A soft sensor builds it from the metered flows and the certificate, as shown in Equation (5.14).

$$\hat{x}_{in} = \frac{F_m x_{f,lab} + W_m + S_m}{F_m + W_m + S_m} \tag{5.14}$$

The estimate $\hat{x}$ passes through two lags of $\tau_{pc}/2$, the preconditioner's two cells at their declared 150 s whatever the run's residence, starts at reset from its own reading of the metered flows, and is exact only while the certificate is right.

Table 5.6 Implemented Loops and Default Tuning. Action $a = +1$ Is Reverse Acting and $a = -1$ Direct Acting in the ISA Sense

Loop	PV	Output	K_c	τ_I (s)	a, limits
FIC-101	F_m	u_F	0.080	4	+, 0–100 %
FFC-102	W_m	u_W	0.50	4	+, 0–100 %
TIC-103	T_{pc}	u_S	1.0	110	+, 0–100 %
TIC-Z2..Z5	$T_{b,z}$	u_Z	8	120	+, ±100 %
TQC	Γ_m	N_{cmd}	5	15	−, N_{SP}–N_{max}
PIC	Δp_m	F_{SP}	100	150	+, 60–F_{base}
QIC	ρ_m	Δx	0.05	240	+, ±4 points
MIC	NIR	$T_{a,SP}$	10	300	−, 90–149 °C
TIC-A	$T_{a,m}$	u_A	1.5	45	+, 0–100 %
TSH-A	$T_{a,m}$	u_A	4	20	+, 0–request

Units of K_c follow the pairing, % per kg/h (FIC, FFC), % per °C (TIC), rpm per % (TQC), kg/h per MPa (PIC), points per g/L (QIC) and °C per point (MIC).

5.5.2 One Proportional-Integral Algorithm

All thirteen controllers share the framework's proportional-integral code, the velocity form of Section 1.6.2 without its derivative term, written in this chapter's symbols as Equation (5.15).

$$\begin{aligned} e_k &= a(\mathrm{SP}_k - \mathrm{PV}_k), \qquad P_k = K_c e_k \\ S_k &= S_{k-1} + (P_k - P_{k-1}), \qquad c_k = S_k + I_{k-1} \\ u_k &= \mathrm{clamp}(c_k, u_{min}, u_{max}) \\ I_k &= I_{k-1} + \left[\frac{K_c}{\tau_I} e_k + \frac{u_k - c_k}{\tau_I}\right] \Delta t_c \end{aligned} \tag{5.15}$$

with $a = \pm 1$ the action and a zero output bias. Here c_k is the output before limiting, v_k of Equation (1.14), and the integral is corrected by back-calculation at a tracking time of τ_I, only while τ_I is finite. The shared code's derivative and output rate limit are not used here. Manual mode and the switch back to automatic are bumpless, as Section 1.6.2 describes. Table 5.6 gives each loop's pairing, tuning and limits.

5.5.3 Regulatory Loops and the Ratio Station

FIC-101 holds the feed with the feeder screw, TIC-103 holds the preconditioner with the steam, and the four zone loops, which share one tuning, hold the barrel metal through their split-range outputs. The water setpoint comes from a ratio station that solves the process-moisture balance for the water, as shown in Equation (5.16).

$$W_{SP} = \frac{x_{SP}(F_m + S_m) - F_m x_{f,lab} - S_m}{1 - x_{SP}} \tag{5.16}$$

The result is clamped to the valve's range and slewed at 60 kg/h per minute before FFC-102 receives it. The ratio station is a feedforward from ingredient data, exact while the certificate is right and silently wrong when it is not, because Equations (5.14) and (5.16) trust the same number. In ratio mode the knife follows the flow, with $n_k = 60\,\hat{v}\,\mathrm{LEI}_{ref}/(n_{knife}\,\ell_{SP})$ and $\hat{v}$ the velocity in a clean hole of the melt flow $F_m(1 - x_{f,lab})/(1 - \hat{x})$, both LEI_{ref} and the melt density at their declared values.

5.5.4 Two Overrides by External Reset

The torque override raises the screw speed above the operator's setpoint when the torque passes its 85 % limit, because a starve-fed screw carries the same flow with less fill and less torque at a higher speed. It is a direct-acting PI whose lower output limit is the speed setpoint, so the selector

$N_{cmd} = \max(N_{SP}, \mathrm{PI})$ is realized by the clamp, the form of Section 1.4.3, and the integral that back-calculation holds on the clamp is the external reset of Section 1.4.4. Below the limit the output rests on N_{SP} and back-calculation holds the integral at N_{SP}, with the unclamped output K_c times the error below it. At the limit the error changes sign and the output rises without a bump. The die-pressure override is the mirror image, a reverse-acting PI on the die pressure whose upper output limit is the feed setpoint F_{base} from the supervisor or the operator, so $F_{SP} = \min(F_{base}, \mathrm{PI})$ above a floor of 60 kg/h. When an override is switched off, in manual or after a trip, it tracks the selected value plus its own proportional action, $F_{base} + K_c(p_{lim} - p_m)$ or $N_{SP} - K_c(\Gamma_{lim} - \Gamma_m)$, the value its back-calculation would hold, so switching it on does not engage it on noise.

5.5.5 The Density Trim and the Dryer Cascade

QIC adds a bias of up to ± 4 points to the process-moisture setpoint from the at-line density, executing Equation (5.15) once per result with $\Delta t_c = 60\,\mathrm{s}$. The setpoint is then clamped to 19 to 28 %, a limit outside the trim's own, so with the recipe moved near either end the setpoint can stop while the trim's integral runs on toward ± 4 points. The supervisor counts either limit as a saturated trim. A denser product lowers the moisture, since a drier dough is stiffer, heats itself more and puffs more. With the lot restart on, a new flour lot resets the trim to zero because the correction it learned belongs to the lot that ran out. In manual or after a trip the trim holds what it learned. The dryer is a cascade. MIC, direct acting, sets the air setpoint from the NIR reading, and TIC-A holds the air with the heater. With the setpoint fixed, MIC tracks the operator's air setpoint. The air setpoint is limited to 90 to 149 °C, 1 K under the air's hard limit. A high-limit override, TSH-A, is a PI on the measured air at 149.5 °C whose upper output limit is the heater request, so the heat is the lower of the two, in automatic and in manual and under either controller. Until the air passes its limit it tracks the request plus its own proportional action. With the heater stepped to 100 % in manual in case 8, the air peaks at 154.0 °C through the thermocouple's 5 s lag and stays at or below 149.8 °C after 120 s, against 175.0 °C without it.

5.5.6 The Throughput Supervisor

The supervisor ramps the feed setpoint toward a production target while every constraint keeps its margin, the back-off of Section 1.4.2, as shown in Figure 5.3. Chapter 4 pushes the throughput of a harvester against engine and rotor overrides through a low selector. Because the preconditioner holds 2.5 min of dough, the torque and die pressure respond to a feed change late, so the supervisor predicts where they will settle at the feed it is asking for, by Equation (5.17).

$$\hat{\Gamma} = \Gamma_m \rho^{0.72} \left(\frac{N_{fin}}{N_m} \right)^{-0.4}, \qquad \hat{p} = p_m \rho^{0.46}$$
$$\rho = \frac{F_{base}}{F_m} \frac{F_{mix}}{\hat{F}_x} \tag{5.17}$$

with $F_{mix} = F_m + W_m + S_m$ the metered flow into the preconditioner, $\hat{F}_x$ the same flow through the same two declared lags, and N_{fin} the top speed when the torque override is on. Each constraint is given a slack in units of its own margin, $[(\Gamma_{lim} - m_\Gamma) - \hat{\Gamma}]/m_\Gamma$ for torque with $m_\Gamma = 3\,\%$, the same form for pressure with 0.3 MPa, $[(T_{max} - 4) - \hat{T}_{ex}]/4$ on the exit temperature predicted from the melt thermocouple and the die pressure, $\hat{T}_{ex} = T_{10,m} + p_m/(\rho_m c_{p,0})$ with the declared ρ_m and $c_{p,0}$ taken at 23.3 % moisture, $\min(\rho_{hi} - 3 - \rho_m,\ \rho_m - \rho_{lo} - 3)/3$ on the at-line density, $(x_{hi} - 0.2 - \mathrm{NIR})/0.2$ on the final moisture in points, and -1 while the trim is saturated. A measured torque above its limit at top speed or a die pressure above its limit forces the slack to -1. With the smallest slack above 0.25 the supervisor raises the feed at up to the ramp rate (10 kg/h per minute), between -0.25 and 0.25 it holds, and below -0.25 it backs off at the ramp rate, down to 60 kg/h. Above the target it comes down at twice that rate. A leaky timer counts the time spent more than 5 kg/h short of the target while not pushing, and at 120 s the target is declared infeasible, with the binding constraint named. No limit is relaxed to reach a target.

