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CHAPTER 7

Water Inertia, Transient Droop and Closure Laws in a Hydroelectric Governor Simulator

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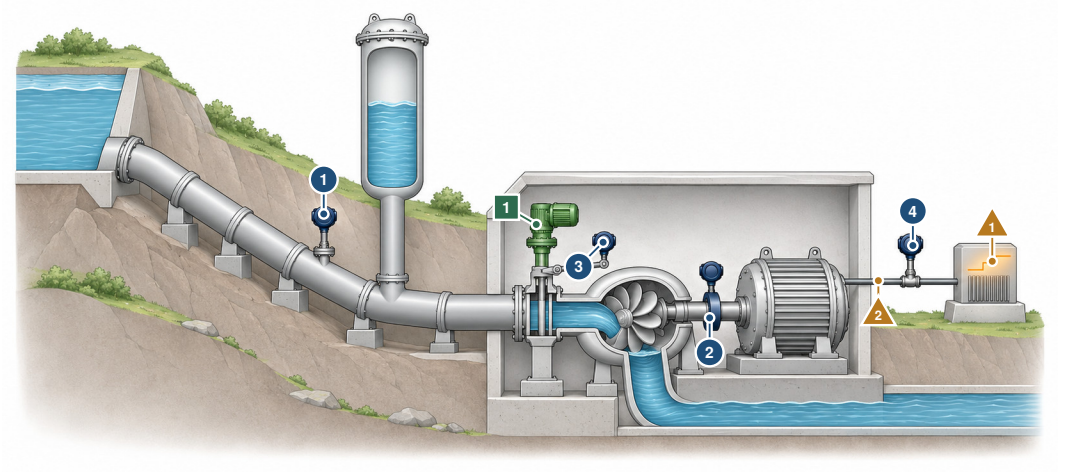
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CHAPTER 7

Water Inertia, Transient Droop and Closure Laws in a Hydroelectric Governor Simulator



The controller moves 1 the gate opening, through the gate servo

The controller reads 1 water pressure in the pipe 2 turning speed of the shaft 3 gate position 4 electrical power to the town

What disturbs it 1 a step in the town's load 2 loss of the whole load when the breaker opens

Notice the long column of water between the reservoir and the gate: it must speed up or slow down before the power can follow the gate. In the chapter's model the surge tank stands at the top of the long penstock, the pressure is read at the turbine inlet and the gates are turned by a hydraulic servomotor. Illustration, not to scale.

IN PLAIN TERMS A Water Turbine That First Answers Backward

The main idea. A hydroelectric plant sends water from a lake down a long pipe to a turbine. A governor opens or closes the gates in front of the turbine to hold its speed, and that speed sets the frequency of a town the plant feeds alone.

The challenge. Opening the gates first lowers the power, because the water in the pipe takes time to speed up. A governor that pushes too hard makes the town's frequency swing. Shutting the gates fast when the town is suddenly cut off sends a pressure surge up the pipe.

Why it matters. A governor must be tuned to the water in its pipe before anything else.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the history of the first hydroelectric plants (Section 7.2), the fact on plant size (Section 7.3), the try-it segments of cases 1 and 5 and the fact on the nadir (Section 7.6) and the key points
Study	45 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 7.6), Technology in Practice (Section 7.7), Limitations (Section 7.8) and the open challenge

The control question Opening the gate of a hydro turbine first lowers its power, because the water in the penstock must be accelerated by head taken from the turbine. How far does that right-half-plane zero limit the governor of a unit that supplies a town alone, and what does the same water column demand of the gate when the load is lost?

Learning objectives

- Derive the gate-to-power zero and pole of the rigid water column, and the ratio of the first response to the final one.
- Compute the frequency at which an islanded town settles at a fixed gate.
- Read the island's gain limit on the stability map and on the running plant, and tune a transient-droop governor by the rule.
- Explain why a surge tank can oscillate under a governor that holds the power, and compare Thoma's area with the closed-loop boundary.
- Choose a gate closure law that trades water hammer against overspeed after a load rejection.

Prerequisites Sections 1.4.2 and 1.4.3 (balances and linear models), 1.4.7 (inverse response and the ideal turbine), 1.6.2 (velocity form) and 1.6.4 (scenarios and the challenge).

The app https://apmonitor.com/apps/hydro-governor/

See also Chapter 6 for an inverse response from swell and shrink and Chapter 8 for droop on a grid with finite inertia. Volume 7, Chapter 13 for a stability boundary that moves with the operating point and Volume 7, Chapter 3 for one tuning that must serve several plants. Volume 17, Chapter 7 for droop governors, shedding and a synchronizer on an island.

7.1 Introduction

A 400 m steel penstock that delivers 80 MW to a Francis turbine on a 100 m head holds about 9400 tonnes of water moving at 3.8 m/s. In the simulator of this chapter, raising the gate from 65.6 to 75 % by hand at that load cuts the turbine power by 4.4 MW within 0.74 s. The power is back above 80 MW only 1.37 s after the move, and the frequency of the islanded town it supplies passes 62 Hz 10.9 s after the move. The extra opening lowered the head at the runner before the column could speed up, which is the inverse response of a water column and the starting point for every hydro governor. It is the plant behind the ideal turbine of Section 1.4.7, and Section 7.6.1 measures it.

Hydropower is an established technology that is still developing, as a recent survey of the emerging technologies of the sector shows [1]. Its governors are tuned against the water rather than against the machine. The standard dynamic models of the turbine, the penstock and the surge tank were collected by an IEEE working group [2], the guide to applying governing systems is an IEEE standard [3], and the classical textbook treatment of the turbine and its governor is Kundur's [4]. The behavior has a wider family. A right-half-plane zero imposes design tradeoffs on any feedback loop around it [5], inverse response was treated as a process control problem in 1962 [6], and the shrink and swell of a boiler drum is the same structure in another plant [7], a structure that Chapter 6 meets in a flash vessel. A reader who has tuned a drum level, a boost converter or a reactor with competing fast and slow effects has met the shape of this problem.

The simulator is built around two misconceptions. The first is that more opening always means more power at once, which the inverse response in the opening paragraph contradicts. The second is that a governor that behaves well on the grid will behave well when the unit supplies a load alone. On the grid the stiff network holds the speed and almost any gain is stable, while on an island the water starting time against the rotor's mechanical starting time sets a hard ceiling. The grid tuning that the challenge ships with, a gain of 8 with a 2 s integral time, sits above the island boundary of 6.04 for that integral time, and the town swings between 56.0 and 62.5 Hz.

A student usually meets the pieces of this problem in separate places, governor stability in power system texts [4, 8], gate closure schedules in hydraulic studies [9, 10], and surge tanks in the literature on hydraulic transients [11, 12]. This chapter works with one browser simulator in which the inverse

response, the gain limit, the transient-droop rule, the load share on a grid, the closure law of a load rejection and the surge tank all act on one plant, and it reports what each does there, with numbers. The model is the IEEE working group's, the governors are the two forms the industry builds, and multi-stage closure laws are established practice. Section 7.2 traces how hydro governing reached its present form. Section 7.3 gives the 26-state model with its optional elastic penstock, and a measurement boundary that keeps the governor away from the simulated flow. Section 7.4 derives the right-half-plane zero of the water column and the stability map, computed from the same equations the engine integrates and checked against the engine's own oscillation onset, and treats the tuning rule, water hammer and the surge tank. Section 7.5 states the two governors, the power controller and the load-rejection sequencer. Section 7.6 measures nine graded cases, from a gate moved by hand to a challenge that islands a town through a load step and a rejection, and checks the code against its own equations. Section 7.7 describes the instruments, gate servomotors and platforms of real governors, Section 7.8 states the limitations and Section 7.9 draws the results together.

7.2 From Dashpot Governors to Closure Laws

Hydro governors have long been tuned in a form peculiar to water, a governor that acts weakly at first, with a large transient or temporary droop, and builds to its full action over a reset time. In 1962 Hovey published a method for adjusting the governors of a utility's hydro units on that basis [13], and five years later Undrill and Woodward revisited the temporary droop calculation with a nonlinear model [14]. The rule that relates the transient droop and the reset time to the water and mechanical starting times reached later engineers through Kundur's textbook, which gives it after Hovey [4]. Interconnection made the governors a system matter as well as a unit matter. The coordination of hydraulic turbine governors for power system operation was addressed in 1966 [15], and governor tests by simulated isolation of hydraulic turbine units were reported two years later [16].

Proportional-integral-derivative governors followed, and the stability of a hydraulic turbine generating unit under a PID governor was analyzed in 1979 [8]. Hydraulic turbine and turbine control models for system dynamic studies were collected by an IEEE working group in 1992 [2], and the application of governing systems to hydroelectric units is now covered by an IEEE guide [3].

The hydraulic side has its own history. The pressure rise when a valve stops a flow faster than a wave can travel the pipe and back is Joukowsky's relation, and the slower closures that hydro plants actually use were treated by the water hammer theory that followed it [17]. A surge tank shortens the water column that the turbine sees, and a governor that holds its power constant can make the tank's slow oscillation grow, which is why tanks are sized against a stability area named after Thoma [11] and why throttling a tank changes governor stability and power control [12]. The closure law, the schedule on which the gate shuts after a load rejection, was treated as an optimal gate closure schedule in 1973 [9], and three-stage closure schemes are now optimized for pumped storage plants [10].

