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CHAPTER 10

Oil Price Versus Market Share in a Cartel's Quota Loop

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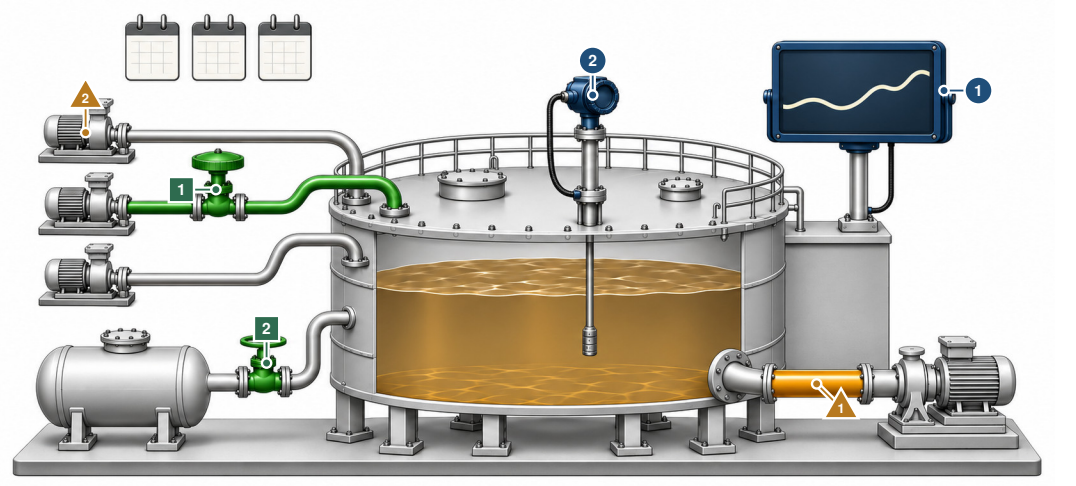
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CHAPTER 10

Oil Price Versus Market Share in a Cartel's Quota Loop



The controller moves 1 the producer group's quota, set at its meetings 2 the consumers' reserve release

The controller reads 1 the oil price 2 the stocks, reported a month late

What disturbs it 1 a change in world demand 2 rival shale wells made cheaper by new technology

Notice that only one of the three supply lines into the tank passes the group's valve, so the rival lines are free to grow while a high price is held. Illustration, not to scale.

IN PLAIN TERMS Pumping Less Raises the Price, for a While

The main idea. A group of oil producers can lift the world price of oil within weeks by pumping less. The oil stored around the world works like one big tank: when less flows in than people burn, the tank drains, and a lower tank means dearer oil.

The challenge. A high price slowly pays rival drillers to pump more. Over years it also pays buyers to use less. So the group must keep cutting to hold its price, and in eight years it can lose about half of its share of the market.

Why it matters. A target that looks easy to hold this month can be out of reach once the slow responses catch up.

THREE WAYS TO READ THIS CHAPTER

Explore	13 min	the first page, the founding agreement (Section 10.2), the fact on the largest cut and the history of the consumers' reserve (Section 10.5), the try-it segments of cases 4 and 7 and the fact on the economic MPC (Section 10.6), and the key points
Study	50 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 10.6), Technology in Practice (Section 10.7), Limitations (Section 10.8) and the open challenge

The control question A cartel that pumps less lifts the oil price within weeks, and the price it then holds pays rivals to drill and consumers to buy efficiency for years. How far can a quota decided at two meetings a year hold the price before the market it protects has gone to the fringe?

Learning objectives

- Write the balance on the world's stocks with its price law, and compute the stocks for a price and the rate at which the stocks alone return to balance.
- Compute the market's short-run and long-run gains from its equations and compare them with a quota step in the app.
- Compute the long-run residual demand at a held price, and explain why no tuning holds both a high price and the cartel's share.
- Explain how the meeting calendar, the quantum and the cap shape a quota, and what a leaking actuator does to an integral loop.
- Compare a PI on price, a PI on stocks and an economic MPC on a ten-year challenge.

Prerequisites Sections 1.4.2, 1.4.3, 1.4.4, 1.6.2 and 1.6.4, and the tank balance of Chapter 2. Section 1.6.3 is needed only for the economic MPC.

The app https://apmonitor.com/apps/opec-oil-market/

See also Volume 6, Chapter 11 for stocks ordered on a calendar in a quantum with a cap, and Volume 6, Chapter 7 for the choice of which flow holds an inventory. Volume 9, Chapter 9 for a specification written as a band that a controller can spend, and Volume 9, Chapter 6 for a price set inside a predictive controller.

10.1 Introduction

A barrel that a cartel withholds is missed at once, because the world cannot stop burning oil quickly, so the price rises within weeks. The same price then pays shale drillers within a year, conventional producers within several years and consumers who buy efficiency over a decade, and the barrels they add or no longer need are the cartel's lost market. In the simulator of this chapter, one more million barrels a day (Mb/d) from the cartel lowers the price by $8.1 within three months and by only $1.6 seven years later. A cartel that holds $100 through a shale revolution keeps the price there, at an average of $98.3 over years 1 to 8, while its share of world supply falls from 30 to 15.27 %.

Oil economics has long described this market in the terms of dynamic optimization and feedback. The economics of exhaustible resources was set out in 1931 [1], a cartel dominating a competitive fringe was posed as a Nash–Cournot problem on the world oil market in 1976 [2], and producers' gains from cartelization were computed from optimal dynamic pricing on small models with lagged demand and supply [3], which is open-loop optimal control. When ten world oil models were compared in 1980 and 1981, most of them set the cartel's price by a rule driven by its capacity utilization, one treated the cartel as an intertemporal wealth maximizer, and every one made it the residual supplier [4]. The first is a feedback law and the second an economic optimizer, which economic model predictive control (MPC) now formalizes [5]. Inventories join the two sides of the market, because prices, production and stocks are set together through a market for storage [6], and inventory data identify the speculative part of the oil price [7]. The plant is a stock with four inflows and one outflow, the actuator is a quota and the controller is a rule or an optimizer, which makes the market a control problem in the ordinary sense of the word.

The belief this plant tests is that an integral controller can hold any setpoint its actuator is able to reach. A price held above the market's reference is itself an input to the plant, because it moves the slow states of the fringe and of the consumers, and those states take the cartel's barrels. In the application's own sweep of three price targets, four tunings and two calendars, every run that averages $95 or more over years 1 to 8 loses at least 49 % of its share, so the infeasibility lies in the plant and not in the tuning. The same structure appears wherever a level is held by metering one of several inflows whose owners, or whose outflow, adapt to the level, such as a reservoir shared with other diverters, a buffer stock whose customers substitute, or a market that entrants join when the price

is high. In control terms it is a self-regulating plant (Section 1.4.4) whose long-run gain is about a ninth of its short-run gain, driven through an actuator that is sampled twice a year, quantized and rate limited. The level loop of Chapter 2 holds a tank against a fixed outflow law, and the central bank of Volume 2, Chapter 11 decides a policy rate through the same calendar module.

The work that explains the cartel's choices uses equilibrium and game models calibrated to history [8, 9], and the simulation models of the 1980s embedded a pricing rule as part of the market [4]. This chapter instead puts a chair, a PI loop, a stock target and an economic optimizer on one tank and one calendar, and measures how far integral control, a stock target and an economic optimizer can hold the oil price before the market they protect has gone to the fringe. The market structure, the PI loop, the inventory target and economic MPC are all standard. The analysis is a set of measurements on one deterministic simulator together with a linearization of the simulator's own equations. The simulator is a ten-state market whose barrel ledger closes to 7.4×10^{-13} Gb over ten years of every case, with a stock report published a month late beside its five-year average and an MPC that relinearizes the market five times at each meeting. Every equation, constant and limit is stated. Nine cases, a balanced reference and a ten-year challenge report price, share and revenue side by side.

Section 10.2 follows the domain from exhaustible-resource theory to the shale decade. Section 10.3 writes the balance on the stocks, the price law and the lagged fringe and demand, and Section 10.4 linearizes them to find the market's two speeds and its two gains. Section 10.5 sets out the meeting calendar, the two PI loops, the price war, the consumers' reserve and the economic MPC. Section 10.6 reports the measured cases and the challenge, Section 10.7 the surveys, targets and meetings of the real market, Section 10.8 the limitations and Section 10.9 the conclusions. The exercises in Section 10.10 close the chapter.

10.2 From Exhaustible Resources to the Shale Decade

The cartel's problem entered economics as a problem of time, through the economics of exhaustible resources [1]. In 1976 the cartel was modeled as a unified enterprise that dominates the other extractors because of its larger reserves, with equilibrium price and sales paths that give neither the cartel nor the competitive fringe an incentive to change its behavior. In that model a disproportionate share of the extra profit from forming the cartel went to non-members, and the cartel's restriction of sales eventually left it the sole supplier [2]. Two years later the gains from cartelizing the world petroleum market were calculated under optimal dynamic monopoly pricing, with small market models that carried short-term lags in demand and supply, by comparing the discounted profits of optimal price paths under competition and under cartelization [3]. The fringe, the lags and a discounted objective are all in the simulator, and the last becomes the objective of its economic MPC.

The forecasting models of the next decade wrote the cartel as a feedback rule. During 1980 and 1981 the Energy Modeling Forum at Stanford University compared ten prominent models of the international oil market. Most were simulation models in which the cartel set its price by a target capacity utilization rule, one let it act as an intertemporal wealth-maximizing price setter, one made its capacity endogenous while the rest held it constant at 34 million barrels a day, and all of them made it the residual supplier, producing the difference between world demand and non-OPEC supply [4]. The simulator's cartel has the same 34 Mb/d of capacity and the same residual role. Its controllers replace the capacity-utilization rule with a PI loop on the price or on the stocks, so that the rule becomes a design choice rather than a fixed part of the market.

The collapse of 1986 is the price war of the fifth case and the leaking valve of the seventh. From 1979 to 1985 the cartel's members restricted output, Saudi Arabia most of all, by more than 60 % of its 1979 level, and a shortfall in demand for the cartel's oil drove Saudi production below 3 million barrels a day in the summer of 1985 [4]. Early in 1986, after years of futile attempts to get its partners to observe their quotas and share the burden of restriction more equitably, Saudi Arabia and its allies produced their "fair share" and let the price fall. From August 1985 to mid-1986 the cartel's output

rose by about 4 million barrels a day, some 25 %, more than half of it Saudi, and in the first half of 1986 crude oil fell to about $12 a barrel, its level of 1974 [4]. A discussant of that analysis described the small members as cheating readily, knowing that the others would cover for them, while the agreement collapses if the largest member cheats [4].

