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CHAPTER 2

Teaching Limit Cycles and Protective Override with a Compressor Surge Simulator

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Edited by John D. Hedengren

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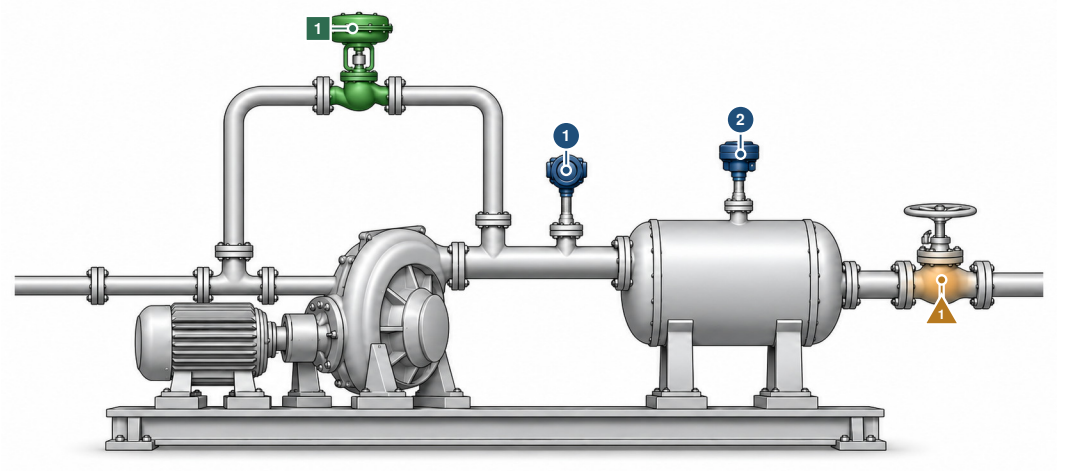
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CHAPTER 2

Teaching Limit Cycles and Protective Override with a Compressor Surge Simulator



The controller moves 1 valve that sends gas back to the inlet

The controller reads 1 flow from the compressor 2 pressure in the tank, for the surge alarm

What disturbs it 1 throttle closed by the users

Notice that the only path the controller can open is the overhead line back to the compressor inlet, while the users' throttle on the right is not its to move. Illustration, not to scale.

IN PLAIN TERMS Keeping a Compressor Out of Surge

The main idea. A compressor pushes gas into a tank and on to the users. If the users take too little gas, the flow through the machine can collapse and run backward, again and again, in a violent cycle called surge.

The challenge. A protective controller opens a valve that sends gas back to the compressor inlet, but only when the flow nears the danger line. If that valve is too slow or too small, no setting of the controller can save the machine.

Why it matters. One surge can damage the bearings and blades, yet gas sent back around the machine wastes the power used to compress it.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the fact on what protection costs (Section 2.1), the two history segments (Section 2.2), the two try-it segments and the fact on valve speed (Section 2.7) and the key points
Study	60 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	20 min	the first page, the results (Section 2.7), Technology in Practice (Section 2.8), Limitations (Section 2.9) and the two open challenges

The control question A centrifugal compressor throttled past the peak of its characteristic does not diverge but settles into a deep-surge limit cycle that reverses the flow through the rotor. How does a protective override whose valve rests shut keep the machine off that orbit, and when do stroke speed, valve capacity or noise defeat it whatever the tuning?

Learning objectives

- Derive the trace and determinant conditions and locate the static and dynamic surge boundaries.
- Tell a limit cycle from a divergence, and an aerodynamic surge cycle from a control-induced one, by experiment.
- Explain why the integral of an override resting on its clamp decays instead of winding up.
- Judge a protective loop by surge margin and recycled flow, not by IAE.
- Decide whether a failed closure is a tuning, a stroke-speed or a capacity problem, and what a throttle signal adds.

Prerequisites Sections 1.4.3 (override), 1.4.4 (external reset), 1.4.5 (saturation and rate limits), 1.6.2 (velocity form) and 1.6.4 (scenarios and exported files). Section 1.6.3 is needed only for the predictive controller.

The app https://apmonitor.com/apps/compressor-surge/

See also Volume 7, Chapter 2 for a Hopf bifurcation and a loop left cycling, and Volume 7, Chapter 4 for a setpoint at the peak of a characteristic. Volume 10, Chapter 11 for override anti-surge loops on parallel machines that share one load. In this volume, Chapter 3 for a compressor protected without a measured surge margin.

2.1 Introduction

Compressor surge is a control problem in which the regulated steady state itself disappears. As the downstream valve closes, the operating point moves toward lower flow along the machine's pressure-rise curve, and near the peak of that curve the steady state the loop is regulating ceases to be attracting. What replaces it is not a new steady state and not an exponential divergence. It is a closed orbit. Flow collapses, the discharge volume blows down backward through a rotor still turning at full speed, flow re-establishes, the volume refills, and the sequence repeats at a period set by the acoustics of the pipework. In the simulator of this chapter that period is 7.5 nondimensional time units and the flow swings from -0.175 to $+0.711$, and the oscillation persists indefinitely because the amplitude is a property of the system rather than of the disturbance that started it.

The mechanism follows from one momentum balance. The compressor accelerates the gas in its duct according to the difference between the pressure rise it can produce at the flow currently passing and the pressure already stored downstream. Where the characteristic slopes down, a dip in flow moves the machine to a part of the curve that delivers *more* pressure rise than the plenum holds, and the flow is pushed back up. Where the characteristic slopes up, the same dip delivers *less*, and the flow falls further. The sign of one derivative determines whether the aerodynamics contribute negative feedback or positive feedback to the compression system. Volume 7, Chapter 4 meets the same reversal of the process gain at the peak of a tire's friction curve.

The consequence for control is unusual. The correct response to an approaching surge line is to open a recycle valve, and the correct response at every other time is to leave that valve shut, because recycled gas is compressor power converted directly into heat. An anti-surge controller therefore spends almost its entire service life at the closed end of its output range and is judged on an event that may occur once a year. That is an override of the kind a process control text treats alongside selectors and constraint loops [1] (Section 1.4.3), not a regulator, and it changes what the usual performance measures mean. Integral of absolute error accumulates distance to a setpoint the loop has no intention of reaching. The fraction of time spent saturated is meaningless at one end of travel and critical at the other. The numbers that matter are the smallest surge margin reached and the quantity of gas recycled.

The model behind the application is built from two conservation statements, a momentum balance on the gas in the duct and a mass balance on the plenum, closed by a static compressor characteristic and two valve laws [2]. The experiments built on it address process dynamics and control principles that a course built on linear plants reaches late or not at all: nonlinear stability and limit cycles, protective override structure, integral anti-windup, actuator rate and capacity limits, and the effect of measurement noise and filtering on a loop with no lag to spare [1, 3]. The behavior matters industrially because a single deep-surge event can damage bearings and blading, while a conservative surge control line recycles gas continuously, so both the protection and its operating cost are plant-level concerns [4].

> **DID YOU KNOW? Protection Paid in Megawatts**
>
> An instrumentation textbook calls surge a violent and potentially self-destructing action of a centrifugal compressor [5]. Avoiding it has a price. On the refrigerant compressor of Volume 10, Chapter 14, turned down to 20 kg/s, the margin costs 10.6 % recycle valve opening and 1.8 MW of shaft power.

This chapter gives every equation, parameter and measured number of the simulated compression system, so that its four states, its characteristic, its two valves and its override can be reimplemented from the text, and it separates three effects that one surge event presents together. The analysis yields a pair of stability boundaries computed from the implemented equations, a geometric one at the peak of the characteristic and a dynamic one two percentage points of throttle travel inside it. At the default parameters the classical static criterion never fails at any throttle position, so the Greitzer parameter B decides the outcome by itself. The model has stroke-limited valves, a windowed surge detector and twelve scenarios. The override's lower clamp and back-calculation anti-windup are given as implemented rather than as intended, and the measured advantage of the predictive controller is divided between its prediction and the throttle signal it reads. The result is a setting where a limit cycle can be measured instead of described, and where two oscillations that look alike on a trend can be distinguished by experiment.

Section 2.2 traces how surge protection reached its present form. Section 2.3 sets out the compression system, its instruments and its assumptions, and Section 2.4 derives the two boundaries and the role of B. Section 2.5 gives the override as implemented and, in Section 2.5.1, the predictive controller. Section 2.6 explains which performance measures mean anything on a protective loop. Section 2.7 reports the twelve scenarios: the staged closure, the deep-surge limit cycle, valve stroke speed and capacity, a control-induced oscillation, measurement noise, the predictive controller and the challenge. Section 2.8 sets the loop on plant hardware, Sections 2.9 and 2.10 state the limitations and conclude, and Section 2.11 holds the exercises.

2.2 Background

Surge was documented as mechanical damage long before it was described as a system dynamic. Industrial blowers and early aircraft engine compressors were reported to hammer, to reverse, and to shed blading when the discharge was restricted, and the practical remedy adopted throughout the first half of the twentieth century was a blow-off valve set by experience to keep the machine away from the region where the damage started. The phenomenon acquired a physical description once compressor instabilities were separated into two distinct objects. A circumferential cell of stalled passages that travels around the annulus at a fraction of rotor speed leaves the annulus-averaged flow steady. A one-dimensional oscillation of the whole compression system moves the annulus-averaged flow and depends on what is bolted downstream. The first is rotating stall and the second is surge.

The decisive step for control was the recognition that the second object is a property of the system rather than of the machine, as the history segment below tells. A smooth cubic representation of the compressor characteristic, valid on both sides of the peak and continued into reverse flow, arrived

with the post-stall transient theory of Moore and Greitzer [6], and it is that cubic which the present application uses.

> ⧖ HISTORY **The Installation Decides Between Stall and Surge, 1976**
>
> A two-part study published in 1976 posed a nonlinear lumped model of a compression system to predict its transient response after a disturbance. It found one nondimensional parameter on which that response depends. Above a critical value the system falls into the large oscillation of surge, and below it the machine moves toward rotating stall at a much reduced flow and pressure ratio [7]. The second part tested the model on a three-stage axial compressor, computed the instantaneous flow from a mass balance on the plenum, and found the predicted criterion in good accord with the data [8]. The 1980 Freeman Scholar Lecture then reviewed the instabilities of pumping systems of many kinds and found the idea of negative damping common to them [9]. That parameter is the B of this chapter.

Industrial practice and research followed separate paths. Plant protection settled on a recycle line around the machine, a fast valve in it, and a control line drawn a fixed margin to the right of the measured surge limit line in a corrected-flow against corrected-head plane, with the valve opened by a controller that acts only when the operating point crosses that line. A technology assessment of centrifugal compressor surge control drew together the sensing options, the valve requirements and the field experience of that arrangement [10], and the practical consequences for gas transmission service, where a station may be tripped or recycled many times a day, are set out with the associated economics [4]. Volume 10, Chapter 11 places three such machines in parallel on one pair of headers, each with its own override anti-surge loop. Research instead asked whether feedback could move the surge line rather than merely respecting it, the line of work the second history segment follows. Active stabilization has not displaced the recycle valve in production plants, so the loop found in industrial service is still the override, and it is the override that this application implements.

> ⧖ HISTORY **Feedback Aimed at the Surge Line Itself, 1989 to 1999**
>
> A strategy proposed in 1989 drove an extra disturbance in a turbomachine from data measured inside it in real time, so that the controlled machine behaved differently from the bare one. For the compressor analyzed, the control widened the stable operating range by 20 percent of the mean flow, at a control power that can be several orders of magnitude below the machine's [11]. In 1993 a low-speed axial research compressor delayed rotating stall with a ring of hot wires and individually driven inlet guide vanes, and lowered its stalling flow by 23 percent [12]. A study of sensor and actuator choices for surge found the actuators most closely coupled to the compressor the most effective [13], and in 1999 a close-coupled valve combined with speed control was given a proof of stability [14].