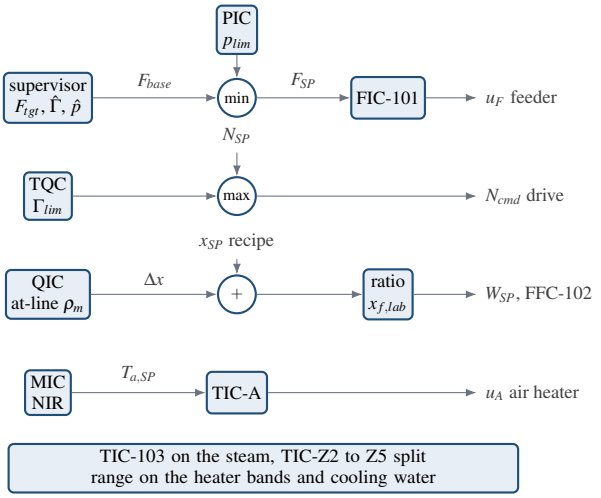


Figure 5.3 Control structure. The supervisor's feed setpoint passes a minimum selector with the die-pressure override, the speed setpoint a maximum selector with the torque override, and both selectors are realized as output limits of the overriding PI. The density trim biases the recipe moisture that the ratio station turns into a water setpoint.

5.5.7 Trips

A torque of 100 % of rating held for 1 s, or a die pressure of 12 MPa, trips the extruder. The drive, the feeder, the water and the steam go to zero, the preconditioner stops discharging and the filled length freezes, while the zone and dryer loops continue, and the run pauses 5 s later. Only the operator's trip reset restarts the line, an action separate from the Reset that starts a case again from $t = 0$.

5.5.8 Simulation, Identified and Internal Models

Three models have to be kept apart. The 105-state plant of Section 5.3 is the simulation model. Step tests on that plant give the identified control model, the gains of Table 5.4 and the dynamics behind them. A one-point step down of the moisture setpoint at 300 s of the first case, taken as the difference from an otherwise identical unstepped run, lowers the cutter density by 11.25 g/L, reaching 10 % of the change after 77 s, 63 % after 187 s and 90 % after 295 s, which a two-point fit reduces to a dead time of 77.5 s and a time constant of 109 s. The at-line check adds its 90 s report delay and its 60 s sampling to that, and the trim's loop gain is $0.05 \times 11.25 = 0.56$. The internal models of the loops and the supervisor are three. The ratio station and the soft sensor are one mass balance built on the certificate. The supervisor's are the exponents of Equation (5.17), which agree with the simulation to within 0.02 at the recipe, where the steady-state solver gives 0.70 to 0.72 for the feed exponent of torque between 285 and 437 kg/h, -0.39 to -0.40 for its speed exponent between 330 and 450 rpm, and 0.46 to 0.47 for the feed exponent of pressure. The knife assumes the design lengthwise expansion. The trim, the overrides, the zone loops and the dryer cascade carry no model. The predictive controller of Section 5.5.9 carries a fourth internal model, identified by step tests at a higher feed than the recipe.

5.5.9 A Predictive Controller on Three Setpoints

A controller option on the supervisor's card switches the line from the loops to a model predictive controller (MPC) in automatic, the option of Section 1.6.3, and the loops stay the default everywhere except case 9. The MPC moves three setpoints that the loops already own, the feed setpoint below the die-pressure override's minimum selector, the process-moisture setpoint that the ratio station of Equation (5.16) turns into water, and the screw speed setpoint below the torque override's maximum

selector. Both overrides, the trips, the zone loops, the preconditioner loop, the ratio station and the dryer cascade act beneath it unchanged, so the overrides remain the protection and the plan only respects their limits, as an optimizer above selectors does in industrial practice [14]. The production target, the ramp rate and the two margins become the plan's own bound, ramp and ceilings.

The controller's model was identified on the plant by stepping the three setpoints in automatic, ±20 kg/h, ±0.5 points and ±20 rpm in both directions with the noise off, at 400 kg/h, 23.3 % and 400 rpm, where the limits bind, rather than at the 300 kg/h recipe. The regulatory loops stay closed during the steps because the MPC moves their setpoints. The feed and the moisture each pass the preconditioner's two cells at the declared $\tau_{pc}/2 = 75$ s and then two parallel lags, and the speed passes one lag and then two more, as Equation (5.18) states.

$$\begin{aligned} \tfrac{1}{2}\tau_{pc}\dot{\phi}_{m,1} &= \Delta u_m - \phi_{m,1}, & \tfrac{1}{2}\tau_{pc}\dot{\phi}_{m,2} &= \phi_{m,1} - \phi_{m,2} \\ \vartheta_{m,j}\dot{\phi}_{m,j} &= \phi_{m,2} - \phi_{m,j}, & \vartheta_{N,1}\dot{\nu}_1 &= \Delta N_{SP} - \nu_1 \\ \vartheta_{N,j}\dot{\nu}_j &= \nu_1 - \nu_j, & \hat{y}_i &= y_{0,i} + \textstyle\sum_m \mathbf{c}_{i,m}^{\mathsf{T}} \xi_m \end{aligned} \tag{5.18}$$

Here $m \in \{F, x\}$ in the first two lines, $j \in \{a, b\}$, Δu is a setpoint's deviation from $\mathbf{u}_0 = (400\,\text{kg/h}, 23.3\,\%, 400\,\text{rpm})$, $\xi_F = (\phi_{F,2}, \phi_{F,a}, \phi_{F,b})$, ξ_x likewise, $\xi_N = (\nu_1, \nu_a, \nu_b)$, and the four outputs $\hat{y}_i$ are the torque, the die pressure, the melt at the die exit and the bulk density at the cutter, with $\mathbf{y}_0 = (80.61\,\%, 7.735\,\text{MPa}, 156.74\,^\circ\text{C}, 140.90\,\text{g/L})$. The lags, fitted by a grid, are $\vartheta_{F,a}, \vartheta_{F,b} = 45$, 90 s, $\vartheta_{x,a}, \vartheta_{x,b} = 30$, 90 s and $\vartheta_{N,1}, \vartheta_{N,a}, \vartheta_{N,b} = 3$, 12, 150 s, and the twelve coefficient vectors $\mathbf{c}_{i,m}$ by least squares, with fit residuals of 0.17 to 2.5 % of each response. The steady gains are +0.145, −6.13 and −0.079 % of torque per 1 kg/h, per point and per rpm, and +0.062, +12.6 and −0.110 g/L of density. The melt output is the melt thermocouple plus the die's pressure work $\Delta p_m/(\rho_m c_{p,0})$ with the declared ρ_m, the supervisor's own estimate. The eleven states are discretized exactly at the decision period. The model is linear at one point, and it omits the amylose share, the wear, the die deposit, the zone heaters' capacity and the dryer.

The NIR analyzer is not one of its outputs, and its 40 s of delay and lag are therefore not in the model. Within the 5 min horizon the three setpoints move the NIR reading by 7.8×10^{-5} points per 1 kg/h and 0.0026 points per point of moisture setpoint, because the product reaches the analyzer only after the 12 min belt and the dryer cascade holds it. The delayed measurement the plan does use is the at-line density, whose cup is sampled every 60 s and reported 90 s later.

A steady-state Kalman filter estimates the model's state from the torque, the die pressure and the melt at every decision, propagating it with the setpoints that were applied over the last period, the feed after the die-pressure override and the speed after the torque override. In its default form the soft sensor's error enters where it arises, with the flour, as an integrating disturbance d_x added to Δu_x in the first line of Equation (5.18), while the die pressure and the melt carry output offsets b_p and b_T and the torque carries none, which is one of the disturbance models that make a plan offset free [19, 20]. A torque change that the planned setpoints do not explain is therefore read first as moisture, and the same model turns that moisture into a predicted density change of 12.6 g/L per point. The measurement variances are the instruments' of Table 5.5, and the random walks per decision are 0.02 points for the cells and for d_x, 0.004 MPa and 0.03 °C. At a new flour lot d_x restarts from zero, because the old lot's error leaves the cells with its dough. The at-line density corrects the predicted density through a bias b_ρ, as Equation (5.19) states.

$$b_\rho \leftarrow \text{clamp}\Big(b_\rho + \lambda\,\big[\rho_{lab} - \hat{\rho}(t_s) - b_\rho\big],\ \pm 40\,\text{g/L}\Big) \tag{5.19}$$

Here $\lambda = 0.4$ and $\hat{\rho}(t_s)$ is the model's density at the time t_s the cup was taken, interpolated from the decision history, so a result that arrives 90 s late is compared with the prediction it can confirm. The correction applies on arrival and enters the next plan. Volume 5, Chapter 10 corrects the bias of a soft sensor from a sampled and delayed laboratory result as well. A second estimate, chosen as an

advanced option, replaces d_x by an output offset on the torque, the ordinary form, and serves as the ablation of Section 5.6.8.

