Modeling and Control of an Aquaponics System

ChEn 593R Project

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April 24, 2018

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Abstract

We simulate and optimize an aquaponics system by integrating three separate models found within the literature, namely a model of lettuce growth in a hydroponics system, of bacteria population dynamics, and of fish dynamics.

We first perform sensitivity and control experiments on the lettuce model, which essentially is the control of a hydroponics system. We found that as long as these parameters are not extreme, plant growth is fairly insensitive to chosen values. Thus we can fix both at nominal values and perform reasonable analyses. We did find, however, that nitrogen uptake was somewhat more sensitive to these values. We also, given fixed temperature and solar radiation levels, use dynamic optimization to choose the optimal fertilizer application rate that maximizes plant growth while minimizing the amount of fertilizer used. With these results, we were able to show that adding all fertilizer at the beginning of the growing horizon, which is a common practice, is nearly-equally optimal.

We then discuss the bacteria and the fish models and how to connect the three together to form a model of a complete aquaponics system. Utilizing previous results, we set temperature and solar radiation to nominal levels and sought to find–given a fixed fish tank size and a fixed planting schedule cycling between three growing beds–the optimal number of plants per growing bed and fish to stock in the tank. We found that 60 plants and 10 fish in the tank produced the best results.

1.1 Literature Review

For this project, we do not work with a physical system. Instead, we be combine existing models in the literature in order to build a computer simulation of an aquaponics system.

In order to model an aquaponics paper, we need a model of the key subcomponents of the overall system, namely a model of the growth of lettuce in a hydroponics system [1], the growth of fish in an aquaculture system [2], and the growth of bacteria which converts the nitrogen between these two systems [3]. We pulled the key papers (i.e. most cited) on each of these models with the criteria that each must include nitrogen dynamics as a part of the model, as the nitrogen dynamics are what connects the three together. To the authors' knowledge, no prior work has been done on combining these three models.

1.2 Project Overview

Chapter 2 contains a brief introduction into aquaponics. We begin our analysis in Chapter 3 by analyzing the lettuce model independently of the other two, as the lettuce model has complexities which make optimization difficult. We validate our simulation and approximations with the data from the original paper, and then we run experiments to determine optimal temperature, solar radiation, and nitrogen application levels to use in order to maximize lettuce growth. Due to difficulties in simulating and optimizing a full aquaponics system, our estimation and dynamic optimization results exist solely within this chapter dealing with the simpler hydroponics model.

We then explore how to model the full aquaponics system with the objective to set two fixed variables to maximize yield and minimize fish mortality. In Chapter 4, we discuss how to extend the lettuce model to describe a farm growing many heads of lettuce spread across multiple growing beds with staggered planting and harvest dates. In Chapter 5, we establish the bacteria growth dynamics. In Chapter 6, we establish the fish growth dynamics. In Chapter 7, we explain how to connect the lettuce farm, bacteria, and fish models together into a comprehensive system. And finally, in Chapter 8, we provide recommendations on the optimal number of fish to stock and plants per growing bed to plant in order to maximize various economic objectives.

2

Introduction

Aquaponics, simply defined, is a farming approach that combines aquaculture (raising fish in a closed environment) and hydroponics (cultivating plants in water).

In aquaculture systems, waste product removal is the main obstacle. Fish are often stocked in high densities, which leads to an inadequate oxygen supply and the buildup of ammonia, a nitrogen compound that is highly toxic to fish, especially when oxygen levels are low. The opposite problem is found in hydroponics, where nitrogen is the limiting factor of plant growth and must be continuously added to the system. In aquaponics, the excess nitrogen produced by the fish is absorbed by the plants' roots. In this symbiotic relationship, the plants receive necessary nutrients, and the fish are provided with clean water.

2.1 Advantages of Aquaponics

The following are strong advantages of using an aquaponics system as an alternative source of food:

- **Efficient Use of Water:** According to estimates by the United States Department of Agriculture, over 80 percent of the nation's consumed water is used by agriculture [4]. For a given amount of food, Aquaponics requires 90% less water to produce that food than traditional agricultural methods [5]. This, at first, may seem counter-intuitive as aquaponics is based entirely around water systems. However, in traditional agriculture, water is lost through runoff, evaporation, and drainage through the soil, whereas in aquaponics, the water is contained in a closed system and is only lost through evaporation.

- **Low Pollution:** Unlike traditional agriculture, aquaponics does not produce nutrient runoff [6]. Also, aquaponics systems do not suffer from weed outbreaks or soil-borne pests, so herbicides and pesticides are not used.

- **Local Food Source:** Unused urban areas can become productive spaces that provide food to local communities [7]. In the agricultural model, food often travels thousands of miles to its destination [8, 9], whereas aquaponics could put an abundant source of meat (fish) and produce in every neighborhood [6].

- **Low Energy Use:** Traditional agriculture requires large amounts of fuel to produce, store, and transport food [10]. In aquaponics systems where natural heat and sunlight are available, power for water pumps is the only energy requirement. Furthermore, since aquaponics are often sold in local and urban markets, less energy is required to transport the food to its final market.
- **Organic Production:** Under some state laws, since aquaponics does not require artificial fertilizer or pesticides, food grown in aquaponics systems can be sold under an organic label, allowing farmers to sell their produce at a premium.

2.2 Disadvantages of Aquaponics

Aquaponics boasts many benefits, but there are also some disadvantages. The most significant is that as a man-made system, aquaponics does not contain the buffers between components that are present in natural ecosystems. Technical failures in the energy supply can easily break the equilibrium of an aquaponics system.

2.3 Control in an Aquaponics System

Not only can aquaponics benefit from simulation and control (which can help resolve the aforementioned robustness issues), it is an environment that is well suited for automatic control. The reason is that it can be easily measured, at least compared to traditional agricultural methods.

In traditional agriculture, in order to measure the water, oxygen, nitrogen, etc. available to the plants, a soil sample must be taken, sent to a laboratory, and evaluated. This is both a time- and resource-intensive process that makes it infeasible to use such measurements in the control of a farm.

For hydroponics and aquaponics systems, however, these measurements can be taken relatively cheaply and in real time by putting a probe in the water to measure the chemical levels. Thus real-time control decisions can be implemented using real-time measurements in order to stabilize, robustify, and optimize the benefits of an aquaponics system.

3

The Lettuce Submodel

We begin with a hydroponics model, which contains only a model of growing a single head of lettuce in a water medium with nitrogen added to the water as fertilizer. This model will be connected in feedback to a simple nitrogen pool model that manages the amount of nitrogen in the water, which will be replaced later with the bacteria submodel.

3.1 The Complete Lettuce Model

The model of single head of lettuce growing in a nutrient solution (i.e. in water with nitrogen and other nutrients as used in aquaponics and hydroponics) is contained in [1]. It should be noted that the original model was designed for a head of lettuce growing in a soil medium. We do need to make the following assumptions:

- Dissolved oxygen and other macro-nutrients in the water are not a limiting factor for growth
- The lettuce is supported by a growing medium that does not limit root growth, nor allow more than soil would

The parameters of the model in [1] have been tuned to fit actual data. As we are not using a physical system, we will not perform our own estimation; instead, we will validate our model against the results contained in the original paper. State variables, manipulated variables, intermediates, and parameters for this model are defined in Tables 3.1, 3.2, 3.3, and 3.4 respectively.

The state variable in the lettuce sub-model that we care about is the dry weight w of the lettuce. The model splits w into a structural pool w_S and a non-structural pool w_G, with

$$w = w_S + w_G. \tag{3.1}$$

The equations governing dry matter accumulation in the structural and non-structural

pools are given by:

$$\frac{dw_G}{dt} = \mu_{max}\frac{w_S}{w_S + w_G}c_{q10,\mu}^{(T-20)/10}w_G, \tag{3.2}$$

$$\frac{dw_S}{dt} = \theta P_G - \frac{1}{Y_G}\frac{dW_G}{dt}. \tag{3.3}$$

Nitrogen uptake is governed by the dynamics

$$\frac{dN_{up}}{dt} = J_{max}\frac{cN}{cN + K}, \tag{3.4}$$

where J_{max} limits nitrogen uptake as a function of age (shoot size), governed by the equation

$$J_{max} = J_{max,0}e^{-\alpha w}. \tag{3.5}$$

The gross canopy photosynthesis rate is given by

$$P_G = A\left(1 - e^{-kLAI}\right)f(N_{up})\frac{\xi I(\sigma C_{CO2} - \beta)}{\xi I + \sigma C_{CO2}}. \tag{3.6}$$

The total leaf area, plant area, and leaf area index are given by

$$LA = -0.0025w^2 + 0.072w, \tag{3.7}$$

$$A = \begin{cases} 0.02w & w \leq 2.4 \\ 0.0484 & w > 2.4 \end{cases}, \tag{3.8}$$

$$LAI = LA/A. \tag{3.9}$$

Finally, $f(N_{up})$ is given by

$$f(N_{up}) = \begin{cases} 0 & (N_{up}/w) < 0.02 \\ \frac{100}{3}\left(\frac{N_{up}}{w} - 0.02\right) & 0.02 \leq (N_{up}/w) \leq 0.05 \\ 1 & (N_{up}/w) > 0.05 \end{cases}. \tag{3.10}$$

In addition to the dynamics given in [1], we wish to also include a simple nitrogen pool model. To do this, we introduce another manipulated variable, N_{add}, which allows us to add nitrogen into the nutrient pool. The nitrogen concentration is then given by the balance equation

$$\frac{dcN}{dt} = \frac{dN_{add}}{dt} - \frac{dN_{up}}{dt}. \tag{3.11}$$

Note, we can run the plant model with or without the nitrogen pool dynamics. If these dynamics are turned off, cN is treated as a manipulated variable set by the user across all time, usually at a constant value.

3.2 An Approximate Lettuce Model

The model, as given, contains many complexities that make simulation and control difficult with APMonitor through Gekko. Furthermore, the LA equations are not sensible for all regions of interest. As such, we relax some of the equations to approximate the behavior in the paper.

Variable	Initial	Units	Description
w	1	g	Total lettuce dry matter
w_G	1/3	g	Non-structural dry matter
w_S	2/3	g	Structural dry matter
cN	0	mmol	Nitrogen concentration in the nutrient solution
N_{up}	0	mmol	Cumulative nitrogen in the shoot (nitrogen uptake)

Table 3.1 State Variables for the Lettuce submodel with their default initial values, units, and descriptions.