2.3 Compression System Model

The application simulates a compressor discharging through a short duct into a plenum, with a throttle taking gas out of the plenum to the process and a recycle line returning gas from the plenum to the compressor inlet through the anti-surge valve. The layout and the symbol for each quantity in the equations below are given in Figure 2.1. Nothing in the model is dimensional. Flow and pressure rise are the usual compressor coefficients and time is scaled by the Helmholtz period of the duct and plenum pair, so the trends and the exported time column both read $\hat{t}$.

2.3.1 Variable Classification

The one controlled variable, the manipulated variable, the disturbances, the four states and the derived outputs worth watching are collected in Table 2.1. The throttle demand requires one clarification. It is an operator-set input that stays active in both modes, but it is not a controlled variable and it is not the controller's actuator. It is the load, and it is the disturbance that the manual experiments of Section 2.7 drive directly.

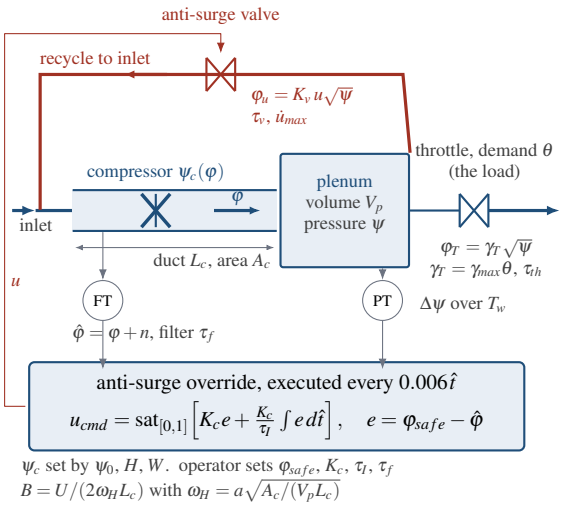


Figure 2.1 Compressor, duct, plenum, throttle and the anti-surge recycle line. Flow is measured on the duct and is the only signal the override uses. Plenum pressure feeds the surge detector and never reaches the controller. The predictive controller of Section 2.5.1 also reads the throttle demand θ.

Table 2.1 Classification of Every Signal in the Model

Symbol	Role	Range	Unit
φ	CV, state	−0.2 to 0.80	—
$\hat{\varphi}$	measured CV	$\varphi + n$	—
φ_{safe}	setpoint	0.500–0.720 (0.575)	—
u_{cmd}	MV, controller output	0–100	%
u	state, valve position	0–1	—
ψ	state, plenum pressure rise	0–0.72	—
γ_T	state, throttle conductance	0–1.40	—
θ	DV, throttle demand	0–100 (70)	%
σ_n	DV, flow transmitter 1σ	0–0.040 (0)	—
$\Delta\theta_b$	DV, restriction drift	−25 to +5 (0)	%
$\psi_c(\varphi)$	output, characteristic	—	—
ψ_c'	output, slope	−3.5 to +1.08	—
φ_T	output, throttle flow	—	—
φ_u	output, recycled flow	—	—
SM	output, surge margin	—	%
$\Delta\psi$	output, detector swing	—	—

2.3.2 Aerodynamic States

The gas in the duct between the compressor and the plenum is treated as a slug of fixed inertia, and Newton's law applied to that slug gives the first state equation, as shown in Equation (2.1).

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\hat{t}} = B\left[\psi_c(\varphi) - \psi\right] \tag{2.1}$$

where φ is the nondimensional flow through the compressor, ψ is the nondimensional pressure rise held in the plenum, $\psi_c(\varphi)$ is the pressure rise the compressor produces at the flow currently passing, and B is the Greitzer parameter. The right-hand side is a difference of two pressures and the equation is a force balance, so the acceleration of the flow is zero whenever the machine and the plenum agree, whatever value they agree on. Nothing in Equation (2.1) restricts φ to positive values, and

the reverse-flow excursion reported later is a solution of this equation rather than an artifact of the numerics.

Conservation of mass for the plenum gives the second state equation, as shown in Equation (2.2).

$$\frac{\mathrm{d}\psi}{\mathrm{d}\hat{t}} = \frac{1}{B}\left[\varphi - \varphi_T(\psi) - \varphi_u(\psi)\right] \tag{2.2}$$

where φ_T is the flow the throttle passes to the process and φ_u is the flow the anti-surge valve recycles to the inlet. The plenum is a capacity and Equation (2.2) is an inventory balance, so the pressure rises when the compressor delivers more than the two valves remove. Opening the recycle valve therefore acts on both states at once. It lowers ψ, which raises the imbalance driving Equation (2.1), and it adds a second outlet, which raises the flow the compressor has to pass.

The compressor characteristic is the Moore-Greitzer cubic, given by Equation (2.3).

$$\psi_c(\varphi) = \psi_0 + H\left[1 + \tfrac{3}{2}\left(\frac{\varphi}{W} - 1\right) - \tfrac{1}{2}\left(\frac{\varphi}{W} - 1\right)^3\right] \tag{2.3}$$

where ψ_0 is the shut-off pressure rise, H is the semi-height of the curve and W is its semi-width. Differentiating gives $\psi_c' = (3H/2W)\left[1 - (\varphi/W - 1)^2\right]$, which vanishes at $\varphi = 0$ and at $\varphi = 2W$, is positive between them and negative beyond. The peak of the curve is therefore at exactly $\varphi = 2W = 0.500$ with $\psi = \psi_0 + 2H = 0.560$, and that peak is the conventional surge line. The cubic is used rather than a fitted rig curve because it is smooth, it is bounded, and its continuation to $\varphi < 0$ turns upward again, which is the feature that closes the deep-surge orbit instead of letting the solution run away.

Both valves are square-root devices with no downstream pressure, as shown in Equation (2.4).

$$\varphi_T = \gamma_T\sqrt{\max(\psi, 0)}, \qquad \varphi_u = K_v\, u\sqrt{\max(\psi, 0)} \tag{2.4}$$

where γ_T is the conductance of the throttle actually in place, K_v is the capacity of the anti-surge valve wide open and u is its position on 0 to 1. The max keeps the square root real and prevents either valve from passing flow backward during the part of the surge cycle in which the plenum pressure falls below zero. The slope of the throttle line in the pressure-against-flow plane follows from the same expression as $\psi_T' = 2\sqrt{\overline{\psi}}/\gamma_T$, and that slope appears in the stability conditions of Section 2.4.

2.3.3 Actuator States

Neither valve moves instantly. Both are first order on their commands, as shown in Equation (2.5).

$$\tau_v\frac{\mathrm{d}u}{\mathrm{d}\hat{t}} = u_{app} - u, \qquad \tau_{th}\frac{\mathrm{d}\gamma_T}{\mathrm{d}\hat{t}} = \gamma_{cmd} - \gamma_T \tag{2.5}$$

where u_{app} is the controller output after rate limiting, τ_v is the anti-surge valve time constant, $\gamma_{cmd} = \gamma_{max}\,\mathrm{clamp}(\theta + \Delta\theta_b/100, 0, 1)$ is the commanded throttle conductance including the unmeasured restriction drift, and τ_{th} is the throttle stroke time constant. After each integration step u is clamped to $[0, 1]$ and γ_T is floored at zero. Rate limiting is separate from τ_v and is described in Section 2.5, because it is applied inside the controller as well as on the applied position.

2.3.4 Instrumentation and Surge Detection

Flow is the only measurement the controller receives, and Equation (2.6) is what reaches it.

$$\hat{\varphi} = \mathscr{F}_{\tau_f}\left[\varphi + \sigma_n\,\xi(\hat{t})\right] \tag{2.6}$$

where ξ is unit-variance noise passed through a first-order coloring filter of time constant $0.02\,\hat{t}$, σ_n is the operator-set transmitter standard deviation, and $\mathscr{F}_{\tau_f}$ is an optional first-order measurement filter that is switched off by default. There is no transmitter bias and no transport delay. The noise sequence is generated by a seeded mulberry32 stream, so a scenario with noise replays identically.

Table 2.2 Parameter Defaults and Adjustable Ranges

Symbol	Meaning	Value	Range	Unit
B	Greitzer parameter	1.50	0.30–3.00	—
H	characteristic semi-height	0.180	0.080–0.300	—
W	characteristic semi-width	0.250	0.150–0.400	—
ψ_0	shut-off pressure rise	0.20	0.00–0.40	—
γ_{max}	throttle wide open	1.40	0.80–2.20	—
τ_{th}	throttle stroke constant	0.50	0.05–4.00	$\hat{t}$
K_v	anti-surge valve capacity	0.60	0.05–1.40	—
τ_v	anti-surge valve constant	0.30	0.02–3.00	$\hat{t}$
$\dot{u}_{max}$	valve stroking rate limit	4.00	0.10–20.0	$1/\hat{t}$
Fixed detector settings				
T_w	detector window	8.0	2–20	$\hat{t}$
$\Delta\psi_{on}$	surge alarm threshold	0.25	0.05–0.60	—
$\Delta\psi_{off}$	alarm clear threshold	0.12	0.02–0.40	—
Derived from the parameters above				
$2W$	surge line flow	0.500	—	—
$\psi_0 + 2H$	pressure rise at the peak	0.560	—	—
φ_z	flow at zero pressure rise	0.8037	—	—

Plenum pressure is instrumented separately and never reaches the controller. It feeds a detector that holds the last $T_w = 8.0\ \hat{t}$ of ψ in a ring buffer sampled on a fixed $0.02\ \hat{t}$ grid and reports the peak-to-peak swing $\Delta\psi$ over that window. The surge alarm latches when $\Delta\psi$ exceeds $\Delta\psi_{on} = 0.25$ or when φ goes negative, and clears when $\Delta\psi$ falls below $\Delta\psi_{off} = 0.12$ with the flow positive. The window is deliberately about one surge period long so that the single transient following an ordinary throttle move does not trip it.

2.3.5 Parameter Values, Start-Up State and Numerics

Every parameter, its default and its adjustable range are given in Table 2.2. The three detector settings are excluded from the adjustable set because they are instrument configuration rather than controller tuning.

Every run begins at an exact steady state. The initializer takes the total downstream conductance $\gamma = \gamma_T + K_v u_0$, with u_0 the operator's valve position in MANUAL and zero in AUTO, and solves $\varphi = \gamma\sqrt{\psi_c(\varphi)}$ by bisection. The bisection interval is split at $\varphi = 2W$ and the branch is chosen by the sign of the residual at the peak, so a requested operating point on the positive-slope side is placed there deliberately rather than by accident. The states are then $[\varphi^*,\ \psi_c(\varphi^*),\ u_0,\ \gamma_T]$, which makes every derivative zero to within 1×10^{-9}, so any response after the first event is attributable to the model rather than to an initialization transient.

The engine and its three clocks are described in Section 1.6.1. Here the states advance by fourth-order Runge-Kutta at the fixed step $\Delta\hat{t} = 0.002$, and the controller executes every $0.006\ \hat{t}$, three physics steps, after a priming execution at $\hat{t} = 0$. The value 0.006 is also the Δt handed to the integral term and the rate limiter. The throttle demand reaches the model through the controller as well, so a throttle move takes effect at the next execution (a move scheduled at $\hat{t} = 8$ reaches the throttle actuator at $\hat{t} = 8.004$). The historian records 50 samples per unit of $\hat{t}$, and the exported file puts those samples in comma-separated rows under a time column named `time_hat`. The default seed is 20260824 and the application starts in AUTO, so every run reported below is reproducible.