Every 5 s, ten executions of the loops, the plan is solved over $N = 60$ steps (5 min), its three setpoints held over ten blocks that grow from one step each to 14 (1, 1, 2, 2, 4, 6, 8, 10, 12, 14), 30 decision variables, for the objective of Equation (5.20).

$$\begin{aligned} J = \sum_k \Big[& Q_\rho (\hat{\rho}_k - \rho_{SP})^2 - w_F F_{SP,k} \\ & + R_N (N_{SP,k} - N_{op})^2 \Big] + \sum_j \Delta \mathbf{u}_j^\top \mathbf{S} \, \Delta \mathbf{u}_j \\ & + \sum_{i,\ell} \left(r_i s_{i,\ell}^2 + q_i s_{i,\ell} \right) \end{aligned} \tag{5.20}$$

The feed earns $w_F = 1$ per 1 kg/h and step, the density is held on its setpoint with $Q_\rho = 5$ per $(\mathrm{g/L})^2$, the speed is drawn toward the operator's setpoint N_{op} with $R_N = 0.005$ per rpm^2, and the block moves are suppressed by $\mathbf{S} = \mathrm{diag}(2, 200, 0.05)$ per $(\mathrm{kg/h})^2$, point squared and rpm^2. The soft rows keep the torque under $\Gamma_{lim} - m_\Gamma = 82\,\%$, the die pressure under $p_{lim} - m_p = 8.7\,\mathrm{MPa}$, the exit melt under $T_{max} - 4 = 166\,^\circ\mathrm{C}$ and the density between 123 and 147 g/L, each with its slack $s_{i,\ell} \geq 0$ priced at $r = (10^3, 10^5, 10^3, 100)$ and $q = (2000, 20\,000, 2000, 200)$ per unit of %, MPa, °C and g/L. The linear price exceeds what the room is worth, which makes the penalty exact when the rows can be met [21]. A percent of torque is worth 6.9 kg/h of feed at the model's gain, and its price of 2000 is 4.8 times the reward that room would earn over all 60 steps. The torque and the die pressure have one slack for each of the first 12 steps (1 min) and one for their worst step beyond, and the melt and the density one for the whole horizon, a choice explained in Section 5.6.9. The hard bounds are the feed between the overrides' 60 kg/h floor and the production target, the moisture setpoint within the trim's ± 4 points and 19 to 28 %, and the speed between N_{min} and N_{max}. The first feed move may differ from the feed in force by the supervisor's ramp times 5 s, and each later block boundary by the ramp times the time between block centers, in both directions.

A solve is accepted only if it is optimal, its plan, predictions, cost and slacks are finite and of the right size, every planned setpoint is inside its bounds and the plan's first move is the move applied. Any other outcome hands the three setpoints back to the loops from where the plan left them, marks the historian and posts a note, and control returns to the plan at the next solve that passes. Under the MPC the supervisor's feed and the trim's integral follow the applied setpoints, so a switch to manual moves no actuator, and with the noise off the return to automatic moved the speed by 0.29 rpm, the water valve by 0.21 % and the feeder by 0.04 %, against 0.09 rpm, 0.08 % and 0.04 %, the largest change in one 0.1 s step of ordinary running from 1400 to 1500 s. The plan reads only the torque, die pressure and melt readings, the at-line results with their cup times, the setpoints applied and on the panel, the limits, and its own state. Its model constants are frozen from the declared parameters, so a parameter the learner changes on the plant becomes model mismatch. A rule written before any search and judged on case 9 only set the weights, and the final 243-setting grid was flat, 212.6 to 212.9 kg. In Chrome a decision takes 2.26 ms on average and 16.1 ms at most over the 360 decisions of 1800 s of case 9, measured under a load average of about 3, which is 0.045 % of a core at real time and 2.7 % at the top speed of 60 times real time. The decision exceeds 2 ms, so a frame guard skips a decision rather than a frame when the browser is behind.

5.6 The Cases as Measured

The cases are taken below in the order of the application's selector, from case 1, the steady recipe with a dye pulse at 120 s, to case 10, the graded challenge. Each has a completion rule of the kind Section 1.6.4 describes, and the challenge is judged on the case as given, so a changed model parameter or disturbance voids the attempt.

Table 5.7 Measured Outcomes, Default Seed

Case	Measured outcome
1, 330/400/450 rpm	residence 36.4/31.6/29.1 s, SME 38.2/43.0/46.1
2, trim off	true 25.9, estimate 23.3 %, torque 51.9 %
2, density crossings	cutter 208.8, at-line 330.0, exit 939.9 s
2, by 1800 s	63.8 kg dense, air 133.7 to 135.6 °C
2, trim on	−2.67 points, below 150 g/L at 540.3 s, 24.3 kg
2, certificate fixed	17.7 kg off-spec
3, trim off	torque 81.3 %, 118.0 g/L, 56.7 kg light
3, trim on	+1.55 points, torque 72.6 %, nothing lost
4, 380 kg/h	torque 77.6 %, 140.1 g/L, 40.3 kWh/t
4, 480 kg/h target	held at 437.2 kg/h, 409 rpm, infeasible 1657 s
5, override on	takes over 740 s, peak 9.40 MPa, 112.1 kg/h
5, override off	trips at 1344.0 s
6, zone 3	132.0 °C at 100 %, melt 155.1 °C
7, 60 % wear	torque 52.2 %, L_f 487.7 mm, 145.6 g/L
8, cascade	peak 3.56 % at 360 s, air 145.3 °C
8, air fixed	4.11 %, 89.5 kg wet

Each case begins from the recipe of Section 5.3, with both overrides on, the dryer in cascade, the trim and the supervisor off and the lot restart on, unless the case says otherwise. A setting changed for a run was changed after the case was selected and before Run. The application treats such a change exactly as the same setting carried through Reset, so each configuration has one run. The numbers in this section were measured headlessly on the engine at the default seed, in seconds of simulated time. Plant events carry the resolution of the 0.1 s step, and a controller action is timed at the execution that made it. Table 5.7 collects the main outcomes.

5.6.1 A Wrong Certificate

The second case loads a lot of 15 % moisture at 60 s whose certificate still says 12 %. The ratio station keeps adding the water the certificate implies, so the dough becomes $300 \times 0.03/344.2 = 2.61$ points wetter, 25.9 % water, while the estimate built on the same certificate reads 23.3 %. The thinner melt needs 51.9 % torque and 5.45 MPa at the die, it runs 150.3 °C instead of 155.7 °C, and it expands to an SEI of 3.07 instead of 4.53. The density at the cutter passes 150 g/L at 208.8 s and settles at 164.6 g/L. The at-line check reports the crossing at 330.0 s, after 31 s of waiting for the next cup and 90 s in the laboratory, and the first off-spec parcel leaves the dryer at 939.9 s. By then 12 min of dense product is already on the belt, as shown in Figure 5.4. The dryer's cascade copes with the extra water by raising the air from 133.7 to 135.6 °C, and the final moisture stays in its window, because a density cannot be dried away. By 1800 s 63.8 kg are off-spec, all of them dense.

> **TRY IT IN THE APP** **2 · Wetter Flour**
>
> Load the scenario and press Run with the density trim off. Watch the process-moisture trend: the estimate stays on its 23.30 % setpoint while the truth climbs to 25.93 %. The true density at the cutter passes 150 g/L at 208.8 s, and the at-line result reports the crossing only at 330.0 s.

With the trim on, the at-line results lower the moisture setpoint by 2.67 points by 1800 s, close to the 2.61-point certificate error expressed in process moisture, the cutter density is back under 150 g/L at 540.3 s, and 24.3 kg are lost. Correcting the certificate at 300 s restores the water exactly, and 17.7 kg are lost. The feedforward is exact but only as good as its data, while the feedback measures the product itself and arrives late, after the preconditioner, the barrel and the laboratory. An error of three points in a certificate is large, but the case serves to separate the two kinds of correction.

Pause. Try Exercise 5.1 before reading on.

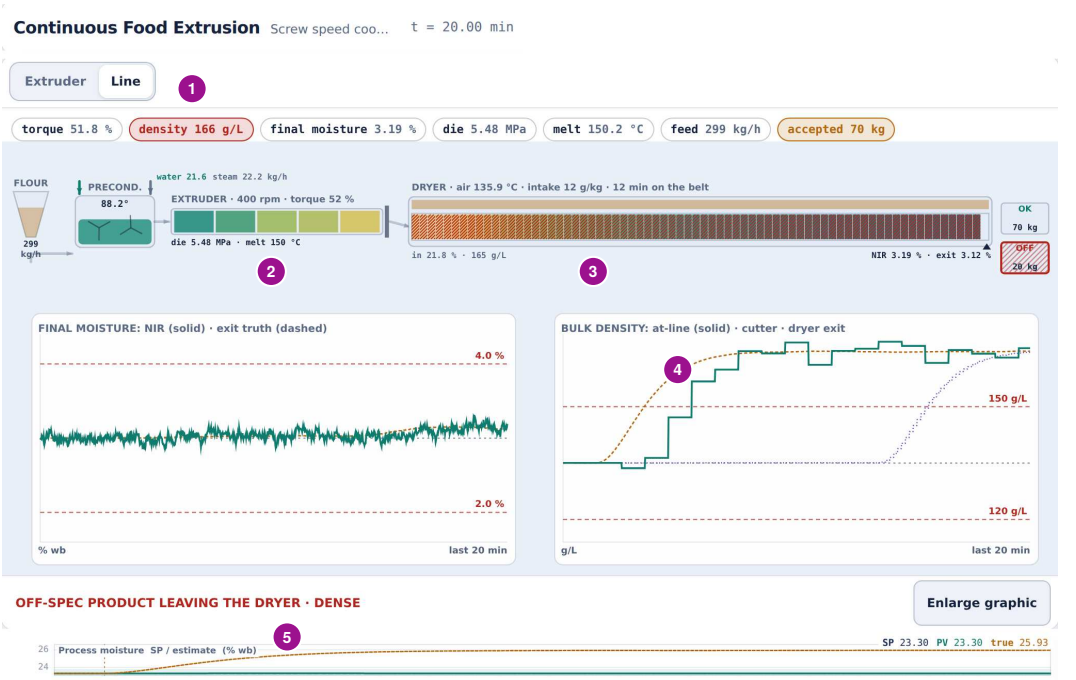


Figure 5.4 The extrusion application in case 2, wetter flour, at $t = 20.00$ min (1200 s) in the Line view, 1140 s after a 15 % lot on a 12.0 % certificate entered the feeder, run with nothing changed and a 20 min trend window. The picture is a composite: the run clock of the header above the Line view, and below it the process-moisture strip of the trends, scaled to the same width. Every parcel on the belt (3) is hatched as too dense, the OK bin holds 70 kg and the OFF bin 20 kg, and the badges read an at-line density of 166 g/L in alarm (1), a torque of 51.8 %, 5.48 MPa at the die and a melt of 150.2 °C, and the extruder bar (2) shows the screws at 400 rpm. The density history (4) shows the cutter truth rising first, the at-line staircase following it and the dryer-exit truth rising 12 min later, while the final moisture, NIR 3.19 % against an exit truth of 3.12 %, stays between its 2.0 % and 4.0 % limits with the air at 135.9 °C. The process-moisture strip (5) holds the estimate on its 23.30 % setpoint while the truth reads 25.93 %, and the status line reports off-spec product leaving the dryer as dense.

