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CHAPTER 14

Defending a Currency Peg With a Finite Vault and Too Few Instruments

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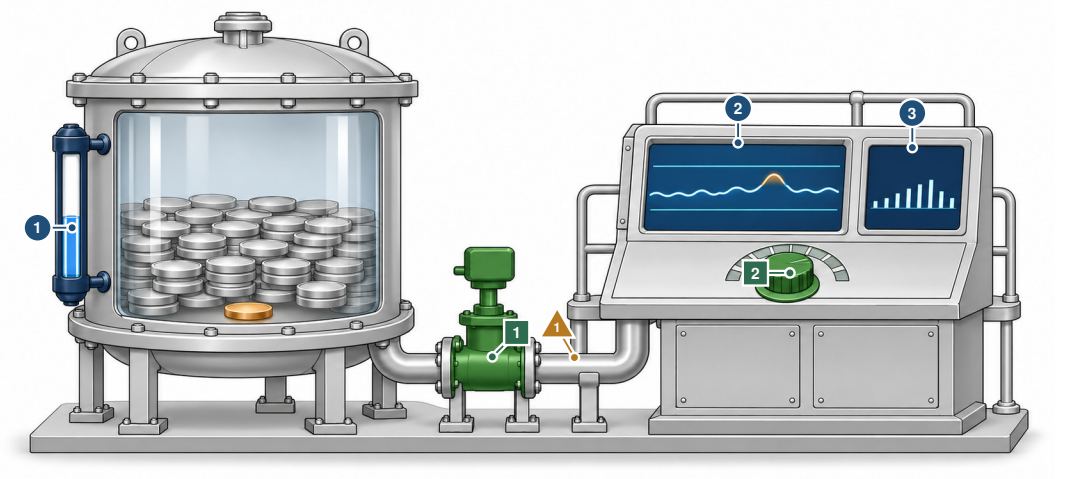
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CHAPTER 14

Defending a Currency Peg With a Finite Vault and Too Few Instruments



The controller moves 1 sales from the reserve vault 2 policy interest rate

The controller reads 1 reserves on the bank's books 2 exchange rate inside its band 3 quarterly output report

What disturbs it 1 money leaving the currency, an outflow or an attack

The vault pays for every sale through the green valve while the dial sets the interest rate, and the peg can end when the reserves fall to the floor marked by the amber disc. Illustration, not to scale.

IN PLAIN TERMS **Holding a Currency Steady With a Limited Vault**

The main idea. A central bank can promise a fixed price for its money in another currency. To keep the promise, it sells from a vault of foreign money when more people sell its money than buy it.

The challenge. The vault is limited. When it runs low, people expect the promise to break, and they pull money out even faster. A plan made once a week is too slow for a sudden attack, so a fast backup rule briefly limits money leaving the country.

Why it matters. Measuring each defense shows what it can buy. When the bank keeps lending to its government, no defense moves the day the promise breaks.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history segment and the fact on the foreign exchange market (Section 14.2), the two try-it segments and the fact on sterilization (Section 14.6) and the key points
Study	60 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	20 min	the first page, the measured cases and the challenge (Section 14.6), Technology in Practice (Section 14.7), Limitations (Section 14.8) and the two open challenges

The control question A central bank holds its exchange rate in a band with a vault of reserves that has a floor, and below a threshold the market's fear of a devaluation makes the vault drain faster. What can a currency board, a PI desk, a capped rate defense and an MPC that weighs defending against floating buy with two handles, and what does no defense change?

Learning objectives

- Explain why a reserve clock that divides the reserves by the outflow is optimistic once expectations feed on the vault.
- Derive the first-generation date of an attack from the shadow rate, and show why no intervention policy or policy rate moves it.
- Compare sterilized and unsterilized defense through the drain law and the overnight rate.
- Measure in the app what a capped rate defense, the MPC's float and a capital-controls override do against a speculative attack.
- Judge a defense by counting its limits against its instruments.

Prerequisites Sections 1.4.3 (selectors and override), 1.4.5 (saturation, rate limits and the calendar), 1.4.9 (actuators that deplete) and 1.6.2 (velocity form). Section 1.6.3 is needed only for the MPC, and Chapter 8 uses the same calendar.

The app https://apmonitor.com/apps/currency-peg/

See also Volume 6, Chapter 2 for two actuators against more variables worth holding and Volume 6, Chapter 11 for the same decision calendar on a chain of inventories. Volume 9, Chapter 7 for a store whose release is decided ahead of a forecast inflow and Volume 9, Chapter 11 for a discrete decision priced by optimization.



14.1 Introduction

A central bank that pegs its currency keeps a vault of foreign reserves and sells from it whenever the market wants more foreign currency than it offers. In the simulator of this chapter, capital that starts leaving at 6 % of gross domestic product (GDP) a year drains a vault that holds 15 % of GDP. A year in, the clock that divides the reserves above the floor by the outflow on the bank's books reads 9.8 months. The peg lasts eight, because below a threshold of 8 % of GDP the market starts to expect a devaluation, and the outflow grows to 9.8 % of GDP a year by year 1.6. When the bank instead lends the government 2 % of GDP a year without sterilizing, the peg ends at year 2.252 under every controller in the application, against a date of 2.25 years that follows from the model's equations alone, and the day before it the shipped intervention desk still holds 12.15 % of GDP.

The defense of a peg has been written as a problem of feedback and optimization for almost fifty years. A government that defends a price ceiling with sales from a stockpile must eventually meet a sudden attack by speculators [1], an argument made first for gold and then for a fixed exchange rate defended while domestic credit grows [2], after which a linear example gave the date of the attack in closed form [3]. An exchange rate held in a target zone by a policy that limits its variation is shaped by that expectation even while it sits inside the zone and nobody is defending it, a problem formally similar to option pricing [4]. Second-generation models center on the government's continuous comparison of the net benefits of devaluing and of defending [5], which is a switching decision, and the interest-rate defense has been analyzed as a policy that delays a crisis up to a point and hastens it beyond that point [6]. The trilemma, a stark tradeoff among exchange stability, monetary independence and open capital markets [7], goes back to stabilization policy analyzed under highly mobile capital [8], and in control terms it counts instruments against targets. Control is the natural language for the problem, since the domain already reasons in it. The band is a hard output constraint, the reserves are an inventory with a floor, the interest rate is a second actuator whose cost lands on another output, capital controls are a third actuator thrown as an override, and abandoning the peg is a decision to stop controlling.

The same shape recurs well outside currencies. A level held above a floor by an inventory that pays for every imbalance, a drain that grows as the level falls, a second actuator that draws the inflow

back at a cost charged to a different output, and a fast override that adds authority for a limited time describe a bank's liquid assets facing depositors, and speculation against a currency has been likened to a run on a bank [5]. They also describe a vessel kept off a low-level trip by a makeup stream whose demand rises as the level falls, and the nitrogen tank of Chapter 12 likewise gives out by its pressure while nitrogen is still in it. The misconception this chapter corrects is that a vault lasts as long as its reserves divided by the outflow, and that a defense tuned well enough can hold any peg its reserves can pay for. The clock of the first case is optimistic by 1.8 months, and in the deficit case no controller moves the attack by a single day, while the defenses that keep the most reserves hand the most to the speculators, 11.44 % of GDP under a hand that holds the policy rate at 8 % against 7.14 % under the shipped desk.

However, the crisis models compute the date of an attack from perfect foresight or from the conditions of an equilibrium, with the policy as a parameter of the analysis [5, 9], so a student seldom has a peg on which a hand, a currency board, a PI desk, a rate rule and an optimizer defend one vault through one calendar against flows that answer the money market. This chapter describes such a plant completely and measures on it what each defense buys and what no defense can change. The economics comes from the literature cited below: the shadow rate, the first-generation date, the second-generation choice and the trilemma. The plant is a continuous-time vault whose weirs conserve reserves against the integrated flows to 1.4×10^{-13} points of GDP in every case, a money market that raises a currency board's overnight rate by exactly $1/\alpha$ per point sold, and a shadow rate that reproduces a linear first-generation date to within a day at five settings. The bank adds an MPC that prices floating against defending at every decision and a capital-controls override, all on the decision-calendar module that the economics applications share. Nine cases and a graded challenge carry the measurements, and every number this chapter quotes from them was measured headless on the application's engine.

Section 14.2 traces the problem from a gold stockpile to a franc floor. Section 14.3 gives the economy, its vault, money market, capital flows and shadow rate, and Section 14.4 the reasons its vault drains faster than its clock and the date that no controller moves. Section 14.5 gives the bank's two handles, its calendar, the PI desk, the rate defense, the MPC that chooses between defending and floating, and the capital-controls override. Section 14.6 reports the nine cases and the challenge, Section 14.7 sets the bank's handles beside the instruments of real central banks, Section 14.8 states the limitations and Section 14.9 summarizes. The exercises in Section 14.10 run the cases.

14.2 From a Gold Stockpile to a Franc Floor

Speculative attacks on fixed rates were once taken for market pathologies that well-functioning markets would avoid, and the work of the late 1970s recast them as the competitive response of such markets to an inconsistent government policy [10]. The first model concerned a government that pegs the price of gold, or defends a ceiling on it, with sales from its stockpile, and it showed that the policy must end in a sudden attack by speculators [1]. The same reasoning was applied to a fixed exchange rate defended by a stock of reserves while domestic credit expands [2], and a linear version gave the date of the attack explicitly [3]. In those classical models a fixed rate contains inflationary pressures that end in a sudden balance-of-payments crisis, which frees the currency to depreciate [5]. A survey of 1991 set out the construction this application follows. The shadow floating rate is the rate that would prevail if the reserves were gone and the currency floated, the fixed rate is safe while the shadow rate stays on the strong side of it, and the regime collapses on the day the two meet, the earlier the larger the interest semi-elasticity of money demand [9].

Europe supplied the experience that tested the theory in the exchange rate mechanism (ERM) of the European monetary system, whose crisis of 1992 the history segment below recounts. In August 1993 most ERM bands were widened from ± 2.25 % to ± 15 % [5].

HISTORY 1992: Unlimited Intervention at the Edge of a Band

The exchange rate mechanism obliged both central banks to intervene when a bilateral rate reached the edge of its band, 2.25 % from the central parity in the normal band, and its founding act made those interventions unlimited. Capital controls had protected central banks' reserves against speculative attacks until the Single European Act mandated their elimination by July 1, 1990, while still allowing emergency controls for no more than six months. On September 16, 1992, the Bank of England reportedly spent as much as $20 billion, half its foreign exchange reserves, raised its discount rate from 10 to 12 % and announced a second increase to 15 %, and sterling left the mechanism at the end of the day. That month the Swedish central bank raised its marginal lending rate from 16 to 75 %, and overnight rates later reached an annualized 500 % [11].

Those attacks struck pegs that were not unsustainable in the sense of the classical models, and a second generation of models showed why. The discomfort a government suffers from speculation shapes the speculators' incentives and can admit more than one equilibrium, so even a sustainable peg may be attacked and broken [5]. Sterilization exposed a gap of a different kind. In practice a monetary authority insulates the monetary base from an attack by sterilizing the reserve outflow, whereas in the standard models a sterilized attack leaves a regime that either works indefinitely or collapses at inception, a contradiction that the Mexican crisis of December 1994 brought forward [10]. In Thailand the baht came under attack in late 1996 and twice more early in 1997, the government reportedly spent $16 billion in its defense, and the baht was cut loose on July 2, 1997. Creditors then withdrew capital from the region with little effort to distinguish among its economies, and by early September the currencies of Indonesia, Malaysia, the Philippines and Thailand had each fallen by 20 % or more [12].

