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CHAPTER 12

Commissioning a Servo Axis Whose Coupling Resonance Caps the Speed Loop

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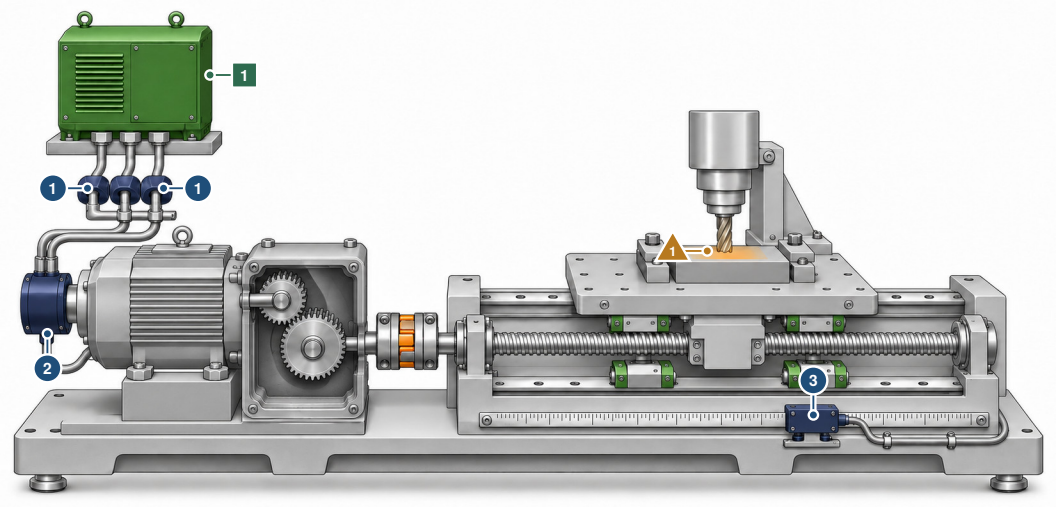
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CHAPTER 12

Commissioning a Servo Axis Whose Coupling Resonance Caps the Speed Loop



The controller moves 1 inverter that sets the motor's current

The controller reads 1 current sensors on the motor leads 2 encoder on the motor
3 position read on the load side (an encoder on the screw in the model)

What disturbs it 1 cutting force on the table

Notice that the motor's encoder and the load-side reading sit on opposite sides of the gears and the springy coupling, so the motor can read on target while the table is not. Illustration, not to scale.

IN PLAIN TERMS **Tuning a Machine Axis Around a Springy Joint**

The main idea. A motor turns a long screw that slides the table of a machine. A controller sets the motor's current, its speed and the table's position, one loop inside the next, and it is tuned from the inside out.

The challenge. The motor reaches the table through gears with a small gap and a coupling that twists like a stiff spring. A sensor on the motor cannot see the gap or the twist, and a speed loop pushed too hard makes the motor and the table ring against each other.

Why it matters. A machine that cuts metal or places parts needs axes that stop where they are told.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history of speed set by voltage (Section 12.2), the fact on error motions (Section 12.3), the try-it segments of cases 3 and 6 and the fact on the hunting integral (Section 12.5) and the key points
Study	65 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	25 min	the first page, the results (Section 12.5), Technology in Practice (Section 12.6), Limitations (Section 12.7) and the open challenges

The control question A servo motor drives a machine table through a flexible coupling and a gear pair with a gap. What caps the gain of its speed loop, what can a motor encoder not see, and what does a relay autotuner measure on such an axis?

Learning objectives

- Compute the anti-resonance, resonance and locked-load frequencies of the two-mass axis from its inertias and stiffness.
- Compute the voltage a motor needs at speed and the base speed at which the inverter's voltage circle runs out.
- Read the gain and phase margins of a speed loop that crosses unity more than once, and use a notch filter to raise its usable gain.
- Explain why Ziegler–Nichols gains from a relay test tune a resonant axis to its coupling.
- Budget the final error of a position loop closed on the motor encoder, and compare feedforward, a load encoder and an explicit MPC on one move.

Prerequisites Sections 1.6.2 (velocity form), 1.4.7 (relay test), 1.4.9 (windup), 1.4.10 (transfer), 1.4.11 (sample time) and 1.4.12 (margins). Section 1.6.3 is needed only for the explicit MPC. Chapter 2 introduces the algorithm on a simpler plant.

The app https://apmonitor.com/apps/servo-axis/

See also Volume 3, Chapter 2 for cascade structure and Volume 3, Chapter 9 for an inner current loop and an explicit MPC. Volume 7, Chapter 12 for a lightly damped mode under a moving drive and Volume 7, Chapter 13 for the cutting forces. Volume 20, Chapter 10 for trajectory feedforward on an arm and Volume 20, Chapter 14 for joint servos and a cable-driven wrist.

12.1 Introduction

A machine-tool feed axis converts the torque of a motor into the travel of a table through a coupling, a screw and a nut, and the design, modeling and control of such feed drives have a review literature of their own [1]. Its drive closes a current loop inside a speed loop inside a position loop, the structure that direct-current drives established [2]. The motor is the well-behaved part of the chain. High-performance servo drives are often limited by mechanical load resonance instead [3], and in drives whose speed-loop bandwidth and resonance both lie above 100 Hz a flexible coupling narrows the range of stable speed gains [4].

On the axis of this chapter a 1 A current step at rest rises from 10 to 90 % in 0.30 ms, while the motor and a table of five times its inertia swing against each other on the coupling at 399.9 Hz. Three expectations that a block diagram of the drive encourages fail on it. A higher speed gain does not buy a stiffer axis until the motor's torque runs out, because the speed loop sings once its gain passes 0.55 A s/rad, at 413.8 Hz for a gain of 0.7, while its q-axis current swings only between −15.9 and 17.5 A, less than 56 % of the drive's 31.7 A peak. A position loop closed on a 20-bit motor encoder, 9.54 nm of table per count, does not place the table to that resolution. With 0.5° of backlash in the gears and a 300 N cut, the table of the graded case stops 56.6 counts of the 16-bit screw encoder (8.6 µm) from its target, 55.0 of them half the gap (45.5) and the twist of the coupling under the cut (9.5). An autotuner does not find the loop's limit and then tune to it. The relay experiment finds a limit, at 432.4 Hz, but the limit belongs to the coupling, and the Ziegler–Nichols gains it implies overshoot a 600 rpm step by 82.2 % against 29.7 % for the commissioned ones.

The axis is one member of a general class. A drive that measures and actuates one inertia while a lightly damped mode joins it to another meets a speed-loop limit set by the mode rather than by its actuator. The damping of a PI speed loop on a two-inertia system depends strongly on the ratio of the load's inertia to the motor's [5]. A prototype rolling-mill drive with three inertias on shafts of low stiffness, resonating at 17.4 and 51.3 Hz like an actual mill, reached a high speed bandwidth with an observer-based two-degree-of-freedom controller, compared in the same study with the conventional PI one [6], and a servo motor whose coupling resonates at 160 Hz needed a modified

speed controller to extend its range of stable gains [4]. The same loops enclose a gap in the gears, among the nonlinearities that limit speed and position control the most [7], and friction, which with integral action makes a position servo hunt [8]. A transfer function from current to speed with a rigid load carries none of these.

However, the methods for resonant loads were compared on physical drives, a 100 Hz resonance in one study [3] and 160 Hz in another [4], where the true position of the table, the twist of the shaft and the state of the gear teeth cannot be shown beside what the encoders report, and no move can be run twice with one setting changed. This chapter works with a simulated axis that can be tuned from the current loop outward, one step at a time, with every step judged against where the table truly is. The dq model, the two-mass model, the notch, the relay experiment and the profile feedforward follow the literature reviewed in Section 12.2. The axis can be rebuilt from this chapter alone. Its drive reads only its phase currents, its bus voltage and two encoders, runs its three loops at three rates in one controller, and draws the frequency response of its own speed loop from the numbers the simulation runs, within 1.6 % in magnitude and 0.3° in phase of a simulated sine sweep between 200 and 500 Hz. Nine cases and a graded move and return each isolate one step of commissioning, and one of them is honestly infeasible. A tenth case sets an explicit model predictive controller (MPC) in place of the position and speed loops, computed offline as the piecewise affine solution of a constrained linear quadratic regulator [9], and measures it against the commissioned cascade on the same move and cut, losses included.

The chapter is organized as follows. Section 12.2 traces how the servo axis acquired its cascade of current, speed and position loops. Section 12.3 states the model from the windings to the table: the motor in its rotor frame, the two masses joined by a spring and a gap, the friction, the thermal image, the encoders and the motor-side transfer function. Section 12.4 describes the drive as implemented, with its three loops, its filters, its profile and feedforward, its relay autotuner and inertia identification, the linear loop behind its Bode view and the explicit MPC. Section 12.5 reports the measurements case by case, from torque applied by hand to the graded move and return, and Section 12.5.10 compares the explicit MPC with the cascade. Section 12.6 sets the axis among the sensors, drives and controllers of practice, Section 12.7 states what the model leaves out, Section 12.8 summarizes the results, and the exercises follow in Section 12.9.

12.2 How the Servo Axis Acquired Its Cascade

Electric motion control began with the direct-current motor, and the history box below tells how, in 1896, its speed came to be set by voltage rather than by resistance [10]. Closing a loop around such a drive asked for a measurement of its speed as well as its position, and by 1952 a tachometer on the shaft supplied the rate signal of a high-performance servomechanism, with designs that traded signal level for low inertia, low noise and low friction [11]. Once static converters fed the armature, the direct-current drive was controlled as the cascade that this simulator keeps, an armature-current loop inside a speed loop inside a position loop, each faster than the loop around it [2]. Volume 3, Chapter 2 compares a cascade with a single loop and with feedforward on a tank level, where the same rule of a faster inner loop applies.

> ⧖ HISTORY **Speed by Voltage, 1896**
>
> On 18 November 1896 H. Ward Leonard read a paper before the American Institute of Electrical Engineers on controlling the speed of a motor by the voltage generated in its armature circuit rather than by a resistance in it. Elevators, printing presses, cranes and the turrets of warships all needed a motor held at any speed from rest to full. In his scheme an engine turned a generator at a practically constant speed, and a rheostat in the generator's field set the voltage fed to the motor. At one tenth of full speed under full torque, throwing off the whole load changed his example motor's speed by 9 revolutions per minute, against 415 under resistance control, nearly 50 times as much. Reversing the rheostat sent current back to the line as the motor braked [10]. The drive of this chapter likewise acts on its motor only through the voltages it applies.

Alternating-current machines did not fit that structure at first. Their dynamic interactions are far more complex than those of direct-current machines and some of the important quantities cannot be measured, and the difficulty was overcome about twenty years before 1991 by field orientation, which controls the machine in moving frames of reference tied to its flux and needs on-line signal processing that only microprocessors made economical [12]. In less than two decades field orientation went from research studies to the accepted method of controlling AC machines for high dynamic performance [12]. Vector control of the permanent-magnet synchronous motor had been modeled and simulated with its inverter and speed controller by 1989 [13], and in 1994 sinusoidal permanent-magnet machines were reviewed as offering extremely smooth torque and extended high-speed operating ranges [14]. Setting the d- and q-axis currents according to the rotor speed, within the voltage and current that the inverter can supply, had been shown in 1990 to expand the operating limits of such motors [15], and the field weakening of case 9 is a simple form of that idea.

With the motor under control, the mechanics became the limit. On a two-inertia system a conventional PI speed loop has a damping that depends strongly on the inertia ratio, and derivative feedback of the motor speed improves it when the ratio is small [5]. Rolling mills were the heavy-industry version of the problem, with resonances of 17.4 and 51.3 Hz on a laboratory mill built to the inertia ratios of an actual one [6], and servo drives met it again above 100 Hz [4]. A comparison of seven methods of resonant-load control on a lightly damped 100 Hz resonance evaluated a notch filter, a bi-quad filter, a low-pass filter, rigid-body observers, observer filtering, active resonance damping and center-of-mass control for their command response and dynamic stiffness [3]. The gap in a gear train and the friction of machine slides each acquired a survey of models, analysis tools and compensation [7, 8].

Automatic tuning arrived from the regulator side. A relay in place of the controller drives a loop into a limit cycle at the frequency where its phase reaches $-180°$, and in 1984 the amplitude and period of that cycle became the basis of tuning simple regulators to specifications on phase and amplitude margins [16]. The ultimate gain and period it measures are the inputs of the Ziegler–Nichols rule [17], which the application applies, as Section 1.4.7 describes for a process loop. Trajectory planning for point-to-point moves came to be designed together with feedforward, bounding the jerk so that feedback is left only what the model does not predict [18]. The application assembles these pieces on one axis, where each can be tried against a measurement that no physical drive offers, the table's true position.

12.3 The Axis From Windings to Table

The simulated axis is a permanent-magnet servo motor on a 48 V drive that turns a leadscrew of 10 mm lead through a flexible coupling and a 1:1 gear pair, which folds the motor beside the screw as on many machine-tool axes, as shown in Figure 12.1. Everything mechanical is referred to the motor shaft. Seven states are integrated, the d- and q-axis currents i_d and i_q, the angle θ_m and speed ω_m of the rotor, the angle θ_l and speed ω_l of the screw, and the drive's thermal image h of the winding. Table 12.1 sorts the signals by their roles in the three loops.

> **+ DID YOU KNOW? Eighteen Error Motions on Three Axes**
>
> A typical machine tool has three linear axes, each with six translational and angular error motions, 18 in all. They are major contributors to the error between the tool and the workpiece, and they grow as guideways and recirculating balls pit, wear and crack [19]. The axis of this chapter keeps one motion, the travel itself.

