# ME 575: Rocket Fin Design 

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The objective of this optimization problem is to design fins for a rocket to minimize drag and improve the performance of the of the rocket. The design of the rocket itself is already determined and only the geometry and dimensions of the fins needs to be determined. The following are values that will be necessary in the calculations and can be seen on the corresponding figure:
$L_{N}=$ length of nose
$d=$ diameter at base of nose
$d_{F}=$ diameter at front of transition
$d_{R}=$ diameter at rear of transition
$L_{T}=$ length of transition
$X_{P}=$ distance from tip of nose to front of transition
$C_{R}=$ fin root chord
$C_{T}=$ fin tip chord
$S=$ fin semispan
$L_{F}=$ length of fin mid-chord line
$R=$ radius of body at aft end
$X_{R}=$ distance between fin root leading edge and fin tip leading edge parallel to body
$X_{B}=$ distance from nose tip to fin root chord leading edge
$\theta=$ sweep angle
$N=$ number of fins


Figure 1: Rocket dimensions used to calclulate center of pressure and drag
For this design the total length of the rocket is 108 in with a nosecone of length $L_{N}=24 \mathrm{in}$. The rocket has no transition section so the diameter stays constant at $d=6.142$ in and all of the transition terms go to zero. The number of fins is set at 3 .

The four terms that can be varied are the root chord length $\left(C_{R}\right)$, tip chord length $\left(C_{T}\right)$, fin height (S), and sweep angle $(\theta)$. The sweep angle is defined as the angle of incidence of the leading edge of the fin from normal. The sweep angle is used to calculate the mid-chord line $\left(L_{F}\right)$ with the equation below.

$$
\begin{equation*}
L_{F}=\sqrt{S^{2}+\left(\frac{1}{2} C_{T}-\frac{1}{2} C_{R}+\frac{S}{\tan \theta}\right)^{2}} \tag{1}
\end{equation*}
$$

The Barrowman equations are used to calculate the stability of the rocket by finding the rocket's center of pressure. A stable rocket design should have the center of pressure below the center of gravity. The distance between the center of gravity and the center of pressure is measured in numbers of diameters, this is called the stability factor. The stability factor can be found by taking the difference of the distance from the tip of the nose to center of pressure and the distance from the nose to the center
of gravity and divide by the length of the diameter. Generally the stability factor should be greater than one. For this problem the distance from the tip of the nose to the center of gravity is 68.631 in and the design should have a stability factor of 1.58 .

The first of the Barrowman equations deal with the rocket's nose shape. For a cone shape $X_{N}=0.666 L_{N}$ and for and ogive shape: $X_{N}=0.466 L_{N}$ The rocket in this design has an ogive shaped nose cone.

The center of pressure of the rocket is found using the equation below and the value is the distance from the tip of the nose to the center of pressure.

$$
\begin{equation*}
\bar{X}=\frac{\left(C_{N}\right)_{N} X_{N}+\left(C_{N}\right)_{T} X_{T}+\left(C_{N}\right)_{F} X_{F}}{\left(C_{N}\right)_{R}} \tag{2}
\end{equation*}
$$

The rocket in the figure has a transition section so the following two equations are necessary:

$$
\begin{gather*}
\left.\left(C_{N}\right)_{T}=2\left[\left(\frac{d_{R}}{2}\right)^{2}-\left(\frac{d_{F}}{d}\right)^{2}\right)\right]  \tag{3}\\
X_{T}=X_{P}+\frac{L_{T}}{3}\left[1+\frac{1-\frac{d_{F}}{d_{R}}}{1-\left(\frac{d_{F}}{d_{R}}\right)^{2}}\right] \tag{4}
\end{gather*}
$$

However the rocket given in this design has no transition section so these two terms should be set to zero.

The following three equations are intermediate equations that are plugged into the given center of pressure equation:

$$
\begin{gather*}
\left(C_{N}\right)_{F}=\left[1+\frac{R}{S+R}\right]\left[\frac{4 N\left(\frac{S}{d}\right)^{2}}{1+\sqrt{1+\left(\frac{2 L_{F}}{C_{R}+C_{R}}\right)^{2}}}\right]  \tag{5}\\
X_{F}=X_{B}+\frac{X_{R}}{3} \frac{\left(C_{R}+2 C_{T}\right.}{\left(C_{R}+C_{T}\right.}+\frac{1}{6}\left[\left(\left(C_{R}+C_{T}\right)-\frac{\left(C_{R} C_{T}\right)}{\left(C_{R}+C_{T}\right)}\right]\right.  \tag{6}\\
\left(C_{N}\right)_{R}=\left(C_{N}\right)_{N}+\left(C_{N}\right)_{T}+\left(C_{N}\right)_{F} \tag{7}
\end{gather*}
$$

The drag caused by the fins is calculated using equations found in the article, Estimating the dynamic and aerodynamic parameters of passively controlled high power rockets for flight simulation by Simon Box, Christopher M. Bishop, and Hugh Hunt. The drag on the fins of a rocket is given by the equation:

$$
\begin{equation*}
C_{D(f)}=2 C_{f(f)}\left(1+2 \frac{T_{f}}{L_{F}}\right) \frac{4 N A_{f p}}{\pi d^{2}} \tag{8}
\end{equation*}
$$

$A_{f p}$ is the fin planform area and is given by $A_{f p}=A_{f e}+\frac{1}{2} d C_{R}$ where $A_{f e}=\frac{1}{2}\left(C_{R}+C_{T}\right) S$ $T_{f}$ is the thickness of the fins and is set at 0.118 in .
$C_{f(f)}$ is the coefficient of viscous friction and is function of the Reynolds number. The Reynolds number is calculated by $R e=\frac{\rho V L}{\mu}$ where $\rho$ is the atmospheric density, $\mu$ is the kinematic viscosity of air, V is the apparent velocity and L is the length of the mid-chord line $L_{F}$. For this problem assume $\mathrm{Re}=50000000$.

$$
\begin{equation*}
C_{f(f)}=\frac{0.074}{R e^{1 / 5}}-\frac{B}{R e} \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
B=\operatorname{Re}_{c}\left(\frac{0.074}{R e^{1 / 5}}-\frac{1.328}{\sqrt{\operatorname{Re}}}\right) \tag{10}
\end{equation*}
$$

To assure a reasonable fin design both the height and the tip chord length must be shorter than the root chord length.