Two regimes mark the ends of the design space. Hong Kong's linked exchange rate system has held 7.8 Hong Kong dollars to the US dollar since October 1983, and in May 2005 it added a convertibility undertaking on the strong side at 7.75 and moved the weak side to 7.85, a symmetric zone that made the system less discretionary and, by the evidence of option prices, more credible [13]. From September 2011 to January 2015 the Swiss central bank defended a floor of 1.20 francs per euro from the other side, by buying foreign currency without limit, and option prices put the floor's three-month survival probability below 75 % in its first months and at 95 % by August 2014 [14]. More than 130 years of interest rates largely bear out the trilemma's constraints [7], although a global financial cycle driven by the center country's monetary policy can bind even a floating rate, which leaves targeted capital controls among the remaining options [15]. The application's weirs are the ERM's and Hong Kong's unlimited interventions at the edges, its desk is intervention inside the band, and its override is a temporary flow measure.

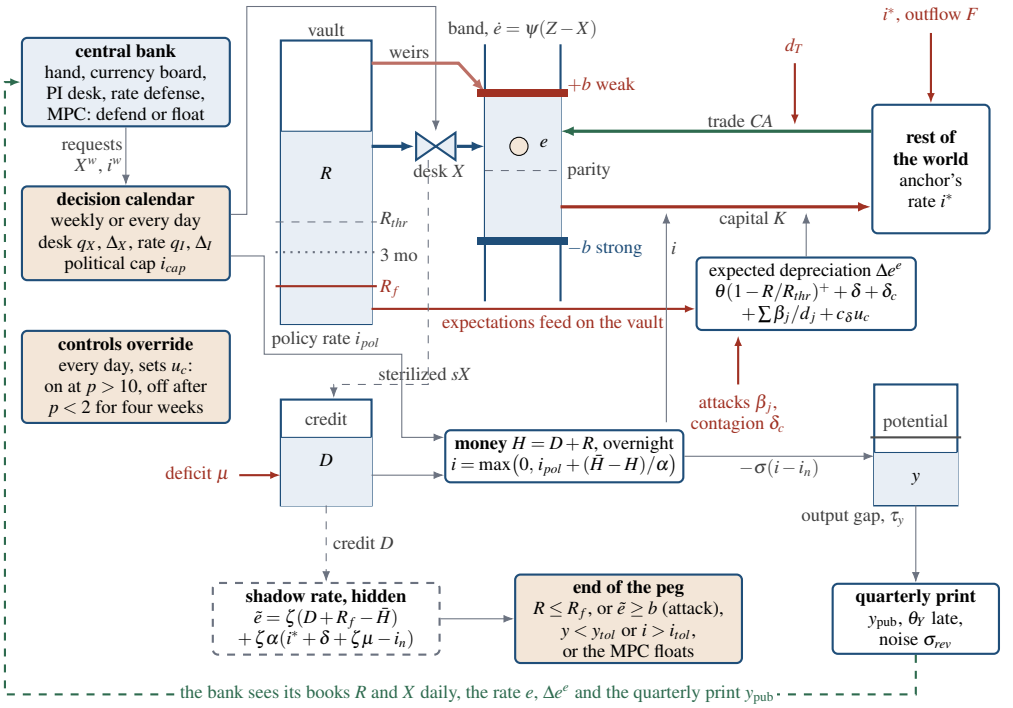


Figure 14.1 The peg as a vault, a band and a money market, not to scale. The vault R pays the desk's sales X and whatever the weirs at $\pm b$ sell or buy, and the float e moves at $\psi(Z - X)$ on the private demand $Z = -(CA + K)$. Domestic credit D grows with the monetized deficit μ and, when the bank sterilizes ($s = 1$), with every sale, money is $H = D + R$, and the overnight rate i rises above the policy rate i_{pol} when H falls below $\bar{H}$. Capital K answers i against the anchor's rate i^* and expected depreciation Δe^e, which grows below the threshold R_{thr}. Output y pays for i. The hidden shadow rate $\tilde{e}$ follows D, and the peg ends at the floor R_f, when $\tilde{e}$ reaches b, past the government's tolerance or by the MPC's choice. Thick arrows that enter from outside the plant (red) are disturbances, thin arrows (gray) carry decisions and signals, the dashed box marks the hidden shadow rate, and the long dashed path along the bottom and the left edge (green) is what the bank sees.

> ✦ **DID YOU KNOW? Seven and a Half Trillion Dollars a Day**
>
> By the triennial survey of central banks, foreign exchange trading averaged $7.5 trillion a day in April 2022, up 14 % from 2019 [16]. The 1992 defense in the history segment above spent as much as $20 billion, and that was half of one central bank's foreign exchange reserves [11].

14.3 A Vault, a Band and a Money Market

The simulated economy is a small open economy pegged to an anchor currency, in years. Stocks are in percent of GDP, flows in percent of GDP a year, interest rates in percent a year, and the exchange rate e in percent above the central parity, so that a positive e is a weaker currency. Figure 14.1 draws the plant as the application does, a vault, a channel with two weirs and arrows for the flows, and Table 14.1 classifies its signals.

Table 14.1 Signals of the Peg by Role, With Their Defaults

Symbol	Role	Default	Range
e	CV, exchange rate from parity, SP e_{sp}	0, SP 0	$\pm b$ pegged
R	CV and state, reserves, floor R_f	15 %	—
i	output, overnight rate, judged in the challenge	5 %	≥ 0
y	output and state, true gap, hidden from the bank	0	—
y_{pub}, $\hat{y}$	measured, quarterly print, and the bank's estimate	0, 0	—
X	MV, desk sales inside the band	0	−20 to 60
i_{pol}	MV, policy rate	5 %	0–25
u_c	MV, capital controls in force	0	off, on, override
F, i^*	DVs, persistent outflow, anchor's rate	0, 3 %	0–20, 0–10
μ, d_T	DVs, monetized deficit, trade shock	0, 0	0–5, −6–3
δ_c, β_j	DVs, contagion premium, attack bets	0	events
D, H	state and output, domestic credit, money	35, 50	—
$\tilde{e}$	output, shadow rate, hidden	−20	—
Δe^e	output, expected depreciation (market price)	2	—
q, I_y	states, integrated flows, integrated gap	0	—
C, P_s, Π	states, defense cost, private holdings, profit	0	—

Stocks in % of GDP, flows in % of GDP a year, rates and e in %.

14.3.1 The Vault, the Float and the Weirs

The bank's reserves R fall by what its desk sells, and the exchange rate floats on the private demand for foreign currency that the desk does not meet, as shown in Equation (14.1).

$$\dot{R} = -X, \qquad \dot{e} = \psi\,(Z - X), \qquad Z = -(CA + K) \tag{14.1}$$

where $\psi = 20$ percent of rate per point of GDP, CA is the trade balance and K the net capital flow. After each integration step the weirs act. A rate past the weak edge b is put back at b and the excess $(e - b)/\psi$ is sold from the vault, and a rate past $-b$ is put back at $-b$ and the shortfall bought into it. The weirs are marginal intervention and the desk is intramarginal intervention, and the vault pays for both. Because the weirs move R and e together, the vault always holds what the flows say it should, as shown in Equation (14.2).

$$R - \frac{e}{\psi} - q = \text{const}, \qquad \dot{q} = CA + K \tag{14.2}$$

The application's check finds the identity held to 1.4×10^{-13} points of GDP in every scenario while its peg stands. The default band is $b = 1\,\%$, a teaching value, with presets for the ERM's narrow band of 2.25 % and Hong Kong's undertakings at 7.75 and 7.85 to the dollar, $\pm 0.641\,\%$ of 7.80.

14.3.2 Money, Sterilization and the Overnight Rate

Money is domestic credit plus reserves, and the overnight rate clears a linear money market around the money held at the peg, as shown in Equation (14.3).

$$\begin{aligned} H &= D + R, \qquad \dot{D} = \mu + sX \\ i &= \max\left(0,\; i_{pol} + \frac{\bar{H} - H}{\alpha}\right) \end{aligned} \tag{14.3}$$

where $\bar{H} = 50\,\%$ of GDP, $\alpha = 1.5$ points of GDP per point of rate, μ is the monetized deficit and s is 1 when the bank sterilizes and 0 when it does not. A weir sale adds s times the sale to D in the same way. Unsterilized, a point of GDP sold takes a point of money out and raises the overnight rate $1/\alpha = 0.67$ points above the policy rate, the automatic adjustment of a currency board, which the application's check confirms on every day of case 2 to 3.6×10^{-15}. Sterilized, the bank buys domestic bonds with the money a sale brings in, so money and the overnight rate stay where they were and domestic credit rises by the sale instead.

14.3.3 Capital, Expectations and Attacks

Capital moves on uncovered interest parity with imperfect mobility, against an expected depreciation that grows as the vault falls below a threshold, as shown in Equation (14.4).

$$\begin{aligned} K &= \kappa_c\,(i - i^* - \Delta e^e) - (1 - c_K u_c)F \\ \Delta e^e &= \theta \max\!\left(0, \frac{R_{thr} - R}{R_{thr}}\right) + \delta + \delta_c(t) + \sum_{j\,\text{open}} \frac{\beta_j}{d_j} + c_\delta u_c \end{aligned} \tag{14.4}$$

where $\kappa_c = \kappa(1 - c_K u_c)$, $\kappa = 1.5$ points of GDP a year per point of return, $\theta = 10\,\%$ a year with the vault empty, $R_{thr} = 8\,\%$ of GDP, $\delta = 2$ points of fundamental premium, and u_c is 1 while capital controls are in force. Below the threshold each point of reserves lost adds $\theta / R_{thr} = 1.25$ points to Δe^e, which the check confirms to 10^{-12}, and the plant's feedback turns positive. A speculative attack j is a bet on a devaluation of $\beta_j = S_j / \kappa_0$ percent within d_j years, with $\kappa_0 = 1.5$, which at the shipped mobility moves S_j points of GDP out of the currency. The three attacks move 5 points in a quarter ($\beta = 3.33$, 20 % of GDP a year), 10 points in a quarter ($\beta = 6.67$, 40 % a year) and 15 points in six weeks ($\beta = 10$, 130 % a year). Contagion δ_c adds 5 points for half a year. Controls cut capital mobility and the persistent outflow F by $c_K = 80\,\%$ at a trade cost $c_T = 1$ point of GDP a year and a credibility cost $c_\delta = 1$ point of premium.

14.3.4 Output and the Trade Balance

The overnight rate, not the policy rate, is what output pays, and the trade balance answers the gap and the position of the rate in the band, as shown in Equation (14.5).

$$\begin{aligned} \tau_y \dot{y} &= -y - \sigma(i - i_n) + \phi\,(\eta e + d_T - c_T u_c) \\ CA &= -m_y y + \eta e + d_T - c_T u_c \end{aligned} \tag{14.5}$$

where $\tau_y = 0.5\,\text{yr}$, $\sigma = 0.7$ points of gap per point of rate, $i_n = 5\,\%$ is the neutral rate, $\phi = 0.5$, $\eta = 0.2$ points of GDP a year per percent of rate and $m_y = 0.3$. At the defaults $i_n = i^* + \delta$, so capital, trade and the vault are all at rest.

14.3.5 The Shadow Rate and the End of the Peg

The shadow rate is where the currency would float if speculators took every reserve above the floor and the bank stopped defending. Money would then be $D + R_f$, the price level would follow money, and money demand would fall with the float's interest rate, the anchor's rate plus the premium plus the trend depreciation $\zeta\mu$, as shown in Equation (14.6).

$$\tilde{e} = \zeta\,(D + R_f - \bar{H}) + \zeta\alpha\,(i^* + \delta + \zeta\mu - i_n), \qquad \zeta = 100/\bar{H} \tag{14.6}$$

where $\zeta = 2$ percent of rate per point of GDP and $R_f = 5\,\%$ of GDP. The shadow rate depends on domestic credit and not on the reserves in the vault, which the application checks. After each step the peg ends on the first of four conditions. When $\tilde{e} \geq b$ with $R > R_f$, the reserves above the floor are free money and are taken in one day, too fast to sterilize, which is the first-generation attack. When $R \leq R_f$, the bank gives up at its floor. When the government's tolerance is on and $y < y_{tol}$ or $i > i_{tol}$, with $y_{tol} = -3\,\%$ and $i_{tol} = 12\,\%$ (each moved halfway to neutral by a political shock), the defense has cost more than the government will bear. When the MPC of Section 14.5 asks to float, the peg ends by choice. At the exit the bank stops selling and sets its rate to i_n, the open attacks close, and the currency jumps to

$$\begin{aligned} e^+ &= \max\left(\tilde{e},\; b + J\right) \\ J &= \textstyle\sum_{j\,\text{open}} \beta_j + \theta \max\left(0, 1 - R/R_{thr}\right) + \delta_c \end{aligned} \tag{14.7}$$

with $J = 0$ at a first-generation attack, after which e drifts at $\zeta\mu$ and the vault is frozen.