2.3.6 Assumptions

The model is a lumped two-state representation of the aerodynamics with two actuator states attached, and each simplification has a regime in which it fails.

One flow and one pressure. The compressor is one duct with one inertia, so a circumferential disturbance pattern cannot be represented at all and rotating stall is absent. Real machines frequently enter rotating stall first and surge afterwards, and a compressor that recovers into a stalled branch rather than into surge exhibits behavior this model cannot represent.

Quasi-steady characteristic. The machine is assumed to deliver its steady-state pressure rise instantly at whatever flow is passing, including during a reversal. The reverse-flow branch of a real characteristic is measured with difficulty on a rig and is only approximately the continuation of the cubic, so the deep-surge orbit reported here has the correct qualitative shape, but its exact amplitude is a property of the cubic.

One speed line. Shaft speed, inlet temperature and inlet pressure are constant, so there is one characteristic and the surge line is fixed at $\varphi = 2W$. An industrial anti-surge controller works in a corrected-flow against corrected-head plane precisely because none of that holds, and the surge control line it uses is a curve fitted across a family of speed lines. Chapter 3 protects a jet engine compressor on corrected schedules of this kind, with no measurement of the surge margin at all.

Incompressible valve law. Both valves obey $\varphi = \gamma\sqrt{\psi}$ with no choking, no downstream pressure and no dead time. A recycle valve on a high-pressure machine is choked for much of its travel, which flattens its installed characteristic, and the transmitter and signal path ahead of it add several milliseconds of delay that this model does not represent.

Cold recycle. Gas returned through the anti-surge valve arrives at the inlet at the conditions it left. A real recycle line returns hot gas, so inlet temperature climbs while the valve is open, the characteristic moves, and a cooler is required. This omission understates the cost of extended operation with the recycle valve open.

Detection by pressure swing. Surge is declared from a windowed peak-to-peak of ψ, which is a plausible detector but a slow one. It is set not to trip on a single transient, an appropriate trade for instruction but not for a machine that must be protected within the first surge cycle.

2.4 Stability Analysis

With the anti-surge valve shut, Equations (2.1) and (2.2) linearized about an equilibrium (φ^*, ψ^*) give the Jacobian of Equation (2.7).

$$J = \begin{bmatrix} B\,\psi_c' & -B \\ \dfrac{1}{B} & -\dfrac{1}{B\,\psi_T'} \end{bmatrix}, \qquad \psi_T' = \frac{2\sqrt{\psi^*}}{\gamma_T} \tag{2.7}$$

where ψ_c' is the slope of the characteristic at the operating point and ψ_T' is the slope of the throttle line through it. Both slopes are pressure rise per unit of flow, so the two off-diagonal terms carry B and $1/B$ and the trace and determinant follow directly, as shown in Equation (2.8).

$$\operatorname{tr} J = B\,\psi_c' - \frac{1}{B\,\psi_T'}, \qquad \det J = 1 - \frac{\psi_c'}{\psi_T'} \tag{2.8}$$

Stability of the equilibrium requires a negative trace and a positive determinant. The determinant condition is the classical static criterion that the throttle line must be steeper than the characteristic, and it is independent of B. The trace condition is dynamic, it involves no comparison of the two curves, and B is the only quantity in it that is not fixed by where the operating point sits. On the negative-slope branch $\psi_c' < 0$ and both conditions hold for any B. On the positive-slope branch the trace turns positive as soon as $\psi_c' > 1/(B^2\psi_T')$, so the machine can lose stability while the static criterion is still satisfied.

At the default parameters that is what happens, and the static criterion never fails at any throttle position. At an equilibrium the throttle-line slope reduces to $\psi_T' = 2\psi^*/\varphi^*$, which is at least 2.24 at every equilibrium on the positive-slope branch, while the largest slope the cubic reaches is $3H/2W = 1.08$ at $\varphi = W$. Sweeping the throttle across its whole travel gives a minimum determinant

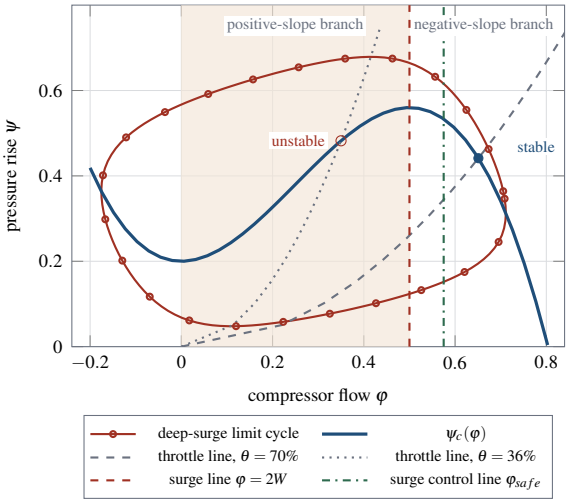


Figure 2.2 Compressor map with both stability boundaries and the measured orbit. The orbit carries open circles, the characteristic is the heavy solid curve, and the throttle lines at 70 % and 36 % are dashed and dotted. The filled marker is the stable point at 70 % throttle and the open marker is the unstable equilibrium at 36 % throttle that the deep-surge cycle encircles.

of 0.639 at $\theta = 31.2\,\%$, where the local slopes are 1.06 for the characteristic and 2.94 for the throttle line. The trace, by contrast, changes sign at $\theta = 45.62\,\%$, where $\gamma_T = 0.6386$, $\varphi^* = 0.4769$, $\psi^* = 0.5578$, $\psi_c' = 0.1900$ and $\psi_T' = 2.3390$. A pair of complex eigenvalues crosses the imaginary axis there with $\det J = 0.9188$, so the linearized oscillation frequency is $\sqrt{\det J} = 0.9585$ and its period is $6.56\,\hat{t}$. The crossing is a Hopf bifurcation, the same event that Volume 7, Chapter 2 locates on the hot branch of an exothermic reactor.

Two boundaries therefore exist and they are not in the same place. The surge line an operator is taught, the peak of the characteristic at $\varphi = 0.500$, is reached at $\theta = 47.73\,\%$. The dynamic boundary at which the equilibrium actually stops attracting is at $\varphi = 0.4769$ and $\theta = 45.62\,\%$, which is 2.11 percentage points of throttle travel further in. The healthy operating point at the default $\theta = 70\,\%$ is far from both boundaries, with $\gamma_T = 0.980$, $\varphi^* = 0.6511$, $\psi^* = 0.4414$, $\psi_c' = -1.700$, $\operatorname{tr} J = -3.042$ and $\det J = 2.254$.

Pause. Try Exercise 2.1 before reading on.

The response past the dynamic boundary is shown in Figure 2.2. The trajectory spirals away from the equilibrium, and the two ends of the cubic then stop the growth. Toward high flow the characteristic falls steeply, so the imbalance driving Equation (2.1) becomes strongly negative. Toward negative flow the cubic turns upward again, so a reversed flow is pushed forward. The result is a closed orbit, whose amplitude is set by the nonlinearity and not by the disturbance that displaced the equilibrium. A throttle closure to 36 % produces an orbit with a period of $7.51\,\hat{t}$, φ running from -0.175 to 0.711 and ψ from 0.048 to 0.679. The flow amplitude changes by 3×10^{-9} between the successive $60\,\hat{t}$ windows that begin at $\hat{t} = 68$ and $\hat{t} = 128$, which is the test that separates a limit cycle from a slow divergence. The application's verification suite applies the same test to the orbit at 40 % throttle with a tolerance of 2 % and measures a change of 7×10^{-6}. The nonlinear period is 15 % longer than the $6.56\,\hat{t}$ predicted by the linearization at onset, because most of the orbit is spent far from the equilibrium where the linear model does not apply.

2.4.1 Physical Interpretation of the Greitzer Parameter

The Greitzer parameter is the ratio of a pressure-rise time scale to a flow-acceleration time scale, and it is built from the geometry of the installation as much as from the machine, as shown in

Equation (2.9).

$$B = \frac{U}{2\,\omega_H L_c}, \qquad \omega_H = a\sqrt{\frac{A_c}{V_p L_c}} \tag{2.9}$$

where U is rotor tip speed, a is the speed of sound, A_c and L_c are the flow area and length of the duct and V_p is the plenum volume. Combining the two gives $B = (U/2a)\sqrt{V_p/(A_c L_c)}$, so B grows with rotor speed and with the square root of the volume bolted to the discharge, and shrinks with duct length. The Helmholtz frequency ω_H is the resonance of that duct and volume acting as a mass and a spring, and $\hat{t} = \omega_H t$ is the nondimensional time used throughout. On a real machine one unit of $\hat{t}$ is typically tens of milliseconds.

The trace condition puts a critical value on B at every operating point, $B_{crit} = 1/\sqrt{\psi_c'\,\psi_T'}$ wherever $\psi_c' > 0$. At $\theta = 36\%$ that value is 0.633, so the default $B = 1.5$ surges and $B = 0.5$ does not. Along the whole positive-slope branch B_{crit} never falls below 0.538, the limit it approaches as the throttle closes completely, so at $B = 0.5$ no throttle position produces an unstable equilibrium. Scenario 8 demonstrates this result. Two machines with identical aerodynamics, identical characteristics and identical final operating points can behave differently depending on the volume connected downstream, because a large discharge drum raises B, and a compressor that is stable on a test stand can surge in the plant installation.

2.5 Controller Design

The override described here is the default, and a model predictive controller (Section 2.5.1) is offered as a second option. The anti-surge loop is a proportional-integral controller with a positive gain and the ordinary sign convention, so a measured flow below the setpoint produces a positive error and opens the valve. Derivative action is set to zero and is not offered. The algorithm is the velocity form of Section 1.6.2, which computes an increment to the valve signal at each control interval $\Delta t = 0.006\,\hat{t}$ rather than the signal itself. For this loop it reads as Equation (2.10), with subscript k the execution index.

$$e_k = \varphi_{safe} - \hat{\varphi}_k \tag{2.10a}$$

$$\Delta u_k^P = K_c\,(e_k - e_{k-1})$$

$$\Delta u_k^I = \frac{K_c}{\tau_I}\,e_k\,\Delta t$$

$$s_k = s_{k-1} + \Delta u_k^P$$

$$c_k = s_k + I_{k-1} \tag{2.10b}$$

$$u_{cmd,k} = \mathrm{slew}\big(\mathrm{sat}_{[0,1]}(c_k),\ u_{cmd,k-1},\ \dot{u}_{max},\ \Delta t\big) \tag{2.10c}$$

$$I_k = I_{k-1} + \Delta u_k^I + \frac{\Delta t}{T_t}\,\big(u_{cmd,k} - c_k\big) \tag{2.10d}$$

$$u_{app,k} = \mathrm{slew}\big(u_{cmd,k},\ u_{app,k-1},\ \dot{u}_{max},\ \Delta t\big) \tag{2.10e}$$

where s_k is the accumulator that carries the proportional increments without limit, c_k is the unclamped output the algorithm demands, built on the integral accumulated through the previous execution, sat clips to the valve travel, slew limits the change to $\dot{u}_{max}\Delta t$ per execution, I is the integral state in valve fraction and $T_t = \tau_I$ is the anti-windup tracking time. In the symbols of Equations (1.13) to (1.15), $s_k = S_k$, $c_k = v_k$ and $u_{cmd,k} = u_k$, with the output bias u_b zero, so $s_k = K_c e_k$ at every execution. Anti-windup is back-calculation, the last term of (2.10d), which drives the integral toward the value that reproduces the command actually issued after both the clamp and the rate limit. Away from both limits the recursion is the positional PI law with the same gains. Every term in (2.10d) carries units of

Table 2.3 Controller Settings and Their Ranges

Symbol	Meaning	Default	Range
φ_{safe}	anti-surge setpoint	0.575	0.500–0.720
K_c	proportional gain, valve fraction per φ	8.00	0–60
τ_I	integral and tracking time ($\hat{t}$)	4.00	0.2–30
τ_f	flow measurement filter ($\hat{t}$)	0	0–1.0

valve fraction, because K_c is valve fraction per unit of flow, e is a flow, τ_I, T_t and Δt are all in $\hat{t}$, and the tracking difference is one valve fraction subtracted from another. The rate limit appears twice, in (2.10c) inside the controller and again in (2.10e) on the position that reaches the valve model, which is why the applied position lags the command even before the first order lag τ_v of Equation (2.5) is applied.