5.6.2 A Stiffer Starch

The third case switches to a flour with 32 % amylose at the same moisture at 60 s. The torque rises from 65.4 to 81.3 %, 3.7 points under its override limit, the die pressure from 6.77 to 8.20 MPa, and the SME from 43.0 to 53.5 kWh/t. The melt reaches the die at 161.3 °C instead of 155.7 °C, and the extra superheat, a fuller cook of 0.971 and the higher die pressure expand the pieces to an SEI of 5.67, and the density falls to 118.0 g/L, under the 120 g/L limit at the cutter from 330 s on. By 1800 s 56.7 kg are off-spec, all too light. The fix is the same handle as for the wetter flour, turned the other way. By 1800 s the trim adds 1.55 points of moisture, the torque comes down to 72.6 % and the density to 134.2 g/L. It acts early enough that the cutter density bottoms at 120.2 g/L at 372 s, just inside the limit, and nothing is lost.

5.6.3 Throughput Against Torque

The fourth case turns the supervisor on and raises its target from 300 to 380 kg/h at 60 s. The feed setpoint reaches 380 kg/h at 539.5 s, the torque settles at 77.6 % and the density rises to 140.1 g/L, because each kilogram now gets 40.3 kWh/t instead of 43.0. At 900 s the target becomes 480 kg/h.

The predicted torque at the top speed reaches its margin at 424.8 kg/h at 1200 s, and from there the supervisor alternates between holding and slow pushing. At 1422.5 s, with the setpoint at 433.0 kg/h, a noise excursion of the torque transmitter first touches 85 % and the override takes its first increment, 129.5 s before the true torque reaches the limit at 1552.0 s. At 1657 s the supervisor declares the target infeasible, with the torque at maximum speed as the binding limit. The line settles with the feed setpoint at 437.2 kg/h, the screws at 409 rpm and the torque at 85.0 %, the worst case over the run being 85.5 % against the 90 % hard limit, and the density at 142.4 g/L. With the trim on as well, the trim lowers the moisture 0.41 points to hold 135.4 g/L at 2400 s, and the torque then caps the feed at 416.0 kg/h. At the top speed the recipe would need 87.6 % torque for 480 kg/h. Half a point more water brings that to 84.4 % but the density to 147.2 g/L, at the supervisor's quality margin, while a drive rated at 600 instead of 540 N m would carry it at 78.9 %.

5.6.4 A Restricted Die

The fifth case grows a deposit in the die holes at 1.2 % of their radius per minute from 60 s. The die pressure passes 8.76 MPa at 699 s and the override takes over at 740 s. It holds the pressure but overshoots its 9.0 MPa limit to 9.40 MPa at 913 s, because a feed change reaches the die through the preconditioner's 2.5 min. By 1500 s the feed is down to 112.1 kg/h. The screws still turn at 400 rpm, so each kilogram takes 68.2 kWh/t, the melt reaches 162.2 °C, the dough stays 50.8 s, and the pieces over-expand to 111.7 g/L at the cutter. The override protects the die, not the product. After the clean plate at 1500 s the feed is back at 300 kg/h by 1545 s. With the override off the pressure reaches the 12 MPa trip at 1344.0 s and the extruder stops.

> **TRY IT IN THE APP** **5 · Restricted Die**
>
> Run the scenario with the Die-pressure override on and watch the die pressure and the feed. The override takes over at 740 s, overshoots to 9.40 MPa and has cut the feed to 112.1 kg/h by 1500 s, while the pieces over-expand to 111.7 g/L. Press Reset, set Die-pressure override to Off on the PIC card and run again: the extruder trips at 1344.0 s.

5.6.5 A Failed Heater and a Worn Screw

In the sixth case zone 3 keeps a quarter of its heater capacity from 60 s on. Its loop pins the output at 100 % from 161.5 s, and the barrel falls from 140 to 132.0 °C and stays there, with back-calculation keeping the integral from winding up. The dough in zone 3 runs 134.0 °C instead of 136.6 °C, which makes it stiffer, the torque rises from 65.4 to 67.0 %, and the melt at the die falls only 0.6 °C, to 155.1 °C, with the density at 136.9 g/L at 1200 s. A saturated loop is not always a loop that matters, since the kneading zone's barrel is a small part of the melt's heat. Raising zone 4 to 145 °C at 300 s, where its loop runs at 24.4 % heat, returns the density to 135.1 g/L.

Pause. Try Exercise 5.2 before reading on.

The seventh case wears the screws to 20, 40 and 60 % at 60, 240 and 420 s. By 1200 s the torque is 52.2 %, the SME 34.3 kWh/t, the melt 150.2 °C, the SEI 3.99 and the density 145.6 g/L, 4.4 g/L under the upper limit. The worn flights build less pressure per filled meter, so the filled length grows from 163.2 to 487.7 mm, reaches the kneading block at 444.2 s, and raises the residence from 31.6 to 43.7 s. The die pressure rises from 6.77 to 7.53 MPa, because the cooler melt is stiffer. The falling torque is the symptom of a machine that has lost shear, not of an easier product. Raising the speed to 450 rpm at 600 s brings back part of the work, 37.9 kWh/t, and the density to 139.5 g/L, but the filled length stays at 442.7 mm.

Pause. Try Exercise 5.3 before reading on.

5.6.6 Humid Dryer Air

The eighth case turns the dryer intake to 50 g/kg at 60 s. Every parcel on the belt dries more slowly and toward a higher equilibrium at once, so the final moisture starts to rise within a minute. The

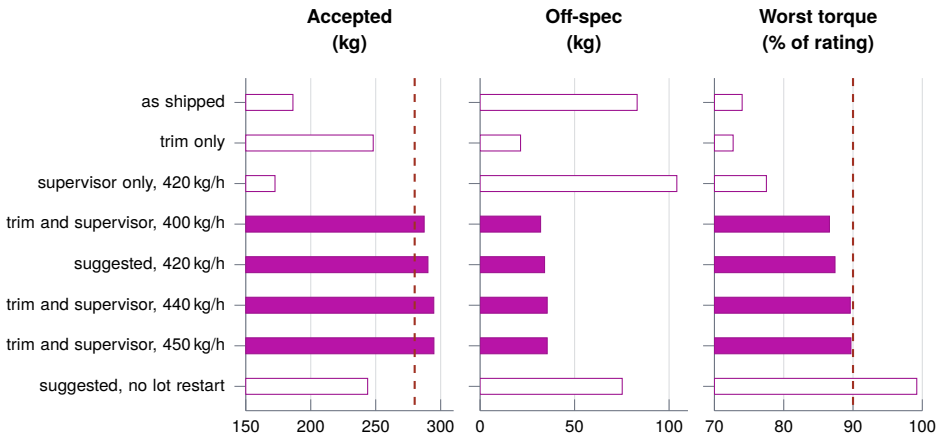


Figure 5.5 Table 5.8 redrawn against the challenge's requirement of at least 280 kg accepted and its 90 % hard limit on torque, dashed. Solid bars pass and white bars fail. Notice that the supervisor alone accepts less than the line as shipped, and that without the lot restart the suggested configuration runs its torque to 99.2 %.

Table 5.8 Challenge Configurations Over 3600 s. Mass in kg, Worst Torque in Percent of Rating

Configuration	Accepted	Off-spec	Torque	Result
as shipped	186.3	83.1	74.0	fail
trim only	248.1	21.4	72.7	fail
supervisor only, 420 kg/h	172.5	104.1	77.5	fail
trim and supervisor, 400 kg/h	287.3	32.0	86.6	pass
suggested, 420 kg/h[a]	290.2	34.1	87.4	pass
trim and supervisor, 440 kg/h	294.7	35.5	89.6	pass
trim and supervisor, 450 kg/h	294.8	35.5	89.7	pass
suggested, no lot restart	243.8	75.2	99.2	fail

[a] trim on with the lot restart, supervisor on at 420 kg/h

cascade raises the air, the final moisture peaks at 3.56 % at 360 s, is back to 3.05 % at 1200 s and stays within 3.04 to 3.06 %, and the air reaches 145.3 °C by 1800 s, 4.7 °C under its limit, with no wet product. With the air fixed at the recipe's 134 °C, the run starting at that air temperature, the final moisture passes 4.0 % at 640 s and settles at 4.11 %, and 89.5 kg are off-spec by 1800 s.

5.6.7 The Shift of Flour Lots

The challenge runs 3600 s through lot B at 600 s, with 14.5 % moisture on a 12.0 % certificate, lot C at 1800 s, a 29 % amylose flour at 12.5 % moisture on a correct certificate, and 50 g/kg dryer air at 2700 s. It is met with at least 280 kg of accepted product, the torque never above 90 %, the die never above 10 MPa, the melt never above 170 °C, the dryer air never above 150 °C and no trip. The measured configurations are listed in Table 5.8, and Figure 5.5 draws them against the requirement.