Table 14.2 Teaching Values. M Marks a Model-Dialog Slider, D a Disturbance, A an Advanced Control and C a Source Constant, and None Is Hidden

Symbol	Meaning	Value	Range
M, model-dialog sliders			
$R(0)$, R_f	reserves at the start, floor (% of GDP)	15, 5	6–25, 0–10
R_{thr}, IM	threshold (% of GDP), imports (%/yr)	8, 30	4–14, 10–60
κ, θ	capital mobility, devaluation expected when empty	1.5, 10	0.25–5, 0–30
δ	fundamental premium (points)	2	0–6
α, $\bar{H}$	money demand slope, money at the peg	1.5, 50	0.25–5, 20–100
i_n	neutral domestic rate (%)	5	1–10
ψ	rate moved (%) per unmatched point of GDP	20	5–100
η, m_y	trade response to e and to y	0.2, 0.3	0–1, 0–1
σ, τ_y	output response to i, gap lag (yr)	0.7, 0.5	0.1–2, 0.1–2
ϕ	output response to trade	0.5	0–1.5
c_K, c_T, c_δ	controls: share stopped, trade cost, premium	0.8, 1, 1	0–1, 0–3, 0–3
y_{tol}, i_{tol}	government's tolerance (%)	−3, 12	−8–−0.5, 6–25
θ_Y, σ_{rev}	print delay (yr), print noise	1/12, 0.3	0–13 wk, 0–1.5
D, disturbances			
F, d_T	persistent outflow, trade shock	0, 0	0–20, −6–3
i^*, μ	anchor's rate (%), monetized deficit	3, 0	0–10, 0–5
—	political shock: tolerance factor	0.5	—
A, advanced control			
b	band half-width (%)	1	0.641, 2.25
C, source constants			
κ_0	mobility that sizes an attack	1.5	—
—	attack sizes, lengths (yr)	5, 10, 15 in 1/4, 1/4, 6/52	—
—	contagion premium, length	5, 0.5 yr	—
Δt	integration step, control period, log	1/365 yr	—
—	seed	20261005	—

14.3.6 What the Bank Can See

The bank reads its reserves, its domestic credit, its sales and the credit it extends to the government every day from its own books, and the exchange rate, the overnight rate, the anchor's rate and expected depreciation as market prices. It sees output only through a quarterly print of the quarter's average gap, published $\theta_Y = 1/12$ year after the quarter ends, as shown in Equation (14.8).

$$y_{\text{pub},j} = 4\int_{(j-1)/4}^{j/4} y\,\mathrm{d}t + \sigma_{rev}\,\nu_j \tag{14.8}$$

where $\sigma_{rev} = 0.3$ points and ν_j is a standard normal deviate from the engine's seeded generator. The shadow rate and the speculators' position are the model's and never the bank's.

14.3.7 Start, Integration and Bookkeeping

Unless a case declares otherwise, it starts at rest with $R = R(0) = 15\,\%$ of GDP, six months of the imports of 30 % of GDP a year, $D = \bar{H} - R(0) = 35$, $e = 0$, $y = 0$ and the policy rate at 5 %. The engine of Section 1.6.1 integrates the nine states by classical fourth-order Runge–Kutta on a step of a day, $\Delta t = 1/365$ year, the controller runs once a day ahead of each step, and every day is logged. The default equilibrium has derivatives of exactly zero and holds for two years with no drift, and at half the step the challenge keeps both verdicts with its lowest reserves within 0.0091 points of GDP. Four integrals keep the books, the flows q of Equation (14.2), the gap I_y for the print, the defense cost $C = \int [y^2 + ((i - i_n)^+)^2]\,\mathrm{d}t$ and the private sector's holdings P_s bought from the vault, with a teaching profit Π that collects the gain on the rate's moves less the carry given up. Noise comes from seed 20261005 by default. Table 14.2 gathers the constants.

14.3.8 Assumptions

Expectations are mechanical. Expected depreciation is a premium, a term that rises as the vault falls, the contagion premium and the scripted bets. Only the first-generation attack looks ahead, through

the shadow rate. A market that learned the bank's rule would change these relations, which is the critique of [17], so the model shows what a defense does to this economy and not what it would do to one that anticipates it.

Flows, not portfolios. A constant gap in returns gives a constant capital flow, as in the textbook flow models, with no portfolio stocks that fill up, and one mobility κ serves every kind of capital. A long attack is bounded only by its scripted length.

One money and one bank. Base money and the deposits it supports are one stock H, the money market is linear about $\bar{H}$, and sterilization is all or nothing. There are no banks with balance sheets, no foreign-currency debt and no default, which leaves out the banking side of crises such as Thailand's, where public money also propped up failed banking institutions [12].

Linear money demand about the peg. The shadow rate of Equation (14.6) turns a point of GDP of money into ζ percent of price level, which is accurate only near the peg.

An ending rule, not a float. After the exit the currency follows Equation (14.7) and a drift, which is a rule for the end of the story and not a model of a floating rate.

Teaching values. Every threshold, elasticity, attack size, tolerance and rule is chosen to expose one mechanism. The real ones come from the crisis literature [2, 3, 5, 6] and from the institutions themselves, such as the ERM's margins [11] and Hong Kong's undertakings [13]. The application uses three months of imports as its rule of thumb for a safe vault, and its floor of 5 % of GDP is two.

14.4 Why the Vault Drains Faster Than Its Clock

14.4.1 A Float That Crosses the Band in a Day

The exchange rate is the fastest state in the plant. During the 10-point attack the unmatched demand is 40 % of GDP a year, which moves the rate at $\psi Z = 800\,\%$ a year, across the two-point band in 0.9 day. Inside the band the trade balance pulls the rate back at $\psi\eta = 4$ per year, a quarter-year time constant, which is slow beside an attack. The float also stores almost nothing. By Equation (14.2) a full crossing of the band is worth $2b/\psi = 0.1$ point of GDP of reserves, so where in the band the bank chooses to sell changes little about how fast the vault empties. A desk that decides once a week cannot hold the float off the weak weir in an attack, and the weir sells what the desk misses. In a tuning sweep of case 4 without sterilization, the float spends 64 % of the time from year 0.1 to year 1 at an edge under the weekly desk at the shipped gain of 3, against 7 % for a desk deciding every day at a gain of 10.

14.4.2 The Drain Law

Once the desk and the weirs meet the whole private demand, the vault absorbs the whole private flow, $\dot{R} = CA + K$, and at a given gap that flow is a static function of the vault. Evaluated with the policy rate at neutral, the trade term at zero and $y = 0$, it is the drain law of Equation (14.9).

$$\dot{R} = \kappa\left[(1-s)\frac{R(0)-R}{\alpha} - \theta\max\left(0, 1-\frac{R}{R_{thr}}\right)\right] - F \tag{14.9}$$

where the first term is the money market drawing capital back as unsterilized sales drain money, since then $H = \bar{H} - (R(0) - R)$. The vault is a depleting actuator whose drain grows as the stock falls, the case of Section 1.4.9. Figure 14.2(a) plots the law from the application's own flow function. Sterilized, the vault is neutrally stable above the threshold and loses $\kappa\theta/R_{thr} = 1.875$ points of GDP a year for every point below it, a positive feedback with a doubling time of 0.37 year. Unsterilized, every point sold draws $\kappa/\alpha = 1$ point of GDP a year back, which is enough to make the drain law positive everywhere below $R(0)$ when nothing else is leaving, so a board refills its vault after an attack ends. A persistent outflow of 6 % of GDP a year shifts both curves down by 6. The sterilized vault then drains at a constant rate to the threshold and faster below it, while the unsterilized one balances at $R(0) - \alpha F/\kappa = 9\,\%$ of GDP with the overnight rate 4 points above neutral. With the float on the weak weir, the trade balance gains $\eta b = 0.2$ point of GDP a year, which moves the sterilized point of no return from 8 to 7.89 % of GDP.

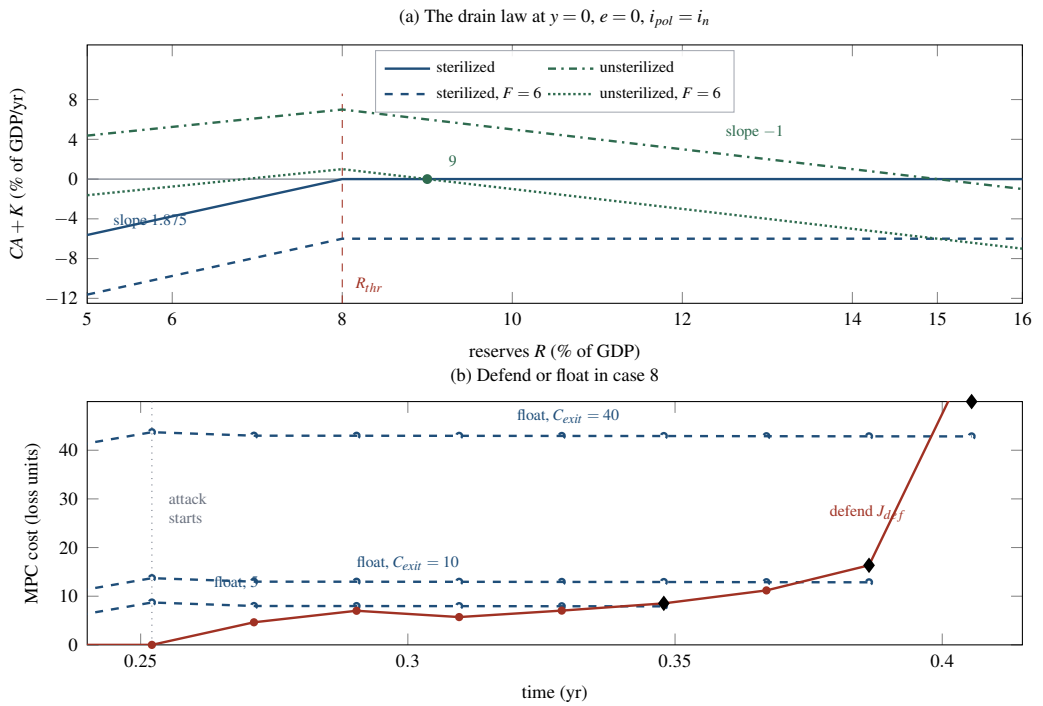


Figure 14.2 Two characteristics from the application's own functions and engine. (a) The drain law, the private inflow to the vault against the reserves held, at $y = 0$ with the float at parity and the policy rate at neutral, sterilized (solid with no outflow, dashed with 6 % of GDP a year leaving) and not (dash-dotted with no outflow, dotted with the outflow). (b) The MPC's cost of its best defense (solid, filled circles) and its cost of floating (dashed, open circles) at each weekly decision of case 8, for costs of floating of 5, 10 (shipped) and 40, with the attack starting at 0.25 year. A diamond marks the decision at which each run floats, the last one off the scale at 2231.

