

Model Predictive Control with Mixed Integer Non-Linear Programming (MINLP)



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What is MINLP?

- Mixed Integer Nonlinear Programming
- Refers to the *Optimization* of an objective where its inputs are both *continuous* and *discrete* variables.
- *Continuous* – Real numbers: 1, 1.2, 1.5, 2
- *Discrete* – Integer number: 0, 1, 2, 3

Math Theory:

$$\begin{aligned} & \text{minimize } f(\mathbf{x}, \mathbf{y}) \\ & \text{with respect to } \mathbf{x}, \mathbf{y} \\ & \text{subject to } \mathbf{g}(\mathbf{x}, \mathbf{y}) \leq \mathbf{0} \\ & \quad \mathbf{h}(\mathbf{x}, \mathbf{y}) = \mathbf{0} \\ & \quad \mathbf{x} \in \mathcal{R}^n \\ & \quad \mathbf{y} \in \mathcal{Z}^m \end{aligned}$$

Applications:

- Friction Stir Welding
- Combined Heat, Cooling, and Power Plants
- Even Bench top Systems

Benchtop System: PCT40



- PCT40 is a basic process control unit produced by Armfield
- Main Devices: Large Process Vessel, CSTR, Hot water Tank
- Sensors: temperature, pressure, and level sensors
- Devices: valves, heating coils, and pumps



How does this work for MINLP Problems?

Objective: Keep the tank level at 100 mm

Continuous: Proportional Solenoid Valve (PSV)

Discrete: On/Off Valve (SOL1)



Interface - ARMConnect

- ARMConnect
 - Custom MATLAB Library that connects to the PCT40
 - Calls MEX functions (C code) that communicate with PCT40
- Full control of ALL 25 sensors and control devices
- Example

```
ARMConnect.setPSV(100); % Sets PSV to 100%
ARMConnect.getF1();    % Gets the Flowrate
ARMConnect.setSOL1(1); % Opens the SOL1 Valve (discrete)
```



Creating the Model

➤ Governing Equation – First Principle

➤ Mass Balance on the process vessel

$$A \frac{dH}{dt} = \dot{q}_{in} - \dot{q}_{out}$$

➤ Inlet Flow- Empirical

INLETFLOW(q.in) =	Empirical Quadratic Fit	+	Constant	-	Empirical Quadratic Fit	+	Disregarded
	PSV FLOW(PSV%) "-0.1566 * (psv ^ 2) + 32.635 * psv - 848.45" R ² = 0.994		SOL1 Flow 770		Interaction(SOL 1,PSV%) y = -0.0484x ² + 9.8846x - 243.62 R ² = 0.9939, x = psv Interaction = Actual - (PSV + SOL1)		HeightFactor(Height) N/A