Later episodes separated the causes of a high price. The run-up of 2007 and 2008 came from strong demand confronting stagnating world production, unlike earlier shocks that came from physical disruptions of supply [10]. A structural model that identifies the speculative component of the price from inventory data attributed the 2003 to 2008 surge to world consumption driven by the business cycle and not to speculation [7], and supply shocks, shocks to the demand for all industrial commodities and shocks specific to the oil market were shown to move the real price in different ways [11]. An overview of every major fluctuation from 1973 to 2014 also explains why the price remains hard to predict [12]. The simulator keeps one channel of all these, the price set by the stocks, and drives it with demand and supply shocks of declared size.

The shale decade made the fringe fast. In November 2014 OPEC announced a strategy aimed at its market share, and crude prices crashed over the following year. An equilibrium model calibrated to the market explains the switch from accommodating shale up to 2014 to defending share afterward by the growth of shale, slower demand, a less cohesive cartel and rising non-OPEC output, with cheaper shale working the other way [8]. Saudi policy in that fall was read as a trade-off between revenue now and market share later, in which uncertainty about the elasticity of shale supply made not cutting the safer choice [9]. Well-level data from North Dakota later put the supply elasticity of shale wells at 0.3 to 0.9 and found no such response in conventional wells [13]. The storage limit had its own episode in April 2020, when a collapse of demand during the pandemic took West Texas Intermediate (WTI) futures below zero [14]. The simulator's sixth case is that tank filling to its top.

> ⧖ HISTORY **A Founding Agreement That Regulates Production, 1960**
>
> Representatives of the governments of Iran, Iraq, Kuwait, Saudi Arabia and Venezuela met at Baghdad from 10 to 14 September 1960 and formed a permanent Organization of the Petroleum Exporting Countries. Their first resolution called on the members to study and formulate a system to ensure the stabilization of prices by, among other means, the regulation of production, with due regard to the interests of the producing and of the consuming nations. The organization was to hold meetings at least twice a year, and more often if necessary [15]. A loop that reads a price, moves production and decides at two meetings a year is the structure this chapter simulates, and twice a year is its shipped calendar.

10.3 One Tank for the World Market

The simulated market is one aggregate stock of commercial crude oil, drawn in Figure 10.1 as the application draws it, with the producers' inflows at the left, the world's demand leaving at the right and the price read off the level. Time is in years, flows in million barrels a day (Mb/d), stocks in billion barrels (Gb), prices in dollars a barrel and revenue in billions of dollars. A flow of 1 Mb/d held for a year is $K = 0.365$ Gb. Table 10.1 classifies the signals.

10.3.1 Stocks, Price and the Quota Valve

The stocks integrate the imbalance between supply and demand, as every inventory balance of Section 1.4.2 does, and the price is a steep, falling function of the stocks, as shown in Equations (10.1) and (10.2).

$$\dot{I} = K(S - D), \qquad S = q_c + q_{sh} + q_{cv} + q_o + r \tag{10.1}$$

$$P = P_0 \exp\left[\Pi_{[-8,5]}\left(-k(I - I_0)/I_0\right)\right](1 + d_r) \tag{10.2}$$

where $P_0 = \$75$ is the price at the normal stock $I_0 = 2.9$ Gb, $k = 9$, d_r is a risk premium and Π clips the exponent far outside the operating range. A 10 % build multiplies the price by $e^{-0.9} = 0.407$, so $100 sits at 2.807 Gb and $50 at 3.031 Gb. The shape follows the role that inventories play in the oil

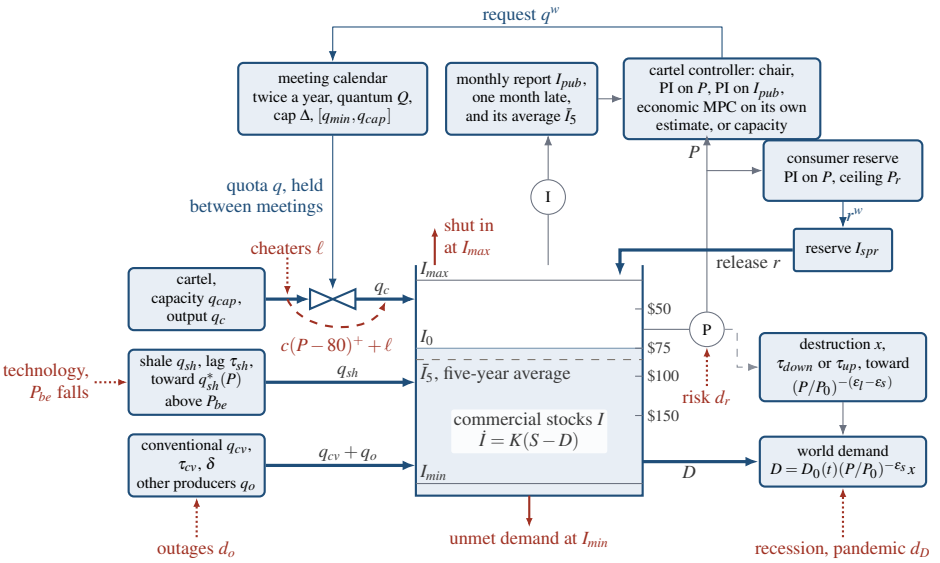


Figure 10.1 The world oil market as one tank, not to scale. Commercial stocks I integrate the cartel's output q_c, shale q_{sh}, conventional supply q_{cv}, other producers q_o and a consumers' release r, less world demand D. The ruler on the wall is the price law of Equation (10.2), cheapest when the tank is fullest, and oil above I_{max} is shut in. The quota q is a valve with a dashed bypass, the overproduction $c(P-80)^+$ and cheaters ℓ. Shale moves toward a price-dependent target above its breakeven P_{be}, and demand carries a destruction state x that falls faster than it recovers. The cartel's controller reads the price, or the monthly report I_{pub} and its five-year average $\bar{I}_5$, and its request q^w passes the meeting calendar's quantum Q, cap Δ and bounds. The consumers' reserve I_{spr} is a second controller on the same level. Densely dotted arrows (red on screen) are disturbances, solid red arrows are the oil shut in or not supplied, and the gray dashed line marks the price reaching the slow responses.

Table 10.1 Signals of the Market by Role

Symbol	Role	Unit
P	CV, oil price, SP P_{sp}	\$/bbl
I_{pub}	CV of the stocks loop, SP $\bar{I}_5 + \Delta I_{sp}$	Gb
q	MV, quota in force, q_{min} to q_{cap}	Mb/d
q^w	request of the chair or a controller	Mb/d
r	MV of the consumers' reserve, 0 to r_{max}	Mb/d
d_D, d_o, d_r	DVs, demand, other supply, risk premium	Mb/d, —
P_{be}, ℓ	DVs, shale breakeven, cheaters' output	\$/bbl, Mb/d
I, I_{spr}	states, commercial stocks and the reserve	Gb
q_{sh}, q_{cv}	states, shale and conventional output	Mb/d
x	state, demand factor, 1 less destruction	—
V	state, cartel's discounted revenue	\$ billion
q_c, S, D	outputs, cartel output, supply, demand	Mb/d
ϕ	output, cartel share of supply	%
$\bar{I}_5$	published average of 60 reports	Gb

market [6, 7], but the coefficients are a teaching closure and not an estimate. The cartel's output is the quota plus what the members add on their own, bounded by capacity, as shown in Equation (10.3).

$$q_c = \Pi_{[0,q_{cap}]}\left(q + c\,(P - P_c)^+ + \ell\right) \tag{10.3}$$

where $q_{cap} = 34\,\text{Mb/d}$, the members overproduce $c = 0.03\,\text{Mb/d}$ for each dollar above $P_c = \$80$, and every scripted cheater adds 1 Mb/d to ℓ. The quota therefore commands the valve, but not the whole flow through it. At \$100 the members add 0.6 Mb/d beyond any quota, enough to raise the CHEATING status, which reads more than 0.25 Mb/d above the quota, and in the fourth case the cartel produces 14.18 Mb/d against a quota of 13.6 at year 8.

10.3.2 A Fringe With Lags

Shale and conventional supply answer the price through first-order lags (Section 1.4.4), as shown in Equations (10.4) and (10.5).

$$\tau_{sh}\dot{q}_{sh} = \Pi_{[0,\,q_{max}]}\big(q_b + s\,(P - P_{be})\big) - q_{sh} \tag{10.4}$$

$$\dot{q}_{cv} = \frac{(1+\tau_{cv}\delta)\,q_{cv0} + s_{cv}(P-P_0) - q_{cv}}{\tau_{cv}} - \delta\,q_{cv} \tag{10.5}$$

where the breakeven P_{be} is the price at which new wells just replace the decline of old ones, so that shale holds at its base q_b, each dollar above it adds $s = 0.1\,\text{Mb/d}$ in the long run up to $q_{max} = 15\,\text{Mb/d}$, and $\tau_{sh} = 0.75\,\text{yr}$. Conventional supply has an investment response $s_{cv} = 0.2\,\text{Mb/d}$ per dollar, a lag $\tau_{cv} = 4\,\text{yr}$ and a decline $\delta = 3\,\%$ a year, so its steady state is $q_{cv0} + s_{cv}(P-P_0)/(1+\tau_{cv}\delta)$, 0.179 Mb/d for each dollar. Other producers supply a fixed $q_o = 10\,\text{Mb/d}$ plus any outage d_o. At \$75 shale gives 10 Mb/d, from $q_b = 7\,\text{Mb/d}$ and $P_{be} = \$45$ by default or from $q_b = 10.5\,\text{Mb/d}$ and $P_{be} = \$80$ in the shale-revolution cases, in which a technology shift lowers P_{be} by \$35 over four years. Read as an elasticity, s is 0.75 at \$75 and 10 Mb/d, inside the range estimated for shale wells [13], although that estimate is for individual wells.

10.3.3 Demand That Is Destroyed and Slowly Restored

Demand answers the price at once through the short-run elasticity and over years through a destruction state, as shown in Equations (10.6) and (10.7).

$$D = D_0(t)\,(P/P_0)^{-\varepsilon_s}\,x \tag{10.6}$$

$$\tau\dot{x} = x^*(P) - x, \qquad \tau = \begin{cases} \tau_{down} & x^* < x \\ \tau_{up} & x^* \geq x \end{cases}$$

$$x^*(P) = \big(\max(1,P)/P_0\big)^{-(\varepsilon_l - \varepsilon_s)} \tag{10.7}$$

where $D_0(t) = [D_0 \mathrm{e}^{gt}\sigma(t) + d_D(t)]^+$ is the demand at the reference price before any destruction, $D_0 = 100\,\text{Mb/d}$, the growth g is zero by default, $\sigma(t)$ is one unless a carbon policy turns the growth to $-1\,\%$ a year, d_D collects the scripted demand shocks, $\varepsilon_s = 0.06$, $\varepsilon_l = 0.35$, $\tau_{down} = 6\,\text{yr}$ and $\tau_{up} = 12\,\text{yr}$. On the model's own equation, a price held at \$100 cuts demand by 1.71 % at once and moves x 63 % of the way to its target of 0.920 in 6.0 yr, half in 4.16 yr, while a price then held at \$50 takes 8.32 yr to move it half of the way to its target of 1.125. Efficiency bought at a high price is therefore sold back at a low one only half as fast, the asymmetry that OECD demand shows between price increases and decreases [16].