14.4.3 The Date No Controller Moves

With a monetized deficit and no sterilization, domestic credit grows as $D(t) = D(0) + \mu t$ whatever the bank does with its vault or its rate, and with $\bar{H} = D(0) + R(0)$ the shadow rate of Equation (14.6) reaches the weak edge b at the date of Equation (14.10).

$$T_K = \frac{R(0) - R_f}{\mu} + \frac{b}{\zeta\mu} - \frac{\alpha\,(i^* + \delta + \zeta\mu - i_n)}{\mu} \tag{14.10}$$

In case 3, $\mu = 2$ points of GDP a year, the shadow rate starts at −8 % and rises at $\zeta\mu = 4\,\%$ a year, and $T_K = 5 + 0.25 - 3 = 2.25$ years. No intervention policy and no policy rate appears in it. The application's check runs the simulation at five settings and finds the attack within a day of the formula each time, at 7.501 against 7.50 years for $\mu = 1$, 2.252 against 2.25 for $\mu = 2$, 0.501 against 0.50 for $\mu = 3$, 0.252 against 0.25 from a vault of 11 at $\mu = 2$, and 0.877 against 0.875 from 20 at $\mu = 4$. The day is the integration step, since the attack falls on the first step at which the shadow rate has reached the edge.

A survey restates the linear first-generation model, in the tradition of the linear examples of [3], with the date $t_c = \theta R_0/\mu - \alpha$ in logarithms, where money is a weighted sum of credit and reserves, θ is the ratio of the weights, reserves are exhausted at zero and α is the semi-elasticity of money demand [9]. Equation (14.10) is that date written in levels of GDP, and it differs in four places. The reserves count down to a floor R_f instead of zero. The attack comes when the shadow rate reaches the

Table 14.3 The Bank's Settings, Defaults and Ranges. A Marks an Advanced Control

Symbol	Setting	Default	Range
Who decides and when			
—	controller in AUTO	PI desk	board, PI, rate, MPC
—	calendar	weekly	weekly, every day
q_X, Δ_X	desk quantum, largest change (A)	0.5, 20	0–5, 1–80
q_I, Δ_I	rate quantum, largest move (A)	0.25, 3	0–1, 0.25–10
s	sterilization	on	off, on
u_c	capital controls	off	off, on, override
—	tolerance, band (A)	on, ±1 %	on, off, 3 bands
The hand			
X^w, i^w	desk and rate requests	0, 5 %	−20–60, 0–25
PI desk			
e_{sp}	band setpoint	0	±1
K_c, τ_I	gain, integral time	3, 0.03 yr	0–5, 0–0.5
Interest-rate defense			
k_r, i_{cap}	rate per point of outflow, political cap	0.25, 12 %	0–1, 5–25
MPC, defend or float			
w_R, w_y	price of reserves spent, weight on the gap	1, 1	0–10, 0–5
C_{exit}	cost of floating	10	0–50
H_p, τ_{att}	horizon, expected fading of an outflow (A)	2, 0.25 yr	0.5–3, 0.05–2

Desk in % of GDP a year, rates in %, k_r in points per % of GDP a year.

weak edge of a band rather than a single parity, which adds $b/(\zeta\mu) = 0.25$ year. The semi-elasticity term is $\alpha\zeta$, the same flight of money divided by the credit growth, and it reduces to the textbook's form only while $i^* + \delta = i_n$, as it does in every shipped case. The fourth difference is in the path to the date. In the textbook model the fixed rate holds money demand constant, so reserves run down in proportion to the growth of credit and the attack takes just the money that demand no longer wants. Here capital is imperfectly mobile and the overnight rate clears the money market, so the vault falls more slowly than credit grows and the attack takes whatever stands above the floor. In case 3 the first-generation count of the vault, $R(0) - \mu t$, stands at 10.6 % at year 2.2, against 12.25 % in the vault under the shipped desk, and the attack takes 7.14 points where the count puts only 5.5 above the floor. The difference is the excess money, 1.65 points above $\bar{H}$ that the market holds at an overnight rate of 3.90 %. The date survives unchanged because it depends on credit alone. Sterilized sales are credit as well and bring the day forward. The standard model cannot represent that channel, because there a sterilized attack leaves a regime that either works indefinitely or collapses at inception [10], and this model keeps it because the final attack is never sterilized.

14.5 Two Handles, a Calendar and an Override

The bank has two handles, the desk's sales X inside the band and the policy rate i_{pol}, and one more switch, capital controls. The controller executes every day. At each execution it updates the bank's estimate of the gap, reads its books and runs the controls override. It then checks the calendar, and only an execution that counts as a decision consults the selected policy. Table 14.3 lists the settings.

14.5.1 Weekly Instants, Quanta and Caps

Both the calendar and the shaping are those of `core/cadence.js`, a module of the framework that the economics applications share. The same shaping governs the cash decisions of Chapter 8 and the policy rate of Volume 2, Chapter 11, and Section 1.4.5 sets it beside the rate limits of other chapters. The weekly calendar places an instant every $1/52$ year from $t = 0$, and a decision happens at the first daily execution that reaches an instant not yet acted on, so decisions fall seven or eight days apart. The continuous calendar makes every daily execution a decision. At a decision each handle's request

is shaped as shown in Equation (14.11).

$$\begin{aligned} \tilde{u} &= q\,\mathrm{round}(u^{w}/q), \qquad \Delta_q = q\max\left(1, \lfloor \Delta/q \rfloor\right) \\ u_k &= \mathrm{clip}_{[u_{min},u_{max}]}\left(u_{k-1} + \mathrm{sat}_{\Delta_q}(\tilde{u} - u_{k-1})\right) \end{aligned} \tag{14.11}$$

where for the desk $q_X = 0.5$, $\Delta_X = 20$ and $[-20, 60]$ points of GDP a year, and for the rate $q_I = 0.25$, $\Delta_I = 3$ and $[0, 25]$ percent. Halves round away from zero, the bounds snap inward to the grid before the clip, a request that is not a number changes nothing, and the political cap i_{cap} trims the rate defense's and the MPC's requests before the shaping, though never the hand's. Between decisions both handles hold. The application's check finds every decision on its grid, inside its cap and on its calendar instant under the PI, the rate defense and the MPC on both calendars. In AUTO each handle's slider follows its controller's request before the quantum and the caps, and a switch of mode or controller holds both handles at the first decision after it, so every controller transfers without a jump.

14.5.2 The Hand and the Currency Board

In MANUAL the hand holds both handles, and its two sliders are requests that the next decision shapes exactly as it shapes a controller's. A helper sets the desk to the outflow the books showed over the last interval. The currency board holds both handles mechanically. Its desk sells nothing, so the weirs do all the selling, its policy rate stays at i_n, and its sales are never sterilized, whatever the switch says, so its money falls one for one with its vault.

14.5.3 The PI Desk

The desk's PI is the velocity-form proportional-integral-derivative (PID) controller of Section 1.6.2, with no derivative term, reverse acting on the rate's position in the band and updated only at a decision, as shown in Equation (14.12).

$$\begin{aligned} \varepsilon_k &= e_k - e_{sp}, \qquad P_k = K_c \varepsilon_k \\ S_k &= S_{k-1} + P_k - P_{k-1}, \qquad \tilde{X}_k = S_k + I_{k-1} \\ X_k &= \mathrm{clip}_{[-20,60]}(\tilde{X}_k) \\ I_k &= I_{k-1} + \left[K_c \varepsilon_k/\tau_I + (X_k - \tilde{X}_k)/\tau_I\right]\Delta t_k \end{aligned} \tag{14.12}$$

where $K_c = 3$ points of GDP a year per percent of rate, $\tau_I = 0.03\,\mathrm{yr}$, Δt_k is the time since the previous decision, and the symbols map onto Equations (1.13) to (1.15) with $\tilde{X}_k = v_k$, no bias and a tracking time $T_t = \tau_I$. The request is the unclamped output $\tilde{X}_k$, which the calendar then shapes, and the integral never learns of the quantum or the cap. A weaker rate asks for more sales. When the PI does not hold the desk it tracks the applied sales, and $\tau_I = 0$ removes the integral.

14.5.4 The Interest-Rate Defense

The rate defense is a proportional controller on the outflow that the books show between decisions, as shown in Equation (14.13).

$$i_k^{w} = \min\left(i_n + k_r \max\left(0, \frac{R_{k-1} - R_k}{\Delta t_k}\right),\ i_{cap}\right) \tag{14.13}$$

where $k_r = 0.25$ points per point of GDP a year and $i_{cap} = 12\,\%$. It holds the policy rate while the hand keeps the desk. Its gain is limited by politics rather than by stability, because the 130 % a year of the large attack would need a rate above 90 % to stop it at the shipped mobility.

14.5.5 Defend or Float as Model Predictive Control

At each decision the MPC plans the policy rate over $H_p = 2$ years in monthly steps on the bank's own model of the defended peg, in which every flow is met from the vault and every coefficient is the

declared value of Table 14.2, as shown in Equation (14.14).

$$\begin{aligned}\dot{R} &= \kappa_c\Big(u + \tfrac{\bar{H}-H}{\alpha} - i^* - \delta - c_\delta u_c - \rho\,\theta\big(1-\tfrac{R}{R_{thr}}\big)\Big)\\ &\quad - m_y\hat{y} + CA_x + d\\ \dot{H} &= \mu + (1-s)\dot{R}\\ \tau_y\dot{\hat{y}} &= -\hat{y} - \sigma\Big(u + \tfrac{\bar{H}-H}{\alpha} - i_n\Big) + \phi\,CA_x\end{aligned} \tag{14.14}$$

where u is the policy rate, $CA_x = \eta e + d_T - c_T u_c$ is frozen at the decision, ρ is 1 in the regime below the threshold and 0 above it, and d is a preview of the outflow the bank cannot explain, its smoothed reading $\hat{d}$ fading as $\hat{d}\,\mathrm{e}^{-(k+1/2)T_s/\tau_{att}}$ with $\tau_{att} = 0.25\,\mathrm{yr}$. The model is linear in each regime. It is solved in the regime the vault is in now and solved once more in the other regime when more than half of the planned months cross the threshold. The model is discretized exactly for a zero-order hold over $T_s = 1/12$ year, with the rate held over a step added as a state so that the overnight rate is an output, and the framework's linear MPC of Section 1.6.3, `core/linmpc.js`, poses the quadratic program (QP) of Equation (14.15). It keeps the move suppression and priced slacks of Equation (1.17) and replaces the tracking term with costs on the gap and the rate.

$$\begin{aligned}\min_{u,\varsigma}\ & \textstyle\sum_{k=1}^{N} T_s\big[w_y\hat{y}_k^2 + w_i(i_k - i_n)^2\big]\\ & + \nu\textstyle\sum_{j=0}^{9}(u_j - u_{j-1})^2 + \sum_{l=1}^{4}\big(r_l\varsigma_l^2 + \lambda_l\varsigma_l\big)\\ \text{s.t. }\ & 0 \le u_j \le i_{cap}, \quad R_k \ge R(t) - \varsigma_1\\ & R_k \ge R_f + 0.25 - \varsigma_2, \quad \hat{y}_k \ge y_{tol} + 0.25 - \varsigma_3\\ & i_k \le i_{tol} - 0.25 + \varsigma_4, \quad \varsigma \ge 0\end{aligned} \tag{14.15}$$

where $N = 24$, the ten move blocks are four of one month, two of two, and then 3, 4, 6 and 3 months, u_{-1} is the rate in force, $w_y = 1$, $w_i = 0.1$, $\nu = 0.05$, $r_1 = 0.02 w_R + 10^{-3}$ and $\lambda_1 = w_R$ price the deepest drawdown of the vault, and $r_l = 10^4$ and $\lambda_l = 10^3$ make the floor and the government's tolerance stiff. One slack serves each band over the whole horizon, so ς_1 is the plan's deepest drawdown. The program has 14 variables and 120 rows, a ridge of $10^{-9}(1 + d_{max})$ keeps it strictly convex, and the dual active-set method of [18] in `core/qp.js` solves it. In case 8 the MPC plans 21 times, at the start and at every weekly decision to the float, and five of the plans after the start are solved a second time in the other regime. The 25 QPs after the start take 13.1 active-set iterations on average and 22 at most, and none fails.

