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CHAPTER 4

Fired Heater Simulator for Cross-Limiting, Oxygen Trim and an Optimum Held by a Constraint

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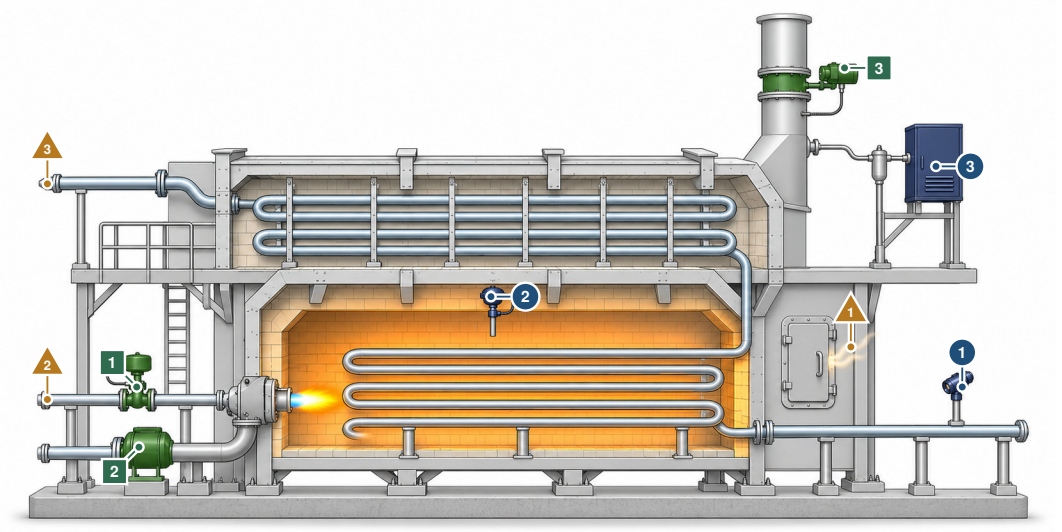
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CHAPTER 4

Fired Heater Simulator for Cross-Limiting, Oxygen Trim and an Optimum Held by a Constraint



The controller moves 1 fuel gas valve 2 combustion air fan, through its flow setpoint 3 stack damper

The controller reads 1 oil outlet temperature 2 draft at the arch 3 stack oxygen and carbon monoxide analyzer

What disturbs it 1 air leaking in at a sight door 2 heating value of the fuel gas 3 oil flow to the heater

Notice that air leaking in at the door reaches the stack analyzer without passing the burner, so the analyzer counts air the flame never had. Illustration, not to scale.

IN PLAIN TERMS **Air Goes First, Then Find the Limit**

The main idea. A fired heater burns gas to warm oil flowing through its tubes. Its fuel valve answers in seconds, but the fan that blows in the air is six times slower.

The challenge. Too little air leaves unburned fuel in a hot box, which can explode, so two simple rules let the air move first. The best mix makes more carbon monoxide, a poison gas, than the permit allows, and a sensor in the chimney that can drift must find that limit.

Why it matters. The same rules guard any process where one mistake is a danger and the other is only a cost.

THREE WAYS TO READ THIS CHAPTER

Explore	11 min	the first page, the history of combustion control and the fact on air leaks (Section 4.2), the try-it segments of scenarios 2 and 8 and the fact on draft (Section 4.5) and the key points
Study	35 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 4.5), Technology in Practice (Section 4.6), Limitations (Section 4.7) and the open challenge

The control question The forced-draft fan of a fired heater answers six times more slowly than its fuel valve, and too little air is a hazard while too much is a cost. How does a pair of selectors keep the burners lean through a load step, and why does a carbon monoxide override, not an oxygen setpoint, find the best legal operating point?

Learning objectives

- Compute the burner and stack air ratios, the stack oxygen and the carbon monoxide from the model, with and without tramp air.
- Explain how the selectors of cross-limiting make the air lead on an increase and follow on a decrease, and what the outlet pays for it.
- Predict the burner air ratio after an unmeasured heating value change and the trim that restores it.
- Locate the efficiency peak and the permit on a walk of the oxygen setpoint, and explain how a high-select override holds the optimum.
- Tell a drifting analyzer and tramp air from a real shortage of air, and judge what a trim clamp and an override each protect against.

Prerequisites Sections 1.4.4 and 1.4.6 (ratio with a trim, cross-limiting), 1.4.3 (static feedforward), 1.6.2 (velocity form) and 1.6.4 (scenarios and exported files).

The app https://apmonitor.com/apps/fired-heater/

See also Volume 4, Chapter 2 and Volume 4, Chapter 6 for protective overrides on a limit with a hazardous side. Volume 9, Chapter 8 for an optimum on a constraint that moves, and Volume 9, Chapter 4 for a heating value no controller measures. Volume 11, Chapter 3, Volume 11, Chapter 14 and Volume 11, Chapter 13 for refinery heaters, their fuel gas and an air trim on a drifting analyzer.

4.1 Introduction

Ratio control is usually taught as a symmetric problem, in which the wild stream moves and the controlled stream follows it at a fixed multiple [1, 2]. On a burner the ratio is not symmetric. Air in excess of what the fuel needs costs efficiency, while air short of it leaves unburned fuel in a hot enclosed space, and a burner management system exists precisely because that condition can end in an explosion [3]. The two flows also differ in speed. A fuel control valve reaches a new flow in about two seconds and a forced-draft fan with its damper takes about twelve, so a ratio that simply slaves the air to the measured fuel runs short of air on every load increase for as long as the fan lags.

The standard answer is cross-limiting. A low selector holds the fuel setpoint to what the measured air can burn and a high selector holds the air setpoint up to what the measured fuel needs, so the air leads on the way up and follows on the way down [3, 4]. Section 1.4.6 compares the three arrangements on the water heater of Chapter 1, and this chapter builds them on a refinery heater. Cross-limiting is a pair of selectors enforcing an inequality, and the same structure reappears one level higher. Efficiency on a heater has an interior maximum in excess air, but on the unit modeled here the maximum sits where carbon monoxide is more than four times the permit. The economic operating point is therefore the lowest excess air at which carbon monoxide stays legal, an active constraint whose position moves with burner condition, fuel and load. Holding a process on an active constraint of unknown position is a selector problem as well [5, 6]. Volume 9, Chapter 8 meets the same situation in a data hall, where the lowest-cost feasible point sits on a constraint that moves with the weather.

The simulator is aimed at two misconceptions. The first is that ratio control is symmetric, which the fuel-first scheme disproves in the first minute of the first load step. The second is that an optimum is a setpoint to be found and held. The class of problem is general: wherever one direction of error is a hazard and the controlled flows respond at different speeds, as in fuel and air on boilers, kilns and furnaces, reactant feeds to exothermic reactors and chemical dosing, the answer is an asymmetric limit rather than a better tuning, and wherever an optimizing loop sits over a safety-relevant constraint the answer is an override rather than a narrower setpoint.

This chapter works with a fired heater simulator in which the combustion scheme, the oxygen trim and its protections can be changed while the heater runs, and it measures the consequence of each change in air ratio, carbon monoxide, outlet temperature and fuel. Cross-limiting, oxygen trim and override control are standard methods. What the model adds is a pair of separate burner and stack air ratios, so that tramp air and analyzer faults behave as they do on a real heater, and a carbon monoxide override written on the logarithm of the concentration with external reset feedback. The scenarios run from a load step that goes fuel-rich to an analyzer that drifts the firebox rich over half an hour. Section 4.2 reviews combustion control on fired equipment. Section 4.3 sets out the model, its two air ratios and its assumptions, and Section 4.4 the temperature loop, the three fuel and air schemes, the trim with its override and the draft loop. Section 4.5 gives the measured results scenario by scenario, through the graded challenge. Section 4.6 describes the real instruments, fan and control platform, Section 4.7 states the limitations, Section 4.8 draws the conclusions, and the exercises follow in Section 4.9.

4.2 Combustion Control on Fired Equipment

Fuel and air were first coupled by a linkage, then metered and trimmed by an analyzer, as the history segment of this section tells. Oxygen tells the operator how much air is left over, while carbon monoxide tells the operator whether the burners were short of it.

