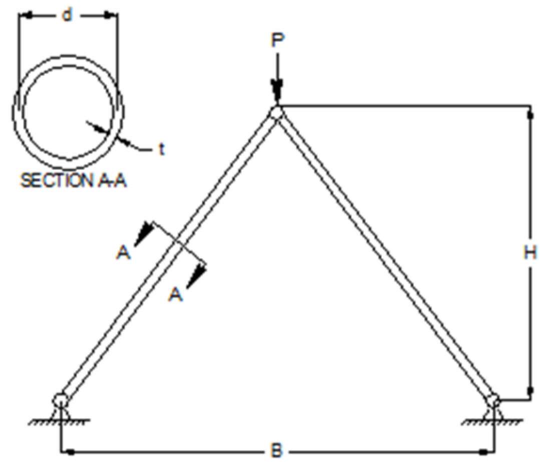
Discrete Optimization

Name______________________

We are revisiting the two-bar truss problem but with design variables allowed only at discrete values instead of continuous values. In many engineering design problems, the design variables are not continuous but are possible only at select levels known as “discrete” values. These levels may be binary (e.g. 0 or 1), integer (e.g. 5, 6, 7, 8), or at real number values (e.g. 0.05, 0.10, 0.20, 0.50). Recall that we are interested in designing a truss that has a minimum weight, will not yield, will not buckle, and does not deflect "excessively”. We created a model that calculates weight, stress, buckling stress and deflection. The model of the truss using explicit mathematical equations is shown below:



$$Weight = \rho \cdot 2 \cdot \pi \cdot d \cdot t \sqrt{\left(\frac{B}{2}\right)^2 + H^2}$$

$$Stress = \frac{P \cdot \sqrt{\left(\frac{B}{2}\right)^2 + H^2}}{2 \cdot t \cdot \pi \cdot d \cdot H}$$

$$Buckling\ \ Stress = \frac{\pi^2 E(d^2 + t^2)}{8[\left(\frac{B}{2}\right)^2 + H^2]}$$

$$Deflection = \frac{P \cdot \left[\left(\frac{B}{2}\right)^2 + H^2\right]^{(3/2)}}{2 \cdot t \cdot \pi \cdot d \cdot H^2 \cdot E}$$

Optimize the values of height, diameter, and thickness to minimize the weight of the two bar truss. The deflection must remain less than 0.25 in, the stress less than 90 ksi, and the stress minus buckling stress less than zero (i.e. no buckling).

1. Optimize the weight of the structure with the design variables allowed at any continuous value between the lower and upper bounds. Record the value of the objective and the values of the diameter, height, and thickness.

2. With the height fixed at 37.161in, generate a contour plot of the weight varying the thickness and the diameter. Include the constraint contours and indicate all of the feasible design points at the discrete values.

3. With the height, thickness, and diameter calculated, make a table that shows:
    a. All of the potential discrete combinations. This is an exhaustive search and requires a re-optimization of the height at every combination of thickness and diameter.
    b. Which combinations are feasible, infeasible, or failed to converge
    c. The objective function (weight) for each of the feasible points

    How much more weight (objective function) is required because discrete variables (versus continuous variables) are required for solving this problem?

4. Use branch and bound to obtain an optimal solution. Compare the optimal objective, computational time, and program complexity with that of exhaustive search.

Summary of Parameters and Variables

Variables (Design Variables)	**Lower**	**Allowable Values**	**Upper**
Diameter, d (in)	1.0	1.0, 1.5, 2.0, 2.5, 3.0	3.0
Height, H (in)	10	Continuous	50
Thickness, t (in)	0.05	0.05, 0.10, 0.15, 0.20	0.20
Parameters (Analysis Variables)		**Value**	
Width of separation at base, B (inches)		60.0	
Modulus of elasticity, E (1000 lbs/in^2)		30,000	
Density, ρ (lbs/in^3)		0.3	
Load (1000 lbs)		66	