12.3.1 The Motor in Its Rotor Frame

The motor has surface magnets, so its d- and q-axis inductances are equal, and in the frame of the rotor its windings and torque obey Equation (12.1), in the form used for permanent-magnet

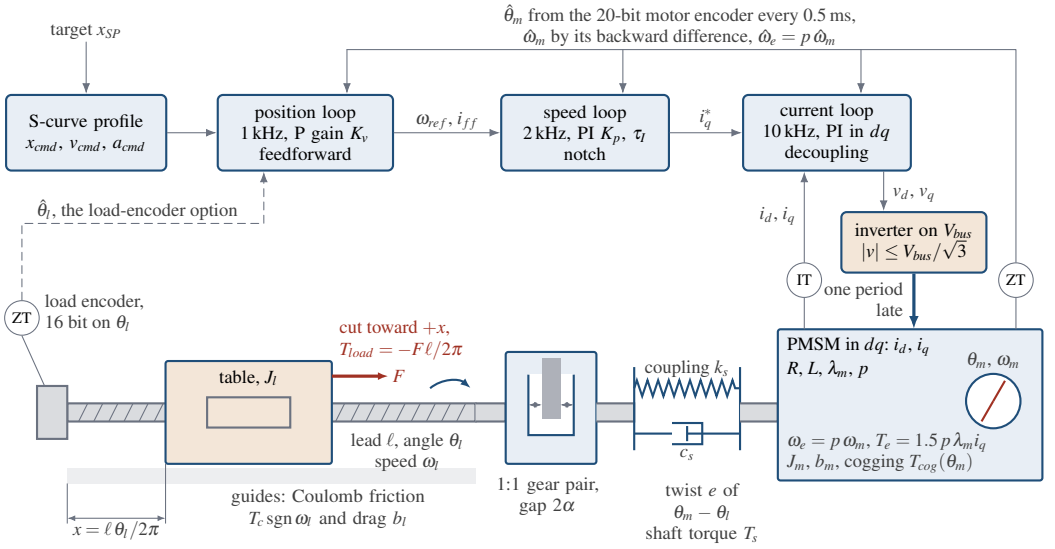


Figure 12.1 One servo axis and its drive, not to scale. The table, of inertia J_l referred to the motor, rides a leadscrew of lead ℓ at position x on guides with Coulomb friction T_c and drag b_l, and a cut F loads the screw with T_{load}. A 1:1 gear pair with a gap 2α and a coupling of stiffness k_s and damping c_s pass the shaft torque T_s of the engaged twist e to the motor, whose rotor of inertia J_m and drag b_m turns through θ_m at ω_m under the torque T_e of its q-axis current and the cogging T_{cog}. The drive reads the phase currents (IT), the 20-bit motor encoder and, as an option, the 16-bit load encoder (ZT), and closes the position, speed and current loops at 1, 2 and 10 kHz. The inverter applies v_d and v_q one 100 µs period late, inside the circle $V_{bus}/\sqrt{3}$.

Table 12.1 Signals of the Axis, Their Roles and Starting Values

Signal	Role	At start	Range
x	CV, position loop, the table	0	−100 to 300 mm
ω_m	CV, speed loop, a state	0	reference ±4500 rpm
i_d, i_q	CVs, current loop, states	0	limit 31.67 A
v_d, v_q	MVs, current loop	0	$\lvert v\rvert \le 27.71$ V
i_q^*	MV, speed loop	0	$\pm i_{lim}$
ω_{ref}	MV, position loop	0	±4500 rpm
θ_m, θ_l, ω_l	states, rotor and screw	0	—
h	state, I^2t image	0	1 folds back
F	DV, cut on the table	0	±1000 N
F_r, f_r	DV, ripple of the cut	0, 163 Hz	0–500 N, 10–1000 Hz
—	DV, workpiece inertia change	0	−80 to 300 %
—	DV, bus sag, friction rise	0, 0	0–50, 0–300 %
—	DV, encoder glitch	—	4096 counts once
$\hat{\theta}_m$, $\hat{\theta}_l$	measured, encoders	20, 16 bit	every 0.5 ms
i_d, i_q, V_{bus}	measured, sensors	ideal	every 0.1 ms

synchronous motor drives [13].

$$\begin{aligned} L\,\dot{i}_d &= v_d - R\,i_d + \omega_e L\,i_q \\ L\,\dot{i}_q &= v_q - R\,i_q - \omega_e\,(L\,i_d + \lambda_m), \qquad \omega_e = p\,\omega_m \\ T_e &= \tfrac{3}{2}\,p\,\lambda_m\,i_q = K_t\,i_q \end{aligned} \tag{12.1}$$

where the transform is amplitude invariant, so i_q is the peak phase current that makes torque, R and L are the resistance and inductance of a phase, λ_m the magnets' flux linkage and $p = 4$ the pole pairs. With $\lambda_m = 0.02\,\text{Wb}$ the torque constant is $K_t = 0.12\,\text{N m/A}$, the rated 1.27 N m takes 10.58 A and the peak 3.8 N m takes 31.67 A, and the back-EMF constant $p\lambda_m$ is 0.08 V s/rad. The inverter is averaged over each 100 µs PWM period and applies the voltages the current loop computed one period earlier, which the controller keeps inside the circle of Equation (12.2).

$$\sqrt{v_d^2 + v_q^2} \le V_{max} = V_{bus}/\sqrt{3} \tag{12.2}$$

The 48 V bus gives $V_{max} = 27.71\,\text{V}$. In steady state with $i_d = 0$ the voltage needed is

$$\sqrt{(Ri_q + \omega_e \lambda_m)^2 + (\omega_e L i_q)^2},$$

so without field weakening the motor runs out of voltage at 3308 rpm with no load, at 2419 rpm with its rated current and at 1008 rpm with its peak, and its nominal rating of 1.27 N m at 3000 rpm would need 33.20 V.

12.3.2 Two Masses, a Spring and a Gap

The rotor and the load, referred to the motor, obey Equation (12.3), with $\dot\theta_m = \omega_m$ and $\dot\theta_l = \omega_l$.

$$\begin{aligned} J_m\,\dot\omega_m &= T_e - T_{cog}(\theta_m) - b_m\,\omega_m - T_s \\ J_l\,\dot\omega_l &= T_s - b_l\,\omega_l - T_c\,\operatorname{sgn}\omega_l - T_{load} \\ T_{cog} &= A_{cog}\sin(n_{cog}\theta_m), \qquad T_{load} = -F\ell/2\pi \end{aligned} \tag{12.3}$$

where A_{cog} is 2 % of the rated torque, 0.0254 N m, at $n_{cog} = 24$ cycles per revolution, the cut F is positive toward $+x$, and the table stands at $x = \ell\theta_l/2\pi$. The compliances of the coupling, the gear mesh and the screw are lumped into one spring and one damper in series with the gap of the gears, which is modeled as a dead zone in the twist, as shown in Equation (12.4).

$$\begin{aligned} e &= \begin{cases} \Delta\theta - \alpha\operatorname{sgn}\Delta\theta, & |\Delta\theta| > \alpha \\ 0, & |\Delta\theta| \le \alpha \end{cases}, \qquad \Delta\theta = \theta_m - \theta_l \\ T_s &= k_s\,e + c_s\,(\omega_m - \omega_l)\min\bigl(1,\ |e|/e_r\bigr), \qquad T_s\,e \ge 0 \end{aligned} \tag{12.4}$$

where 2α is the whole gap and $e_r = 20\,\mu\text{rad}$ fades the damping in as the teeth meet, so the torque is continuous at engagement. The last condition sets T_s to zero whenever the damper would make a separating flank pull, since teeth push and never pull. With no gap the shaft is the plain spring and damper, and the default gear pair is preloaded. At the rated torque the spring of $k_s = 526\,\text{N m/rad}$ twists by 2.414 mrad, 3.84 µm of table travel. A dead zone is the simplest of the backlash models in use for control [7].

12.3.3 Friction, Heat and the Encoders

The guides' Coulomb friction, $T_c = 0.08\,\text{N m}$, is handled as a hybrid. The sliding direction is latched for each integration step so that the right-hand side is smooth inside it, and after the step the screw is stopped where its speed crosses zero and held still while the shaft and the cut pull on it by no more than T_c, $|T_s - T_{load}| \le T_c$. There is no static friction above the Coulomb level and no Stribeck dip. The drive protects the winding with the thermal image of Equation (12.5).

$$\tau_{th}\,\dot h = (i_d^2 + i_q^2)/I_{rated}^2 - h \tag{12.5}$$

where $\tau_{th} = 10\,\text{s}$. When h reaches 1 the current limit folds back from the peak to the rated current until h has fallen to 0.9, and at 31.6 A with the screw held the image trips after 1.191 s. Every 0.5 ms the drive samples both encoders as $\hat\theta = q\,\text{round}(\theta/q)$, with $q_m = 2\pi/2^{20}$ on the rotor, 9.54 nm of table per count, and $q_l = 2\pi/2^{16}$ on the screw, 0.1526 µm per count, so that a 1° gap is 182.0 load counts. The drive never reads ω_m, ω_l, e or x themselves.

Table 12.2 Constants of the Model. V Marks a Show Model Slider, H a Hidden Parameter, A an Advanced Control and C a Value Fixed in the Code

Symbol	Meaning	Value	Class
Motor and drive			
R	phase resistance	0.5 Ω	V
L	phase inductance, $L_d = L_q$	1.0 mH	V
λ_m	magnet flux linkage	0.020 Wb	V
p	pole pairs	4	H
J_m	rotor inertia	1.0 kg cm^2	V
b_m	bearing drag	1.0×10^{-4} N m s/rad	H
T_{rated}	continuous torque, the I^{2}t base	1.27 N m	V
T_{peak}	peak torque, the current limit	3.8 N m	V
V_{bus}	bus voltage	48 V	V
τ_{th}	time constant of the thermal image	10 s	V
Mechanics, referred to the motor			
J_l	table, screw and workpiece	5.0 kg cm^2	A
k_s	coupling and mesh stiffness	526 N m/rad	A
c_s	coupling and mesh damping	0.020 N m s/rad	V
2α	gear backlash, the whole gap	0, to 2°	A
b_l	screw and guide drag	2.0×10^{-4} N m s/rad	H
T_c	guide Coulomb friction	0.080 N m	V
ℓ	leadscrew lead	10 mm per rev	H
A_{cog}	cogging amplitude	2 % of rated	A
n_{cog}	cogging cycles	24 per rev	H
e_r	damping fade-in at contact	20 µrad	C
Instruments and detectors			
N_m, N_l	motor and load encoders	20, 16 bit per rev	H
—	singing detector, band and Q	420 Hz, 1	C
—	detector on and off, 10 ms RMS	0.30, 0.15 N m	C
—	encoder glitch	4096 motor counts	C

12.3.4 The Motor-Side Transfer Function

Without the gap, the friction and the drags, the response of the motor speed to the motor torque is the two-inertia transfer function of Equation (12.6) [5].

$$G_m(s) = \frac{\omega_m}{T_e} = \frac{J_l s^2 + c_s s + k_s}{s\left[J_m J_l s^2 + (J_m + J_l)(c_s s + k_s)\right]} \tag{12.6}$$

Its zeros lie at the anti-resonance $f_{ar} = \sqrt{k_s/J_l}/2\pi = 163.2$ Hz, where the load rings against a motor that stands still, and its poles at the resonance $f_r = \sqrt{k_s(1/J_m + 1/J_l)}/2\pi = 399.9$ Hz, where motor and load swing against each other with a damping ratio of $c_s/2\sqrt{k_s J_m J_l/(J_m + J_l)} = 0.048$. Against the rigid-body line $1/(J_m + J_l)s$ the damped response dips to 0.047 at 163.15 Hz and peaks at 52.3 times at 401.15 Hz. With the load held still the motor alone would ring at $\sqrt{k_s/J_m}/2\pi = 365.0$ Hz, the locked-load frequency. Above the resonance the motor feels only its own inertia, so the gain of the loop there is $(J_m + J_l)/J_m = 6$ times what a rigid axis would give.

12.3.5 Starting Point and Numerics

The constants are those of Table 12.2. Every case starts at rest at $x = 0$ with no current, the teeth centered in their gap, the screw held by its guides and a cold winding, $h = 0$. The seven states advance by the framework's fourth-order Runge–Kutta method (Section 1.6.1) on a fixed 100 µs step, 25 steps a period of the resonance, and the friction latch, the scorecard and the singing detector act after each step, never inside the derivative function. The singing detector passes the shaft torque through two band-pass sections at 420 Hz with $Q = 1$ and reports RESONANCE while their RMS over about 10 ms exceeds 0.30 N m, until it falls below 0.15 N m. The historian logs at 2 kHz, so that the resonance shows in the trends as more than a smear. Halving the physics step, with the loops kept at their rates, moves the frequency at which case 3 sings from 413.733 to 413.708 Hz and

the position at which case 6 settles by 0.001 of a load count, and the graded case run twice gives bit-identical states. The model has no random terms, so quantization is its only noise and every case replays exactly, which is why the exercises of this chapter need no seed. All 57 of the application's verification checks pass.

12.3.6 Assumptions

One torsional mode. The coupling, the mesh and the screw are one spring, so the axis has one resonance where a machine has several, among them the screw's axial mode, its bearings and the frame, and the next one up often limits a notched loop.

An averaged inverter. The voltages arrive one period late and clipped to the circle, with no switching ripple, dead time or diode drops, so above the no-load speed without field weakening the current simply follows the back-EMF.

An ideal surface-magnet motor. The inductances are equal and constant, the back-EMF is sinusoidal, iron saturates nowhere and loses nothing, and the thermal image stands in for a thermal model.

Coulomb friction only. The guides hold the table whenever it is pulled by less than T_c, with no static peak above it, no Stribeck dip and no pre-sliding spring, so the last micrometers of a move are a staircase of sticks and slips rather than the creep of a real guide.

Perfect quantizers. The encoders quantize and do nothing else, the current sensors are exact, and nothing is noisy, whereas real encoders carry interpolation error and a linear scale grows with temperature.

Teaching values. The motor, the mechanics and every threshold, the 20 mm following-error window, the 0.30 N m singing threshold, the two counts and the 6 dB of the graded case, are teaching values rather than a drive maker's data, and a real axis takes them from its drive's commissioning guide and its machine's accuracy class.

12.4 The Drive as Implemented

The drive reads its phase currents and its bus voltage every 100 µs and its two encoders every 0.5 ms, and nothing else. One controller runs every physics step. It first hands the inverter the voltages computed on the step before, then on every fifth step samples the encoders, on every tenth step runs the position loop, on every fifth the speed loop, and on every step the current loop, which computes the voltages for the next period. Table 12.3 gives each setting with its default and range.