> ⧖ HISTORY **From a Shared Linkage to a Carbon Monoxide Trim, 1961 and 1984**
>
> Simple burners have long kept fuel and air in proportion mechanically. One rotary actuator moves the fuel gas valve through a cam and the air damper through a linkage, and changing the ratio is a task for a skilled mechanic. A more automated approach puts a flow loop on one line and a flow transmitter on the other [7]. Trimming the ratio needed a measurement of what the flame achieved. In 1961 a high-temperature galvanic cell was described whose voltage is nearly linear in the logarithm of the oxygen pressure between 600 and 750 °C [8]. By 1984 a steamship operator reported automatic combustion air trim by measuring carbon monoxide, unburned hydrocarbons and opacity in the flue gas, with fleetwide fuel savings of 3.5 percent [9].

Process heaters add their own problems to the boiler practice. The heat is delivered mostly by radiation in the firebox and partly by convection in a separate bank [10]. The draft is natural, held by a stack damper, so air leaks in through sight doors and casing seams, and that tramp air reaches the stack oxygen analyzer without ever passing a burner. Field practice treats a heater that reads normal oxygen with high carbon monoxide as a leaking heater before it treats it as a control problem [11]. Efficiency depends on excess air, stack temperature and ambient conditions in ways that have been measured on operating heaters [12], and the value of each percent of excess oxygen removed is what pays for the analyzers. Volume 11, Chapter 3 places a fired heater at the end of a fouling preheat train, where the fuel valve saturates during a cleaning outage and the crude charge rate has to give way.

> ✦ DID YOU KNOW? **A Leak Can Cost a Tenth of the Fuel**
>
> The draft of a furnace draws cold air in through cracks and badly closed doors, and that air leaves at flue gas temperature. In a high-temperature furnace that is not properly maintained, the extra fuel can reach 10 % of the fuel input [13].

4.3 Process Model

The heater warms 100 kg/s of oil from 253 to 360 °C, absorbing 29.96 MW. At design it fires 0.785 kg/s of fuel gas (2828 kg/h) with 15.36 kg/s of air, a burner air ratio of 1.150, and with 3 % tramp air the stack reads 3.00 % wet oxygen. Carbon monoxide is 41 ppm, the stack is at 355 °C, the bridgewall at 794 °C, and the efficiency is 81.16 %. The arrangement is shown in Figure 4.1.

The variables are classified in Table 4.1. Six states are integrated: the fuel flow F_f, the burner air flow F_a, the heat reaching the process Q_f, the bridgewall temperature T_{bw}, the outlet temperature T_{out}

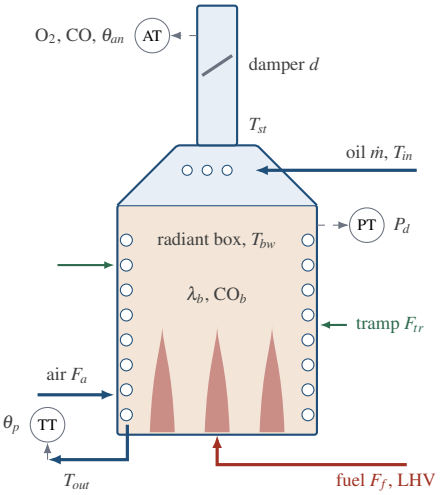


Figure 4.1 Natural-draft process heater. Fuel F_f of heating value LHV and air F_a from the forced-draft fan burn at a burner air ratio λ_b in the radiant box, whose bridgewall temperature is T_{bw}. Tramp air F_{tr} leaks in through sight doors and casing and joins the flue gas. The oil enters the convection bank at T_{in} with flow $\dot{m}$ and leaves the radiant coil at T_{out}, measured by TT after a delay θ_p. The stack damper d holds the arch draft P_d measured by PT, and the analyzer AT reads stack oxygen and carbon monoxide after a sample-line delay θ_{an}.

Table 4.1 Variables at the Design Point

Symbol	Role	Nominal	Range
T_{out}	CV, outlet temperature	360 °C	SP 330–390
O_2	CV, stack oxygen (wet)	3.0 %	SP 0.5–6
CO	constraint, stack	41 ppm	limit 200
P_d	CV, arch draft	−25 Pa	SP −60 to −5
D	MV, firing demand	73.8 %	0–100
ρ	MV, air ratio trim	0 %	±8 %
d	MV, stack damper	44 %	5–100
F_f, F_a	states, flows	0.785, 15.36 kg/s	—
Q_f, T_{bw}, T_{out}, P_d	states	—	—
$\dot{m}$	DV, process flow	100 %	60–120
T_{in}	DV, inlet temperature	253 °C	230–275
LHV	DV, heating value	100 %	85–115
F_{tr}	DV, tramp air at −25 Pa	3 %	0–15
b, $\dot{b}$	DV, analyzer bias, drift	0	±2, ±0.2/min
φ	DV, burner fouling	0	0–1
w	DV, wind gusts (1σ)	0 Pa	0–30

and the arch draft P_d. Everything about the combustion itself is algebraic in the two flows and the draft.

4.3.1 Two Air Ratios

The burners see only the air that passes through them, while the stack analyzer sees the air that reaches the stack. The two ratios are defined in Equation (4.1).

$$\lambda_b = \frac{F_a}{F_f\,\mathrm{AFR}}, \qquad \lambda_t = \frac{F_a + F_{tr}}{F_f\,\mathrm{AFR}} \tag{4.1}$$

where AFR is the stoichiometric air per kilogram of fuel. For gaseous hydrocarbons the air needed per megajoule is nearly constant, so both LHV and AFR scale with the heating value disturbance,

$\mathrm{AFR} = 17.0\,(\mathrm{LHV}/47.0)$. A richer gas therefore needs more air per kilogram, and a ratio set in kilograms for the old gas is short of air on the new one. Tramp air flows through fixed openings and scales with the square root of the draft, Equation (4.2).

$$F_{tr} = \frac{\tau_{tr}}{100}\,(15\,\mathrm{kg/s})\sqrt{\frac{\max(0,-P_d)}{25\,\mathrm{Pa}}} \tag{4.2}$$

with τ_{tr} the leak in percent of design air at the design draft.

4.3.2 Carbon Monoxide, Heat Release and Stack Loss

Carbon monoxide leaving the burners follows an exponential correlation above a knee, Equation (4.3).

$$\mathrm{CO}_b = \min\left[50000,\ \mathrm{CO}_0 + 1000\exp\left(\frac{\lambda_k + 0.08\varphi - \lambda_b}{w}\right)\right] \tag{4.3}$$

with the floor $\mathrm{CO}_0 = 30\,\mathrm{ppm}$, the clean-burner knee $\lambda_k = 1.04$, the width $w = 0.025$ and burner fouling φ moving the knee up. The fraction of fuel energy not released is the fuel that finds no oxygen plus a small carbon monoxide penalty, and the stack loss is the flue gas heated from ambient to stack temperature, Equations (4.4) to (4.6).

$$u_b = \min[1,\ \max(0, 1-\lambda_b) + k_{CO}\min(\mathrm{CO}_b, 20000)] \tag{4.4}$$

$$Q_{rel} = \mathrm{LHV}\,F_f\,(1-u_b)$$

$$T_{st} = T_{in} + \Delta T_{ap}\left(\frac{F_{fl}}{F_{fl,0}}\right)^{0.6} \tag{4.5}$$

$$Q_{abs} = Q_{rel} - F_{fl}c_{p,g}(T_{st} - T_{amb}) - \varepsilon_c\,\mathrm{LHV}\,F_f \tag{4.6}$$

where $F_{fl} = F_f + F_a + F_{tr}$ is the flue gas flow, $F_{fl,0}$ its design value, ΔT_{ap} the stack approach at design flow and ε_c the casing loss. The efficiency is $\eta = Q_{abs}/(\mathrm{LHV}\,F_f)$. Below the knee, carbon monoxide and unburned fuel cost efficiency, and above it every kilogram of extra air is heated to stack temperature, so the efficiency has an interior maximum. Carbon monoxide at the stack is diluted by tramp air, $\mathrm{CO}_{st} = \mathrm{CO}_b\,(F_f + F_a)/F_{fl}$.