The lower clamp in (2.10c) is what makes this an override rather than a regulator, and the tracking term in (2.10d) is what makes the clamp usable. At the default operating point the flow is 0.6511 against a setpoint of 0.575, so $e = -0.0761$, the proportional term is -0.61 and the unclamped output is far below zero. A textbook integrator fed that error would integrate downward without bound for as long as the machine ran well, and would then need the same time to unwind before the valve could move at all. With the command resting on the lower clamp, $u_{cmd,k} = 0$ and $c_k = K_c e_k + I_{k-1}$, and because $T_t = \tau_I$ the error terms in (2.10d) cancel exactly, leaving $I_k = (1 - \Delta t/\tau_I) I_{k-1}$ [15]. This is the external reset of Equation (1.4) with the lower clamp as the applied output. The integral therefore decays toward zero with time constant τ_I whatever the size of the error, and the demand rests on the proportional term alone. In scenario 1 the integral starts at zero and stays there, so the demand is –0.609 for the whole run. At the wide-open clamp the same algebra relaxes I toward 1, so the command leaves either stop as soon as the error changes sign. The resulting behavior is the one a protective loop requires. The valve stays shut while the flow is above the setpoint, and the command leaves zero at the first execution after the measured flow crosses φ_{safe}, with no dead period. In scenario 3 the flow falls below 0.575 at $\hat{t} = 9.124$ and the command opens at the execution at $\hat{t} = 9.126$.

🕮 **Pause.** Try Exercise 2.3 before reading on.

In MANUAL the integral state is seeded, $I_k = u_{man} - s_k$, so the transfer in either direction is bumpless, as in Section 1.6.2. On request in MANUAL the application also sets the valve to the opening that holds the declared machine at the setpoint in steady state with the throttle where it is set, the opening u_{ss} of Section 2.5.1 at $\hat{d} = 0$. The required opening changes as the throttle moves.

Two defenses that a real installation carries are deliberately absent. There is no feedforward from the rate of throttle movement, so the loop cannot begin opening before the flow has already started to fall, and there is no close-coupled valve or speed manipulation of the kind studied in the active stabilization literature [13, 14]. The setpoint itself is the third defense and it is the one left open to adjustment, because placing φ_{safe} above the surge line provides anticipation that no tuning can supply. The default $\varphi_{safe} = 0.575$ is a surge margin of 15.0 %, and the surge control line it draws is the vertical line in Figure 2.2.

Table 2.3 lists the default settings of the four operator-adjustable loop quantities.

2.5.1 Model Predictive Control

The second option replaces the proportional-integral law by a receding-horizon optimization on the same valve [16, 17], the option set out in general in Section 1.6.3. In AUTO, at every third execution, $T_s = 3\Delta t = 0.018\ \hat{t}$, the controller plans the valve command over the next $N = 111$ steps (2.0 $\hat{t}$) in eight move blocks of 1, 2, 3, 6, 12, 24, 30 and 33 steps, and the first move of each plan is applied and held for one decision period. Its model is the compression system of Section 2.3 with the declared

defaults of Table 2.2, marked with an overbar, and never with the values of the running plant, as shown in Equation (2.11).

$$\begin{aligned} \frac{\mathrm{d}\varphi}{\mathrm{d}\hat{t}} &= \bar{B}\big[\bar{\psi}_c(\varphi) - \psi\big], & \bar{\tau}_v \frac{\mathrm{d}u}{\mathrm{d}\hat{t}} &= u_c - u, \\ \frac{\mathrm{d}\psi}{\mathrm{d}\hat{t}} &= \frac{\varphi - (\gamma_T + \bar{K}_v u)\sqrt{\psi} + d}{\bar{B}}, & \bar{\tau}_{th} \frac{\mathrm{d}\gamma_T}{\mathrm{d}\hat{t}} &= \bar{\gamma}_{max}\theta_m - \gamma_T \end{aligned} \tag{2.11}$$

where u_c is the valve command, $\bar{\psi}_c$ is the cubic of Equation (2.3) with the declared ψ_0, H and W, d is an unmeasured flow imbalance held constant over the horizon, and θ_m is the commanded throttle demand, or the design value of 0.70 when the throttle signal to the controller is switched off. The restriction drift is not part of θ_m and reaches the model only through d. At each decision the state $x = [\varphi,\ \psi,\ u,\ \gamma_T,\ d]^T$ is linearized about the current estimate, with the remainder carried as an affine term, and sampled by an exact zero-order hold over T_s, so the model follows the slope ψ_c', which changes sign at the peak, and the valve-to-flow gain, which step tests on the plant put between 0.151 and 0.238 over the operating range.

The model is checked rather than identified. At the 70 % design point its Jacobian agrees with central differences of the plant's derivatives to 1.5×10^{-10}, and its steady valve-to-flow gain is 0.1769 against 0.1697 from a 0.05 valve step on the plant, to which a first-order-plus-dead-time fit (0.170, 1.15 $\hat{t}$, 0.55 $\hat{t}$), never used by the controller, leaves a residual of 1.4 % of the step. On scenario 11 the root-mean-square error of the predicted flow is 4.5×10^{-4} one decision ahead and 0.011 at 2 $\hat{t}$. Each plan solves Equation (2.12).

$$\begin{aligned} \min_{u_0^b,\ldots,u_7^b,\ s} \ & \sum_{i=0}^{N-1} \ell\, u_i + S \sum_{j=0}^{7} \big(u_j^b - u_{j-1}^b\big)^2 + \rho s^2 + \sigma s \\ \text{s.t.}\ \ & \varphi_i \geq \varphi_{safe} - s, \quad i = 1,\ldots,N, \qquad s \geq 0, \\ & 0 \leq u_j^b \leq 1, \qquad \big|u_j^b - u_{j-1}^b\big| \leq \dot{u}_{max} T_s, \\ & u_7^b \geq \min\big(u_{ss},\ u_{-1}^b + 8\,\dot{u}_{max} T_s\big) \end{aligned} \tag{2.12}$$

where u_j^b is the command of block j, u_i is the command in force at step i, u_{-1}^b is the command applied when the plan is made, φ_i is the predicted flow, $\ell = 0.01$ is the price of valve opening per step, $S = 10$ is the move weight, $\rho = 10^6$ and $\sigma = 10^3$ are the prices of the slack s, and $u_{ss} = [(\varphi_{safe} + \hat{d})/\bar{\psi}_c(\varphi_{safe})^{1/2} - \bar{\gamma}_{max}\theta_m]/\bar{K}_v$, clipped to $[0, 1]$, is the opening that holds the declared model at φ_{safe} in steady state. The recycle is a linear cost, so the plan opens the valve only as far as the predicted path requires, and the setpoint becomes a soft floor with one slack for the horizon (one per step changed no minimum margin on scenarios 6, 7 and 11 at 41 to 53 times the solve time). The valve travel and the stroking rate the override slews at are hard, the terminal row stops the final block from closing the valve where its effect would fall past the horizon, plenum pressure carries no weight, and $S = 10$ is the smallest move weight within 10 % of the least command travel found on scenario 11 (85.6 % against 79.1 % of stroke).

The estimator is updated at each execution regardless of which controller has the valve, as given by Equation (2.13).

$$\hat{x}_k^- = \Phi\big(\hat{x}_{k-1},\, u_{app,k-1},\, \theta_m\big), \qquad \hat{x}_k = \hat{x}_k^- + L\big(\hat{\varphi}_k - \hat{x}_{k,1}^-\big) \tag{2.13}$$

where Φ is one fourth-order Runge-Kutta step of Equation (2.11) over Δt with the applied command, $\hat{x}_{k,1}^-$ is the predicted flow, and L is the steady-state Kalman gain [3] of the model linearized at 70 % throttle with the valve shut, for a process noise covariance of $\mathrm{diag}(10^{-7}, 10^{-7}, 10^{-9}, 10^{-9}, 10^{-6})$ and a measurement variance of 10^{-5}, which gives $L = [0.0957,\ -0.161,\ 6.5 \times 10^{-6},\ 2.14 \times 10^{-5},\ -0.301]^T$. The gain is fixed, because recomputing it at every decision costs 6.35 to 6.42 ms in Node, about 35 % of a processor core at real time, so the estimator is a constant-gain extended

Kalman filter. The imbalance d is an input disturbance model that makes the controller offset free [18], and on a plant with $K_v = 0.45$, $B = 1.2$ and $H = 0.19$ the flow rests 1.8×10^{-9} from φ_{safe} at $\hat{t} = 60$.

A plan is accepted only if the solver reports an optimal solution, the first move equals the first entry of the plan, every planned command lies in $[0, 1]$, and the predictions, cost and slack are finite and of the expected sizes. Otherwise the override, which tracks the applied command with no preload, takes the valve with a step of 0.25 % of stroke when forced in the middle of the closure of scenario 11, until a later plan is accepted. Tracking leaves the override's demand at zero, so a fallback forced from $\hat{t} = 8$ to 20 opens at the first fall in flow and reaches +7.70 % rather than −8.6 %. Under the plan the override's gain and integral time have no effect, while φ_{safe} and τ_f stay in force.

Three models are kept apart, the plant of Equations (2.1) to (2.5) with the run's parameters and the drift, the instrument of Equation (2.6), which is the only measurement, and the controller's model of Equation (2.11), which takes the throttle command as an operator setting and not as a measured position. At normal speed, one unit of $\hat{t}$ a second, the controller solves 55.6 plans a second, and in Chrome 157 its whole execution took 1.30 to 2.00 % of one processor core and 5.19 to 6.49 % at the fastest speed of four times, with a worst execution of 2.1 ms.

2.6 Performance Metrics for Override Control

The application reports the five standard control-performance measures that every framework app shows on its scorecard (Section 1.6.4) together with three measures specific to this loop, and four of the five standard measures require careful interpretation on an override.

The integral of absolute error is the clearest case. In healthy operation the valve is clamped shut, the flow sits well above the anti-surge setpoint, and the accumulating error is simply the distance to a target the controller has no intention of reaching. Standing still at the default operating point for 29.9 $\hat{t}$ produces an IAE of 2.28 while the valve has not moved once and the surge margin has never fallen below 30.2 %, and a longer run would produce a larger number for the same perfect behavior. Every IAE quoted in this chapter is reported with the run length beside it and is never used to compare one tuning against another.

The saturated fraction is reported for the wide-open end of travel only. A regulatory loop that sits on either limit has lost authority, but a protective override resting shut is doing its job, so counting the closed end would report 100 % saturation for a machine running perfectly. Saturation is therefore counted at the wide-open limit only, and these percentages are not comparable with those of the other apps, where both limits count.