Variable	Initial	Units	Description
T	25	deg C	Temperature
I	5×10^6	J/m^2/day	Solar radiation
N_{add}	0	mmol / day	Nitrogen added through fertilizer

Table 3.2 Manipulated Variables for the Lettuce submodel with their default initial values, units, and descriptions.

Variable	Units	Description
J_{max}	mmol/day/g DM	Limit to nitrogen uptake as a function of dry matter weight
P_G	g(CO_2)/day	Gross canopy photosynthesis rate
A	m^2	Ground cover area per plant
LA	m^2	Leaf area
LAI	-	Leaf area index
$f(N_{up})$	-	Coefficient in $[0, 1]$ controlled by nitrogen concentration in the shoot

Table 3.3 Intermediate equations for the Lettuce submodel with their units, and descriptions.

Parameter	Value	Units	Description
θ	0.68	-	Factor to convert CO_2 to dry matter
$c_{q10,\mu}$	1.6	-	The Q_{10} factor for growth
k	0.9	-	
ξ	14×10^{-6}	g(CO_2)/J	Leaf light use efficiency
β	0.36/24	g(CO_2)/m^2/day	CO_2 compensation point to account for photorespiration
σ	7.2/24	m/day	Leaf conductance to CO_2 diffusion
Y_G	0.08	-	Conversion efficiency
μ_{max}	0.01/24	1/day	Saturation growth rate at 20 deg C
C_{CO2}	0.8	g(CO_2)/m^3	CO_2 concentration in the air
$J_{max,0}$	0.0374×24	mmol/day/g DM	The value of J_{max} when $w = 0$
α	0.151	1/g DM	Coefficient of J_{max}

Table 3.4 Parameters for the Lettuce submodel with their values, units, and descriptions.

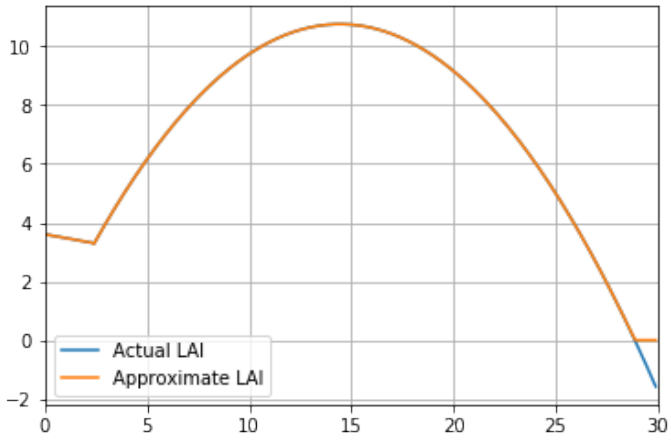


Figure 3.1 Modeled LAI in [1] as a function of w (blue) and a more realistic approximation (orange).

3.2.1 Approximating LAI

In [1], solar radiation varied between 0 at night and full intensity during the day. However, for simplicity in our early experiments, we assume that solar radiation maintains a constant intensity throughout the day, which generates a roughly double growth rate than that presented in the paper and a saturation reached at about 30 g, twice the value reached in the 30-day growing period shown in the paper.

As can be seen in Figure 3.1, as w approaches 28-29 g, LAI becomes negative, which is unreasonable. Therefore, one way to fix this equation would be to require LAI to remain positive. However, it is also unreasonable to assume that LAI goes to 0 as the plant gets too large, and so this may also be an unrealistic approximation. Another experiment, not shown here, did not allow LAI to decrease as the plant grew larger; however, such a model did not saturate and allowed the lettuce to grow infinitely, which is unreasonable.

However, LAI is only used once in the model, and that is in Equation (3.6), where a section of the function is computed with $(1 - e^{-kLAI})$. From the structure of this equation, it is reasonable to assume that e^{-kLAI} is bounded between 0 and 1. As can be seen in Figure 3.2, e^{-kLAI} grows towards infinity as w approaches 30, however, when LAI is bounded to be non-negative, e^{-kLAI} is bounded between 0 and 1 as desired.

That said, LAI is modeled as a discontinuous function, whether or not it is bounded to be non-negative. As such, e^{-kLAI} may result in severe sensitivities that make simulation, estimation, and control very difficult. To resolve these difficulties and the discussion above, we utilize a very rough approximation of e^{-kLAI} using the following logistics curve:

$$e^{-kLAI} \approx \frac{1}{1 + e^{-2(w-27.5)}}. \tag{3.12}$$

In Figure 3.3, a simulation was run where cN was set to 0.1 across all time (no nitrogen pool dynamics), which ensures that nitrogen was not limited. The original dynamics given in Section 3.1 and the approximate dynamics using e^{-kLAI} given by Equation (3.12) are compared side-by-side. The dynamics of both are nearly identical, uptaking roughly the

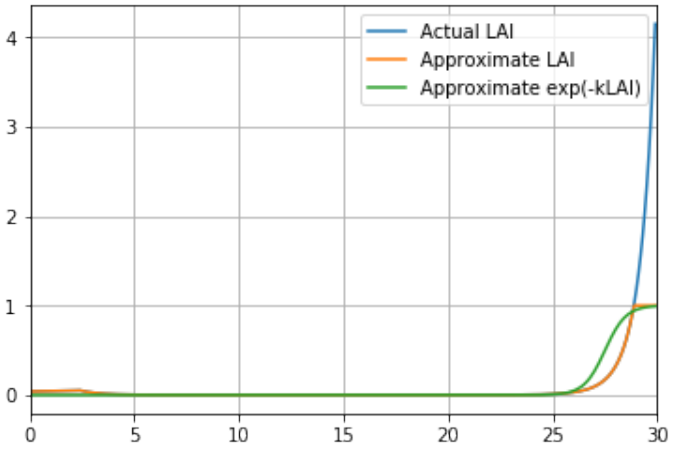
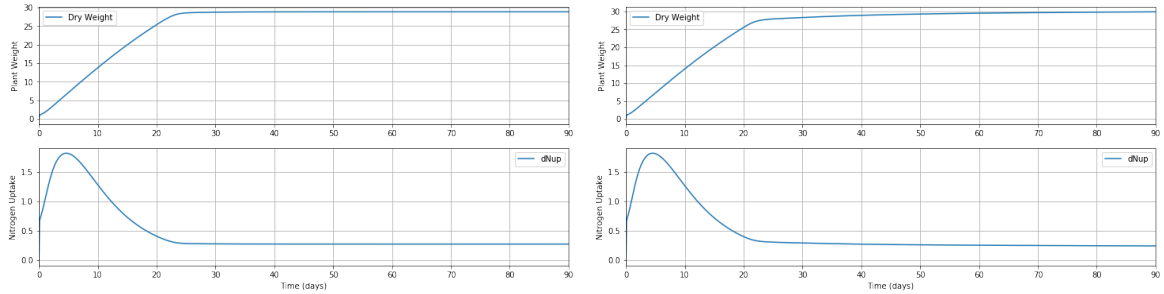


Figure 3.2 e^{-kLAI} using the modeled (blue) and approximate LAI (orange) given in Figure 3.1, and approximated using the logistics curve in Equation (3.12) (green).



(a) Dynamics with LAI and e^{-kLAI} according to Section 3.1.

(b) Dynamics with e^{-kLAI} given by Equation (3.12).

Figure 3.3 Comparison of modeled and approximate dynamics of LAI and e^{-kLAI} when the nitrogen pool cN is set to be 0.1 across all time.

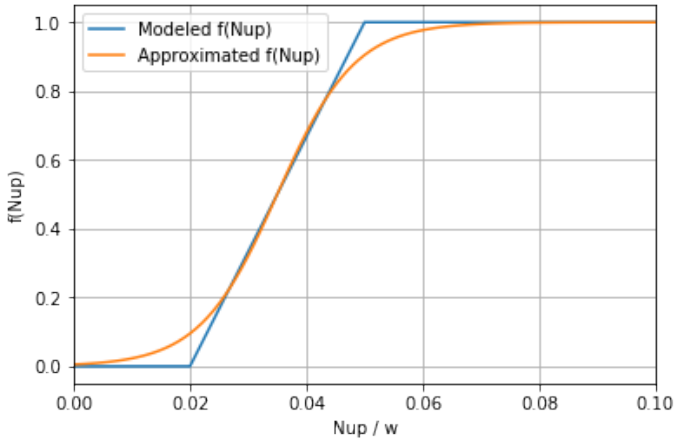


Figure 3.4 Modeled $f(N_{up})$ as given in Section 3.1 (blue) and as approximated by Equation (3.13) (orange).

same amount of nitrogen and kinking at saturation at roughly the same time. The key difference is that the convergence to saturation of the approximate dynamics after the kink is slightly slower than with the original.

3.2.2 Approximating $f(N_{up})$

In the original model, $f(N_{up})$ is given as a piece-wise-linear function. However, it is more likely that this function in nature follows a logistics curve. Furthermore, such a curve is continuous and continuously differentiable, and with a shallow slope, the second derivative shouldn't be hard to compute either, making it a more attractive function to use in the optimizer. Thus, we modify $f(N_{up})$ to the following equation:

$$f(N_{up}) = \frac{1}{1 + e^{-150(N_{up}/w - 0.035)}}. \tag{3.13}$$

As shown in Figure 3.4, these approximate dynamics fit the original very well.

3.2.3 Approximating A

Like $f(N_{up})$, the leaf area A is modeled using a discontinuous function, which we approximate with a logistics curve given by

$$A = \frac{0.2(2.4)}{1 + e^{-2(w-1.2)}}. \tag{3.14}$$

As shown in Figure 3.5, this too is approximated fairly well by a logistics curve.