As shipped the line accepts 186.3 kg, 93.7 kg short of the requirement, and all 83.1 kg of its off-spec product is lot B's dense product, with every hard limit respected. The trim alone accepts 248.1 kg. The supervisor alone accepts 172.5 kg, less than doing nothing. It pushes toward 420 kg/h just as lot B arrives and makes 104.1 kg of dense product, and in the last part of the hour its quality condition backs the feed from 249.8 kg/h at 2250 s to 79.8 kg/h at 3600 s while the at-line density sits between 117.9 and 124.7 g/L, although less feed puts more work into each kilogram and makes the pieces lighter still. The suggested configuration, the trim on with its lot restart and the supervisor

Table 5.9 Case 9 and the Challenge at 420 kg/h Under Five Structures, Default Seed

Structure	Case 9, wet lot, 2400 s					Challenge, 3600 s				
	Acc. (kg)	Off (kg)	Feed (kg/h)	Γ_{max} (%)	ρ_{max} (g/L)	Acc. (kg)	Off (kg)	Feed (kg/h)	Γ_{max} (%)	$t_{>85}$ (min)
Loops, supervisor and trim	169.4	36.8	389.3	84.7	171.2	290.2	34.1	383.9	87.4	12.5
Loops retuned once	182.3	26.0	389.6	78.6	166.4	302.0	23.1	388.0	83.5	0.0
Loops with the MPC's estimate[a]	186.0	20.9	388.2	84.9	161.5	310.8	20.7	396.1	84.4	0.0
MPC, bias estimate	177.6	35.4	401.7	82.0	156.1	308.4	18.3	384.3	87.3	4.9
MPC, moisture estimate	212.9	0.0	401.6	82.3	147.8	336.9	0.0	401.5	84.3	0.0

Accepted and off-spec mass at the dryer exit, mean feed, worst true torque, peak true density at the cutter and minutes with the true torque above the 85 % override limit. Every row respects every hard limit and none trips. Each MPC run makes 480 or 720 decisions with no fallback.
[a] At 400 rpm on case 9 and 450 rpm on the challenge, the better of the two speed setpoints measured for each (172.5 and 34.0 kg at 450 rpm on case 9, 300.7 and 27.8 kg at 400 rpm on the challenge).

aimed at 420 kg/h, accepts 290.2 kg, 10.2 kg above the requirement, with 34.1 kg off-spec, a worst torque of 87.4 %, the die at 8.36 MPa, the melt at 162.9 °C and the air at 147.2 °C. Its trim reaches −2.78 points for lot B by 1800 s. Without the lot restart that correction carries into lot C, whose stiffer starch needs more water rather than less, and the torque reaches 99.2 %, less than a point from the coupling's trip, and 44.1 kg come out too light. A target of 440 kg/h passes at a worst torque of 89.6 %, and every target from 445 kg/h up gives one and the same run, 294.8 kg at 89.7 %, because the supervisor's margins then set the rate and hold the feed setpoint at or below 440.9 kg/h. Over six seeds the suggested configuration accepts 290.2 to 293.5 kg, the 450 kg/h target reaches a worst torque of 89.1 to 89.7 %, and the supervisor alone accepts 170.4 to 177.0 kg, always less than the 186.3 kg of the shipped line.

> ✦ **DID YOU KNOW?** **Pushing for Throughput, Accepting Less**
>
> From the simulator: through the hour of flour lots, the throughput supervisor alone accepts 172.5 kg of product, less than the 186.3 kg of the line as shipped. With the density trim and its lot restart added, the same supervisor aimed at 420 kg/h accepts 290.2 kg (Table 5.8).

5.6.8 Predictive Control Through a Wet Lot

Case 9 asks for 420 kg/h from the recipe's 300 and at 600 s refills the feeder from a lot of 15 % moisture whose certificate says 12 %, the error of case 2 at full rate. It runs 2400 s under the MPC, and the comparison runs it again on the same seed under the loops with the supervisor and the trim on, as the case pins them. Table 5.9 adds two stronger classical rows. The first is the loops retuned once by the MPC's own rule over every setting they have, 864 combinations of the ramp, both margins, the trim's gain and integral time and the speed setpoint, which kept the ramp and the torque margin, narrowed the die margin to 0.1 MPa, doubled the trim's gain and raised the speed setpoint to 450 rpm. The second gives the loops the MPC's own moisture estimate, the same filter on the same instruments and applied setpoints, with d_x subtracted from the moisture setpoint ahead of the trim and restarted at each lot. No signal the MPC uses is withheld from the loops.

The wet dough reaches the screws through the preconditioner, and the torque falls at the same feed and speed, to 71.2 % under the MPC and 68.5 % under the loops. The MPC's moisture estimate passes one point at 720 s and reaches 3.09 points by the end. By 755 s the plan has cut the moisture setpoint from 22.8 to 19.9 %, it runs the screws to their 450 rpm top and holds the feed near 420 kg/h, and the density at the cutter peaks at 147.8 g/L at 771 s, inside the 150 g/L limit, as shown in Figure 5.6. The first at-line result to show the wet lot arrives at 810 s under either structure, and under the MPC it only confirms the estimate. The loops read the same torque fall as room, the supervisor pushes the feed into it, and the trim waits for the at-line density, so the cutter density passes 150 g/L at 712 s and peaks at 171.2 g/L at 876 s, and 36.8 kg come out dense.

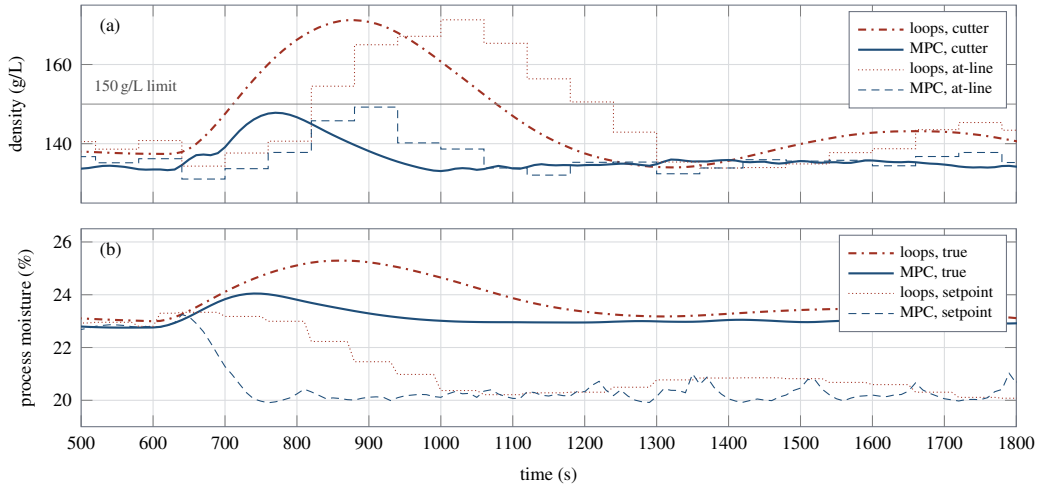


Figure 5.6 Case 9 from 500 to 1800 s, the MPC against the loops with the supervisor and the trim on. (a) True bulk density at the cutter and the at-line result, which lags it by the cup's sampling and the 90 s report. (b) True process moisture and its setpoint. The MPC's lines are thick solid (truth) and dashed (at-line result or setpoint), the loops' dash-dotted and dotted, and the thin gray line in (a) is the 150 g/L limit. The MPC lowers the setpoint within a minute of the wet dough's arrival, before any at-line result shows it.

The ablation in Table 5.9 locates the gain. With the bias estimate, the same plan, horizon and weights make 35.4 kg of dense product against the loops' 36.8 kg, because an offset on the torque explains the symptom and leaves the predicted density unchanged, so on the lot change the product gain is the estimator, not the optimizer. Given the same estimate, the loops reject 20.9 kg, 43 % less than as shipped, and the remaining 20.9 kg of off-spec product and 13.4 kg/h of mean feed belong to moving the three handles together against predicted rather than measured limits. The claim holds for the lot change only. On the challenge, whose third change is a stiffer starch with a correct certificate, the bias-estimate MPC also beats the shipped loops, 308.4 against 290.2 kg. The MPC with the moisture estimate accepts 336.9 kg on the challenge with nothing off-spec and no minute above the torque override's limit, 26.1 kg more than the strongest classical row. With the shipped 300 kg/h target it accepts 269.5 kg, also with nothing off-spec, against the shipped line's 186.3 kg.

5.6.9 One Slack for the Whole Horizon

The first build of the controller judged the torque with one slack for the whole horizon and crossed the 90 % hard limit. With the torque override off and a 32 % amylose lot at 600 s and 420 kg/h, the stiffer starch raises the torque, the estimator reads it as a drier dough (d_x reaches -2.73 points by 900 s), and an excess over the 82 % ceiling within the next minute is already unavoidable. Paid once for the horizon, the excess left the rest of the plan free under it, and the plan lowered the screw speed from 410 to 305 rpm to hold the density while the torque rode at 89.9 to 90.9 % from 780 to 960 s. The worst torque was 91.1 %, 3.1 min above the hard limit, with 28.6 kg of dense product. Judging the torque and the die pressure at every step of the first minute, where an excess may already be on its way through the preconditioner, and once beyond, removes the breach. On every row of Table 5.11 but the two die-deposit trips, where the dense product differs by 2.1 kg, it gives the same result as a slack at all 60 steps, at 1.8 to 2.8 ms a decision in Node against 16 to 21 ms. The same run then keeps the speed between 394 and 450 rpm and the torque at or below 85.3 %, as Table 5.10 shows.

