K-T Conditions, LaGrange Multipliers

1. (10) Solve the following problem using K-T conditions:

\[ f = x_1^2 - 2x_1x_2 + 4x_2^2 \]

\[ 0.1667x_1 - x_2 = 2 \]

Plot the equality constraint on your paper and show the optimum point. Does your calculated optimum agree with a graphical optimum?

2. (10) Change the constraint to be,

\[ 0.1667x_1 - x_2 = 2.1 \]

Solve again for the optimum. Does the Lagrange multiplier from part 1 accurately predict the change in the objective for a change in the constraint right hand side? Compare the actual change to the predicted change.
3. For the problem:

$$\text{Min } f(x) = x_1^2 + x_2$$

$$g_1(x) = x_1^2 + x_2^2 - 9 = 0$$

$$g_2(x) = x_1 + x_2^2 - 1 \leq 0$$

$$g_3(x) = x_1 + x_2 - 1 \leq 0$$

A contour plot of this problem looks like:

![Contour Plot](image)


Using the K-T equations (constraints should be considered satisfied within acceptable round-off):

a. (10) Verify that the point $[-2.3723, -1.8364]$ is a local optimum

b. (10) Verify that the point $[-2.5000, -1.6583]$ is not a local optimum

c. (15) Drop the equality constraint from the problem. Using the contour plot to see where the optimum lies, solve for the optimum using the K-T conditions.