The stack oxygen analyzer sees λ_t. For a methane-like fuel burning with λ times stoichiometric air, the wet oxygen fraction is given by Equation (4.7).

$$y_{O_2} = \frac{200\,(\lambda_t - 1)}{1 + 9.52\,\lambda_t}\ \% \tag{4.7}$$

4.3.3 Dynamics

The fuel and air flow loops are represented by their closed-loop responses. The fuel path is a 2 s lag, and the air path is a 12 s lag behind a 2 s damper dead time with a stroking limit of 5 % of fan capacity per second, Equations (4.8) and (4.9).

$$\tau_f\frac{\mathrm{d}F_f}{\mathrm{d}t} = F_f^{SP} - F_f \tag{4.8}$$

$$\frac{\mathrm{d}F_a}{\mathrm{d}t} = \mathrm{clamp}\left(\frac{F_a^{SP}(t-\theta_a) - F_a}{\tau_a},\ \pm 0.05\,F_{a,max}\right) \tag{4.9}$$

The heat reaches the process and the bridgewall through a firebox and refractory lag, and the outlet temperature follows an energy balance on the coil, Equations (4.10) and (4.11).

$$\tau_{fb}\frac{\mathrm{d}Q_f}{\mathrm{d}t} = Q_{abs} - Q_f, \qquad \tau_{fb}\frac{\mathrm{d}T_{bw}}{\mathrm{d}t} = T_{bw}^{*} - T_{bw} \tag{4.10}$$

$$C_{th}\frac{\mathrm{d}T_{out}}{\mathrm{d}t} = Q_f - \dot{m}\,c_p\,(T_{out} - T_{in}) \tag{4.11}$$

Table 4.2 Model Parameters, With Hidden Parameters Not Exposed in the Interface

Symbol	Meaning	Value	Class
LHV_0, AFR_0	design fuel gas	47.0 MJ/kg, 17.0	visible
Q_{max}	fired duty at 100 %	50 MW	visible
$\dot{m}_0$, c_p	process flow, heat capacity	100 kg/s, 2.8	visible
τ_p, θ_p	coil lag, outlet delay	90, 15 s	visible
τ_{fb}	firebox and refractory lag	40 s	visible
f_r	radiant fraction	0.60	visible
$c_{p,g}$	flue gas heat capacity	1.15 kJ/(kg K)	visible
ΔT_{ap}	stack approach	100 K	visible
ε_c	casing loss	1.5 %	visible
T_{amb}	ambient	20 °C	visible
λ_k, w	CO knee, width	1.04, 0.025	visible
CO_0	CO floor	30 ppm	visible
k_{CO}	heat lost per ppm CO	4.5×10^{-6}	hidden
τ_f	fuel loop response	2 s	visible
τ_a, θ_a	air loop response, dead time	12, 2 s	visible
—	fan stroking rate	5 %/s	visible
$F_{a,max}$	fan capacity	25 kg/s	visible
θ_{an}, τ_{an}	analyzer delay, lag	12, 6 s	visible
D_0	stack draft	360 Pa	visible
$P_{c,0}$, $P_{d,0}$	convection, damper loss	150, 125 Pa	visible
τ_D	draft response	2 s	visible
—	CO permit	200 ppm	visible

with $C_{th} = \tau_p \dot{m}_0 c_p$ and $T^*_{bw} = T_{amb} + Q_{rel}(1 - f_r)/(F_{fl} c_{p,g})$. The arch draft is the stack draft less the convection and damper losses, each proportional to the square of the flue gas flow ratio $r = F_{fl}/F_{fl,0}$, Equation (4.12).

$$\tau_D \frac{\mathrm{d}P_d}{\mathrm{d}t} = -D_0 + \left[P_{c,0} + P_{d,0}\, e^{4.5(0.5 - d/100)}\right] r^2 + P_w - P_d \tag{4.12}$$

The damper loss falls exponentially with opening, so a percent of stroke is worth the same fraction of the loss, 4.5 %, anywhere in its range. A plain butterfly characteristic with loss proportional to the inverse square of the open area, matched at 50 %, has 2.6 times the gain at 30 % as at 50 %, each taken at the flue gas flow that the opening holds at −25 Pa, against 1.5 times for the exponential characteristic. An earlier version of the model with the butterfly characteristic made the draft loop unstable at turndown. P_w is a colored wind disturbance.

The outlet thermocouple reads T_{out} after a 15 s delay with colored noise of 0.1 °C. The analyzers read after a 12 s sample-line delay and a 6 s lag. These two delays and the 2 s damper dead time are sampled at each controller execution, so a delayed value is held for up to one 0.5 s scan. The oxygen reading carries its bias, a drift that accumulates at the drift rate, and 0.03 % of noise. The fuel-rich flag is raised whenever $\lambda_b < 1$ and a positive-pressure flag whenever $P_d > 0$.

The parameters are listed in Table 4.2. Each scenario starts from a true steady state computed for its own load, inlet temperature, tramp air and trim setting, so no transient is left over from initialization.

4.3.4 Assumptions

The whole firebox is one burner air ratio. Real heaters have many burners, and the worst-adjusted burner sets the carbon monoxide.

Carbon monoxide is an exponential correlation above a knee rather than a kinetic model. The knee on a real heater moves with burner condition and must be found by test, which is why the economic optimum is approached with an override rather than computed [10].

The fuel is methane-like, with stoichiometric air proportional to heating value. Hydrogen-rich refinery fuel gas breaks that proportionality, because hydrogen needs less air per megajoule than methane.

The fuel and air flow loops are closed-loop lags. Valve characteristics are not modeled, and only the stack damper has a characteristic.

No burner management system is modeled. The fuel-rich flag marks a condition on which a real system would trip the heater, and the simulation continues so that the event can be studied.

Analyzer faults are a bias, a drift and noise. Cell failures, calibration checks and cross-sensitivity are outside the model.

4.4 Implemented Control System

The controller runs every 0.5 s and the physics is integrated by fourth-order Runge-Kutta at 0.1 s, on the three clocks of Section 1.6.1.

4.4.1 Outlet Temperature and Feedforward

The temperature controller TIC is the framework's velocity-form PID of Section 1.6.2, executed every $\Delta t = 0.5\,\mathrm{s}$ on the error $e_k = T_{SP} - T_{out,m}$, Equation (4.13).

$$\begin{aligned} s_k &= s_{k-1} + \Delta u_k^P + \Delta u_k^D \\ c_k &= D_{ff} + s_k + I_{k-1}, \qquad D_k = \mathrm{clamp}(c_k, 0, 100) \\ I_k &= I_{k-1} + \frac{K_c}{\tau_I}\, e_k \Delta t + \frac{\Delta t}{\tau_t}\,(D_k - c_k) \end{aligned} \tag{4.13}$$

where $\Delta u_k^P = K_c(e_k - e_{k-1})$ is the proportional increment and Δu_k^D the change in the derivative term. In the symbols of Equations (1.10) to (1.12), s_k is S_k, c_k is v_k, the applied firing demand D_k is u_k and the feedforward D_{ff} is the output bias u_b. The demand is clipped to 0 to 100 % of the 50 MW fired duty, and only the integral is corrected, by back-calculation with tracking time $\tau_t = \tau_I$. The derivative, off by default, acts on the measurement through a 5 s filter. The defaults are $K_c = 1.0\,\%/^{\circ}\mathrm{C}$ and $\tau_I = 110\,\mathrm{s}$, with 1200 s at the top of the slider meaning proportional only. The fuel setpoint clamp and the cross-limiting low selector act downstream of D_k and are not fed back, so the integral keeps integrating while the fuel waits for the air. The feedforward is the static inverse of the heat balance as a deviation from design, the static feedforward of Section 1.4.3, Equation (4.14).

$$D_{ff} = D_0 \left[\frac{(\dot m/\dot m_0)(T_{SP} - T_{in})}{360 - 253} - 1 \right] \tag{4.14}$$

with $D_0 = 73.3\,\%$ the design demand. It enters as the output bias, and switching it on or off moves the integral by the same amount so the demand does not bump.