> ⧖ HISTORY **Water Turbines Begin to Make Electricity, 1880 to 1909**
>
> In 1880 a water turbine drove a dynamo that lit a theater and a storefront in Grand Rapids, Michigan, and in 1881 a turbine in a flour mill at Niagara Falls powered street lights. Both ran on direct current, which serves only short distances. Alternating current carried power farther, and the Redlands plant in California, the first commercial hydropower facility in the United States, came online in 1893 [18]. In the arid West, the Bureau of Reclamation first put small generators to work building the Theodore Roosevelt Dam in Arizona, lifting stone blocks into place, and sold the surplus to the nearby community. A 4500 kW plant followed, and by 1909 five generators were pumping irrigation water and supplying the Phoenix area [19].

7.3 The Unit as Modeled

The simulated plant is one generic 100 MW Francis unit with synthetic teaching values, shown in Figure 7.1 in the same arrangement as the application's Plant view. Water leaves a reservoir 100 m above the tailwater through a headrace tunnel, passes a surge tank, and falls through a 400 m penstock

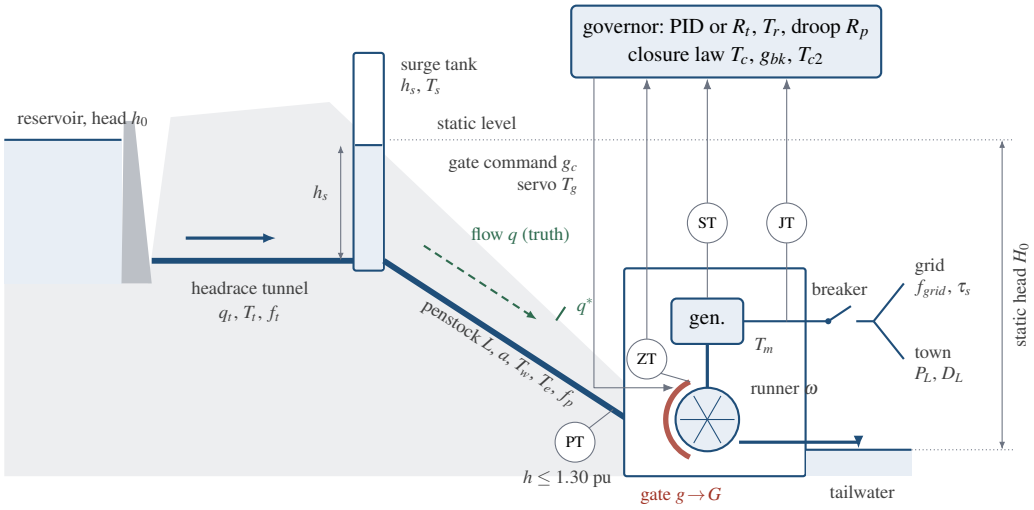


Figure 7.1 Hydroelectric unit as modeled. The reservoir at head h_0 feeds the headrace tunnel (flow q_t, starting time T_t, loss f_t) and the surge tank (head h_s, filling time T_s), which feeds the penstock of length L, wave speed a, water starting time T_w, travel time T_e and loss f_p. The penstock flow q is teaching truth (dashed), and the tick marks q^*, the flow the present gate passes once the column has caught up. The servo moves the stroke g, the ideal gate is G, the runner turns at ω with mechanical starting time T_m, and the breaker connects the generator to the grid or to the town load P_L with frequency sensitivity D_L. PT reads the inlet head h against its 1.30 pu teaching limit, and the speed (ST), gate (ZT) and power (JT) measurements feed the governor and its closure law. Not to scale.

to the turbine's wicket gates. The runner drives a synchronous generator whose breaker connects it either to a stiff grid or to an islanded town. The governor reads the speed, the gate position and the electrical power, and moves the gate through a hydraulic servo. The penstock flow has no meter in the plant and appears in the application only as teaching truth.

> ✦ DID YOU KNOW? **A Powerhouse of 6.8 Million Kilowatts**
>
> A federal pamphlet names Grand Coulee Dam on the Columbia River as the largest single electrical generating complex in the United States, with a capacity of more than 6.8 million kilowatts. A modern hydroelectric plant turns about 90 % of the energy of its falling water into electricity [19].

Every quantity is in per unit of the static head $H_0 = 100\,\text{m}$, of the flow $Q_0 = 115\,\text{m}^3/\text{s}$ through the fully open gate at that head, of the generator rating of 100 MW and of the synchronous speed at 60 Hz. Table 7.1 classifies the variables. At the design point of the island cases, the town draws 80 MW, the gate stands at 65.6 % of its stroke and the inlet head is 0.964 pu, or 9.45 bar above the tailwater.

7.3.1 Water Column and Turbine

The rigid penstock is a column of water accelerated by the difference between the surge tank's head and the head at the turbine, and the turbine passes flow in proportion to the gate and the square root of its head, as shown in Equation (7.1).

$$T_w \frac{\mathrm{d}q}{\mathrm{d}t} = h_s - h - f_p\, q|q|, \qquad h = \left(\frac{q}{G}\right)^2 \tag{7.1}$$

where $T_w = LV_0/(gH_0) = 2.0\,\text{s}$ is the water starting time at the full-gate velocity $V_0 = 4.905\,\text{m/s}$, so that the penstock's area is $23.45\,\text{m}^2$ and its diameter 5.46 m, and $f_p = 0.02$ is its head loss at full flow.

Table 7.1 Variables by Role, With Their Values at 80 MW on the Island

Variable	Role	Value	Range
f	CV, speed probe	60.00 Hz	band 58–62 Hz
P_e	CV on the grid, power transducer	80 MW	dispatch 0–100
g	MV, gate stroke	65.6 %	0–100 %
P_L	DV, town load	80 MW	0–100 MW, +10 % step
f_{grid}	DV, grid frequency	60 Hz	59.5–60.5, −0.2 Hz event
Δh_0	DV, reservoir level	0 m	−20 to 10 m
breaker	DV, load rejection	closed	opens on the event
$\Delta\omega$	state, speed deviation	0	—
q, q_t	states, penstock and tunnel flow	0.778 pu	truth only
h_s	state, surge tank head	0.976 pu	tank level, measured
q_i, h_i	states, elastic sections	mirrored	10 flows, 10 heads
$\Delta\omega_g$	state, grid speed	0	—
G	algebraic, ideal gate	79.2 %	—
h	algebraic, inlet head	0.964 pu	limit 1.30 pu
P_m	algebraic, turbine power	80 MW	—

The ideal gate G is the effective flow area. It follows the servo's stroke g through a declared concave characteristic, and the turbine's mechanical power follows the working group's form [2] as public lecture notes reproduce it (the working group's report itself was not read), as shown in Equation (7.2).

$$\begin{aligned} G &= \max\!\left(\frac{g(1+c)}{1+cg},\; G_{leak}\right) \\ P_m &= A_t\, h\,(q - q_{nl}) - D_t\, G\,\Delta\omega \\ A_t &= \frac{1}{q_1^2(q_1 - q_{nl})}, \qquad q_1 = \frac{1}{\sqrt{1+f_t+f_p}} \end{aligned} \tag{7.2}$$

where $c = 1$ makes a percent of stroke cut more flow near closed than near open, $G_{leak} = 0.01$ is the leakage of the closed wicket gates, $q_{nl} = 0.08$ is the flow that only spins the runner, and $A_t = 1.1893$ makes the power exactly 1 pu at full gate on the static head with both losses included. The term $D_t G\Delta\omega$ with $D_t = 1.0$ is the turbine's loss of power with speed, chosen so that the runaway speed at full gate is near 2 pu. The curvature c and D_t are teaching values, not a measured wicket-gate curve or runner characteristic.

7.3.2 Rotor, Network and Surge Tank

The rotor integrates the mismatch between turbine and electrical power, and the electrical power depends on what the breaker connects, as shown in Equation (7.3).

$$\begin{aligned} T_m \frac{\mathrm{d}\Delta\omega}{\mathrm{d}t} &= P_m - P_e - D\,\Delta\omega \\ P_e &= P_L\,(1 + D_L\Delta\omega) \quad \text{(island)}, \qquad 0 \quad \text{(open)} \\ P_e &= P_m - D\Delta\omega + \frac{T_m}{\tau_s}(\Delta\omega - \Delta\omega_g) \quad \text{(grid)} \\ T_{g,grid}\frac{\mathrm{d}\Delta\omega_g}{\mathrm{d}t} &= \frac{f_{grid}}{60} - 1 - \Delta\omega_g \end{aligned} \tag{7.3}$$

where $T_m = 2H = 8\,\mathrm{s}$, the unit damping D is zero, the town's load rises 1 % for each percent of frequency ($D_L = 1$), and on the grid the rotor is pulled to the grid's speed in $\tau_s = 0.1\,\mathrm{s}$ while the grid follows its own frequency setting with $T_{g,grid} = 2\,\mathrm{s}$. The grid case therefore omits the electromechanical swing of the machine against the network. Chapter 8 models the frequency of a whole grid with finite inertia, where droop and load damping set the offset. The surge tank and the

headrace tunnel close the hydraulic circuit, as shown in Equation (7.4).

$$T_s \frac{\mathrm{d}h_s}{\mathrm{d}t} = q_t - q, \qquad T_t \frac{\mathrm{d}q_t}{\mathrm{d}t} = h_0 - h_s - f_t\, q_t |q_t| \tag{7.4}$$

where $T_s = A_s H_0 / Q_0 = 250\,\mathrm{s}$ is the tank's filling time (a tank of about $290\,\mathrm{m}^2$), $T_t = 10\,\mathrm{s}$ is the tunnel's starting time, $f_t = 0.04$ its head loss and h_0 the reservoir head, 1 unless the reservoir slider moves it. At a fixed gate the tank and tunnel oscillate with a period near $2\pi\sqrt{T_s T_t} = 314.2\,\mathrm{s}$.

