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CHAPTER 5

Air-Levitated Ball in a Tube Whose Height No Valve Angle Holds

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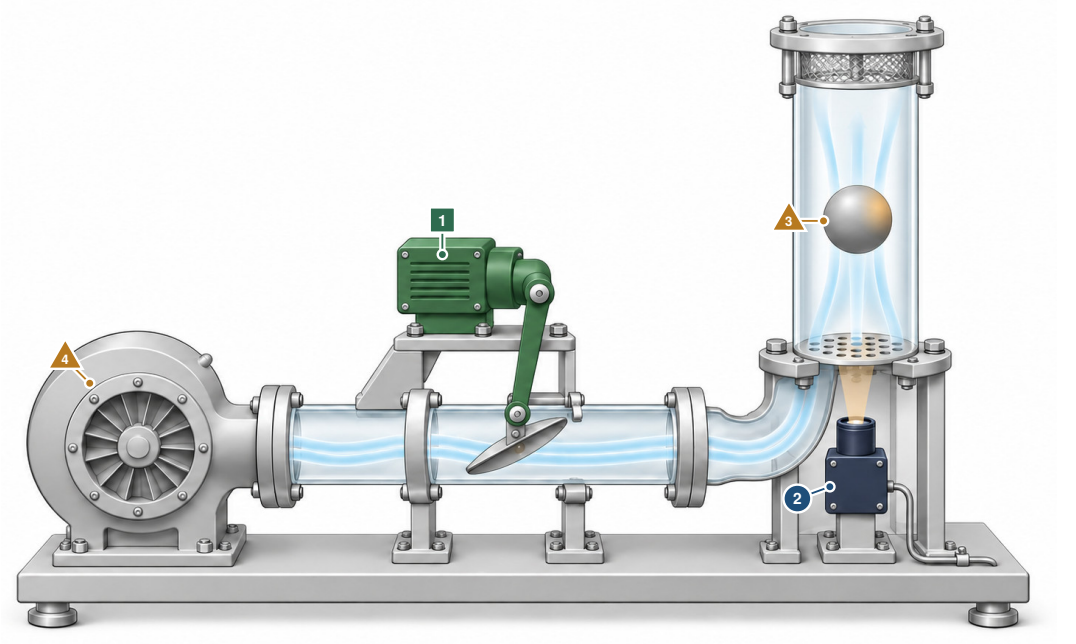
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CHAPTER 5

Air-Levitated Ball in a Tube Whose Height No Valve Angle Holds



The controller moves 1 the flap, turned by a servo, that sets how much of the blower's air reaches the ball

The controller reads 2 the light ranger below the seat that reads the ball's height

What disturbs it 3 a ball heavier or lighter than declared 4 a brief sag in the blower's supply

Notice that the blower runs at full power and the flap, not the blower's speed, sets the air that lifts the ball, which on the rig nearly fills a tube far longer and narrower than drawn. Illustration, not to scale.

IN PLAIN TERMS A Ball That Floats on a Stream of Air

The main idea. A light ball floats inside a clear upright tube on air blown from below. A controller reads the ball's height and turns a small flap that lets more or less air through.

The challenge. The air moves as fast at the top of the tube as at the bottom, so no fixed flap setting holds the ball still. Left alone, the ball drifts up to the top or down to the bottom, so the controller must keep correcting.

Why it matters. Tank levels and gas pipelines in plants drift in the same way, and the same lessons about valves and sensors apply to them.

THREE WAYS TO READ THIS CHAPTER

Explore	12 min	the first page, the fact on a rig's cost (Section 5.3), the history of a tube built for remote students (Section 5.2), the try-it on hovering by hand, the fact on a slower sensor and the try-it on the valve with no series restriction (Section 5.6) and the key points
Study	45 min	the whole chapter with its three pause points and the twelve exercises at its end
Practice	15 min	the first page, the results (Section 5.6), Technology in Practice (Section 5.8), Limitations (Section 5.9) and the open challenge

The control question A ball that nearly fills a vertical tube floats on air throttled by a servo flap, and because the air is as fast at the top of the tube as at the bottom, its height integrates any error in the valve. What do the momentum balance, a drop test and two step tests say about that integrator, and how does an orifice in series with the valve change what the loop can do?

Learning objectives

- Derive the hover speed, the ball's lag and the integrating gain from the momentum balance and the static map of the valve.
- Compute the pressure split and the hover angle of the flap with and without a series restriction, and explain why the restriction widens the usable band.
- Identify the integrating gain from a drop test and step tests in both directions, and tell a chord of the valve curve from its tangent.
- Predict the offset of a proportional loop under a load and the tuning that the dead time of a slower sensor requires.
- Separate a tuning fault from an equipment fault such as a servo dead band.

Prerequisites Sections 1.4.2 and 1.4.3 (balances and linearization), 1.4.4 (integrating processes), 1.4.6 (dead time and the half rule) and 1.4.8 (step tests and least squares). Sections 1.6.2 and 1.6.4 are needed for the controller and the cases, and Chapter 4 for a twin run beside its device.

The app https://apmonitor.com/apps/ball-levitate/

See also Volume 2, Chapter 5 for tuning an integrating loop from an identified slope, Volume 2, Chapter 6 for a gain that moves with the operating point and Volume 2, Chapter 4 for a sticking valve. Volume 4, Chapter 3 for a rate limit on an actuator command and integrators that track it at a limit.

5.1 Introduction

A ball floating on a column of air is one of the few control plants whose controlled variable can be read from across a lecture hall, and laboratories have built it from a beach ball over a blower, from a light ball in a fan-driven tube and as a shield for a microcontroller board [1–3]. The version simulated here departs from them in two ways. A 22.9 mm styrofoam ball rides in a 25.4 mm bore, a diameter ratio of 0.902, so it behaves less like a body in a stream than like a leaky piston, floating on 0.38 L/s squeezed through a 1.25 mm gap on the nominal table. And the blower runs at full power while an SG90 hobby servo throttles it with a flap, so the actuator is a valve with a seat, a curve and a dead band rather than a fan's spin-up.

The plant belongs to the class of integrating processes (Section 1.4.4). Linearized about hover, the ball's velocity follows the air's with a lag of 38 ms and its height integrates that velocity, so a valve 0.5° away from the hover angle drifts the ball at 8.4 mm/s and carries it from 200 mm to the retainer in 20 s. The level of a tank with a pump on its outlet, the pressure in a gas header and the inventory in a surge drum have the same shape, and Chapter 2 meets the same integrator when it closes the outlet valve of its tank. Integrator-plus-dead-time plants have tuning rules written for them [4, 5]. The misconception the rig corrects is that a valve set exactly right will hold the ball. Switched to Manual at the controller's last command, the twin's ball reaches its seat or its retainer within 60 s in 323 of 364 trials (case 1 below), because a command that jitters with the sensor's noise is never left at the hover angle and nothing in a uniform tube pulls a drifting ball back.

The rig also teaches a lesson of valve sizing. On the nominal table a blower that makes 295 Pa at the hover flow against a ball that needs 7.15 Pa leaves the flap to drop 97.5 % of the pressure, and the flap's whole usable band is then 1.19° wide. A fixed orifice in series takes 71 % of the pressure instead, which moves the hover angle to 23.96° and widens the usable band to 22.34°. This is the valve-authority argument of process plants, measured on a desktop. Stiction and backlash in valves are a familiar cause of oscillating loops, and data-driven models of them have been proposed [6], so the servo's dead band joins the lesson as its own case. Volume 2, Chapter 4 includes a sticking steam valve among its cases.

Each surveyed air-levitation rig whose actuator is described drives the speed or voltage of its fan or blower, none floats a ball as close to its bore, and none runs a simulation and its hardware under one controller in one page. This chapter works with a twin of a close-fitting ball behind a throttling valve. It measures on the twin the integrating behavior, the valve's usable band with and without a series restriction, and the limits that the sensor's sample period and the servo's dead band set, and it states what the physical rig should show if the twin's nominal parameters hold. The integrator, the drag law, the shared velocity-form PID and the SIMC rules are standard. The central result is the series restriction: a band that widens 18.8 times and an open-valve air speed that falls from 7.08 to 1.17 times the hover speed, measured on the twin and awaiting the bench. Section 5.2 reviews levitated-ball laboratories. Section 5.3 writes out the model with each equation and constant, and Section 5.4 linearizes it about hover, compares the valve with and without the restriction and tunes the loop. Section 5.5 describes a controller that reads only the sensor, the bench tests that identify the rig, and a hardware session whose firmware, protocol and protections were checked without hardware. Section 5.6 measures ten cases and a graded challenge, and Section 5.7 states the predictions for the hardware. Section 5.8 sets the rig's devices beside their industrial counterparts, and Sections 5.9 and 5.10 give the limitations and the conclusions.

5.2 Floating Balls in Teaching Laboratories

The early air-levitation rigs were free jets. A beach ball above a dc blower, its height tracking a reference through the blower voltage, was built as a remote laboratory for a distance course, with MATLAB/Simulink and xPC Target as the controller platform and video conferencing as the link to students at a second campus [1]. In a survey the distance students rated the remote experience as favorably as the local students, and the course grades of the two groups were similar [1]. A jet holds its own height, because the air slows as it spreads, so a ball that rises meets slower air and falls back. The comparative literature on hands-on, simulated and remote laboratories had by then argued that the three serve different objectives [7], which is the case for running a twin beside a rig rather than choosing between them.

The tube came next, in a design for teaching on the bench and over the internet that the history segment below describes [2]. Fan-driven tubes followed as low-cost builds whose sensor and fan were chosen for the purpose [8], as a ball-and-pipe setup controlled through an Arduino and a real-time Simulink toolbox with an infrared ranger [9], and as a blower-fed plant with an ultrasonic ranger on which a modified PID structure was compared with the classic one [10].

> ⧖ HISTORY **A Ball in a Tube Goes Online, 2017**
>
> A design published in 2017 for teaching on the bench and over the internet floated a light object in a methacrylate tube above a computer fan. An infrared ranger, which updates about every 40 ms, read its position, and a single-board computer closed the loop. The model was a quadratic drag on the air's speed past the ball. Because it holds a pure integrator, the plant is unstable in open loop, and a step input is likely to drop the ball or send it to the top. The designers therefore identified it in closed loop under a proportional controller with a 100 ms period and fitted a fourth-order model with an 85 % fit. A servo-driven flap under the fan served only to disturb the flow, the role this chapter's flap reverses [2].