10.3.4 Revenue, Share and the Monthly Report

The cartel's objective, its share and the consumers' reserve close the state equations, as shown in Equation (10.8).

$$\dot{V} = KPq_c\,\mathrm{e}^{-\rho t}, \qquad \phi = \frac{q_c}{\max(1,\,S-r)}, \qquad \dot{I}_{spr} = -Kr \tag{10.8}$$

where V is the discounted revenue at $\rho = 5\,\%$ a year in billions of dollars, the share leaves the reserve's release out of the denominator, and the reserve starts with 0.7 Gb. Four running totals, oil

Table 10.2 Scripted Shocks, With ς the Years Since the Event

Event	Effect
recession	$d_D = -3\varsigma$ for $\varsigma < 1$, then $-3(3-\varsigma)/2$ to $\varsigma = 3$
pandemic	$d_D = -20$ for $\varsigma < 0.25$, then $-20(1.75-\varsigma)/1.5$ to $\varsigma = 1.75$
outage	$d_o = -1.5$ for a year, or -5.7 for two weeks
technology	P_{be} falls by $35\min(\varsigma/4, 1)$
cheater	ℓ rises by 1, for good
carbon policy	$\sigma = \mathrm{e}^{(-0.01-g)\varsigma}$
risk spike	d_r rises by 0.2 for three months
reserve release	r rises by 1 for six months

Flows in Mb/d, prices in $/bbl.

supplied, oil demanded, oil shut in and demand unmet, keep the barrel balance auditable. At every month end the stocks are recorded, and the report of month m is published at the end of month $m + L_r$, with $L_r = 1$, so that

$$I_{\text{pub}} = I(t_m), \qquad \bar{I}_5 = \frac{1}{60}\sum_{i=0}^{59} I_{\text{pub},m-i} \tag{10.9}$$

where t_m is the end of the latest month reported and the average spans the last five years of reports. The price, by contrast, is read without delay.

10.3.5 Shocks, Initial State and Numerics

The scripted shocks of Table 10.2 are pure functions of the time since the event fired, so the derivatives stay pure, and each event takes effect on the earliest step that reaches its time. Three advanced sliders add constant shifts to demand (−20 Mb/d to 10), to other supply (−5 Mb/d to 5) and to the risk premium (0 to 0.5). The model draws no random number, so every run is deterministic.

Every case starts balanced at $75 unless it declares otherwise. Shale starts at its target, conventional supply at q_{cv0}, $x = 1$, and the quota at $D_0 - q_{sh} - q_{cv0} - q_o = 30\,\text{Mb/d}$, so that supply equals demand at I_0 and the sixty reports behind $t = 0$ all read I_0. The third case instead starts at 3.15 Gb, 0.25 Gb above I_0, after a glut that built linearly over three years, which the report history remembers as an average of 2.977 Gb. The physics advances by fourth-order Runge–Kutta on a step of a week, $\Delta t = 1/52$ year, and the controller runs every week before each step (Section 1.6.1). After each step, stocks above I_{max} are clipped and the excess booked as shut in, stocks below the working minimum $I_{min} = 0.1$ Gb are lifted and the deficit booked as unmet demand, the month-end record and the report queue are updated, and the status flags are set. The application's checks find every derivative at the balance below 1.8×10^{-15} and the reference unchanged after ten years, and over ten years of every case the stocks equal their start plus supply, less demand and shut-in oil, plus unmet demand, to 7.4×10^{-13} Gb. At half the step the challenge's suggested run keeps its verdict and its revenue to $0.1 billion. The application passes its own 23 checks and the 17 that the framework runs on every application.

10.3.6 Assumptions

One tank and one price. The world's commercial stocks are one vessel and the price is a function of their level alone. Real stocks and prices move together only loosely, through grades, regions, freight and a futures curve that the model leaves out, and a price shock can come from supply, from aggregate demand or from demand specific to oil [11].

A single decision maker. The cartel decides as one, its members do not bargain, and shale does not anticipate the cartel, so this is feedback against a price-taking fringe and not the game of [2, 8]. Cheating is an assumed slope above $80 and a set of scripted cheaters, and the loyal members' floor is a stated number.

Table 10.3 Teaching Values and Where Each Is Set: A Among the Advanced Model Parameters or Controls, C on a Controller Card, S in the Source. No Constant Is Hidden From the Reader

Symbol	Meaning	Default	Range	
D_0, g	demand (Mb/d), growth (/yr)	100, 0	70–130, −0.02–0.03	A
ε_s, ε_l	short-, long-run elasticity	0.06, 0.35	0.01–0.2, 0.2–0.7	A
τ_{down}, τ_{up}	destruction, recovery (yr)	6, 12	2–10, 6–20	A
I_0, I_{max}	normal stocks, storage (Gb)	2.9, 4.5	2–4, 2.6–6	A
P_0, k	reference price ($), sensitivity	75, 9	40–110, 3–15	A
L_r	report delay (months)	1	0–3	A
q_b, P_{be}	shale base (Mb/d), breakeven ($)	7, 45	3–12, 20–90	A
s, q_{max}	shale slope (Mb/d/$), ceiling	0.1, 15	0.02–0.3, 10–25	A
τ_{sh}	shale lag (yr)	0.75	0.25–2	A
q_{cv0}, s_{cv}	conventional (Mb/d), response	50, 0.2	35–60, 0–0.4	A
τ_{cv}, δ	investment lag (yr), decline (/yr)	4, 0.03	2–8, 0–0.1	A
q_o	other producers (Mb/d)	10	0–20	A
K	Gb per Mb/d for a year	0.365	—	S
P_c, I_{min}	overproduction price ($), working minimum (Gb)	80, 0.1	—	S
—	status limits: x, over quota, spare (Mb/d)	0.95, 0.25, 0.5	—	S
q_{cap}, q_{min}	capacity, quota floor (Mb/d)	34, 0	28–40, 0–30	A
c	overproduction (Mb/d/$)	0.03	0–0.1	A
ρ	discount rate (/yr)	0.05	0–0.15	A
$I_{spr}(0)$	strategic reserve (Gb)	0.7	0–1.5	A
P_{sp}	PI price target ($)	75	40–120	C
K_c, τ_I	price PI (Mb/d/$, yr)	0.05, 0.4	0.01–0.3, 0.1–5	C
ΔI_{sp}	stocks target above $\bar{I}_5$ (Gb)	0	±0.3	C
K_s, τ_{Is}	stocks PI (Mb/d/Gb, yr)	10, 1	1–40, 0.1–5	C
P_{lo}, P_{hi}	announced band ($)	60, 100	30–90, 70–150	C
ϕ_{min}, w, H	MPC share floor, move weight, horizon (yr)	0.27, 1, 8	0–0.4, 0.1–10, 2–10	C
P_r, K_r, τ_{Ir}	reserve ceiling ($), PI gain, time (yr)	90, 0.05, 0.5	50–150, 0.01–0.3, 0.1–3	C
Δ, Q, r_{max}	cap, quantum, largest release (Mb/d)	1.5, 0.1, 1	0.5–5, 0–0.5, 0.1–2	A
—	meeting calendar	twice a year	monthly, weekly	C
Δt, Δt_c	physics step, control period (yr)	1/52, 1/52	—	S
—	challenge: months in $60–100, share, V ($bn)	0.9, 0.3, 5950	—	S

First-order fringe. Shale and conventional supply relax toward price-dependent targets. There is no well-by-well decline, financing, bankruptcy or capacity lost to a shut-in, which matters most in the price war, where a real fringe would also lose firms.

No growth and no history. Demand growth is zero by default. With demand growing 1 % a year and fringe supply curves that do not grow, seven of the eleven shipped cases reach the cartel's capacity within the decade, at year 7.0 while a PI holds $75 and at 9.5 in the second case, so growth is an advanced parameter. No historical price series is drawn, and the cases echo 1986, 2008, 2014 and 2020 in shape and not in any figure.

Teaching values. Every price, elasticity, lag, capacity and threshold is chosen to expose a mechanism. Real values come from estimates of demand elasticities [17], of the asymmetric response of demand [16], of shale supply [13] and of the role of inventories [7], which the application anchors on but does not reproduce.

10.4 Two Speeds of the Market

The market regulates itself, because a fuller tank is cheaper oil and cheaper oil is burned faster. Linearized at the balance (Section 1.4.3), with the state (I, q_{sh}, q_{cv}, x), the stock row of the Jacobian is $(-6.80, 0.365, 0.365, -36.5)$ in the units of Table 10.1 per year, so the stocks alone would return to equilibrium at 6.80/yr. The price couples them to the fringe, because near the balance the price falls by $232.8 per Gb of extra stock and the shale and conventional targets fall with it. The four eigenvalues are a damped pair at $-3.85 \pm 3.47\,j$ per year and two slow modes at –0.651/yr and –0.225/yr when the price rises, and $-3.90 \pm 3.10\,j$, –0.560/yr and –0.142/yr when it falls. The two

sets differ because the destruction law of Equation (10.7) has a different time constant on each side of its target, so the linearization holds on one branch at a time.

Pause. Try Exercise 10.2 before reading on.

The quota's grip on the price fades between the two speeds. With the fringe and x frozen, an extra 1 Mb/d must be burned by consumers alone, a static short-run gain of $P_0/(\varepsilon_s D_0) = \$12.5$ per Mb/d. Once shale, conventional supply and demand have all adapted, the market absorbs 0.745 Mb/d for each dollar (s plus 0.179 Mb/d of conventional supply plus $\varepsilon_l D_0/P_0$), a long-run gain of \$1.34 per Mb/d, about a ninth of the short-run one. The measured step lies between the two, as shown in Figure 10.2(a). A quota of 31 Mb/d decided at the meeting of year 1 lowers the price to \$70.2 four weeks later and to its lowest, \$66.87, 0.27 year later, because shale has already begun to answer. The price is back to \$72.0 after a year, \$73.0 after three and \$73.4 after seven, with shale at 9.84 Mb/d, and it reaches \$73.67 after sixty. A cut to 29 Mb/d gives a peak of \$83.76 at 0.25 year, larger than the dip because the price law is convex, and \$76.40 after seven years.

What a held price leaves the cartel in the end does not depend on the controller. With every slow state at its target for a price P, the cartel supplies the long-run residual demand of Equation (10.10).

$$R_{lr}(P) = D_0\left(\frac{P}{P_0}\right)^{-\varepsilon_l} - q^*_{sh}(P) - q^*_{cv}(P) - q_o \tag{10.10}$$

where q^*_{sh} and q^*_{cv} are the steady states of Equations (10.4) and (10.5). After the technology shift of the fourth case it is 11.0 Mb/d at \$100 (13.5 before the shift), 16.1 at \$90 and 26.5 at \$75, a long-run share of 12.1 % at \$100 against 26.5 % at \$75. A PI controller that holds \$100 sees the fringe's growth and the destruction of demand as a disturbance that keeps rising like a ramp, and it can only follow the price by giving the barrels away. The 24 runs of Figure 10.2(b) lie along one frontier whatever their tuning, which runs close to the long-run curve, above it at high prices and below it at low ones, because eight years is not yet the long run.