12.4.1 One Shared Law at Three Rates

All three loops run the velocity-form PID of the framework, which Section 1.6.2 gives in general, with the clamp on the emitted output. With no derivative action and no bias it reduces to Equation (12.7).

$$\begin{aligned} p_k &= K_c\,(r_k - y_k), \qquad S_k = S_{k-1} + (p_k - p_{k-1}) \\ \hat{u}_k &= S_k + I_{k-1}, \qquad u_k = \mathrm{clamp}(\hat{u}_k,\ u_{min},\ u_{max}) \\ I_k &= I_{k-1} + \left[(K_c/\tau_I)(r_k - y_k) + (u_k - \hat{u}_k)/\tau_I\right] T \end{aligned} \tag{12.7}$$

where T is the loop's own period. In the notation of Equations (1.23) to (1.26), p_k is the proportional term P_k and $\hat{u}_k$ is the output before limiting v_k. The running sum S_k is never clamped, so a change of gain reaches the whole proportional term at once. Only the integral is wound back against what was applied (Section 1.4.9), with a tracking time equal to τ_I, and a loop without integral action, such as the default position loop, back-calculates nothing. Each loop has its own manual station, the q-axis voltage for the current loop, the current command for the speed loop and the speed reference, the jog, for the position loop. A loop in manual tracks its station by solving Equation (12.7) for I, every loop outside it tracks too because it cannot act through an inner loop that is not listening, and the header's MANUAL and AUTO switch throws the speed and position loops together, copying their outputs

Table 12.3 Drive Settings, Defaults and Ranges

Symbol	Setting	Default	Range
Position loop, 1 kHz			
—	position and speed control	cascade	or MPC
—	MPC evaluation, preview	lookup, on	online, off
K_v	position gain	30 s^{-1}	5–200
$\tau_{I,x}$	position integral time	off	0–500 ms
—	position feedback	motor encoder	or load
ϕ	profile feedforward	off	on, off
$\hat{J}$	inertia it assumes	6.0 kg cm^2	0.5–25
$\hat{T}_c$	friction it assumes	0.080 N m	0–0.4
e_{max}	following-error window	20 mm	0.5–50
$2\hat{\alpha}$	backlash compensation, its gap	off, 0	0–2°
—	dwell between queued moves	50 ms	0–500
v_{max}	profile speed	300 mm/s	10–700
a_{max}	profile acceleration	5 m/s^2	0.5–10
j_{max}	profile jerk	200 m/s^3	20–2000
Speed loop, 2 kHz			
K_p	speed gain	0.30 A s/rad	0.02–3
τ_I	speed integral time	20 ms	1–100
—	notch filter	off	on, off
f_n	notch center	400 Hz	100–900
Δf	notch width	200 Hz	20–400
d	notch depth	−30 dB	−40 to −3
τ_f	current-command low-pass	off	0–2 ms
—	speed feedback	motor encoder	or load
d_r	relay amplitude	2 A	0.2–10
Current loop, 10 kHz			
$K_{p,i}$	current gain	3 V/A	0.5–10
$\tau_{I,i}$	current integral time	2 ms	0.2–10
—	field weakening	off	on, off

into their stations once the run has started, so every transfer is bumpless (Section 1.4.10), and AUTO returns them to the case's own arrangement.

12.4.2 The Current Loop and the Voltage Circle

The current loop runs a PI on each axis with the cross-coupling and the back-EMF fed forward from the measured currents and the encoder speed, as shown in Equation (12.8).

$$\begin{aligned} v_d &= \mathrm{PI}_d(i_d^* - i_d) - \hat{\omega}_e L i_q \\ v_q &= \mathrm{PI}_q(i_q^* - i_q) + \hat{\omega}_e (L i_d + \lambda_m) \end{aligned} \tag{12.8}$$

with $|v_d| \le V_{max}$ and $|v_q| \le \sqrt{V_{max}^2 - v_d^2}$, where $\hat{\omega}_e = p\,\hat{\omega}_m$ and each PI part is Equation (12.7) with $K_c = 3\,\mathrm{V/A}$ and $\tau_I = L/R = 2\,\mathrm{ms}$, a pole-zero cancellation, its clamp shifted by the feedforward so that the total voltage stays inside the circle. The d axis is served first and may take the whole circle, and each PI back-calculates against what was applied. The linear analysis puts the small-signal bandwidth of the loop at 926 Hz, and a 1 A step at rest rises from 10 to 90 % in 0.30 ms. With field weakening on, the d-axis reference is the root nearest zero of Equation (12.9) that is not positive.

$$\begin{aligned} &\left(R i_q + \hat{\omega}_e (L i_d^* + \lambda_m)\right)^2 \\ &\quad + \left(R i_d^* - \hat{\omega}_e L i_q\right)^2 = (0.95\, V_{max})^2 \end{aligned} \tag{12.9}$$

Below the base speed $i_d^* = 0$, the reference is bounded by $-0.8\,I_{peak} = -25.3\,\mathrm{A}$, and where no i_d^* reaches the circle the one that comes closest is used. At the nominal 3000 rpm with the rated current it asks for −8.65 A. Without field weakening $i_d^* = 0$.

12.4.3 The Speed Loop, Its Filters and Its Limit

The speed is the backward difference of the motor encoder over one speed period, $\hat{\omega}_m = (\hat{\theta}_{m,k} - \hat{\theta}_{m,k-1})/T_\omega$ with $T_\omega = 0.5\,\text{ms}$, an average over the last period. The PI of Equation (12.7), $K_p = 0.30\,\text{A s/rad}$ and $\tau_I = 20\,\text{ms}$, acts on $\omega_{ref} - \hat{\omega}_m$, and the torque feedforward i_{ff} of the position loop is added to its output inside the limit $\pm i_{lim}$, $i_{lim} = \sqrt{I_{max}^2 - i_d^{*2}}$, where I_{max} is the peak current, or the rated current after an I^2t trip. The command then passes a first-order low-pass, exact for a held input and off by default, and the notch of Equation (12.10), and is clamped to $\pm i_{lim}$ again. The integral is wound back against the PI's own clamp, before the filters, so a filtered command that only this second clamp cuts is not fed back to it.

$$N(s) = \frac{s^2 + 2\zeta_z\omega_0 s + \omega_0^2}{s^2 + 2\zeta_p\omega_0 s + \omega_0^2} \tag{12.10}$$

where $\omega_0 = 2\pi f_n$, $\zeta_p = \Delta f/2f_n$ sets the width and $\zeta_z/\zeta_p = 10^{d/20}$ the depth at the center. The notch is discretized by the bilinear transform prewarped at its center, $s \to K(1-z^{-1})/(1+z^{-1})$ with $K = \omega_0/\tan(\omega_0 T_\omega/2)$, so that it sits exactly where it is asked to, and it runs at 2 kHz in transposed direct form II. It filters the command at all times, so switching it on in a running axis uses a filter that is already settled. The relay autotuner can stand in for the PI, and the inertia identification can add a chirp to ω_{ref}.

12.4.4 The Position Loop, the Profile and Feedforward

A move is a rest-to-rest S-curve of seven segments whose jerk is $\pm j_{max}$, solved exactly before it starts and lowered in its peaks when the distance is too short to reach them, and evaluated in closed form, so that x_{cmd} lands on the target to the last bit and never passes it. A target asked for during a move waits for that move to end and for the dwell, 50 ms by default, after it. A 100 mm move at the defaults takes 418.0 ms. The position loop gives the speed reference and the torque feedforward of Equation (12.11).

$$\begin{aligned} \omega_{ref} &= K_v\,\frac{x_{cmd} - \hat{x}}{\ell/2\pi} + \phi\,\frac{v_{cmd}}{\ell/2\pi} \\ i_{ff} &= \phi\,\frac{\hat{J}\,a_{cmd}\,2\pi/\ell + \hat{T}_c\,\operatorname{sgn} v_{cmd}}{K_t} \end{aligned} \tag{12.11}$$

where $\hat{x}$ is the motor encoder's position or the load encoder's, ϕ is the feedforward switch, and ω_{ref} is limited to $\pm 4500\,\text{rpm}$. Without feedforward a proportional loop needs a following error of v/K_v to move at the speed v, 10 mm at 300 mm/s and $30\,\text{s}^{-1}$. If $|x_{cmd} - \hat{x}|$ ever exceeds the window e_{max}, the drive faults and switches its torque off until Reset. Backlash compensation, for the motor-encoder loop only, moves the motor's aim by $\hat{\alpha}$ toward the last commanded direction at 20 rad/s, so that each reversal is a quick step of the assumed gap, as a numerical control's backlash compensation does.

12.4.5 The Drive's Own Tools

The relay autotuner replaces the speed PI by the relay of Equation (12.12), about the current that was holding the axis.

$$\begin{aligned} i_{cmd} &= i_0 \pm d_r, & K_u &= |U_1|/|E_1| \\ K_p &= 0.45\,K_u, & \tau_I &= T_u/1.2 \end{aligned} \tag{12.12}$$

where the sign follows the speed error with a hysteresis of two encoder counts of speed, 0.024 rad/s, the period T_u is the mean of eight cycles counted after four are skipped, and U_1 and E_1 are the fundamentals at $1/T_u$ of the relay's output and of the speed error over the counted cycles. The classic estimate $4d_r/\pi a$, Equation (1.16) with no switching band, reads the oscillation's peaks a, but at 2 kHz a cycle near 430 Hz has four or five samples, and the sampled peaks make it 9.4 % high in case 7 (0.701 against 0.641 A s/rad). The gains are then set by the Ziegler–Nichols PI rule [17] of Table 1.2.

The inertia identification sweeps the speed reference with a chirp from 3 to 30 Hz of $\pm 25\,\mathrm{rad/s}$ for 0.5 s, filters the commanded torque $K_t i_q^*$ and the encoder speed through the same 60 Hz low-pass, and fits $T = J\dot{\omega} + b\,\omega + T_c \operatorname{sgn}\omega$ by least squares, $\dot{\omega}$ being the difference of the filtered speed. The fitted J is the rigid-body inertia and goes to $\hat{J}$. The friction and drag fitted beside it do not, because the sweep cannot separate them (Section 12.5).

12.4.6 The Loop the Bode View Draws

The application computes the open speed loop, broken at the PI, from the same parameters the simulation runs, as shown in Equation (12.13).

$$L(j\omega) = C(z)\,F_{lp}(z)\,N(z)\,H(j\omega)^2\,T_i(j\omega)\,K_t\,\tilde{G}_m(j\omega) \tag{12.13}$$

where $z = e^{j\omega T_\omega}$, $H(j\omega) = (1 - e^{-j\omega T_\omega})/j\omega T_\omega$, $C(z) = K_p + (K_p/\tau_I)T_\omega z^{-1}/(1 - z^{-1})$ is the PI as Equation (12.7) runs it, H appears twice for the held current command and the averaged speed estimate, T_i is the closed current loop at 10 kHz with its period of delay, and $\tilde{G}_m$ is the plant of Equation (12.6) with the drags b_m and b_l and with the damping the back-EMF adds through the current loop, since the back-EMF is fed forward from the 2 kHz speed estimate and does not cancel at the mode. The margins are those of Equation (1.19), read with care because the loop can cross unity more than once. The crossover reported is the first frequency where $|L(j\omega)| = 1$, the phase margin is the worst over every crossover, and stability is judged by counting the crossings of the negative real axis to the left of -1. The gain margin is the ratio of the ultimate gain of the proportional loop with the filters to K_p, which differs from the margin of the PI loop by no more than 0.11 dB in any configuration quoted in this chapter. Each hold acts much as the added dead time of Section 1.4.11. At 400 Hz the hold, the speed estimate, the current loop and the PWM delay together cost 116.9° of phase, against 86.4° at 300 Hz. Against a simulated sine sweep of the plant at 300 rpm, the model's magnitude agrees within 1.56 % and its phase within 0.29° from 200 to 500 Hz and within 1.0 % from 20 to 150 Hz, but at the anti-resonance the simulated axis responds seven times more than the model's deep zero, and the error grows to 4.0 % at 600 Hz and 9.3 % at 700 Hz, because aliasing near the 1 kHz Nyquist frequency is left out, and its stability limit for case 3, a gain of 0.592, lies between the 0.55 at which the simulated axis is quiet and the 0.60 at which it sings.

12.4.7 Three Models

The simulation model is Equations (12.1) to (12.5). Equation (12.13) is the linear model behind the Bode view and the margins, derived from the parameters rather than identified, and blind to the gap, the Coulomb friction, the cogging, the quantization and aliasing. The controller's internal models are the decoupling of Equation (12.8), with the true L, λ_m and p, the feedforward's $\hat{J}$ and $\hat{T}_c$, the notch's center, which is 400 Hz against the plant's 401.15 Hz, and the compensation's assumed gap. Two models are identified while the axis runs, the relay's ultimate point and the chirp's inertia, and the comparison of each with the linear model is part of what the cases teach. The explicit MPC of Section 12.4.8 adds one more internal model, the two-mass axis at the declared machine.

12.4.8 Explicit Model Predictive Control

PI cascades, state-space control and model-based predictive control have been compared on drives with elastically coupled loads [20], and the application offers the predictive alternative as a second structure for the two outer loops, beside the cascade, never as the default and selected only in case 10. Section 1.6.3 describes the MPC option of the apps in general. This one differs in that its law is computed offline and looked up, and in that it replaces two loops of a cascade at once. Volume 3, Chapter 9 stores an explicit MPC as a lookup table in the same way, beside a converter cascade with an inner current loop. Wherever the position loop would act, with every loop in automatic, no fault and no relay test or chirp under way, one decision on each tick of the position loop sets the torque command u for the next millisecond, and the speed PI, its low-pass and its notch are bypassed. The current loop with its voltage circle and field weakening stays underneath, and the current limit with

its I^2t foldback and the following-error window stay above. With the position loop in manual, as in cases 1 to 4 and 7, the speed loop runs as shipped under either structure.