Overshoot and settling time are step-response measures, and the application reports them only after a setpoint move. No scenario moves φ_{safe}, so both read blank throughout, which is the correct outcome for an override. Computed instead against the standing difference between the initial flow and the setpoint, 0.0761 at the default operating point, scenario 4's excursion to −0.175 would read as an overshoot near 986 %, which restates the minimum surge margin rather than judging the tuning. Settling has a second defect on this loop, because a surging flow passes through any settling band twice per cycle. The application commits a settling time only after the flow has stayed inside the band for a dwell, which keeps a limit cycle from reporting a finite value, but the loop still has no target to settle to. Neither measure appears in Table 2.5.

Three measures are meaningful without qualification. Minimum surge margin, $100(\varphi_{min} - 2W)/2W$, is negative if and only if the operating point crossed the surge line at any moment. Cumulative recycled flow, $\int \varphi_u \, d\hat{t}$, is the gas compressed and thrown straight back to the inlet and is the running cost of protection. Peak detector swing is the largest windowed $\Delta\psi$ reached and is the quantity the surge alarm acts on. Those three and the total variation of the controller output are what the scenarios below are judged by.

Table 2.4 The Twelve Scenarios and the Quantity Each One Moves

#	Name	Setting
1	Stable operation	AUTO at 70 % throttle, valve shut, nothing scheduled
2	Close the throttle by hand	MANUAL, operator steps the throttle down
3	Rapid closure, controller on	AUTO, $\tau_{th} = 0.12$, $\theta \to 36\,\%$ at $\hat{t} = 8$
4	Same closure, controller off	MANUAL, $\tau_{th} = 0.12$, same event
5	Fast anti-surge valve	$\tau_v = 0.08$, $\dot{u}_{max} = 14$, $\varphi_{safe} = 0.60$, $\tau_{th} = 0.25$, $\theta \to 36\,\%$ at $\hat{t} = 8$
6	Slow anti-surge valve	$\tau_v = 2.2$, $\dot{u}_{max} = 0.5$, $\varphi_{safe} = 0.60$, $\tau_{th} = 0.25$, same event
7	Undersized valve	$K_v = 0.18$, $\tau_{th} = 0.40$, $\theta \to 30\,\%$ at $\hat{t} = 8$
8	A smaller plenum	MANUAL, $B = 0.5$, $\tau_{th} = 0.40$, $\theta \to 50, 44, 38, 34\,\%$ at $\hat{t} = 8, 22, 36, 50$
9	Noisy flow transmitter	$\sigma_n = 0.014$, $\tau_{th} = 0.40$, $\theta \to 42\,\%$ at $\hat{t} = 8$
10	Aggressive PI tuning	$K_c = 34$, $\tau_I = 0.8$, $\tau_{th} = 0.40$, $\theta \to 40\,\%$ at $\hat{t} = 8$
11	Predictive control against the PI	As 3, predictive controller
12	Challenge	$K_c = 1.5$, $\tau_I = 15$, $\varphi_{safe} = 0.55$, $\tau_{th} = 0.40$, $\theta \to 38\,\%$ at $\hat{t} = 10$

2.7 Results

The twelve scenarios are given in Table 2.4. A scenario sets the quantities under study together with the throttle stroke constant τ_{th} of its closure (0.12 to 0.40 against the default 0.50) and leaves every other setting at its default. Every scenario that ends in surge begins at a stable point on the negative-slope branch and is driven across the boundary by a scheduled throttle event, because an unstable equilibrium is still an equilibrium and a run that merely started there would sit motionless. Each scenario also carries a completion rule (Section 1.6.4), and several of the exercises at the end of the chapter follow those rules.

The measured performance of each run with the scenario as supplied, read once it had reached the $\hat{t}$ shown, is collected in Table 2.5. Scenario 2 is driven by hand and has no scheduled event, so it carries no row and is described separately below, and scenario 11 is compared with the override in Section 2.7.6. MV travel is the total variation of the controller output in percent of valve travel, not of the valve position, because the sixth scenario exists to show that those are different signals. The overshoot and settling measures read blank for the reasons given in Section 2.6. Figure 2.3 sets the minimum margin and the recycled flow of that table side by side.

2.7.1 Staged Manual Throttle Closure

The second scenario locates the stability boundary by measurement. Stepping the throttle down by one percent every 20 $\hat{t}$ from 70 %, with the anti-surge valve locked shut, leaves the plenum pressure swing immeasurably small down to 54 %, then growing slowly in the last few steps of the settled part of the run. Twelve to twenty units after each step the swing is 0.0014 at 48 % and 0.0100 at 46 % and still decaying, 0.046 at 45 % and growing, and 0.537 at 44 %, where the surge alarm latches 12.0 $\hat{t}$ after the step. The last position that holds and the first that does not are one percent of throttle travel apart, and the analysis of Section 2.4 puts the boundary between them at 45.62 %. Crossing the peak

Table 2.5 Measured Performance of Each Scenario

#	$\hat{t}$	IAE	MV travel	Sat. (%)	Min margin (%)	Recyc. flow	Peak $\Delta\psi$
1	29.9	2.276	0	0.0	+30.2	0.00	0.000
3	30.0	1.138	624	2.0	−8.6	4.19	0.222
4	59.7	17.808	0	0.0	−135.1	0.00	0.631
5	29.9	0.717	182	0.0	+3.3	4.75	0.171
6	59.3	2.649	387	5.2	−105.4	11.23	0.624
7	59.9	16.394	1497	56.2	−129.0	3.60	0.645
8	95.5	17.117	0	0.0	−43.8	0.00	0.211
9	30.5	0.987	4617	0.0	+1.0	2.95	0.164
10	59.9	2.207	3500	27.7	+6.0	9.01	0.170
12	60.0	4.604	402	0.0	−93.5	5.56	0.595

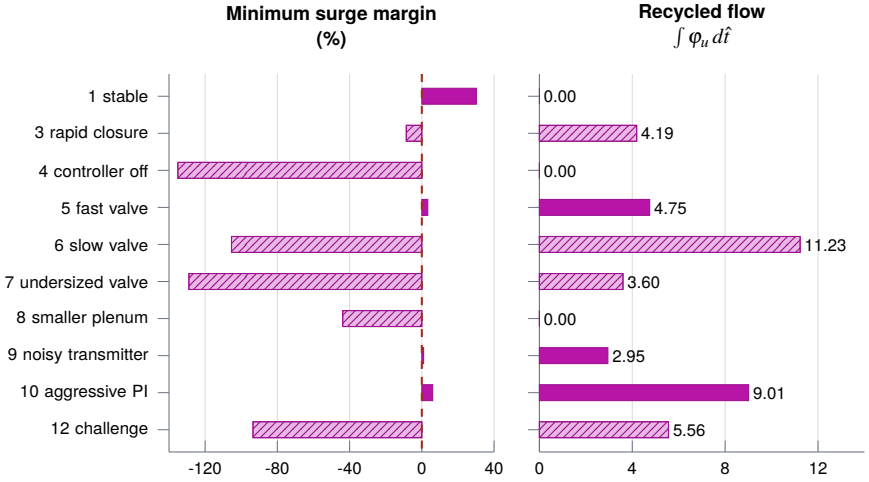


Figure 2.3 Table 2.5 redrawn: minimum surge margin and recycled flow of the ten scenarios with a row. Solid bars keep a positive margin, hatched bars cross the surge line (dashed, zero margin). Notice that recycling more does not by itself protect the machine: the slow valve of scenario 6 recycles the most gas and still surges, while the fast valve of scenario 5 holds with less than half as much.

of the characteristic at 47.73 % produced no visible response, which demonstrates that the static surge line and the dynamic stability boundary are different objects.

2.7.2 The Deep-Surge Limit Cycle

Scenario 4 is the central demonstration, and it is the run captured in Figure 2.5. The throttle steps from 70 % to 36 % in about a tenth of a Helmholtz period with the valve locked shut, and the machine settles into the deep-surge orbit of Figure 2.2. The surge alarm latches at $\hat{t} = 11.21$, the instant the flow first reverses. After 59.7 $\hat{t}$ the minimum margin is −135.1 %, meaning the flow reached −0.175, and the detector swing is 0.631 against an alarm threshold of 0.25. The flow trend shows a clean periodic waveform with negative excursions once per cycle and the pressure trend shows ψ and $\psi_c(\varphi)$ separating, which is the visual signature that the two states are no longer in balance at any instant.

TRY IT IN THE APP 4 · Same Closure, Controller Off

Load the scenario and press Run. Watch the Compressor flow trend. After the throttle closes at $\hat{t} = 8$ the flow falls, reverses to about −0.175 and recovers, again and again with the same amplitude. The surge alarm latches at $\hat{t} = 11.21$, the instant the flow first reverses. Nothing grows without bound.

The oscillation neither settles nor grows. That combination is what makes the object a limit cycle rather than an instability in the usual sense. It has no dependence on the size of the disturbance that started it, it cannot be described by a growth rate, and it returns to the same orbit from any nearby initial condition. The eighth scenario is the control against which that claim is tested. With B lowered to 0.5 and the throttle stepped to 34 % in four stages, the machine oscillates and decays after each step, dips past the surge line on the way, and then settles toward the equilibrium at $\varphi = 0.3209$, reading 0.3207 at $\hat{t} = 95.5$ with a residual swing of ± 0.002. The minimum margin over the run is −43.8 % and the peak detector swing is 0.211, below the alarm threshold. Identical aerodynamics, identical final flow, and no surge, because B_{crit} at that operating point is 0.596 and the plenum is now too small to sustain the oscillation. Run again with $B = 1.5$, the same sequence trips the surge alarm at $\hat{t} = 34.3$, 12.3 units after the step to 44 %, and ends in deep surge with a minimum margin of −138.0 %.

2.7.3 Effect of Valve Stroke Speed and Capacity

Scenarios 3, 5, 6 and 7 take a fast closure and change the final element. With the standard valve and a closure to 36 % at $\tau_{th} = 0.12$, about a tenth of a Helmholtz period, the controller is too late, reaching a minimum margin of −8.6 % after spending 624 points of command travel and 4.19 of recycled flow. Scenario 5 gives the valve a stroke constant of 0.08 instead of 0.30 and a rate limit of 14 instead of 4, moves the setpoint out to 0.60, and strokes the throttle with $\tau_{th} = 0.25$. It holds the closure at +3.3 % minimum margin for 182 points of travel and 4.75 of recycle. Both changes to the loop are needed on that closure, since the standard valve at a setpoint of 0.60 reaches −1.03 % and the fast valve at 0.575 reaches −3.03 %. The slower throttle is not what saves it, because the fast valve with $\tau_{th} = 0.12$ still holds +2.95 %.

The reverse change is the more instructive one. Scenario 6 has the same capacity, the same tuning and the same setpoint as scenario 5, and only the stroke speed is different at $\tau_v = 2.2$ and $\dot{u}_{max} = 0.5$. The controller acts correctly. Its command leaves zero 1.0 $\hat{t}$ after the closure and reaches 100 % at $\hat{t} = 11.0$, when the valve is 34 % open, and the machine surges anyway. The alarm latches at $\hat{t} = 11.7$, the flow reverses once, to a minimum margin of −105.4 % at $\hat{t} = 12.4$, and the alarm clears at $\hat{t} = 24.6$ once enough recycle is open. No proportional gain and no integral time changes that outcome, because the decision was correct while the actuator had not reached the required position in time. This is why anti-surge valves are specified by stroke time as well as by capacity [10].