3.3 Model Validation

In order to check our implementation of the model in [1], we ran empirical tests in conditions similar to those found in the original work. The largest deviation in our model to the original

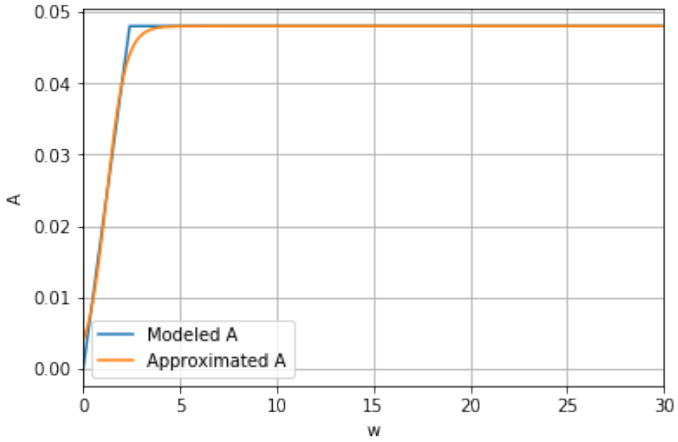


Figure 3.5 Modeled A as given in Section 3.1 (blue) and as approximated by Equation (3.14) (orange).

specifications was with respect to the source of light intensity for the growing environment. The environment of [1] utilized natural light that varied over the experimental growing period and entered into a greenhouse where the plants were grown. For the purposes of our model, we assume that our environment will be contained in a structure outfitted with lighting to replace natural light. This will enable continual growth which would enable the plants to reach their maximum sizes in a shorter amount of time.

To check our implementation against the original data given in [1], we modified our lighting source to simulate a daily cycle. This consisted of turning the lights off for a 12 hour period, followed by a period of light for another 12 hours, which repeated over the entire 90 day window. As can be seen in Figure 3.6, the growth of the plant as well as the change in nitrogen uptake are visible as the source of light follows a pattern of night and day.

In comparison of Figures 4, 5, and 6 from [1], we felt that the model accurate follows the growth patterns exhibited by the aforementioned figures. Given that the simulation under similar conditions follows that of [1], we feel comfortable utilizing this model with estimation and control.

3.4 Estimation of Lettuce Model Parameters

After validation of the model dynamics in comparison with [1], we continued development of our implementation by attempting to perform estimation of three key parameters for lettuce growth. These were $J_{max,0}$, K, and α, which are the maximum nitrogen uptake rate per shoot dry matter, the semi-saturation constant for nitrogen uptake, and a system coefficient, respectively.

The original paper for the lettuce growth model conducted a series of experiments at varying intensities of average light (quantities given in Table 3 of [1]) and then used equations within the model to solve for the parameters, which are also given in the previously mentioned table. Figure 3.7 is from [1] as well, and is where the underlying data for this

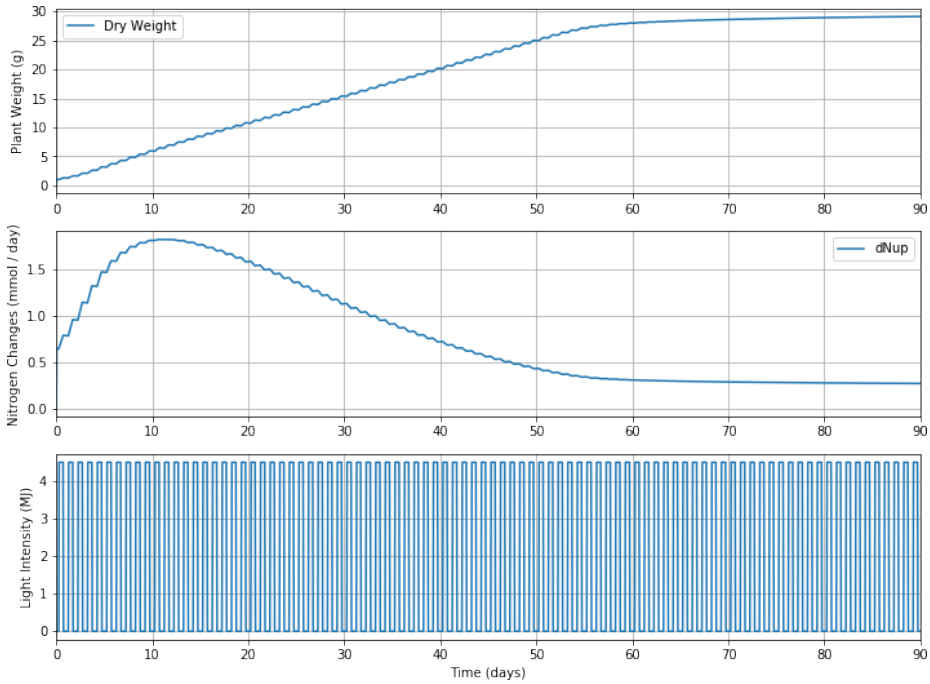


Figure 3.6 Plant model simulation using daily cycles of sunlight

estimation was sourced from. Unfortunately, since the actual data used from the original paper was not given or could be found anywhere else, we performed a rudimentary graphical analysis of this figure. This analysis consisted of calibrating a graphical tool with the axes of the figure and then fitting a computer drawn curve to the first experiment's corresponding line. Given that the alternative was sourcing the data manually by hand, we felt that this would give a much more accurate estimate of the original data set, though we would of course need the original data set to actually verify that claim.

As seen in Figure 3.7, the growth of the plants, correlated to the dry matter weight, follows the pattern of light visible to the plant beds and rises in proportion to the average intensity of the light. For our estimation, we used the light intensity corresponding to the first experiment, 7.1 megajoules per square meter per day.

Using the GEKKO library, we performed estimation by means of Moving Horizon Estimation for the three parameters of the lettuce model. Initial efforts for concurrently estimating all three parameters failed to reasonably converge, so while holding two of the parameters at a fixed value, the third parameter was estimated. The initial values used were those given by the Table 3 of the source paper (3.7), and then subsequent estimated values from the moving horizon estimator implemented via GEKKO. The values that we then converged on can be seen in Table 3.5.

The results of the system estimation show that the model was insensitive to the parameters being estimated. As can be seen in Figure 3.8, the output of the model is compared to the actual data sourced from [1]. The results seen here remained the same or very similar with either no, some, or complete use of the estimated parameters derived from the process above. We note that utilizing control of the nitrogen input of the model allows for tracking

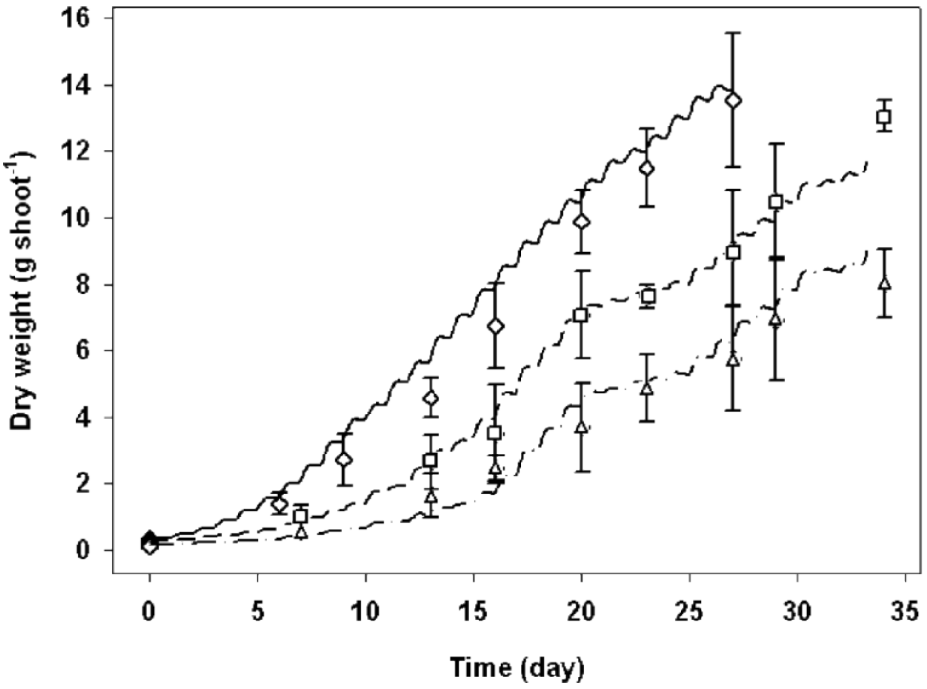


Figure 3.7 Original figure from [1] of Nitrogen uptake throughout lettuce growth under various experimental conditions

Parameter	Final value	Units
$J_{max,0}$	$0.0505 \cdot 24$	mmol h^{-1} g^{-1} DM
K	0.04	mM
α	0.161	g^{-1} DM

Table 3.5 Estimated parameters for the lettuce growth model.

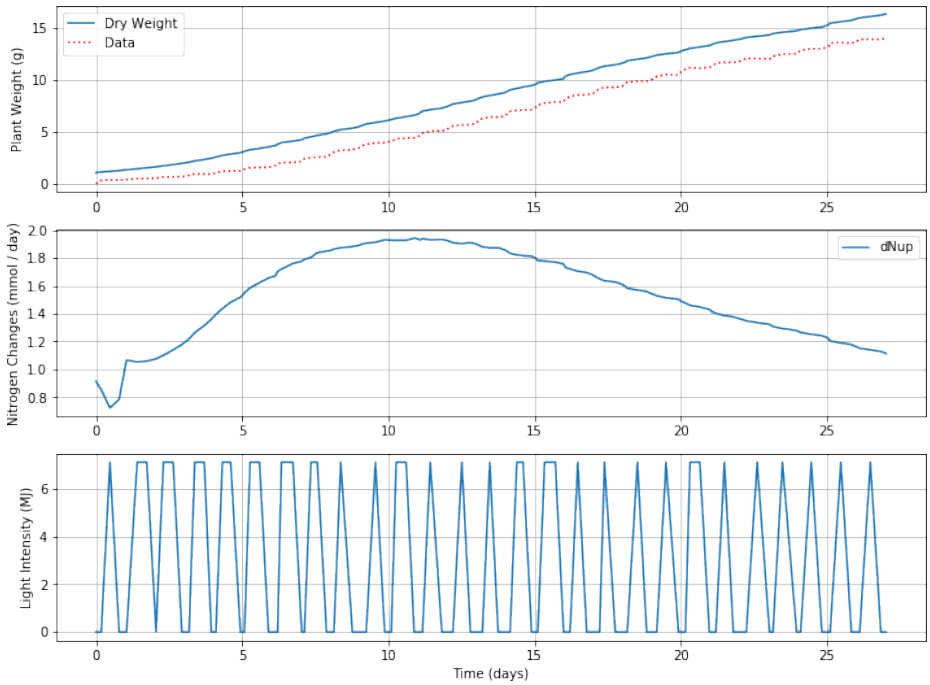


Figure 3.8 Results of the parameter estimation for lettuce growth using 7.1 MJ for the average light intensity

the original data completely.