➤ Outlet Flow – Semi-Empirical

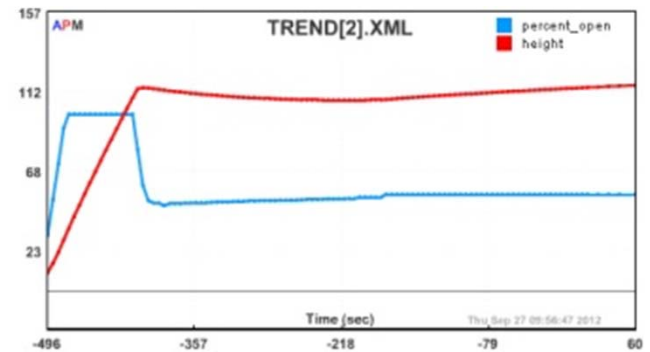
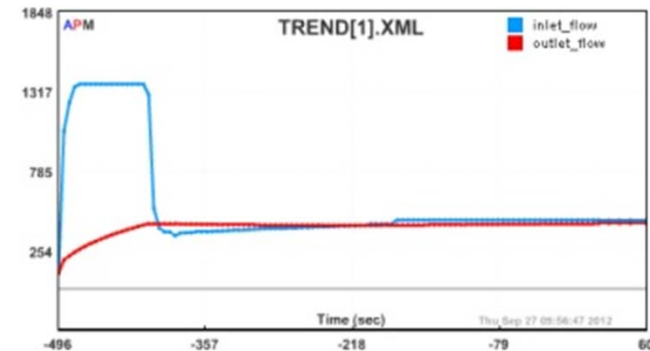
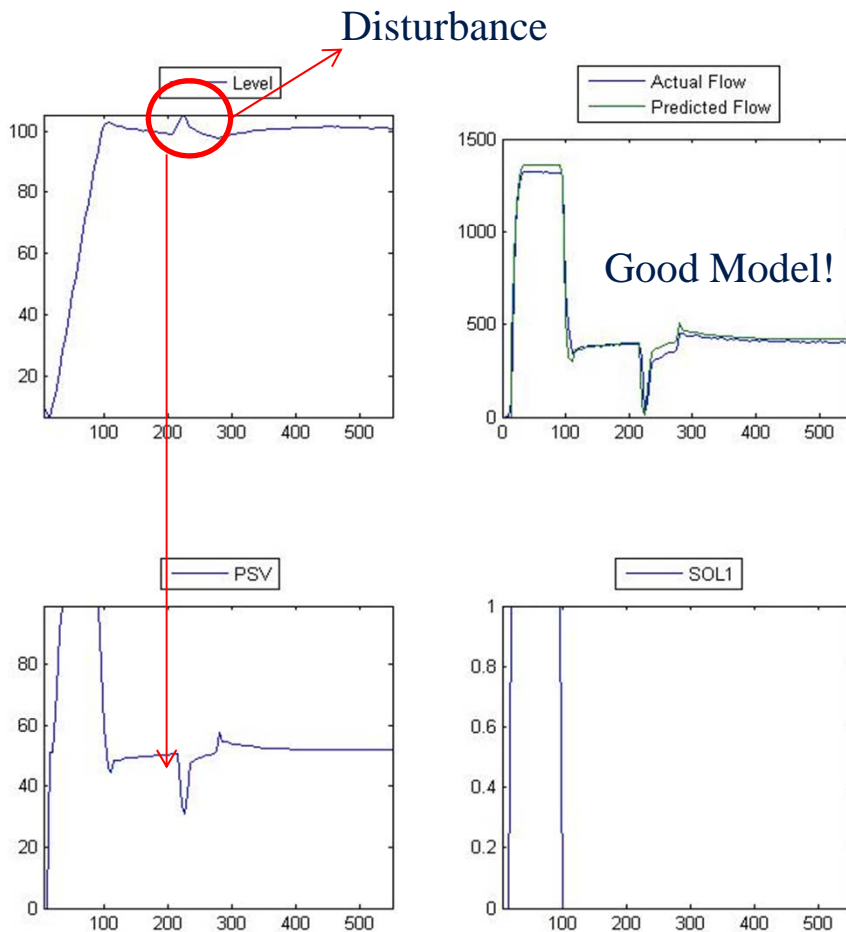
Semi Empirical- Bernoulli's

OUTLET FLOW(q.out)= 35.648*SQRT(height)+50

All model constants fit after 3 test runs.