Open-source shields then packaged the plant. FloatShield is an air-levitation device built as an Arduino shield from off-the-shelf parts and 3D-printed mounts, programmable from the Arduino IDE as well as from MATLAB and Simulink [3]. Its hardware documentation lists a 30 mm cork ball in a 35.5 mm tube, a diameter ratio of 0.85, over a 40 mm axial fan, read by a VL53L0X time-of-flight ranger, the sensor family this rig uses. It belongs to an initiative of take-home shields with a shared programming interface [11]. The Temperature Control Lab had carried the board-sized plant into heat transfer earlier, as a benchmark in which the board's cycle time, its sampling, the serial link's

overhead and the gap between model and device are deliberate features [12], and Chapter 4 runs that device beside its twin.

Control on these rigs has grown past the single PID. A laboratory plant with its own board and firmware was modeled parametrically, fitted by nonlinear least squares and controlled by optimal state feedback with a Kalman filter [13]. A disturbance-rejection design with an extended state observer, needing only the sign of the plant's control coefficient, was tested on a ball driven by a blower voltage [14], and a fan-driven rig was given a honeycomb upstream of its fan to promote laminar flow and reduce the Magnus effect on the ball [15].

The physics behind the lift is older. A quadratic drag law holds for Reynolds numbers from about 10^3 to 2×10^5 and a linear one below 1 [16], and the wall effects on a sphere moving along a tube are collected in the particle literature [17]. A ball that nearly plugs its tube sits at the edge of both. In free air the 0.30 g ball would fall at 5.38 m/s with a drag coefficient of 0.47, a ball Reynolds number near 7100, while at hover the air squeezes through the gap at 4.0 m/s, a Reynolds number of about 290 on the gap's width. The rig differs from the surveyed ones in four respects. It throttles a constant blower rather than modulating a fan. Its ball is at a diameter ratio of 0.90, against 0.85 for the shield whose dimensions are published. Its sensor looks up through the inlet from below the seat. And its twin and its hardware run under one controller object in a browser page, the TCLab pattern carried to a mechanical plant.

5.3 The Rig as Modeled

The rig is a clear polycarbonate tube 16 in tall with a 1 in bore, a 0.9 in styrofoam ball, a small blower at full power on a 5 V, 2 A supply blowing along the base into an elbow, a flap on an SG90 servo, an orifice plate, a VL53L0X time-of-flight sensor under the seat and an Arduino Leonardo, as shown in Figure 5.1. The ball's height z is the height of its underside above the seat, positive upward. Every parameter in Table 5.2 is a nominal rig awaiting bench identification. No rig has yet been weighed, calipered or flown, and the Model dialog and the Instructions page of the application both carry that label. Table 5.1 classifies the signals, and the application's Rig view draws the same parts with the ball's true and read heights, as shown in Figure 5.2.

> ✦ DID YOU KNOW? **A Levitation Rig for Under 100 Euros**
>
> The designers of an open air-levitation rig published in 2017 estimated 200 hours for its software, 60 hours for the design of its structural parts and 10 hours with under 100 euros of parts for its assembly. Their times for replication were the mean of five people given the design files [2].

5.3.1 The Ball and Its Lift

One translational degree of freedom carries the ball, and a momentum balance on it in the form of Equation (1.1) gives Equation (5.1).

$$\begin{aligned} m\dot{v} &= F_{lift}(u-v) - mg - F_{wall}(v), \qquad \dot{z} = v \\ F_{lift}(w) &= \tfrac{1}{2}\rho\, C_{eff} A_b\, w\,|w|^{n-1}, \qquad u = Q_f/A_t \end{aligned} \tag{5.1}$$

where $w = u - v$ is the air's speed past the ball, $A_b = \pi d^2/4 = 411.9\,\text{mm}^2$, $A_t = \pi D^2/4 = 506.7\,\text{mm}^2$, ρ the air density, n the lift exponent (2 by default) and C_{eff} a lift coefficient that stands for the drag coefficient divided by the square of a wall factor. With the mass change, $m = m_0(1+\Delta m/100)$. Nothing on the right depends on z. The hover speed follows from $F_{lift} = mg$ as shown in Equation (5.2).

$$u_{eq} = \left(\frac{2mg}{\rho\, C_{eff} A_b}\right)^{1/n}, \qquad \tau_v = \frac{u_{eq}}{n\,g} \tag{5.2}$$

For the 0.30 g ball that is 0.753 m/s, or 0.382 L/s, with $\tau_v = 38.4$ ms and a pressure under the ball of $mg/A_b = 7.15$ Pa. The coefficient $C_{eff} = 24$ is bracketed by two design estimates. A Newton-regime

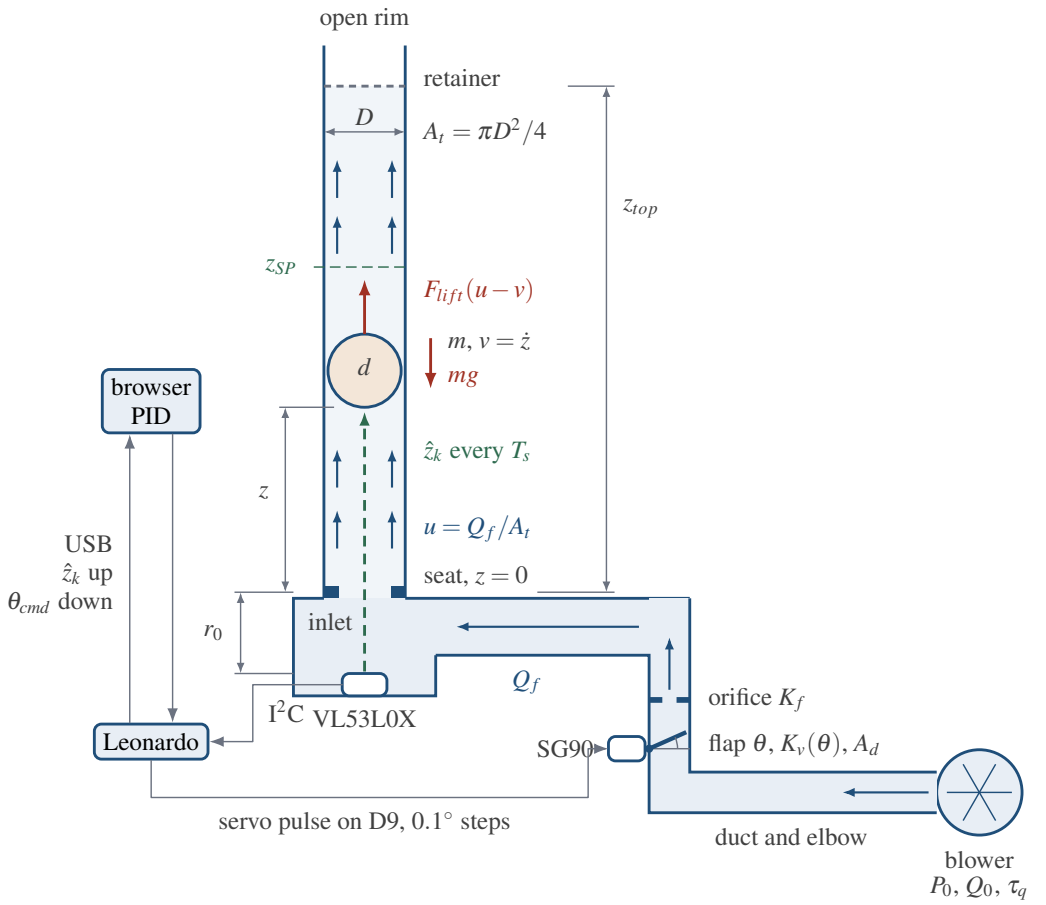


Figure 5.1 The rig as modeled, not to scale. The ball of diameter d and mass m rides in a tube of bore D with its underside at z above the seat, between the seat and a retainer that stops it at z_{top}, 10 mm below the open rim. The blower (P_0, Q_0, lag τ_q) blows along the duct into the elbow, through the flap at angle θ (resistance $K_v(\theta)$ in a duct of area A_d, seated at θ_0, open at θ_1) on the SG90's horn, then through the orifice plate K_f into the inlet below the seat. The flow Q_f rises at $u = Q_f/A_t$ and lifts the ball with $F_{lift}(u-v)$ against its weight. The VL53L0X sits r_0 below the seat and delivers $\hat{z}_k$ every T_s. The Leonardo passes readings to the PID in the browser and its valve requests to the servo.

wall factor $1-\lambda^{1.5}$ at $\lambda = d/D = 0.902$ scales the free-air terminal speed to 0.775 m/s, and the annulus treated as an orifice with a discharge coefficient of 0.65 passing 7.15 Pa gives 0.449 m/s. The correlation is used only as a bracket, so C_{eff} is a fitted parameter, and the drop test of Section 5.5 fits it. Wall friction is off by default and is the regularized Coulomb law $F_{wall} = F_c \tanh(v/v_\varepsilon) + c_v v$ with $v_\varepsilon = 10\,\text{mm/s}$ and $c_v = 1\,\text{mN s/m}$, which keeps the fourth-order Runge–Kutta stage stable at 1 ms up to 2 mN on the lightest ball.

5.3.2 Blower, Flap, Orifice and Tube as One Network

The air below the ball holds a few pascals in about a tenth of a liter, which relaxes in well under a millisecond, so the flow network is algebraic. A parabolic fan curve $P = P_0(1-(Q/Q_0)^2)$ meets the series resistances and the pressure the ball holds up, and the flow that results reaches the ball through

Table 5.1 Signals by Role

Symbol	Role	Nominal	Range
z	CV and state, true height	200 mm	0 to 370 mm
v	state, ball velocity	0	—
Q_f	state, air flow past the ball	0.382 L/s	0 to Q_0
θ	state, flap angle	24.0°	0 to 90°
$\hat{z}$	measured, delivered reading	z	1 mm steps
θ_{cmd}	MV, valve command	holds z	θ_0 to 90°
z_{SP}	setpoint	200 mm	50 to 350 mm
Δm	DV, mass change	0	−20 to 60 %
σ_n	DV, sensor noise at 33 ms	3 mm	0 to 8 mm
T_s	DV, timing budget	33 ms	20, 33, 66, 200
δ	DV, servo dead band	0	0 to 3°
F_c	DV, wall friction	0	0 to 2 mN
—	events: tap, add mass, sag, dropout	—	—
u, u_{eq}, K_i	outputs, air speed, hover speed, gain	—	—

one lumped lag, as shown in Equation (5.3).

$$\begin{aligned} Q &= Q_0\sqrt{\frac{1-\Delta P_b/P_0}{1+Q_0^2\,(K_v(\theta)+K_f+K_t)/P_0}} \\ \tau_q\dot{Q}_f &= Q-Q_f, \qquad K_v(\theta)=\frac{\rho}{2\,(C_cA_d\,a(\theta))^2} \\ a(\theta) &= a_{min}+(1-a_{min})\,\mathrm{clamp}\Big(\frac{\theta-\theta_0}{\theta_1-\theta_0},0,1\Big)^p \end{aligned} \tag{5.3}$$

where $\Delta P_b = F_{lift}/A_b$ is the ball's pressure drop (limited to $0.9P_0$ in the code), K_f the orifice, K_t the tube, C_c a discharge coefficient, A_d the duct area at the flap and $a(\theta)$ the opening fraction. The flap seats at $\theta_0 = 8°$ with 1 % leakage and opens fully at 90° as $(\Delta\theta)^{1.5}$, its inherent characteristic. A supply sag multiplies P_0 and Q_0 by 0.85 for 0.3 s. The resistances are in Pa s^2/m^6, and $K_f = 1.44\times10^9$ is a 6.1 mm hole at $C_c = 0.65$.