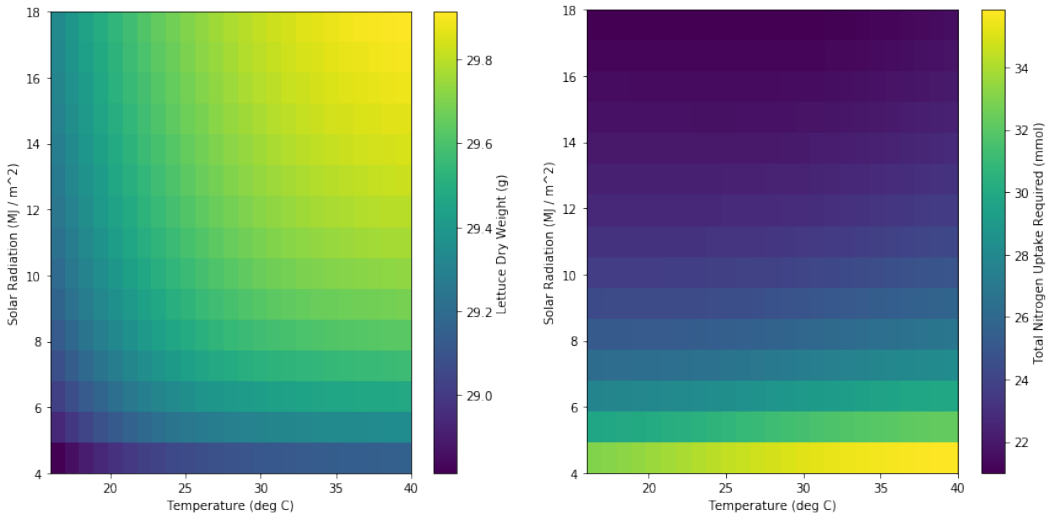
3.5 Controlling the Lettuce Model

We now seek to choose nitrogen, temperature, and/or solar radiation in order to control lettuce growth. As will be shown, the lettuce will grow until saturation unless limited, and so the control usually takes the form of manipulating these variables as little as possible in order to not limit growth.

Note that these controls are *optimal* in the sense that, assuming that the models are perfect, they maximize economic potential as much as possible. However, they are not *robust* in the sense that if the model is wrong (and it likely will be), then considerable performance loss may be seen. A common practice employed by farmers growing many different crops in many different mediums in order to increase robustness is to apply much more nitrogen than is needed to ensure saturation is reached.

3.5.1 Controlling Temperature and Solar Radiation

One question we may ask is, assuming the lettuce is not nitrogen limited, what is the optimal temperature and solar radiation profile over time in order to maximize yield. To do this, we set the model so that the plant is able to uptake as much nitrogen as it wants whenever it wants (which is done by setting the nitrogen concentration in the water to a constant that is larger than the maximum uptake the plant sees at any time).



(a) Gridded Plant Weight. (b) Gridded Nitrogen Uptake.

Figure 3.9 Plant weight (left) and total nitrogen uptake (right) as a function of constant temperatures and solar radiations over time.

Unfortunately, adding a degree of freedom to either temperature or solar radiation, even when upper and lower limits are provided along with costs to changing either parameter, causes the optimizer to wander into infeasible regions from which it can never return, preventing a complete answer of this question.

Therefore, as a proxy, we first choose to hold temperature and solar radiation constant across the entire horizon (again, this may be unrealistic as natural and artificial lighting tends to follow a night/day cycle, which is needed by the lettuce but not modeled here). We then grid these temperature and solar radiation choices across reasonable values, run a simulation over 45 days, and measure the final weight of lettuce to see where the optimum lies.

As shown in Figure 3.9, the lettuce will increase in weight as both temperature and solar radiation increase, with the maximum found at the maximum allowed temperature and solar radiation. However, the difference between the maximum lettuce weight and the minimum lettuce weight in this range is 1.1 grams, or 3.8%, small enough to be considered negligible.

However, if we look at the total nitrogen uptake over the same grid points (see the right side of Figure 3.9), we see that nitrogen uptake is more sensitive to temperature and solar radiation than lettuce weight, and increases as solar radiation decreases but temperature increases, though is far less sensitive to temperature than solar radiation.

In summary, yield (plant weight) remains fairly constant across temperature and solar radiation; thus if the objective is to maximize yield, the cheapest option would be to keep temperature uncontrolled (assuming we are not in extreme winter conditions) and to use the minimum solar radiation profile possible given our grow lights (supposing an indoor environment with no natural sunlight). However, as we reduce solar radiation, the amount of nitrogen needed to support plant growth increases, potentially increasing costs. Thus we

encounter a trade-off between energy costs and fertilizer costs.

3.5.2 Controlling Nitrogen

We now fix temperature to 25 degrees C (which is roughly the average temperature in Utah from May to June), and solar radiation to 5 MJ/m^2/day (as a tradeoff between minimizing solar radiation and nitrogen requirements) and attempt to optimize the nitrogen addition profile over time. Fortunately, the optimizer is able to run this experiment.

With N_{add} as our manipulated variable, we choose the objective function:

$$\min_{N_{add}} \sum_t \|100N_{add}(t) - w(t)\|_1. \tag{3.15}$$

The variable N_{add} is weighted heavily as we want to add as little fertilizer as possible across time. The variable w is included because we want to not only reach the maximal weight possible, but we want to do it as soon as possible so that we can harvest the lettuce and replant, thus maximizing profits.

The results of this simulation are shown in Figure 3.10. The strategy is to add nitrogen into the pool at the same rate that the plant is taking it up. This way, the plant is not nitrogen starved, but we are not adding more than necessary, leaving the final nitrogen pool at 0. Furthermore, the plant has consumed all of its nitrogen by day 5, so all of the nitrogen can be added at the very beginning of the growing horizon. In fact, given that the nitrogen uptake required by the plant is slightly larger than 1 mmol, an optimal strategy might be to add this full amount at the very beginning of the growing period. This strategy is shown in Figure 3.11, where 1.1 mmol of nitrogen is added to the nutrient pool at the initialization of the simulation. The growth profile is nearly identical to the growth profile of the lettuce, even though the uptake profile is very different. Contrast this to Figure 3.12, where only 0.8 mmol of nitrogen is added at the beginning of the simulation and the final mass of the lettuce has been reduced.

3.6 Conclusions

In conclusion, we have found that the ability to simulate and optimize the dynamic plant model is extremely sensitive to temperature, solar radiation, and the objective function used. However, as long as enough nitrogen is added given a constant temperature and solar radiation over time, the plant is capable of growing to saturation, and this saturation point is, for the most part, insensitive to temperature and solar radiation.

When connecting the plant model to the fish and bacteria models (described in subsequent sections), we are no longer able to control for nitrogen. This, instead, is computed as a function of fish excrement and bacteria dynamics. We can still control for temperature and solar radiation; however, as shown above, final yield is fairly invariant to both. Uptake is more sensitive; however, it is not clear whether more or less uptake is desirable. Too much uptake will cause our plants to starve. Too little uptake will leave nitrogen in the water that poisons the fish. As such, for simplicity, we will hold temperature and solar radiation constant and tune the other variables around the constant point chosen.

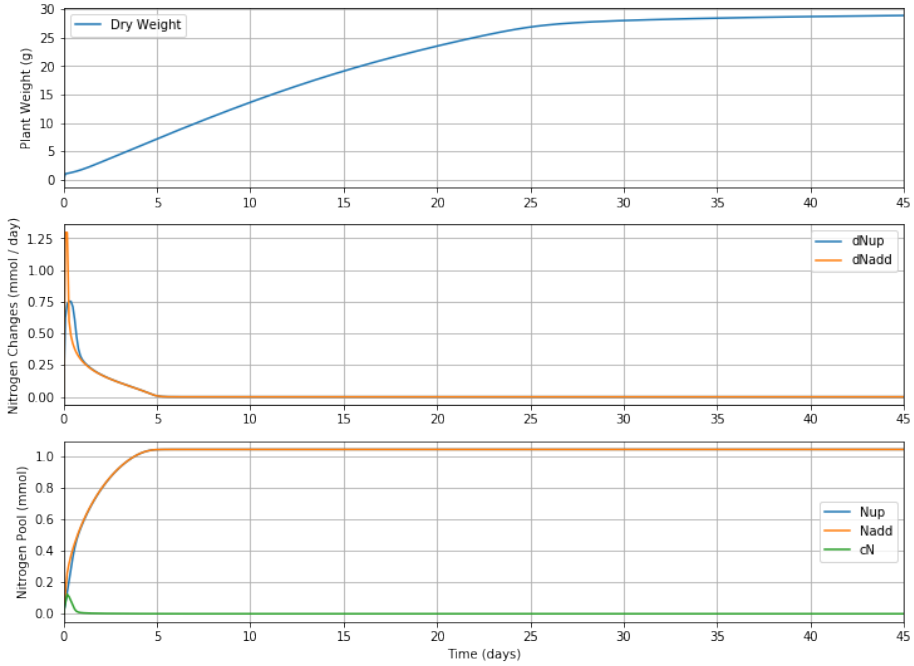


Figure 3.10 Experiment where the optimal nitrogen profile is chosen by the optimizer.