10.5 Deciding at Meetings

The controller executes every week, $\Delta t_c = 1/52$ year, and an execution becomes a decision only when the cartel's calendar says so. At each execution the consumers' reserve sets its release, the selected controller forms a request q^w, and on a meeting the request is shaped into the quota that will hold until the next meeting. The chair and every automatic controller share that last step.

10.5.1 The Calendar and the Shape of a Quota

Both pieces come from the framework module `core/cadence.js`. The calendar meets twice a year, at every whole and half year from $t = 0$, as shipped, or monthly, or at every weekly execution, and a meeting falls at the first execution at or after its instant. A month is not a whole number of weeks, so monthly meetings fall four or five weeks apart, twelve a year, while the twice-yearly ones fall every 26 weeks. A meeting turns the request into the quota in force by Equation (10.11).

$$\begin{aligned} \tilde{q} &= Q\,\mathrm{round}(q^w/Q), \qquad \Delta_Q = Q\max\left(1, \lfloor \Delta/Q \rfloor\right) \\ q_k &= \Pi_{[q_{min},\, q_{cap}]}\left(q_{k-1} + \mathrm{sat}_{\Delta_Q}(\tilde{q} - q_{k-1})\right) \end{aligned} \tag{10.11}$$

where $Q = 0.1\,\mathrm{Mb/d}$ and $\Delta = 1.5\,\mathrm{Mb/d}$, a tie is rounded away from zero, the bounds are first moved onto the grid, and a non-finite request leaves the quota where it was. In the application's check, which runs the challenge for six years on each of the three calendars and every case for ten years, all 916 decisions sit on the 0.1 grid, move by no more than the cap, stay within the bounds and fall on their calendar instants. No other rate limit acts on the quota, and neither the quantum nor the cap is fed back to a controller.

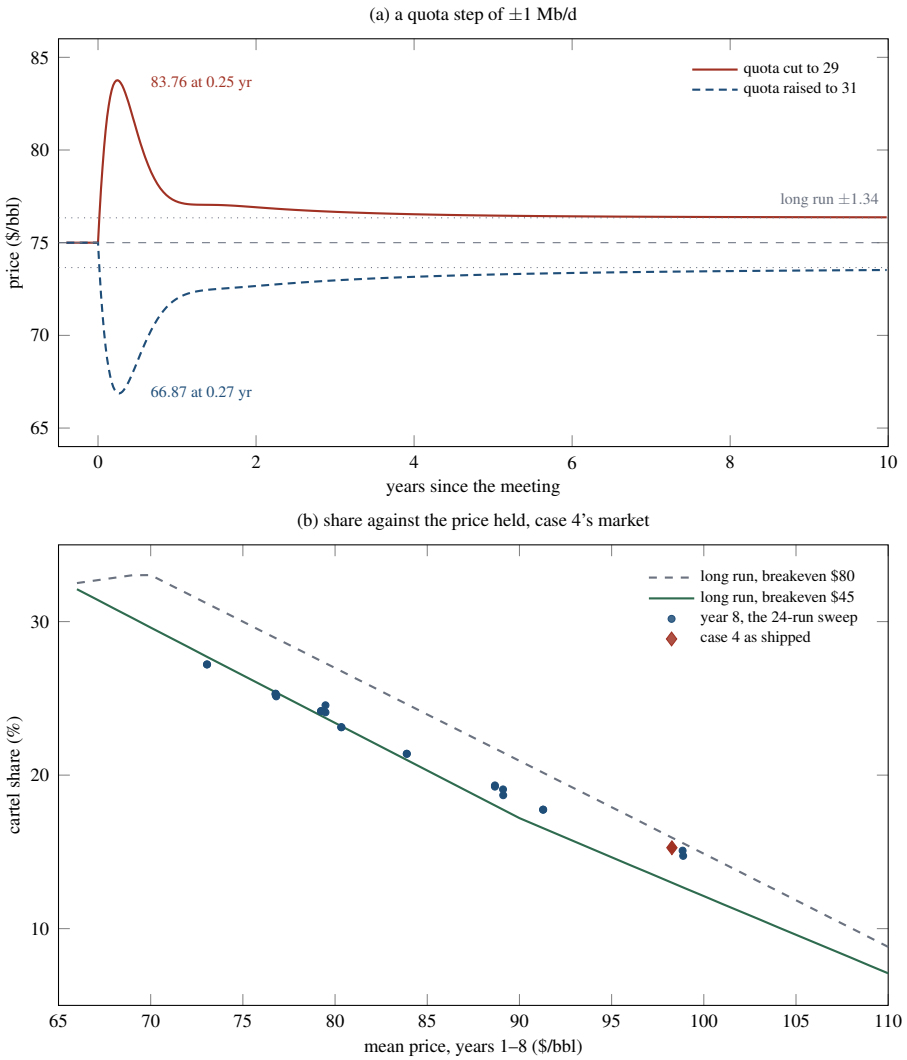


Figure 10.2 Two characteristics of the market from the application's own engine and functions. (a) The balanced reference with the quota cut (solid) or raised (dashed) by 1 Mb/d at the meeting of year 1, against the long-run gain of Section 10.4 (dotted). (b) Case 4's market, the share at year 8 against the mean price over years 1 to 8 for the 24 runs of the application's sweep (circles, the shipped case a diamond), with the long-run share, R_{lr} of Equation (10.10) over the long-run demand, before (dashed) and after (solid) the technology shift, capped where the cartel would need more than its capacity.

DID YOU KNOW? A Cut of 9.7 Million Barrels a Day, 2020

On April 15, 2020 the producer group and 10 partner countries agreed to cut crude oil output by an initial 9.7 million barrels a day from May. The group's production fell by 6.0 million barrels a day from April to May, its largest monthly decline since 1993 [18]. The simulator's cap allows 1.5 Mb/d a meeting.

10.5.2 Two PI Loops on One Valve

Each cartel loop is the framework's velocity-form PID of Section 1.6.2 with its derivative time set to zero, as Equation (10.12) writes it.

$$\begin{aligned} e_k &= a(y_k^{sp} - y_k), \qquad \Sigma_k = \Sigma_{k-1} + K_c(e_k - e_{k-1}) \\ \tilde{u}_k &= u_b + \Sigma_k + \mathscr{I}_{k-1}, \qquad u_k = \Pi_{[q_{min}, q_{cap}]}(\tilde{u}_k) \\ \mathscr{I}_k &= \mathscr{I}_{k-1} + \Delta t_c \left[K_c e_k / \tau_I + (u_k - \tilde{u}_k)/\tau_I \right] \end{aligned} \tag{10.12}$$

where Σ_k and $\mathscr{I}_k$ are the running sum S_k and the integral I_k of Equations (1.25) and (1.27), renamed because S is supply and I the stocks here, the tracking time equals τ_I, u_b is the starting quota and the request is $q^w = u_k$. The price loop has $a = -1$, $y = P$ and $y^{sp} = P_{sp}$, so a price above target raises the quota, with $K_c = 0.05\,\text{Mb/d}$ per dollar and $\tau_I = 0.4\,\text{yr}$. The stocks loop has $a = +1$, $y = I_{\text{pub}}$ and $y^{sp} = \bar{I}_5 + \Delta I_{sp}$, so a report above target cuts the quota, with $K_s = 10\,\text{Mb/d}$ per Gb and $\tau_{Is} = 1\,\text{yr}$. A stock target is a price target seen through Equation (10.2) and filtered by a sixty-month moving average of the controlled variable itself, which the application prints as the implied price on its card.

The loops run at every weekly execution, so their integrals keep collecting the error between meetings while the quota stands still, and a request is shaped only when a meeting falls. A loop that is not deciding tracks the quota in force, so a change of controller, or MANUAL to AUTO, is bumpless, and the first meeting after such a switch during a run holds the quota in force. A setting changed before the first step, a switch included, restarts the run with it, as Reset does (Section 1.6.1), so the meeting at $t = 0$ is decided with the learner's settings. In AUTO the chair's slider follows the request, not the shaped quota. SATURATED is read from the command, the loop's unclipped output beyond the floor or the capacity, or the cap binding at the last meeting.

10.5.3 The Chair, the Price War and the Consumers

In MANUAL the chair's slider, 0 Mb/d to 40 in steps of 0.1, is the request, shaped at the next meeting exactly as a controller's is. The price war is open loop, a request equal to q_{cap}. The consumers' strategic reserve is a second, independent controller on the same tank, off as shipped, a manual release or a PI on the price of Equation (10.12) with $a = -1$, $y^{sp} = P_r$, $K_r = 0.05\,\text{Mb/d}$ per dollar, $\tau_{Ir} = 0.5\,\text{yr}$ and its output clipped to $[0, r_{max}]$. The release is further limited to what the reserve holds, $r \le I_{spr}/(K\Delta t_c)$, and the reserve is never refilled.

> ⧖ HISTORY **A Reserve Born of an Embargo, 1973 to 1977**
>
> A national oil stockpile had been proposed in the United States since 1944, but the embargo of 1973 and 1974, which cut off the oil flowing to the country from the Arab members of the producer group, made the case. The Energy Policy and Conservation Act, signed on December 22, 1975, declared it national policy to establish a reserve of up to one billion barrels of petroleum. Existing salt caverns were acquired in April 1977, and on July 21, 1977 the first oil, about 412,000 barrels, was delivered [19]. The chapter's consumers' reserve is such a stock under a second controller on the same tank.