5.3.3 Servo

The command is rounded to the firmware's 0.1° and handed to the servo only when it moves more than the dead band from the angle the servo was last given, as shown in Equation (5.4).

$$\begin{aligned} \theta_{req} &= 0.1\,\mathrm{rnd}\big(10\,\mathrm{clamp}(\theta_{cmd},0,90)\big) \\ \theta_{held} &\leftarrow \theta_{req} \text{ only if } |\theta_{req}-\theta_{held}|>\delta \\ \dot{\theta} &= \mathrm{clamp}\big((\theta_{held}-\theta)/\tau_a,\ \pm\omega_{max}\big) \end{aligned} \tag{5.4}$$

with $\tau_a = 25$ ms and $\omega_{max} = 600°/\mathrm{s}$. The dead band δ stands for the SG90's pulse dead band and gear backlash. The firmware maps 0 to 180° onto 544 to 2400 µs, so 0.1° is about 1.03 µs of pulse and a dead band of 10 µs is about 1°.

Pause. Try Exercise 5.3 before reading on.

5.3.4 Sensor

Every T_s a ranging measurement starts, and its reading arrives one T_s later, as shown in Equation (5.5).

$$\begin{aligned} r_k &= \max\big(0,\ 10^{-3}\,\mathrm{rnd}(10^3(z(t_k)+r_0+b+\sigma\,n_k))\big) \\ \hat{z}_k &= r_k - r_0 \text{ delivered at } t_k+T_s \\ n_k &= \mathrm{e}^{-1}n_{k-1}+\sqrt{1-\mathrm{e}^{-2}}\,\xi_k \end{aligned} \tag{5.5}$$

Table 5.2 Nominal Rig Awaiting Bench Identification. V Marks a Model-Dialog Slider and H a Hidden Constant. The Provenance Column Names the Bench Measurement That Replaces Each Value

Sym.	Meaning	Value	Range	Provenance
m_0	ball mass	0.30 g	0.10–0.80	V weigh
d	ball diameter	22.9 mm	20–24	V given
D	tube bore	25.4 mm	24–32	V caliper
z_{top}	travel to retainer	370 mm	300–380	V measure
ρ	air density, Provo	1.05 kg/m^3	0.9–1.3	V site
C_{eff}	lift coefficient	24	5–60	V drop test
n	lift exponent	2	1–2	V step tests
P_0	dead-head pressure	300 Pa	50–1000	V manometer
Q_0	free delivery	3.0 L/s	0.5–10	V anemometer
τ_q	flow lag	30 ms	0–150	V step tests
A_d	duct area at flap	507 mm^2	50–600	V measure
θ_0, θ_1	seat, full open	8°, 90°	—	V hover map
p	opening exponent	1.5	1–3	V hover map
a_{min}	seated leakage	0.01	0–0.05	V drop test
K_f	orifice	1.44×10^9	0–1×10^{10}	V orifice size
C_c	discharge coefficient	0.65	0.5–0.9	V fixed
K_t	tube friction	2×10^6	—	H —
τ_a	servo lag	25 ms	5–100	V bench step
ω_{max}	servo rate	600 °/s	100–1000	V bench step
r_0	sensor below seat	40 mm	0–80	V calibration
b	residual bias	0	±10	V calibration
g	gravity	9.81 m/s^2	fixed	H —
u_{eq}, τ_v	hover speed, lag	0.753 m/s, 38.4 ms	derived	
θ_{hov}, K_i	hover angle, gain	23.96°, 16.88	derived	

where heights are in meters, $r_0 = 40$ mm is the sensor's distance below the seat, b a residual bias (0 by default), ξ_k a standard normal draw from the sensor's own random stream, seeded from the case's seed, and σ the noise of the budget in force, σ_n scaled by 5/3, 1, 1/2 and 1/3 at 20, 33, 66 and 200 ms, so the default 3 mm gives 5, 3, 1.5 and 1 mm. A raw range under 45 mm is flagged FLOOR and one within 5 mm of the retainer CEILING, and both are still delivered. A dropout event flags five readings invalid, and each repeats the last valid reading. A change of budget drops the reading in flight, so the command holds until the first reading of the new period.

5.3.5 Stops, Initial State and Numerics

The seat ($z = 0$) and the retainer ($z_{top} = 370$ mm) are perfectly inelastic stops applied after each step, and each arrival is counted. With the retainer parameter off, a ball that reaches the rim faster than 0.5 m/s is ejected and the run pauses. A run starts with the ball at rest at the case's height, the flow at the network's value and the flap at the 0.1° step nearest the true hover angle in Auto, or at the manual station in Manual. The four states are integrated by the framework's fourth-order Runge–Kutta step (Section 1.6.1) at $\Delta t = 1$ ms, the controller is called every step and runs once per delivered reading, and the historian logs at 50 Hz. Halving the step moves the challenge's IAE by 6.8×10^{-9} of itself and its peak height by 1.3×10^{-7} mm. The application passes its 49 verification checks, among them the inversion of the static map, the drop speed, the band ratio and a grep-level test that the controller's source never reads the true height, the true mass or the stop flags.

5.3.6 Assumptions

A nominal rig. The blower curve, the ball's mass, the flap's shape and the orifice are design proposals. They break down as soon as the rig is measured, and Section 5.7 gives the procedure that replaces

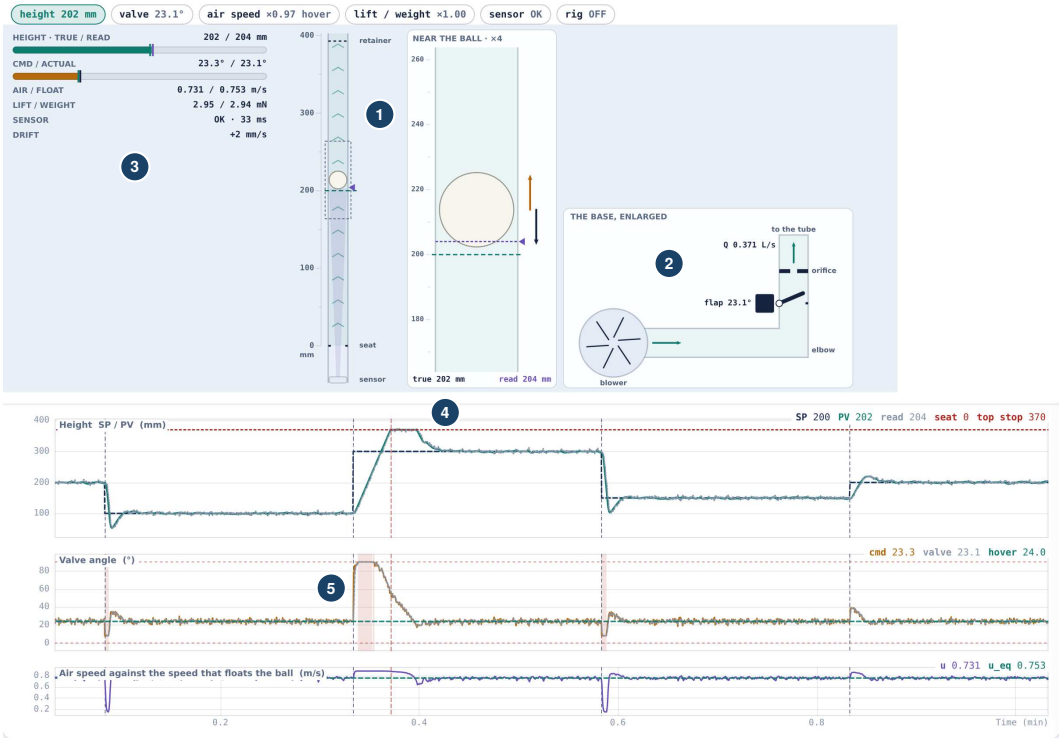


Figure 5.2 The application in case 10, *A setpoint program, on the twin and on the rig*, as shipped at $t = 62$ s on the Rig view with a 1 min trend window and no rig connected, cropped to a composite of the Rig view above the trends. (1) The tube and the zoom near the ball show the ball at a true height of 202 mm read as 204 mm against the 200 mm setpoint. (2) The enlarged base shows the flap at 23.1° and a flow of 0.371 L/s. (3) The readouts give a command of 23.3°, an air speed of 0.731 against 0.753 m/s and a lift of 2.95 against 2.94 mN. (4) The height trend shows the four steps and the rise to 300 mm reaching the retainer. (5) The command is shaded where the PID asks past the open flap during the rise, as it does past the seat after the down steps, and the air speed drops on each down step. The scorecard, cut from this crop, reads an IAE of 554.9 mm s, a valve travel of 3358° in both tiles, one retainer contact and 2.4 % of the time saturated, and its overshoot and settling tiles, 36.5 % and 9.56 s, describe the last step from 150 to 200 mm.

them.

One degree of freedom. No spin, no sideways motion and no rattling against the wall. A fan-driven rig was given a honeycomb to reduce the Magnus effect on its ball [15], a force this model leaves out.

A drag law with a fitted coefficient. At a gap Reynolds number near 290 the flow is below the quadratic range [16], so the true law may lie nearer linear, which the exponent n covers. The linearized gain ng/u_{eq} is what the fits see either way.

Quasi-static air. No pressure capacitance below the ball, one lag for the air column and the blower, a parabolic fan curve at one speed, and a flap whose opening has a fixed form. The supply sag is the only blower change.

A plain sensor. Rounding, a bias, colored noise and one sample of latency. The 25° cone of the VL53L0X is 133 mm wide at 300 mm and always sees the wall, and the cross-talk this adds beyond the bias, ambient light and range-dependent noise are not modeled.