7.3.3 Gate Servo and Closure Law

The servo follows the governor's command with a first-order lag, opens no faster than 10 % of stroke per second, and closes no faster than the closure law allows, as shown in Equation (7.5).

$$\begin{aligned} \frac{\mathrm{d}g}{\mathrm{d}t} &= \mathrm{clip}\Big(\frac{g_c - g}{T_g},\ -r_c(g),\ r_{open}\Big) \\ r_c(g) &= \begin{cases} 1/T_{c2} & \text{two slopes and } g < g_{bk} \\ 1/T_c & \text{otherwise} \end{cases} \end{aligned} \tag{7.5}$$

where $T_g = 0.2\,\mathrm{s}$, $r_{open} = 0.10\,\mathrm{pu/s}$, T_c is the time the fast slope would take for a full stroke (6 s as shipped) and the cushion below the breakpoint g_{bk} closes a full stroke in T_{c2}. The stroke is held inside 0 to 1.

7.3.4 Elastic Penstock

An advanced switch replaces the rigid column with ten lumped sections, a staggered method of lines on the water hammer equations with a flow in each section and a head at each node, as shown in Equation (7.6).

$$\begin{aligned} T_w \Delta\xi\, \frac{\mathrm{d}q_i}{\mathrm{d}t} &= h_{i-1} - h_i - f_p \Delta\xi\, q_i |q_i|, \quad h_0 \equiv h_s \\ \frac{T_e^2}{T_w} \Delta\xi\, \frac{\mathrm{d}h_i}{\mathrm{d}t} &= q_i - q_{i+1}, \quad i < N \\ \frac{T_e^2}{T_w} \frac{\Delta\xi}{2}\, \frac{\mathrm{d}h_N}{\mathrm{d}t} &= q_N - G\sqrt{h_N} \end{aligned} \tag{7.6}$$

where $N = 10$, $\Delta\xi = 1/N$ and $T_e = L/a = 0.4\,\mathrm{s}$ for a wave speed of 1000 m/s. A closure faster than $2T_e = 0.8\,\mathrm{s}$ then raises the head by the Joukowsky amount $Z\Delta q$ with $Z = aV_0/(gH_0) = T_w/T_e = 5.0$. The rigid column's sections are mirrored from its flow and a linear friction line, so the switch can be made during a run.

7.3.5 Measurements, Initialization and Numerics

The governor sees seven instruments, read once per 20 ms scan: the speed probe, the bus frequency on the grid side of the breaker, the gate position, the generator power, the inlet pressure, the surge tank level and the head pond level. Each carries band-limited noise from a first-order filtered Gaussian with a 0.05 s correlation time, with standard deviations of 10^{-4} pu for the two speeds, 5×10^{-4} of stroke for the gate, 0.2 MW for power, 2×10^{-3} pu for pressure and 0.02 m for both levels, scaled by an instrument-noise slider. The penstock and tunnel flows are logged only in columns prefixed `truth_`, and a test fails if the controller source reads anything but the measurement packet.

Every case starts lined out. In automatic mode a bisection finds the gate that carries the town's load, or the grid dispatch, at 60 Hz, and in manual mode the speed is set to the deviation at which the manual gate balances the town. The largest derivative at the initial state is 3.2×10^{-17}. The 26 states (five plant states, twenty elastic section states and the grid's speed) advance by fourth-order Runge-Kutta steps of $\Delta t = 10\,\mathrm{ms}$, the governor executes every 20 ms, and a row is logged every 0.1 s, in the engine of Section 1.6.1. A load rejection and a breaker closure are applied once, after a step,

Table 7.2 Model Parameters. A: Adjustable in the Parameters Panel, F: Fixed, H: Hidden, D: Derived

Symbol	Meaning	Value	Unit	
H_0, Q_0	static head, full-gate flow	100, 115	m, m^3/s	F
L, a	penstock length, wave speed	400, 1000	m, m/s	F
T_w	penstock water starting time	2.0	s	A
T_e	wave travel time L/a	0.4	s	A
f_p	penstock head loss at $q=1$	0.02	pu	A
T_t, f_t	tunnel starting time, head loss	10, 0.04	s, pu	A
T_s	surge tank filling time	250	s	A
q_{nl}	no-load flow	0.08	pu	A
D_t	turbine damping	1.0	pu	A
c	gate curvature	1.0	—	A
G_{leak}	closed-gate leakage	0.01	pu	H
A_t	turbine gain	1.1893	—	D
T_g, r_{open}	servo lag, opening limit	0.2, 0.10	s, pu/s	A
T_m	mechanical starting time $2H$	8	s	A
D, D_L	unit damping, load sensitivity	0, 1.0	pu	A
τ_s	grid coupling time	0.1	s	H
$T_{g,grid}$	grid frequency lag	2	s	F
N	elastic switch, sections	off, 10	—	A
h_{max}	inlet head limit	1.30	pu	F
ω_{max}	overspeed limit	1.50	pu	F

with a mark on the trends. The pressure and overspeed limits are annunciated rather than tripped, so that the whole peak a closure law produces is visible.

Table 7.2 lists every parameter. The values make a consistent design point for a generic unit and are not fitted to any plant.

7.3.6 Assumptions

The column is rigid unless the elastic switch is on, which is accurate for the governor's seconds-long motions and for closures slower than $2L/a$. The turbine passes $q = G\sqrt{h}$ at every speed, so the Francis runner's S-characteristic, its efficiency hill and the draft tube are absent, and the overspeed after a rejection is a teaching estimate rather than a prediction for a real runner. The gate characteristic is a declared smooth curve whose concavity gives a cushioned closure a reason to exist. The surge tank has no throttle and cannot overflow, and the reservoir and tailwater are fixed except for the reservoir slider. The stiff grid removes the generator's 1 to 2 Hz swing against the network, voltage and excitation. The pressure limit of 1.30 pu (12.75 bar), the overspeed limit of 1.50 pu and the island band of 60 ± 2 Hz are teaching values, and a real plant takes its limits from the penstock's design pressure, the generator's overspeed rating and the frequency standard it operates under, with the governing system specified as the IEEE guide describes [3].

7.4 Why the Water Limits the Governor

7.4.1 The Right-Half-Plane Zero

Linearizing Equations (7.1) and (7.2) at a constant head upstream, with the ideal gate as the input and the mechanical power as the output, as Section 1.4.3 linearizes a tank, gives Equation (7.7).

$$\frac{\Delta P_m}{\Delta G} \propto \frac{h - 2f_p q(q - q_{nl}) - (q - q_{nl})T_w s}{T_w s + 2h/q + 2f_p q} \tag{7.7}$$

With $f_p = q_{nl} = 0$ at rated flow this is the textbook ideal turbine $(1 - T_w s)/(1 + T_w s/2)$ of Section 1.4.7, whose zero lies at $+1/T_w$ and whose pole lies at $-2/T_w$, so the first response to a gate step is -2 times the final one. The design plan for the application placed the zero at $2/T_w$, which is the pole's magnitude, and the application follows the derivation instead. On the application's plant the framework's eigenvalue routine, applied to the linearized column, returns the zero of Equation (7.7)

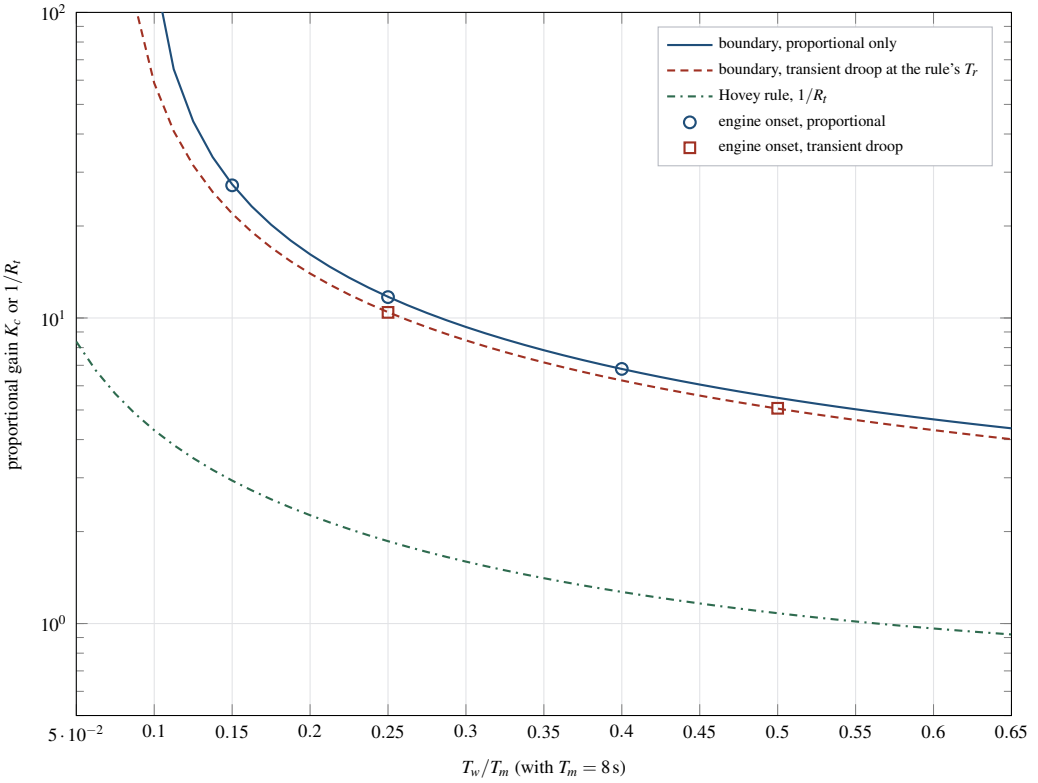


Figure 7.2 Island stability boundary against T_w/T_m at 80 MW with 5 % permanent droop, from the application's map function: the proportional governor (solid), the transient-droop governor at the rule's own reset time at each T_w (dashed) and the gain $1/R_t$ of the rule (dash-dotted). The circles (proportional) and squares (transient droop) are the engine's oscillation onset found by bisection.

within 10^{-6} at every load tested: 1.143, 0.675 and 0.510 rad/s at 50, 80 and 100 MW, with poles at -1.952, -1.254 and -0.991 rad/s. The first response is therefore 1.71 to 1.94 times the final one, and of the opposite sign. Section 7.6.1 shows the dip on the running plant. With the surge tank and tunnel left active the zero at 80 MW moves only to 0.672 rad/s and a slow pair appears near $\pm 0.020i$ rad/s. Doubling T_w to 4 s halves the zero to 0.337 rad/s at 80 MW.