10.5.4 An Economic MPC by Successive Linearization

The economic MPC plans the quota at each meeting by maximizing discounted revenue over a horizon of $H = 8\,\text{yr}$, built with the framework's condensed linear MPC, `core/linmpc.js`, which Section 1.6.3 outlines and Volume 9 develops. Its prediction step is the twice-yearly meeting interval, $T_s = 0.5\,\text{yr}$, so $N = 16$ (on the monthly and weekly calendars T_s is a nominal month of 1/12 year), and the inputs are held over nine move blocks, four of one step, two of two, one each of three and four, and a last block of one step. At a nominal state $\bar{x}$ and quota $\bar{q}$ the four slow states of Equations (10.1) to (10.7) are linearized, by central differences in the states and a one-sided difference in the quota, backward at capacity and forward elsewhere, and discretized exactly for a zero-order hold, with the affine remainder and the previous quota carried as two more states. The price is linear in the stocks

with slope $p_I = -kP(\bar{x})/I_0$, and the share linear in the fringe and the quota. The quadratic program (QP) is Equation (10.13).

$$\begin{aligned}
\min_{u,\sigma} \; & \sum_{j=1}^{N} \Big[-KT_s \mathrm{e}^{-\rho t_j} \left(\bar{q}_c P_j + \bar{P} u_j \right) + 10^{-4} P_j^2 \Big] \\
& + R \sum_{j=1}^{N} (u_j - \bar{q})^2 + w \sum_{b} (u_b - u_{b-1})^2 \\
& + 30\sigma_1^2 + 10\sigma_1 + 10^5 \sigma_2^2 + 10^4 \sigma_2 \\
\text{s.t. } & \underline{u} \le u_j \le \bar{u}, \\
& |u_b - u_{b-1}| \le \Delta, \quad P_{lo} - \sigma_1 \le P_j \le P_{hi} + \sigma_1, \\
& \phi_N \ge \phi_{min} - \sigma_2, \quad \sigma \ge 0
\end{aligned} \tag{10.13}$$

where u_j is the quota over the step that ends at $t_j = t + jT_s$, the bounds are $\underline{u} = \max(q_{min}, \min(q, \bar{q} - 6))$ and $\bar{u} = \min(q_{cap}, \max(q, \bar{q} + 6))$, $\bar{q}_c$ and $\bar{P}$ are the cartel's output and the price at the nominal point, so the first term is revenue linearized in price and output, $w = 1$ billion dollars per $(\mathrm{Mb/d})^2$ suppresses moves between blocks, with $u_0 = q$ the quota in force, and $R = KT_s |p_I B_{d,I}| \mathrm{e}^{-\rho(t+NT_s/2)}$ restores the curvature that a linear revenue leaves out, one more barrel a day for a step lowering the price by $|p_I B_{d,I}|$. The bounds hold the quota within 6 Mb/d of the nominal, a trust region that always contains the quota in force, the price band is soft at every step and the share floor $\phi_{min} = 0.27$ soft at the last. The program has 11 variables and 71 constraint rows, and `core/qp.js` solves it by the numerically stable dual method [20] with a ridge of 10^{-9} times one plus the largest diagonal entry. After each solve the plan is rolled out on the nonlinear model in weekly Euler substeps, and the next linearization is taken at the rollout's mean state and the plan's mean quota. Five such solves make one meeting's plan, and the plan's first quota is the request that the meeting then rounds and caps. In the eighth case the 100 solves of ten years take 5.10 active-set iterations on average and 9 at most, with no failure, and across that case and the challenge no planned price leaves the band in the QP's prediction and the nonlinear rollouts leave it by at most \$0.83.

An earlier version of the application had one defect in this linearization, since corrected. A forward difference in the quota reads zero once the cartel's output sits at its capacity, where one more barrel meets the clip of Equation (10.3), so the QP saw no effect of the quota on the price and the MPC never left capacity. The difference is now taken from below there. The logged runs of all eleven scenarios over thirty years and the challenge's suggested run are byte-identical before and after that correction, and two learner experiments changed. With the eighth case run to year 9 and the floor then raised to \$80, the MPC used to stay at 34 Mb/d with the price at \$64.8 six years later, and it now cuts to 25.8 Mb/d and holds \$78.2. Selected at year 5 of the fifth case, it used to hold capacity at \$56.2 a year later, below its own \$60 floor, and it now holds \$60.9 at 32.5 Mb/d. A new check in the application runs both experiments.

10.5.5 Market, Prediction and Literature

The simulation model is Equations (10.1) to (10.9) with every clip, the storage limits, the monthly report and the shocks. The PI loops hold no model at all. The MPC's internal model is the same four slow states with every coefficient at its declared default, whatever the run's parameters, linearized afresh at each meeting, with no storage limit inside the QP and no report delay. Each plan starts from the cartel's own estimate. The declared model runs forward week by week with the quota and the reserve's release, its stocks are read from the price through Equation (10.2) whenever the two disagree, and the barrels a day that the model cannot account for are held over the horizon as one unexplained supply, with no shock to come foreseen. The destruction state x is never read and follows the declared equation. At the declared defaults with no shock the estimate equals the simulated market at every step. The models these stand for are the cartel-and-fringe models [2, 8] and the econometric

Table 10.4 The Cases and the Challenge

#	Change from the defaults	Teaching point
1	MANUAL, recession at year 2	the market regulates
2	$P_{sp} = \$80$	share erodes every year
3	PI on stocks, start after a three-year glut	a target that drifts
4	$P_{sp} = \$100$, $K_c = 0.2$, $\tau_I = 0.2$, monthly, shale shift from year 1	honestly infeasible
5	case 4 from year 0, valve at capacity from year 3	a war against a lag
6	$I_{max} = 3.5$ Gb, pandemic at year 1	the tank fills
7	$K_c = 0.12$, $\tau_I = 0.3$, monthly, band \$65–85, $q_{min} = 26$, a cheater a year from year 2	a leaking valve
8	economic MPC, shale shift from year 1	what a band costs
9	$P_{sp} = \$90$, reserve PI at \$80, outage at year 2	two controllers
ch	$P_{sp} = \$90$, recession and shale shift at year 2	choose the price
ref	overproduction off, nothing moves	the balance

Table 10.5 Measured Cases

Case	Result
1, hands off	low \$59.1 at yr 3.04, overshoot \$80.1 at 5.02
1, PI at \$75	low \$63.9 at yr 3.00
2	mean \$78.33 over yr 2–10, share 29.75, 28.81, 28.15 % at yr 1, 5, 10
3	implied target \$58.2, 59.5, 68.7 at yr 1, 4, 10, mean \$66.2 over yr 1–5
3, PI at \$75	mean \$73.7 over yr 1–5, share 30.24 % at yr 10
4	mean \$98.3 over yr 1–8, share 30 to 15.27 % at yr 8
5	\$23.5 at yr 3.94, shale 14.14 to 10.11 Mb/d at 4.58, V \$5603 billion
6	full yr 1.115–1.500, 0.687 Gb shut in, spike \$123.5 at 2.77
7	floor at yr 6.10, band broken at 8.12, \$61.1 at 9.25
8	mean \$62.1 over yr 1–10, share 32.64 %, V \$5973 billion
8, $P_{lo} = \$80$	mean \$78.0, share 24.70 %, V \$5951 billion
8, $P_{lo} = \$70$	mean \$68.9, share 29.18 %, V \$6047 billion
9	reserve empty at yr 4.33, \$86.3 at yr 10 against \$86.1 without
ref	largest derivative 1.8×10^{-15}, no drift in ten years

links between stocks, demand and price [6, 7, 17], whose coefficients are not copied here. Nothing in the application fits a model to data.

10.6 Ten Years at a Time

Table 10.4 lists the cases. Each starts from the defaults of Table 10.3 with the PI on price at \$75, the twice-yearly calendar, a 1.5 Mb/d cap and a 0.1 Mb/d quantum, and changes only what its row lists. The values quoted below were measured headless on each case as the dropdown loads it, with any variation applied before the first step, which gives the same run as the briefings' Reset, and Table 10.5 gathers the main ones. A mean over a span of years averages the weekly rows from its first year to its last, both included, and a quota decision is timed at its meeting. The scorecard's integrated absolute error is left out, since it grows with the length of the run.

10.6.1 A Chair, a Recession and a Held Price

In the first case the learner chairs the cartel while a recession takes 3 Mb/d of demand away over a year from year 2 and returns it over the next two. Left at 30 Mb/d, the quota sees \$65.2 at the meeting of year 2.5, the price bottoms at \$59.1 at year 3.04 and then recovers by itself, because consumers burn more at the lower price and shale falls to 9.01 Mb/d by year 3.33. When demand returns and shale has not, the price overshoots to \$80.1 at year 5.02. A cut to 28.5 Mb/d asked at year 2.1 takes effect at the meeting of year 2.5, keeps the price above \$65.2 and, if never given back, lifts it to \$82.6 after the recession, while asking for 30 Mb/d again at year 3.6, in force from the meeting of year 4, keeps it between \$67.9 and \$79.5 from then on. In AUTO the PI on price holds it above \$63.9.

The second case holds \$80 with the shipped PI. The price is \$76.8 at year 1 and averages \$78.33 over years 2 to 10, never more than \$2.85 from the target after year 2, while the share falls every year, 29.75, 28.81 and 28.15 % at years 1, 5 and 10. By year 10 the quota is 27.8 Mb/d, shale has gained 0.37 Mb/d, conventional supply 0.58 Mb/d, and 1.00 Mb/d of demand has been destroyed. The constant shortfall is a PI following a ramp. The calendar caps the gain, because the price settles within a season of a quota change. At $K_c = 0.06$ the quota reverses direction at 7 of the 20 meetings, at 0.07 at 14 and at 0.08 at 18, where the mean error grows to \$5.15.

Pause. Try Exercise 10.1 before reading on.

The third case starts after a three-year glut, with the stocks at 3.15 Gb, the price at \$34.5 and a five-year average of 2.977 Gb that implies \$59.0. The glut drains within months, to \$72.7 at year 0.25, but the PI on stocks aims the report at an average that still holds the glut. Its implied target is \$58.2 at year 1, \$59.5 at year 4, \$66.6 at year 6 and \$68.7 at year 10, when the price is \$68.7, against \$75.1 for the PI on price at \$75. The lower price buys share, 33.18 % against 30.24 % at year 10, for \$39 billion less revenue (6424 against 6463). The case's briefing says that the cartel's communiqués have stated their aim this way since 2017, a statement the application makes and this chapter does not test.

Pause. Try Exercise 10.3 before reading on.

10.6.2 The Shale Revolution and the Price War

The fourth case is the honestly infeasible one. A monthly PI with $K_c = 0.2$ and $\tau_I = 0.2$ yr holds \$100 while the breakeven falls from \$80 to \$45 between years 1 and 5. The price is \$96.3 at year 1, averages \$98.3 over years 1 to 8 and is \$99.5 at year 8, and the share falls from 30 to 15.27 %, a loss of 49 %, with the quota at 13.6 Mb/d, shale at its 15 Mb/d ceiling, conventional supply at 53.74 Mb/d and 5.49 Mb/d of demand destroyed. The demand factor first falls below 0.95 at year 6.85. In the application's sweep of targets of \$80, \$90 and \$100, four tunings and monthly and weekly calendars, every run that averages \$95 or more loses at least 49 % of its share, and the runs that lose less than a third average at most \$83.9. At a \$75 target the PI keeps 26.67 % at year 8. The state at year 8 is shown in Figure 10.3.

> **TRY IT IN THE APP** **4 · The Shale Revolution: Hold \$100**
>
> Load the case and press Run. Watch the share badge while the price sits near its \$100 target. Over eight years the price averages \$98.3, shale climbs to its 15 Mb/d ceiling, and the cartel's share falls from 30 to 15.27 %.