Hard stops. The seat and the retainer absorb all energy. Temperature and humidity are constant.

5.4 An Integrator Behind a Valve

Linearized about hover in deviation variables (Section 1.4.3), Equation (5.1) gives $\tau_v\,\delta\dot{v} = \delta u - \delta v$ with $\delta\dot{z} = \delta v$, and the valve adds its slope, so the plant from valve angle to height is the transfer function of Equation (5.6).

$$G(s) = \frac{Z(s)}{\Theta(s)} \approx \frac{K_i\,\mathrm{e}^{-\theta_d s}}{s\,(\tau_v s + 1)}, \qquad K_i = \frac{1}{A_t}\frac{dQ}{d\theta}\bigg|_{\theta_{hov}} \tag{5.6}$$

where $\theta_d = T_s + \tau_a + \tau_q = 88\,\mathrm{ms}$ at the 33 ms budget lumps the sample, the servo and the flow lag as dead time (Section 1.4.6), and $K_i = 16.88\,\mathrm{mm/s}$ per degree at the hover angle of 23.96°. The coefficient $1/\tau_v = ng/u_{eq}$ is the $2g/v_{eq}$ of the fan-driven tube [2]. There is no restoring term. A valve 0.5° off hover drifts the ball at 8.4 mm/s, and even the firmware's 0.1° step nearest hover, 24.0°, drifts it upward at 0.75 mm/s.

🕮 **Pause.** Try Exercise 5.1 before reading on.

The gain is not one number, as shown in Figure 5.3. K_i is 30.4, 16.8, 5.2 and 1.0 mm/s per degree at 20, 24, 32 and 46°, because the opening grows as $(\Delta\theta)^{1.5}$ while the blower, orifice and tube take a rising share of the pressure. A ball 20 % heavier hovers at 31.14°, where K_i is 5.78, 2.92 times less, which is case 6. The usable band, defined as the travel from the hover angle to the angle that gives $1.15u_{eq}$, is 22.34° wide, from 23.96 to 46.29°. The fully open valve gives $1.17u_{eq}$, so the ball rises at 0.125 m/s at most.

5.4.1 The Series Restriction

Without the orifice the same blower must be throttled by the flap alone. At the hover flow the flap then drops 287.7 Pa of the blower's 295.1 Pa, 97.5 %, its opening is 0.0495 against the 0.0100 of a seated flap, and the hover angle falls to 17.57°, 9.57° above the seat. The band shrinks to 1.19°, about the width of the SG90's dead band, and K_i at hover rises to 92.5 mm/s per degree, 5.48 times the restricted rig's. With the orifice the plate takes 209.7 Pa (71 %) and the flap 78.0 Pa, the opening at hover is 0.095, and the open valve gives $1.17u_{eq}$ instead of $7.08u_{eq}$. The band widens 18.8 times, and the cap on the open valve's air speed also bounds how fast a windup can throw the ball at the retainer. This is the result the bench has to confirm.

🕮 **Pause.** Try Exercise 5.2 before reading on.

5.4.2 Tuning an Integrator With a Lag

The half rule of Equation (1.16) folds half of τ_v into the dead time, $\theta' = \theta_d + \tau_v/2 = 0.107\,\mathrm{s}$, and the SIMC rules for an integrating process with $\tau_c = \theta'$ give Equation (5.7) [5]. Volume 2, Chapter 5 builds the tuning of an integrating level loop from the same rules.

$$K_c = \frac{1}{K_i(\tau_c + \theta')} = \frac{1}{2K_i\theta'}, \qquad \tau_I = 4(\tau_c + \theta') = 8\theta' \tag{5.7}$$

which gives 0.276 °/mm and 0.858 s. The application adds a derivative time equal to the lag the half rule folded away, $\tau_D = \tau_v = 0.0384\,\mathrm{s}$, with a filter of $\tau_D/2$, and rounds the four to the sliders, 0.28 °/mm, 0.86 s, 0.038 s and 19 ms. On an integrating plant a PI loop overshoots a setpoint step by design, because the integral must carry the valve past the hover angle to stop the ball. With the noise off this tuning overshoots a 50 mm step by 19.9 mm and settles in 1.97 s, and a 10 mm step by 3.1 mm in 0.23 s, so an overshoot of 5 % estimated before the twin was built does not hold. On case 4's step, integral times of 0.3, 0.4 and 0.5 s give three, two and one crossings of the new setpoint, so the SIMC value is 1.7 times the 0.5 s at which the ball first crosses only once. A sensor at 200 ms makes $\theta' = 0.274\,\mathrm{s}$, and the reading's age alone adds 0.167 s, which is case 7.

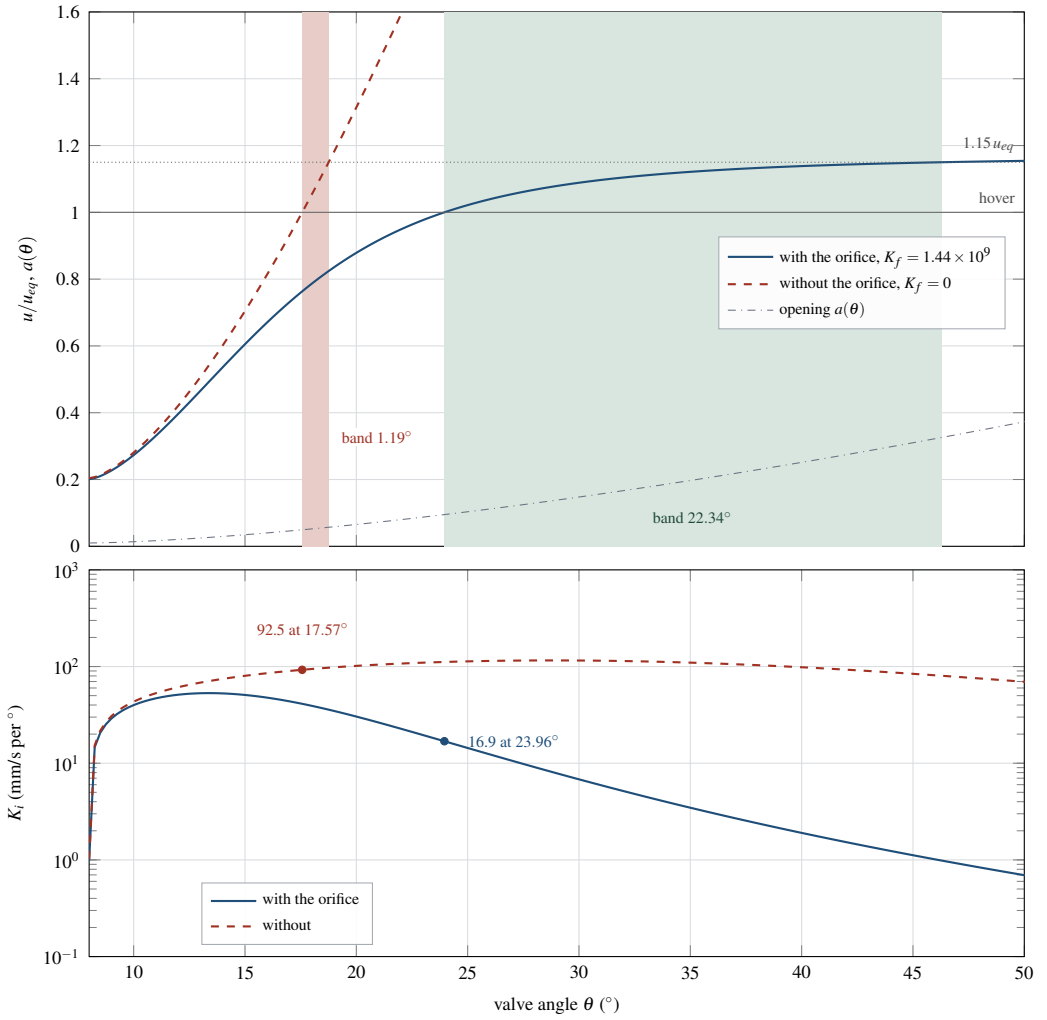


Figure 5.3 The static map of the nominal rig with the 0.30 g ball, from the application's own network functions. Top: air speed as a ratio to the hover speed with the orifice (solid) and without it (dashed), and the flap's opening $a(\theta)$. The shaded bands run from the hover angle to $1.15u_{eq}$, 22.34° with the orifice and 1.19° without. Bottom: the integrating gain $K_i(\theta)$ on a logarithmic axis, with each hover angle marked.

5.5 The Controller as Coded

The controller reads the delivered reading $\hat{z}_k$, its status, the setpoint and its own state. It never reads the true height, the true mass, the true flow or the stop flags, and a test in the application's suite reads its source to enforce that. Table 5.3 lists its settings.

5.5.1 The Shared PID Once per Reading

The engine calls the controller every 1 ms step, and the law runs only on the step at which a reading is delivered, with a period of T_s, so the sample period follows the timing budget without changing the engine's own control period. The law is the shared velocity-form PID of Section 1.6.2, written in Equation (5.8) in this chapter's symbols: $\hat{z}_k$ is the PV, θ_b the bias u_b, $\hat{u}_k$ the output v_k before the

Table 5.3 Controller Settings on the ZC Faceplate

Symbol	Setting	Default	Range
z_{SP}	height setpoint	200 mm	50 to 350
θ_{man}	manual valve station	24.0°	0 to 90, 0.1°
K_c	proportional gain	0.28 °/mm	0 to 2
τ_I	integral time, 0 is off	0.86 s	0 to 10
τ_D	derivative time	0.038 s	0 to 0.2
τ_f	derivative filter (Advanced)	19 ms	0 to 100
r_{SP}	setpoint ramp, 0 is off (Adv.)	0	0 to 300 mm/s
—	hover-angle feedforward (Adv.)	on	on, off

limits, and the action a is $+1$.

$$\begin{aligned} e_k &= z^*_{SP,k} - \hat{z}_k, \qquad P_k = K_c e_k \\ D_k &= -K_c \tau_D \left(\hat{z}_{f,k} - \hat{z}_{f,k-1}\right)/T_s \\ S_k &= S_{k-1} + (P_k - P_{k-1}) + (D_k - D_{k-1}) \\ \hat{u}_k &= \theta_b + S_k + I_{k-1}, \qquad \theta_{cmd} = \mathrm{clamp}(\hat{u}_k,\ \theta_0,\ 90) \\ I_k &= I_{k-1} + (K_c/\tau_I)\, e_k T_s + (\theta_{cmd} - \hat{u}_k)\, T_s/\tau_I \end{aligned} \tag{5.8}$$

where $\hat{z}_f$ is the reading through the exact first-order lag $\hat{z}_{f,k} = a\hat{z}_{f,k-1} + (1-a)\hat{z}_k$ with $a = \mathrm{e}^{-T_s/\tau_f}$, the gains act on errors in meters with K_c stored in degrees per meter, and the integral is updated only when $\tau_I > 0$. The action is direct, more valve for a ball below its setpoint. The back-calculation's tracking time is τ_I.