4.4.2 Three Fuel and Air Schemes

The demand is converted to a fuel mass rate $F_{dem} = D\,Q_{max}/(100\,\mathrm{LHV}_0)$ at the design heating value, because the controller does not know the actual one. The ratio applied is $R = \mathrm{AFR}_0\,\lambda_0\,(1 + \rho/100)$, with $\lambda_0 = 1.15$ and the trim ρ. The three schemes are given by Equations (4.15) to (4.17).

$$\text{fuel leads:} \quad F_f^{SP} = F_{dem}, \quad F_a^{SP} = R\,F_{f,m} \tag{4.15}$$

$$\text{air leads:} \quad F_a^{SP} = R\,F_{dem}, \quad F_f^{SP} = F_{a,m}/R \tag{4.16}$$

$$\begin{aligned} \text{cross-limited:} \quad F_f^{SP} &= \min[F_{dem},\,(1+\delta)F_{a,m}/R] \\ F_a^{SP} &= \max\left[R\,F_{dem},\,(1-\delta)R\,F_{f,m}\right] \end{aligned} \tag{4.17}$$

where the subscript m marks a measured flow and $\delta = 2\,\%$ is the margin by which fuel may run ahead of air. The cross-limited scheme is Equation (1.8) written in kilograms, with the trimmed ratio R in

place of λ_0. The fuel setpoint is clamped to 12 to 100 % of its maximum and the air setpoint to 20 to 100 % of fan capacity.

🕮 **Pause.** Try Exercise 4.1 before reading on.

4.4.3 Oxygen Trim and the Carbon Monoxide Override

The trim multiplies the ratio, as Section 1.4.4 recommends. It is a high select between an oxygen PI controller and a carbon monoxide PI controller, written in external reset form [14]. Each controller's integral is a first-order filter of the trim actually applied, with its own integral time, Equations (4.18) to (4.20).

$$
\begin{aligned}
F_{O_2} &\leftarrow F_{O_2} + \frac{\Delta t}{\tau_{I,O}}\left(\rho_{k-1} - F_{O_2}\right) \\
\rho_{O_2} &= K_{c,O}\, e_{O_2} + F_{O_2} \qquad (4.18) \\
F_{CO} &\leftarrow F_{CO} + \frac{\Delta t}{\tau_{I,CO}}\left(\rho_{k-1} - F_{CO}\right) \\
\rho_{CO} &= K_{c,CO} \ln\frac{\mathrm{CO}_m}{\mathrm{CO}_{SP}} + F_{CO} \qquad (4.19) \\
\rho_k &= \mathrm{clamp}[\max(\rho_{O_2}, \rho_{CO}), \pm\rho_{max}] \qquad (4.20)
\end{aligned}
$$

where $e_{O_2} = \mathrm{O}_{2,SP} - \mathrm{O}_{2,m}$. The selected controller behaves as an ordinary PI, the unselected one sits below it by the difference in proportional terms, and neither winds up because both filter the clamped trim that was applied. This form replaced a version built from two tracking PID objects that deadlocked with a steady oxygen error, because tracking wiped the integral increment of whichever controller had been selected on the previous scan. The override acts on the logarithm of carbon monoxide because carbon monoxide is exponential in the air ratio, Equation (4.3), so one e-fold of carbon monoxide is always about the same change in air ratio and the loop gain is nearly constant. The defaults are $K_{c,O} = 2.0$ percent of ratio per percent oxygen, $\tau_{I,O} = 60\,\mathrm{s}$, $K_{c,CO} = 2.0$ percent of ratio per e-fold, $\tau_{I,CO} = 60\,\mathrm{s}$, an override setpoint of 150 ppm below the 200 ppm permit, and a trim authority $\rho_{max} = 8\,\%$. Because the selection is a maximum, carbon monoxide can only ever add air. In normal operation the carbon monoxide controller is the unselected one and waits below the oxygen trim, much as the protective override of Volume 4, Chapter 2 rests on its clamp through healthy operation.

🕮 **Pause.** Try Exercise 4.2 before reading on.

4.4.4 Draft

The draft controller PIC is a reverse-acting PI on the arch pressure, opening the damper when the arch rises toward atmospheric, with $K_c = 0.4\,\%/\mathrm{Pa}$, $\tau_I = 3\,\mathrm{s}$, a setpoint of −25 Pa and an output range of 5 to 100 %, on the same algorithm as the TIC with $\tau_t = \tau_I$. The characterized damper holds the process gain between 12.6 and 5.4 Pa per percent of stroke from 60 to 120 % of design flue gas flow, where a butterfly would span 28 to 3.9 Pa per percent.

4.5 Results

Table 4.3 lists the scenarios and Table 4.5 the measured outcomes, each read from the app at its default seed (Section 1.6.1). Steady-state values are ten-minute averages of the logged run, and peaks, crossing times and fuel-rich times are taken at the 0.1 s physics step. The exported file of a run holds both air ratios, the analyzer readings beside their true values and the selected trim controller (Section 1.6.4). Table 4.4 gives the requirement that completes each scenario. The comparisons in cases 3, 5, 7 and 8 start from Reset, which keeps the reader's settings, and a setting changed before Run gives the same run as that setting carried through Reset.

Table 4.3 Scenario Sequence

#	Scenario	Teaching point
1	Fire it by hand	the thermal lag
2	Fuel leads	a load step goes rich
3	Cross-limited	the safe direction
4	Trim toward the peak	an optimum on a constraint
5	A fuel-gas swing	an unmeasured disturbance
6	Drift, wide-open trim	trusting an analyzer
7	Drift, clamped, overruled	bounding the trim
8	Tramp air	two air ratios
9	Draft in the wind	margin to atmosphere
10	Challenge	graded run

Table 4.4 Scenarios and Completion Requirements

#	Scenario	Kind	Required to complete
1	Fire it by hand	Experiment	In Manual, hold the firing demand at 73.8 % for 1 min, step it to 78.8 % and hold it for 5 min, then return the outlet to 360 °C by hand and hold it within ±0.5 °C for 1 min.
2	Fuel leads	Demonstration	Run with fuel leading through the +20 % process flow step at 60 s until the burners go fuel-rich ($\lambda_b < 1$), and on to 3 min.
3	Cross-limited	Experiment	Run 4 min cross-limited through the same step, Reset and run 4 min with air leading, then step the process flow down 20 % and follow the fuel and air for 30 s.
4	Trim toward the peak	Experiment	With the override off, hold the oxygen setpoint at 2.0 % for 3 min and then at 1.0 % or below for 3 min, switch the CO override on with the setpoint below 1.5 %, and keep the override in control for 5 min.
5	A fuel-gas swing	Experiment	Run 10 min through the +6 % heating value step at 60 s with the oxygen trim on, then Reset and run 5 min with the trim off.
6	Drift, wide-open trim	Demonstration	Run with the analyzer drifting 0.1 % oxygen per minute from 60 s until the reading exceeds the true oxygen by 1 %, and on to 30 min.
7	Drift, clamped, overruled	Experiment	With the override off, run until the trim reaches its −8 % clamp and on to 30 min, then Reset and run 30 min with the CO override on.
8	Tramp air	Experiment	With the sight doors opened at 60 s, keep the CO override in control for 5 min, then Reset and run 15 min with the override off.
9	Draft in the wind	Experiment	In 30 Pa gusts, run 1 min at −25 Pa, then hold the arch draft setpoint at −10 Pa for 10 min, at −5 Pa for 5 min and at −40 Pa for 5 min.
10	Challenge	Challenge	Through the load step at 2 min, the fuel-gas swing at 10 min and the analyzer drift from 15 min: never fuel-rich, CO never above 200 ppm, a run-average efficiency of at least 80.5 %, and the outlet within ±1 °C over the final 10 min, judged at 30 min with the model parameters and disturbances as given and no extra disturbance button pressed.[a]

[a] With the shipped settings (fuel leads, no override, ±25 % trim) the run fails, with 3.7 s fuel-rich and 118 s over the permit, although its 80.70 % efficiency and 0.22 °C worst outlet error meet their criteria. With the suggested settings (cross-limited, override on, ±8 % trim) it passes with no time rich or over the permit, a 173 ppm peak 27 ppm under the permit, 80.70 % efficiency 0.20 points above the target and a 0.21 °C worst outlet error.