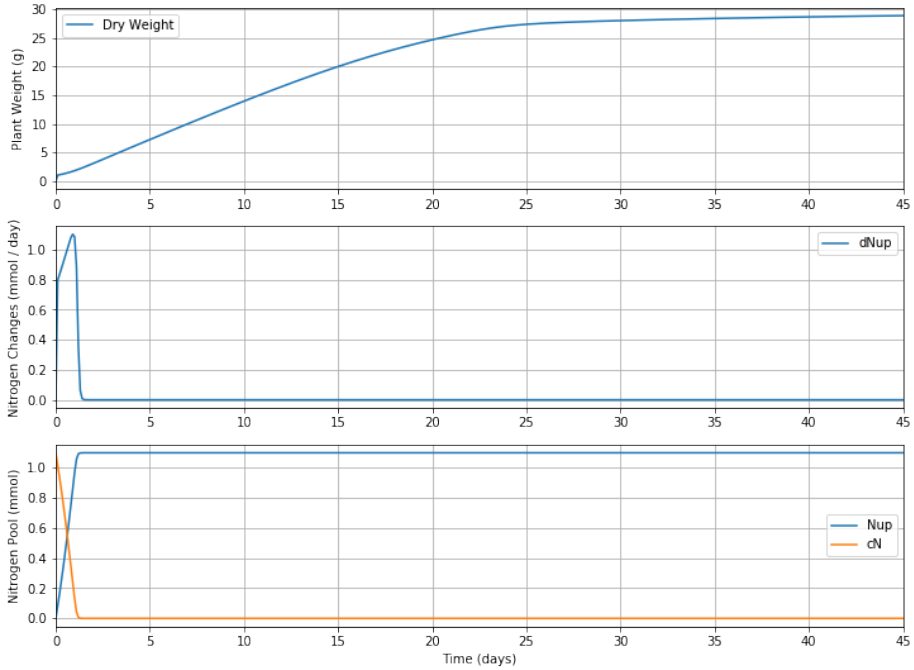


Figure 3.11 Experiment where cN is initialized to 1.1 mmol.

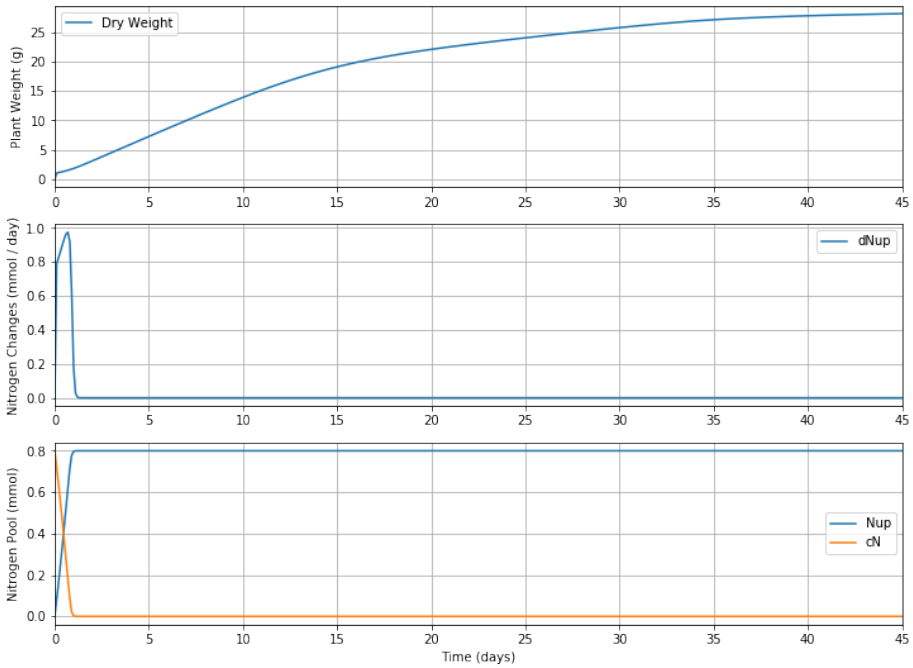


Figure 3.12 Experiment where cN is initialized to 0.8 mmol.

4

The Hydroponics System Submodel

We now extend the plant model discussed in the previous chapter in order to simulate the growth of multiple heads of lettuce in the hydroponics system. We do this by modeling two things: (1) a single growing bed, and (2) multiple growing beds with staggered planting and harvest dates.

Note that since the dynamics here do not change from the previous chapter, other than scaling and delay effects, we will not perform identification or control experiments on this submodel.

4.1 Single Growing Bed

To model a single growing bed, we introduce a new fixed variable, *ppb* (plants per bed), which represents the number of plants that grow in the bed. Then, the total plant dry matter weight w and the total uptake N_{up} by the entire bed is simply *ppb* multiplied by these variables for a single plant.

Note that, for an aquapoics model, *ppb* is one of the most important decision variables available to the farmer. If *ppb* is too large, then the fish do not produce enough nitrogen to support plant growth. If *ppb* is too small, then the plants can't clean the nitrogen out of the water fast enough, and the fish are poisoned.

4.2 Staggered Growing Beds

Recall from the previous chapter that the plant typically consumes most of its nitrogen in the early days of the lettuce growing season. However, the fish are constantly producing nitrogen that needs to be cleaned out of the system. In order to maintain a semi-constant nitrogen uptake to keep the fish healthy, we connect multiple growing beds to a single tank and stagger the planting and harvest dates so that we have multiple sequential peaks of nitrogen uptake.

To do this, for a single growing bed, let *plantingdate* be the time index of when the bed is planted and *harvestdate* be the time index of when the bed is harvested. With t as

a variable representing the time index (in days), we create a new intermediate called *on*, defined as follows:

$$on = \begin{cases} 1 & \text{if } plantingdate \leq t \leq harvestdate \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

We approximate *on* with the Gekko *switch* extension (see Appendix B) as

$$on_{harvest} = switch(1, 0, t, harvestdate), \tag{4.2}$$
$$on = switch(0, on_{harvest}, t, plantingdate). \tag{4.3}$$

We then use *on* to turn on and off plant growth and nitrogen uptake. With w_G, w_S, and N_{up} defined with the following dynamics from the previous chapter:

$$\frac{dw_G}{dt} \triangleq DWG, \tag{4.4}$$
$$\frac{dw_S}{dt} \triangleq DWS, \tag{4.5}$$
$$\frac{dN_{up}}{dt} \triangleq DNUP, \tag{4.6}$$

we redefine the new plant dynamics to be

$$\frac{dw_G}{dt} = DWG \cdot on, \tag{4.7}$$
$$\frac{dw_S}{dt} = DWS \cdot on, \tag{4.8}$$
$$\frac{dN_{up}}{dt} = DNUP \cdot on, \tag{4.9}$$

Then, to compute the total plant weight and nitrogen uptake in the entire hydroponics system, we simply add the respective variables from each of the beds.

4.3 Validation

We now validate the functionality of the hydroponics system by comparing it to the functionality of the plant model in a sequence of experiments, each growing in complexity. For each experiment, we simulate over 90 days with an unconstrained nitrogen pool available to each plant. These are also simulated at 25 degrees C with 5 MJ/m^2/day solar radiation.

4.3.1 Experiment 1

For this first experiment, we run the hydroponics system with only one growing bed and one plant per bed, with Day 0 as the planting date and day 30 as the harvest date. We would expect that the results would be identical to that found in the previous chapter, but with nitrogen uptake cutting of at day 30 since that is when we harvest. As shown in Figure 4.1, this is indeed the case.

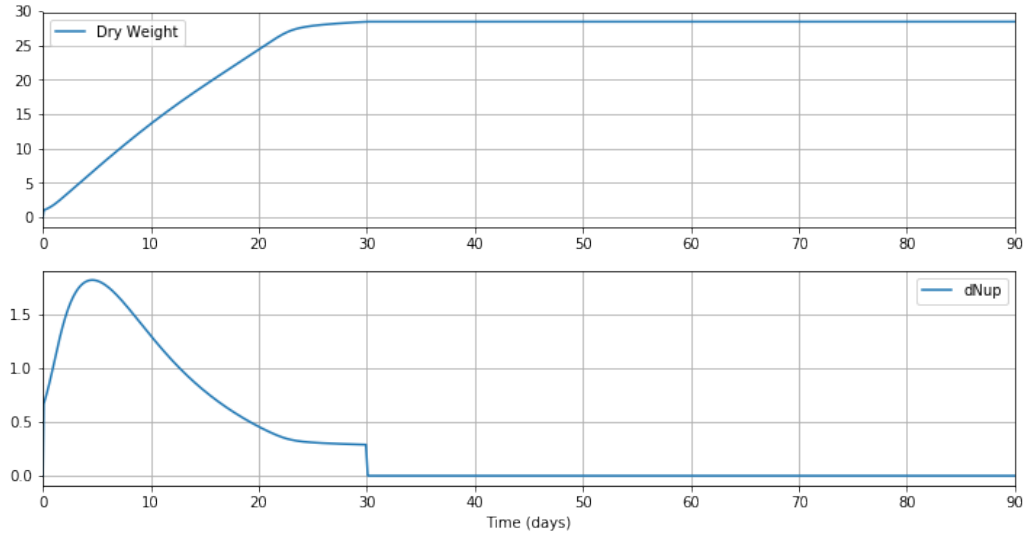


Figure 4.1 The results of Experiment 1 on the Hydroponics System (1 bed, 1 plant, plant on day 0, harvest on day 30).

4.3.2 Experiment 2

This experiment is a repeat of Experiment 1, but with 30 plants on the single bed instead of 1. We would expect the same results as Experiment 1, but scaled by 30. As shown in 4.2, this is indeed the case.

4.3.3 Experiment 3

This experiment is a repeat of Experiment 2, but where the planting day is on day 30 and the harvest day on day 60. As shown in Figure 4.3, uptake and growth do not begin until day 30, and terminate on day 60, as expected. Furthermore, the final cumulative plant weight is still the same as in Experiment 2, also as expected.

4.3.4 Experiment 4

In this experiment, we introduce multiple growing beds, each growing 30 plants. One starts on day 0 and is harvested on day 30, the second starts on day 30 and ends on day 60. The third starts on day 60 and ends on day 90. The results shown in Figure 4.4 are exactly what would be expected (note that the plant weight shown is cumulative weight of all beds from the beginning of the simulation).

4.3.5 Experiment 5

This experiment is a variation of Experiment 4, this time with 6 growing beds, each with a 30 day harvest window, and each planted 15 days after the previous. Again, as shown in Figure 4.5, the simulation proceeds as expected.