The controller's model is the two-mass axis of Equation (12.3) without the gap, the cogging and the Coulomb term, written at the motor shaft in the states $x = (\theta_l, \omega_l, \varepsilon, \dot{\varepsilon}, \tau)$, where $\varepsilon = \theta_m - \theta_l$ is the twist and τ the torque that the closed current loop delivers, as shown in Equation (12.14).

$$\begin{aligned} J_l\,\dot{\omega}_l &= k_s\varepsilon + c_s\dot{\varepsilon} - b_l\,\omega_l - d \\ J_m(\dot{\omega}_l + \ddot{\varepsilon}) &= \tau - k_s\varepsilon - c_s\dot{\varepsilon} - b_m(\omega_l + \dot{\varepsilon}) \\ \tau_c\,\dot{\tau} &= u - \tau \end{aligned} \tag{12.14}$$

where d is the load torque on the screw, the cut and the guides' friction together. Each constant takes the declared value of Table 12.2 ($J_m = 1.0\,\mathrm{kg\,cm^2}$, $c_s = 0.020\,\mathrm{N\,m\,s/rad}$, b_m and b_l) or the declared machine ($J_l = 5.0\,\mathrm{kg\,cm^2}$, $k_s = 526\,\mathrm{N\,m/rad}$), frozen when the application loads, so a machine or motor that a reader changes is a mismatch for the controller and never a new model. The lag $\tau_c = 0.25\,\mathrm{ms}$ stands in for the closed current loop. It was fitted to that loop's response to a 3 A step at rest, whose mean over the first millisecond is 0.756 of the step. A delay of 0.15 ms followed by a lag of 0.154 ms fits the step best, the single lag with the same mean over the ten samples is 0.30 ms, and at 1 ms the adopted lag predicts 0.982 of the step against a measured 1.000. The model keeps the resonance at 399.9 Hz and the anti-resonance at 163.2 Hz. An exact zero-order hold over $T_s = 1\,\mathrm{ms}$ gives $x_{k+1} = \Phi x_k + \Gamma u_k + E\,d_k$. Torque becomes current through the K_t that the shipped feedforward of Equation (12.11) uses, so a changed flux linkage moves the torque conversion of both structures alike.

A decision plans four torque moves of one millisecond each and applies only the first, as Equation (12.15) states.

$$\begin{aligned} &\min_{u_0,\ldots,u_3} \sum_{k=1}^{3} \left[q_x\,\tilde{\theta}_{l,k}^2 + q_\varepsilon\,\tilde{\varepsilon}_k^2 + q_\tau\,\tilde{\tau}_k^2 \right] + \tilde{x}_4^T P\,\tilde{x}_4 + r\sum_{j=0}^{3} u_j^2 \\ &\text{subject to } x_{k+1} = \Phi x_k + \Gamma u_k + E\,d_k, \quad x_0 = \hat{x} \\ &\qquad -U \le u_j \le U, \qquad U = K_t\,i_{lim} \end{aligned} \tag{12.15}$$

where $\tilde{x}_k = x_k - x_k^*$ is the predicted error against the reference state k periods ahead, $\hat{x}$ is the estimate, and P solves the discrete algebraic Riccati equation for Φ, Γ, $\mathrm{diag}(q_x, 0, q_\varepsilon, 0, q_\tau)$ and r, the cost-to-go that closes the horizon in the usual stabilizing construction [21]. Bryson's rule on 10 µm of table (6.283 mrad of screw), on the twist that holds the rated 1.27 N m and on that torque itself gives $q_x = 2.533 \times 10^4\,\mathrm{rad^{-2}}$, $q_\varepsilon = 1.715 \times 10^5\,\mathrm{rad^{-2}}$ and $q_\tau = 0.620\,(\mathrm{N\,m})^{-2}$, and the command is priced at 10^{-4} of the torque, $r = 6.2 \times 10^{-5}$ in the same unit. With no bound active the first move is the LQR's, $u = -K\tilde{x}_0$, with gains of 81.59 N m/rad on the table's angle, 0.3108 N m s/rad on its speed, −134.1 N m/rad on the twist, 0.055 54 N m s/rad on its rate and 0.2009 on the torque, which closes the rigid-body motion at 80.9 Hz with a damping ratio of 0.712, damps the coupling's mode to 0.200 at 461.8 Hz against its passive 0.048, and leaves the lag's pole at 2130 Hz. The weights were fixed before any closed-loop run.

The reference comes from the move profile evaluated k periods ahead, as shown in Equation (12.16), with $\bar{v}$, $\bar{a}$ and $\bar{j}$ the profile's speed, acceleration and jerk at the screw in radians.

$$\begin{aligned} \theta_{l,k}^* &= 2\pi x_{cmd}/\ell, \qquad \omega_{l,k}^* = \bar{v}, \qquad \dot{\varepsilon}_k^* = (J_l\bar{j} + b_l\bar{a})/k_s \\ \varepsilon_k^* &= (J_l\bar{a} + b_l\bar{v} + d_k - c_s\dot{\varepsilon}_k^*)/k_s \\ \tau_k^* &= (J_m + J_l)\bar{a} + (b_m + b_l)\bar{v} + d_k \end{aligned} \tag{12.16}$$

where $d_k = \hat{d} + T_c\,\mathrm{sgn}\,\bar{v}$ adds the declared friction in the profile's direction to the estimated load torque $\hat{d}$, which is the target-shift form of offset-free predictive control [22]. The plan therefore puts

in the twist that an acceleration needs before the table can follow. The preview reaches the move in progress 1 to 4 ms ahead, while the cascade's feedforward reads the present profile, and a queued move enters neither before it starts. With the preview off the present command is held with no motion or friction ahead, the knowledge of the cascade without feedforward.

Only the torque box constrains the plan, and its rows are hard. Its bound U is the current limit as the speed loop clamps it (the peak current, folded back to the rated one by the I^2t image, less the share of the d axis) converted by K_t, and it enters the problem as a parameter. There is no speed row, because the profile bounds the speed, and no voltage row, because the circle stays with the current loop. Collecting the estimate, the four references, the four load torques, U and a constant into a parameter ϑ of 31 entries turns Equation (12.15) into $\min \frac{1}{2} z^T G z + z^T F \vartheta$ over the moves z subject to $C_q z \geq b$, and an active set $\mathscr{A}$ of rows gives the law of Equation (12.17).

$$\begin{aligned} z_u &= -G^{-1} F \vartheta, \qquad & W_{\mathscr{A}} &= \left(C_{\mathscr{A}} G^{-1} C_{\mathscr{A}}^T \right)^{-1} \\ \lambda_{\mathscr{A}} &= W_{\mathscr{A}} \left(b_{\mathscr{A}} - C_{\mathscr{A}} z_u \right), \qquad & z &= z_u + G^{-1} C_{\mathscr{A}}^T \lambda_{\mathscr{A}} \end{aligned} \tag{12.17}$$

The law is optimal wherever $\lambda_{\mathscr{A}} \geq 0$ and every row holds. With four moves in a box each move is free, at its floor or at its ceiling, so there are $3^4 = 81$ candidate sets, and because $G^{-1}F$ has rank 4 the unconstrained moves cover all of $\mathbb{R}^4$ and every one of the 81 sets is optimal on a region with an interior. The table of 81 regions is thus the complete partition, nothing is truncated, and a finite parameter always finds its region. The search scales z_u by U and takes its tolerances relative to $\max(1, \|z_u\|_\infty / U)$, because a large following error makes z_u thousands of newton meters, where fixed tolerances miss. In 46,909 decisions over 56 closed-loop runs, every case of the application, the worst cases and the comparison of Section 12.5.10, 30 of the 81 regions occur, and 38 once 5,085 states at region boundaries and 20,000 states sampled from the physical range are added. Against the online solver that the apps share, on the same program, the lookup's moves differ by at most 2.5×10^{-13} of $\max(1, U)$ in closed loop and 7.9×10^{-11} at the boundaries, with no miss. In Node.js a lookup takes a median of 0.18 µs and a 99th percentile of 2.06 µs, against 13.4 and 27.2 µs for the online solve. Played in Chrome at real time, the top speed of 1000 decisions per second, the whole controller takes 1.06 and 0.76 % of one core in cases 10 and 6 under the lookup, 3.23 and 3.27 % under the online solve and 0.66 and 0.84 % under the cascade, at 60 frames per second, while other computations loaded the same host. The online solve is offered as a switch beside the table and gives the same moves.

A steady Kalman filter at the encoders' 2 kHz estimates x and $\hat{d}$ from the motor encoder, the measured q-axis current as torque, and the load encoder when a loop reads it, with a seventh state then for the load encoder's slow offset from the model's load angle (the gap or the screw's own wind-up), so that the fast motion is estimated through the collocated motor encoder. It predicts with the declared model from the torque commanded over the last 0.5 ms, assuming the quantization over $\sqrt{12}$ and 0.01 N m on the torque as measurement noise, and the cogging amplitude on the rotor, 0.02 N m on the load, 0.005 N m per read on $\hat{d}$ and 2 µrad per read on the offset as process noise. The offset state was added after the first results, in which the six-state filter left the challenge's program without its gap never in position, 2.1 counts off at 1.6 s, where the seven-state filter has it in position at 0.907 s. A motor-encoder read more than 10 mrad from the prediction is set aside once, as the glitch is, and the two reads after it are always taken, since setting every other read aside makes the error of the motor-encoder filter grow by 1.039 a read where two taken reads make it shrink by 0.879. Two large reads in a row reject the next decision as a model mismatch. The filter runs under both structures and writes nothing the cascade reads.

A decision is accepted only if a region is found, or the online solve is optimal with a plan of the right shape, and every planned move is finite and inside $\pm U$. Otherwise the cascade, whose speed PI has tracked the applied current and whose proportional position loop has tracked its own output, takes the torque without a bump, the historian marks the fallback, and the MPC takes it back over 20 ms after 20 acceptable decisions. A trip forced in the middle of case 10's move steps the current

command by 0.332 A, where the largest step the MPC itself makes in that window is 0.855 A, and a trip inside the blend back by 0.000 A.

A supervisor sits above the decision, because the model has no gap. While the teeth are apart the load is disconnected, and a law that prices the twist acts on a spring that is no longer there. With k_s and c_s set to zero the gain K as state feedback has a spectral radius of 2.08 per period, against 0.70 on the declared shaft. The supervisor rejects decisions, and the cascade holds the axis, while the estimate disagrees with an encoder between decisions by more than 5.6 mrad on the motor or 3.7 mrad on the load, while the estimated twist exceeds $2U/k_s$, while the estimated motor speed exceeds the cascade's 4500 rpm, or while the following error exceeds a quarter of the window e_{max}. The gates were sized before the worst cases were measured, from sixteen runs with no gap and no voltage loss, as twice the largest innovations (2.80 and 1.86 mrad), against the largest twist of 1.18 U/k_s, the largest speed of 1981 rpm and the largest following error of 2.6 % of the window. After a trip the MPC decides again only after 50 decisions back inside the gates, then the re-entry and its blend, and a second trip leaves the axis to the cascade until Reset. Without that lockout a machine the model does not fit cycles between the two structures. On case 6's move with the workpiece 80 % lighter the axis changes hands 15 times, at intervals of about 80 ms, and ends 5497 counts from its target, against 1.5 counts with the lockout. The supervisor was added after the first build ended the challenge as shipped in a following-error fault at 0.639 s with the motor at 3426 rpm. A reconstruction of that build shows the fault arising while the cascade held the axis after model-mismatch hand-overs, driven by a position-loop bias left from tracking the MPC and a diverging estimate, both since removed, rather than under the MPC's own law, which with the gap rings but did not run away.

The simulated plant, its instruments and the MPC's model are kept separate. The simulation integrates Equations (12.1) to (12.5) with whatever the run carries, the MPC predicts and estimates with the frozen declared values, and it reads only what the cascade reads, the two encoders, the measured current, the profile, K_t and the current limit. Because its estimator writes nothing the cascade uses, each earlier case gives the same logged values to the last bit on each column recorded before the MPC was added.

12.5 Ten Cases and a Move and Return

The cases of Table 12.4 start from the defaults of Tables 12.2 and 12.3, in automatic mode unless the row says otherwise, and depart from them only as the row states. Each number in this section comes from the engine itself, run headless as the browser runs it, with every setting a case changes made before Run, and the main ones are in Table 12.5. The same settings carried through Reset give the same run. Table 12.7 lists what the application requires before it credits each case, the completion rules of Section 1.6.4. A case counts as done once the changes it lists have been made and the simulated axis has reached the state it names.

12.5.1 Torque by Hand, and the Current Loop at Speed

Case 1 opens the speed and position loops and steps the current command, the drive's torque request, to 3 A, −3 A and zero. Torque integrates to speed and speed to position. After 0.1 s at 3 A, 0.36 N m against 6 kg cm^2, the motor turns at 429.8 rpm and the table has gone 3.58 mm. The reversed current has the guides' friction on its side, so it stops the axis before 0.25 s and drives it back to −163.7 rpm, and with the current at zero the friction brings the table to rest at 3.54 mm. Each step also rings the coupling. A damped sine fitted to the first 20 ms after the step swings at 398 Hz with a damping ratio of 0.054, about a mean of 0.31 N m late in the push with a peak of 0.57 N m. That is above the shaft's own 0.048 and close to the 0.056 of the linear model of Section 12.4, which includes the back-EMF that the current loop does not cancel at the mode. The speed loop will have to live with this mode.