Pause. Try Exercise 7.1 before reading on.

7.4.2 The Gain Limit and the Stability Map

The application's stability map linearizes the island loop by central differences of its own right-hand side (rigid column, servo without rate limits, rotor, town load and the governor in continuous form) with the surge tank held at its steady head, and computes the eigenvalues with the framework's balanced Hessenberg QR routine. The map plots the damping of the least-damped mode over the proportional gain and T_w/T_m, with T_m held and T_w varied, at the load the town draws at the frequency the speed probe has averaged over the last 30 s. The boundary falls steeply with the ratio, as shown in Figure 7.2. For a proportional governor with 5 % droop at 80 MW the boundary gain is 27.4 at $T_w/T_m = 0.15$, 11.73 at 0.25 and 6.81 at 0.40, and the engine's own oscillation onset, found by bisection on the growth rate of a small disturbance with the noise off, lies within 0.75 % of each.

The boundary also depends on the operating point. After case 2's load step a gain of 4 leaves the island near 58.7 Hz, where the town draws about 86 MW instead of the 88 MW it would draw at

60 Hz. At the load the map then uses, the boundary reads 13.04 at 30 s while the averaging window still contains pre-step data, and 12.89 once that window is entirely post-step, against 13.36 at 88 MW and 60 Hz and 11.73 at 80 MW, and the engine's onset is 13.14. The map is a linear prediction of where a small swing begins to grow. It ignores the servo's rate limits and the tank, so it says nothing about the size of a limit cycle or the gain at which a large one dies, which is why the application asks for a margin of about 10 %. A stability boundary that moves with the operating point is also the subject of Volume 7, Chapter 13, where the depth of cut at which a mill chatters moves with the spindle speed.

🕮 **Pause.** Try Exercise 7.2 before reading on.

7.4.3 Transient Droop and the Rule

The transient-droop governor of Section 7.5 acts first with a gain of $1/(R_p + R_t)$ and relaxes over the reset time T_r to its permanent droop. The rule the application suggests is Equation (7.8), which Kundur gives after Hovey [4, 13].

$$R_t = \big[2.3 - 0.15(T_w - 1)\big]\frac{T_w}{T_m}, \qquad T_r = \big[5 - 0.5(T_w - 1)\big]T_w \tag{7.8}$$

The equation was not checked against a primary copy. Kundur's book has no open copy and Hovey's paper is behind a paywall, so the form was taken from two editions of a public university lecture that quote it as Kundur's equations 9.41 and 9.42, and its worked example ($T_w = 1.3$ s, $T_m = 8$ s, giving 0.366 and 6.31 s) is reproduced by the application's function. Undrill and Woodward later proposed an improved calculation of the temporary droop [14], so the application calls Equation (7.8) a teaching rule and nothing more. At the design values it gives $R_t = 53.75$ % (54 % on the slider) and $T_r = 9$ s, and at $T_w = 4$ s it gives 92.5 % and 14 s. The rule sits far inside the linear boundary. At $T_w/T_m = 0.25$ its gain $1/R_t = 1.86$ is 5.6 times below the transient-droop boundary of 10.45 at the same reset time, and at 0.5 the factor is 4.7, as shown in Figure 7.2.

7.4.4 Water Hammer and the Closure Law

Stopping the column raises the head at the gate. For a closure faster than $2L/a = 0.8$ s the rise is the Joukowsky value, $aV_0/g = 500$ m for the full velocity, five times the static head. The governor's closures last seconds, so the rigid column applies and the rise is roughly T_w times the rate at which the flow is cut. Closing slowly lets the water drive the runner with no load, so the closure time trades the inlet pressure against the overspeed. The concave gate characteristic makes the trade uneven along the stroke, because near full opening a percent of stroke cuts little flow and near closed it cuts a lot, and a two-slope law closes fast while the gate is wide open and slowly in the cushion below a breakpoint.

7.4.5 The Surge Tank Under a Governor

When the tank's level falls, the head at the turbine falls with it. A governor that holds the power then opens the gate and draws still more water from a tank that is already low, which is negative damping on the tank's slow mode. Thoma's stability area answers how large a tank must be for that mode to decay anyway [11]. In per unit it is $T_{s,th} = T_t/(2 f_t h_s)$, 128.1 s at 80 MW and 128.8 s at 88 MW, against the plant's 250 s. The linearized closed loop of this plant, with the rule's transient droop and no permanent droop at 88 MW, puts the boundary lower, at $T_s = 85.2$ s. Removing terms one at a time explains the gap. With $D_L = 0$ the boundary rises to 112.6 s, with $D_t = 0$ to 112.4 s, with both zero to 130.4 s (close to Thoma's value), and with $q_{nl} = 0$ to 95.2 s, while $f_p = 0$ lowers it to 81.1 s. Thoma's derivation assumes the power is held exactly, while here the town's and the turbine's frequency sensitivity and the no-load flow let the power give way over the tank's three-minute period, and the penstock's own head loss takes a little of that damping back.

7.5 The Governors as Implemented

Both governors execute every $\Delta t_c = 20\,\text{ms}$ on the measurement packet and act on a load reference g_{ref}, the gate at which the unit runs at its speed setpoint. The permanent droop R_p acts on the measured gate position, as it does in the dashpot governor, with a default of 5 %. The form that is not in use tracks the gate command every scan, so switching forms and switching between Manual and Auto are bumpless.

7.5.1 PID Governor

The PID governor is the framework's velocity-form PID of Section 1.6.2, run on the per-unit speed with the droop folded into the measurement, as shown in Equation (7.9).

$$\begin{aligned} PV_k &= \Delta\omega_{m,k} + R_p\,(g_{m,k} - g_{ref}), \qquad e_k = \Delta\omega_{SP} - PV_k \\ P_k &= K_c e_k, \qquad D_k = -K_c \tau_D \dot{PV}_{f,k} \\ s_k &= s_{k-1} + (P_k - P_{k-1}) + (D_k - D_{k-1}) \\ \hat{u}_k &= u_0 + s_k + I_{k-1}, \qquad g_{c,k} = \min(1, \max(0, \hat{u}_k)) \\ I_k &= I_{k-1} + \left[\frac{K_c}{\tau_I} e_k + \frac{g_{c,k} - \hat{u}_k}{\tau_I}\right]\Delta t_c \end{aligned} \tag{7.9}$$

where $\Delta\omega_{SP} = f_{SP}/60 - 1$ and the derivative is taken on the measurement through a 0.1 s filter. The running sum and the back-calculated integral are those of Equations (1.25) to (1.27), with $s_k = S_k$, $\hat{u}_k = v_k$, $g_{c,k} = u_k$, $u_0 = u_b$ and the tracking time constant equal to τ_I. An integral time of zero removes integral action. The loop is direct acting in this sign convention, so a speed below the setpoint opens the gate. The defaults are $K_c = 3$, $\tau_I = 3\,\text{s}$ and $\tau_D = 0$.

7.5.2 Transient-Droop Governor

The transient-droop governor is the electronic equivalent of the dashpot governor, as shown in Equation (7.10).

$$\begin{aligned} g_{c,k} &= \text{clip}\left(\frac{(\Delta\omega_{SP} - \Delta\omega_{m,k}) + R_p\, g_{ref} + R_t\, z_k}{R_p + R_t},\ 0,\ 1\right) \\ z_{k+1} &= z_k + \frac{\Delta t_c}{T_r}\,(g_{c,k} - z_k) \end{aligned} \tag{7.10}$$

where z is the dashpot state. A speed error first moves the gate by $1/(R_p + R_t)$ per unit, and z then follows the gate so that the action grows over T_r toward the permanent droop $1/R_p$. With $R_p = 0$ the law is a PI controller with $K_p = 1/R_t$ and $\tau_I = T_r$, which is why the stability map plots $1/R_t$ for this form. With an ideal servo the structure equals the textbook dashpot loop exactly, and the transfer function $(1 + sT_r)/[R_p(1 + sT_rR_t/R_p)]$ that the lecture notes quote from Kundur is its approximation for $R_t \gg R_p$. The defaults are the rule's 54 % and 9 s.