The fifth case opens the valve. After three years of holding \$100 while the breakeven falls, the price is \$97.2, the quota 19.3 Mb/d, the share 20.6 % and shale 14.14 Mb/d. The switch to the price war holds the quota at the meeting of year 3, and from the next meeting it climbs 1.5 Mb/d at each monthly meeting to capacity, reached at the meeting of year 3.85. The price halves by year 3.46 and bottoms at \$23.5 at year 3.94, when the stocks peak at 3.274 Gb. Shale falls half of its eventual fall by year 3.75 and bottoms at 10.11 Mb/d at year 4.58, then grows again, because the price, \$49.8 at year 5 and \$63.1 at year 10, is above the new breakeven. The war wins a year of shut-ins and not the fringe's exit. It still earns more over ten years, \$5603 billion, than case 4 earns by holding \$100, \$5269 billion, with the share back to 33.0 % at year 5.

10.6.3 A Full Tank and a Leaking Valve

The sixth case cuts storage to 3.5 Gb and takes 20 Mb/d of demand away for three months from year 1. Its default capacity follows normal inventory plus 0.6 Gb, unless the learner explicitly edits capacity. On the twice-yearly calendar the tank is full from year 1.115 to 1.500, the price sits at the full-tank \$11.7 and 0.687 Gb is shut in. Monthly decisions shut in 0.309 Gb and weekly ones 0.169 Gb, both with the 1.5 Mb/d cap, and only weekly decisions with a 5 Mb/d cap and $K_c = 0.3$,



Figure 10.3 Case 4, the shale revolution, at $t = 8.00$ yr in the Fringe and demand view, in AUTO as loaded, cropped to the plant view. (1) The badges read price \$99.5, share 15.3 %, stocks less the five-year average −0.002 Gb, spare capacity 19.8 Mb/d, discounted revenue \$4635 billion and 5.49 Mb/d of demand destroyed. (2) The shale panel shows today's supply curve above the \$45 breakeven beside the dashed curve it started on, with output at its 15 Mb/d ceiling. (3) The demand panel shows the steep short-run curve through the operating point and the flatter long-run curve. (4) The last panel shows what is left for the cartel at \$100, 14.2 Mb/d now and 11.0 Mb/d in the long run. (5) The status line reads DEMAND DESTRUCTION.

with τ_I left at 0.4 yr, keep the tank from filling, at a lowest share of 15.9 %. On every calendar the tank fills six to eight weeks after the shock, at year 1.115, 1.135 or 1.154. When demand returns, the cartel's cuts and shale's shut-ins, shale down to 6.17 Mb/d at year 1.85, are still in force, and the price spikes to \$123.5 at year 2.77 (\$107.7 monthly, \$107.4 weekly). A storage limit saturates the plant itself, and a controller too slow on the way down is too slow on the way up.

The seventh case holds \$75 inside an announced band of \$65 to \$85 with a monthly PI while one more member overproduces 1 Mb/d every year from year 2, and the loyal members will not go below a quota of 26 Mb/d. The first cheater takes the price to \$69.4 at year 2.10, and the integral has it back to \$74.6 by year 2.9. With four cheaters producing, the price is still \$74.2 at year 5.9 with the quota at 26.7 Mb/d. The quota reaches its floor at the meeting of year 6.10, after which each cheater is uncompensated, and the price leaves the band at year 8.12 and reaches \$61.1 at year 9.25, while the cartel's total output climbs to its 34 Mb/d and its share to 33.4 %. With $\tau_I = 5$ yr the band breaks at year 5.10, and on the twice-yearly calendar at year 3.12. Integral action is indifferent to why the output is wrong, which compensates the leak until the actuator runs out of travel.

> **TRY IT IN THE APP** **7 · The Cheater: An Actuator That Leaks**
>
> Load the case and run ten years. Watch the applied quota on the trend. Each new cheater pulls the price down and the integral cuts the quota, until the quota reaches its floor of 26 Mb/d at the meeting of year 6.10. From then on each cheater goes uncompensated, and the price leaves the band at year 8.12.

10.6.4 What a Band Costs

The eighth case gives case 4's market to the economic MPC with a \$60 to \$100 band and a 27 % share floor. It holds the price near the bottom of the band, at a mean of \$62.1 over years 1 to 10 and never below \$60.1, with the quota between 33.0 and 33.9 Mb/d from year 1 and at capacity from year 8, and ends with 32.64 % and \$5973 billion. Raising the floor to \$80 is the price of a band, a mean of \$78.0, 24.70 % and \$5951 billion, while case 4's PI holding \$100 in the same market keeps 14.39 % and earns \$5269 billion, case 4's own result. The MPC is a local optimizer and not the best policy. A floor of \$70 earns \$6047 billion, more than its own choice, and a PI at \$75 earns \$6044 billion and at \$70 \$6031 billion. The linearized revenue overrates one more barrel, and the internal model holds the extra supply it has inferred so far over eight years without knowing that the breakeven will keep falling. A share floor of 0.35 changes little (32.62 %, \$5978 billion).

> **DID YOU KNOW?** **The Optimizer's Own Choice Is Not Its Best**
>
> From the simulator: in case 8 the economic MPC settles near the bottom of its band, at a mean of \$62.1, and earns \$5973 billion. Raising its price floor to \$70 gives a mean of \$68.9 and earns \$6047 billion, more than its own choice, because the optimizer is local (Table 10.5).

10.6.5 Two Controllers on One Tank

In the ninth case the cartel's PI holds \$90 and the consumers' reserve, with 0.7 Gb, releases up to 1 Mb/d when the price is above \$80, while other producers lose 1.5 Mb/d for a year from year 2. The reserve takes the outage's peak from \$93.8 to \$90.7, but it fights the cartel the whole time. The cartel's integral cuts the quota for every barrel released, the price averages \$81.79 over years 0.5 to 4 against \$82.78 without the reserve, and the reserve is empty at year 4.33. At year 10 the price is \$86.3 with the reserve and \$86.1 without, and the cartel has earned \$6306 billion against \$6349 billion. Two integrators with different setpoints on one level fight until one saturates. With a ceiling of \$95 the reserve never releases and keeps its 0.7 Gb.

10.6.6 A Shale Decade

The challenge runs ten years with a recession at year 2, the breakeven falling from \$80 to \$45 between years 2 and 6, twice-yearly meetings and the 1.5 Mb/d cap. It asks for at least 90 % of month-end prices inside \$60 to \$100, a share of at least 30 % at year 10 and at least \$5950 billion of discounted revenue. Table 10.6 compares nine configurations. The shipped PI at \$90 never reaches its target, ends at \$84.9 and keeps every month in the band, but it cuts the quota to 20.8 Mb/d and ends with 21.61 % and \$5868 billion, short on share by 8.39 points and on revenue by \$82 billion. The suggested MPC with its floor raised to \$65 passes with 91.7 % of months, 31.77 % and \$5982 billion, margins of 1.77 points and \$32 billion, and so do a PI at \$65 and the PI on stocks 0.02 Gb above its average. At the shipped \$60 floor the unforeseen recession takes the price to \$50.7, and the MPC passes with no month to spare, 108 of 120 inside the band. The \$5 of margin in the suggested floor is back-off against a disturbance the MPC does not predict, and it buys two months. PI loops at \$70 and \$75 earn more than any passing run, \$6011 billion and \$6021 billion, and fail on share. The challenge is judged on the case as given, so a changed model parameter or disturbance, or an extra shock fired, voids an attempt. Figure 10.4 draws the nine settings against the three limits.

A setting chosen before the first step gives the same run as the setting carried through Reset, so the suggested run as Reveal applies it is the one in Table 10.6.

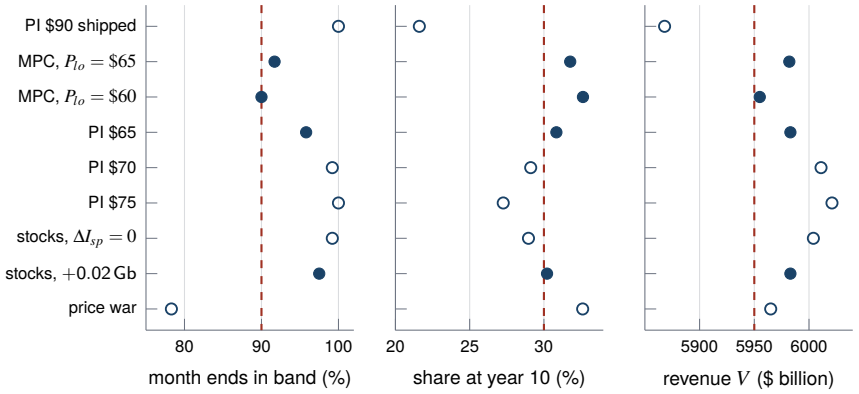


Figure 10.4 The ten-year challenge in nine settings, from Table 10.6: month ends inside $60–100, the share at year 10 and the discounted revenue, each against its limit (dashed line). Filled circles pass all three limits, open circles fail at least one. Notice that the PI loops at $70 and $75 earn the most and fail on share, and that every passing setting sits just above the share and revenue limits.

Table 10.6 The Ten-Year Challenge in Nine Configurations

Setting	band (%)	months in band	share (%)	V ($bn)	
PI $90, shipped	100.0	120	21.61	5868	fail
MPC, $P_{lo} = \$65$	91.7	110	31.77	5982	pass
MPC, $P_{lo} = \$60$	90.0	108	32.63	5955	pass
PI $65	95.8	115	30.84	5983	pass
PI $70	99.2	119	29.11	6011	fail
PI $75	100.0	120	27.26	6021	fail
stocks, $\Delta I_{sp} = 0$	99.2	119	28.96	6004	fail
stocks, +0.02 Gb	97.5	117	30.21	5983	pass
price war	78.3	94	32.61	5965	fail

Band: month ends in $60–100, of 120.

In the balanced reference the derivatives vanish to 1.8×10^{-15} and the price stays at $75 for ten years with the overproduction turned off.

10.7 Technology in Practice

The model of Section 10.3 reads a price and a stock report and moves a quota, signals made by markets, surveys and meetings. Figure 10.5 draws the loop with them in place of the schematic's symbols.

What Measures It

The price is a market price, and the spot price and the futures prices answer the stocks through the incentive to store oil [21]. The stocks are read by surveys. The industrialized countries usually publish stock statistics on a regular basis, while data for other countries, including fast-growing developing consumers and major producers, come late or not at all, and crude oil is also stored on ships at sea [21]. Even in the United States weekly report some figures are a model's output: weekly domestic crude oil production is estimated from the statistical agency's short-term outlook model and daily pipeline data, and most of the uncertainty of the estimate comes from that model, the more so when production changes quickly [22]. The chapter's price loop reads the true price at once, and its stocks loop reads a report published a month late beside the average of sixty reports.

TECHNOLOGY Sensor: Surveys of Commercial Stocks

No instrument reads the world's stocks. In the United States a statistical agency surveys operators each week, adding companies from the largest down until the sample covers about 90 % of the volume of each item and region. The week runs from Friday to Friday, and the figures appear the following Wednesday. Missing reports are filled with exponentially smoothed means of the company's recent values, and a company must resubmit an error greater than 5 %. The weekly figures are leading indicators of a complete monthly survey, the most accurate data, and stocks are the most accurate of the weekly estimates [22]. Its faults are delay and missing coverage rather than noise: outside the industrialized countries stock data come late or not at all [21].