4.5.1 A Load Step in the Firebox

The thermal side is slow. A 5 % firing step in manual gives 6.9 °C at the outlet once settled, about 1.4 °C per percent, of which 6.5 °C has arrived after five minutes. The outlet barely moves for 20 s, and 63 % of the change has arrived after about 143 s. The 2 s fuel valve is faster than that chain by a factor of about seventy.

The combustion side is fast, and the scheme decides what happens in it. With the air slaved to the measured fuel, a 20 % process flow step with the feedforward on sends the fuel ahead of the fan. Burner λ falls to 0.990, the firebox is fuel-rich for 3.7 s and carbon monoxide peaks near 7300 ppm.

Table 4.5 Measured Results

Case and condition	Result
1, 5 % firing step	6.9 °C, 63 % after 143 s
2, fuel leads, +20 % load	λ_b 0.990, 3.7 s rich, CO 7270 ppm
3, cross-limited, same step	$\lambda_b \geq 1.136$, CO 50 ppm
2 and 3, outlet dip	4.6 against 5.8 °C
3, air leads, outlet dip	6.3 °C
4, O_2 3.0 / 2.5 / 2.0 %	η 81.16 / 81.73 / 82.23 %
4, O_2 2.0 %	CO 210 ppm
4, O_2 1.2–1.5 %	η 82.51–82.55 %, CO $\geq$ 660 ppm
4, override on	O_2 2.15 %, CO 150 ppm, η 82.09 %
5, LHV +6 %, no trim	λ_b 1.085, CO 190 ppm
5, LHV +6 %, trim	trim +6.0 %, λ_b 1.150
6, drift, ±25 %	over permit at 12 min, rich at 26 min
6, end of run	true O_2 0.3, reading 3.2 %, 187 s rich
7, drift, ±8 % clamp	λ_b 1.055, CO 500 ppm
7, clamp and override	CO 150 ppm, reading 5.1 %
8, tramp air, override off	O_2 3.0 %, λ_b 1.061, CO 420 ppm
8, tramp air, override on	O_2 3.4 %, λ_b 1.090, CO 150 ppm
9, 30 Pa gusts, SP −25 / −10 / −5 Pa	positive 0 / 2 / 86 s

Cross-limited, the same step never takes burner λ below 1.136 and carbon monoxide peaks at 50 ppm. The price is paid in temperature, because the heat now waits for the fan: the outlet dips 5.8 °C against 4.6 °C for fuel leads (Table 4.5), and Figure 4.2 shows the heater 65 s after the step. Air leads is safe as well, with a 6.3 °C dip, but on a 20 % load decrease, pressed at 240 s to return the flow to design, it makes the fuel wait for the fan too. Over the next ten minutes its temperature error integrates to 1725 °C s, about twice the 810 and 898 °C s of fuel leads and cross-limiting. Cross-limiting behaves as air leads on the way up and as fuel leads on the way down, which is the rule stated as two selectors.

> **TRY IT IN THE APP** **2 · Fuel Leads, and a Load Step Goes Rich**
>
> Load the scenario and press Run. At the 60 s marker the process flow rises 20 % and the fuel moves before the air. Watch the burner line of the Burner air ratio λ strip dip below one, then read the Time fuel-rich and Peak CO tiles: about 3.7 s rich and a peak near 7300 ppm. Scenario 3 runs the same step cross-limited.

4.5.2 The Optimum and the Permit

Walking the oxygen setpoint down with the override off, the steady-state efficiency, averaged over ten minutes, is 81.16 % at 3.0 % oxygen, 81.73 % at 2.5 and 82.23 % at 2.0, where carbon monoxide is already 210 ppm, over the permit. The efficiency peak, 82.55 % at 1.37 % oxygen with carbon monoxide near 890 ppm, is flat within 0.04 points between 1.2 and 1.5 % oxygen, where carbon monoxide is 660 ppm and above, so the true economic optimum lies on the carbon monoxide constraint and not at the efficiency maximum. With the override on and the oxygen setpoint pushed to 1.5 % or below, the heater settles at 2.15 % oxygen, 150 ppm and 82.09 %, burning about 32 kg/h less fuel than at 3 %. The operating point is found by the constraint, and it moves when the constraint moves, which a fixed oxygen setpoint cannot do.

4.5.3 An Analyzer That Cannot Be Trusted

A heating value increase of 6 % is invisible to every measurement except the temperature and the oxygen. Without trim the burner ratio settles at 1.085 with carbon monoxide near 190 ppm, close enough to the permit that thermocouple noise, passed to the fuel by the temperature loop, carries it over for 70 s in the first ten minutes. With trim the trim rises about 6.0 % and restores $\lambda_b = 1.150$. Either way the transient dips λ_b to about 1.08 before the analyzer and trim respond. Volume 11, Chapter 14 follows such swings to their source, a refinery fuel gas header, and feeds the Wobbe index of the gas forward to the burners.

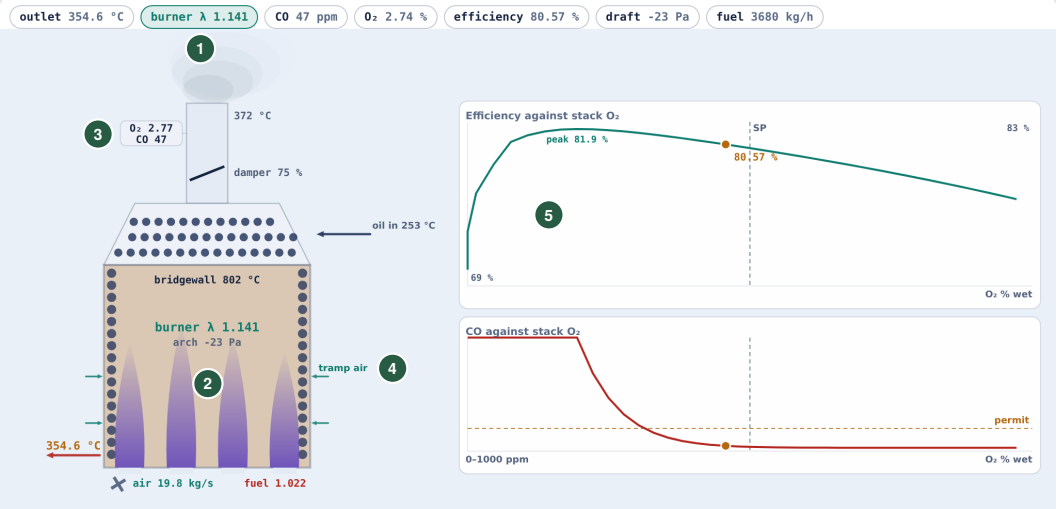


Figure 4.2 The fired heater running scenario 3 with the fuel and air cross-limited, 65 s after a 20 % process flow increase at 60 s, cropped to the plant view. (1) The row of readings, with the outlet at 354.6 °C, the burner air ratio at 1.141 and carbon monoxide at 47 ppm. (2) The four burners, with flames colored by the burner air ratio, between the wall coils. (3) The stack with its damper and the analyzer's oxygen and carbon monoxide readings. (4) Tramp air entering at the sight doors. (5) Efficiency against stack oxygen at the present conditions, with the operating point, the oxygen setpoint and the efficiency peak left of the setpoint, above carbon monoxide against stack oxygen, which rises steeply below the setpoint and crosses its permit.