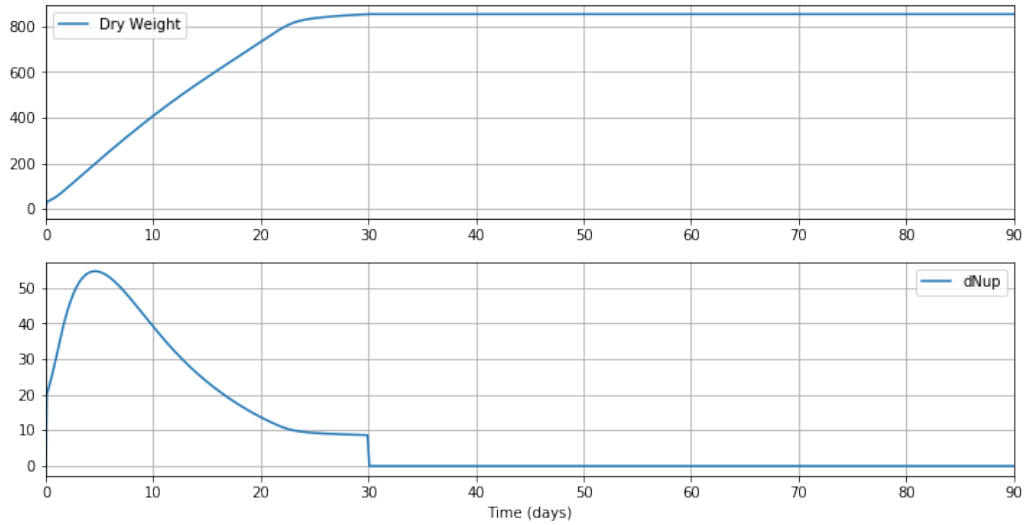


Figure 4.2 The results of Experiment 2 on the Hydroponics System (1 bed, 30 plants, plant on day 0, harvest on day 30).

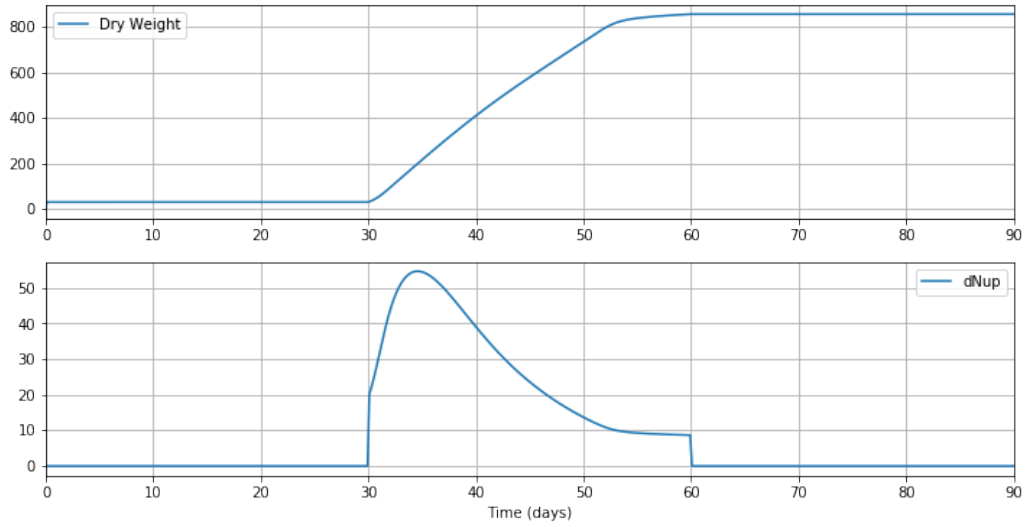


Figure 4.3 The results of Experiment 3 on the Hydroponics System (1 bed, 30 plants, plant on day 30, harvest on day 60).

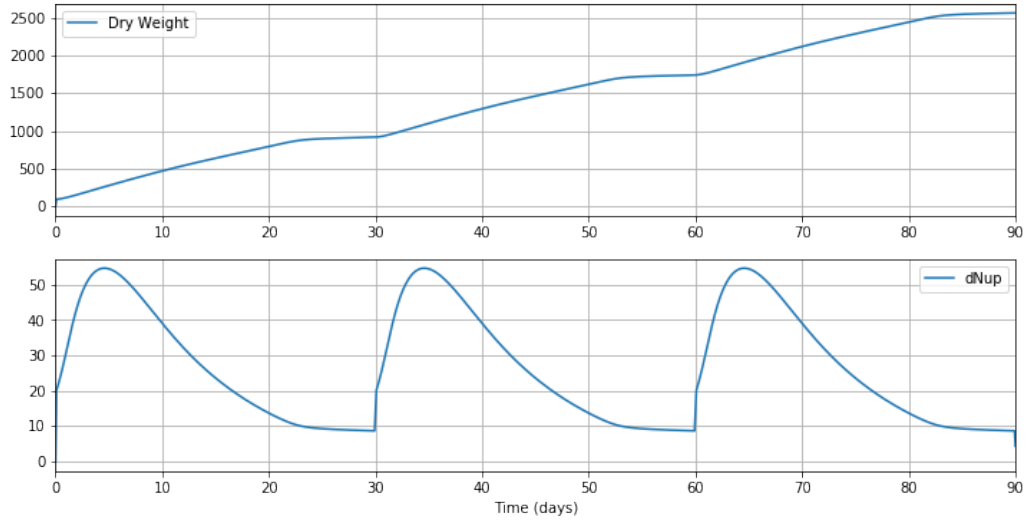


Figure 4.4 The results of Experiment 4 on the Hydroponics System (3 beds, 30 plants, staggered planting days).

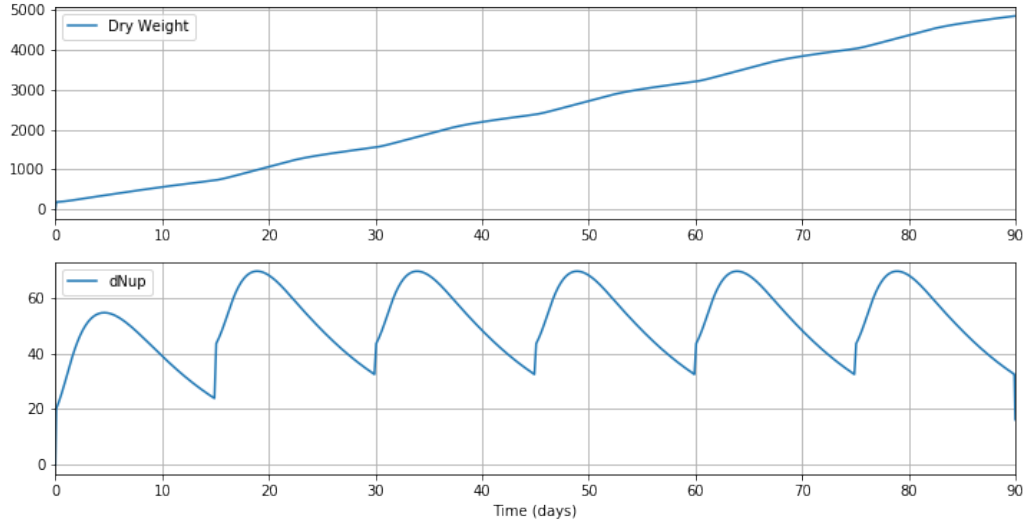


Figure 4.5 The results of Experiment 5 on the Hydroponics System (6 beds, 30 plants, staggered planting days).

5

Bacteria Submodel

Bacteria are an important component in an aquaponics system, oxidizing ammonia (NH_3) in fish excrement and fertilizer into nitrites (NO_2) and nitrates (NO_3) that the lettuce uptakes. This section specifically focuses on *Nitrosomonas* and *Nitrobacter* and the relationship between colony growth and oxidation of NH_3 supplied to the system.

5.1 The Complete Bacteria Model

From an article [3] on determining the kinetic constants for nitrifying bacteria using Michaelis-Menten kinetics, we can obtain differential equations describing the growth of bacteria cultures. Both *Nitrosomonas* and *Nitrobacter* cultures conform closely to these equations and we assume that only these two bacteria species are present. The parameters found in [3] fit experimental data, as such we will not perform our own parameter estimation, but will validate our model against the original results in the paper. State variables, manipulated variables, intermediates, and parameters for this model are defined in Tables 5.1, 5.2, 5.3, and 5.4 respectively. Using Michaelis-Menten [3] kinetics we obtain the following two equations:

$$\frac{dC_m}{dt} = \frac{k_m C_m x}{x + X}, \tag{5.1}$$

$$\frac{dC_b}{dt} = \frac{k_b C_b y}{y + Y}, \tag{5.2}$$

Where C_m represents the concentration of *Nitrosomonas* and C_b represents the concentration of *Nitrobacter*. Here k_m, X, k_b and Y are given by the following equations:

$$k_m = 10^{0.0413T - 0.944}, \tag{5.3}$$

$$X = 10^{0.051T - 1.158} \tag{5.4}$$

$$k_b = 10^{0.0255T - 0.492}, \tag{5.5}$$

$$Y = 10^{0.063T - 1.149} \tag{5.6}$$

Variable	Initial	Units	Description
C_m	0.0025	mg / L	Concentration of *nitrosomonas*
C_b	0.0005	mg / L	Concentration of *nitrobacter*
x	0.0	mg / L	Concentration of NH_3
y	0.0	mg / L	Concentration of NO_2
z	0.0	mg / L	Concentration of NO_3

Table 5.1 State Variables for the bacteria submodel with their default initial values, units, and descriptions.

Variable	Initial	Units	Description
T	25	deg C	Temperature
x_a	0	mg / L	NH_3 added to the model
y_a	0	mg / L	NO_2 uptake by lettuce
z_{up}	0	mg / L	NO_3 uptake by lettuce

Table 5.2 Manipulated Variables for the bacteria submodel with their default initial values, units, and descriptions.

If the oxidation of unit mass of ammonia produces a dry mass E_m of *Nitrosomonas* then, at any time,

$$C_m - C_{m0} = E_m(x_0 + x_a - x), \tag{5.7}$$

where x is the concentration of ammonia (mg/L), C_{m0} and x_0 are the initial values of C_m and x, respectively, and x_a is the total ammonia added through outside sources (mg/L). Likewise, the concentration of *Nitrobacter* is given by the equation

$$C_b - C_{b0} = E_b(y_0 - y_a + f_m(x_0 + x_a - x) - y), \tag{5.8}$$

where y is the concentration of nitrite (mg/L), C_{b0} and y_0 are the initial values of C_b and y, f_m is the ratio of the mass of nitrite formed to that of ammonia oxidized, y_a is the total nitrate removed by outside sources (mg/L), and where the oxidation of unit mass of nitrite produces a dry mass E_b of *Nitrobacter* [3]. From these equations, the nitrate concentration z (mg/L) can then be computed as the total amount of nitrite that has been oxidized, given by

$$z = z_0 - z_{u_p} + f_n(y_0 - y_a + f_m(x_0 + x_a - x) - y), \tag{5.9}$$

where z_0 is the initial nitrate concentration (mg/L).