The change cost one row. With die deposits growing at 2 % of the radius per minute at 420 kg/h, both versions trip at 2329 s, and the dense product before the trip grows from 26.5 to 33.8 kg. Case 9

Table 5.10 The MPC With One Torque Slack for the Horizon and With a Slack per Step Over the First Minute. Accepted/Off-Spec kg, Worst Torque %

Case	One slack		First minute	
Amylose at 420, torque override off	178.6/28.6	91.1	210.2/0.0	85.3
Challenge 420, torque override off	325.9/9.1	86.9	336.9/0.0	84.3
Challenge 420	336.1/0.0	85.7	336.9/0.0	84.3
Case 3, amylose at 300	131.1/4.9	84.5	136.0/0.0	85.1
Amylose at 420	210.6/0.0	87.3	210.2/0.0	85.3
Drier lot at 420	215.2/0.0	85.9	216.1/0.0	84.3
Die deposits 2 %/min, trip	168.8/26.5	82.9	162.3/33.8	82.4

moved only in its mean feed, from 401.3 to 401.6 kg/h, and its tuning rule gave the same weights. Judging the rows only at every second or third step degrades the plan, with 22.3 kg of dense product on case 9 at every second step and a trip with the torque override off at every third.

Two other parts of the design were changed after the worst cases had been run, and both are stated here with the results without them. The speed setpoint was first unpriced once the feed reached its target, and the QP's small ridge, which acts on the absolute inputs, drifted it toward the torque ceiling. Without the pull toward N_{op} case 3 reaches a worst torque of 89.2 % and case 5 a die pressure of 9.02 MPa. The die pressure was first judged with one slack for the horizon too, and without a slack per step case 5 peaks at 9.58 MPa and spends 8.9 min over its 9.0 MPa limit, against 8.94 MPa and no minute with it. Without both changes, and with the move suppression of the first grid, case 9 still accepts 212.9 kg with nothing off-spec and the challenge 337.9 kg, so the headline does not depend on either change.

5.6.10 Worst Cases Under Every Structure

Table 5.11 runs every case and every disturbance the application offers under the loops as the case ships them, the loops with the supervisor and the trim on, and the MPC with either estimate, with every limit checked. The melt never exceeds 165.3 °C against its 170 °C limit, the dryer air never exceeds 147.4 °C against 150 °C, and only case 7 backs the filled length into the kneading block, for 12.6, 11.8 and 12.0 min under the loops, the MPC and its bias form. After the change of Section 5.6.9, no MPC run spends any time above the 90 % torque limit or the 10 MPa die limit outside the die-deposit trips, with either estimate. The loops cross the torque limit with the override on in the amylose lot at 420 kg/h, at 90.8 % for 1.9 min, and reach 93.3 % with the override off, and the supervisor and trim reach 90.9 % for 2.2 min on the challenge with the torque override off.

The MPC is worse in one row. With die deposits at 2 % per minute every structure trips, because the overrides' and the plan's feed sits at its 60 kg/h floor for more than five minutes before the pressure reaches 12 MPa, from 1980 s under the MPC and 1860 s under the loops, so no handle can prevent the trip. Before it, the moisture estimate reads the deposits as a drier dough, d_x reaches −5.49 points and the moisture setpoint sits at its 27.3 % ceiling, so the MPC makes 33.8 kg of dense product where the loops make 20.3 kg of light product and the bias form none. The same misreading appears without a loss in case 3, where the stiffer starch is taken for a dough 2.83 points drier and the cutter density climbs to 149.3 g/L against the loops' 135.1 g/L. Amylose, dryness and deposits move torque, die pressure and melt in the same directions, and only the delayed density tells them apart.

The model was identified at 400 kg/h. Stepped again on the plant, its steady gains agree within 2.1 % at 400 kg/h, while at 300 kg/h the plant's differ from them by 0.4 to 18.6 % of the model's value. The model's torque per point of moisture, −6.13 %, is then 21.7 % above the plant's −5.04 %, the value of Table 5.4, and its torque per rpm 22.9 % above. Run on case 9 with targets of 150, 200, 300, 360, 420, 480 and 540 kg/h, the MPC makes no off-spec product and spends no time above an override limit at any of them, while the loops make 8.1 to 38.3 kg off-spec. With the recipe's moisture

Table 5.11 Worst Cases Under Every Structure. Accepted/Off-Spec kg (Worst Torque %, Worst Die Pressure MPa). A Dagger Marks a Die-Pressure Trip

Case	Loops as shipped	Loops, supervisor and trim	MPC, moisture	MPC, bias
2, wetter flour, 300	70.3/63.8 (65.4, 6.77)	106.8/23.4 (65.4, 6.77)	133.9/0.0 (66.7, 6.93)	114.3/19.8 (65.6, 6.79)
3, amylose, 300	79.4/56.7 (81.5, 8.20)	134.8/0.0 (79.0, 7.97)	136.0/0.0 (85.1, 8.29)	133.7/0.0 (85.6, 8.39)
4, targets 380 and 480	211.5/0.0 (85.5, 8.08)	210.1/0.0 (85.6, 8.14)	211.8/0.0 (82.3, 7.96)	211.5/0.0 (82.3, 7.95)
5, die deposits	129.1/5.0 (70.8, 9.40)	135.0/0.0 (68.8, 9.33)	135.4/0.0 (69.6, 8.94)	133.7/0.0 (74.1, 9.04)
5, die override off	101.2/0.0† (74.7, 12.00)	135.2/0.0 (68.8, 9.66)	135.4/0.0 (69.6, 8.94)	133.7/0.0 (74.1, 9.04)
Challenge, 420	186.3/83.1 (74.0, 7.56)	290.2/34.1 (87.4, 8.36)	336.9/0.0 (84.3, 8.06)	308.4/18.3 (87.3, 8.33)
Challenge, 420, torque override off	186.3/83.1 (74.0, 7.56)	285.5/34.5 (90.9, 8.60)	336.9/0.0 (84.3, 8.06)	308.3/18.3 (87.3, 8.33)
Case 9 lots at 420 kg/h: wet, 15 %	—	169.4/36.8 (84.7, 8.08)	212.9/0.0 (82.3, 7.91)	177.6/35.4 (82.0, 7.87)
amylose, 32 %	—	182.4/20.5 (90.8, 8.67)	210.2/0.0 (85.3, 8.16)	202.5/0.0 (85.5, 8.19)
amylose, torque override off	—	198.9/0.0 (93.3, 8.87)	210.2/0.0 (85.3, 8.16)	202.5/0.0 (85.5, 8.19)
drier, 10 %	—	185.1/21.0 (87.0, 8.32)	216.1/0.0 (84.3, 8.09)	208.6/0.0 (84.4, 8.12)
die deposits 2 %/min	—	153.6/20.3† (85.5, 12.00)	162.3/33.8† (82.4, 12.00)	190.2/0.0† (83.0, 12.00)
same, die override off	—	103.8/0.0† (85.5, 12.00)	144.8/0.0† (82.4, 12.00)	151.0/0.0† (83.0, 12.00)
humid air	—	215.2/0.0 (85.6, 8.14)	216.4/0.0 (82.5, 7.95)	216.3/0.0 (82.5, 7.96)

Cases 1 to 8 over their own duration, the challenge over 3600 s, the lots of case 9 changed at 600 s and run 2400 s. Case 9 ships with the supervisor and the trim on. The challenge's "as shipped" column is its 300 kg/h default, and its other columns aim the supervisor or the MPC at 420 kg/h. A lot at 480 kg/h gives the same rows as at 420. Cases 6, 7 and 8 are ties, 90.7 to 90.8, 89.8 to 89.9 and 136.4 kg with nothing off-spec under every structure, and case 9 with both overrides off or a 3 % feeder noise gives the MPC's rows unchanged and the loops 168.3/37.3 and 169.4/36.8.

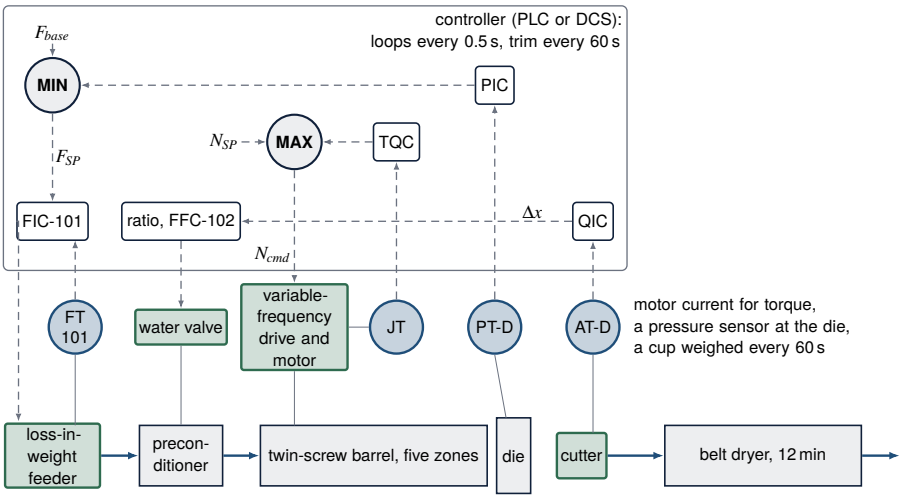


Figure 5.7 The overrides and the density trim on real hardware. The torque JT is read from the main drive's motor current and feeds TQC, whose output meets the speed setpoint in the maximum selector ahead of the variable-frequency drive. The die pressure PT-D feeds PIC, whose output meets the supervisor's feed in the minimum selector ahead of the feeder loop FIC-101. The at-line density AT-D trims the moisture that the ratio station turns into water. Notice that both selectors sit in the controller beside the loops they limit.

set to 21 or 26 %, outside the identified neighborhood, both structures make off-spec product, the MPC 57.9 to 91.2 kg and the loops 81.4 to 107.8 kg.