Case 2 pushes the current loop against its bus. At rest the step to 10 A rises in 0.40 ms, but at the 1391 rpm of the scan that commands 20 A the back-EMF already takes 11.7 V of the 27.71 V circle, the step from 10 to 20 A needs 1.30 ms from 11 to 19 A, and 20 A holds only up to 1777 rpm before

Table 12.4 Scenario Settings and Teaching Points

#	Change from the defaults	Teaching point
1	speed and position loops manual, i_q^* 3, −3 and 0 A at 0.05, 0.15 and 0.25 s	torque, speed, position
2	as 1, i_q^* 10, 20, −20 and 0 A at 0.02, 0.10, 0.16 and 0.24 s	the voltage circle
3	position loop manual at 300 rpm, $K_p = 0.7$	the resonance caps the gain
4	position loop manual, $J_l = 1\,\text{kg cm}^2$, $K_p = 0.4$, $\tau_I = 15\,\text{ms}$, steps to 500 rpm and back twice, a workpiece of +400 % at 0.28 s	tuned at 1:1, run at 5:1
5	gap 1°, load encoder, $\tau_{I,x} = 50\,\text{ms}$, move to 10 mm	an integral that hunts
6	move to 100 mm	following error
7	as 3 at $K_p = 0.3$, relay at 0.2 s, 600 rpm at 0.5 s	what the relay measures
8	$K_p = 0.5$, feed to 200 mm at 50 mm/s, ripple 100 N at 425 Hz	a force at the mode
9	feedforward on, 250 mm at 600 mm/s and 2.5 m/s^2	field weakening
10	MPC in place of the position and speed loops, load encoder, feedforward, notch, $K_p = 0.8$, $\tau_I = 12\,\text{ms}$, $K_v = 70\,\text{s}^{-1}$, move to 100 mm, 300 N cut at 0.6 s, 1 s	a model of the axis
C	gap 0.5°, 100 mm out and back with a 300 N cut on the return, 1.6 s	everything at once

Table 12.5 Headless Measurements of the Nine Cases

Case, setting	Measured
1, after 0.1 s at 3 A	429.8 rpm, 3.58 mm, shaft rings at 398 Hz, $\zeta = 0.054$
2, at rest and at 1391 rpm	a 10 A step in 0.40 ms, 10 to 20 A in 1.30 ms, 20 A held to 1777 rpm
2, bus sagged 25 %	the step to 20 A reaches 13.6 A, at 1464 rpm
3, K_p 0.7, no notch	sings at 413.8 Hz, PM −8.4°, GM −1.4 dB
3, quiet limit, no notch, notch	0.55 and 1.75 (model 0.592 and 1.821)
3, gain margin 6 dB	0.297 without the notch, 0.904 with it
4, 1:1 and 5:1	bandwidth 58.1 and 22.6 Hz, PM 63.6 and 27.8°, GM 8.1 and 3.4 dB
4, 5:1 at K_p 1.2	unstable, or 55.5 Hz at 61.2° with the notch
5, K_v 10 to 80	hunts 994 to 271 counts, 381 with no gap
5, $\tau_{I,x}$ 200 ms or none	settles, within 0.4 counts with no integral
5, motor encoder	stops 58.7 counts short, 1.6 with $2\hat{\alpha} = 0.8°$
6, feedforward off and on	worst error 9.990 and 0.035 mm, in position 554 and 343 ms after the profile
7, relay, no notch and notch	432.4 Hz, K_u 0.641, and 500.0 Hz, 1.817
7, 600 rpm step	overshoot 82.2 (relay), 58.2 (relay, notch), 29.7 (shipped), 24.7 % (notch, K_p 1.2)
8, 425 Hz, K_p 0.5, 0.3, notch	motor ±2.80, ±0.79, ±0.35 µm
8, 163 Hz, any loop	motor ±0.76 µm, table ±0.69 to ±0.81 µm
9, field weakening off and on	top 3209 and 3609 rpm, error 15.19 and 0.02 mm, $\lvert i\rvert$ 32.3 and 14.7 A

the voltage runs out, with VOLTAGE LIMIT showing for 55.5 ms of the run. With the bus sagged by 25 % the step to 20 A reaches only 13.6 A, at 1464 rpm, and falls from there as the speed rises. A bandwidth of 926 Hz is a small-signal number, and a large step at speed is limited by the voltage left over.

12.5.2 The Gain That Sings, and the Notch

Case 3 runs the axis at 300 rpm under the speed loop alone with its gain raised from the commissioned 0.3 to 0.7 A s/rad. At 0.3 the open loop's resonance peak stands 0.91 dB below unity. At 0.7 it stands 6.45 dB above, and $|L(j\omega)|$ crosses unity three times, at 22.9 Hz with 64.3° of phase margin, upward at 360.8 Hz, and downward again at 441 Hz, where the phase is already 8.4° past −180°. The gain margin is −1.4 dB, as shown in Figure 12.2. The axis sings from 3.0 ms, at 413.8 Hz. The shaft torque swings between −10.6 and 10.3 N m, more than eight times the motor's rating, while the q-axis current, once the singing has settled, stays between −15.9 and 17.5 A. Switched on while the axis sings, the notch quiets it 84.9 ms later. Every run of the case opens with the step to 300 rpm at $t = 0$, which steps the current command by K_p times the 31.4 rad/s error, up to its limit, and rings the shaft, so even a quiet setting shows RESONANCE at first, for 49 to 55 ms with the notch at gains from 1.0

to 1.6, 113 ms at 1.75 and 218.7 ms without the notch at 0.55. Quiet here means quiet once that ring has died away, as it has by 0.6 s. Without the notch the axis is quiet up to a gain of 0.55 and sings from 0.60, where the frequency response puts the limit at 0.592, and with the notch it is quiet up to 1.75 and sings again from 1.80, at 489.8 Hz, where the loop above the notch runs out of phase (the model says 1.821). The quiet limits leave almost no margin, 0.65 dB at 0.55 and 0.26 dB at 1.75, and at an equal gain margin of 6 dB the notch still triples the usable gain, from 0.297 to 0.904, and the bandwidth from 16.7 to 41.3 Hz.

> **TRY IT IN THE APP** **3 · The Speed Gain Until the Axis Sings**
>
> Load the case and press Run. Within the first milliseconds the status line reads RESONANCE, the shaft sings at 414 Hz, and the shaft-torque trend swings far past the motor's rating. Then switch the Notch filter on in the speed-loop card while it sings, and watch the ringing die away 84.9 ms later.

12.5.3 Tuned at 1:1, Run at 5:1

Case 4 tunes the speed loop at $K_p = 0.4$ and $\tau_I = 15\,\text{ms}$ with a light fixture of $1\,\text{kg}\,\text{cm}^2$, steps to 500 rpm and back, clamps on a workpiece five times heavier at 0.28 s and steps again with the same gains. At 1:1 the step rises in 5.40 ms with 16.9 % overshoot, at a bandwidth of 58.1 Hz with 63.6° of phase margin and 8.1 dB of gain margin, the mode sitting at 522.1 Hz. At 5:1 the same gains rise in 12.20 ms with 31.4 % overshoot. The bandwidth has fallen to 22.6 Hz, and the resonance, now at 401.1 Hz and much taller, crosses unity again near 413 Hz and leaves 27.8° and 3.4 dB, so the shaft rings at a band RMS of 0.60 N m on each step against 0.24 N m before. On a frequency grid finer than the Bode view's 260 points that margin is 27.1°. A sine sweep of the simulated loop measures the same bandwidths, 58.1 and 22.6 Hz. Getting the bandwidth back needs the gain raised with the inertia, to about 1.2, which without the notch is unstable (−31.4°, −6.1 dB) and with it gives 55.5 Hz at 61.2°, though with only 3.5 dB of gain margin. A workpiece five times heavier clamped on between two speed steps while the drive holds the axis would not happen on a real machine, but it isolates what the inertia ratio does to a fixed tuning.

12.5.4 A Gap, Friction and an Integral That Hunts

Case 5 puts 1° of backlash, 182.0 load counts or 27.8 µm of table travel, in the gears and closes the position loop on the load encoder with a 50 ms integral added, so that the table itself is measured and every last micrometer is supposed to be corrected. It never settles. Over the last 2 s of a 4 s run the table hunts 482 counts (73.5 µm) peak to peak, reversing 69 times and crossing the gap 10 times, and no position gain removes it, 994 counts at $10\,\text{s}^{-1}$ and 271 at $80\,\text{s}^{-1}$. With the gap closed it still hunts, 381 counts, because the cycle is the integral winding up while the guides' friction holds the table, then driving it past, the hunting that the friction literature describes for integral action against friction [8]. The gap only turns each reversal into a rattle across the teeth. Removing the cause works. A 100 ms integral still hunts 245.9 counts over the last 6 s of a 12 s run, a 200 ms one stops it, and so does none at all, because the speed loop's own integral already holds the table, which then settles within 0.4 counts of its target with or without the gap. On frictionless guides the gap alone makes the same loop without an integral hunt 249.8 counts, the textbook backlash limit cycle [7], which the friction here suppresses. On the motor encoder there is nothing to hunt, but the table stops 58.7 counts short in the direction of every move, where the encoder cannot see it. Backlash compensation set to the gearbox's 1° overcorrects by 14.8 to 16.5 counts, because the table floats into the gap as it decelerates, and set to 0.8°, found by trying, it leaves at most 1.6.

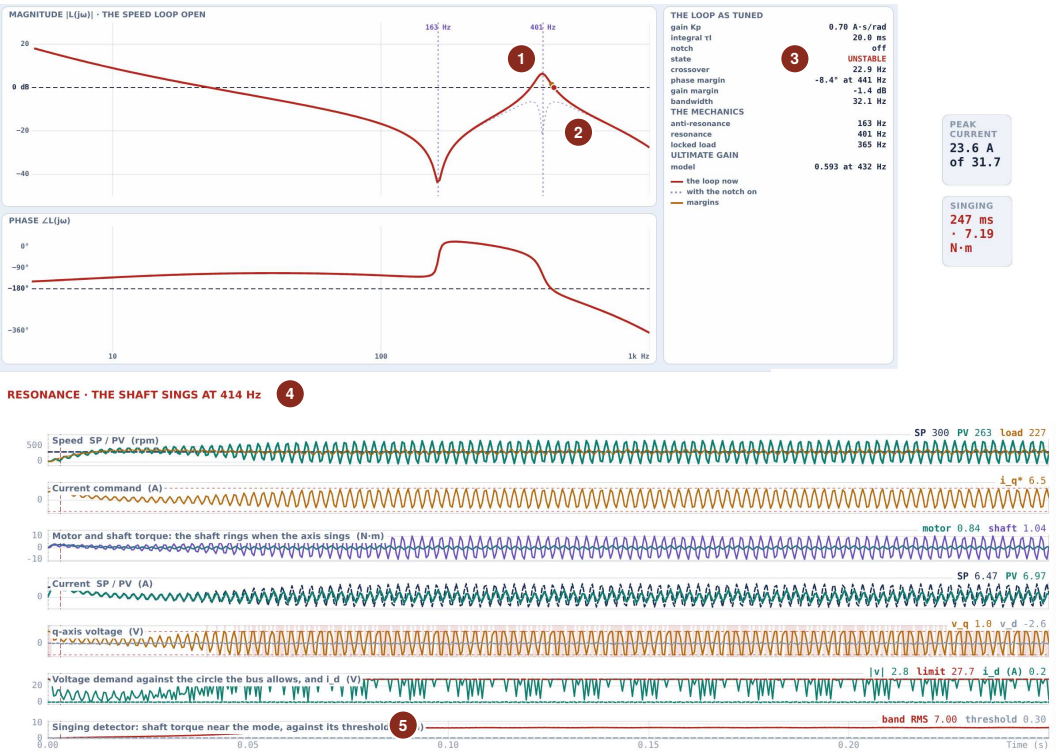


Figure 12.2 The speed loop of case 3, *The speed gain until the axis sings*, at $t = 0.25$ s with no control changed. The figure is a composite of the Bode view, two scorecard tiles set at its right and the trend strips below it. The open loop at $K_p = 0.70$ A s/rad (the solid curve, red on the screen) lifts its resonance peak at 401 Hz above 0 dB (1) and keeps more than unit gain near 432 Hz, where its phase passes $-180°$, and the dotted curve is the same loop with the 400 Hz notch on (2). The column beside the plots (3) reads $\tau_I = 20.0$ ms, the notch off, the state UNSTABLE, the crossover at 22.9 Hz, a phase margin of −8.4° at 441 Hz, a gain margin of −1.4 dB, a bandwidth of 32.1 Hz, the anti-resonance at 163 Hz, the resonance at 401 Hz, the locked load at 365 Hz and the model's ultimate gain of 0.593 at 432 Hz. The status line (4) reads RESONANCE · THE SHAFT SINGS AT 414 HZ. Over the 0.25 s window the trends show the speed, the current command and its tracking, the motor and shaft torques and the q-axis voltage oscillating at the mode, and at the last sample the speed at 263 rpm against 300 rpm, the shaft torque at 1.04 N m, the voltage at 2.8 V of 27.7 V and the singing detector (5) at 7.00 N m against its 0.30 N m threshold. The scorecard tiles count 247 ms of singing with a band RMS of up to 7.19 N m and a peak current of 23.6 A of 31.7 A.

> **✦ DID YOU KNOW? The Integral That Causes the Hunt**
>
> From the simulator: with an integral in the load-encoder position loop of case 5, the table hunts 994 to 271 counts at position gains from 10 to 80 s^{-1}, and still 381 counts with the gap closed. With no position integral at all it settles within 0.4 counts (Table 12.5).

12.5.5 Feedforward From the Profile

Case 6 moves the table 100 mm on the S-curve with the commissioned gains and a proportional position loop. Without feedforward the table runs 9.990 mm behind at cruise, is still 2.61 mm short when the 418.0 ms profile ends and comes within two counts of the target 554 ms later. With the feedforward the worst error is 0.035 mm, 285 times smaller, the table is 18.7 µm short at the end

of the profile and in position 343 ms later, with no overshoot, as feedforward from a jerk-limited profile intends [18]. Volume 20, Chapter 10 applies the same idea, torque feedforward computed along a planned trajectory, to a two-link arm. Both terms of Equation (12.11) matter. Without the friction term the speed integral carries the guides' friction through the move and pushes the table 152.5 counts past its target when it stops, and an inertia estimate taken at 1:1, 2 kg cm^2, leaves a 0.734 mm error and a 2683-count overshoot. Run on the speed loop of case 3 at the commissioned gain of 0.3, which is case 7 without its relay test and speed step, the chirp identification recovers the inertia, 6.010 to 6.068 kg cm^2 for mean speeds of the sweep from 0 to 600 rpm, within 1.1 % of the 6.000 kg cm^2 of the rigid body. Its friction estimate is another matter, 18.4 to 213.3 mN m against the guides' 80 over the same speeds, with the drag at −1.78 to 1.00 mN m s/rad against 0.30, which is why the drive passes only the inertia to the feedforward.

> **TRY IT IN THE APP** **6 · S-Curve Feedforward**
>
> Press Run on the Profile view and read the following-error strip: the table runs 9.990 mm behind its command at cruise. Press Reset, set Profile feedforward On in the position card and run again. With no gain changed, the worst error falls to 0.035 mm, and the table is in position 343 ms after the profile ends instead of 554 ms.

12.5.6 What the Relay Measures

Case 7 replaces the speed PI at 300 rpm by a 2 A relay for 27.5 ms from 0.2 s. The loop oscillates at 432.4 Hz, just above the resonance, where its phase passes −180°, with an ultimate gain of 0.641 A s/rad, 8 % above the 0.593 that the linear model gives at the same 432.4 Hz and within the 10 % that the application's check allows. The Ziegler–Nichols rule turns that into $K_p = 0.288$ and $\tau_I = 1.93$ ms, an integral time tuned to a 2.31 ms period that belongs to the coupling rather than to the rigid-body motion the integral has to correct, near a crossover of 11.5 Hz. The step to 600 rpm on those gains overshoots 82.2 % with 8.6° of phase margin, against 29.7 % for the commissioned gains put back after the test. With the notch on before the test the relay finds 500.0 Hz and 1.817, the model 1.804 at 495.1 Hz, and the same rule gives 1.67 ms and a 58.2 % overshoot. A notch design by hand, $K_p = 1.2$ with $\tau_I = 15$ ms entered after the test, steps in 5.5 ms with 24.7 % overshoot and 61.2° of phase margin. The relay measures the ultimate point correctly, and what fails is the rule's assumption that the oscillation it finds is the loop's dominant dynamics.