Implemented in APM and MATLAB

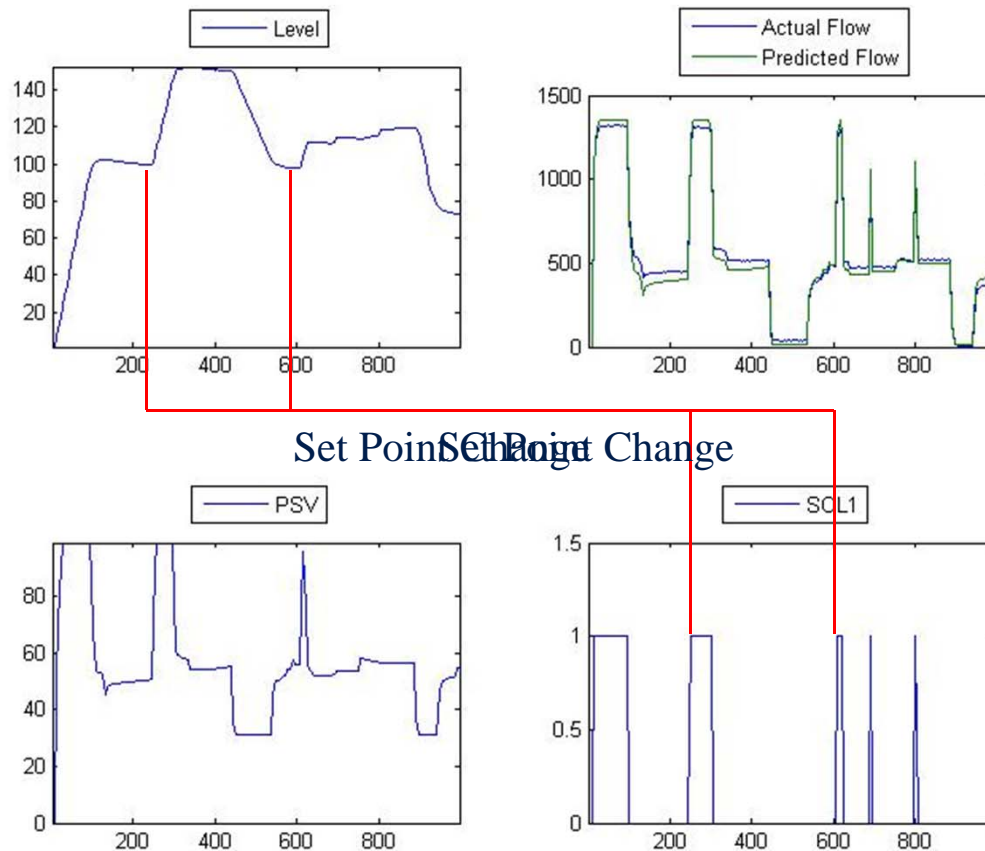
Results – Test 1



CPU Time: 1.6 – 2.4 s
Device Poll, Upload, Solve,
Download, Device Write
Solve Iterations: 4 – 7



Results – Test 2



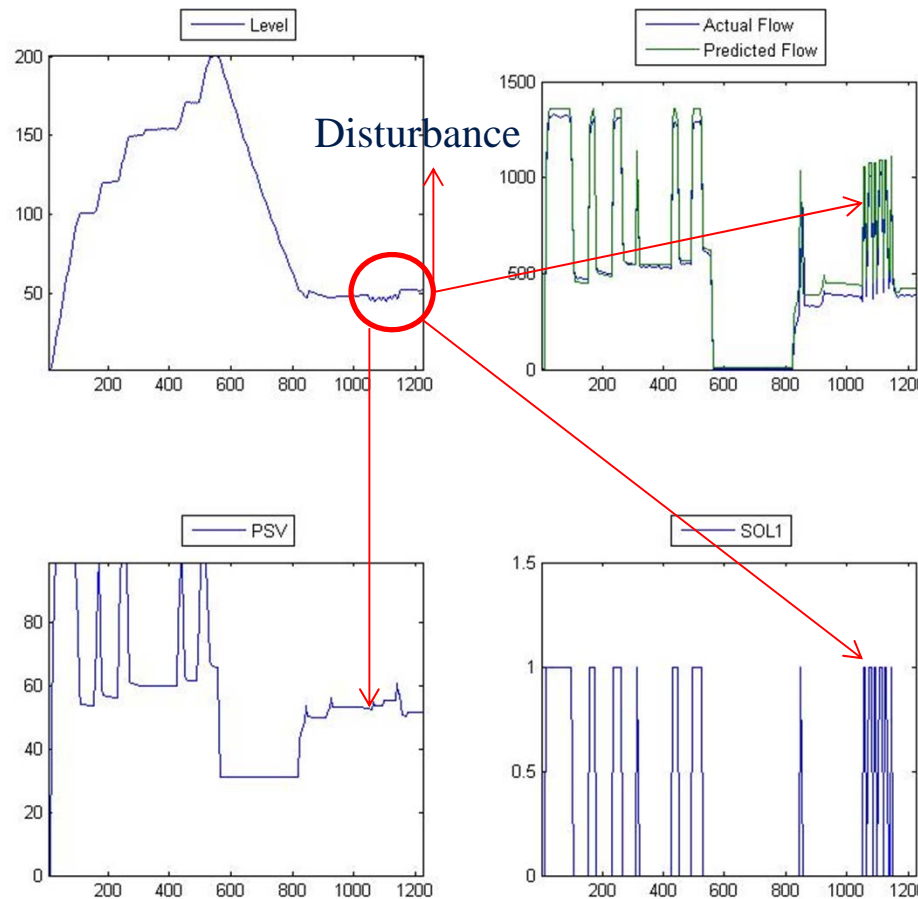
Set Point Changes

- Large (50) – ON
- Medium (20) – ON
- Small (5) – ON
- Very Small – OFF

Needs another constraint to limit SOL1 use.



Results – Test 3, Disturbance



Disturbance –
Opened up an exit
valve which is not
part of the model

Need to help the
solver learn how to
handle
disturbances.