A trim that follows its analyzer follows its analyzer's faults. With the authority at 25 % and no override, an oxygen reading drifting 0.1 % per minute takes carbon monoxide past the permit about 12 min after the drift begins, and the burners first go rich about 26 min after it begins. By the end of the half hour the true oxygen is 0.3 % against a reading of 3.2 %, and the heater has spent 187 s fuel-rich. Clamping the trim at ±8 % bounds the damage: the trim reaches the clamp about 17 min in, burner λ bottoms at 1.055 and carbon monoxide settles near 500 ppm. The override removes it. Carbon monoxide takes the trim about 11 min in and holds near its 150 ppm setpoint, at 145 ppm after 30 min, while the oxygen reading climbs to 5.1 %. A drifting analyzer takes the Claus plant of Volume 11, Chapter 13 outside its permit in the same way, with its air demand reading on target.

Tramp air produces a different signature from the same instruments. With the sight doors opened and the override off, the stack oxygen settles at 3.0 %, its setpoint, while the burner ratio falls to 1.061 and carbon monoxide settles near 420 ppm, because the trim has removed burner air to make up for the air leaking in. With the override on, carbon monoxide holds at 150 ppm, λ_b at 1.090 and the stack oxygen reads 3.4 %, each averaged over the last ten minutes of a half hour. High oxygen, normal carbon monoxide and an active override are the combination that identifies a leak, which is how field troubleshooting describes it [11].

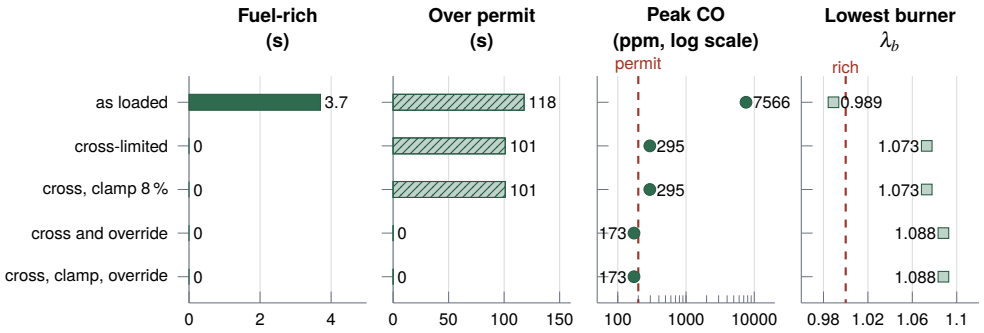


Figure 4.3 Table 4.6 redrawn: the five challenge configurations over 30 min, with time fuel-rich and time over the permit in bars, and peak carbon monoxide (round dots, logarithmic scale) and lowest burner air ratio (square dots) against the dashed permit and stoichiometric lines. Notice that cross-limiting alone ends the fuel-rich time but leaves carbon monoxide over the permit, and only the rows with the override come to rest left of the permit line.

Table 4.6 Challenge Outcomes Over 30 min

Configuration	Rich (s)	Over permit (s)	Peak CO	min λ_b
as loaded	3.7	118	7566	0.989
cross-limited	0	101	295	1.073
cross, clamp 8 %	0	101	295	1.073
cross and override	0	0	173	1.088
cross, clamp, override	0	0	173	1.088

TRY IT IN THE APP **8 · Tramp Air the Burners Never See**

Run the scenario as loaded for half an hour. The sight doors open at 60 s, the burner and stack lines part on the λ strip, and the CO override holds carbon monoxide near 150 ppm. Then Reset, switch CO override off and run again. The stack oxygen sits on its 3.0 % setpoint while carbon monoxide settles near 420 ppm.

With 30 Pa wind gusts the arch at a −25 Pa setpoint peaks at −10.5 Pa in fifteen minutes. At −10 Pa it goes positive for about 2 s, and at −5 Pa for about 86 s, counted at the physics step. Five other noise seeds give 4 to 12 s and 71 to 97 s, so the ordering is robust and the counts are not.

DID YOU KNOW? **The Last Few Pascals of Draft**

From the simulator: in 30 Pa gusts the arch spent no time above atmospheric with its draft setpoint at −25 Pa, about 2 s at −10 Pa and about 86 s at −5 Pa (Table 4.5). Other noise seeds change the counts but not their order.

4.5.4 The Graded Challenge

The challenge runs 30 min with a 20 % load step at 2 min, a 6 % fuel-gas swing at 10 min and an analyzer drifting from 15 min, starting with fuel leading, no override and a 25 % trim. It is met with no time fuel-rich, carbon monoxide never over the permit, a run-average efficiency of at least 80.5 % and the outlet within 1 °C for the last ten minutes. Table 4.6 reports the measured outcomes, and Figure 4.3 draws them.

Cross-limiting removes the fuel-rich excursion but not the drift. The clamp alone changes nothing within the half hour, because the drift drives the trim down from the +5.3 % the fuel-gas swing required, and it has reached only −0.3 % when carbon monoxide crosses the permit at 28 min and

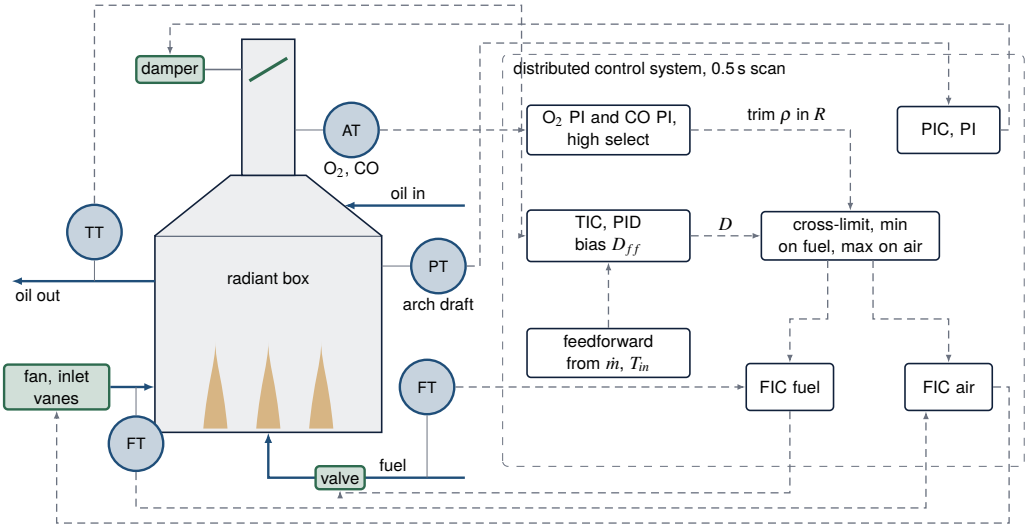


Figure 4.4 The combustion loops on real hardware. The thermocouple TT feeds TIC, whose output bias carries the feedforward from the process flow and inlet temperature. Its demand D enters the cross-limiting selectors, which also read the measured flows and set the fuel and air flow loops on the fuel gas valve and on the fan's inlet vanes. The analyzer AT drives the trim, a high select between the oxygen and carbon monoxide controllers, and PIC holds the arch draft with the stack damper. Solid arrows are process streams and dashed gray arrows are signals. Notice that the trim and the selectors only move setpoints of the two flow loops, so on a load increase the slow fan sets the pace.

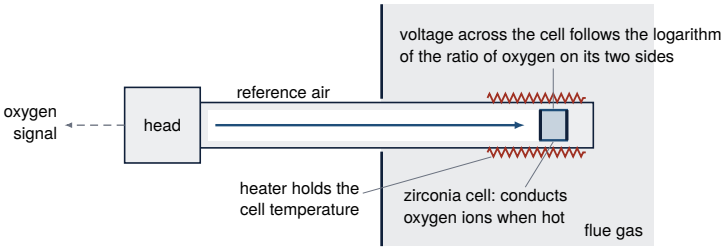


Figure 4.5 The zirconia oxygen cell. Notice that the voltage compares the oxygen at the probe with the reference air. Nothing in the cell knows whether that oxygen passed through a burner, so tramp air reads as excess air, the lesson of scenario 8.