The lower limit is the flap's seat angle $\theta_0 = 8°$ of the fitted map, not 0°. Below the seat the flap is already closed, and with a limit at 0° the integral would wind through 8° of dead travel before the back-calculation acted, which deepened the overshoot of case 10's down steps until the application's technical review moved the limit. The saturation flag compares $\hat{u}_k$ with the two limits in Auto and is cleared every sample.

The bias is the static map's hover angle at a declared mass, $\theta_b = \theta_{hov}(0.30\,\mathrm{g}) = 23.955°$, computed on the parameters in force but never from the ball's true mass, so after a mass change the feedforward is wrong by a known amount and the integral finds the rest. The setpoint ramp rate-limits the setpoint the PID sees, $z^*_{SP,k} = z^*_{SP,k-1} + \mathrm{clamp}(z_{SP} - z^*_{SP,k-1}, \pm r_{SP} T_s)$, in Auto only. In Manual the PID tracks the station at every reading, so the return to Auto is bumpless (Section 1.6.2), and in Auto the station mirrors the command, which lets a change to Manual leave the valve where it was. The shared PID's slew limit is not used.

5.5.2 The Bench Tests as Helpers

Three helpers run in Manual. *Set valve to hover angle* applies θ_b rounded to 0.1°. *Step test* $\pm 2°$ needs a ball that has flown at least 1 s in Manual, steps the station and fits up to 3 s of readings before the step and 6 s after it, stopping 30 mm short of either stop, by least squares on the simulation (output) error of Section 1.4.8 with Equation (5.9).

$$y(t) = y_0 + v_0 t + K_i \Delta\theta \left[(t - t_d) - \tau\left(1 - \mathrm{e}^{-(t-t_d)/\tau}\right) \right]_{t > t_d} \tag{5.9}$$

where v_0 absorbs the drift the ball already had. For each (τ, t_d) the three linear parameters follow from linear least squares, and τ in 3 to 600 ms and t_d up to 400 ms come from a grid refined by a coordinate search. The fit reports K_i, τ, t_d and the SIMC tuning they imply with $\theta' = t_d + \tau/2$. *Drop test* opens the valve until the reading passes 300 mm, closes it and fits a straight line to the fall from 0.25 s after the close until 30 mm above the seat. With the valve closed only the seated leakage flows,

Table 5.4 Firmware Protocol, One Line per Command

Command	Reply	Meaning
`VER`	identity	Ball Levitation Firmware 1.0.0 Leonardo
`Z`	`h,s`	latest height (mm) and status
`V 24.3`	`24.3`	valve request, clamped to 10 to 90°, the angle given
`S 1/S 0`	on/off	stream `t_ms,h,s,theta,flags` per sample
`P 33`	`33`	timing budget, 20, 33, 66 or 200 ms
`L 360`	`360`	ceiling, 50 to 400 mm
`C`	`r0=41`	record the resting range as r_0 in EEPROM
`X`	`Closed`	valve closed, stream off
`A`	`Start`	clear the watchdog and ceiling latches

so $u_{eq} = v_{fall} + u_{leak}$, with $u_{leak} = 0.151\,\mathrm{m/s}$ from the fitted map, and $C_{eff} = 2mg/(\rho A_b u_{eq}^n)$ at the declared mass.

The models stay apart, as Section 1.4.9 keeps the simulation, the identified model and the controller's model apart. The simulation runs the true rig, the PID's bias and the drop test use the declared 0.30 g ball on the fitted map, the linearization of Equation (5.6) is the tuning model, and a fourth model, an integrator behind a lag about one angle, comes out of each step test.

5.5.3 The Hardware Session

The same page drives the physical rig over Web Serial from a Chromium-based browser on HTTPS or localhost, as the TCLab application does. The rig follows the app: the mode, the setpoint, the station, the tuning, the ramp and the timing budget are shared, while the rig's PID is a second instance built by the twin's factory with its own integral and filter state, its lower limit raised to the firmware's closed angle of 10°. Its loop is driven by the wall clock. Each stream line from the firmware runs the law once with the period from the firmware's own clock and sends one valve request, a newer request replacing one still waiting. On connect the PID starts in tracking with the valve closed and the ball on its seat, so the first move is the controller's, and while the twin is paused the rig holds the valve closed. Rows are logged on the twin's clock, the speed is held at 1×, and the rig's CSV carries the twin's columns without the true height and with the firmware's clock and a flag word added. The helpers run on the rig's readings as well. Table 5.4 gives the protocol at 115 200 baud.

The firmware protects the rig whatever the browser does. The servo's pulse is set to the closed angle before it is attached, so the valve starts closed, and closed means 10°, 2° short of the seat, so the servo never stalls against it. In stream mode a command must arrive at least once a second, and after 1000 ms without one the valve closes and every stream line carries `WD` until `A`, so a browser that stalls and recovers does not reopen the valve. Two valid readings in a row above the ceiling close the valve and set `CEIL`, which releases 20 mm below it, and an invalid reading neither trips nor releases the catch. The shipped image is the I^2C build, 14 244 bytes of program, 49 % of the 28 672 below the boot loader, and 541 bytes of RAM, 21 % of 2560, compiled with arduino-cli 1.5.1 on the Arduino AVR core 1.8.8 with an open-source VL53L0X library (version 1.3.1, MIT license) included unmodified. The page installs it through the Leonardo's boot loader with a write-and-verify flasher whose three transport files are copied byte for byte from the TCLab application.

5.6 Ten Cases and a Challenge, Measured

Table 5.5 sets out the cases and Table 5.6 what the engine measured. Each run was stepped outside the browser one 1 ms step at a time at the app's default seed, 20261025, with any change the text names made before Run, which gives the same run as a change carried through Reset (Section 1.6.1). Each case also carries a completion rule judged on the twin, so no reader needs a rig, and the exercises of Section 5.11 use the twin alone. Heights are the true heights. A settling time runs from the change until the ball stays within ±5 mm for at least 1 s, as the scorecard's tile reads it, valve travel sums the

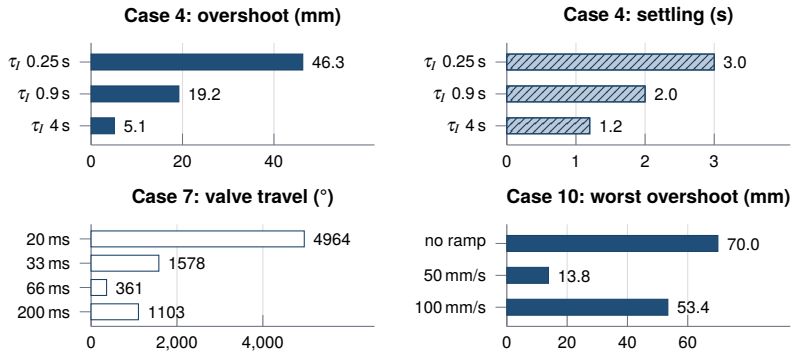


Figure 5.4 Four rows of Table 5.6 as bars: the overshoots of case 4 at three integral times and of case 10's setpoint program at three ramps (solid), the settling of case 4 (hatched) and the valve travel of case 7 at four timing budgets of the sensor (open). Notice that the longest integral time is the fastest to settle, that the 66 ms budget moves the valve least, and that a 50 mm/s ramp gives a smaller worst overshoot than no ramp or a faster one.

Table 5.5 Scenario Sequence. Every Case Starts at 200 mm Under the Suggested Tuning Unless Its Row Says Otherwise

#	Setup	Teaching point
1	Auto, 3 mm noise, switch to Manual	no angle holds
2	Manual at 24.0°, noise off, drop and step tests	u_{eq} and K_i
3	P with τ_D 0.04 s, noise off, SP 250 at 5 s, +20 % mass at 20 s	offset under load
4	τ_I 0.25 s, noise off, SP 250 at 5 s	slow ringing
5	K_c 0.80 °/mm, τ_I 1.5 s, noise off, SP 220 at 5 s, tap at 15 s	derivative as damping
6	+20 % mass, noise off, SP 220 at 5, 200 at 20 s, sag at 35 s	gain falls with the angle
7	3 mm noise, budgets 20 to 200 ms, SP 250 at 5, tap at 15 s	period against noise
8	dead band 1°, noise off	limit cycle
9	Manual, $K_f = 0$ at 17.6°, noise off, Valve view	a one-degree band
10	3 mm noise, SP 100, 300, 150, 200 at 5, 20, 35, 50 s	ramp against windup
C	P with τ_D, 3 mm noise, SP 150, 250 at 5, mass at 18, 200 at 30, tap at 45 s	everything at once

moves of the Auto command, and an IAE is the height loop's from $t = 0$ over the window named. Figure 5.4 draws four of the table's rows as bars.

5.6.1 By Hand, and Why the Valve Cannot Be Left

Case 1 opens on the ball hovering in Auto with the rig's 3 mm of noise at 33 ms, which the loop turns into a command with a standard deviation of 1.58° from 2 to 30 s. Switched to Manual at 3 s, the valve stays at 22.56°, 1.40° under hover, and the ball comes down on its seat 8.1 s later. In ten switches between 3 and 15 s the valve was left 0.04 to 3.92° from hover, and nine balls reached a stop 3.3 to 23.2 s later, while the tenth, left at 23.99°, was still flying 40 s later. Over 364 switches, one at every reading from 3 to 15 s, half reached a stop within 10.4 s, 300 within 30 s and 323 within 60 s, and the rest crept at 0.8 to 2.7 mm/s. One press of the arrow key, 0.5° the way a creeping ball went, sent each of those to a stop 14 to 20 s later.