The first move becomes the rate request, and the plan then prices both choices, as shown in Equation (14.16).

$$\begin{aligned}J_{def} &= \textstyle\sum_k T_s\big[w_y\hat{y}_k^2 + w_i(i_k - i_n)^2\big] + \nu\sum_k \Delta u_k^2\\ &\quad + w_R \max_k\big(R(t) - R_k\big)^+ + 10^4\bar{\varsigma}^2 + 10^3\bar{\varsigma}\\ J_{flt} &= C_{exit} + w_E J_e^2\\ &\quad + \textstyle\sum_{k=1}^{N} T_s w_y\big[\hat{y}_\infty + (\hat{y} - \hat{y}_\infty)\mathrm{e}^{-kT_s/\tau_y}\big]^2\end{aligned} \tag{14.16}$$

where $\bar{\varsigma}$ is the plan's largest shortfall against the floor and the tolerance, $J_e = \max(0, b + J - e)$ is the jump the market prices by Equation (14.7), $w_E = 0.05$, $C_{exit} = 10$, and $\hat{y}_\infty = \phi(CA_x + \eta J_e)$ is the gap the cheaper currency would support with the rate back at neutral. The MPC floats when $J_{flt} < J_{def}$, which is the second-generation comparison [5] made with a number on each side. A discrete choice priced by two costs is also the core of the integer pump schedules of Volume 9, Chapter 11. If a solve fails, the rate in force stays and the failure is counted.

14.5.6 The Capital-Controls Override

Controls are off, on, or an override that decides by itself. The override reads the outflow the bank cannot explain with its own model, the attack, a portfolio shift or contagion, as shown in Equation (14.17).

$$\begin{aligned} g &= \frac{R_k - R_{k-1}}{\Delta t} + m_y \hat{y} - CA_x \\ &\quad - \kappa_c \Big(i - i^* - \theta \big(1 - \tfrac{R}{R_{thr}}\big)^+ - \delta - c_\delta u_c \Big) \\ \hat{d} &\leftarrow \hat{d} + (g - \hat{d}) \min(1, \Delta t / \tau_f) \\ p &= -\hat{d} / \max(0.05,\, 1 - c_K u_c) \end{aligned} \tag{14.17}$$

where $\tau_f = 1/52$ year and p is the pressure behind the outflow, with what the controls stop added back. Controls come on when $p > 10$ points of GDP a year and lift once $p < 2$ has held for four weeks. The override runs at every daily execution, outside the calendar, as a fast protective layer under the weekly policy, and the same reading $\hat{d}$ is the MPC's preview.

14.5.7 The Estimate, and the Three Models

The bank's estimate $\hat{y}$ is the first line of Equation (14.5) run forward every day with the overnight rate in force. At each print it is corrected by $0.6\,(y_{\text{pub}} - \bar{\hat{y}})\,\mathrm{e}^{-a/\tau_y}$, where $\bar{\hat{y}}$ is the bank's own average for the quarter and a the time since the quarter's middle, a teaching gain rather than a Kalman design. Equations (14.1) to (14.8), with the weirs, the four endings and the scripted events, are the simulated economy. The MPC's internal model is Equation (14.14), which sends every flow straight to the vault without a float or weirs, drops the overnight rate's floor at zero, foresees no attack the bank has not yet seen, and reads the gap only through its estimate. The models it stands for are the crisis models of [2, 3, 5] and the analysis of the interest-rate defense in [6], whose coefficients are not copied here. The PI, the board and the rate defense use no model, and nothing in the application is fitted to data. The bank's estimate, its reading $\hat{d}$ and the MPC keep the declared values of Table 14.2, and so does the neutral rate that the board and the rate defense set. A parameter changed by the learner therefore changes the economy and not the bank's models. The plan starts from the reserves and the credit on the bank's balance sheet, and the reserve floor and the government's tolerance are read as set.

14.6 Nine Defenses, Measured

Table 14.4 lists the cases. Each begins at the defaults of Tables 14.2 and 14.3, loaded through the dropdown, alters only what its row names, and is run over its listed length on the default seed. A variant changes one more setting before the run starts, and the application gives the same run whether that setting is chosen before the first Run or carried through Reset, as the briefings direct, so it is in force from the first day. Section 1.6.4 describes the briefings, the scorecard and the exported file from which these values are read. Table 14.5 collects the outcomes.

14.6.1 The Clock and the Equilibrium

In the reference the flows balance because $i - i^* - \delta = 5 - 3 - 2 = 0$, and the vault holds 15 % of GDP exactly for two years. Raised by hand to 6 %, the policy rate draws capital in at 1.5 % of GDP a year, the float goes to the strong weir, which buys it, and the vault is at 16.38 % a year later, while the gap is –0.32 % after a quarter and –0.69 % after a year. Selling 5 % of GDP a year from the desk into the balanced market does nothing lasting, because the strong weir buys back 4.74 of the 5 points within the year.

In case 1 capital leaves at 6 % of GDP a year from month 1 and the bank sterilizes. With the desk at zero the weak weir sells all ten points. The vault is 12.64 % at year 0.5 and 9.75 % at year 1, when the books show 5.8 points a year leaving and the clock reads 9.8 months. It crosses the threshold at year 1.304 and three months of imports at 1.384, after which the outflow is 7.9 % of GDP a year at year 1.5 and 9.8 at 1.6, and the peg falls at the floor at year 1.666, eight months after the clock's

Table 14.4 Cases 1 to 9, Challenge C and Reference R

#	Change from the defaults, run length	Teaching point
1	MANUAL, outflow of 6 from month 1, 2 yr	the clock is optimistic
2	currency board, 10-point attack at 0.25 yr, 1.5 yr	automatic adjustment
3	$\mu = 2$, unsterilized, 3 yr	a date no one moves
4	PI desk, 10-point attack at 0.25 yr, 1.5 yr	sterilization
5	rate defense, $i_{cap} = 15$, $\Delta_I = 10$, 15-point attack, 1 yr	a gain capped by politics
6	i^* from 3 to 6 % at 0.25 yr, 2 yr	importing the anchor's policy
7	$R(0) = 11$, contagion at 0.25 yr, 2 yr	a basin of attraction
8	MPC, unsterilized, 10-point attack at 0.25 yr, 1 yr	the choice to float
9	controls on, 10-point attack at 0.25 yr, 1.5 yr	the third instrument
C	unsterilized, 10-point attack at 0.5 yr, 1.5 yr	three limits, two handles
R	exact prints, nothing happens, 1 yr	the equilibrium

Table 14.5 How Each Peg Ended, the Lowest Vault, the Highest Overnight Rate and the Worst Gap Over Each Case's Own Run

Case and configuration	End (yr)	R_{low}	i_{max}	y_{min}
1, weirs only, as loaded	floor 1.666	5.00	5.00	0.00
1, desk sells 6	floor 1.608	5.00	5.00	−0.04
1, unsterilized	held	10.37	8.09	−1.77
1, policy rate 7 %	held	10.54	7.00	−1.27
2, currency board	held	6.22	10.85	−2.47
3, PI desk, as loaded	attack 2.252	12.14	5.00	0.00
3, policy rate 8 % by hand	attack 2.252	15.00	8.00	−0.95
3, sterilized	attack 1.321	13.14	5.00	0.00
4, sterilized, as loaded	floor 0.501	4.97	5.00	0.00
4, unsterilized	held	6.12	10.92	−2.44
5, cap 15 %, as loaded	tolerance 0.274	12.26	15.00	−0.02
5, cap 12 %	floor 0.337	4.70	12.00	−0.54
5, controls on	held	11.44	11.75	−1.16
6, policy rate 5 %, as loaded	attack 1.589	8.99	5.00	0.00
6, policy rate 8 %	held	15.00	8.00	−2.06
6, unsterilized	held	11.50	7.33	−1.37
6, rate defense	held	9.66	6.00	−0.40
7, $R(0) = 11$, as loaded	floor 1.441	5.00	5.00	−0.01
7, $R(0) = 12$	held	8.19	5.00	−0.01
7, unsterilized	held	8.05	6.97	−0.85
8, MPC, as loaded	float 0.389	10.32	11.17	−0.54
8, $C_{exit} = 5$	float 0.351	11.50	10.83	−0.27
8, $C_{exit} = 40$	float 0.408	9.73	11.17	−0.67
8, $\tau_{att} = 1$ yr	float 0.293	13.56	11.96	−0.11
8, 5-point attack	held	10.96	7.77	−0.67
9, controls on, as loaded	held	11.20	5.00	−0.48
9, override	held	12.25	5.00	−0.27
9, controls off	floor 0.501	4.97	5.00	0.00
R, reference	held	15.00	5.00	0.00

R_{low} in % of GDP, rates and gap in %, on the true states, as the scorecard reads them. R_{low} and i_{max} include the day that ends a peg, at an attack before the speculators' take. The five-point row replaces the scripted attack, and the reference case reproduces it with the MPC, sterilization off, print noise 0.3 and the small attack fired at 0.25 yr.

reading. The currency then falls 3.76 points, the devaluation the market had been pricing. Selling 6 % a year from the desk from the start holds the float just strong of parity, −0.19 % on average from year 0.2 to 1.2, and brings the floor forward to year 1.608, because a stronger rate costs a little trade balance. The same sales requested when the float first reaches the weak weir, at year 0.096, leave it there, and the floor comes at 1.630. Unsterilized, the same outflow leaves the vault at 10.37 % after two years with the overnight rate at 8.09 % and a gap of −1.77 %, the drain law's balance on its way, and a policy rate of 7 % holds it at 10.54 %.

> **TRY IT IN THE APP** **1 · Sell Reserves by Hand and Read the Clock**
>
> Load the scenario and press Run without touching the desk, so the weak weir does all the selling. Watch the clock under the vault: a year in it reads 9.8 months, yet the vault reaches the floor at year 1.666, eight months later, because the outflow grows once the vault is below the threshold.

Pause. Try Exercise 14.1 before reading on.

14.6.2 A Board, a Sterilizing Desk and the Trilemma's Center

In case 2 speculators bet on a devaluation of 6.67 % within three months and move ten points of GDP out, and the currency board meets them with its weirs alone. The overnight rate rises by itself to 10.85 % at year 0.504, the end of the attack, with the vault at 6.22 %, 2.5 months of imports, and below the threshold from year 0.449. The high rate draws capital back, the strong weir starts to buy, and by year 1.5 the vault is 11.78 % with the rate at 7.15 %. The bill is output, a gap of −2.47 % at year 1.10, long after the attack, and the speculators lose 0.62 % of GDP, the carry they paid for a devaluation that did not come. The Hong Kong band gives nearly the same run, 6.19 % and 10.87 %.

Case 4 meets the same attack with the PI desk. Sterilized, the overnight rate stays at 5 %, the outflow keeps running at 40 % of GDP a year, and the vault reaches the floor at year 0.501, the last day of the attack, after which the currency falls 10.46 points. The sterilized sales have also driven the shadow rate from −20 % to 0.07 %. Unsterilized, the overnight rate climbs to 10.92 % at year 0.507, the vault bottoms at 6.12 % and recovers to 11.89 % by year 1.5, and the gap reaches −2.44 % at year 1.06. At year 0.45 the sterilized vault holds 7.10 % against 7.86 %, and the check finds the sterilized vault lower at every instant before it ends. A desk deciding every day at a gain of 10 keeps the float off the weir and loses the peg one day sooner, at year 0.499. Sterilized intervention keeps a fixed rate, free capital and an independent rate together, the trilemma's empty center, and the vault pays for it until it cannot.