−1.3 % at the end of the run, far inside the ±8 % limit. Only the override keeps carbon monoxide legal, with a peak of 173 ppm, and the clamp then never binds, so the suggested configuration of the last row runs identically to cross-limiting with the override alone. The run average efficiency is 80.70 to 80.71 % in every configuration, and the outlet stays within 0.22 °C over the last ten minutes in every configuration, so the challenge lies entirely in the firebox. Over five other noise seeds the run as loaded spends 3.3 to 3.7 s fuel-rich and 108 to 117 s over the permit, and the suggested configuration never goes rich or over the permit.

🕮 **Pause.** Try Exercise 4.3 before reading on.

4.6 Technology in Practice

Figure 4.4 draws the loops of Section 4.4 with their hardware.

What Measures It

Each flow is measured by its own transmitter, because a flow loop on one line and a transmitter on the other is what turned a mechanical ratio into one a controller can compute [7]. The model hides both meters inside the closed-loop lags of Equations (4.8) and (4.9). The outlet thermocouple TT reads after 15 s. The stack gas is read by an analyzer for oxygen and carbon monoxide, and the oxygen cell is drawn in Figure 4.5. The 12 s delay and 6 s lag of AT stand for the sample line and the cell, and its bias and drift stand for a cell between calibrations.

> **TECHNOLOGY** **Sensor: A Zirconia Cell at the End of a Sample Line**
>
> A common flue gas oxygen detector is a sandwich of platinum electrodes on a solid zirconium oxide electrolyte, held hot, with the flue gas on one side and reference air on the other. Its voltage follows the logarithm of the ratio of the two oxygen activities, and a microcomputer inverts that function [7]. An oxygen analyzer for flue gas spans zero to the oxygen of air and is calibrated against air at 20.9 % and against nitrogen at zero. An extractive system adds a probe and a heat-traced sample line, and analyzer systems need much more upkeep than other instruments [7]. A slow drift looks on a trend like a real change in excess air.

What Moves It

The firing demand reaches the burners through the fuel gas control valve, which the model treats as a 2 s lag. The air path is the slow one.

> **TECHNOLOGY** **Final Element: A Forced-Draft Fan and Its Inlet Vanes**
>
> A forced-draft fan pushes the combustion air into the furnace [13]. Its flow is commonly set by inlet vanes, which swirl the entering air in the direction of the impeller and so lower the fan's output, or by dampers, which add backpressure and move the fan along its curve. A fan throttled far back can run unstably, its airflow surging back and forth. Vanes and dampers move through mechanical linkages, and dirt built up on a linkage impairs its travel [15]. The model's 2 s dead time, 12 s lag and stroking limit stand for that actuator and the fan. Under cross-limiting, air that cannot rise holds the fuel back with it, so a sticking linkage shows as an outlet temperature that sags.

The stack damper holds the draft. A damper is a multi-element device that throttles large flows of air at low pressure, built with parallel or radial vanes, and furnace and boiler draft control is one of its common uses [7]. The model gives it the exponential characteristic of Equation (4.12).

Where the Controller Runs

Control loops in a plant are often nested or cascaded, with cycle times from milliseconds to minutes, and a distributed control system uses a supervisory loop to mediate a group of localized controllers [16]. In the app every loop runs on a 0.5 s scan, against a fuel lag of 2 s and a coil lag of 90 s. On such a platform TIC is a PID function block whose output bias carries the feedforward. The ratio is a multiplying block, cross-limiting is a low and a high selector on the two flow setpoints, and the trim is a high select over two PI blocks in external reset form. The burner management system is a separate safety system that watches for low or high fuel pressure and loss of flame and shuts off the fuel [7].

> **IN PRACTICE** **The Annual Tune-Up: Burner, Ratio, Then Carbon Monoxide**
>
> A United States emission rule requires an annual tune-up of each boiler or process heater of 10 million Btu per hour or more. The burner is inspected and cleaned, the flame pattern adjusted and the air-to-fuel ratio control checked for calibration. Carbon monoxide is then optimized, and carbon monoxide in parts per million and oxygen in percent are measured before and after the adjustments. The annual frequency does not apply to units with a continuous oxygen trim that holds an optimum air-to-fuel ratio [17].

What Is Modernizing It

The first development is the fan drive. Fan speed adjustment is in general the most efficient way to control airflow, and in dirty air an adjustable-speed drive avoids the fouled linkages of vanes and dampers [15]. The second is where the gas is read. A single-beam infrared analyzer of the gas filter correlation kind suits carbon monoxide among carbon dioxide and water vapor, and it can pass its beam across the diameter of the stack instead of through a sample cell [7]. A path across the stack has no sample line to wait for. The third is the fuel itself. A federal roadmap counts large fired heaters among the larger greenhouse gas sources of a refinery and judges that the most cost-effective capture would likely be a central reformer with carbon capture supplying hydrogen to the process fired heaters [18], the subject of the open challenge below.

4.7 Limitations

The single burner air ratio hides burner-to-burner variation, which on a real heater makes the carbon monoxide knee both earlier and less sharp.

The carbon monoxide correlation is not kinetic, and its knee has been placed so that the efficiency peak falls beyond the permit. That placement is typical but not universal, and on a heater with a sharper knee the optimum and the constraint can coincide.

Tramp air enters at one point with a square-root law. Real leaks are distributed and some enter above the burners, where they contribute to the convection section but not to combustion.

The fuel and air flow loops are lags, so valve nonlinearity, fan surge and air density changes with ambient temperature are absent [12].

There is no burner management system, no flame supervision and no startup or shutdown sequence. The fuel-rich flag marks the condition on which such a system would trip.

> **OPEN CHALLENGE** **Firing Refinery Heaters on Hydrogen**
>
> Hydrogen burned in air gives a flame near 2100 °C in special burners. For refining, a federal roadmap calls for the capability to burn hydrogen for high-temperature process heat and lists the needs that remain: applicability at furnaces with higher hydrogen in blends, temperature control and management, embrittlement, unified standards for retrofits and moisture after combustion [18]. The ratio of this chapter is set for a methane-like fuel (Section 4.3), and the constraint its override holds already moves with burner condition, fuel and load. Progress would be heaters run on rising hydrogen fractions with their ratio, trim and override shown to hold.

4.8 Conclusion

A fired heater teaches that the safe structure and the economic structure are made of the same parts. In the simulator of this chapter, a 20 % load step with the air following the fuel takes the burners to $\lambda = 0.990$ for 3.7 s with carbon monoxide near 7300 ppm, and the same step cross-limited never goes below 1.136, at a cost of 5.8 °C of outlet dip against 4.6 °C. The efficiency optimum lies beyond the carbon monoxide permit, so the best operating point is the permit itself, reached by a logarithmic carbon monoxide override at 2.15 % oxygen and 82.09 % efficiency against 81.16 % at the conventional 3 %. The same override detects an analyzer drifting 0.1 % per minute that would otherwise make the firebox fuel-rich within half an hour, and it separates tramp air from a real shortage of air. The model uses one air ratio for the whole firebox, a correlated rather than kinetic carbon monoxide, and closed-loop lags for the flow loops. A fuller model would add multiple burners, hydrogen-rich fuel gas, air preheat and a burner management sequence. Within those limits the heater shows how control structures are built from selectors.

KEY POINTS

- On a burner, too little air is a hazard and too much is only a cost, and the fan is much slower than the fuel valve.
- With the air following the fuel, a 20 % load step takes the burners to $\lambda = 0.990$. Cross-limited, the same step never goes below 1.136, at a cost of 5.8 °C of outlet dip against 4.6 °C.
- The efficiency optimum lies beyond the carbon monoxide permit, so the best operating point is the permit itself, held by a logarithmic override.
- High oxygen, normal carbon monoxide and an active override are the combination that identifies a leak.

Where else this applies. Volume 4, Chapter 2 and Volume 4, Chapter 6 hold protective overrides on a limit with a hazardous side, Volume 9, Chapter 8 holds an optimum on a constraint that moves, and Volume 15, Chapter 11 cross-limits coal and air on a kiln.

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4.9 Exercises

4.1 Concept check · 2 min

The fuel and air are cross-limited, and the process flow steps up by 20 % with the feedforward on. Which quantity sets the fuel flow setpoint in the seconds after the step?