What Moves It

The final element is a target for each member's production. The agreement of 2020 set an initial cut and then a taper through April 2022, and three members were exempt from it [18]. Production in the group is mainly in the hands of national oil companies, while producers outside it decide independently, as price takers that usually produce at or near full capacity [23]. The chapter's quota, shaped to a quantum of 0.1 Mb/d and a cap of 1.5 Mb/d a meeting, is this target, and its price-taking fringe is the rest of the market.

TECHNOLOGY Final Element: Production Targets and Spare Capacity

The producer group acts by setting production targets, limits on how much oil each member may produce. Members do not always adhere to them, and outages can cut their output further. The room to move up is spare capacity, defined by the United States statistical agency as production that can be brought online within 30 days and sustained for at least 90 days, and the group's members hold almost all of it. When spare capacity is low, prices tend to carry a rising risk premium, and the targets themselves follow a changing market with a lag [24]. The chapter's overproduction, cheaters, capacity and risk premium stand for these traits.

Where the Controller Runs

The controller is no plant computer but a conference of the members' governments, which the founding agreement asked to meet at least twice a year and more often if necessary [15]. The application executes its controller every week, $\Delta t_c = 1/52$ year, and turns a request into a quota only at a meeting through the framework's calendar module. Its PI loops are the velocity-form block of Section 1.6.2, and its MPC replans eight years at each meeting.

IN PRACTICE Stocks Are Read Against Past Years

Analysts do not read a stock level alone. Stocks are most usefully judged against the levels of prior years in the same calendar quarter, because demand for some products is seasonal [21], and the weekly report plots each stock against the highest and lowest levels of the same week over the prior five years [22]. They also watch the futures spread, the contract 12 months ahead less the one a month ahead. An expected shortage raises futures prices and pays companies to build stocks, while a sudden loss of supply lifts the spot price and draws stocks down [21]. The chapter's stocks loop holds its report against a five-year average in the same spirit, without the seasons.

What Is Modernizing It

Three changes speed up the measurement. Six international organizations launched a joint oil data exercise in April 2001, a monthly questionnaire on 42 key data points, and within a year more than 70 countries, with 90 % of world supply and demand, took part. It became a permanent initiative with a database open to all, whose stated challenge is wider coverage, shorter delays and better quality [25]. Administrative records now replace some surveys: weekly United States export figures come from

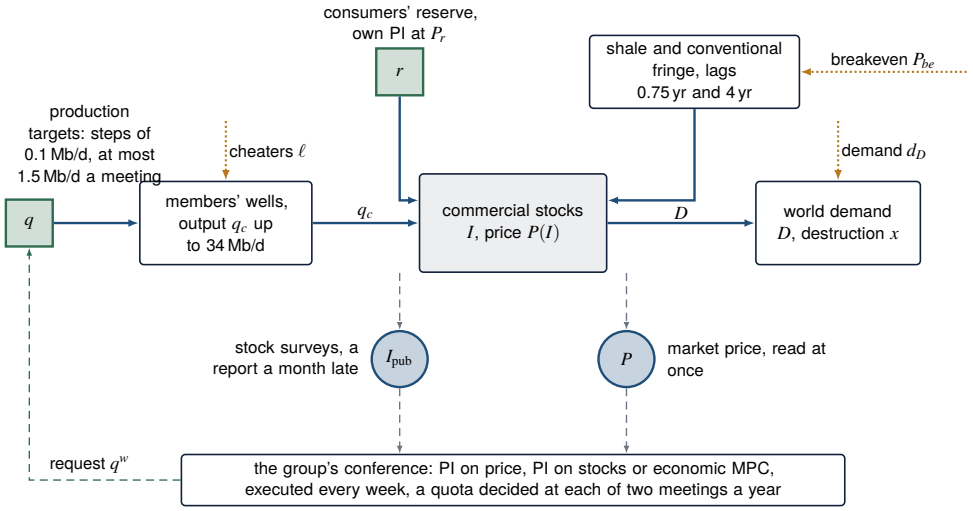


Figure 10.5 The quota loop with the institutions of the real market in place of the schematic's symbols. Surveys give the published stocks I_{pub} a month late and the market gives the price P at once, the conference forms a request each week, and only a meeting turns it into a step of the production targets q. The consumers' reserve r is a second controller on the same tank. Notice that the fringe answers the price with lags of years while the quota can move only 1.5 Mb/d a meeting, and that cheaters leak past the valve.

the electronic filings that exporters make with the customs agency, where the official monthly figures arrive about six weeks after the month ends. And model-based estimates fill the weeks between complete surveys, compared with the survey data and rebenchmarked when the two differ widely [22]. The chapter's MPC estimates the unexplained supply from the price in the same way.

10.8 Limitations

The market is one tank with one price, one decision maker and a first-order fringe, without grades, regions, futures or strategic play, and every coefficient is a teaching value. Demand growth is zero, the fringe's supply curves do not grow, and shut-in oil costs nothing later. The price controller reads the true price at once, the stocks loop a report a month old, and the MPC the price through its declared model, from which it estimates the rest of the market. The MPC is a local solution that a $70 band floor beats by $75 billion. The challenge's limits, like every other threshold here, were set for teaching.

> **OPEN CHALLENGE** **No Forecast Beats a Price That Stays Put**
>
> A plan for quotas needs the future price. A benchmark study with data through the collapse of 2008 found that at longer horizons the forecast that the real price stays where it is typically has the lowest mean squared error. Forecasts from futures prices could not be recommended, and the United States statistical agency's own forecasts had at best modest success in the short run and none at longer horizons. Results in the literature were often sensitive to the sample period [26]. The chapter's MPC plans eight years ahead and foresees no shock. Progress would be forecasts that beat the no-change forecast out of sample over the years that the fringe and the consumers take to answer.

10.9 Conclusion

The simulator writes a cartel's dilemma as a level loop whose setpoint pays the competing inflows and narrows the outflow. An extra 1 Mb/d moves the price $8.1 in a season and $1.6 in seven years,

and its static gains differ by a factor of 9.3. A PI holding \$80 loses share every year, from 29.75 to 28.15 %, and holding \$100 through a shale revolution costs 49 % of the share in eight years, with no tuning in the 24-run sweep doing better than 49 % at a mean of \$95 or more. A price war halves the price within half a year and wins a year of shut-ins, a full tank sets the price at \$11.7 on every calendar under the 1.5 Mb/d cap, and an integral compensates four cheaters before the floor binds. The economic MPC holds about \$62 for 32.64 % and \$5973 billion, and a \$65 floor passes the challenge that the shipped \$90 PI fails at 21.61 %. The market is one aggregate tank with teaching coefficients. A fuller model would add a strategic fringe, regional stocks and an estimator for the destruction state. The tank makes the price a cartel can hold a property of the market, measured in barrels and years.

KEY POINTS

- Oil held back lifts the price within weeks, but the high price pays rivals and thrifty buyers for years, and their barrels become the cartel's lost market.
- An extra 1 Mb/d moves the price \$8.1 in a season and \$1.6 in seven years, and the static gains differ by a factor of 9.3.
- Holding \$100 through a shale revolution costs 49 % of the share in eight years, whatever the tuning.
- An integral compensates four cheaters before the floor binds, and a \$65 floor on the economic MPC passes the challenge that the shipped \$90 PI fails.

Where else this applies. Volume 6, Chapter 11 orders stocks on a calendar in a quantum with a cap, and Volume 2, Chapter 11 decides a policy rate through the same calendar module.

References

[1] H. Hotelling, "The economics of exhaustible resources," *Journal of Political Economy*, vol. 39, no. 2, pp. 137–175, 1931.

[2] S. W. Salant, "Exhaustible resources and industrial structure: A Nash-Cournot approach to the world oil market," *Journal of Political Economy*, vol. 84, no. 5, pp. 1079–1093, 1976.

[3] R. S. Pindyck, "Gains to producers from the cartelization of exhaustible resources," *The Review of Economics and Statistics*, vol. 60, no. 2, pp. 238–251, 1978.

[4] D. Gately, "Lessons from the 1986 oil price collapse," *Brookings Papers on Economic Activity*, vol. 1986, no. 2, pp. 237–284, 1986, with comments by M. A. Adelman and J. M. Griffin.

[5] M. Ellis, H. Durand, and P. D. Christofides, "A tutorial review of economic model predictive control methods," *Journal of Process Control*, vol. 24, no. 8, pp. 1156–1178, 2014.

[6] R. S. Pindyck, "The dynamics of commodity spot and futures markets: A primer," *The Energy Journal*, vol. 22, no. 3, pp. 1–29, 2001.

[7] L. Kilian and D. P. Murphy, "The role of inventories and speculative trading in the global market for crude oil," *Journal of Applied Econometrics*, vol. 29, no. 3, pp. 454–478, 2014.

[8] A. Behar and R. A. Ritz, "OPEC vs US shale: Analyzing the shift to a market-share strategy," *Energy Economics*, vol. 63, pp. 185–198, 2017.

[9] B. Fattouh, R. Poudineh, and A. Sen, "The dynamics of the revenue maximization–market share trade-off: Saudi Arabia's oil policy in the 2014–15 price fall," *Oxford Review of Economic Policy*, vol. 32, no. 2, pp. 223–240, 2016.

[10] J. D. Hamilton, "Causes and consequences of the oil shock of 2007–08," *Brookings Papers on Economic Activity*, vol. 2009, no. 1, pp. 215–261, 2009.

[11] L. Kilian, "Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market," *American Economic Review*, vol. 99, no. 3, pp. 1053–1069, 2009.

[12] C. Baumeister and L. Kilian, "Forty years of oil price fluctuations: Why the price of oil may still surprise us," *Journal of Economic Perspectives*, vol. 30, no. 1, pp. 139–160, 2016.

[13] H. C. Bjørnland, F. M. Nordvik, and M. Rohrer, "Supply flexibility in the shale patch: Evidence from North Dakota," *Journal of Applied Econometrics*, vol. 36, no. 3, pp. 273–292, 2021.

[14] S. Corbet, J. W. Goodell, and S. Günay, "Co-movements and spillovers of oil and renewable firms under extreme conditions: New evidence from negative WTI prices during COVID-19," *Energy Economics*, vol. 92, p. 104978, 2020.

[15] United Nations, "Agreement concerning the creation of the organization of the petroleum exporting countries (OPEC), done at Baghdad, on 14 september 1960," United Nations Treaty Series, vol. 443, No. 6363, 1962. [Online]. Available: https://treaties.un.org/doc/Publication/UNTS/Volume%20443/volume-443-I-6363-English.pdf

[16] D. Gately and H. G. Huntington, "The asymmetric effects of changes in price and income on energy and oil demand," *The Energy Journal*, vol. 23, no. 1, pp. 19–55, 2002.