Scenario 7 fails for a third reason. Holding the flow at $\varphi_{safe} = 0.575$ requires a total downstream conductance of 0.7874, and at 30 % throttle the throttle itself supplies only $\gamma_T = 0.420$, so the recycle valve must contribute 0.3674. At $K_v = 0.18$ the most it can contribute wide open is 0.18. The command pins at 100 % for 56.2 % of the run, the reachable equilibrium at $\varphi = 0.444$ is past the dynamic boundary on the positive-slope branch, and the machine surges to −129.0 % minimum margin. A saturated protective actuator is a sizing statement, not a tuning statement (Section 1.4.5).

DID YOU KNOW? Same Valve Size, Opposite Outcomes

From the simulator: scenarios 5 and 6 share valve capacity, tuning and setpoint and differ only in how fast the valve strokes. The fast valve holds the closure at a minimum surge margin of +3.3 %. The slow one lets the flow reverse, to −105.4 % (Table 2.5).

2.7.4 Control-Induced Oscillation

Scenario 10 raises the gain to $K_c = 34$ and shortens the integral time to $\tau_I = 0.8$, and the first flow dip is shallower. Running the identical closure to 40 % throttle with the default $K_c = 8$ and $\tau_I = 4$ gives a minimum margin of −2.13 % for 476 points of command travel, and the aggressive tuning gives +5.97 % for 3500 points. The added margin costs a factor of seven in valve activity.

What the aggressive tuning leaves behind is a sustained oscillation, and it is not surge. The peak detector swing is 0.170 against a threshold of 0.25, so the alarm never trips. The flow cycles between 0.534 and 0.627 with a period of 2.92 $\hat{t}$, entirely above the surge line and astride the setpoint, against the 7.51 $\hat{t}$ and the excursion to −0.175 of the aerodynamic cycle. Period and amplitude are set here by the loop gain and the valve lag rather than by the Helmholtz dynamics, and one diagnostic separates the two cases, switching the loop to MANUAL. Aerodynamic surge continues and a control-induced cycle stops.

Pause. Try Exercise 2.2 before reading on.

TRY IT IN THE APP 10 · Aggressive PI Tuning

Run the scenario and watch the flow after the closure to 40 % throttle. It cycles between 0.534 and 0.627, astride the setpoint and above the surge line, with a period of 2.92 $\hat{t}$, and the surge alarm never trips. Then switch the loop to Manual. The chapter's test is that a control-induced cycle stops there while aerodynamic surge would continue.

2.7.5 Effect of Measurement Noise

Scenario 9 adds 0.014 of transmitter noise, about 3 % of the surge flow, and closes the throttle to 42 %. Over 60.0 $\hat{t}$ the controller output travels 11 234 points with the noise present and 399 points with the noise removed, so the same duty costs twenty-eight times as much command activity. The travel with noise accumulates linearly with run length and the travel without it does not, so that ratio is only meaningful with the run length attached. Adding a measurement filter of 0.10 $\hat{t}$ brings the travel back to 3571 and costs 1.78 points of minimum surge margin, from +1.03 % to −0.74 %, because the filter adds phase lag to a loop that has none to spare.

The valve position travels far less, 1090 points with the noise against 352 without it, a factor of 3.1 rather than 28, because the rate limit and the valve lag absorb most of the chatter. This is why the failure mode is hard to see from the process side until the positioner is worn out. A noisy protective loop is judged on the controller output rather than on what the process ends up seeing.

2.7.6 Predictive Control Against the Override

Scenario 11 repeats the closure of scenario 3 under the predictive controller, as shown in Figure 2.4. The plan starts opening the valve at its first decision after the throttle command changes ($\hat{t} = 8.01$), strokes the command to 64.5 % by $\hat{t} = 9.43$ and eases it to 47.2 % by $\hat{t} = 20$, and the flow comes down onto the floor at $\hat{t} = 14.50$ without crossing it, a minimum margin of +15.00 %. The override opens only once the measured flow crosses its setpoint and reaches −8.61 % at $\hat{t} = 10.33$, after 1.12 $\hat{t}$ under the surge line. To $\hat{t} = 20$ the plan recycles 2.459 against 2.114, with 85.6 % of command travel against 529.4 %.

The comparison over nine cases is collected in Table 2.6, every row of a case with the same seed, duration, instruments and noise, and the predictive controller at scenario 11's settings throughout. Two classical rows receive the throttle command the plan reads, measured outside the application on

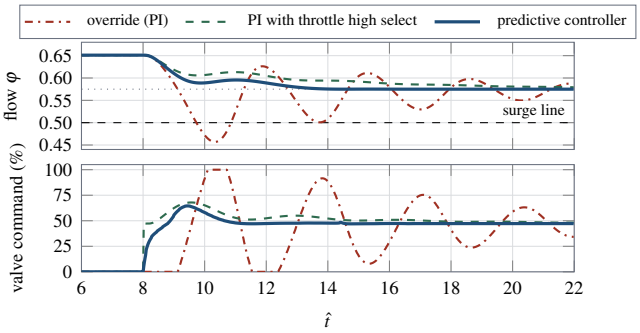


Figure 2.4 Flow and anti-surge valve command on the rapid closure of scenario 11 under three structures. The throttle is commanded from 70 % to 36 % at $\hat{t} = 8$. The override is dash-dotted, the override with the throttle high select dashed and the predictive controller thick solid. The thin dashed line is the surge line and the dotted line is the anti-surge setpoint at 0.575.

the same engine. As a feedforward of u_{ss} at $\hat{d} = 0$ added to the override's output it raises the margin on the closure only to +3.77 %, because the proportional term on a flow above the setpoint cancels it, while as a minimum opening through a high select, the open-loop step of industrial anti-surge systems [10], it gives the second group of Table 2.6.

Most of the margin on the closure is the throttle signal. Without it the plan bottoms at +4.67 %, near the additive feedforward's +3.77 %, while the high select holds +16.16 % at 8.6 % more recycle than the plan to $\hat{t} = 20$ and 1.4 % more over 90 $\hat{t}$, so what the plan adds is recycle and travel. The override alone needs a setpoint of 0.661 to hold +15 %, with 58 % more recycle over 90 $\hat{t}$ than the plan (26.762 against 16.946), and three retunings without the signal reach −10.23 % to +1.85 %. Where the command says nothing, the prediction from the flow still helps, at +11.67 % against +3.81 % for an unannounced drift (the most aggressive retuning reaches +8.97 %) and +15.00 % against +12.10 % for deep surge caught at $\hat{t} = 30$, with less travel in both. With a command that overstates the opening by 10 points, the plan holds +11.86 % and the high select +8.36 %.

The plan loses on recycle wherever its floor binds and the override's does not, by 2.0 % over 90 $\hat{t}$ of scenario 3, where the override saves gas by letting the flow sag under its setpoint, and by 16.7 % over 60 $\hat{t}$ of scenario 9, where the floor binds on the low excursions of a noisy estimate. On the slow valve of scenario 6 the plan (−4.09 %) and the high select (−3.91 %) avoid the alarm the override trips but cannot hold the surge line, because no plan strokes the valve faster than $\dot{u}_{max} = 0.5$ allows, and the undersized valve of scenario 7 defeats every structure (−118.03 % against −128.96 % over 90 $\hat{t}$). With the challenge's supplied settings the plan meets the challenge's target, +10.00 % with no alarm and 7.72 of recycle, because it does not use the weak gains.

2.7.7 Challenge Scenario

The challenge closes the throttle from 70 % to 38 % at $\hat{t} = 10$ and stops itself at $\hat{t} = 60$ so that the two scoring measures of different runs can be compared. The challenge is judged on the case as given, so changing a model parameter or a disturbance voids the attempt. The supplied tuning is deliberately weak at $K_c = 1.5$ and $\tau_I = 15$ on a loop whose work is over within two or three units, and it fails, reaching −93.5 % minimum margin with the surge alarm on from $\hat{t} = 14.1$ to 36.1. The target is a minimum margin at or above zero, no surge alarm, and cumulative recycled flow below 12.0. The suggested answer moves the setpoint to 0.60, raises K_c to 12 so that most of the valve travel happens inside the first unit, and leaves τ_I at 4. It finishes at +5.34 % minimum margin with a peak detector swing of 0.162, 10.59 of recycled flow (1.41 under the limit) and 544 points of command travel.

The two requirements pull against each other by design. A conservative setpoint makes the closure easy to survive and recycles gas every hour the plant runs, while a tight setpoint is inexpensive and

Table 2.6 Predictive Controller Against the Override and the Override Given the Throttle Signal

		Override (PI)			PI with throttle high select			Predictive controller				Predictive, signal off		
Case	$\hat{t}$	SM	Rec.	Trav.	SM	Rec.	Trav.	SM	Rec.	Trav.	Plans	SM	Rec.	Trav.
Rapid closure, sc. 3 and 11	0–20	−8.61	2.114	529.4	+16.16	2.670	96.4	+15.00	2.459	85.6	1112	+4.67	2.268	167.6
Rapid closure, sc. 3 and 11	0–90	−8.61	16.612	638.1	+15.00	17.190	98.2	+15.00	16.946	85.6	5000	+4.67	16.720	167.9
Signal 10 points wrong, sc. 3	0–90	−25.80†	24.757	738.9	+8.36	25.260	243.9	+11.86	25.333	140.8	5000	−3.55	25.100	192.2
Unannounced drift, sc. 1	0–90	+3.81	9.121	288.0	+3.81	9.121	288.0	+11.67	9.275	206.5	5000	+11.67	9.275	206.5
Deep surge caught, sc. 4	30–90	+12.10	12.339	197.0	+15.00	12.782	97.8	+15.00	12.266	57.3	3334	+12.28	12.200	181.6
Slow valve, sc. 6	0–90	−105.43†	18.593	387.5	−3.91	19.603	154.8	−4.09	19.282	234.8	5000	−96.64†	18.134	376.8
Noisy flow, sc. 9	0–60	+1.03	7.226	11234	+22.39	11.017	11731	+15.10	8.433	6205	3334	+11.47	8.257	6276
Challenge, supplied	0–60	−93.49†	5.556	402.3	+12.12	8.831	51.6	+10.00	7.718	49.6	3334	+3.04	7.633	200.9
Challenge, suggested	0–60	+5.34	10.588	544.2	+20.00	11.062	77.1	+20.00	10.861	59.3	3334	+11.83	9.995	185.1

SM is the minimum surge margin in the window (%), Rec. the recycled flow $\int \varphi_{rt}\, d\hat{t}$ in the window, Trav. the command travel (% of stroke), Plans the decisions solved. †Surge alarm. The wrong signal is a restriction drift of −10 % from $\hat{t} = 0$, the unannounced drift a step of −25 % at $\hat{t} = 8$, and the caught deep surge scenario 4 switched to AUTO at $\hat{t} = 30$, its earlier alarm unmarked. No predictive run had a fallback, and the mean solve time was 122 to 184 µs a plan in Node.