5.2 Model Validation

In order to check our implementation of the model in [3], we ran empirical tests in conditions similar to those found in the original work. The largest deviation in our model to the original specifications was with respect to the initial concentration of bacteria. Our initial concentrations are about three orders of magnitude greater than the concentration given in the original work. Though our initial concentration is different the behavior of the model

Variable	Units	Description
k_m	1 / day	*nitrosomonas* growth constant given by Eq. 5.3
k_b	1 / day	*nitrobacter* growth constant given by Eq. 5.5
X	mg / L	*nitrosomonas* saturation constant given by Eq. 5.4
Y	mg / L	*nitrobacter* saturation constant given by Eq. 5.6

Table 5.3 Intermediate equations for the bacteria submodel with their units, and descriptions.

Parameter	Value	Units	Description
E_m	0.05	-	Production rate of dry mass of *nitrosomonas* from NH_3
E_b	0.02	-	Production rate of dry mass of emphnitrobacter from NO_2
f_m	0.99	-	Ratio of mass of NO_2 formed to NH_3 oxidized
f_n	0.99	-	Ratio of mass of NO_3 formed to NO_2 oxidized

Table 5.4 Parameters for the bacteria submodel with their values, units, and descriptions.

is the same (see 5.1) and this figure in comparison with [3] is very similar. Therefore we use this bacteria model in the simulation and control of the aquaponics system.

The ability of the bacteria model to simulate step impulses was also tested by adding NH_3 on day 10 to reach an ammonia concentration of 3 mg/ml. This is shown in Figure 5.2.

As mentioned previously NH_3 and NO_3 are linking variables such that fish will be adding NH_3 to the system and the plants will be removing NO_3 from the aquaponics system while the bacteria is oxidizing NH_3 to NO_3. Thus another important test to simulate was the removal of NO_3 from the system. Figure 5.3 shows the simulated NO_3 removal starting at day 12.

These tests illustrate the functionality of the bacteria submodel and its capacity to respond to disturbances from the other submodels or to changes in set points of ammonia and nitrate levels. The simulation also reveals that the bacteria will quickly use up whatever levels of ammonia are available.

5.3 Conclusion

The bacteria component of the aquaponics system will not be directly controlled. Its function is to manage the nitrogen pools and incorporate a realistic delay between NH_3 from fish waste to a form of nitrogen that the lettuce heads will uptake. In our experience the bacteria concentration grows to the saturation point quickly for whatever level of NH_3 is supplied. Therefore we are confident that the bacteria component can respond to control needed for the plant and fish components of the aquaponics system, and have incorporated terms x_a, y_a and z_{up} to allow for such control. The concentration of NH_3 and the temperature will be determined by the control needed by the plant model through feedback control.

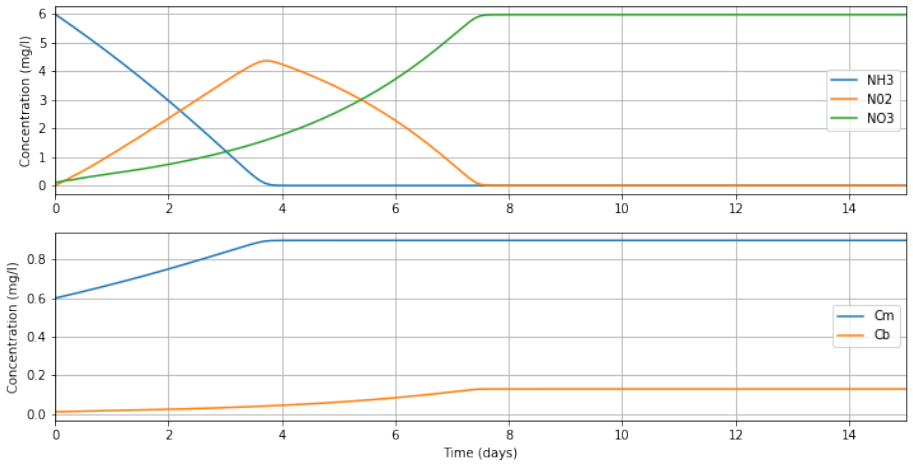


Figure 5.1 Concentration of nitrogen compounds with initial values of 6 mg/ml of ammonia and 0.01 mg/ml of nitrite.

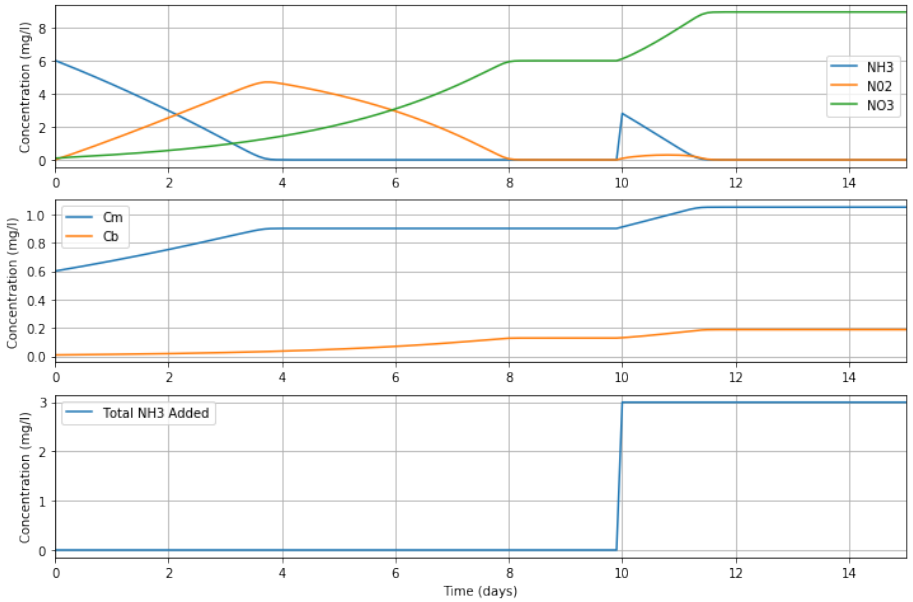


Figure 5.2 Impulse test at t = 10 days.

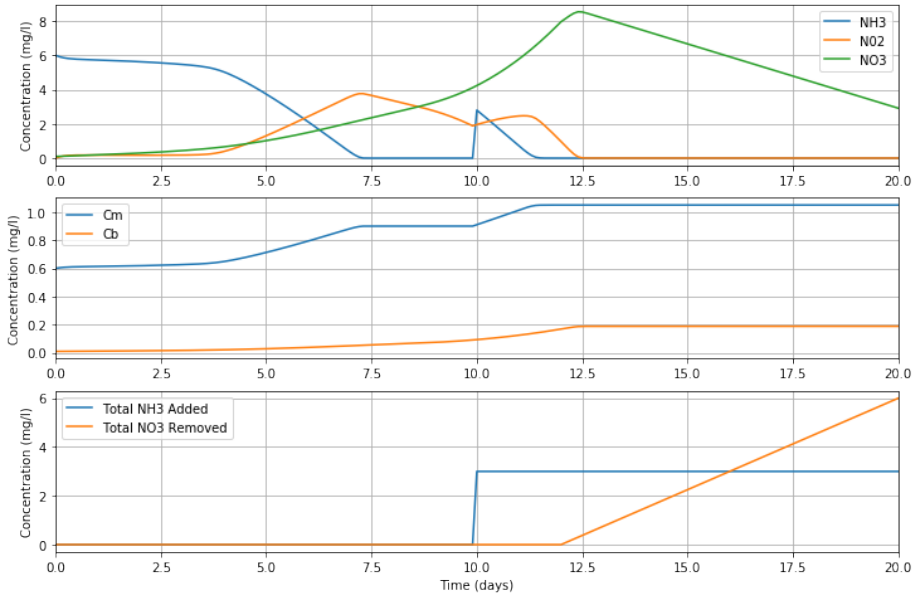


Figure 5.3 Testing NO_3 removal at t = 12 days.

6

Fish Submodel

We simulate a population of Nile Tilapia (*Oreochromis niloticus*) residing in a 1,800 liter tank. The fish model tracks fish population and biomass, and the fish waste contributes to the nitrogen pool. This model is used to provide nitrogen to the aquaponics model.

6.1 The Complete Fish Model

The model for Nile Tilapia population is taken from [2], which describes the feedback interaction between two subsystems in the tank–the fish and the alternate food sources in the tank. The model along with parameter and intermediate values are used with slight modification in the aquaponics system. The primary difference includes approximating switch functions with sigmoids and using fractions of a fish to make a continuous objective, and allow the solver to optimize manipulated variables. Allowing the fish population to be any real number instead of integers only can be interpreted as having the characteristic value of fish with the mantissa being the probability of an additional fish. The original model in [2] is experimentally verified and we will validate our model with their results. Note that this model does not take ammonia concentration as an input. Rather, it takes total nitrogen (ammonia + nitrite + nitrate) and assumes that a fixed percentage (36.5% to be precise) of that concentration is toxic. This fixed percent may be greater than or less than the ammonia produced as an output by the bacteria model causing fish to die more or less frequently than in reality. However, the use of this constant may be desirable since nitrites and nitrates can also be toxic to tilapia, though only in higher concentrations [11, 12]. State variables, manipulated variables, intermediates, and parameters for this model are defined in Tables at the end of the chapter.

6.1.1 Model's structure and Mathematical Representation

Due to the large fish model size the model was split into separate submodels as follows: fish anabolism, fish autotrophic, fish catabolism, fish growth, fish heterotrophic and fish nutrient submodels. Because of the large size of the model, the naming convention for parameters was kept as simple as possible. Fish anabolism components are typically represented by the capital letter A and catabolism with the captial letter C etc. Naming conventions

followed the original work done by [2]. The representation outlined here goes through each of the submodels, starting with the equation that is most directly related fecal waste, or to fish population and fish biomass. After introducing the key equation each subsection than defines each term that appears in the equation, and in subsequent equations.