[17] C. A. Dahl, "Measuring global gasoline and diesel price and income elasticities," *Energy Policy*, vol. 41, pp. 2–13, 2012.

[18] U.S. Energy Information Administration, "OPEC+ agreement to reduce production contributes to global oil market rebalancing," Today in Energy, 2020, september 23, 2020. [Online]. Available: https://www.eia.gov/todayinenergy/detail.php?id=45236

[19] U.S. Department of Energy, Office of Petroleum Reserves, "SPR origins," Web page, 2022, last updated September 9, 2022. [Online]. Available: https://www.energy.gov/hgeo/opr/spr-origins

[20] D. Goldfarb and A. Idnani, "A numerically stable dual method for solving strictly convex quadratic programs," *Mathematical Programming*, vol. 27, no. 1, pp. 1–33, 1983.

[21] U.S. Energy Information Administration, "What drives crude oil prices: Balance," Web page, undated, accessed November 21, 2026. [Online]. Available: https://www.eia.gov/finance/markets/crudeoil/balance.php

[22] ——, "Weekly petroleum status report, appendix B: Explanatory notes and detailed methods report," Frame table dated August 2026, accessed November 21, 2026. [Online]. Available: https://www.eia.gov/petroleum/supply/weekly/pdf/appendixb.pdf

[23] ——, "What drives crude oil prices: Supply non-OPEC," Web page, undated, accessed November 21, 2026. [Online]. Available: https://www.eia.gov/finance/markets/crudeoil/supply-nonopec.php

[24] ——, "What drives crude oil prices: Supply OPEC," Web page, undated, accessed November 21, 2026. [Online]. Available: https://www.eia.gov/finance/markets/crudeoil/supply-opec.php

[25] Joint Organizations Data Initiative, "History," Web page, undated, accessed November 21, 2026. [Online]. Available: https://www.jodidata.org/about-jodi/history.aspx

[26] R. Alquist, L. Kilian, and R. J. Vigfusson, "Forecasting the price of oil," Board of Governors of the Federal Reserve System, International Finance Discussion Papers 1022, Jul. 2011.

10.10 Exercises

10.1 Concept check · 2 min

In case 2 the PI holds the price within \$2.85 of its \$80 target after year 2, yet the cartel's share falls every year, from 29.75 to 28.15 % between years 1 and 10. Which explanation does the chapter give?

(a) The twice-yearly calendar caps the gain, so the PI cannot reject the steady disturbance that a price above \$75 creates

(b) The members overproduce above \$80, so the quota leaks barrels to cheaters while the price is held at the target

(c) The held price keeps moving shale, conventional supply and demand, a disturbance that grows like a ramp, and the PI answers by cutting the quota

(d) The monthly stock report is a month late, so the PI acts on old data and gives away barrels at every meeting

10.2 Concept check · 2 min

Linearized at the balance, the market has the eigenvalues $-3.85 \pm 3.47\,j$, –0.651 and –0.225/yr when the price rises, and $-3.90 \pm 3.10\,j$, –0.560 and –0.142/yr when it falls. Why are there two sets?

(a) The price law is exponential, so its slope at the balance is steeper for a rise than for a fall

(b) Overproduction begins above \$80, so the cartel's output answers a rising price and not a falling one

(c) Shale is clipped at its 15 Mb/d ceiling when the price rises, which removes one of its states

(d) The demand destruction state moves with $\tau_{down} = 6\,\text{yr}$ toward a lower target and with $\tau_{up} = 12\,\text{yr}$ toward a higher one

10.3 Concept check · 2 min

In case 3 the PI on stocks ends year 10 with the price at \$68.7, while the PI on price, run on the same start, ends at \$75.1. Why does the stocks loop settle lower?

(a) Its setpoint is the five-year average of its own reports, which carries the glut forward and then follows wherever the stocks have been

(b) The report is a month late, and a delayed measurement leaves an integral loop with a steady offset

(c) The stocks loop is direct acting, so its integral pushes the price down where the price loop pushes it up

(d) Its one-year integral time is too long for the integral to remove the error within ten years

10.4 Theory and calculation · 6 min

Use the price law of Equation (10.2) with $P_0 = \$75$, $I_0 = 2.9\,\text{Gb}$, $k = 9$ and no risk premium, and the demand law of Equation (10.6) with $x = 1$ and $D_0 = 100\,\text{Mb/d}$.

(a) Find the stocks at which the price is \$100 and \$50. *Checkpoint:* 2.807 Gb and 3.031 Gb.

(b) Find the slope $\mathrm{d}P/\mathrm{d}I$ at the balance. *Checkpoint:* −\$232.8 per Gb.

(c) With the fringe and x frozen, find the diagonal entry of the stock row of the Jacobian of Equation (10.1), and the time constant of the stocks alone in weeks. *Checkpoint:* −6.80/yr, a time constant of 0.147 yr, about 7.7 weeks.

10.5 Theory and calculation · 6 min

Use the defaults of Table 10.3 at the balance, $P_0 = \$75$ and $D_0 = 100\,\text{Mb/d}$.

(a) With shale, conventional supply and x frozen, find the static gain from the cartel's output to the price. *Checkpoint:* \$12.5 per Mb/d.

(b) With every slow state at its target, find how many Mb/d the market absorbs for each dollar, the long-run gain and the ratio of the two gains. *Checkpoint:* 0.745 Mb/d per dollar, \$1.34 per Mb/d, a ratio of 9.3.

(c) A quota raised by 1 Mb/d lowers the price by \$8.1 in three months and by \$1.6 after seven years. Use the slowest eigenvalue for a falling price to explain why the seven-year reading still exceeds the long-run gain. *Checkpoint:* −0.142/yr, a time constant of 7.0 yr.

10.6 Theory and calculation · 7 min

Case 4's market has $q_b = 10.5\,\text{Mb/d}$, $q_{max} = 15\,\text{Mb/d}$ and a breakeven that falls from \$80 to \$45. Use Equation (10.10) with the other defaults of Table 10.3.

(a) After the shift, find the long-run demand, the shale and conventional steady states and the residual demand R_{lr} at \$100. *Checkpoint:* 90.42 Mb/d, 15 Mb/d (at the ceiling), 54.46 Mb/d and 11.0 Mb/d.

(b) Find the long-run share R_{lr} over the long-run demand, and compare it with the 15.27 % that case 4 reaches at year 8. *Checkpoint:* 12.1 %, below the year-8 share.

(c) Find R_{lr} at \$75 after the shift, and the barrels a day that holding \$100 instead of \$75 costs the cartel in the long run. *Checkpoint:* 26.5 Mb/d, a loss of 15.5 Mb/d.

10.7 App challenge · 8 min

Load Reference · The balanced market. Before Run, switch to Manual and set Quota request to 31 Mb/d, so the meeting at $t = 0$ applies it. Run eight years at the default seed and export the CSV.

(a) Read the price, column `P`, at 0.25, 1 and 7 years, and the lowest price and its time. *Checkpoint:* \$66.88, \$71.99 and \$73.43, lowest \$66.87 at year 0.27, within \$0.05.

(b) Reload the case, switch to Manual, set Quota request to 29 Mb/d before Run and repeat the run. Read the price at 0.25 and 7 years. *Checkpoint:* \$83.76 and \$76.40, within \$0.05.

(c) From the two runs, give the price change per Mb/d at three months and at seven years for each direction, and explain why the cut moves the price more than the rise. *Checkpoint:* \$8.12 and \$8.76 at three months, \$1.58 and \$1.40 at seven years.

10.8 App challenge · 8 min

Load 4 · The shale revolution: hold \$100 and run eight years at the default seed.

(a) Read the cartel share badge, the quota in force on the quota card, and on the Fringe and demand view the long-run figure in the note of the LEFT FOR THE CARTEL panel. *Checkpoint:* 15.27 %, 13.6 Mb/d and 11.0 Mb/d, the share within 0.05 points.

(b) Set Price target to 80, press Reset and run eight years. Read the same three values. *Checkpoint:* 24.18 %, 23.8 Mb/d and 22.9 Mb/d, the share within 0.05 points.

(c) The long-run shares at \$100 and \$80 are 12.1 % and 23.4 %. Say which run is further from its long run at year 8 and why. *Checkpoint:* The \$100 run, by about 3.2 points against 0.8.

10.9 App challenge · 8 min

Load 7 · The cheater: an actuator that leaks. Run ten years at the default seed and export the CSV.

(a) Find the first row at which the quota, column `q`, reaches its floor of 26 Mb/d, the first row with the price, column `P`, below the \$65 band floor, and the lowest price after that. *Checkpoint:* Year 6.12 (the meeting of year 6.10), year 8.12, and \$61.06 at year 9.25, the times within 0.02 years.

(b) Set Price PI integral time to 5 yr, press Reset and run six years. Find the first row with the price below \$65. *Checkpoint:* Year 5.10, within 0.02 years.

(c) Explain why the shipped loop holds \$75 through four cheaters and then loses the band, and why the slower integral loses it three years sooner. *Checkpoint:* The integral compensates the leak until the quota floor saturates the actuator.

10.10 Application area · 4 min

In case 6, demand falls by 20 Mb/d for three months from year 1, the storage limit is 3.5 Gb, 0.6 Gb above the normal stocks, and the PI on price decides at its twice-yearly meetings with a 1.5 Mb/d cap.

(a) At a net inflow of 20 Mb/d and nothing else changing, how long does the headroom last? The chapter reports the tank full at year 1.115. Give the reason for the difference from the price law and the short-run elasticity. *Checkpoint:* 0.082 yr, about 4.3 weeks, against six weeks.

(b) Which meeting is the first that can cut, and by how much? What does this case say about sizing a buffer and choosing a sample period on a process plant? *Checkpoint:* The meeting of year 1.5, a cut of at most 1.5 Mb/d, after the tank has been full for 20 weeks.

10.11 Application area · 4 min

In case 9 the cartel's PI holds \$90 while the consumers' reserve, with its own PI, releases oil whenever the price is above \$80.

(a) Give the year the reserve runs empty, the price at year 10 with and without the reserve, and what the reserve cost the cartel in discounted revenue. *Checkpoint:* Year 4.33, $86.3 against $86.1, $43 billion (6306 against 6349).

(b) Name two ways a plant avoids two integrating controllers with different setpoints on one inventory, and the one this case offers. *Checkpoint:* One controller per inventory, or a split of the range with a gap between setpoints. Here a reserve ceiling of $95.

10.12 Review the agent · 5 min

An AI assistant reviews the challenge: “The shipped PI fails because it is too sluggish on twice-yearly meetings and never reaches $90. Set Price PI gain to 0.2 and Price PI integral time to 0.2 yr. The price will then reach the $90 target, every barrel will earn more, and both the share and the revenue will clear their limits.” Find the flaw, and say how to check it in the app and with the chapter’s model.