Pause. Try Exercise 12.1 before reading on.

12.5.7 A Cut at the Mode and at the Anti-Resonance

Case 8 feeds the table at 50 mm/s with the speed gain at 0.5, stable with 1.5 dB of gain margin, while the cut ripples ±100 N at 425 Hz, as a cutter's teeth would at that tooth-passing frequency. Volume 7, Chapter 13 treats the cut itself, where each tooth meets the surface the previous one left. The loop feeds the mode. The motor shakes ±2.80 µm and the table ±0.44 µm, and RESONANCE shows from 44.7 ms for the rest of the run. At a gain of 0.3 the motor moves ±0.79 µm and nothing rings, and with the notch at 0.5 the motor moves ±0.35 µm and the table ±0.12 µm. At 163 Hz, the anti-resonance, torque at the motor cannot move the motor, so the motor shakes ±0.76 µm whatever the loop does, while the table shakes ±0.69 µm at a gain of 0.3, ±0.71 µm at 0.5 and ±0.81 µm at 1.5 with the notch, because the torque the loop answers with goes into the table. At 365 Hz, the locked-load frequency, the table barely moves (±0.05 µm) while the motor shakes for it (±0.26 µm), as a tuned absorber would. A notch removes the controller's gain at one frequency and leaves every other frequency to the mechanics.

Pause. Try Exercise 12.3 before reading on.

12.5.8 Field Weakening for a Rapid

Case 9 programs a 250 mm rapid at 600 mm/s, 3600 rpm at the motor, with the feedforward on and field weakening off. The voltage runs out first. The motor tops out at 3209 rpm, the following error

Table 12.6 The Move and Return in Nine Configurations

Configuration	In (s)	End	GM (dB)	$\|i\|$ (A)	Sings (ms)
as shipped	never	56.6	5.92	21.7	0
suggested, motor encoder	never	55.5	7.06	22.5	120
and compensation 0.4°	never	18.7	7.06	22.4	124
suggested, no feedforward	1.066	0.4	7.06	20.1	53
suggested, no notch	never	739.95	−2.60	31.5	1569
no notch, K_p 0.25, τ_I 20 ms, K_v 80	1.041	0.3	7.50	24.7	121
no notch, K_p 0.45, τ_I 8 ms, K_v 80	0.9795	−0.2	2.40	24.8	275
the first with no dwell	0.989	0.3	7.50	24.8	116
suggested	0.989	0.3	7.06	22.4	166

End is the table's final distance from zero in load counts. The suggested configuration is the load encoder, the feedforward, the notch, $K_p = 0.8$, $\tau_I = 12$ ms and $K_v = 70\,\mathrm{s}^{-1}$. Only it meets the whole target, and the run with no dwell is in time but is not the programmed move.

grows to 15.19 mm and VOLTAGE LIMIT shows for 389 ms. When the axis then brakes, the d-axis decoupling its braking current needs, up to 38.8 V at 2957 rpm, is more than the circle holds, so for 19.3 ms, from 3078 down to 1988 rpm, the voltage sits on the circle, and for 18.1 ms of it the d axis takes the whole circle, v_q is left at zero and the back-EMF rather than the drive sets the currents. The d-axis current swings to −11.8 A that nobody commanded, and the q-axis current overshoots its −31.67 A limit as the voltage comes back, so $|i|$ reaches 32.3 A, past the drive's peak. With field weakening the axis follows the profile to 3609 rpm with 0.02 mm of following error and never reaches the voltage limit, at the cost of a d-axis current down to −11.28 A that makes no torque, while $|i|$ peaks at 14.7 A. Slowing the rapid to 500 mm/s instead keeps the error to 0.34 mm with the limit touched for 76 ms.

12.5.9 The Move and Return

The graded case moves the table 100 mm, dwells 50 ms and brings it back while a 300 N cut resists the return and stays on at the end, with 0.5° of backlash and the 5:1 load. The target asks for the table within two load counts (0.305 µm) of zero by 1.02 s and staying there, never more than two counts past either target, no following-error fault, the current within the 31.7 A peak and at least 6 dB of gain margin in the speed loop, with the programmed profile, dwell and machine. The target is judged on the machine and the cut as given, so a changed model parameter or disturbance, or an extra disturbance button, voids the attempt. Table 12.6 compares configurations, Figure 12.3 charts three of its columns, and Figure 12.4 shows four returns. As shipped, on the motor encoder with no notch and no feedforward, the following error reaches 9.991 mm and the table ends 56.6 counts from zero, mostly the half gap of 45.5 counts and the twist of 9.5 under the cut, which a motor encoder cannot see, and the gain margin of 5.92 dB also falls short. The suggested gains on the motor encoder still end 55.5 counts off, and backlash compensation of 0.4° only brings that to 18.7 while overshooting the first target by 48.8. On the load encoder with the feedforward, the notch, the speed loop at $K_p = 0.8$ and $\tau_I = 12$ ms and a position gain of $70\,\mathrm{s}^{-1}$, the table is in position at 0.989 s with no overshoot, a peak current of 22.4 A and 7.1 dB of gain margin. Each piece is needed. Without the feedforward the table arrives at 1.066 s, without the notch the same gains sing, and the best found without the notch inside the 6 dB margin arrives at 1.041 s, while a quicker one at $K_p = 0.45$ arrives at 0.9795 s with only 2.4 dB of margin.

🕮 **Pause.** Try Exercise 12.2 before reading on.

The suggested configuration passes, but not quietly. The singing detector reports RESONANCE for 166 ms of its run, in five bursts, at the start and in the deceleration of each move and once just after the return, each following a crossing of the gap, with a band RMS of up to 0.58 N m. The teeth are apart for 77.8 ms in all, and with the gap closed the same gains never ring, which is the limit of a notch on a geared axis, since every strike rings the mode afresh. The target counts the time from the start of the run, so the dwell is part of the program. With the dwell set to zero, the best

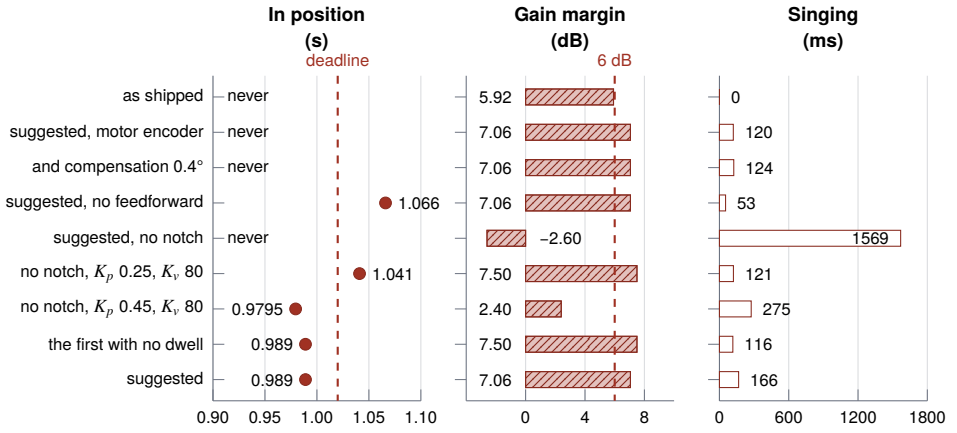


Figure 12.3 The move and return of Table 12.6 in its nine configurations: the time at which the table comes to stay within two load counts (dots, with the 1.02 s deadline dashed), the gain margin of the speed loop (hatched bars, with the 6 dB floor dashed) and the time the shaft sings (open bars). Notice that only the suggested configuration comes in on time with at least 6 dB on the programmed move, that the one quicker run pays for it with 2.40 dB of margin, and that every configuration but the shipped one, which never comes into position, sings for part of the run.

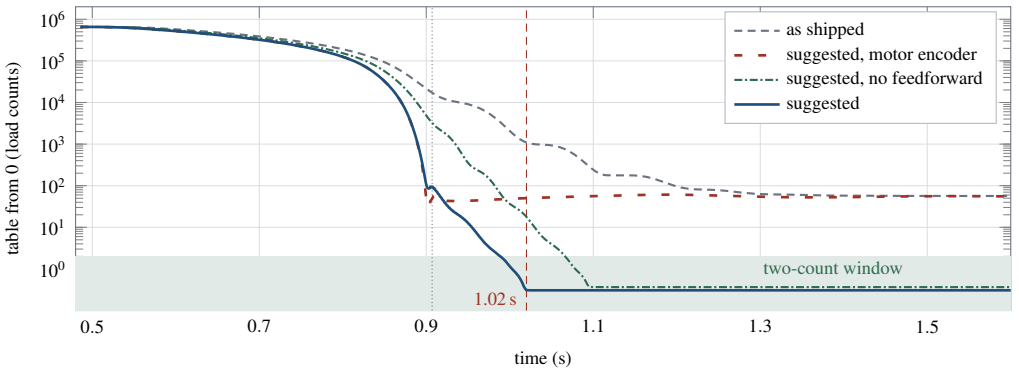


Figure 12.4 The return of the graded case, the table's true distance from zero in load counts. The motor-encoder runs, as shipped (densely dashed) and at the suggested settings (loosely dashed), settle 56.6 and 55.5 counts out, where the encoder cannot see the gap and the twist. The load-encoder runs enter the two-count window at 0.989 and 1.066 s with the feedforward (solid) and without it (dash-dotted). The vertical dotted line marks the end of the return's profile at 0.907 s and the vertical dashed line the deadline.

configuration without the notch is in position at 0.989 s, and the application scores such a run as not the programmed move (Table 12.6).

12.5.10 Explicit MPC Against the Cascade

Case 10 runs the 100 mm move of case 6 on the load encoder with no gap, then a 300 N cut that arrives at 0.6 s with the table at rest, and pins the MPC with the cascade beside it at the challenge's suggested tuning. Table 12.8 runs that program and a harder move, 10 m/s^2 and 2000 m/s^3, which asks for more than the drive's peak torque, on both encoders, one engine per row, with the same events, plant, quantization and duration in every row. The model has no random term, so every row replays exactly. The cascade appears as shipped, with its feedforward, at the suggested tuning

Table 12.7 Scenarios and Completion Requirements

#	Scenario	Kind	Required to complete
1	Current by hand	Experiment	Run through the scripted current steps of 3, −3 and 0 A, then from Reset drive the current command by hand until the motor turns faster than 100 rpm for 20 ms.
2	Current loop against the bus	Experiment	Hold the 20 A command on the full bus until $\|v\|$ has sat on the voltage circle for 20 ms, then from Reset, with the bus sagged by at least 20% (25% is suggested), run until the current stays below 18 A against its 20 A command for 10 ms.
3	Speed gain until the axis sings	Demonstration	Let the shaft sing at $K_p = 0.7$ without the notch for 0.1 s, switch the notch on while it sings, then with the notch on raise K_p to 1.0 or more and run 0.2 s with no resonance.
4	Tuned at 1:1, run at 5:1	Experiment	Run to 0.45 s with the shipped gains through the heavier workpiece at 0.28 s, then from Reset raise K_p to 1.0 or more and switch the notch on before 0.30 s, so that the linear loop keeps a bandwidth of at least 45 Hz and a positive phase margin for the first 0.1 s of the 5:1 step.
5	Backlash and an integral position loop	Demonstration	With the position integral on, see the table hunt more than 100 load counts peak to peak between 1 and 2 s, again from Reset at a position gain from 10 to 80 s^{-1} other than 30, then from Reset with the integral off run at least 1.5 s until the table is within one count of its target.
6	S-curve feedforward	Experiment	Run the 100 mm move without feedforward until the table is in position, then from Reset with the feedforward on until it is in position again with a worst following error below 0.1 mm.
7	Relay autotuner	Experiment	Run to 0.6 s through the relay test and the 600 rpm step with the notch off, then from Reset with the notch on before Run, again to 0.6 s.
8	Cutting force at the resonance	Experiment	Run 0.15 s against the 100 N ripple at 425 Hz with $K_p \geq 0.45$ and no notch, then from Reset with the notch on run 0.15 s against the same ripple with no resonance, then move the ripple to 163 ± 10 Hz and run 0.15 s more.
9	Field weakening for a rapid	Experiment	Run the 250 mm rapid without field weakening until its profile ends, then from Reset with field weakening on until the profile ends with a worst following error below 1 mm.
10	Explicit MPC against the cascade	Experiment	Run the move and the 300 N cut under the explicit MPC to 0.9 s, then from Reset switch the position and speed control to the cascade and run the same move and cut to 0.9 s.
11	Move, return and settle	Challenge	The table back within two load-encoder counts of 0 by 1.02 s and staying there to 1.6 s, never more than two counts past either target, no following-error fault, the current within 31.7 A and at least 6 dB of gain margin, with the programmed profile and dwell and the machine and cut as given.[a]

[a] With the shipped settings the table never comes within two counts, ending 56.6 counts from 0, and the gain margin of 5.92 dB is short of 6 dB. With the suggested settings (load encoder, feedforward, notch, $K_p = 0.8$, $\tau_I = 12$ ms, $K_v = 70\ s^{-1}$) the table is in position at 0.989 s, 31 ms inside the deadline, with no overshoot, a peak current of 22.4 A and 7.06 dB of gain margin.

and retuned once by a rule fixed before measuring, the largest speed gain on a 0.01 grid that keeps 6 dB of gain margin with the notch and $\tau_I = 12$ ms, then K_v at a quarter of the resulting bandwidth ($K_p = 0.90$, $K_v = 71\ s^{-1}$), with the feedforward. The MPC runs at case 10's settings throughout, and two counterfactual rows remove its torque rows or its preview. Every MPC row makes one decision per millisecond (1001 on case 10, 601 on the hard move) with no fallback and no supervisor trip, at a mean logged time of 0.4 to 2.3 µs with the lookup in Node.js. The online solve reproduces every MPC row, except on the harder move with the motor encoder, where it differs by 0.1 ms and 0.2 counts.