Table 5.6 Outcomes Read From the Running Engine

Case and condition	Result
1, switch at 3 s	valve 22.56°, on its seat 8.1 s later
1, 364 switches, 3 to 15 s	median 10.4 s to a stop, 323 within 60 s
2, drop test	fall 0.592 and u_{eq} 0.744 m/s, C_{eff} 24.6
2, step tests + / −2°	K_i 14.3–14.5 / 19.6–19.8, lag 118–134 ms
3, P only, mass step	settles 25.45 mm low
3, τ_I 0.9 s	mass peak 19.9 mm, back in 2.0 s
4, τ_I 0.25 / 0.9 / 4 s	over 46.3 / 19.2 / 5.1 mm, 3.0 / 2.0 / 1.2 s
5, τ_D 0.04 / 0, noise off	travel 646 / 931°, tap 0.21 s / never
5, 3 mm, τ_D 0.04 / 0	travel 4786 / 2334° by 30 s
6, up / down / sag, shipped	2.64 / 1.75 / 2.88 s (0.30 g: 1.05 / 0.64 / 0.55)
6, twice / three times K_c	1.78 / 0.77 / 1.91 s; over 9.4 / 25.5 mm
7, 20 / 33 / 66 / 200 ms	IAE 83 / 76 / 72 / 1880, 4964° / 1578 / 361 / 1103
7, 200 ms at 0.17 °/mm	settles 8.3 s, within 8 mm for 13.4 s
8, dead band 1 / 0.5 / 0°	±2.49 / 0.85 / 0.57 mm
9, Manual at 17.6 / 17.7 / 18.1°	retainer at 58.2 / 14 / 3.5 s
9, Auto at 0.28 / 0.05 °/mm	saturated 46.9 / 0 %, error 99.1 / 1.3 mm
10, ramp 0 / 50 / 100 mm/s	worst over 70.0 / 13.8 / 53.4 mm, IAE 555 / 868 / 618
C, shipped / suggested	worst 27.9 / 5.5 mm, 3262° / 1371, fail / pass

TRY IT IN THE APP **1 · Hover by Hand (Start Here)**

Load the case and press Run. The ball hovers at 200 mm in Auto. Switch to Manual while it hovers, so the valve stays at the controller's last command, and watch the ball. Switched at 3 s, the valve stays at 22.56° and the ball comes down on its seat 8.1 s later. At other moments it may rise to the retainer or only creep.

5.6.2 A Drop and Two Steps

Case 2 hovers in Manual at 24.0° with the noise off. The drop test's fall fits 0.592 m/s where the ball truly falls at 0.601 m/s at 150 mm, and with the leakage it gives $u_{eq} = 0.744\,\mathrm{m/s}$ and $C_{eff} = 24.6$, within 1.3 % and 2.6 % of the twin's 0.753 and 24. Over three step instants the +2° step fits 14.3 to 14.5 and the −2° step 19.6 to 19.8 mm/s per degree, chords on either side of the hover angle, and their mean, 17.0, lies within 1 % of the tangent, 16.9. The total lag $\tau + t_d$ fits 118 to 134 ms against the 126 ms of $T_s + \tau_a + \tau_q + \tau_v$, and the reading's 1 mm steps decide how it splits between lag and dead time. With 3 mm of noise over ten seeds in both directions the fitted gain stays within 12 % of its chord while the lag ranges from 67 to 210 ms, so only the gain is identified on a noisy sensor.

5.6.3 Proportional, Then Integral

Case 3 runs 0.28 °/mm with $\tau_D = 0.04$ s and no integral. At 19 s the ball is at 250.4 mm, since the plant's own integrator removes a setpoint offset. After the mass step it settles 25.45 mm low, because the heavier ball hovers at 31.14°, 7.19° above the feedforward, and 7.19° at 0.28 °/mm is 25.7 mm of error. With $\tau_I = 0.9$ s the mass step costs a peak of 19.9 mm and the ball is back within 5 mm 2.0 s later, while the setpoint step now overshoots 19.2 mm. Case 4 shortens the integral time to 0.25 s. After the 50 mm step the ball swings 46.3 over, 21.5 under, 13.3 over, 6.5 under and 3.7 mm over, about 1.2 s from the first to the third swing, and settles in 3.0 s. At 0.9 s it overshoots 19.2 mm once and settles in 2.0 s, and at 4 s 5.1 mm and 1.2 s, the slow cycle of two integrators in a loop giving way as the integral time grows [4].

5.6.4 Derivative, and the Noise It Buys

The ball's lag is short against the loop's dead time, so at the suggested gain the derivative changes the tap's dip by only 0.2 mm (9.0 against 8.8 mm on case 7's setup) while more than doubling the valve travel, 1578 against 691°. Case 5 therefore raises the gain to 0.80 °/mm with $\tau_I = 1.5$ s and the noise off. A tap's 9.8 mm dip is gone in 0.21 s and the valve travels 646° by 30 s, while without the derivative the loop never settles, swinging up to 6.4 mm from 20 to 30 s for 931°. With the rig's 3 mm

of noise this gain is too high either way, 15.3 and 27.6 mm off with and without the derivative, and the derivative doubles the travel, 4786 against 2334°, or 5701° with the filter off.

5.6.5 A Heavier Ball on a Flatter Valve

Case 6 starts 20 % heavy on the suggested tuning with the noise off. The 20 mm step up settles in 2.64 s, the step down in 1.75 s and a supply sag in 2.88 s, against 1.05, 0.64 and 0.55 s for the 0.30 g ball. Twice the gain restores 1.78, 0.77 and 1.91 s, while three times, close to the full ratio of 2.92, overshoots the step down by 25.5 mm against 9.4 mm, because below the hover angle the valve is steep again. Volume 2, Chapter 6 meets a gain that moves too far for one tuning and schedules it.

5.6.6 The Sensor's Bargain

Case 7 steps 50 mm at 5 s and taps at 15 s with noise of 5, 3, 1.5 and 1 mm at 20, 33, 66 and 200 ms. Over 30 s at the suggested tuning the IAE is 83, 76 and 72 mm s at the first three budgets with 4964, 1578 and 361° of valve travel, so the quieter 66 ms budget wins both. At 200 ms the loop oscillates onto its stops six times for an IAE of 1880, because the reading's age alone has grown θ' from 0.107 to 0.274 s. At 0.17 °/mm the 200 ms loop settles the step in 8.3 s after 47.5 mm of overshoot, costs 9.2 mm on the tap and holds within 8 mm for 13.4 s.

> **DID YOU KNOW? A Slower Sensor That Wins**
>
> From the simulator: in case 7 at the suggested tuning, the 66 ms timing budget beat the fastest one, 20 ms, on both counts over 30 s. Its quieter readings gave an IAE of 72 against 83 mm s and moved the valve 361° against 4964° (Table 5.6).

5.6.7 Dead Band

Case 8 gives the servo a 1° dead band and nothing else. Between 15 and 30 s the ball cycles ±2.49 mm every 1.06 s. Half the gain makes it ±6.35 mm every 4.6 s, an integral time of 2 s ±2.70 mm and twice the gain ±1.52 mm, so retuning moves the cycle around. A 0.5° dead band halves it to ±0.85 mm, and none leaves the ±0.57 mm that the firmware's 0.1° steps make. Adding 0.5 mN of wall friction to the 1° dead band turns the cycle into a stick-slip with one swing every 7.2 s.

5.6.8 No Series Restriction

Case 9 takes the orifice out and opens on the Valve view of Figure 5.3. In Manual at 17.6° the ball takes 58.2 s to reach the retainer, at 17.7° 14 s and at 18.1° 3.5 s. In Auto the shipped gain drives the valve against its seat or its open stop 46.9 % of the first 20 s and the ball swings up to 99.1 mm off its setpoint, while 0.05 °/mm, about 16.88/92.5 of the shipped gain, holds the bare rig within 1.3 mm.

> **TRY IT IN THE APP 9 · A Valve With No Series Restriction**
>
> Load the case, which opens on the Valve view with the orifice taken out, and compare the shaded band with and without it. Press Run and move the Valve angle slider a tenth of a degree at a time. On the twin, 17.6° takes the ball to the retainer in 58.2 s, 17.7° in 14 s and 18.1° in 3.5 s.

5.6.9 A Setpoint Program

Case 10 runs setpoints of 100, 300, 150 and 200 mm at 5, 20, 35 and 50 s with the rig's noise, as shown in Figure 5.2. The steps overshoot 46.9, 70.0, 46.9 and 19.2 mm, and the 200 mm rise strikes the retainer once, for an IAE of 555 mm s over 62 s. A 50 mm/s ramp keeps every overshoot to 13.8 mm or less at an IAE of 868, the tracking error the ramp costs. A 100 mm/s ramp cuts the down steps to 16.0 and 18.3 mm but leaves the rise at 53.4 mm, because the open valve lifts the ball at 0.125 m/s at most and a faster ramp still winds the integral at the open flap. Over ramps of 20, 35, 50 and 70 mm/s at the case's seed and two others the worst overshoot is 28.4 mm.

Table 5.7 Predictions of the Nominal Twin for the Bench

Measurement	Twin
first angle that lifts the ball	24° (hover 23.96°)
seat to retainer at 25 / 46 / 90°	22.75 / 3.32 / 3.00 s
band to $1.15u_{eq}$, with / without orifice	22.34° / 1.19°
drop test: fall, u_{eq} (m/s), C_{eff}	0.592, 0.744, 24.6
step tests + / −2°, K_i	14.3–14.5 / 19.6–19.8 mm/s per °
lag $\tau + t_d$, noise off / 3 mm	118–134 / 67–210 ms
case 10, overshoots, no ramp	46.9, 70.0, 46.9, 19.2 mm

5.6.10 The Challenge

The graded case runs 150, then 250 mm at 5 s, a 20 % mass step at 18 s, 200 mm at 30 s and a tap at 45 s with the rig's noise. It judges the true height every millisecond outside the 3 s after each event and the first 3 s, asking for ±10 mm, no stop contact and valve travel under 2000° by 60 s, on the case's own rig. As shipped, proportional with derivative and no integral, the ball sags 27.9 mm after the mass step (worst at 25.4 s) and the valve travels 3262°, and it fails at the case's seed and at seeds 1 to 10. The suggested configuration, PI at 0.28 °/mm with $\tau_I = 0.86$ s, no derivative and a 50 mm/s ramp, keeps the worst error to 5.5 mm at 8.2 s and the travel to 1371°, and it passes at seeds 1 to 10 with 4.6 to 8.5 mm and 1347 to 1401°. The cases' tuning with its derivative holds the band at 6.9 mm but runs the valve 3224°, and the suggested PI without the ramp passes at the case's seed with 7.0 mm and nine of ten other seeds.

5.7 The Rig Beside Its Twin

No physical rig was available when this chapter was written, so this section states predictions as predictions. The twin's numbers are what the bench procedure should find if the nominal table holds, and a mismatch beyond the acceptance limits below is a finding to report rather than a reason to tune the twin until it agrees.

