Name $\qquad$

# Exam \#2, Practice Exam 

Chemical Engineering 436 (Section 1)<br>Professor John Hedengren<br>Closed Book, 2 Pages of Notes, Single Sided or 1 Page Front/Back

Time Limit: 50 minutes
Hints on getting more points:

1. If you do not get all the way through a problem, please indicate the exact equations and solution method that you would use.
2. More points will be given if your answers are legible and easy to read.
3. Circle or box your final answer.
4. Pace yourself with recommended times for each problem
5. The temperature control lab is a hands-on lab where an objective is to develop a transient model between the heater power and temperature. That model is used to get initial PID tuning parameters for a controller. An energy balance describes the transient temperature response of a single heater with one temperature sensor. The heater is adjusted to a power output between 0 and 1 W . The mass of the heater and temperature sensor is 2 gm with a lumped heat capacity of $500 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$. The surface area of the heater and sensor is $5.0 \mathrm{~cm}^{2}$. A convective heat transfer coefficient for quiescent air is approximately $10 \mathrm{~W} / \mathrm{m}^{2}-\mathrm{K}$. The heater and temperature sensor are assumed to be at the same temperature. You can assume that conduction is negligible
 and that the only heat transferred is through convection to the surrounding air. The heater is initially off and the heater and sensor are initially at ambient temperature.

| Quantity | Value | Value (SI Units) |
| :--- | :--- | :--- |
| Initial temperature $\left(T_{0}\right)$ | $23^{\circ} \mathrm{C}$ | 296.15 K |
| Ambient temperature $\left(T_{\infty}\right)$ | $23^{\circ} \mathrm{C}$ | 296.15 K |
| Heater Output $(Q)$ | 0 to 1 W | 0 to 1 W |
| Heat Capacity $\left(C_{p}\right)$ | $500 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ | $500 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ |
| Surface Area $(A)$ | $5.0 \mathrm{~cm}^{2}$ | $5 \times 10^{-4} \mathrm{~m}^{2}$ |
| Mass $(m)$ | 2 gm | 0.002 kg |
| Overall Heat Transfer Coefficient $(U)$ | $10 \mathrm{~W} / \mathrm{m}^{2}-\mathrm{K}$ | $10 \mathrm{~W} / \mathrm{m}^{2}-\mathrm{K}$ |

Starting with the following energy balance equation with enthalpy $(h)$ :

$$
m c_{p} \frac{d T}{d t}=U A\left(T_{\infty}-T\right)+\mathrm{Q}
$$

a) Linearize the energy balance equation and put the linearized equation into a form to find the two gains ( $K_{P I}$ and $K_{P 2}$ ) as well as the time constant $\left(\tau_{p}\right)$ for the system. The prime denotes deviation variables.

$$
\begin{gathered}
\tau_{p} \frac{d T^{\prime}}{d t}=-T^{\prime}+K_{P 1} T_{\infty}^{\prime}+K_{P 2} Q^{\prime} \\
T^{\prime}=T-T_{0} \\
T_{\infty}^{\prime}=T_{\infty}-T_{\infty_{0}} \\
Q^{\prime}=Q-Q_{0}
\end{gathered}
$$

Report the following values with the associated units:

| $\tau_{p}=$ | units: |
| :--- | :--- |
| $K_{p 1}=$ | units: |
| $K_{p 2}=$ | units: |

b) Calculate ITAE set point tracking (not IMC) PI parameters based on the gain and time constant information above. Although dead-time is zero from the derivation, assume a dead-time value of $\theta p=10 \mathrm{sec}$ so that the controller gain is not infinity.

$$
\begin{array}{ll}
K_{C}= & \text { units: } \\
\tau_{I}= & \text { units: }
\end{array}
$$

See https://apmonitor.com/pdc/index.php/Main/ProportionalIntegralControl

## ITAE Tuning Correlations

Different tuning correlations are provided for disturbance rejection (also referred to as regulatory control) and set point tracking (also referred to as servo control).