7.5.3 Power Controller, Sequencer and Synchronizer

On the grid an optional power controller integrates on the load reference so that the unit follows its dispatch P_{SP} and keeps its droop share of a grid event, as shown in Equation (7.11).

$$g_{ref,k+1} = \text{clip}\left(g_{ref,k} + \Delta t_c K_{pc}\left[\frac{P_{SP}}{100} + \frac{e_k}{R_p} - \frac{P_{m,k}}{100}\right],\ 0,\ 1\right) \tag{7.11}$$

where P_m here is the measured generator power in megawatts, e_k/R_p is the frequency bias and $K_{pc} = 0.05\,\text{pu}$ of gate per per-unit power per second. A load-rejection sequencer acts in both modes because it is protection. When the breaker opens it commands the gate closed, and the servo closes along the closure law of Equation (7.5). Once the speed has peaked and fallen below 1.05 pu, it hands back with the load reference at the no-load gate for the measured reservoir head. A Synchronize helper closes the breaker only when the speed is within 1 % of the network's, and sets the load reference to the gate that carries the present load.

Table 7.3 The Nine Cases as Shipped

#	Setting and scripted events	Teaching point
1	Manual on the island, gate 65.6 %	inverse response, overspeed
2	Proportional, $K_c = 4$, load +10 % at 10 s	gain limit, the map
3	Transient droop 10 %, 3 s, load step at 10 s	the rule, why hydro is slow
4	Grid, PID 8 and 2 s, grid −0.2 Hz at 20 s	droop share, power control
5	Rule, 88 MW, rejection at 10 s, single 6 s	closure time and law
6	Rule with $R_p = 0$, $T_s = 60$ s, load step	surge tank instability
7	Rule, load step at 10 s	nadir and settling
8	$T_w = 4$ s, droop 25 %, 3 s, load step	a longer penstock
9	Challenge: grid PID, single 6 s, step at 10, rejection at 90 s	the whole problem

Table 7.4 Measured Results. Frequencies in Hz, Heads and Speeds in Per Unit

#	Setting	Quantity	Value
1	gate 65.6 to 75 % at 2 s	power dip, at	4.38 MW, 0.74 s
		peak f, f at 120 s	62.30, 61.96
2	map, engine at 80 MW	boundary gain	11.73, 11.70
	$K_c = 14$ at 30 s	limit cycle	4.6–5.5 p-p, 10.0 s
3	10 %, 3 s (shipped)	limit cycle, last 30 s	5.26 p-p
3, 7	rule, 54 %, 9 s	nadir, settle ±0.3	58.40, 28 s
	30 %, 4 s	nadir, settle ±0.3	58.52, 8.3 s
4	5 % droop, 40 s after	gate, power	6.65 %, 4.65 MW
	10 % droop	gate, power	3.33 %, 2.53 MW
	5 %, power control	peak, at 900 s	7.29 MW, 6.78 MW
5	single 6 s	head, speed	1.787, 1.366
	single 15 s	head, speed	1.277, 1.604
	two slopes 6/40 %/15 s	head, speed	1.286, 1.439
6	Auto, swing peak to trough	236 s, 836 s	6.28 m, 11.06 m
	Manual from 491 s	493 s, 877 s	8.01 m, 0.79 m
8	shipped 25 %, 3 s	range	52.57–65.52
	rule for 4 s, rule for 2 s	settle ±0.3	116 s, 41 s

7.5.4 Plant, Map and Governor Models

The three models of Section 1.4.9 take a particular form here. Section 7.3 defines the plant. The stability map is a second model, the island loop linearized with the tank frozen and the servo's limits removed, and it is used only for display, never by a controller. Neither governor carries an internal model. The gate-to-power slope of 0.74 at 80 MW and the gate-to-power zero are properties of the plant that a learner identifies, not parameters the governors use.

7.6 Nine Cases, Measured

The cases appear in Table 7.3, and Section 1.6.4 describes how a case, its completion rule and the challenge work in every app. The measurements below were taken without a display, on the same engine and seed the browser uses and with the same fixed steps, and any change a sentence mentions is made at the time it states, once the run is under way. A headless run of the case regenerates each value. Table 7.4 collects the headline measurements.

7.6.1 The Gate by Hand

Case 1 holds the gate at 65.6 % in Manual with the town at 80 MW. Raising it to 75 % at 2 s drops the turbine power by 4.38 MW, with the minimum 0.74 s after the move, and the power is back above 80 MW 1.37 s after it. Figure 7.3 shows the plant during the dip. This is the inverse response of Section 7.4.1 on the running plant. The surplus then spins the rotor up. Nothing but the town's frequency sensitivity and the turbine's own damping pulls the speed back, so the frequency passes 62 Hz 10.9 s after the move, peaks at 62.30 Hz and reads 61.96 Hz at 120 s. Over the following minutes the surge tank's swing carries it between 62.30 and 62.38 Hz. The case's completion rule

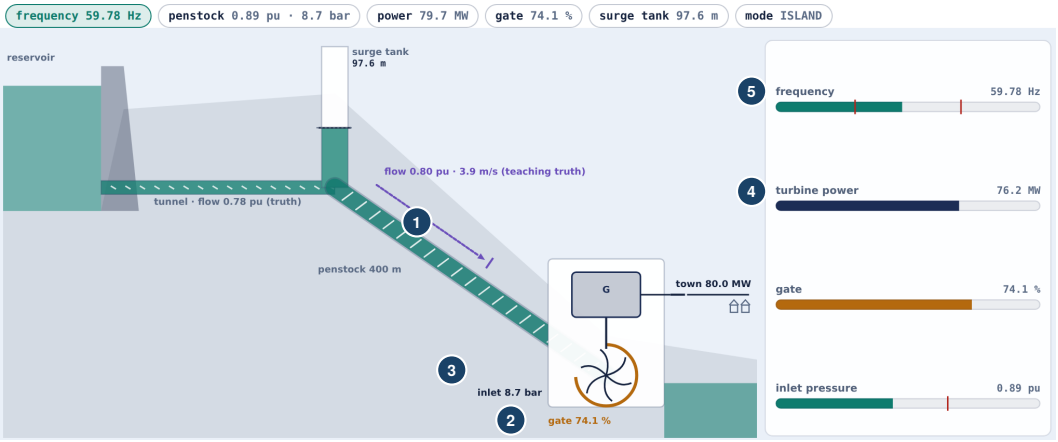


Figure 7.3 The inverse response of case 1 on the Plant view, cropped from the application window, at $t = 2.9$ s (the clock is in the cut header), 0.9 s after the gate station was raised from 65.6 to 75 % in Manual on the island. The servo has reached 74.1 % (2), but the penstock flow (1, dashed, teaching truth) is still 0.80 pu at 3.9 m/s. The tick ahead of its arrow marks the flow the new opening will pass once the column has caught up, 0.83 pu, a value the figure does not print. The inlet pressure has fallen to 8.7 bar (3), the turbine gives 76.2 MW (4) against the 79.7 MW the town draws at that frequency, and the frequency has dipped to 59.78 Hz (5) before the faster water reverses it. The power had passed a minimum of 75.6 MW at 2.74 s, which the cropped view does not show and Table 7.4 gives as a dip of 4.38 MW 0.74 s after the move.

asks for the gate at 74 % or more after Run, then the turbine power below 79 MW, then the frequency above 62 Hz. The gate is moved after Run because a setting changed before Run becomes part of the initial state instead of a step.

> **TRY IT IN THE APP** **1 · Open the Gate by Hand on the Island**
>
> Load the case and press Run. Once the clock passes 2 s, raise Gate opening from 65.6 to 75 % and watch the turbine power on the Power strip. It falls first, by 4.38 MW at its lowest, and is back above 80 MW only 1.37 s after the move. Then keep watching the frequency: with nobody governing it, it climbs past 62 Hz 10.9 s after the move.

7.6.2 The Gain Limit

Case 2 runs a proportional governor with a gain of 4 and 5 % droop through a 10 % load step. Raised to 14 at 30 s, the gain sits above the settled post-step boundary of 12.89, and the island settles into a limit cycle with a period of 10.0 s whose amplitude wanders between 4.6 and 5.5 Hz peak to peak from 60 to 600 s as the surge tank moves the head. The servo's rate limits hold it there. A gain of 13, about 1 % below the engine's onset of 13.14, decays at first (0.74 Hz peak to peak at 160 to 180 s) and then grows into the same limit cycle (4.73 Hz at 580 to 600 s), while 12.5 dies away to 0.02 Hz. Recovering from a running limit cycle needs more margin than the onset. With the gain lowered from 14 at 90 s, the swing falls below 0.2 Hz peak to peak in 110 s at 12 and 60 s at 11, and never at 13 or 12.5, and lowered at 200 s the times are 170 s and 75 s.

7.6.3 Transient Droop, the Nadir and a Longer Penstock

Case 3 ships a transient droop of 10 % with a 3 s reset, the way a steam governor would be set. The boundary at that reset time is $R_t = 12.4$ %, so the island limit-cycles after the load step, 5.26 Hz peak

to peak over the last 30 s of 120 s. The rule's 54 % and 9 s give a nadir of 58.40 Hz 5.4 s after the step with no overshoot, and the frequency stays within 0.3 Hz of its value at 120 s from 28 s after the step. A faster 30 % and 4 s reach a nadir of 58.52 Hz and settle in 8.3 s. The nadir barely moves because it is set in the first five seconds by the rotor and the water. Over 366 calm PID and transient-droop tunings the best nadir is 58.87 Hz, from a PID with a gain of 3, an integral time of 5 s and a derivative time of 2 s, and with the gate held fixed the frequency falls to 57.13 Hz by 15 s. Case 7 repeats the rule's run and adds that over 40 to 900 s the surge tank moves the island between 59.33 and 59.78 Hz, inside the settling band around the 120 s value of 59.61 Hz.