Pause. Try Exercise 14.3 before reading on.

14.6.3 The Date No Controller Moves

In case 3 the bank lends the government 2 % of GDP a year and its desk does not sterilize. At year 2.2 the vault still holds 12.25 % of GDP and loses only 1.5 % a year, and the clock reads 58 months, as shown in Figure 14.3. The shadow rate is then 0.80 % against an edge of 1. At year 2.252 the shadow rate reaches the edge and the speculators take the 7.14 points above the floor in a day. Every controller loses the peg on that day, and only the size of the last attack differs. The board leaves 7.30 points for it, the rate defense 7.72, or 8.53 at a gain of 1, the MPC, which raises the policy rate to 8 %, 10.03, and the hand holding 8 % 11.44, at the cost of a gap of −0.95 %. A rate that defends harder fills the vault with reserves that the attack then takes. Under the PI desk, sterilizing brings the day forward to year 1.321, because sterilized sales are credit too, while the board, which never sterilizes, keeps the attack at 2.252. This is the application's honestly infeasible case, and its check sweeps the hand at two policy rates, the board, two PI desks, two rate defenses and the MPC and finds every one of them ending within a day of T_K.

Pause. Try Exercise 14.2 before reading on.

> **DID YOU KNOW?** **Sterilizing Brings the Attack Forward**
>
> From the simulator: in case 3 the PI desk without sterilization loses the peg to the attack at year 2.252, and so does a hand that holds the policy rate at 8 %, whose vault never falls below 15.00. The same desk sterilizing loses it at year 1.321, because sterilized sales add to domestic credit (Table 14.5).

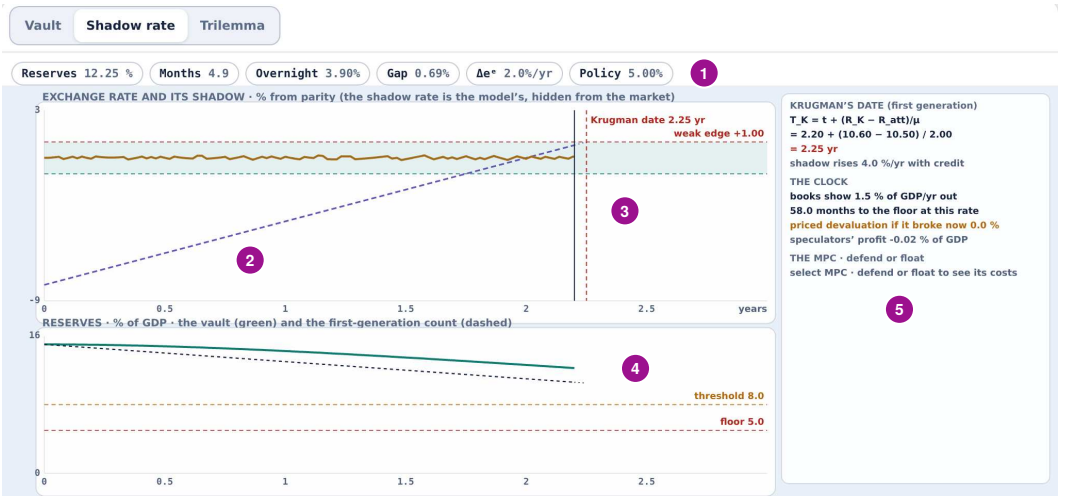


Figure 14.3 Case 3, a monetized deficit of 2 % of GDP a year, at $t = 2.20$ yr in the Shadow rate view, in AUTO as loaded under the PI desk, cropped to the view: the header, the control panel, the trends and the learning panel are cut. (1) The badges read reserves 12.25 %, 4.9 months of imports, an overnight rate of 3.90 % below the 5.00 % policy rate, a gap of 0.69 % and expected depreciation of 2.0 % a year. (2) The upper plot shows the exchange rate near parity and the hidden shadow rate (dashed) rising in a straight line from −8 % toward the weak edge at +1.00, and (3) Krugman's date is marked at 2.25 years. (4) The lower plot shows the vault against its dashed first-generation count. (5) The side panel computes $T_K = 2.20 + (10.60 - 10.50)/2.00 = 2.25$ years, says the shadow rate rises 4.0 % a year with credit, and gives the clock as 1.5 points of GDP a year out and 58.0 months to the floor. The attack comes 19 days later.

14.6.4 A Capped Rate and an Imported Tightening

In case 5 speculators bet on a 10 % devaluation within six weeks and move capital out at 130 % of GDP a year from year 0.25. The rate defense first acts at the weekly decision of year 0.271, a week into the attack, and asks for far more than its 15 % cap, which one decision of up to ten points reaches. The next day the overnight rate is past the government's 12 % and the peg ends with 12.26 % of GDP still in the vault, and the currency falls 10 %, the devaluation the speculators bet on. With the cap at 12 % the rate sits at the cap inside the tolerance, and seven points draw back only 10.5 % of GDP a year against 130, so the vault reaches the floor at year 0.337 and the currency falls 14.12 points. With controls on, the peg holds with the vault at 11.44 % at its lowest and a gap of −1.16 %. The scorecard counts the day that ends the first run, so its highest overnight rate reads the 15 % that broke the peg.

In case 6 the anchor raises its rate from 3 to 6 % at year 0.25 for reasons of its own. With its own rate at 5 % and sterilized sales, the bank loses 4.5 % of GDP a year, from 13.89 % at year 0.5 to 9.39 % at 1.5, and the sterilized sales carry the shadow rate to the band, where speculators take the last 3.99 points at year 1.589. Importing the tightening keeps the peg. At a policy rate of 8 % the flows balance, the vault rises to 17.08 % and the gap is −2.06 % after two years. Unsterilized at 5 %, the money market imports it for the bank, with the overnight rate drifting to 7.33 % and the gap to −1.37 %. The rate defense stops at 6 %, with a gap of −0.40 % and a vault still draining at 9.66 %.

14.6.5 Contagion and the Basin of a Sound Peg

Case 7 has no deficit and no overvaluation. Its vault starts at 11 % of GDP, and a crisis next door adds 5 points to the premium from year 0.25 to 0.75, which alone moves $\kappa \cdot 5 \cdot 0.5 = 3.75$ points of GDP out. The vault crosses the threshold at year 0.655 and holds 7.22 % when the contagion ends,

Table 14.6 The Challenge in Eighteen Configurations, as the Application Scores It

Configuration	Months	i_{max}	y_{min}	Controls	End
PI unsterilized, shipped	2.45	10.92	−2.44	0	held
PI sterilized	1.99	5.00	0.00	0	floor 0.751
policy rate 6 %	2.81	11.31	−2.59	0	held
policy rate 7 %	3.16	11.73	−2.82	0	held
policy rate 8 %	3.58	12.03	−1.81	0	tolerance 0.745
policy 10 %, sterilized	4.43	10.00	−3.00	0	tolerance 0.967
currency board	2.49	10.85	−2.47	0	held
rate defense	5.39	12.01	−0.14	0	tolerance 0.545
rate, $k_r = 0.5$, cap 10 %	4.77	12.05	−0.55	0	tolerance 0.597
MPC	4.13	11.17	−0.54	0	float 0.638
MPC sterilized	4.18	11.75	−0.66	0	float 0.638
MPC, $C_{exit} = 40$	3.89	11.17	−0.67	0	float 0.658
controls on throughout	4.79	7.02	−1.63	1.501	held
override, suggested	4.97	6.72	−0.88	0.384	**met**
override, sterilized	4.90	5.00	−0.27	0.386	**met**
override, MPC	4.95	5.25	−0.15	0.378	**met**
override, rate defense	5.10	8.48	−1.09	0.375	**met**
override, board	5.02	6.63	−0.88	0.378	**met**

Limits 3 months, 10 %, −3 %, 0.5 yr of controls. Lowest months of imports, rates and gap in %, controls in years, end in years. Each configuration is chosen before the run starts. The scorer counts the day that ends a peg.

and from there the drain law carries on alone, to 6.69 % at year 1.0, 5.89 % at 1.25 and the floor at 1.441, eight months after the cause has gone. A vault that starts at 12 % survives with 8.19 % at its lowest, one that starts at 11.75 dips to 7.87 %, just below the threshold, and keeps sliding, and one at 11.5 is at 5.25 % after two years. The basin's edge is therefore the threshold plus what the contagion drains, near 11.8 %. At 11 %, letting the money market tighten holds the peg at 8.05 % with a gap of −0.85 %, and a policy rate of 7 % holds it at 9.64 %.

14.6.6 Defend or Float

Case 8 repeats the ten-point attack, unsterilized, with the MPC deciding every week. It raises the policy rate to 8.5 %, and the drain takes the overnight rate to 11.17 %. Each week that the outflow keeps coming, the projected bill of defending grows, from 7.01 against 12.98 for floating at the decision of year 0.290 to 8.53 against 12.92 at 0.348 and 11.19 against 12.88 at 0.367, as shown in Figure 14.2(b). At the decision of year 0.386 defending would cost 16.35 and floating 12.86, and the peg ends by choice at year 0.389, seven weeks into the attack, with 10.32 % of GDP in the vault and a fall of 6.67 points, the devaluation priced. A cost of floating of 5 moves the date to year 0.351, and one of 40 defends one week longer and floats at 0.408, when no plan can avoid the floor or the tolerance and the cost of defending passes 2000. Expecting the outflow to last a year, $\tau_{att} = 1$, it floats two weeks in, at year 0.293. Against the five-point attack it defends throughout, and over attacks of 2 to 16 points from month 6 in the challenge's economy it defends 2, 4 and 6 and floats at years 0.715, 0.638, 0.581, 0.562 and 0.542 for 8, 10, 12, 14 and 16, earlier the larger the attack, as the application's check requires, and sterilizing leaves the date unchanged at year 0.389, with a policy rate that reaches 11.75 %.

14.6.7 The Third Instrument

In case 9 the ten-point attack meets the sterilizing desk with controls on, so only a fifth of it moves. Kept on, the controls hold the peg with the vault at 12.40 % at year 0.5, but they keep costing after the attack is over, and the vault slides to 11.20 % by year 1.5 with a gap of −0.48 %. The same controls in the reference case, with no attack, cost 1.8 points of reserves in 1.5 years and the same gap. With the switch on the override, the controls start lifted, come on at the execution of year 0.260, three days into the attack, and lift at 0.644, which holds the peg with the vault at 12.25 % at its lowest and a gap

of −0.27 %, controls in force for 0.384 year, while with controls off the vault reaches the floor at year 0.501.

> **TRY IT IN THE APP 9 · Capital Controls as an Override, With the Bill**
>
> Run the scenario on the Trilemma view with controls on and read the lowest reserves, 11.20 % of GDP. Set Capital controls to Override, press Reset and run again. The controls come on three days into the attack and lift at year 0.644, and the vault's lowest is 12.25 %.

The challenge fires the ten-point attack at month 6 against the PI desk unsterilized at a policy rate of 5 %, weekly. Throughout 1.5 years the vault must stay at or above three months of imports, 7.5 % of GDP, the overnight rate at or below 10 % and the gap at or above −3 %, with the peg standing and any controls in force for at most six months. The limit on the overnight rate is judged rather than on the policy rate, because output pays the overnight rate. Table 14.6 compares 18 configurations. As shipped, the vault falls below three months at year 0.712, eleven weeks into the attack, and bottoms at 6.12 %, 2.45 months, with the overnight rate at 10.92 %. No setting without controls passes. A higher policy rate trades the vault against the overnight rate, which passes 10 % from a policy rate of 6 % on and breaks the tolerance at 8 %. The rate defense and the MPC end the peg, sterilizing loses it at the floor, and a sterilized policy rate of 10 % holds 4.43 months until the gap reaches −3 % at year 0.967. Three limits and two handles leave no setting inside all three, the count of instruments against targets that Volume 6, Chapter 2 meets with two actuators and more variables worth holding.