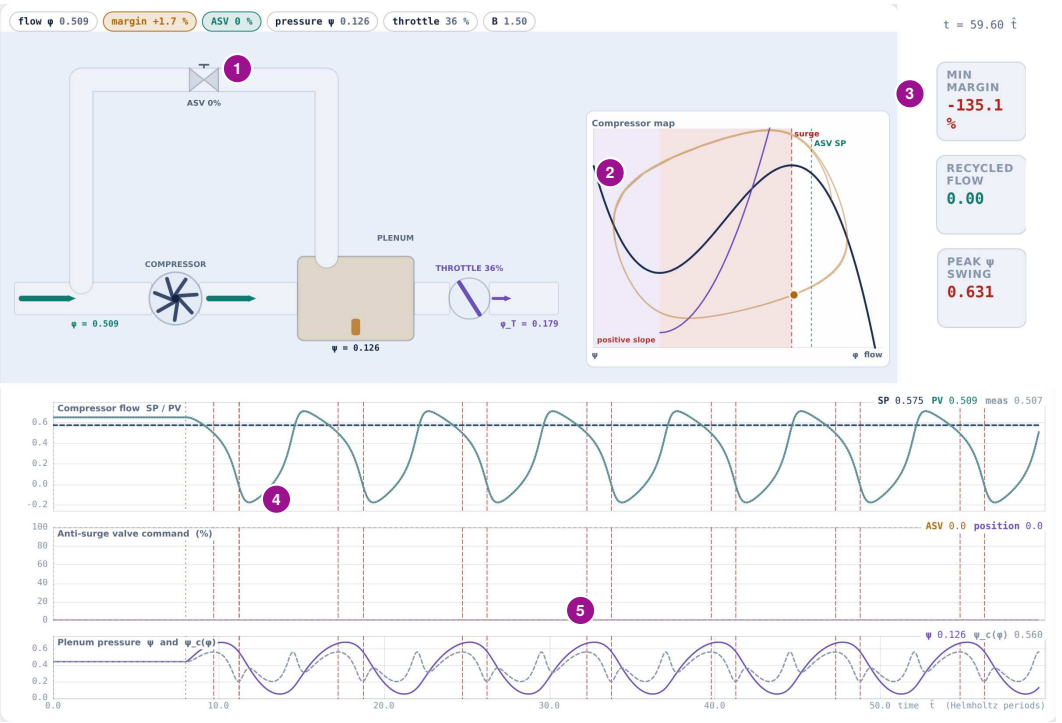


Figure 2.5 Deep surge after a throttle slam from 70 % to 36 % with the anti-surge valve locked shut, in a composite of the application window: the plant view, with the run clock and three scorecard tiles moved beside it, above the three trend strips. (1) The anti-surge valve rests at 0 %. (2) The compressor map shows the closed orbit crossing into reverse flow. (3) The tiles give a minimum surge margin of −135.1 % and a peak detector swing of 0.631. (4) The flow strip shows the periodic collapse and recovery over the whole run. (5) The valve command strip stays at zero on its 0 to 100 % scale. Constant actuator traces retain both declared limits, while moving traces can autoscale to show small movements. The pressure strip shows ψ and $\psi_c(\varphi)$ separating. This is scenario 4 at $\hat{t} = 59.6$.

leaves no margin when the load moves. The same trade-off drives anti-surge setpoint selection in operating plants, and the recycled-flow measure carries the operating-cost side of it that a study of tuning alone omits.

2.8 Technology in Practice

An anti-surge loop in a plant reads the same flow and moves the same valve as the override of Section 2.5. Figure 2.6 draws that loop with field devices in place of the schematic's symbols.

What Measures It

Flow is the most direct sign of surge, which is a breakdown of stable flow, but a 2025 review notes that accurate and fast flow measurement in the harsh compressor environment is demanding and that flow sensors often need an intrusive installation. Pressure-based monitoring is mature and easy to implement, though its accuracy depends on where the sensor sits and it may suffer from noise [19]. The model keeps the two apart as Section 2.3 does: the flow transmitter carries only the noise σ_n and the filter τ_f, with no bias and no transport delay, and the plenum pressure reaches only the detector.

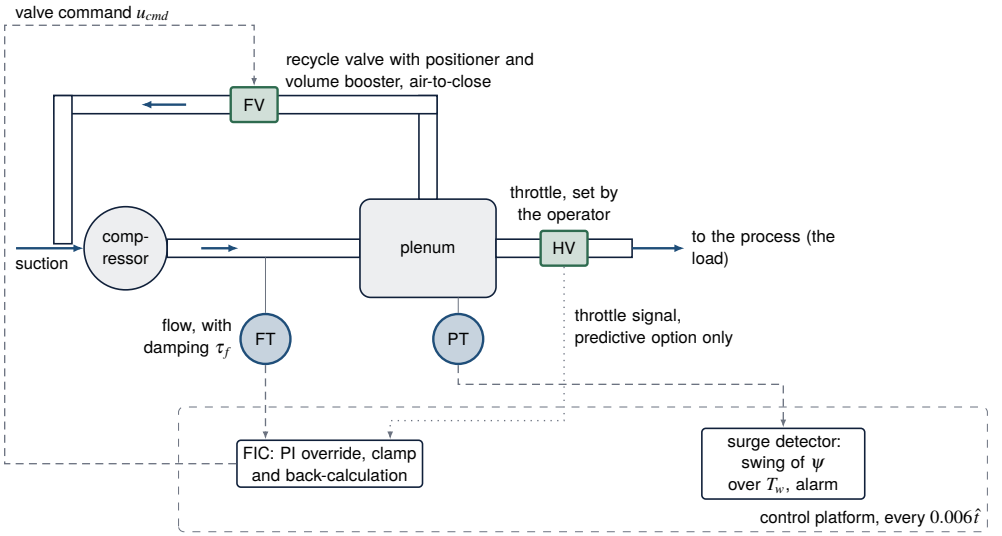


Figure 2.6 The anti-surge loop of Figure 2.1 on plant hardware. A flow transmitter (FT) with its damping feeds the override (FIC) on the control platform, which moves the recycle valve (FV) through its positioner and volume booster. The plenum pressure transmitter (PT) feeds only the surge detector, and the throttle (HV) is the load. Notice that the override never sees pressure, so the detector can report surge but cannot prevent it, and that only the predictive option reads the throttle (dotted).

TECHNOLOGY Sensor: A Flow Transmitter and Its Damping

Nearly every modern process transmitter, the flow transmitter of an anti-surge loop among them, carries a damping setting: a low-pass filter in line with its signal that keeps turbulence noise, such as that at a machine's discharge, out of the control system. Too little damping passes noise as ragged trends and erratic control. Too much makes the transmitter report a process that changes more slowly than it does, and an aggressively tuned loop then overshoots or oscillates while its own trend looks smooth [5]. The rule given is to use as little damping as good control needs. No open source read gives a range, accuracy or speed for such a transmitter. In the model the damping is the filter τ_f, and at 0.10 it cost scenario 9 1.78 points of minimum margin.

What Moves It

The final element is the recycle valve from the discharge back to the suction. Opening it keeps enough flow through the compressor and unloads the pressure across it, and a textbook loop diagram makes it air-to-close, so that a loss of air opens it [5]. The model gives the valve a capacity K_v, a lag τ_v and a stroking rate limit $\dot{u}_{max}$, and scenarios 6 and 7 show that stroke speed and capacity each bound what any tuning achieves.

TECHNOLOGY Final Element: A Recycle Valve Built to Stroke Fast

A pneumatic valve moves only as fast as air can fill and vent its actuator, and a large actuator is slow to fill. A volume booster repeats the signal pressure one to one with far greater air flow, which shortens the delay, although no finite flow removes it. A positioner places the stem precisely and also sources and vents high air flow, so it gives a faster stem as well [5]. These are the means of giving a recycle valve the stroke speed this chapter asks of it. A slow valve fails as scenario 6 does on its trend: the command reaches 100 % while the valve is 34 % open, and the machine surges. In the model τ_v and $\dot{u}_{max}$ stand for the actuator and its air supply.

Where the Controller Runs

Plant control loops run with cycle times from milliseconds to minutes [20]. Compressor surge control is given as an industrial example of a safety function that must respond within a set time, which asks for a network with hard determinism [5]. The application executes the override every 0.006 $\hat{t}$, and on a real machine one unit of $\hat{t}$ is typically tens of milliseconds. In a function-block system the override's selector is a control selector block whose back-calculation output tells a deselected controller to stop integrating, the role external reset played in pneumatic controllers [5]. Here the lower clamp and the tracking term of Equation (2.10d) play that part.

IN PRACTICE A Soft Override With a Hard Trip Behind It

Practice separates two layers of protection. A hard override, such as a switch that stops a well pump on low level, protects the machine at the cost of the service. A soft override is a controller whose output a selector passes only when its limit nears, so the service is curtailed rather than cut. A prudent design keeps the hard trip behind the soft loop, in case a setpoint is badly set or the tuning is poor, with the soft loop's setpoint placed so that it acts before the trip does. A controller with integral action that the selector deselects needs its windup limited [5]. The anti-surge loop of this chapter is the soft layer, and its surge alarm only reports.

What Is Modernizing It

Three developments bear on the loop. The first is predictive control. Nonlinear and simplified predictive controllers for anti-surge and process control have been timed on an industrial programmable logic controller and outperformed standard industrial control on surge margin, tracking and actuation without a large increase in computation time [21], the comparison Section 2.7.6 makes on scenario 11. The second is faster hardware: research is exploring high-speed pressure sensors, solenoid valves and piezoelectric actuators, because sensor delay and slow valve response often limit what a control law can do. The third is fusion and integration: pressure, vibration and temperature read together, data-driven methods combined with classical ones, and surge control coordinated with the combustion and cooling around the machine [19].

2.9 Limitations

The limitations of the model are stated in Section 2.3.6 and are not repeated. Four limitations belong to the application rather than to the physics.

The override sees a perfect throttle position in the sense that no feedforward path exists to use it, so under the override the application demonstrates the inadequacy of purely reactive protection without offering the standard cure, and the minimum valve opening computed from the throttle command, which holds +16.16 % on scenario 3, was measured outside the application. A rate-of-closure feedforward term would change the outcome of scenario 3 under the override and is the most valuable classical extension.

The surge margin is computed against a fixed line at $\varphi = 2W$. On a real machine the surge limit line is a curve in a corrected plane that moves with speed, suction temperature and gas composition, and much of the engineering in a commercial anti-surge system is in constructing and maintaining that curve. Nothing in this application exercises that part of the problem.

⚠ OPEN CHALLENGE Seeing Surge Before It Arrives

Surge monitoring aims at precursors seen early enough to intervene, not at the surge itself. A 2025 review finds those precursors weak, noisy and different from one machine and operating condition to the next, with no universally accepted set of features that reliably foretells surge. A change of fuel flow or of speed can raise pressure swings that resemble surge, and sensor bandwidth, filtering and data transmission add delay [19]. The chapter's detector, a windowed pressure swing set not to trip on one transient, shows the same trade. Progress would be a warning that holds across machines and arrives before the first reversal.

The pressure pulse event adds 0.04 directly to the plenum state rather than acting through an inlet or a valve, which is convenient for probing the basin of attraction near the peak but is not a physical disturbance. The application labels it a non-physical probe and offers it only among the advanced settings for that reason.

The predictive controller's model is the declared machine, linearized at each decision, so it knows neither a parameter the operator changes nor a drifted restriction, which the estimated imbalance absorbs only after the fact, and it reads the throttle command, not the position, which strokes in $0.12\,\hat{t}$ on scenario 3 against the model's 0.50. Its estimator gain is fixed at the 70 % design point, and the signal carries no preview of a closure before it is commanded.

⚠ OPEN CHALLENGE Active Control Has Yet to Leave the Test Bench

Active stabilization widens the stable flow range of research compressors [13], yet engineering hurdles remain. Sensors may suffer noise, delay or drift. Valves, guide vanes or pistons must answer fast enough, and the fastest add cost and complexity. Most strategies have been shown on simplified test benches or in simulation and must still be validated on full-scale machines, with a real-time platform and safety mechanisms for control failures [19]. A close-coupled valve also costs a pressure drop that may become unacceptably large when the valve is used as a steady device [14]. Progress would be a full-scale demonstration whose gain in range outweighs those losses.