Case 8 doubles T_w to 4 s. The tuning that settled the 2 s penstock in 11.7 s, a transient droop of 25 % with a 3 s reset, now sits beyond a boundary of 35.0 % and drives the island between 52.57 and 65.52 Hz. The rule for 4 s settles within 0.3 Hz in 116 s, and the rule for 2 s still settles, in 41 s. A faster tuning on one water column tolerates less of another.

> ✦ DID YOU KNOW? **A Faster Governor Barely Lifts the Nadir**
>
> From the simulator: after the 10 % load step on the island, the rule's transient droop of 54 % with a 9 s reset settles in 28 s, and a faster 30 % with 4 s settles in 8.3 s. The lowest frequency moves only from 58.40 to 58.52 Hz (Table 7.4).

7.6.4 Droop on the Grid

Case 4 synchronizes the unit to the grid at 80 MW with the grid PID, and the grid falls 0.2 Hz at 20 s. With 5 % droop the gate opens 6.65 % of stroke against the $(0.2/60)/0.05 = 6.67\,\%$ the droop asks for, and 40 s after the event the power is up 4.65 MW, not the 6.67 MW the ratio would give in power, because the droop acts on the gate and the turbine's gate-to-power slope at 80 MW is 0.74. A 10 % droop halves the gate move to 3.33 % and gives 2.53 MW. With the power controller on from the start at 5 %, the power carries the droop's share itself, overshooting to 7.29 MW at 239 s and reading 6.78 MW at 900 s, with the surge tank still moving it between 6.41 and 6.99 MW over the last five minutes.

7.6.5 Load Rejection and the Closure Law

Case 5 opens the breaker at 88 MW, where the penstock carries 98.2 m^3/s at 4.19 m/s. A single 6 s closure raises the inlet head to 1.787 pu (17.5 bar) at 14.5 s while the speed peaks at 1.366 pu. A 15 s closure holds the head at 1.277 pu but lets the speed reach 1.604 pu. No single slope between 1 and 40 s meets both teaching limits at this load, and the closest with the head under 1.30 pu, 14 s, overspeeds to 1.583 pu. The two-slope law the case suggests (6 s above a 40 % breakpoint and a 15 s cushion below it) gives 1.286 pu and 1.439 pu. Halving the time step moves these peaks by less than 1×10^{-3} pu. The case's completion rule asks for the two-slope law and a run past the rejection with both peaks inside their limits. Figure 7.4 sets the three laws against the two limits.

> ⌖ TRY IT IN THE APP **5 · Load Rejection: Closure Time Against Pressure and Overspeed**
>
> Load the case and press Run. The breaker opens at 10 s, and the single 6 s closure drives the inlet head to 1.787 pu while the speed peaks at 1.366 pu. Switch Closure law to Two slopes, press Reset and run again: the head now peaks at 1.286 pu and the speed at 1.439 pu, both inside their limits.

From 100 MW the result changes in an instructive way. The servo offers one or two slopes, and on the plant equations, which match the engine to 1×10^{-3} pu at 88 MW, no single slope from 1 to 40 s in steps of 0.1 s meets both limits, and none of 56,610 two-slope laws (fast 1 to 12 s, breakpoint 5 to 95 %, cushion 2 to 80 s) does either. The closest two-slope law, 5.5 s, 42.5 % and 14 s, holds 1.297 pu with 1.541 pu of speed. The water and the rotor do not forbid it. Of 46,032 three-slope laws, 656 meet both limits, such as 4 s for a full stroke above 60 %, 8 s down to 25 % and 14 s below (1.296 pu and 1.491 pu), and a gate that held the head exactly at 1.30 pu would limit the overspeed to 1.383 pu. The

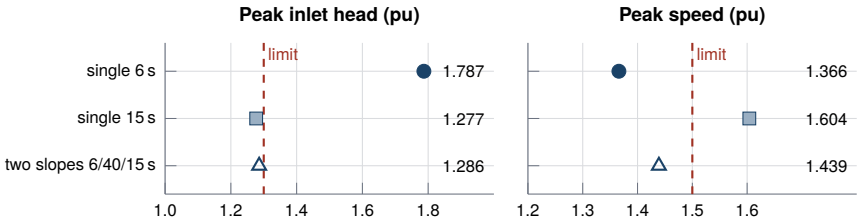


Figure 7.4 The closure laws of case 5 from Table 7.4, rejecting 88 MW: a single 6 s closure (filled circles), a single 15 s closure (squares) and the two-slope law with a 40 % breakpoint and a 15 s cushion (open triangles), with each value printed at the right. The dashed lines are the teaching limits. Notice that each single slope breaks one limit, the fast one the head and the slow one the speed, while the two-slope law stays under both.

infeasibility therefore belongs to the law family the servo offers, not to the plant, and the application says so. Optimal closure schedules were studied in 1973 [9] and three-stage closure schemes are optimized for pumped storage [10], so the counterexample is a measured property of this teaching plant that agrees with that practice.

The same computation sets the limits the application uses. The design plan proposed an overspeed limit of 1.2 pu, but at 88 MW the head-holding gate bounds the overspeed at 1.307 pu, so 1.2 pu with a 1.30 pu head is out of reach for any closure law at $T_w = 2$ s and $T_m = 8$ s. The limit was recalibrated to 1.50 pu, which 1188 of the 56,610 two-slope laws meet at 88 MW, the best at 1.425 pu. The design plan's island band of ± 1 Hz was widened to ± 2 Hz because the best of 366 calm tunings reaches only 58.87 Hz after the 10 % step.

7.6.6 The Surge Tank

Case 6 shrinks the tank to $T_s = 60$ s, an undamped period of $2\pi\sqrt{T_s T_t} = 154$ s, and runs the rule's governor isochronously. After the load step the level swing grows about 8 % each half cycle at a period near 171 s, from 6.28 m peak to trough at 236 s to 11.06 m at 836 s, which the linearized loop matches with a growth of 1.086 per half cycle and a period of 171.0 s. The case's completion rule asks for a swing larger than 8 m under the governor, first true at 491 s, followed by 300 s with the gate held. Switched to Manual at 491 s, the fixed gate passes less water as the head falls, and the swing decays from 8.01 m to 0.79 m by 877 s, about 40 % less each half cycle after the first. On the engine the boundary lies between $T_s = 70$ s, which grows, and 90 s, which decays, consistent with the linear 85.2 s of Section 7.4. The plant's own tank of 250 s decays under the same governor.

Pause. Try Exercise 7.3 before reading on.

7.6.7 The Challenge

The challenge islands the town at 80 MW, raises its load 10 % at 10 s and opens the breaker at 90 s. Its target, judged on the true speed and head every physics step, is 60 ± 2 Hz until the breaker opens, within 0.3 Hz of the final value from 40 s after the step, and a head at or under 1.30 pu and a speed at or under 1.50 pu over the 150 s. Table 7.5 lists four configurations. The verdicts do not change at seeds 1 through 5 or at a time step of 5 ms, and the logged rows are identical when the run is played at $1\times$ and at $60\times$.

7.6.8 Code Verification, Agreement and What Is Not Claimed

Code verification checks that the program solves its own equations. The integral of $P_m - P_e - D\Delta\omega$ over 60 s of case 7 equals T_m times the change in $\Delta\omega$ to 5.8×10^{-6} pu s. At fixed gates of 40, 65 and 90 % the engine's power equals Equation (7.2) evaluated at the column's steady flow within the test's tolerance of 0.1 %. A 2 % gate step at a fixed gate starts a tank oscillation with a period of 313.7 s

Table 7.5 The Challenge Over 150 s at the Default Seed

Configuration	f range (Hz)	settle (Hz)	head (pu)	speed (pu)	verdict
grid PID, single 6 s (shipped)	56.009–62.479	4.394	1.782	1.226	fails
rule, single 6 s	58.396–60.001	0.116	1.776	1.355	fails
PID 4, 3 s, two slopes	58.535–60.002	0.018	1.275	1.431	passes
rule, two slopes (suggested)	58.396–60.001	0.116	1.276	1.428	passes

against $2\pi\sqrt{T_s T_t} = 314.2$ s. On the elastic penstock a closure of 10 % of stroke in 0.3 s raises the head by 0.1383 pu against the Joukowsky estimate of 0.1380 pu from the turbine's own law, a difference of 0.13 %. An independent N-section solver gives errors of 7.0, 0.70, 0.35, 0.13, 0.14 and 0.16 % for 2, 3, 5, 10, 20 and 40 sections, unchanged at time steps of 5 and 2.5 ms, which is why the switch uses ten sections and not the two the design plan proposed. Case 7's frequency at 15 and 60 s is the same to 1×10^{-9} Hz at time steps of 10, 5 and 2.5 ms. A 900 s case costs 134 ms headless, and case 6 plays its fifteen minutes in about 15 s at 60× in a browser.

Agreement with published relations is a separate and weaker claim. With $f_p = q_{nl} = 0$ the application's linearization reproduces the ideal turbine's zero at 0.500 rad/s, pole at −1.000 rad/s and initial to final ratio of −2.000 at rated flow [2], the rule's worked example is reproduced as quoted, and Thoma's area and the Joukowsky constant follow from the declared geometry. No plant data were used, so nothing here validates the model against a real unit, and no learning gain is claimed because no learning study has been made.

7.7 Technology in Practice

The governor of Section 7.5 reads seven instruments and moves one gate servo. Figure 7.5 draws its loop with plant devices in place of the schematic's symbols.