$$
\begin{aligned}
& K_{c}=\frac{0.586}{K_{p}}\left(\frac{\theta_{p}}{\tau_{p}}\right)^{-0.916} \tau_{I}=\frac{\tau_{p}}{1.03-0.165\left(\theta_{p} / \tau_{p}\right)} \quad \text { Set point tracking } \\
& K_{c}=\frac{0.859}{K_{p}}\left(\frac{\theta_{p}}{\tau_{p}}\right)^{-0.977} \tau_{I}=\frac{\tau_{p}}{0.674}\left(\frac{\theta_{p}}{\tau_{p}}\right)^{0.680} \quad \text { Disturbance rejection }
\end{aligned}
$$

2. A PI controller has the following closed loop response.

a) Calculate the controller performance with quantifiable terms. Time is shown on the plot in seconds.

Rise time: $\qquad$

Peak time: $\qquad$

Overshoot ratio: $\qquad$

Decay ratio: $\qquad$
b) Suppose that the objective is to limit the overshoot ratio to 0.10 . What changes would you suggest to the PID tuning constants $K_{c}, \tau_{I}, \tau_{D}$ (circle one for each)?
$K_{C}$ : Increase / Decrease
$\tau_{I}$ : Increase / Decrease
$\tau_{D}$ : Increase / Decrease
c) Is this controller reverse or direct acting (circle one)?

Reverse / Direct / Undetermined
3. A first-order linear system takes 5 minutes for the process response $(x)$ to get to $50 \%$ (not $63.2 \%$ ) of the way to the final value after a step change of 3 in the input $(u)$. Dead-time, $\theta_{P}$, is 2 minutes and the process gain, $K_{p}$, is 10.0 in the linear first-order equation. (Review for those who didn't get it before).
$\tau_{p} \frac{d y}{d t}=-y+K_{p} u\left(t-\theta_{p}\right)$
For a step change, $\Delta u$, the analytical solution for a first-order linear system (no dead-time) is:
$\tau_{p} \frac{d x}{d t}=-x+K_{p} u(t)$
$x(t)=K_{p}\left(1-\exp \left(-\frac{t}{\tau_{p}}\right)\right) \Delta u$
With dead-time, the solution to the step response becomes:
$y(t)=x\left(t-\theta_{p}\right) S\left(t-\theta_{p}\right)=K_{p}\left(1-\exp \left(-\frac{t-\theta_{p}}{\tau_{p}}\right)\right) \Delta u S\left(t-\theta_{p}\right)$
where $S\left(t-\theta_{p}\right)$ is a step function that changes from zero to one at $t=\theta_{p}$. In this case, $x(t)$ is the solution without dead-time and $y(t)$ is the solution with dead-time.

Given this information, what is the process time constant ( $\tau_{P}$ ) for this system?
4. A cooling fluid flows through a jacketed reactor and is controlled by a valve. The cooling fluid flow rate through the pump obeys the following relation:
$\Delta P_{\text {pump }}=10$ bar
where the pressure is in bar and the flow rate, $q$, is in $\mathrm{m}^{3} / \mathrm{min}$. The pressure drop across the system (cooling jacket and other associated plumbing) is described with the following equation:
$\Delta P_{\text {system }}=0.1 q^{2}$ bar


It is suggested that a linear valve be used. The specific gravity, $g_{s}$, of the fluid is 1.0 . Derive a relationship between volumetric flow and lift for a linear valve $f(l)=l$ for the valve installed with the pump and the system. There should only be two variables in the expression including lift $(l)$ and volumetric flow $(q)$, but not valve pressure drop $\left(\Delta P_{v}\right)$. For this part, assume $C_{v}$ is a constant.

The steady-state cooling fluid flow rate is designed to be $4 \mathrm{~m}^{3} / \mathrm{min}$ when the valve is $90 \%$ open. Determine the size of valve needed $\left(C_{v}\right)$. Units for $C_{v}$ are set to whatever makes the correlation consistent because it is an empirical parameter.
$C_{v}$ : $\qquad$