The suggested override passes. It engages at the execution of year 0.510, three days into the attack, lifts at 0.893, and keeps the vault at 4.97 months at its lowest, the overnight rate at 6.72 % and the gap at −0.88 %, with controls in force 0.384 year (4.6 months). The override also rescues the sterilized desk, the MPC, the rate defense and the board, and Figure 14.4 draws every configuration against the three limits. Left on for the whole run, the controls hold the peg but cost 1.5 years of controls and a gap of −1.63 %. Over six seeds, the default and 1 to 5, the shipped run is identical to six decimals, because the seed reaches only the GDP prints and the desk's PI never reads them, and the suggested override passes on all six, with seed 5 alone lifting the controls a day earlier.

The override's speed is part of that verdict. It runs every day, outside the weekly calendar that binds the desk and the rate. In a counterfactual copy of the application's controller with the override checked only at the weekly decisions, it engages at year 0.521 and, because the weekly readings of the pressure at years 0.923 and 0.943 land above 2, it lifts only at 1.041, so controls stay 0.521 year and the suggested run fails the six-month limit. The same copy with a release level of 3 instead of 2 lifts at 0.904 and passes, with 4.84 months and controls for 0.384 year. A protective layer that acts faster than the policy it protects is how an override is normally built (Section 1.4.3), and the challenge's statement, the briefings and the Model dialog say that the override is checked every day, outside the calendar.

14.6.8 What the Calendar Changes

Deciding every day instead of every week moves the float more than the vault. The daily desk keeps the float off the weirs and changes the end of case 4 by one day. The MPC deciding every day floats at year 0.381 instead of 0.389, with 10.67 % of GDP in the vault, and in the challenge at 0.630 instead of 0.638. The rate defense deciding every day moves to its cap one day into the attack and breaks the tolerance by the end of that day, at year 0.258 instead of 0.274, while at the 12 % cap it reaches the floor on the same day, 0.337, on either calendar. A daily PI at a gain of 10 still fails the challenge with 2.42 months. The lag that matters here is not the calendar's but the vault's, which a week of decisions neither adds nor removes.

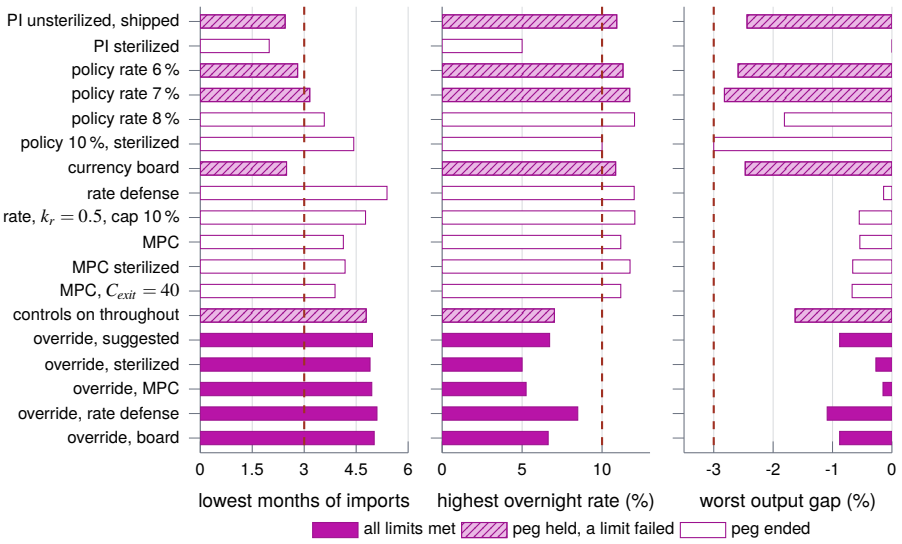


Figure 14.4 The challenge's three measures for the 18 configurations of Table 14.6, which also prints the values and the time in controls. Dashed lines are the limits: at least three months of imports, an overnight rate of at most 10 % and a gap of at least −3 %. Dark bars met every limit, hatched bars held the peg but failed a limit, and white bars lost the peg. The hatched bar of controls on throughout failed the limit on time in controls. Notice that every configuration without controls fails at least one limit, while each of the five with the override, which acts every day, clears all three.

14.7 Technology in Practice

A real central bank reads the same stocks and moves the same handles as the bank of Section 14.5, but its instruments are its books, the market's prices and its dealing room. Figure 14.5 draws the desk, the rate and the override with those instruments in place of the schematic's symbols.

What Measures It

The controlled variables are the exchange rate and the vault. Hong Kong's monetary authority publishes the details of its monetary base every day, and since March 1999 a monthly account that sets the backing assets beside the monetary base [19]. By a 2013 review, most central banks of emerging market economies had increased their market monitoring, particularly of cross-border portfolio flows [20]. The model's bank reads its reserves and its credit every day from its own books and the rate as a market price, and it sees output only through a late quarterly print, as Section 14.3 sets out.

> **TECHNOLOGY Sensor: Reserves on the Books and the Rate on the Screen**
>
> A central bank reads its reserves from its own books and the exchange rate from the market. Since Hong Kong's real-time gross settlement began in December 1996, interbank transactions settle across the books of its monetary authority, which publishes the monetary base every day and, in real time, the forecast change in the banks' aggregate balance caused by its foreign exchange transactions [19, 21]. Dealers post prices through brokers and electronic platforms, and some central banks survey dealers' open positions every day [22]. Readings can mislead: by June 1997 the Thai authorities reportedly had, beyond the reserves already spent, $23 billion in forward swaps outstanding, and the available reserves had fallen far below the short-term debts owed to international banks [12]. The model's bank reads R exactly every day, and its vault carries no forward commitments.

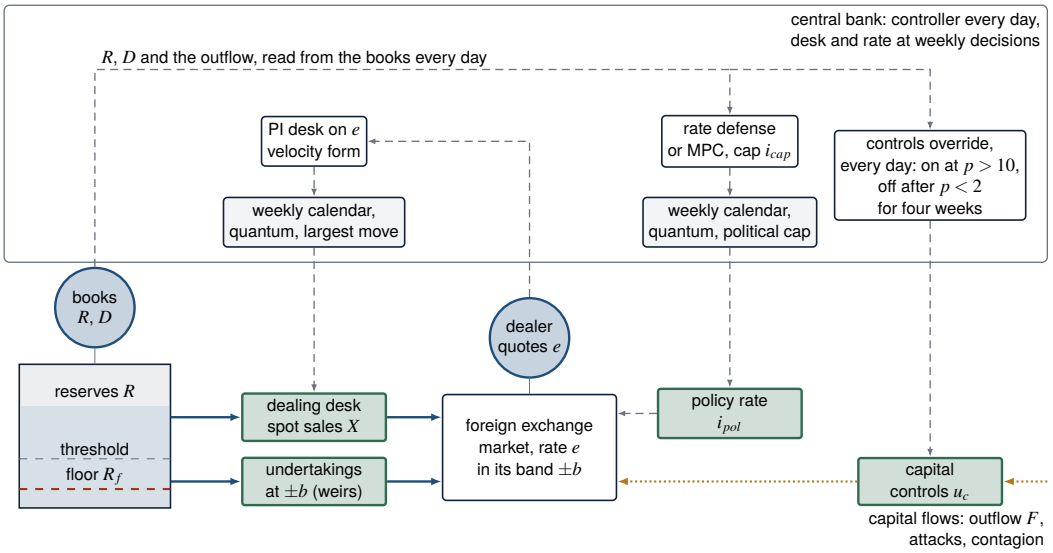


Figure 14.5 The desk, the rate and the override of Section 14.5 with a central bank's instruments. Green boxes are the final elements: spot sales from the dealing desk, the undertakings at the band edges, the policy rate and capital controls. Blue circles are the measurements: the bank's own books and the dealers' quotes. Solid arrows carry reserves, dashed arrows signals, and the dotted arrows the private capital flows that the controls slow. Notice that the desk and the rate wait for the weekly calendar, while the override and the undertakings act without it.

What Moves It

Beside the desk's spot sales, the interest rate is the second handle, and its cost lands on borrowers. Where mortgage payments are indexed to money market rates, a defense that holds those rates high for long periods can raise mortgage rates dramatically, and authorities that doubt a quick success may hesitate to use it [11]. Capital controls took a variety of forms, from taxes on holdings of foreign currency assets to restrictions on the ability of banks to lend abroad [11]. The model folds them into one switch that cuts capital mobility and the persistent outflow by c_K, and its political cap i_{cap} stands for the hesitation over rates.

> **TECHNOLOGY** **Final Element: Undertakings at the Edges and a Dealing Desk**
>
> Hong Kong's monetary authority undertakes to sell its currency to banks at 7.75 to the US dollar and to buy it at 7.85. At the weak side it buys, the banks' aggregate balance contracts and interest rates rise, an automatic adjustment that pushes the rate back inside the zone [23]. In a survey of emerging market central banks, most intervention was a spot transaction in the onshore wholesale market, mostly in normal business hours, when the market is deepest. It is dealt directly with market-makers, through voice or electronic brokers, on trading platforms or by auction [22]. In the model the undertakings are the weirs at $\pm b$, which act after every step, and the dealing desk is the desk X, which moves only at a decision.

Where the Controller Runs

In a survey of central banks, most intervene mainly in their own time zone and during normal business hours. A currency board is the exception: its intervention is passive and automatic, and Hong Kong supports its arrangement on a 24-hour basis through offices in other financial centers [22]. The application's controller executes every day. Its desk and rate move only at the decisions of the weekly calendar of `core/cadence.js`, while the weirs act after every integration step and the

capital-controls override runs at every daily execution, outside the calendar. That fast protective layer under a slower policy is the override structure of Section 1.4.3.

> **IN PRACTICE** **Reserve Adequacy by Three Ratios**
>
> Central banks judge a vault by ratios. The main indicator is reserves against short-term external debt, the debt maturing within one year. A country is typically considered prudent if its reserves cover that debt, a test known as the Greenspan–Guidotti rule. Reserves against imports give the months a country could go on importing if all other capital flows stopped, and a rule of thumb asks for three to four months. Reserves against broad money gauge how much capital residents could take out [24]. The application's safe vault is three months of imports, the challenge's limit, and its floor is two.

What Is Modernizing It

Three developments bear on this loop. The first is rules. Some central banks sell option contracts under well publicized auction rules, which transparently adds an element of automatic stabilization and gives the private sector a more complete set of hedging instruments [22]. The second is a wider set of handles. Under heavy pressure in 2013, several emerging market economies sold reserves, raised interest rates and provided the private sector with insurance against exchange rate risk, and targeted macroprudential and capital flow management measures are discussed as a further way to reduce their vulnerability [20]. The third is disclosure: a real-time forecast of the change in the banks' balances lets the market anticipate liquidity and adjust interbank rates more efficiently [19].

14.8 Limitations

The economy is a teaching model. Its expectations are mechanical and look backward, apart from the shadow rate's trigger, so the objection to evaluating policy with relations fitted under another policy [17] applies in full, and the multiple equilibria of self-fulfilling crises [5] appear only as the threshold of a fixed expectation term, not as a game among speculators. Capital flows on a level difference without portfolio stocks, one mobility serves all capital, money is one stock with a linear demand, and there are no banks, no foreign-currency debt and no float after the exit, only a rule. The first-generation date matches the linear textbook model only in its timing, as Section 14.4 sets out. Every coefficient, threshold, tolerance and attack size is a teaching value, and none of them describes a particular country.

> **OPEN CHALLENGE** **How Large a Vault Is Enough?**
>
> No single ratio settles the question: the most popular is not the only one, and their values depend on the database used [24]. Some central bankers note that the level of reserves perceived as adequate can vary considerably over time, particularly in crisis periods, and others that the financial costs of intervention can be relatively large as a percentage of GDP in emerging market economies [20]. The model fixes its threshold R_{thr}, where expectations turn, as a teaching value. A threshold read from the market as it moves, rather than declared in advance, would give the override a better limit.