What Measures It

A governor is built to see small changes of speed. In a mechanical governor the pilot valve is made very small so that it responds to the minimal forces of minor speed changes [20]. The gate position comes from a position transducer on the servomotor, calibrated so that it reads 0 to 100 % of stroke, and a lost transducer signal must raise an alarm at the controller [20]. At commissioning the speed is often read a second way, by a proximity probe on a temporary target on the shaft, and the two signals are trended together [21]. The chapter's speed probe, with its band-limited noise of 10^{-4} pu read every 20 ms, stands for the toothed wheel and its probes. No range, accuracy or response time is given here, because none was found in an open source that names no maker.

> **TECHNOLOGY** **Sensor: Speed From a Toothed Wheel**
>
> A mechanical governor senses speed with flyweights in a ball head, spun by a motor fed from a permanent magnet generator on the unit's shaft. A digital governor counts the pulses of magnetic probes facing a toothed wheel. The probe's air gap is set to the maker's specification, and metal filings on a probe are cleaned off [20]. A sensor that misses pulses is replaced, and a frequency generator checks the speed channels and the overspeed trip during pre-commissioning [21]. A damaged drive pin on the permanent magnet generator shows on a trend as gates that move a little every revolution while the grid holds the speed, called on-line hunting [20].

What Moves It

The gate servo of Equation (7.5) follows its command with a lag of 0.2 s, opens no faster than 10 % of stroke per second and closes along the closure law. A plant sets the closing rate with an orifice in the servomotor's hydraulic circuit, and some plants have more than one rate limiter in each direction [21], which is what the chapter's two-slope law with its cushion represents. The closing rate sets both the

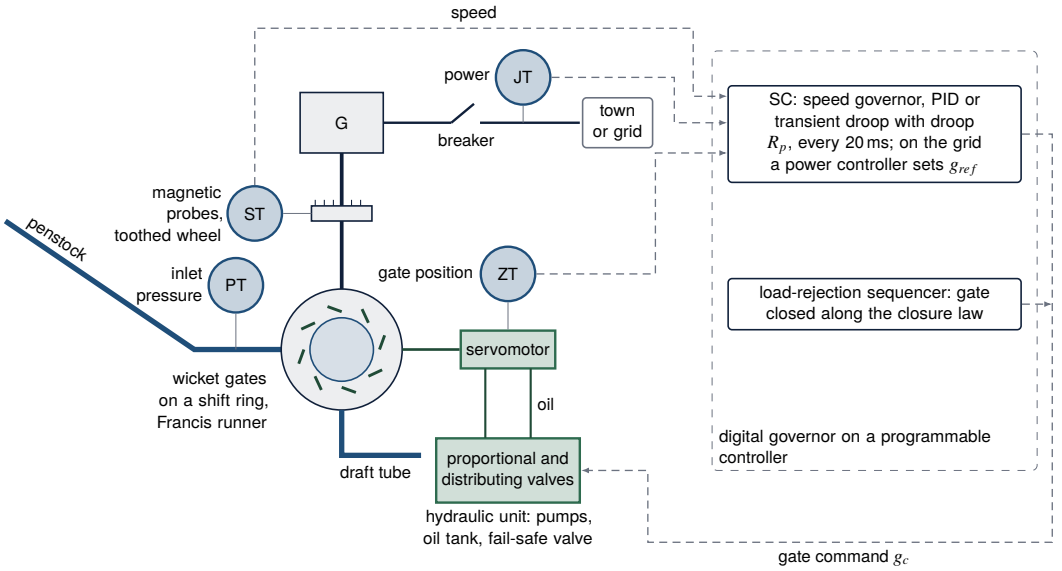


Figure 7.5 The governor loop of the chapter with plant devices in place of the schematic's symbols. ST reads the speed from a toothed wheel on the shaft, ZT the gate position on the servomotor, JT the generator power and PT the inlet pressure. On the digital governor the speed governor, or after a load rejection the sequencer, sets the gate command, which proportional and distributing valves turn into oil flow to the servomotor that turns the wicket gates. Notice that every path from the gate to the speed runs through the water column, the turbine and the rotor.

water hammer and the overspeed after a load rejection, and practice makes it as fast as the design pressure of the penstock or the spiral case allows [20].

> **TECHNOLOGY** **Final Element: Wicket Gates and Their Servomotors**
>
> Hydraulic servomotors drive a shift ring that turns all the wicket gates together. A small pilot valve, sensitive to minute forces, steers a large main distributing valve that meters oil to the servomotors. The oil tank is usually sized so that the gates can still close if the pumps fail. Near the end of the closing stroke the pistons cover their drain ports and a cushioning valve slows them, so that the gates do not slam shut. A digital governor adds a spring-loaded valve that closes the gates at the fastest allowed rate when its controller stops working [20].

Where the Controller Runs

A generating facility is run by a distributed control system that communicates with the grid's supervisory system, and its control loops run with cycle times from milliseconds to minutes [22]. The controller of a digital governor can be a programmable logic controller [20], and the chapter's 20 ms execution sits at the fast end of that range. The transient-droop form of Equation (7.10) began as a hydraulic dashpot whose temporary droop compensates the inertia of the unit and of the water column. On a large grid a bypass lets the dashpot re-center quickly so that load changes go faster, and when the plant is separated to carry a small system alone the bypass is not used [20]. That is the chapter's lesson that grid tuning fails on an island.

IN PRACTICE **Proving a Closure Law by Load Rejection**

The closing rate is set by an orifice in the servomotor circuit and timed from 25 to 75 % of stroke, then doubled, to leave out the cushion. At commissioning the breaker is opened at 25 % power, then at 50, 75 and 100 %, recording the peak speed and the peak penstock pressure each time. After each rejection IEEE Std 125 allows one 5 % underspeed deviation and at most one 5 % overspeed deviation. A pressure above its design limit stops the series, and one close to it goes to the designers before the next step [21].

What Is Modernizing It

Digital governors remove the ball head, the dashpot and the restoring linkage, though their proportional valves need cleaner oil [20]. A review of emerging hydropower technology estimates that digitalizing the world's 1225 GW of hydropower could add 42 TWh of production a year. Variable-speed units, fed through a power converter, control active and reactive power better, at an estimated extra cost of 7 to 15 % of a fixed-speed plant. Flywheels or supercapacitors beside a hydro unit could answer the fastest frequency changes and spare the turbine some wear [1].

7.8 Limitations

All values are synthetic and describe a consistent generic unit, not a measured one. The turbine law omits the S-characteristic and the efficiency hill, so the overspeed numbers of case 5 and the challenge are teaching estimates, and a real runner's flow change with speed would move them. The gate characteristic, the turbine damping and the tank size are declared teaching values. The stiff grid removes the swing of the generator against the network, the governor has no deadband, and the town has no under-frequency load shedding, which Volume 17, Chapter 7 adds to an islanded campus grid. A clean 10 % load step, a town with a fixed frequency sensitivity and a grid held stiff within 0.1 s would not occur on a real system, but each isolates one mechanism. The wave speed of 1000 m/s needs a steel wall near 50 mm on a 5.46 m pipe by Korteweg's formula (30 mm gives 865 m/s), and case 8 doubles T_w without doubling T_e, which matters only with the elastic switch on. The tuning rule was quoted through lecture notes because its primary sources could not be read, and real governors are commissioned by test. The stability map is linear and the limits are annunciated, so neither replaces a transient study or a protection scheme, and a simulated pass is not a certification.

OPEN CHALLENGE **Flexible Duty From Machines Designed for Steady Load**

Most hydropower plants were designed decades ago for conditions unlike today's. Wind and solar generation lower the inertia of the grid, so hydro units are asked for fast frequency reserves and black starts, while hydrodynamic phenomena limit their operating range and frequent starts and stops add wear. Very little has been achieved in modeling and controlling those flow phenomena from standstill to full load. Progress would be a validated digital model of each unit that predicts the stress, wear and fatigue of a start, a stop or a fast change before the unit is asked for it [1].

7.9 Conclusion

This chapter studied a hydroelectric unit, a plant whose actuator first moves its output the wrong way, in a simulator that makes that visible with the IEEE working group's water column, a surge tank, a declared gate characteristic and a rotor on an island or a stiff grid. A 9.4 % gate opening at 80 MW cuts the power by 4.38 MW before the water catches up. The zero sits at 0.675 rad/s near the textbook $1/T_w$ rather than at the $2/T_w$ the design plan proposed, and the island's gain limit of 11.73 from the computed map agrees with the engine's onset to about 0.26 %.

The transient-droop rule settles a 10 % load step in 28 s with a nadir of 58.40 Hz, a governor tuned on the grid oscillates between 56.0 and 62.5 Hz on the island, and a surge tank that Thoma's estimate would call unstable below 128 s is stable under this governor down to about 85 s. Rejecting full load,

no one- or two-slope closure law keeps the head under 1.30 pu and the speed under 1.50 pu, while 656 three-slope laws do.

The model is lumped, its gate and runner are declared rather than measured, and its grid is stiff. A fuller model would add a runner characteristic with an S-shaped flow curve, a generator swing against a finite grid, a governor deadband and a surge tank throttle. The simulator shows that a hydro governor is tuned against its water column before it is tuned against anything else.

KEY POINTS

- Opening a hydro turbine's gate first lowers its power, because the water column must be accelerated by head taken from the turbine.
- On an island the water and mechanical starting times set a gain limit, 11.73 for a proportional governor at $T_w/T_m = 0.25$ and 80 MW.
- The transient-droop rule settles a 10 % load step in 28 s, while a governor tuned on the grid oscillates between 56.0 and 62.5 Hz on the island.
- After a load rejection the closure time trades the inlet pressure against the overspeed, and at 88 MW a two-slope law meets both limits where no single slope does.