2.10 Conclusion

This chapter specifies a four-state Greitzer compression system and the protective override that guards it, at the level of detail a second implementation needs. The two aerodynamic states carry a Moore-Greitzer cubic whose peak at $\varphi = 2W = 0.500$ is the surge line, and linearizing about the equilibrium gives a trace condition that fails at $\theta = 45.62\,\%$ and $\varphi = 0.4769$ while the static determinant condition never fails at any throttle position, so the Greitzer parameter B decides the outcome alone. Past that boundary the trajectory does not diverge but closes into a limit cycle of period $7.51\,\hat{t}$ with the flow running from −0.175 to 0.711, and lowering B from 1.5 to 0.5 removes the cycle at the same final flow. The default controller is a velocity-form proportional-integral override with back-calculation anti-windup against a lower clamp, which drives the integral toward zero while the valve rests shut, and the scenarios show that its outcome is bounded by valve stroke speed at −105.4 % minimum margin and by valve capacity at −129.0 %, neither of which any tuning repairs. A predictive controller holds the closure of scenario 3 at +15.00 % against −8.61 %, although the override given its throttle signal as a minimum opening holds +16.16 % at 8.6 % more recycle. Reported integral error on this loop measures distance to a setpoint the controller never intends to reach and is quoted only with its run length attached, while minimum surge margin and cumulative recycled flow carry the comparison. A fuller model would add a rate-of-closure feedforward path, a family of speed lines with a surge limit line fitted across them, a hot recycle temperature state, and a close-coupled valve for comparison with active stabilization. Within those limits the simulator lets a limit cycle be measured, and a protective override be operated, before either is met in a plant.

KEY POINTS

- Closed too far, a compressor does not run away. It settles into a repeating cycle in which the flow reverses through the machine.
- The dynamic boundary at $\theta = 45.62\,\%$ lies 2.11 points of throttle travel inside the static surge line, and the Greitzer parameter B alone decides it.
- Back-calculation against the lower clamp lets the integral decay while the valve rests shut, so the valve opens as soon as the flow crosses its setpoint.
- Valve stroke speed and capacity bound the outcome, at −105.4 % and −129.0 % minimum margin, and no tuning repairs either.
- A protective loop is judged by minimum surge margin and recycled flow, not by IAE.

Where else this applies. Volume 10, Chapter 11 runs the same override on three machines that share one load, and Chapter 3 protects a jet engine compressor with limiters that vote on one fuel valve.

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2.11 Exercises

2.1 Concept check · 2 min

With the default $B = 1.5$ and the anti-surge valve shut, the throttle is closed slowly from 70 %. Which stability condition of Equation (2.8) fails first, and where?

(a) The determinant condition, at the peak of the characteristic, 47.73 % throttle
(b) Both conditions together, at the peak, where the slope ψ_c' changes sign
(c) The trace condition, at $\varphi = 0.4769$ and 45.62 % throttle
(d) The determinant condition, at its minimum of 0.639 near 31.2 % throttle

2.2 Concept check · 2 min

In scenario 10, Aggressive PI tuning, the flow keeps oscillating long after the closure, and an operator reports that the machine is surging. Which single test tells a control-induced cycle from aerodynamic surge?

(a) Read the Peak ψ swing tile: any swing above zero means the machine surges
(b) Compare the IAE tile with scenario 4's: the larger IAE belongs to surge
(c) Close the throttle 5 points further: surge grows and a tuning cycle dies out
(d) Switch the loop to Manual: surge continues and a tuning cycle stops

2.3 Concept check · 2 min

The machine runs at 70 % throttle with the valve shut and the flow 0.0761 above the anti-surge setpoint. What does the integral state I of the override do?

(a) It decays toward zero with time constant τ_I, so the demand rests on $K_c e$
(b) It integrates the negative error without bound, so the valve opens late
(c) It is held just below the lower clamp, so the valve opens a little early
(d) It is frozen at its last value until the flow falls below the setpoint

2.4 Theory and calculation · 6 min

Use Equations (2.3) and (2.8) with the defaults of Table 2.2, so $B = 1.5$.

(a) At the healthy point, $\varphi^* = 0.6511$ and $\psi^* = 0.4414$, find ψ_c', the throttle-line slope $\psi_T' = 2\psi^*/\varphi^*$, $\operatorname{tr} J$ and $\det J$. *Checkpoint:* -1.700, 1.356, -3.042 and 2.254.

(b) At the trace crossing, $\varphi^* = 0.4769$ and $\psi^* = 0.5578$, check that $\psi_T' \approx 2.339$. With $\psi_c' = 0.1900$, show that the trace is zero to three decimals and find $\det J$. *Checkpoint:* $1.5 \times 0.1900 = 0.285$ and $1/(1.5 \times 2.339) = 0.285$, so $\operatorname{tr} J \approx 0$. $\det J = 0.919$.

(c) Find the frequency and the period of the linearized oscillation there, and compare the period with the 7.51 $\hat{t}$ of the deep-surge orbit. *Checkpoint:* 0.9585 and 6.56 $\hat{t}$. The orbit is about 15 % slower.

2.5 Theory and calculation · 6 min

Scenarios 3 and 4 close the throttle to 36 %, where $\gamma_T = 1.40 \times 0.36 = 0.504$.

(a) Show that $\varphi^* = 0.350$ is an equilibrium, $\varphi^* = \gamma_T \sqrt{\psi_c(\varphi^*)}$, on the positive-slope branch, and find ψ^*. *Checkpoint:* $\psi^* = \psi_c(0.350) = 0.4822$, and $0.504 \times \sqrt{0.4822} = 0.3500$.

(b) Find ψ_c', ψ_T' and $B_{crit} = 1/\sqrt{\psi_c' \psi_T'}$. Do $B = 1.5$ and $B = 0.5$ surge there? *Checkpoint:* 0.907, 2.756 and 0.633. $B = 1.5$ surges and $B = 0.5$ does not.

(c) With rotor speed and duct unchanged, by what factor must the plenum volume shrink to bring B from 1.5 down to 0.5? *Checkpoint:* A factor of 9.

2.6 Theory and calculation · 5 min

Scenario 1 holds the default point for 29.9 $\hat{t}$ with $e = -0.0761$ at every execution, $K_c = 8$ and $\tau_I = 4$.

(a) Find the demand c_k with the shipped back-calculation, and the IAE the scorecard shows at $\hat{t} = 29.9$. *Checkpoint:* -0.609 and about 2.28.

(b) A positional PI with no anti-windup sums the same error. Find its integral term and its demand at $\hat{t} = 29.9$. *Checkpoint:* -4.55 and -5.16.

(c) A closure then holds the flow at the surge line, $\varphi = 0.500$. How long does the demand of part (b) take to climb back to zero so that the valve can move? Compare with the deep-surge period. *Checkpoint:* About 26 $\hat{t}$, three and a half surge periods.

2.7 App challenge · 8 min

Load scenario 4, Same closure, controller off. Change nothing, run to $\hat{t} = 59.7$ at the default seed and export the CSV file.

(a) Read the Min margin and Peak ψ swing tiles and find the lowest flow from the margin. In the file find the first row with `in_surge` equal to 1 and the times of the minima of `flow_phi` after $\hat{t} = 15$. *Checkpoint:* $-135.1\,\%$, 0.631 and $\varphi = -0.175$. Alarm at $\hat{t} = 11.22$ and minima 7.51 $\hat{t}$ apart, both within 0.02.

(b) Load scenario 8, A smaller plenum (B = 0.5), run to $\hat{t} = 95.5$, and read the same two tiles and the last `flow_phi`. Then set Greitzer parameter to 1.5 in the Parameters tab, press Reset, run to $\hat{t} = 60$ and find the first row with `in_surge` equal to 1. *Checkpoint:* $-43.8\,\%$, 0.211, no alarm and 0.3207 within 0.0005. With $B = 1.5$ the alarm comes at $\hat{t} = 34.32$ within 0.02, and Min margin reads $-137.9\,\%$.

Explain why the run at $B = 0.5$ dips past the surge line but never trips the alarm.

2.8 App challenge · 8 min

Load scenario 5, Fast anti-surge valve, run to $\hat{t} = 29.9$ at the default seed and read the Min margin, Recycled flow and MV travel tiles. Then load scenario 6, Slow anti-surge valve (it does not make it), run to $\hat{t} = 59.3$, read the same tiles and export the CSV file.

(a) Report the three tiles for both scenarios. *Checkpoint:* Scenario 5: $+3.3\,\%$, 4.75 and 182. Scenario 6: $-105.4\,\%$, 11.23 and 387. Travel within 1.

(b) In the scenario 6 file find the first row where `asv_cmd` is above zero, the first where it reaches 100, the `asv_position` in that row, and the rows where `in_surge` turns on and off. *Checkpoint:* 9.02 and 11.02, with the valve about 34.5 % open (within 1). Alarm from 11.74 to 24.62, within 0.02.

Explain why no change of K_c or τ_I would save the scenario 6 closure.

2.9 App challenge · 8 min

Load scenario 11, MPC against the anti-surge PI (rapid closure), and use the default seed. For each run below go to $\hat{t} = 20$ and read the Min margin, Recycled flow and MV travel tiles.

(a) Run the scenario as loaded. *Checkpoint:* $+15.0\,\%$, 2.46 and 85.6.

(b) Under Advanced set Throttle signal to the MPC to Off, press Reset and run again. *Checkpoint:* +4.7 %, 2.27 and 168.

(c) Set Controller to PI, press Reset and run again. *Checkpoint:* −8.6 %, 2.11 and 529.

How much of the predictive controller's margin does the throttle signal supply? The PI given the same signal as a minimum valve opening through a high select, measured outside the app, holds +16.16 % to $\hat{t} = 20$ but recycles 8.6 % more than the MPC. What does the plan add?

2.10 Application area · 4 min

Scenario 7 closes the throttle to 30 % with an anti-surge valve of capacity $K_v = 0.18$.

(a) Holding $\varphi_{safe} = 0.575$ needs a total conductance $\gamma = \varphi_{safe} / \sqrt{\psi_c(\varphi_{safe})}$. Find it, the throttle's share, the valve's share, the smallest K_v that supplies that share wide open, and the opening that the standard $K_v = 0.60$ needs. *Checkpoint:* 0.7874, 0.420 and 0.3674. $K_v \approx 0.37$, and 61 % open.

(b) The command of scenario 7 sits at 100 % for 56.2 % of the run. What should an engineer conclude, and what are the remedies?

2.11 Application area · 4 min

After the challenge's closure to 38 % throttle the PI settles with the flow on its setpoint, so the recycle valve carries $\varphi_u = \varphi_{safe} - \gamma_T \sqrt{\psi_c(\varphi_{safe})}$.

(a) Find φ_u and the fraction of the compressor flow recycled for the setpoints 0.575 and 0.60. *Checkpoint:* 0.1865 and 0.2197, or 32.4 % and 36.6 % of the flow. The wider margin recycles about 18 % more.

(b) The model returns the recycle cold (Section 2.3.6). What does hot recycle change in the cost of part (a), and in the plant needed?

2.12 Review the agent · 5 min

An AI assistant reviews a run of scenario 6, Slow anti-surge valve, and replies: "The controller is too sluggish. Its command does not leave zero until 1.0 $\hat{t}$ after the closure, and the machine surges. Raise the proportional gain to 34 and cut the integral time to 0.8, as in scenario 10, and the valve will open in time." Find the flaw and say how you would check it.