5.7.1 Three Kinds of Evidence

Checks of the code, comparisons with published work and classroom use are three separate matters, and each supports different claims. Verification is the application's 49 checks, the 12 completion-rule walks, an identical trajectory for every case whether it is loaded first, from the selector or after Reset, a second implementation of the static map by hand that matches the hover angle, K_i, the bands and the leakage, and the step-halving result of Section 5.3. Agreement with the literature is limited to form: the linearization has the coefficient of the fan-driven tube [2], and the nominal hover speed lies inside the design bracket of 0.449 to 0.775 m/s. No measured rig confirms a number in this chapter. Educational use has not been measured, and no learning gain is claimed.

5.7.2 What the Bench Should Find

Table 5.7 lists the predictions. For the hover map the servo steps from 10° in 1° increments with the ball on its seat. On the twin the ball does not lift at 23° and lifts at 24°, the first step above 23.96°, drifting up at 0.75 mm/s. From the seat it reaches the retainer in 22.75 s at 25°, 12.53 s at 26°, 3.32 s at 46°, the top of the band, and 3.00 s with the valve open. The drop and step tests are the helpers of Section 5.5 run on the rig's own readings. The acceptance limits set for the bench are a hover angle within 2°, a drop speed within 15 % and a step-test K_i within 25 % of the twin's after the twin is refitted.

5.7.3 What Was Checked Without Hardware

The hardware half was checked against software stand-ins, and its 26 tests pass. The firmware's protocol, watchdog and ceiling logic are plain C compiled on the host, where the watchdog does not

trip at 1000 ms and trips at 1001 ms, and a model of the firmware used by the transport and session tests agrees with that C state for state over a 600-action random script. The transport frames lines split across chunks and keeps stream lines apart from replies, the session's connect sequence closes the valve before streaming, the rig's law sends identical commands when the twin's truth is tampered with, and the installer is exercised against a model of the boot loader, writing and verifying the image. The shipped HEX file is 40 084 bytes with SHA-256 `2ceaa311...71f4a`, the build records report it byte-identical on clean rebuilds, and the three flasher files compare equal to the TCLab application's.

5.7.4 Not Run Without a Rig

The installation from the browser on a real Leonardo, the watchdog closing the valve within 1.2 s of a pulled cable, the ceiling catch at a set height, a servo stalled by hand without a board reset, a ten-minute hover without a dropped sample, a clean disconnect mid-run and the twin-to-rig comparison of case 10 are not run. The procedure for each is recorded with the application, and no hardware number appears in this chapter.

5.8 Technology in Practice

Figure 5.5 draws the loop of Section 5.3 with the devices of the rig.

What Measures It

The ranger below the seat is an optical time-of-flight device, an echo instrument that times a pulse of light to the ball and back [18]. Equation (5.5) stands for it with a reading rounded to 1 mm, colored noise, a residual bias and one sample of latency, and a FLOOR flag marks a raw range under 45 mm. Echo instruments can be misled by structures near the beam, and the Assumptions note that the ranger's cone always sees the tube wall, which the model leaves out beyond a constant bias. No range, accuracy or response time is given here, because none was found in an open source that names no maker.

> **TECHNOLOGY** **Sensor: An Optical Time-of-Flight Ranger**
>
> An echo instrument times a wave's trip to a surface and back. Pulsed laser light does this for distances between objects, and mass-produced precision electronics have made it affordable. The target must reflect enough light, dust or vapor in the path weakens the echo, structures near the path can return false echoes, and a target too close to the head falls in a dead zone [18]. The reflective infrared ranger of one levitation rig could be affected by the color, shape and motion of the ball, and reflections from the tube folded its calibration curve back on itself until colored strips were fitted inside the tube [2].

What Moves It

The blower runs at full power and a flap throttles it, so the final element is a small damper on an electric position actuator, not a fan whose speed changes. Its opening $a(\theta)$ in Equation (5.3) seats at 8° with 1 % leakage, the tight shutoff a butterfly disk lacks without a soft seat [18]. Equation (5.4) gives the servo its lag, its rate limit and its dead band, and on a lost command stream the firmware drives the flap to its closed angle (Section 5.5).

> **TECHNOLOGY** **Final Element: A Flap Turned by a Position Servo**
>
> A butterfly valve is a disk that turns across the flow: edge-on it barely obstructs, square-on it blocks, and a tight shutoff is hard to obtain without a soft seat. Dampers, with several vanes moved together, throttle large flows of air at low pressure in furnace and boiler draft and in building ventilation. Electric actuators reduce a fast motor through gears, and one that throttles needs a sensor of its own position, such as a potentiometer, a variable differential transformer or a pulse encoder [18]. A hobby servo is a small actuator of this kind, and its gear backlash and pulse dead band are the δ of Equation (5.4).

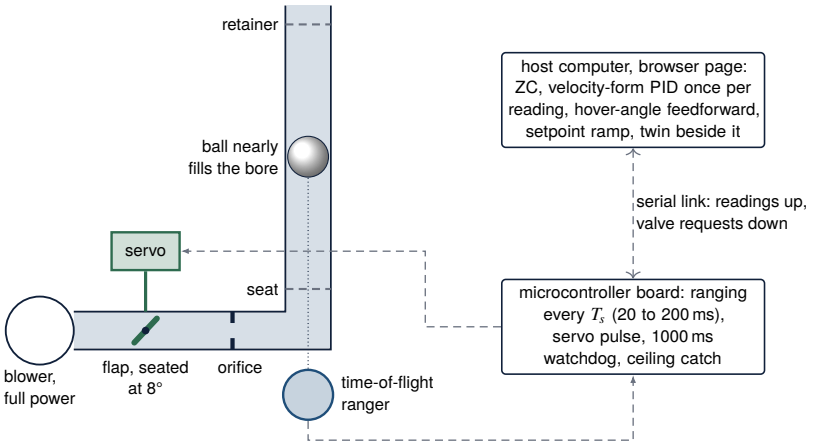


Figure 5.5 The height loop on the rig. The ranger below the seat times light to the ball and back, each reading goes from the board to the controller in the browser page, which returns one valve request, and the servo turns the flap between the full-power blower and the orifice. Notice that nothing in the uniform tube pulls a drifting ball back: the controller's only handle is the share of the blower's air that the flap and the orifice let through.

Where the Controller Runs

The control law runs in a browser page on a host computer. The microcontroller board below the tube times the ranging, sets the servo pulse and keeps its own watchdog and ceiling catch, and each reading it streams over the serial link runs the law once and returns one valve request. That sequence is the cycle time of a standards glossary, in which sensor signals are read into memory, the algorithm is executed and the actuator signals are sent [19]. Plant loops run with cycle times from milliseconds to minutes [19]. Here the cycle is the sensor's timing budget, 20 to 200 ms, while the engine steps every 1 ms, and case 7 of Section 5.6 shows how much the budget alone changes. In a plant the velocity-form PID of Equation (5.8), with its lower limit at the seat, would be the PID function block of a programmable controller or a distributed control system.

> **IN PRACTICE** **Judge a Valve by Its Installed Curve**
>
> A valve tested in a laboratory with a constant pressure drop shows its inherent characteristic. Installed, it sees less pressure drop as the flow rises, so its curve droops: steep near closed and flat near open. A loop on such a valve over-reacts at low flows, where it may oscillate, and under-reacts at high flows. The pressure drop that causes this generally cannot be removed, so the remedy is chosen in the hardware, for example a trim whose inherent curve complements the droop [18]. The orifice of this rig is a different hardware remedy for the same steep start.

What Is Modernizing It

Three developments reach this rig. The first is cheap optical ranging: mass-produced precision electronics made laser distance measurement affordable enough that consumers could buy laser tape measures in 2008 [18]. The second is open hardware: a levitation rig built as a compact shield for a microcontroller board from off-the-shelf parts and 3D-printed parts, with its design files and software published for anyone to replicate [3]. The third is the digital twin, which joins sensor data to physics-based or data-based models. About 40 % of the 31 studies in one review of the process industry combined the two kinds of model [20]. Running a twin beside its own rig under one controller is the subject of the open challenge after Section 5.9.

5.9 Limitations

Every number here is a prediction about a rig, not a measurement of one, because the parameter table is nominal until the hover map, the drop test and the step tests replace it. The lift law is a drag law with a fitted coefficient, not a derivation of the annular flow, and the ball has one degree of freedom with no spin or contact with the wall. The fan curve is a placeholder parabola at constant speed, and the flap's opening has a fixed form fitted at two points. The sensor model omits ambient light, the wall seen inside the cone beyond a constant bias and noise that changes with range, and the serial and pulse-width readers the firmware also carries are untested against a real module. The wall friction has a regularized sign, so a ball under it creeps rather than sticks outright. Every threshold is a teaching value, the ± 5 mm settled band, the challenge's ± 10 mm, 3 s windows and 2000° among them. Web Serial confines the hardware session to Chromium-based browsers on a desktop.

> **OPEN CHALLENGE** **A Twin and a Rig With No Settled Norms**
>
> A survey of control education reports that turning a classic control experiment into an interactive web laboratory is not yet an easy task, nor an art with established norms. Virtual and remote laboratories are commonly accepted to work better when offered together, yet a simulation cannot fully replace real equipment in control engineering, where the behavior of the real elements is decisive [21]. Progress would be a twin and rig pair whose agreement is measured on the bench and whose effect on learning is measured in a course.

5.10 Conclusion

The twin of this chapter floats a ball at a diameter ratio of 0.902 on 0.382 L/s of air throttled by a servo flap, and it measures what that geometry does to a loop. The height integrates the air's mismatch with a 38.4 ms lag and a gain of 16.88 mm/s per degree, so in 323 of 364 switches to Manual the ball reached a stop within a minute. An orifice in series widens the valve's usable band from 1.19 to 22.34° and caps the open valve at 1.17 times the hover speed. The SIMC tuning overshoots a 50 mm step by 19.9 mm, a ball 20 % heavier works on a valve 2.92 times flatter, a 200 ms sensor loses the ball that a 66 ms one holds, a 1° dead band produces a tuning-dependent limit cycle (± 2.49 mm at the shipped tuning, between ± 1.52 and ± 6.35 mm on retuning), and a 50 mm/s ramp holds a program's overshoots to 13.8 mm where steps reach 70 mm. None of these numbers has met the hardware yet. What remains is a weighed ball and a measured blower, the hover map, drop test and step tests on the rig, a refitted table, the twin-to-rig comparison of case 10, and a lift law tested at exponents between 1 and 2. The rig is a desktop integrator whose twin and hardware run under one controller.

KEY POINTS

- No fixed valve angle holds the ball: left in Manual at the controller's last command, it reached its seat or the retainer within a minute in 323 of 364 switches.
- The height integrates the air's mismatch with a 38.4 ms lag and a gain of 16.88 mm/s per degree at hover.
- An orifice in series widens the valve's usable band from 1.19 to 22.34° and caps the open valve at 1.17 times the hover speed.
- A 200 ms sensor loses the ball that a 66 ms one holds, and a 1° dead band produces a limit cycle that retuning only moves.