(a) The firing demand of the TIC, as a fuel rate at the design heating value
(b) The measured air flow divided by the ratio, with the margin δ added
(c) The measured fuel flow multiplied by the ratio, with the margin δ taken off
(d) The stack oxygen reading, acting through the trim on the air-to-fuel ratio

4.2 Concept check · 2 min

Why does the carbon monoxide override act on $\ln(\mathrm{CO}_m/\mathrm{CO}_{SP})$ rather than on the difference $\mathrm{CO}_m - \mathrm{CO}_{SP}$?

(a) The logarithm filters the analyzer noise that the sample line adds
(b) The permit is written as a logarithmic limit on the stack concentration
(c) CO is exponential in λ_b, so an e-fold of CO is a fixed change in λ_b
(d) A logarithm turns negative below the setpoint and lets CO remove burner air

4.3 Concept check · 1 min

In the challenge, cross-limited without the override, a trim authority of ± 8 % leaves 101 s over the permit, the same as ± 25 % (Table 4.6). Why does the clamp change nothing?

(a) The clamp limits the fuel setpoint, so it cannot act on the air
(b) The margin δ of the selectors cancels any limit set on the trim
(c) The temperature loop restores the air that the clamp holds back
(d) The drift has carried the trim only to about -1.3 % by 30 min

4.4 Theory and calculation · 6 min

At the design point of Table 4.1 the heater burns $F_f = 0.785\,\mathrm{kg/s}$ with $F_a = 15.36\,\mathrm{kg/s}$, AFR $= 17.0$, and a tramp leak of 3 % at −25 Pa.

(a) Find the stoichiometric air, λ_b, the tramp air of Equation (4.2), λ_t and the wet stack oxygen of Equation (4.7). *Checkpoint:* 13.35 kg/s, 1.15, 0.45 kg/s, 1.185 and 3.0 %.

(b) The sight doors open and the leak rises to 11 % at −25 Pa. The trim holds the stack oxygen at 3.0 %. With F_f kept at 0.785 kg/s, find λ_b. *Checkpoint:* 1.060.

(c) Find the carbon monoxide at the burners and at the stack for that λ_b. *Checkpoint:* About 470 ppm and 425 ppm.

4.5 Theory and calculation · 6 min

The fuel gas heating value rises by 6 %, and AFR rises with it as in Section 4.3. The O_2 trim is off with $\rho = 0$, so the controller applies $R = \mathrm{AFR}_0\lambda_0$, and the TIC returns the outlet to its setpoint.

(a) Find the burner air ratio once the heater has settled. *Checkpoint:* 1.085.

(b) Find the trim ρ that restores $\lambda_b = 1.150$, and say whether the default authority allows it. *Checkpoint:* +6.0 %, inside the ± 8 % authority.

(c) With the trim off, estimate the carbon monoxide at the burners and at the stack and the stack oxygen. Take the tramp air as 0.45 kg/s, the stoichiometric air as 13.35 kg/s and the fuel as $0.785/1.06$ kg/s. *Checkpoint:* About 196 and 190 ppm, and 2.0 %.

4.6 Theory and calculation · 6 min

Scenario 3 starts with the firing demand at 73.8 %, the air at 15.36 kg/s and $R = 19.55\,\mathrm{kg/kg}$. At 60 s the process flow steps to 120 % with $T_{SP} = 360\,^{\circ}\mathrm{C}$ and $T_{in} = 253\,^{\circ}\mathrm{C}$.

(a) Find the change in D_{ff} from Equation (4.14), the demand just after the step, and the fuel rate F_{dem} it asks for. *Checkpoint:* 14.66 %, 88.5 % and 0.941 kg/s.

(b) Find the cross-limited fuel and air setpoints just after the step. *Checkpoint:* 0.801 kg/s and 18.4 kg/s.

(c) With fuel leading instead, suppose the fuel reached 0.941 kg/s before the air had moved at all. Find λ_b. *Checkpoint:* 0.960.

4.7 App challenge · 9 min

Load scenario 2, Fuel leads, and a load step goes rich. Change no setting, run to 3 min at the default seed, read the Time fuel-rich, Peak CO and Lowest burner λ tiles, and export the CSV file. Load scenario 3, The same step, cross-limited, run to 4 min, read the same tiles and export. Then set Fuel and air to Air leads, press Reset, run to 4 min and export again.

(a) Report the three tiles for scenarios 2 and 3. *Checkpoint:* Scenario 2: 3.7 s, 7268 ppm, 0.990. Scenario 3: 0.0 s, 50 ppm, 1.136. Peak CO within 20 ppm.

(b) In each of the three files find the lowest `tout_C` after 60 s and the dip below 360 °C. *Checkpoint:* Fuel leads 4.6, cross-limited 5.8 and air leads 6.3 °C, within 0.1 °C.

Explain the order of the three dips.

4.8 App challenge · 8 min

Load scenario 4, O_2 trim toward the efficiency peak, and use the default seed.

(a) Before Run set Stack O_2 setpoint to 2.0 %. Run to 40 min and export the file. Average `efficiency_pct` and `co_ppm` over the rows from 30 to 40 min. *Checkpoint:* 82.23 % within 0.03, and 210 ppm within 5 ppm.

(b) Press the reload button, set Stack O_2 setpoint to 1.0 % and CO override On before Run, and run to 40 min. Average `o2_true_pct`, `co_ppm`, `efficiency_pct` and `co_selected` over the same rows. *Checkpoint:* 2.15 %, 150 ppm, 82.09 % and 1.

Explain why the heater in run (b) does not go to the 1.0 % it was asked for.

4.9 App challenge · 8 min

Load the challenge, Half an hour of trouble, and use the default seed. For each run below go to 30 min and read the Time fuel-rich, Time CO over permit, Peak CO and Lowest burner λ tiles.

(a) Run the challenge as loaded. *Checkpoint:* 3.7 s, 118 s, 7566 ppm, 0.989.

(b) Set Fuel and air to Cross-limited, press Reset and run again. *Checkpoint:* 0.0 s, 101 s, 295 ppm, 1.073.

(c) Also set CO override On, press Reset and run again. Read the Efficiency, run average tile and the challenge verdict as well. *Checkpoint:* 0.0 s, 0 s, 173 ppm, 1.088, 80.70 %, passed.

Say which of the three scripted events each change answers.

4.10 Application area · 4 min

With the override on, the heater of Section 4.5.2 settles at 2.15 % oxygen and 82.09 %, burning about 32 kg/h less fuel than at 3 % oxygen and 81.16 %.

(a) Find the fuel saved in a year of continuous operation, in tonnes and in terajoules at 47.0 MJ/kg, and check the 32 kg/h against the two efficiencies and the design fuel rate of 2828 kg/h. *Checkpoint:* About 280 t and 13.2 TJ. A fuel ratio of 81.16/82.09 saves 1.13 %, or 32 kg/h.

(b) The Limitations section says that burner-to-burner variation makes the knee of a real heater earlier and less sharp. What does that mean for the saving, and for the choice between a fixed oxygen setpoint and the override?

4.11 Application area · 3 min

The arch draft setpoint trades tramp air against the risk of a firebox above atmospheric.

(a) With the design leak of 3 %, find the tramp air of Equation (4.2) at -25, -10 and -5 Pa. *Checkpoint:* 0.45, 0.28 and 0.20 kg/s.

(b) In 30 Pa gusts the arch goes positive for 0, about 2 and about 86 s in fifteen minutes at these setpoints, and for 4 to 12 s and 71 to 97 s at the two higher setpoints over five other seeds. Which setpoint would you run in this wind, and why does the worst gust set it?

4.12 Review the agent · 5 min

An AI assistant reviews a run of scenario 8, Tramp air the burners never see, with the CO override off, and replies: "The stack O_2 is on its 3.0 % setpoint, so the burners have their 15 % excess air. Carbon monoxide near 420 ppm with correct air means the burners are fouled. Raise the O_2 setpoint to 4.0 % until CO is under the permit, and schedule a burner cleaning." Find the flaw and say how you would check it.