The bank's estimate uses a fixed teaching gain, its reading of the unexplained outflow is smoothed over a week, and the MPC's model sends every flow straight to the vault and foresees no attack it has not yet seen, while its cost of floating is a declared number on which its date depends, from year 0.351 to 0.408 between costs of 5 and 40. With the economy's capital mobility doubled to 3 in case 8, the MPC still plans with 1.5, and the government's tolerance ends the peg at year 0.293. The override acts every day, outside the weekly calendar, and the suggested pass depends on that.

OPEN CHALLENGE Telling a Passing Attack From a Lasting Shift

The override of this chapter acts on the outflow that the bank cannot explain with its own model. Central banks find it difficult to know in advance why international investors move, or to identify the short-term component of capital flows. The effect of an intervention is hard to measure too, because nobody sees the exchange rate that would have prevailed without it. Senior central bankers meeting in 2013 reached no consensus on how effective intervention is [20]. An estimator that separated a passing attack from a lasting shift, and said how sure it was, would be progress.

14.9 Conclusion

This chapter wrote a currency peg as a band held by two weirs, a vault with a floor that pays every flow, a money market that tightens when sales go unsterilized, and an expectation term that turns the plant's feedback positive below a threshold. Its reserve clock reads 9.8 months a year into a steady outflow and the peg lasts eight, a currency board's overnight rate rises by itself to 10.85 % and holds the vault through a ten-point attack, and the same attack empties a sterilizing desk's vault on its last day. Without sterilization a monetized deficit sets the attack at year 2.252 against a derived 2.25 under every controller, which decide only how much of the vault the speculators take, from 7.14 to 11.44 points of GDP. A rate defense capped at 15 % ends the peg the day after it acts, and an MPC floats seven weeks into an attack with 10.32 % of GDP still in the vault. In the challenge twelve configurations without controls fail and a capital-controls override passes with 4.97 months of imports, an override that acts every day. The expectations are mechanical and every coefficient is a teaching value. A fuller model would add a market that learns the bank's rule, portfolio stocks and banks with foreign-currency debt, and a float after the exit. The vault makes the arithmetic of a peg, its date and its missing instrument something a student measures rather than accepts.

KEY POINTS

- A bank that fixes its exchange rate pays for every unmatched flow from a vault with a floor, and the vault drains faster once the market expects a devaluation.
- A year into a steady outflow the reserve clock reads 9.8 months, and the peg lasts eight.
- Without sterilization a monetized deficit sets the attack at year 2.252 against a derived 2.25 under every controller, which decide only how much of the vault the speculators take.
- In the challenge twelve configurations without controls fail, and a capital-controls override that acts every day, outside the weekly calendar, passes with 4.97 months of imports.

Where else this applies. Chapter 8 defends a cash floor with an override that gives way in a fixed order, and Volume 6, Chapter 2 meets the same count of two actuators against more variables worth holding.

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14.10 Exercises

14.1 ○ Concept check · 2 min

In case 1 capital leaves at 6 % of GDP a year and the bank sterilizes. A year in, the reserves above the floor divided by the outflow on the books give a clock of 9.8 months, yet the peg ends eight months later. Why is the clock optimistic?

(a) It divides by the outflow of the first month, which is smaller than the outflow a year later

(b) Below the threshold each point of reserves lost raises expected depreciation, so the outflow grows

(c) The weak weir sells at a worse rate than the desk would, so each point of outflow costs more

(d) The weekly calendar delays the sales, and the weirs then sell the backlog at the edge of the band

14.2 Concept check · 1 min

In case 3 a monetized deficit of 2 % of GDP a year meets an unsterilizing bank. The hand at a policy rate of 8 %, the currency board, the rate defense and the MPC all lose the peg at year 2.252. Why does none of them move the date?

(a) All of them hold the vault at the same level, so it reaches the floor on the same day

(b) None of them raises the policy rate early enough to draw capital back before the attack

(c) The date is set by the shadow rate, which follows domestic credit, and none of them changes credit

(d) The government's tolerance ends every defense at the same overnight rate of 12 %

14.3 Concept check · 2 min

In case 4 the PI desk meets the ten-point attack. Sterilized, the vault reaches the floor at year 0.501. Unsterilized, the same desk keeps the peg with the vault at 6.12 % at its lowest. What makes the difference?

(a) Unsterilized, the desk holds the float off the weak weir, so the weirs sell nothing

(b) Unsterilized, the speculators see the rate rise and bet a smaller devaluation

(c) Unsterilized sales add to domestic credit, which pushes the shadow rate away from the band

(d) Each point sold takes a point of money out, raises the overnight rate and draws capital back

14.4 Theory and calculation · 6 min

The first-generation date of Equation (14.10) uses $\zeta = 100/\bar{H} = 2$, $\alpha = 1.5$, $i^* = 3\,\%$, $\delta = 2$, $i_n = 5\,\%$, $R_f = 5\,\%$ and $b = 1\,\%$.

(a) For case 3, $R(0) = 15$ and $\mu = 2$ with $D(0) = \bar{H} - R(0)$, find the shadow rate at the start from Equation (14.6), its rate of rise and T_K. *Checkpoint:* $-8\,\%$, 4 % a year and $T_K = 2.25\,\text{yr}$.

(b) Find T_K for $\mu = 1$ from $R(0) = 15$, and for $\mu = 2$ from $R(0) = 11$. *Checkpoint:* 7.5 yr and 0.25 yr.

(c) At year 2.2 of case 3 the first-generation count $R(0) - \mu t$ gives 10.6 %, the vault holds 12.25 % and the overnight rate is 3.90 % under a policy rate of 5 %. Use Equation (14.3) to account for the difference.

14.5 Theory and calculation · 6 min

Use the drain law of Equation (14.9) with $\kappa = 1.5$, $\theta = 10$, $R_{thr} = 8$, $\alpha = 1.5$ and $R(0) = 15$.

(a) Sterilized with no outflow, find the slope of the law below the threshold and the time in which a deviation below the threshold doubles. *Checkpoint:* 1.875 points of GDP a year per point, 0.37 yr.

(b) Unsterilized, with a persistent outflow $F = 6$, find the reserves at which the law balances and the overnight rate there, with the policy rate at neutral. *Checkpoint:* 9 % of GDP and 9 %, 4 points above neutral.

(c) In case 1 (sterilized, $F = 6$ from month 1) the float first reaches the weak weir at year 0.096, and with the float on the weir the trade balance returns $\eta b = 0.2$ point of GDP a year. Estimate when the vault crosses the threshold. *Checkpoint:* 1.30 yr; the run gives 1.304.

14.6 Theory and calculation · 5 min

An attack j is a bet of $\beta_j = S_j/\kappa_0$ percent within d_j years, with $\kappa_0 = \kappa = 1.5$. It enters Δe^e as β_j/d_j.

(a) Find the capital flow of the ten-point attack ($S = 10$, $d = 1/4$) and of the fifteen-point attack ($S = 15$, $d = 6/52$). *Checkpoint:* 40 % and 130 % of GDP a year.

(b) With the vault above the threshold, find the overnight rate at which the fifteen-point attack draws no net capital out. *Checkpoint:* 91.7 %.

(c) In case 5 at a 12 % cap the rate sits 7 points above neutral. Find the inflow those 7 points draw against the fifteen-point attack. Then find the time the ten-point attack takes to carry the float across the two-point band at $\psi = 20$ with nothing sold or drawn against it. *Checkpoint:* 10.5 % of GDP a year, and 0.9 day.

14.7 App challenge · 8 min

Load scenario 1, Sell reserves by hand and read the clock, at the default seed. Leave the Intervention request at zero, so the weirs do all the selling, and press Run. The run pauses when the peg ends, about 1.7 yr in, which takes about seven seconds at the default speed of 0.25 yr a second. Export the CSV file.

(a) In the row at year 1.000, read the reserves `R` and the outflow on the books `outflow_read`, and compute the clock in months. *Checkpoint:* 9.75 %, 5.80 and 9.8 months (±0.1).

(b) Read the Peg tile. How many months did the peg last after year 1, and how optimistic was the clock? *Checkpoint:* Abandoned at 1.67 yr: 8.0 months, so 1.8 months optimistic.

(c) Find in the file the year at which `R` first falls below 8 %, and the `outflow_read` in the rows at years 1.5 and 1.6. Explain the growth with Equation (14.4). *Checkpoint:* Year 1.304, outflow 7.94 and 9.84 (±0.05).

14.8 App challenge · 9 min

Load scenario 3, Krugman: a monetized deficit and its date, at the default seed, on the Shadow rate view. The PI desk does not sterilize. Each run lasts up to 3 yr and pauses at the attack, about ten seconds at the default speed of 0.25 yr a second.

(a) Press Run and read the Peg tile, the Lowest reserves tile and Krugman's date on the Shadow rate view. How many points of GDP does the attack take? *Checkpoint:* Abandoned at 2.25 yr, lowest reserves 12.14 %, Krugman's date 2.25 yr, so 7.14 points.

(b) Set Controller to Currency board, press Reset and run again. Read the same tiles. *Checkpoint:* Abandoned at 2.25 yr, lowest reserves 12.30 %, so 7.30 points.

(c) Press the reload button, set Sterilization to On and run. Read the Peg tile and explain why the date moves now. *Checkpoint:* Abandoned at 1.32 yr.

14.9 App challenge · 9 min

Load the challenge, Survive the attack, at the default seed. The ten-point attack starts at year 0.5 and the run lasts 1.5 yr, six seconds at the default speed of 0.25 yr a second. Imports are 30 % of GDP a year, so a month of imports is 2.5 % of GDP. After each run, export the CSV file.

(a) Run the challenge as loaded. Read Lowest reserves, Highest overnight rate, Worst output gap and the Challenge tile, and convert the lowest reserves to months of imports. *Checkpoint:* 6.12 %, 2.45 months; 10.92 %; −2.44 %; not met.

(b) Set Capital controls to On, press Reset and run. Read the same tiles, and count the rows of the `controls` column that read 1. *Checkpoint:* 11.98 %, 4.79 months; 7.02 %; −1.63 %; 549 rows, 1.5 yr; not met.

(c) Set Capital controls to Override, press Reset and run. Read the same tiles and the time the controls were in force. *Checkpoint:* 12.41 %, 4.96 months (±0.02); 6.72 %; −0.88 %; 140 rows, 0.384 yr; met.

14.10 Application area · 3 min

Central banks often judge a vault in months of imports.

(a) With imports of 30 % of GDP a year, express the starting vault of 15 %, the threshold of 8 %, the floor of 5 % and case 7's starting vault of 11 % in months. *Checkpoint:* 6, 3.2, 2 and 4.4 months.

(b) Case 7 starts at 4.4 months, above the common three-month rule, and a six-month contagion still ends the peg. What should a reserve target be set against, in the language of Section 1.4.9?

14.11 Application area · 4 min

Capital controls in this model cut capital mobility and the persistent outflow by 80 %, at a trade cost of 1 point of GDP a year and a premium of 1 point.

(a) In the reference case, with no attack, controls kept on for 1.5 years cost 1.8 points of reserves and a gap of −0.48 %. Name the two channels of that bill in Equations (14.4) and (14.5).

(b) In the challenge the override checks the outflow every day. A copy that checks it only at the weekly decisions keeps controls 0.521 year and fails. Why should a protective override run faster than the policy it protects?

14.12 Review the agent · 5 min

An AI assistant reviews case 5, where the rate defense capped at 15 % ends the peg at year 0.274, and writes: "The rate defense failed because its gain $k_r = 0.25$ is too timid. Raise k_r to 1 and the political cap to 25 %, so the rate can climb high enough to stop the outflow while the vault still holds 12 % of GDP." Find the flaw, and say how you would check it in the app.