Where else this applies. Volume 2, Chapter 5 tunes an integrating level loop from an identified slope, Volume 11, Chapter 14 holds a header pressure with no self-regulation, and Volume 10, Chapter 6 meets a gas valve with no authority left when its header pressure rises.

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5.11 Exercises

5.1 Concept check · 2 min

Case 1 hovers the ball in Auto with 3 mm of sensor noise. The reader switches to Manual, and the valve stays at the controller's last command. In most trials the ball reaches its seat or the retainer within a minute. Which statement explains why?

(a) The servo's dead band leaves the flap up to a degree from the command, and the ball drifts at the speed that error gives

(b) The air speed in the uniform tube does not depend on the ball's height, so a valve off the hover angle drifts the ball and nothing pulls it back

(c) The air jet slows as the ball rises, so a ball above its hover height falls back until it comes to rest on the seat

(d) Manual mode switches off the hover-angle feedforward, so the flap closes by the 23.96° that the bias was adding

5.2 Concept check · 1 min

Case 9 takes out the orifice plate that sits in series with the flap. What does the plate do on the rig that has it?

(a) It narrows the usable band, because it takes pressure that the flap would otherwise use to set the flow

(b) It leaves the band as it is and only moves the hover angle further from the seat, to 23.96°

(c) It widens the usable band from 1.19 to 22.34° and lowers the gain at hover from 92.5 to 16.88 mm/s per degree

(d) It widens the band by raising the air speed of the fully open valve from 1.17 to 7.08 times the hover speed

5.3 Concept check · 2 min

Case 8 gives the servo a 1° dead band, and the ball cycles ± 2.49 mm every 1.06 s. An engineer halves the proportional gain to calm the loop. What does the run show?

(a) The cycle halves to about ± 1.2 mm, since a gentler loop moves the valve less at every reading

(b) The cycle stops, since the integral alone walks the command slowly to an angle inside the dead band

(c) The cycle is unchanged, since its amplitude is set by the dead band and not by the tuning

(d) The cycle grows to ± 6.35 mm every 4.6 s, since the error must double before the command crosses the dead band

5.4 Theory and calculation · 5 min

The nominal ball of Table 5.2 has $m_0 = 0.30\,\text{g}$, $d = 22.9\,\text{mm}$, $C_{eff} = 24$ and $n = 2$, in air of density $1.05\,\text{kg/m}^3$ inside a tube of bore 25.4 mm, with $g = 9.81\,\text{m/s}^2$.

(a) Find A_b, A_t, the hover speed u_{eq} of Equation (5.2) and the air flow that holds the ball. *Checkpoint:* 411.9 mm^2 and 506.7 mm^2, 0.753 m/s, 0.382 L/s.

(b) Find the lag τ_v and the pressure mg/A_b under the ball. Repeat for a ball 20 % heavier, with its new hover speed. *Checkpoint:* 38.4 ms and 7.15 Pa, then 0.825 m/s, 42.0 ms and 8.57 Pa.

(c) The integrating gain at hover is $K_i = 16.88\,\mathrm{mm/s}$ per degree. At what speed does a valve 0.5° above the hover angle lift the ball, and how long does the ball take from 200 mm to the retainer at 370 mm? *Checkpoint:* 8.4 mm/s and 20.1 s.

5.5 Theory and calculation · 8 min

Use the network of Equation (5.3) with the nominal values of Table 5.2: $P_0 = 300\,\mathrm{Pa}$, $Q_0 = 3.0\,\mathrm{L/s}$, $K_f = 1.44 \times 10^9$ and $K_t = 2 \times 10^6$ (both in Pa s^2/m^6), $C_c = 0.65$, $A_d = 507\,\mathrm{mm}^2$, $\rho = 1.05\,\mathrm{kg/m}^3$, $\theta_0 = 8°$, $\theta_1 = 90°$, $p = 1.5$ and $a_{min} = 0.01$. The ball hovers on $Q = 0.3816\,\mathrm{L/s}$ and holds up $\Delta P_b = 7.15\,\mathrm{Pa}$.

(a) Find the blower's pressure at the hover flow and the pressure drops across the orifice and the tube. *Checkpoint:* 295.1 Pa, 209.7 Pa and 0.29 Pa.

(b) Find the flap's pressure drop and its share of the blower's pressure, with the orifice and without it. *Checkpoint:* 78.0 Pa and 26 %, then 287.7 Pa and 97.5 %.

(c) Find the opening a and the hover angle in both cases. *Checkpoint:* 0.0950 and 23.96°, then 0.0495 and 17.57°.

5.6 Theory and calculation · 5 min

The tuning of Section 5.4 uses the half rule and the SIMC rules of Equation (5.7), with $\tau_a = 25\,\mathrm{ms}$, $\tau_q = 30\,\mathrm{ms}$ and $\tau_v = 38.4\,\mathrm{ms}$.

(a) Find θ_d, θ', K_c and τ_I at the 33 ms budget with $K_i = 16.88\,\mathrm{mm/s}$ per degree. *Checkpoint:* 88 ms, 0.107 s, 0.276 °/mm and 0.858 s.

(b) Repeat at the 200 ms budget. *Checkpoint:* 0.274 s, 0.108 °/mm and 2.19 s.

(c) A ball 20 % heavier hovers at 31.14°, where $K_i = 5.78\,\mathrm{mm/s}$ per degree and $\tau_v = 42.0\,\mathrm{ms}$. Find K_c at 33 ms and its ratio to the shipped 0.28 °/mm. Case 6 runs three times the shipped gain: what does it find on its step down, and why? *Checkpoint:* 0.79 °/mm, 2.8 times. An overshoot of 25.5 mm.

5.7 App challenge · 9 min

Load case 2, Drop test and step tests: the integrating gain. It hovers the ball in Manual at 24.0° with the noise off. Use the default seed and 1× speed.

(a) Press Drop test and then Run. When the fit lands, about 2 s later, read the Fall speed tile. Add the seated leakage speed of 0.151 m/s to find u_{eq}, and find C_{eff} from u_{eq} at the declared 0.30 g. *Checkpoint:* 592 mm/s within 2 mm/s, $u_{eq} = 0.744\,\mathrm{m/s}$ and $C_{eff} = 24.6$, each within 0.1.

(b) Press the reload button, press Run, and press Step test +2° between 1.5 and 3 s. Read the Fitted K_i tile when it appears, about 5 s later. Repeat from a fresh reload with Step test −2°. *Checkpoint:* 14.3 to 14.5, then 19.6 to 19.8 mm/s per degree.

(c) Compare the mean of the two fits with the live K_i that Show Model lists, and say why the two fits differ. *Checkpoint:* A mean of 17.0 against 16.88, within 1 %.

5.8 App challenge · 8 min

Load case 3, Proportional only: no offset to a setpoint, an offset under load. It runs $K_c = 0.28\,°/\mathrm{mm}$ with $\tau_D = 0.04\,\mathrm{s}$ and no integral, steps the setpoint to 250 mm at 5 s and adds 20 % to the ball's mass at 20 s, with the noise off. Run to 40 s at the default seed and export the CSV file.

(a) Read the true height, column `height_mm`, at 19 and 40 s. *Checkpoint:* 250.4 mm and 224.55 mm, each within 0.2 mm.

(b) Read `hover_deg` and `bias_deg` at 40 s. Predict the error the proportional term needs to hold the heavier ball, and compare it with (a). *Checkpoint:* 31.14° and 23.96°, an error of 25.7 mm against 25.45 mm.

(c) Press Reset, set Integral time to 0.9 s and run to 40 s again. Find the lowest true height after the mass step and read the Since the added mass tile. *Checkpoint:* 230.15 mm, a dip of 19.9 mm, settled in 1.98 s within 0.05 s.

5.9 App challenge · 8 min

Load case 7, The sensor's bargain: 20, 33, 66 and 200 ms. It steps the setpoint 50 mm at 5 s and taps the tube at 15 s, with 3 mm of noise at the 33 ms budget, under the suggested tuning. Use the default seed and 2× speed. Run to 30 s, read the IAE tile and export the CSV file. Press Reset, set Sensor timing budget (Advanced) to 66 ms and repeat. Repeat at 200 ms, and finally at 200 ms with Proportional gain 0.17 °/mm.

(a) At 33 and 66 ms, give the IAE at 30 s and the valve travel, column `valve_travel_deg` at 30 s. *Checkpoint:* 76 and 72 mm s, each within 2, and 1578 and 361°, each within 5°.

(b) At 200 ms with the shipped gain, give the IAE, the stop contacts in column `top_contacts` and the travel at 30 s, and explain them with θ'. *Checkpoint:* About 1880 mm s within 5 %, 6 contacts, 1103° within 5°.

(c) At 200 ms and 0.17 °/mm, give the IAE and the travel at 30 s. *Checkpoint:* 174 mm s within 2 and 89.5° within 2°.

5.10 Application area · 3 min

A control valve on a pumped line passes its design flow only a few percent open, because the pump makes far more head than the line needs. Case 9 is the same situation on the rig.

(a) Name two consequences for the loop that case 9 measures. *Checkpoint:* A band about one dead band wide, and a gain that bangs the valve between its stops.

(b) Name the fix the chapter applies and what it costs. *Checkpoint:* A fixed restriction in series. It burns pressure and caps the largest flow.

5.11 Application area · 4 min

A rig is built to the nominal table, and its drop test fits a fall of 0.48 m/s. The acceptance limits of Section 5.7 allow the drop speed of rig and twin to differ by 15 %.

(a) Is the fall within 15 % of the nominal twin's 0.592 m/s? What u_{eq} and C_{eff} does the helper report at the declared 0.30 g, with the seated leakage of 0.151 m/s? *Checkpoint:* No, 19 % slow. $u_{eq} = 0.631\,\mathrm{m/s}$ and $C_{eff} = 34.2$.

(b) The ball is then weighed at 0.36 g. What C_{eff} does the same fall imply, and what does a drop test identify on its own? *Checkpoint:* 41.0. Only u_{eq}, the ratio of the mass to C_{eff}.

5.12 Review the agent · 5 min

An AI assistant reads the results of case 9 and writes: "The orifice plate only rescales the gain. Without it K_i at hover is 5.48 times larger, and case 9 shows that 0.05 °/mm, about 16.88/92.5 of the shipped gain, holds the bare rig within 1.3 mm. Remove the plate, save the 209.7 Pa it burns, and divide the gain by 5.5." Find the flaw and say how you would check the claim in the app.