5.7 Technology in Practice

A real extrusion line reads the same signals and moves the same handles as the overrides of Section 5.5. Figure 5.7 draws the two overrides and the density trim with line hardware in place of the schematic's symbols.

What Measures It

An extension overview counts the residence time, the back pressure, the motor torque and the specific mechanical energy among the system parameters of an extruder: they can be measured, but the operator can change them only through the raw material, the feed rate, the barrel temperatures and the screw speed [5]. Those are the two constraints of this chapter, the torque on the drive and the pressure in front of the die. The melt temperature is harder. Accurate monitoring of the melt temperature remains one of the main challenges of hot-melt extrusion, and wall-mounted thermocouples in a die captured its variations poorly in a polymer study [22]. The model's TT-M reads the bulk melt exactly apart from a 2 s lag and its noise, which Assumptions already flags.

> **TECHNOLOGY Sensor: Drive Torque From the Motor**
>
> Extruders carry standard monitoring of barrel and die temperatures, throughput, screw speed, torque and drive amperage, melt pressure and melt temperature. Torque needs no change to the barrel or the die, and it follows properties that cannot be measured directly during extrusion, such as the rheology of the melt [22]. It is usually read from the drive, because the current of a motor is usually proportional to its torque [23]. In a polymer extruder the screw load torque inferred from the motor current was dominated by the friction of the solids and did not follow melting fluctuations well [22]. No open source read gives a range or an accuracy. The model's JT is a 0.5 s lag with noise of 0.4 % of rating.

What Moves It

Three handles carry the chapter's structure. The feeder's screw meters the flour, the water valve takes its setpoint from the ratio station, and the main drive turns the screws. Feeders deliver gravimetrically or volumetrically, and more than one may feed one extruder [5]. The torque override acts on the main drive.

> **TECHNOLOGY Final Element: A Variable-Frequency Screw Drive**
>
> The motor of an extruder provides the energy that turns the screws [5]. An alternating-current motor is run by a variable-frequency drive, which sets the speed through the frequency of the power it feeds the motor and the torque through the voltage and the current. Such drives offer speed, current and torque limiting, limits on acceleration and deceleration and shutdown on overcurrent, and a host controller can update these settings over a network while the motor runs. The drive must first be programmed with the motor's nameplate data, or it may put out more current than the motor is rated for [23]. The model's drive follows its speed command through a 1.5 s lag.

Where the Controller Runs

Programmable logic controllers serve as local controllers within a supervisory scheme, loops are often nested so that the setpoint of one comes from another, and cycle times range from milliseconds to minutes [24]. In the application the loops, both overrides and the supervisor execute every 0.5 s, the density trim once per at-line result every 60 s, and the predictive controller every 5 s. The selectors are standard function blocks. In one fieldbus standard the control selector block picks among controller outputs and carries back-calculation facilities that modify the response of the deselected controllers, which makes it the block for limit and override strategies [23]. The clamps of Section 5.5, with back-calculation at a tracking time of τ_I, do the same work inside one PI.

What Is Modernizing It

Two developments bear on the overrides and the trim. The first is in-line measurement. Spectroscopic techniques have been explored for monitoring and control of hot-melt extrusion, because control on the conventional temperatures, pressures and torque is in many cases not sufficient to predict the quality of the extrudate [22]. The second is the data-driven model. A review of automated food processing names extrusion as a process that has challenged food technologists for decades,

with operators struggling to relate product quality to settings such as the barrel temperature, and describes machine learning as a way to capture those relations from data [25]. A review of expanded snacks proposes a staged digital twin, from curated data through state variables and inline sensors to optimization or model predictive control that adjusts the inputs toward the product target [26], the role the predictive controller of Section 5.5.9 plays on the simulated line.

> ⚠ **IN PRACTICE** **Limits a Food Safety Plan Writes Down**
>
> United States rules for human food require preventive controls. Process controls ensure the control of parameters during operations such as heat processing, and they name the parameters associated with the hazard and the maximum or minimum value to which each must be controlled (21 CFR 117.135). Verification includes calibration of the process monitoring instruments, or checks of their accuracy, and a review of the monitoring and corrective action records within 7 working days after they are made (21 CFR 117.165) [27]. Such limits come from the hazard, not from the machine. The torque and die-pressure limits of this chapter protect the drive and the die, and its model carries no lethality.

5.8 Limitations

The correlations for cook, expansion and bulk density are calibrated teaching proxies. They give the reported directions but have not been fitted to any flour or extruder, and there is no protein texturization. The thermal treatment is not a food-safety lethality model, and no microbial kinetics are modeled, while a real kill step is validated by experiment [16]. Volume 8, Chapter 13 follows the thermal history of every parcel of a certified heat treatment.

The barrel is ten mixed cells with algebraic fills, so a pulse reaches the die without the delay and with wider tails than a real screw [4], and the melt is a power-law fluid with one index. The instruments are lags with colored noise and fixed delays, the melt thermocouple reads the bulk melt, and there are no failed-instrument modes beyond the wrong certificate. The feeder never refills, the piece length is displayed but not measured, and a trip restarts only through the trip reset, with no purge or die-change procedure.

The supervisor's quality condition is two-sided, so it backs off on a light product, which lowering the feed makes lighter. The application keeps this behavior by choice and records it with its defects. The melt condition guards the exit temperature on which the hard limit is scored, 2.3 °C above the thermocouple at the recipe and about 3 °C above it at 9 MPa, through a prediction that uses the heat capacity of the recipe's moisture whatever the moisture is.

The predictive controller's model is linear at 400 kg/h, 23.3 % and 400 rpm, and at the 300 kg/h recipe its torque gain per point of moisture is 21.7 % above the plant's. It carries no amylose, wear, die deposit or dryer, so every torque change it did not cause is first read as moisture, which is wrong for a stiffer starch (149.3 g/L at the cutter in case 3) and for die deposits (33.8 kg of dense product before the trip). The final moisture is left to the dryer cascade, and no lot change is announced to the plan in advance.

OPEN CHALLENGE **Reading the Product While It Is Made**

A 2026 review of models that link extrusion conditions to the quality of expanded snacks finds quality measurement not standardized: expansion, density, texture and crispness are measured by different protocols. Melt rheology and the behavior of water under real extrusion moisture, temperature and shear remain poorly quantified, and die-exit expansion models rarely bring die geometry, pressure drop and bubble growth into one validated framework. Its proposed path ends with inline sensors and then model predictive control that closes the loop [26]. In this chapter the density still comes from a cup reported 90 s late. Progress would be a model validated across extruders and ingredient lots, fed by a sensor that reads the pieces as they leave the die.

5.9 Conclusion

A cooking extruder is a plant whose heat source is the viscosity of what it carries, and the simulator of this chapter keeps that property exact by making the shaft power and the heat released in the melt one quantity. At the recipe the screws deliver 14.8 kW while the barrel exchanges a net −0.06 kW with the dough, one point of moisture moves the density by 11.48 g/L, and the kneading zone's barrel moves the melt at the die by 0.07 °C per degree. A certificate three points wrong keeps the moisture estimate on 23.3 % while the density climbs to 164.6 g/L, and a trim on the at-line density cuts the loss from 63.8 to 24.3 kg. The torque override and the supervisor hold a 480 kg/h request at 437.2 kg/h and say why, and the die-pressure override saves the die by cutting the feed to 112.1 kg/h and over-expanding the product. An optional predictive controller that moves the feed, the moisture setpoint and the speed together accepts 212.9 kg through a wet lot with nothing off-spec, and its bias-estimate form, at 177.6 and 35.4 kg against the loops' 169.4 and 36.8 kg, shows that the gain lies in where its estimator places the unknown. The hour of flour lots goes from 186.3 to 290.2 kg of accepted product with a trim that restarts at each lot and a supervisor aimed at 420 kg/h. The proxies for cook and expansion are illustrative, and the thermal treatment is not a lethality model. A fuller model would add a plug-flow component to the residence distribution, an extensional viscosity for the expansion, feeder refills, a sign-aware quality condition in the supervisor and microbial kinetics for the thermal treatment. What the application offers is a browser plant, open to anyone, on which the loops of a self-heating process can all be right while its product is wrong.

KEY POINTS

- The work of the screws, not the barrel heaters, heats the dough, so a wetter or stiffer flour moves the torque, the die pressure and the product together.
- The torque and die-pressure overrides realize a maximum and a minimum selector as output limits of the overriding PI.
- A certificate three points wrong keeps the moisture estimate on 23.3 % while the density climbs to 164.6 g/L, and a trim on the at-line density cuts the loss from 63.8 to 24.3 kg.
- The die-pressure override saves the die by cutting the feed to 112.1 kg/h and over-expanding the product: it protects the die, not the product.