On the load encoder the MPC brings the table within two counts 416.1 ms after the move begins, before the profile itself ends at 418.0 ms, against 476.0 ms for the suggested cascade, and it never strays more than 0.69 µm from the command during the move, against 12.78 µm, at a lower peak torque, 2.030 against 2.119 N m. When the cut arrives it yields 39.4 counts (6.0 µm) against 361.3

Table 12.8 Explicit MPC Against the Cascade on the Move, the Cut and a Move Past the Peak Torque

Structure	In (ms)	Past (cnt)	Worst (μm)	T_e (N m)	Cut (cnt)	Back (ms)	End (cnt)
Case 10, load encoder							
Cascade, shipped	—	0	9990	2.012	1625	—	32.8
with feedforward	—	0	38.48	2.090	1818	—	23.2
Cascade, suggested	476.0	0	12.78	2.119	361.3	90.8	0.3
Cascade, retuned	474.3	0	11.37	2.117	316.3	88.8	0.3
MPC	416.1	0	0.69	2.030	39.4	13.8	0.0
no torque rows	416.1	0	0.69	2.030	39.4	13.8	0.0
preview off	469.5	3.4	1527	2.026	42.8	13.5	0.4
Case 10, motor encoder							
Cascade, suggested	509.6	0	14.60	2.120	366.7	—	11.0
Cascade, retuned	502.1	0	13.16	2.119	321.4	—	11.1
MPC	—[a]	2.6	0.60	2.035	54.8	15.9	−2.1
Move past the peak torque, load encoder							
Cascade, suggested	512.8	544.5	471.8	3.806	—	—	0.2
Cascade, retuned	501.4	511.6	467.6	3.807	—	—	0.2
MPC	378.6	1.6	194.5	3.807	—	—	0.2
no torque rows	378.9	0.8	204.6	3.807	—	—	−0.7
Move past the peak torque, motor encoder							
Cascade, suggested	554.5	567.7	476.0	3.806	—	—	1.7
MPC	—[a]	4.6	192.5	3.805	—	—	1.9

In is the time after the move's start at which the table comes to stay within two load counts of the target, Past the largest excursion beyond it, Worst the largest distance between the table and the command at the position loop's instants during the move, T_e the peak motor torque of the move, Cut the excursion under the cut, Back the time after the cut at which the table comes to stay within ten counts (1.5 μm), and End the error at the end of the run (1 s, or 0.6 s for the harder move), all from the true state at every 0.1 ms step. [a]The table enters the two-count band 50 times before the cut on case 10 and 70 times before the end on the harder move, the last at 577.8 and 580.0 ms, without staying.

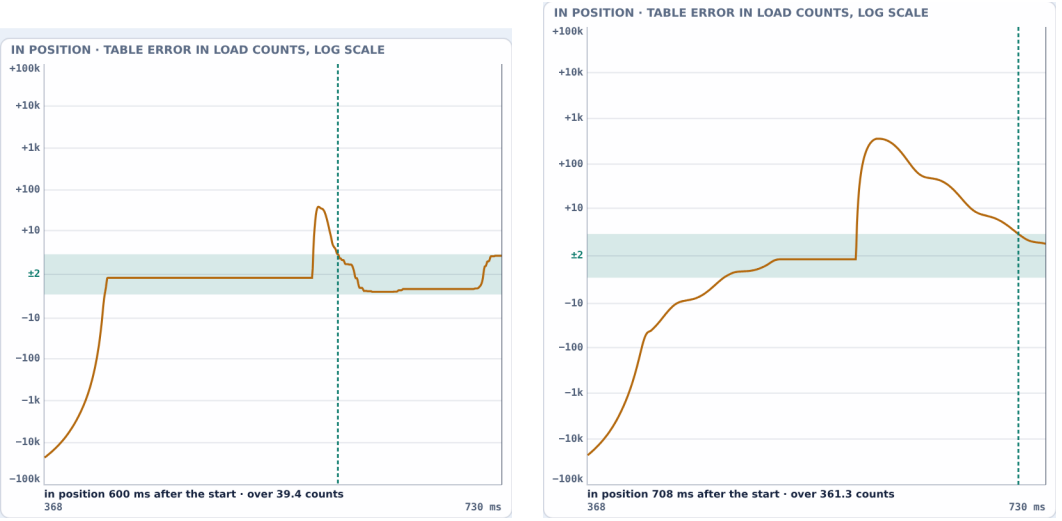
and is back within ten counts in 13.8 against 90.8 ms. The retuned cascade recovers less than 2 ms of the 60 ms (474.3 ms) and 45 counts of the excursion. Within two counts rather than ten the recovery reads 20.4 against 128.4 ms, but that figure turns on the last count at the guides' friction edge, and with the physics step halved it becomes 264.5 against 128.3 ms, while the in-position times, 416.8 and 475.9 ms, and the excursions, 43.0 and 361.4 counts, keep their order. The cut costs the MPC a higher peak torque than the cascade, 0.811 N m against 0.628, the price of holding the table closer.

The two counterfactual rows locate the gain. Without its torque rows the MPC reproduces every number on case 10, because the bound never binds on that move, and on the harder move it comes in at 378.9 ms against 378.6 with 0.8 against 1.6 counts past the target, so the drive's own clamp would serve nearly as well. The faster move and the stiffer hold come from the model, which puts in the twist an acceleration needs, from the load-torque estimate and from the preview, not from the constraint. Without the preview the MPC lags the profile by 1.5 mm, as a loop without feedforward must, and still comes in at 469.5 ms, ahead of the suggested cascade with its feedforward, with 3.4 counts past the target. The cascade's own trouble on the harder move is windup. With the current at its limit the speed integral winds, and the table passes the target by 544.5 counts, where the MPC passes it by 1.6 and is in position at 378.6 ms against 512.8.

The motor encoder reverses part of the result. There the MPC never settles inside two counts before the cut, entering the band 50 times between the move and the cut, and it ends the cut 2.1 counts off, because the table's position and the twist under the cut are then only the filter's estimate. The suggested cascade is in position at 509.6 ms but ends the cut 11.0 counts off and never returns within ten counts. On the harder move the MPC passes the target by 4.6 counts and is still crossing the band at the end of the run, where the cascade passes it by 567.7 counts and is in position at 554.5 ms.

The two runs of case 10 as loaded and with the cascade selected are shown at 0.75 s in Figure 12.5, on the profile view's strip of the table's true error in load counts.

Table 12.9 sets the worst value of each safety-relevant output under the MPC against the cascade over 45 runs, every existing case (the challenge also at its suggested settings) and case 6's move with every machine setting, parameter and disturbance at its extremes, the events and the move shapes.



(a) Explicit MPC, as loaded

(b) Cascade at the suggested tuning

Figure 12.5 Case 10, *Explicit MPC against the cascade: a move and a cut*, at $t = 0.75$ s on the profile view, the table's true error in load counts on a signed logarithmic scale from 368 to 730 ms after the move began, with the two-count band shaded. (a) The MPC enters the band before the cut, the cut lifts the table to 39.4 counts, and the readout gives in position 600 ms after the start (the dashed line), 20 ms after the cut. (b) The cascade enters the band later, the cut lifts the table to 361.3 counts, and the readout gives in position 708 ms after the start, 128 ms after the cut.

Table 12.9 Worst Case of Each Safety-Relevant Output Over 45 Runs

Output	Cascade	MPC
Following-error faults	step move	step move
Top speed (rpm)	5268 ($c_s = 0$)	3209 (case 9)
Peak measured $\lvert i \rvert$ (A)	32.96	32.98
Runs with $\lvert i \rvert$ above the peak	10	18
Largest band RMS (N m)	91.28 ($c_s = 0$)	7.21 (case 3)
Peak shaft torque (N m)	128.8 ($c_s = 0$)	10.60 (case 3)
I^2t trips	0	0

The step move has no profile and faults under both structures. The peak current of both is reached with the load at 20 kg cm^2. Case 3 runs the speed loop alone, so its singing is the cascade's under either selection.

The MPC never faults or overspeeds where the cascade does not, and its top speed of 3209 rpm is the cascade's own in case 9, while the cascade with the coupling's damping at zero is unstable, reaching 5268 rpm and a band RMS of 91.3 N m, a case the MPC holds quiet. The commanded current never passes the drive's limit under either structure, but the measured current passes the peak in 18 runs under the MPC against 10 under the cascade, by at most 1.31 A (4 %, with the load at 20 kg cm^2), because the current loop overshoots a command held at the limit. The MPC is worse than the cascade in 26 of the 45 runs on at least one of singing, peak current, shaft torque or end error, and better in 15. It rings with the gear gap, in case 5 for 187.2 ms against 81.4, and with a machine its model does not describe, for 1.06 s against none with the shaft stiffened to 2000 N m/rad, 706.8 ms with the guides' friction at 0.4 N m and 1.18 s with a 500 N ripple at the 163 Hz anti-resonance. At the voltage limit of case 9 the supervisor trips once, on the following error, and the MPC sings for 42.1 ms where the cascade does not. It is better where the mode, not the model, decides. A cut rippling at 425 Hz in case 8 makes the cascade sing for 1.56 s and the MPC not at all, and both glitches leave the MPC quiet where the cascade rings for 36 and 48 ms.

12.6 Technology in Practice

The axis of Section 12.3 reads its phase currents and two encoders and acts through an inverter. Figure 12.6 draws its three loops with real devices in the place of the schematic's symbols.

What Measures It

Motor speed was once read from a tachogenerator, a small direct-current generator whose voltage is proportional to speed, and an optical encoder does the same work by chopping a light beam into pulses [23]. The motor-side loops rest on such an encoder, described below. A linear encoder can also read the table of a machine tool directly, and an enclosed one keeps out dust, coolant and chips [24]. The 20-bit motor encoder and the 16-bit screw encoder of Section 12.3 quantize exactly and do nothing else, so the mounting errors of a real encoder are what the model leaves out.

> **TECHNOLOGY Sensor: The Optical Rotary Encoder**
>
> A code disc turns with the motor shaft. Its transparent and opaque, or reflective, segments interrupt a light beam, and a stationary index grating and photodetectors turn the passing pattern into signals. Single-chip sensors now carry the light source, the grating, the detectors and an interpolator that divides each line further [25]. An incremental encoder counts pulses and must return to zero after power-up, while an absolute encoder reads a coded pattern and keeps its position with the power off [24]. Mounting sets the accuracy. A disc 5 µm off center on a 65 mm grating reads up to about 32 arcseconds wrong, an error that repeats once every revolution, and one test head measured a span of about 206 arcseconds, mostly from that cause [25].

What Moves It

The final element is the power stage of the drive. Its inverter turns the voltages that the current loop computes into pulses of the bus voltage. In 1993 a motion controller typically sent its motor amplifiers a velocity reference of plus or minus 10 volts, and an open controller of that year also supported a pulse-width modulated one [26]. The averaged inverter of Equation (12.2) stands for the bridge without its switching ripple, dead time or braking circuit, and the bus sag of case 2 is the one fault of it that the model carries.

> **TECHNOLOGY Final Element: The Drive's Inverter**
>
> A drive rectifies its supply into a direct-current bus, smooths the bus with a capacitor and switches it through a bridge of power transistors. The duty cycle of the transistors varies rapidly, so that the pulses build the alternating voltage the motor needs. Braking reverses the flow of energy. Current from the motor charges the bus capacitor and raises the bus voltage, and with nowhere to go that energy would destroy the capacitor. A braking transistor and resistor load the bus in proportion to its over-voltage, or an active front end sends the energy back to the line [23]. On a trend, a braking axis shows the bus voltage climbing above its normal level until the clamp takes hold.

Where the Controller Runs

A machine controller synchronizes drives electronically instead of through a mechanical linkage, and its loops are nested, with cycle times from milliseconds to minutes [27]. In the open machine controller that NIST laid out in 1993, a trajectory generator sends setpoints at an interpolation interval to a servo level, which computes actuator commands at a higher rate with PID or other laws [26]. The profile and feedforward of this chapter belong to the first, and the position, speed and current loops at 1, 2 and 10 kHz to the second. Servo loops are so fast that analog controllers were long favored for them [23]. The velocity form of Equation (12.7) runs all three loops at their own periods in one digital controller.

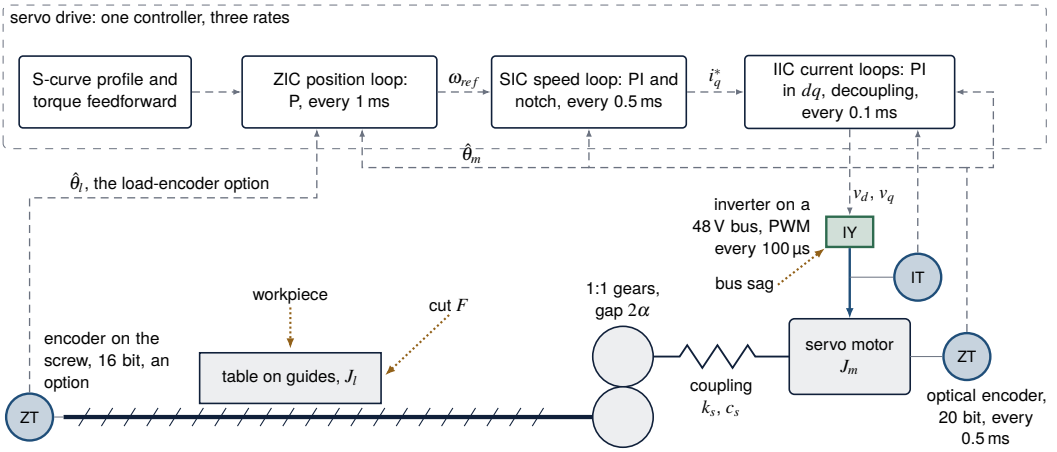


Figure 12.6 The loops of the chapter with real devices in place of the schematic's symbols. Phase-current sensors IT feed the current loops IIC, the optical encoder ZT on the motor feeds the speed loop SIC, the position loop ZIC and the decoupling, and an encoder on the screw can replace it for ZIC. The inverter IY is the final element. Notice that every measurement but the optional screw encoder sits on the motor side of the coupling and the gear gap, so the cut and the twist reach the motor-side loops only through the motion of the motor.

> ⚠ **IN PRACTICE** **Commissioning From the Inside Out**
>
> Motion control is a classic cascade of position, velocity and motor current loops. When a cascade is first commissioned or later retuned, the inner loop is tuned first, because the outer loop depends on it and not the other way round. With the inner loops closed, the position controller no longer faces a runaway process but an integrating one [23]. Errors that repeat are often corrected statically, and lead screw compensation tables were already common in machine controllers in 1993 [26]. The backlash compensation of case 5 is such a static correction, set by trying.