Where else this applies. Chapter 6 meets an inverse response from swell and shrink, and Volume 17, Chapter 7 runs droop governors and a synchronizer on an island.

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7.10 Exercises

7.1 Concept check · 2 min

In case 1 the unit supplies the town alone and the gate is held by hand. The gate is raised from 65.6 to 75 %, and for about a second the turbine power falls. Why?

(a) The head at the runner falls at once, while the water in the penstock needs time to reach the larger flow

(b) The governor sees the frequency fall and closes the gate before the servo has finished opening it

(c) The rotor speeds up at once, and the turbine's speed damping $D_t G \Delta\omega$ takes the power away

(d) The surge tank level drops at once, so the penstock is fed from a lower head than before

7.2 Concept check · 2 min

The island's stability map gives a boundary gain of 11.73 for a proportional governor with 5 % droop at $T_w/T_m = 0.25$ and 80 MW. Which single change lowers that boundary?

(a) A heavier rotor, with T_m raised from 8 to 16 s and T_w unchanged

(b) A penstock twice as long, with T_w raised from 2 to 4 s and T_m unchanged

(c) A surge tank twice as large, with T_s raised from 250 to 500 s

(d) A town load of 88 MW at 60 Hz instead of 80 MW

7.3 Concept check · 2 min

Case 6 runs a small surge tank, $T_s = 60$ s, under the rule's governor with no permanent droop. After the load step the tank's swing grows about 8 % each half cycle. Switched to Manual, the same swing dies away. What makes the difference?

(a) The tank is smaller than Thoma's area, so its swing grows whatever the gate does

(b) The governor's integral action winds up on the gate's rate limit and overshoots each half cycle

(c) The governor holds the power, so it opens the gate as the head falls and draws more water from a tank that is already low

(d) The water hammer wave in the penstock reflects at the tank and pumps its level each half cycle

7.4 Theory and calculation · 6 min

The gate-to-power transfer function of the rigid column is Equation (7.7), with $T_w = 2$ s.

(a) For the ideal turbine, $f_p = q_{nl} = 0$ and $q = h = 1$, find the zero, the pole and the ratio of the first response to a gate step to the final one. *Checkpoint:* $+0.5\,\text{rad/s}$, $-1.0\,\text{rad/s}$ and -2.

(b) At 80 MW the column carries $q = 0.778$ at $h = 0.964$, with $f_p = 0.02$ and $q_{nl} = 0.08$. Find the zero, the pole and the same ratio. *Checkpoint:* 0.675 rad/s, $-1.25\,\text{rad/s}$ and -1.86.

(c) Find the zero at 80 MW when T_w is doubled to 4 s, and say what it means for the governor. *Checkpoint:* 0.337 rad/s.

7.5 Theory and calculation · 6 min

The plant's surge tank has $T_s = 250\,\text{s}$ and its tunnel $T_t = 10\,\text{s}$ and $f_t = 0.04$, on the bases $H_0 = 100\,\text{m}$ and $Q_0 = 115\,\text{m}^3/\text{s}$. At 80 MW the tank head is $h_s = 0.976$.

(a) Find the tank's cross-section and its undamped period at a fixed gate. *Checkpoint:* $287.5\,\text{m}^2$ and 314.2 s.

(b) Find Thoma's filling time at 80 MW, the matching cross-section, and the plant's margin over it. *Checkpoint:* 128.1 s, about $147\,\text{m}^2$, a factor of 1.95.

(c) Case 6 shrinks the tank to $T_s = 60\,\text{s}$. Find its undamped period, and say whether Thoma's area and the chapter's closed-loop boundary each predict a growing swing under the isochronous governor. *Checkpoint:* 153.9 s. Both predict growth.

7.6 Theory and calculation · 8 min

In case 1 the gate is fixed and the unit supplies the town alone, with $h_0 = 1$, $f_t = 0.04$, $f_p = 0.02$, $q_{nl} = 0.08$, $c = 1$, $A_t = 1.1893$, $D_t = D_L = 1$, $D = 0$ and the town at $P_L = 0.8\,\text{pu}$ at 60 Hz.

(a) From Equations (7.1), (7.2) and (7.4) at steady state, show that a fixed ideal gate G passes $q = 1/\sqrt{1/G^2 + f_t + f_p}$, and check the design point at 65.6 % of stroke. *Checkpoint:* $G = 0.792$, $q = 0.778$, $h = 0.964$ and $P_m = 0.800\,\text{pu}$ at rated speed.

(b) Find the same quantities after the gate is raised to 75 % and the water has settled. *Checkpoint:* $G = 0.857$, $q = 0.839$, $h = 0.958$ and $P_m = 0.864\,\text{pu}$ at rated speed.

(c) With no governor, find the speed deviation and the frequency at which the rotor balances the town, and the power the town then draws. *Checkpoint:* $\Delta\omega = 0.0389$, 62.33 Hz and 83.1 MW.

7.7 App challenge · 7 min

Load scenario 1, Open the gate by hand on the island, at the default seed and the default playback speed. Press Run, pause once the clock has passed 2 s, and note that time as t_m. Raise Gate opening from 65.6 to 75.0 %, run on to 130 s at 5×, and export the CSV file. Read the columns `mech_MW` and `freq_Hz`.

(a) Find the lowest turbine power after the move, its time after t_m, and the dip below the starting 79.99 MW. *Checkpoint:* 75.62 MW about 0.7 s after the move, a dip of 4.38 MW, within 0.05 MW and 0.1 s.

(b) Find how long after t_m the turbine power is back above 80 MW, and how long until the frequency passes 62 Hz. *Checkpoint:* 1.4 s and 11.0 s, each within 0.1 s.

(c) Read the peak frequency on the Frequency nadir · peak tile and the frequency at 130 s, and compare them with the steady state of the previous exercise. *Checkpoint:* 62.30 Hz and 62.00 Hz, each within 0.01 Hz.

7.8 App challenge · 8 min

Load scenario 2, A proportional governor and its gain limit, at the default seed, and keep the Stability map view. The governor is proportional, $K_c = 4$, and the town's load rises 10 % at 10 s.

(a) Run at 1×, pause at 5 s and read the boundary printed beside your marker on the map. Run on, pause again between 30 and 31 s, and read it again. *Checkpoint:* 11.72 and 13.04, each within 0.1.

(b) Set Proportional gain (PID) to 14 at that pause and run to 600 s at 20×. From the trend or the CSV column `freq_Hz`, read the peak-to-peak swing and the time between successive peaks after 300 s. *Checkpoint:* 4.6 to 5.5 Hz, and 10.0 s within 0.2 s.

(c) Press the reload button, run at 1× to a pause between 30 and 31 s, set the gain to 13, and run to 600 s at 20×. Read the peak-to-peak swing over 160 to 180 s and over 580 to 600 s. *Checkpoint:* 0.74 Hz within 0.05 Hz, and 4.7 Hz within 0.1 Hz.

7.9 App challenge · 8 min

Load scenario 5, Load rejection: closure time against pressure and overspeed, at the default seed. The unit carries 88 MW on the island, and the breaker opens at 10 s. For each run below, run to 40 s and read the Peak pressure · peak speed tile. The teaching limits are 1.30 pu of head and 1.50 pu of speed.

(a) Run as shipped, with a single 6 s closure. *Checkpoint:* 1.79 pu and 1.37 pu, each within 0.01 pu.

(b) Set Closure time to 15 s, press Reset and run. *Checkpoint:* 1.28 pu and 1.60 pu.

(c) Set Closure time back to 6 s and Closure law to Two slopes, leaving Cushion breakpoint at 40 % and Cushion closure time at 15 s. Press Reset and run. *Checkpoint:* 1.29 pu and 1.44 pu.

Say which limit each run breaks, and why the third law meets both.

7.10 Application area · 4 min

In case 5 the penstock carries 4.19 m/s when the breaker opens. The penstock is 400 m long with a wave speed of 1000 m/s, on a static head of 100 m.

(a) Find the head rise an instant stop of that flow would cause, in meters and in per unit, and the closure time above which the rigid column applies. *Checkpoint:* 427 m, 4.27 pu, and 0.8 s.

(b) From full load no setting of the servo's one- or two-slope law keeps both teaching limits. Name three remedies a plant has, and how the choice is confirmed on the real unit. *Checkpoint:* A shaped law with more slopes, a pressure relief valve, more rotor inertia or a shorter water column, confirmed by load-rejection tests.

7.11 Application area · 4 min

In case 4 the unit carries 80 MW on a stiff grid with a permanent droop of 5 % acting on the gate position. The grid frequency falls 0.2 Hz.

(a) Find the gate move the droop asks for, and estimate the power change from the gate-to-power slope of 0.74 at 80 MW. Compare both with the measured 6.65 % of stroke and 4.65 MW, and with the 6.67 MW a dispatcher would expect from the droop. *Checkpoint:* 6.67 % of stroke and about 4.9 MW.

(b) Explain what the frequency-biased power controller changes for the dispatcher, and what it costs. *Checkpoint:* The power, not the gate, carries the share: 6.78 MW at 900 s, after a peak of 7.29 MW.

7.12 Review the agent · 5 min

The penstock of a unit like the chapter's is to be extended, which raises T_w from 2 to 4 s with $T_m = 8$ s. An AI assistant advises: "Keep the transient droop of 25 % with the 3 s reset. On the present penstock it settles a 10 % load step in 11.7 s, and its gain $1/R_t = 4$ is far below the stability map's boundary of 11.73 for this unit, so the margin covers the change. The rule's settings for 4 s would only make the governor slower." Find the flaw and say how you would check the advice before the unit runs on an island.