Where else this applies. Chapter 4 pushes a harvester's throughput against engine and rotor overrides, and Volume 5, Chapter 10 corrects a soft sensor from a delayed laboratory result.

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5.10 Exercises

5.1 Concept check · 2 min

In scenario 2, Wetter flour, with the density trim off, the process-moisture trend holds the estimate on its 23.30 % setpoint for the whole run, and every heated barrel zone stays within 0.91 °C of its setpoint over the first 20 min. Meanwhile the density at the cutter climbs from 135 to 164.6 g/L. What does the estimate on its setpoint show?

(a) Only that the metered water matches what the certificate implies, since the soft sensor and the ratio station trust the same certificate

(b) That the dough entering the extruder holds 23.3 % water, within the noise of the flow transmitters

(c) That the density error must come from the barrel zones, since the process moisture is held on its setpoint

(d) That the at-line density is in error, since it reports 90 s after its cup is filled

5.2 Concept check · 2 min

In scenario 6, Heater limitation, zone 3, the kneading block, keeps a quarter of its heater capacity. Its loop pins the output at 100 % and the barrel falls from 140 to 132.0 °C, yet the melt at the die falls only 0.6 °C. Why so little?

(a) Back-calculation keeps zone 3's integral from winding up, so its loop makes up the lost heat further along the barrel

(b) The melt's heat comes mostly from viscous dissipation, and the cooler, stiffer dough in zone 3 dissipates more

(c) The melt thermocouple reads the barrel metal of zone 5, which its own loop still holds at 140 °C

(d) The preconditioner's steam loop raises the steam to supply the heat that zone 3 can no longer give

5.3 Concept check · 1 min

In scenario 7, Screw wear, the torque falls from 65.4 to 52.2 % by 1200 s at the same feed and screw speed. What does the falling torque tell the operator?

(a) The drive has gained headroom, so the feed can rise until the torque returns to its old value

(b) The dough has become wetter, since a thinner melt needs less torque

(c) The worn flights shear less, so less work goes into each kilogram and the melt is cooler and expands less

(d) The torque override has raised the screw speed, which lowers the torque at the same flow

5.4 Theory and calculation · 5 min

Scenario 2 feeds 300 kg/h of flour that holds 15 % moisture on a 12 % certificate. The recipe adds 22.48 kg/h of water and 21.71 kg/h of steam, for 344.2 kg/h of dough at a process-moisture setpoint of 23.3 %. Take the steam as unchanged.

(a) How much more water enters with the flour than the ratio station assumes, and how far above 23.3 % does the true process moisture run? *Checkpoint:* 9 kg/h, 2.61 points

(b) With the density gain of Table 5.4, estimate the density at the cutter and compare it with the 164.6 g/L measured. *Checkpoint:* about 165.0 g/L

(c) Use Equation (5.16) to find the water setpoint once the certificate is corrected to 15.0 %. Why does the water fall by more than the 9 kg/h of part (a)? *Checkpoint:* 10.75 kg/h, 11.73 kg/h less

5.5 Theory and calculation · 6 min

The supervisor predicts the torque at the feed it asks for by Equation (5.17). At steady state the two lagged flows agree, so $\rho = F_{base}/F_m$. At the recipe the torque is 65.4 % at 300 kg/h and 400 rpm, the torque limit is 85 % and the margin m_Γ is 3 %.

(a) Predict the steady torque at 380 kg/h and 400 rpm, and compare it with the 77.6 % of case 4. *Checkpoint:* 77.5 %

(b) Predict the torque at 480 kg/h with the screws at their 450 rpm top speed, and compare it with the 87.6 % of the steady-state solver. What is the supervisor's torque slack there, and what does the supervisor do? *Checkpoint:* 87.5 %, slack -1.84

(c) Taking the recipe as the base, with the screws at their 450 rpm top speed as in part (b), find the feed at which the slack falls to 0.25, where the supervisor stops raising the feed, and the feed at which it reaches zero. Compare them with case 4. *Checkpoint:* about 433 and 439 kg/h

5.6 Theory and calculation · 5 min

QIC executes Equation (5.15) once per at-line result, with $\Delta t_c = 60$ s, $K_c = 0.05$ points per g/L and $\tau_I = 240$ s. The step test of Section 5.5 found 11.25 g/L per point of moisture, 10 % of the change after 77 s and 63 % after 187 s, and the at-line check reports 90 s after its cup is filled, one cup a minute.

(a) Find the loop gain of the trim. *Checkpoint:* 0.56

(b) After a run of results on the setpoint, a result arrives 10 g/L above it. How far does that execution move the moisture setpoint, and how much of the move is proportional and how much integral? *Checkpoint:* 0.625 points down

(c) After a move of the moisture setpoint, about how many trim executions pass before an at-line result shows 63 % of its effect? What does that say about a larger gain? *Checkpoint:* about four

5.7 App challenge · 8 min

Scenario 2, Wetter flour, default seed, played at 60 times real time unless stated.

(a) Run the case as loaded, with the density trim off, to 1800 s. Export the CSV file and find the first rows in which `rho_cutter_gL` and `rho_atline_gL` exceed 150. Read the Off-spec product tile at 1800 s. *Checkpoint:* 209 s and 331 s, 64 kg

(b) Press Reset, set *Density trim of the moisture setpoint* On before Run, and run to 1800 s. Read the Off-spec product tile and the trim on the QIC card, and compare the trim with Exercise 5.4(a). *Checkpoint:* 24 kg, -2.67 points

(c) Reload the case. At 20 times real time run to about 300 s, pause, set *Flour moisture on the lot certificate* to 15.0 %, then run to 1800 s and read the Off-spec product tile. *Checkpoint:* 18 kg

Tolerance: 2 s on the crossings, 1 kg on the tiles and 0.02 points on the trim.

5.8 App challenge · 8 min

Scenario 4, Feed-rate increase, default seed, played at 60 times real time. Run each setting to 2400 s and export the CSV file.

(a) Run the case as loaded. Find the first row with `ctrl_infeasible` at 1 and the binding limit in the `limit` column, and at 2400 s read `feed_sp_kgh`, `screw_rpm` and the Worst torque tile. *Checkpoint:* about 1657 s, torque at maximum speed, 437.2 kg/h, 409 rpm, 85.5 %

(b) In the same file, find the first row with `torque_override_active` at 1 and the first row with the true torque `torque_pct` at or above 85. Why does the override act first? *Checkpoint:* 1423 s and 1552 s

(c) Reload the case, set *Density trim of the moisture setpoint* On before Run, and read `feed_sp_kgh` and the trim at 2400 s. Why does a trim that holds the density lower the feed the torque allows? *Checkpoint:* 416.0 kg/h, −0.41 points

Tolerance: 2 s, 0.5 kg/h, 1 rpm, 0.1 point of torque and 0.02 points of trim.

5.9 App challenge · 9 min

The challenge, a shift of flour lots, default seed, played at 60 times real time. Run each setting for the full 3600 s, then change the settings, press Reset and run again. Leave every parameter and disturbance as shipped.

(a) Run as shipped. Read the verdict, the Accepted and Off-spec product tiles and the Worst torque tile. *Checkpoint:* 186 kg, 83 kg, 74.0 %, fails

(b) Set *Density trim of the moisture setpoint* On, *Throughput supervisor* On and *Production target* 420 kg/h. *Checkpoint:* 290 kg, 34 kg, 87.4 %, passes

(c) Keep (b) and set *Restart the trim at a new lot*, an advanced option on the QIC card, Off. Read the same tiles, and the dense and light shares of the off-spec product in the CSV columns `metric_off_dense_kg` and `metric_off_light_kg`. Explain the result. *Checkpoint:* 244 kg, 75 kg with 44 kg light, 99.2 %, fails

Tolerance: 1 kg and 0.1 point.

5.10 Application area · 4 min

Where an extruder is the kill step of a food process, the plant must show that every particle received a lethal treatment. In case 1 at 400 rpm, measured from the middle of the dye pulse, the dye reaches 1 % of its peak rate after 7.7 s, peaks at 27.4 s and is 90 % out by 46.2 s, with a mean residence of 31.6 s.

(a) What fraction of the mean residence does the earliest dough spend in the barrel? Why is a treatment judged on the mean residence and the melt thermocouple not a lethality record? *Checkpoint:* about 0.24

(b) Name two ways in which this model could misstate what the fastest real particle sees, and say what a plant does instead.

5.11 Application area · 4 min

At the 450 rpm top speed the recipe needs 87.6 % of the 540 N m rating for 480 kg/h. Half a point more water brings that to 84.4 % but the density to 147.2 g/L, and a drive rated at 600 N m would carry it at 78.9 %.

(a) What torque and shaft power does 480 kg/h need at the top speed? Check the 78.9 %. *Checkpoint:* about 473 N m, 22.3 kW

(b) Compare the three ways to 480 kg/h: run at 87.6 % with the override limit raised, add water, or buy the larger drive. Which would you put to a plant manager, and what would you measure first?

5.12 Review the agent · 5 min

An AI assistant reads Table 5.9 and writes: “The MPC accepts 212.9 kg through the wet lot with nothing off-spec because it optimizes the feed, the moisture setpoint and the screw speed together over a five-minute horizon. The estimator is a detail, since any offset-free estimate would do. To match it with the loops, replace the supervisor and the trim by a multivariable optimizer.” Find the flaw and say how you would check it in the application.