What Is Modernizing It

Encoders are the first development. Absolute encoders, which need no homing and keep their position without power, are replacing incremental ones in high-end motion control [24], and a modular encoder that averages several read heads around its disc calibrates out its own mounting error, by more than 90 % with a second head [25]. Diagnosis is the second. An inertial measurement unit carried on the moving carriage detects micrometer-level and microradian-level degradation of a linear axis, a step toward machine tools that diagnose themselves [19], and digital twins pair physics-based models of a feed drive with models learned from data [28]. Networks are the third. Drives that share one motion network depend on synchronized clocks, which NIST counts as critical for motion control as well as for security [27]. The explicit MPC of Section 12.4.8 is a fourth, computed offline so that a lookup fits the position loop's period.

12.7 Limitations

The axis has one torsional mode, an averaged inverter, an ideal surface-magnet motor, Coulomb friction alone and encoders that do nothing but quantize, as Section 12.3.6 states, so it shows the first resonance of an axis and not the second, which on a real machine is often what limits a notched loop. The cascade's speed loop runs at 2 kHz on a backward difference of the encoder, while observers that estimate the speed or the load can move the limit [3] and appear here only inside the MPC, and derivative feedback of the motor speed, which improves the damping of a two-inertia loop at small inertia ratios [5], is not offered. The linear model behind the Bode view leaves out aliasing and the

nonlinearities, and while it agrees with a simulated sweep within 1.6 % from 200 to 500 Hz, it is 9.3 % off at 700 Hz. The gain margin it reports is that of the proportional loop, within 0.11 dB of the PI loop's, and its phase margin is read on a grid of 260 frequencies, 0.7° high in case 4 at 5:1. The move profiles, a trapezoid and the jerk-limited S-curve, are the only shaping of the command. Input shaping, which Volume 7, Chapter 12 applies to a swinging load, is another way to keep a move from exciting a lightly damped mode, and it is not offered.

> **OPEN CHALLENGE** **A Notch That Follows the Load**
>
> A notch filter is a simple and effective way to suppress a lightly damped resonance, but a load that changes mass moves the resonant frequency. On one piezo-driven positioning stage a shift of only 1 % can make a high-gain loop with a fixed notch unstable. An adaptive notch moves its center as the resonance moves, but identifying the plant inside a closed loop is hard, because the loop yields less information than an open test and the estimate is sensitive to noise [29]. On this axis the heavier workpiece of case 4 moves the mode from 522.1 to 401.1 Hz. Progress would be a notch that follows the mode safely while the machine cuts.

The explicit MPC's model leaves out the gap, the voltage circle, the Coulomb friction beyond its expected direction, the cogging and the quantization, it stands a 0.25 ms lag in for the current loop, and it keeps the declared machine whatever the run carries. It has no speed or voltage row, so at the voltage limit of case 9 the supervisor, not the plan, hands the axis back. The supervisor prevents a runaway but does not make the law robust, and on the motor encoder, with the gap and on a changed machine the cascade remains the better controller. A model of the gap, or a law that stops pricing the twist while the teeth are apart, lies outside this design. Its timings were taken while other computations loaded the host.

Every threshold is a teaching value. The two counts, the 6 dB, the 20 mm window and the 0.30 N m at which the shaft is said to sing were chosen for teaching, and a real axis takes its in-position window and its margins from its machine's accuracy class and its drive maker's commissioning guide. No number in this chapter describes an industrial drive.

> **OPEN CHALLENGE** **Knowing When an Axis Wears**
>
> Machine tool axes degrade in service, and judging how far is usually a manual, time-consuming job whose measurements interrupt production [19]. A ball screw loses its preload with fatigue and wear, which costs precision and opens a backlash [28]. Methods that diagnose from data need long histories that are rarely available in the field, and their features shift with speed and operating conditions [28, 30]. Progress would be an axis that reports a growing gap or a softening coupling from its own encoders and currents during normal moves, before its parts go out of tolerance.

12.8 Conclusion

This chapter commissioned a servo axis from the current loop outward, and the simulator made each step of it measurable against the table's true position. A 1 A current step rises in 0.30 ms at rest, but the current loop holds 20 A only to 1777 rpm, because the back-EMF fills the voltage circle. The two-mass resonance at 399.9 Hz caps the speed gain at 0.55 without the notch and 1.75 with it, and at an equal 6 dB of margin the notch still triples the gain. Gains tuned at 1:1 lose more than half their bandwidth at 5:1, from 58.1 to 22.6 Hz. An integral in a load-encoder position loop hunts 482 counts with a gap and 381 without one, and removing the integral removes the hunt. Feedforward cuts the worst following error from 9.990 to 0.035 mm, the relay tunes to the coupling's 432.4 Hz, and the graded move and return is met at 0.989 s only with the load encoder, the feedforward and the notch, while the gap still rings the shaft for 166 ms. On the declared machine an explicit MPC with four moves and 81 regions is in position at 416 ms against 476 ms and yields 39 counts to the cut against 361, a gain that comes from its model, estimate and preview rather than its torque limit, while the motor encoder, the gear gap and a changed machine leave it behind the cascade.

The simulator's model keeps one mode, an averaged inverter and ideal sensors. A fuller model would add a second mode, static friction, an observer-based speed estimate, an adaptive notch that follows the workpiece, and a load-side encoder with its own errors. The axis shows that a drive is tuned around its coupling rather than its motor.

KEY POINTS

- A machine axis is tuned around the springy coupling between motor and table, not around the motor.
- The two-mass resonance at 399.9 Hz caps the speed gain at 0.55 without the notch and 1.75 with it, and at an equal 6 dB of margin the notch still triples the gain.
- An integral in a load-encoder position loop hunts 482 counts with a gap and 381 without one, and removing the integral removes the hunt.
- Feedforward cuts the worst following error from 9.990 to 0.035 mm.
- The graded move and return is met at 0.989 s only with the load encoder, the feedforward and the notch.

Where else this applies. A fast inner current loop under a slower outer loop, with an explicit MPC beside it, returns in the converter of Volume 3, Chapter 9, and a lightly damped mode under a moving drive in the swinging load of Volume 7, Chapter 12.

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12.9 Exercises

12.1 ○ Concept check · 2 min

In case 7 the relay autotuner finds an ultimate gain of 0.641 A s/rad at 432.4 Hz, and the Ziegler–Nichols gains it sets overshoot the 600 rpm step by 82.2 %, against 29.7 % for the commissioned gains. Which explanation is right?

(a) The 2 A relay is too weak to excite the loop, so the measured ultimate gain is far too low

(b) The sampled peaks of the cycle overstate the ultimate gain, so the rule sets a gain that is too high

(c) The backward-difference speed estimate adds a delay that the relay test cannot see in the loop

(d) The relay finds the phase crossing at the coupling's mode, and the rule tunes the integral to that period

12.2 ○ Concept check · 1 min

In the graded move and return as shipped, the position loop on the motor encoder ends with the table 56.6 load counts from its target, while the motor encoder reads the motor on its target. Which change removes the cause of this error?

(a) Close the position loop on the load encoder, so that it measures the table itself
(b) Raise the position gain K_v from 30 to 80 s^{-1}, so that the loop corrects faster
(c) Add a position integral, so that the loop removes the error that remains
(d) Set backlash compensation to the whole 0.5° gap of the gear pair

12.3 Concept check · 2 min

A cutting force ripples on the table at 163 Hz, the anti-resonance of the axis. What do the speed gain and the 400 Hz notch do to the table's vibration?

(a) The notch removes it, because it cuts the loop's gain at every frequency of the mechanics
(b) Little, because the motor barely moves there and the torque the loop applies goes into the table
(c) A higher speed gain reduces it in proportion, because a stiffer loop holds the axis at every frequency
(d) The vibration grows without bound, because a force at the anti-resonance destabilizes the loop

12.4 Theory and calculation · 7 min

The axis has a rotor inertia $J_m = 1.0\,\mathrm{kg\,cm^2}$, a load inertia referred to the motor $J_l = 5.0\,\mathrm{kg\,cm^2}$, a coupling stiffness $k_s = 526\,\mathrm{N\,m/rad}$ and a coupling damping $c_s = 0.020\,\mathrm{N\,m\,s/rad}$.

(a) Find the anti-resonance, the resonance and the locked-load frequency. *Checkpoint:* 163.2, 399.9 and 365.0 Hz.

(b) Find the damping ratio of the mode and the factor by which the gain of the loop above the resonance exceeds that of a rigid axis. *Checkpoint:* 0.048 and 6.

(c) A softer coupling halves k_s. Predict the three frequencies and the damping ratio. *Checkpoint:* 115.4, 282.7 and 258.1 Hz, and 0.068.

12.5 Theory and calculation · 6 min

The motor has $p = 4$ pole pairs, a flux linkage $\lambda_m = 0.020\,\mathrm{Wb}$, $R = 0.5\,\Omega$ and $L = 1.0\,\mathrm{mH}$, a rated torque of 1.27 N m and a peak torque of 3.8 N m, on a 48 V bus.

(a) Find the torque constant and the currents for the rated and the peak torque. *Checkpoint:* 0.12 N m/A, 10.58 A and 31.67 A.

(b) Find the radius of the voltage circle and the motor speed at which the back-EMF alone fills it. *Checkpoint:* 27.71 V and 3308 rpm.

(c) Find the voltage needed at 3000 rpm with the rated current and $i_d = 0$, and say whether the drive can deliver the rated torque there. *Checkpoint:* 33.20 V, so it cannot without field weakening.

12.6 Theory and calculation · 6 min

In the graded case the position loop runs on the motor encoder. The screw has a lead of 10 mm, the load encoder has 16 bits per revolution, the gear pair has a whole gap of 0.5°, the coupling has $k_s = 526\,\mathrm{N\,m/rad}$, and a 300 N cut resists the return.

(a) Find the table travel of one load count and the half gap in load counts. *Checkpoint:* 0.1526 µm and 45.5 counts.

(b) Find the torque the cut puts on the screw, the twist it causes and the twist in load counts. *Checkpoint:* 0.477 N m, 0.908 mrad and 9.5 counts.

(c) Add the two terms and compare the sum with the measured final error of 56.6 counts. *Checkpoint:* 55.0 counts, 8.4 µm.

12.7 App challenge · 8 min

Load scenario 7, The relay autotuner, and change no setting. The relay runs from 0.2 s, and the speed reference steps from 300 to 600 rpm at 0.5 s. The model has no random terms, so no seed applies.

(a) Run to 0.6 s. Read the autotuner's note: the frequency of the cycle, the ultimate gain and the PI gains it sets. Compare the frequency with the resonance on the Bode view. *Checkpoint:* 432 Hz, $K_u = 0.641$, $K_p = 0.288$ and $\tau_I = 1.93$ ms, above the 401 Hz resonance.

(b) Export the CSV file. Find the peak of the column `w_motor_rpm` after 0.5 s and the overshoot as a share of the 300 rpm step. *Checkpoint:* About 845 rpm and 82 %, within 1 percentage point.

(c) Press Reset, set the Notch filter On, run to 0.6 s again and repeat (a) and (b). *Checkpoint:* 500 Hz, $K_u = 1.817$, $\tau_I = 1.67$ ms, and an overshoot of 58 %, within 1 percentage point.

12.8 App challenge · 6 min

Load scenario 6, S-curve feedforward. The table moves 100 mm on an S-curve at up to 300 mm/s with $K_v = 30\,\mathrm{s}^{-1}$, and the profile lasts 418.0 ms.

(a) Change no setting and run to 1.2 s. Read the Worst following error and Last move tiles, and compare the first with v/K_v. *Checkpoint:* 9.990 mm and in position 972 ms after the move began, against 10 mm.

(b) Press Reset, set Profile feedforward On and run to 1.2 s again. Read the same two tiles. *Checkpoint:* 0.035 mm and 761 ms.

12.9 App challenge · 9 min

Load the Challenge, Move, return and settle. The target is the table within two load counts of 0 by 1.02 s and staying there, with no fault, the current within 31.7 A and at least 6 dB of gain margin. Run each case the full 1.6 s.

(a) Run the case as shipped. Read the gain margin on the Bode view and, from the exported file, the column `x_table_m` at 1.6 s in load counts of 0.1526 µm. *Checkpoint:* 5.9 dB and 56.6 counts, target not met.

(b) Press Reset and set Position feedback to the load encoder, Profile feedforward On, the Notch filter On, $K_p = 0.80$, $\tau_I = 12$ ms and $K_v = 70\,\mathrm{s}^{-1}$. Run again and read the challenge result, the Peak current tile and the gain margin. *Checkpoint:* In position at 0.989 s, 22.4 A, 7.1 dB, target met.

(c) Press Reset, set Position feedback back to the motor encoder and run again. Read the final error. *Checkpoint:* 55.5 counts, target not met.

12.10 Application area · 4 min

A four-flute end mill cuts on the table. Its tooth-passing frequency is the spindle speed in revolutions per second times the number of flutes.

(a) Find the spindle speeds that put the tooth-passing frequency at the 425 Hz of case 8 and at the 163 Hz anti-resonance. *Checkpoint:* 6375 and 2445 rpm.

(b) For each, say whether the drive's tuning can reduce the vibration, and what the engineer can do if it cannot. *Checkpoint:* Yes at 425 Hz, no at 163 Hz.

12.11 Application area · 4 min

The drive protects the winding with the thermal image of Equation (12.5), with $\tau_{th} = 10$ s and a rated current of 10.58 A, starting cold.

(a) An axis is stalled against a hard stop with 31.6 A in the winding. Find the time until the image reaches 1 and the limit folds back. *Checkpoint:* About 1.19 s.

(b) Field weakening holds the rated torque at 3000 rpm with $i_q = 10.58$ A and $i_d = -8.65$ A. Find the value toward which the image settles and the time to the fold-back. *Checkpoint:* 1.67 and about 9.2 s.

12.12 Review the agent · 5 min

An AI assistant is asked to recover the speed loop of case 4 after the workpiece raises the inertia ratio to 5:1, and replies as follows. “Raise K_p from 0.4 to 1.2 with $\tau_I = 15\,\text{ms}$ and switch the 400 Hz notch on. The speed-loop card then reads a bandwidth of 55.5 Hz and a phase margin of 61.2°, as well damped as the 63.6° of the 1:1 tuning, so the loop is robust and also meets the 6 dB rule of the challenge.” Find the flaw in the proposal and say how it could be checked.