Fish Anabolism Submodel

Fish biomass accumulation depends on fish growth rate, which is the difference between anabolism and total catabolism [2]. Anabolism is dependent on food consumption, food and water quality, fish size and temperature. The fish anabolism submodel provides the bacteria pool with ammonia which is contained in the fish fecal waste. The two components that we need are fecal waste, and total anabolism. The fish anabolism is given by:

$$FA = k_{max,a} TFC f(FQ), \tag{6.1}$$

where $k_{max,a}$ is a maximum assimilation coefficient given in Table 6.5-6.7, and the total food consumption *TFC* is calculated by summing the autotrophic, heterotrophic and supplementary consumption rate (*AFC*, *HFC*, *SFC*, respectively):

$$TFC = AFC + HFC + SFC. \tag{6.2}$$

The autotrophic and heterotrophic consumption rates can be found using Eqs. 6.12 and 6.34, respectively. The effect of fish food quality (*f(FQ)*) is calculated as:

$$f(FQ) = \begin{cases} 1.0 & PE \geq PE_{opt} \\ e^{-k_{PE}[(PE_{opt}-PE)/(PE_{opt}-PE_{min})]^{0.85}} & PE_{min} < PE \leq PE_{opt}. \\ e^{-k_{PE}} & PE < PE_{min} \end{cases} . \tag{6.3}$$

The food quality is expressed in a protein to energy level represented by PE, k_{PE} is a coefficient on food assimilation and PE_{min} and PE_{opt} are the minimum and optimal PE ratio for tilapia growth. PE can be calculated by:

$$PE = (AFC k_{pl} + HFC k_{p2} + SFC k_{p3})/TFC. \tag{6.4}$$

Values of k_{pl}, k_{p2} and k_{p3} can be found in Tables 6.5=6.7. The other state variable of interest, fecal matter (FW), is given by:

$$FW = SFA - SFC + TFC - FA. \tag{6.5}$$

SFC is the minimum between SFA and required supplementary feed (*RSF*). SFA is the supplementary feed availability and is assumed to be equal to the required supplementary feed:

$$RSF = \begin{cases} FAPP - (AFC + HFC) & FAPP > AFC + HFC \\ 0 & FAPP \leq AFC + HFC. \end{cases} . \tag{6.6}$$

FAPP, the fish appetite can be calculated with the following equation:

$$FAPP = FAPP_{max} FBf_{sm} f(WQ) \tag{6.7}$$

where FB is the fish biomass calculated with Eq. 6.31, f_{sm} can be calculated as a function of mean fish size, FB_m, and FB_s, which is the average fish size at which f_{sm} is normalized at 1. The factor f(WQ) accounts for loss of appetite due to fish poisoning, and is related to the toxicity index (TI).

$$f_{sm} = (FB_m/FB_s)^m, \tag{6.8}$$

$$f(WQ) = e^{-k_{TI}TI^2} \tag{6.9}$$

Here K_{TI} is a parameter with its value given in Tables 6.5-6.7 and TI is calculated from dissolved oxygen content and inorganic nitrogen:

$$TI = k_{DOT}(1 - f_1(DO)) + k_{NHT}INCk_{NH}. \tag{6.10}$$

The parameters k_{DOT}, k_{NHT} and k_{NH} can be found in Tables 6.5-6.7. $f_1(DO)$ is calculated as follows:

$$f_1(DO) = \begin{cases} e^{-k_{DO}(4-DOC)^2} & DO < 4 \\ 1 & DO \geq 4. \end{cases} . \tag{6.11}$$

The values of k_{DO} and DOC can also be found in Tables 6.5-6.7.

6.1.2 Fish Autotrophic Submodel

In a pond ecosystem, autotrophic producers convert elementary nutrients (e.g. C, N, P) into food nutrients (energy, protein etc.) [2]. The autotrophic submodel provides the anabolism submodel with the autotrophic food consumption rate, which allows the fish model to predict fish biomass growth and fecal waste. Autotrophic production capacity is influenced by incident radiation, turbidity, water temperature and elementary nutrients. The Autotrophic food consumption rate (AFC) is calculated as:

$$AFC = FBh_a f_a f_{sm} f_2(T) f(WQ). \tag{6.12}$$

In Eq. 6.12, h_a is the atuotrophic food consumption coefficient, f_a is the autotrophic food availability, f_{sm} accounts for fish size and is calculated using Eq. 6.8. The two remaining functions account for effects of temperature of the water and the water quality. Water quality ($f(WQ)$ is calculated from Eq. 6.9. The autotrophic food availability is modelled with:

$$f_a = 1 - e^{-s(AF_e/FB)^{2.2}}. \tag{6.13}$$

In Eq. 6.13, s is a proportionality coefficient of food nutrient quantity to fish biomass and AF_e is the autotrophic food amount in terms of energy. The dynamics of autotrophic food nutrients (in terms of both energy and protein) are described by the following equations:

$$\frac{dAF_e}{dt} = AFG - AFC - AFR - AFM \tag{6.14}$$

$$AFG = \mu_{max} AF_e f(N,P) f(I) f_1(T) \tag{6.15}$$

$$AFR = AF_e k_r \tag{6.16}$$

$$AFM = AF_e k_{ml} \tag{6.17}$$

$$\frac{dAF_p}{dt} = \frac{dAF_e}{dtk_{pl}} \tag{6.18}$$

AF_e and AF_p are autotrophic food quantity in terms of energy and protein. AFC and AFR are autotrophic food loss rate due to tilapia grazing and phytoplankton respiration, AFM is a rate of autotrophic food entering heterotrophic food pool (a result of phytoplankton mortality and harvest by secondary producers. All values of the parameters can be found in Tables 6.5-6.7. The three functions in order of appearance in Eq. 6.15 represent limiting functions on elementary nutrients, solar radiation and water temperature to phytoplankton growth.

$$f(N,P) = min[(INC/(INC + h_n), IPC/(IPC + h_p)] \tag{6.19}$$

$$f(I) = I_0 e^{-(aAF_e + bHF_e)} \tag{6.20}$$

$$f_1(T) = \begin{cases} e^{-k_{T1}(T-T_{opta})^2} & T \leq T_{opta} \\ e^{-k_{T2}(T_{opta}-T)^2} & T > T_{opta} \end{cases}. \tag{6.21}$$

Coefficients and parameters are explained and listed with their default values in Tables 6.5-6.7. The remaining term in Eq. 6.12 is the effect of temperature on autotrophic food consumption $f_2(T)$:

$$f_2(T) = V^x e^{x(1-V)}, \tag{6.22}$$

where the following relationships are used to calculate V and x:

$$V = (T_{maxf} - T)/(T_{maxf} - T_{optf}) \tag{6.23}$$

$$x = [S_1^2(1 + (1 + 40/S_2)^{0.5})^2]/400 \tag{6.24}$$

$$S_1 = ln(Q_{10}(T_{maxf} - T_{optf})) \tag{6.25}$$

$$S_2 = ln(Q_{10}(T_{maxf} - T_{optf} + 2)). \tag{6.26}$$

Here T_{maxf} and T_{optf} are the maximum and optimum temperature for tilapia and Q_{10} is a term to express the relative increase in the rate of a biological activity with an increase in temperature of 10 K.

6.1.3 Fish Catabolism Submodel

Catabolism can be identified into two categories: fasting catabolism and feeding catabolism. Fasting catabolism is affected by body weight, size temperature and dissolved oxygen whereas feeding catabolism is assumed as a fraction of consumed feed [2]. The catabolism submodel returns the catabolism rate, and the difference between the anabolism and catabolism rate is the fish growth. Fish catabolism can be calculated by:

$$FC = TFCk_{feed} + FBk_{fast} f_{sn} f_2(DO) f_3(T). \tag{6.27}$$

TFC is found in Eq. 6.2 and fish biomass is found in Eq. 6.31. The effect of fish size is represented by f_{sn}:

$$f_{sn} = (FB_m/FB_s)^n, \tag{6.28}$$

the effect of dissolved oxygen is represented by $f_2(DO)$:

$$f_2(DO) = \begin{cases} e^{-k_{DOf}(DO_{crit}-DOC)^2} & DO < DO_{crit} \\ 1 & DO \geq DO_{crit} \end{cases}. \tag{6.29}$$

and the effect of temperature is represented by $f_3(T)$:

$$f_3(T) = c + dT. \tag{6.30}$$

Values of the coefficients and their meaning can be found in Tables 6.5-6.7.

6.1.4 Fish Growth Submodel

Fish growth submodel provides the mathematical model for two state variables of interest, namely the fish population and the fish biomass. Fish growth is affected by the fish population and the fish growth rate, or the difference between anabolism (FA) and catabolism (FC). According to [2] fish biomass and fish population in a cultural pond can be expressed as:

$$\frac{dFB}{dt} = FP_s FB_i + FA - FC - FPk_{m2}FB_m, \tag{6.31}$$

$$\frac{dFP}{dt} = FP_s - INT(FPk_{m2}), \tag{6.32}$$

where FB is the fish biomass and FP is the fish population. The coefficient k_{m2} is the fish mortality coefficient, FB_m is mean fish biomass, and FP_s and FB_i are stocking fish number and individual fish biomass during fish stocking. *INT* is a mathematical function which gives the largest integer less than or equal to its argument. The values of FA and FC can be calculated from Eqs. 6.1 and 6.27, respectively. The fish mortality coefficient is calculated from:

$$k_{m2} = 1/(1 + e^{-6(k_{nh}INC-1.25)}), \tag{6.33}$$

where k_{nh} can be found in Table 6.5-6.7 and INC can be calculated using Eq. 6.40.

6.1.5 Fish Heterotrophic Submodel

Heterotrophic food refers to all living and non-living heterotrophic components of particulate organic matters in a pond that can be grazed by tilapia [2]. Heterotrophic food availability is influenced by decomposition, sedimentation, grazing by fish and the respiration of living heterotrophic components. This submodel assumes that heterotrophic food loss per unit of heterotrophic respiration is equal to that of per unit non-living heterotrophic matter decomposition. The heterotrophic submodel gives the heterotrophic food consumption rate, *HFC*, to the fish anabolism model which is used to calculate fish biomass change and

fecal waste production. *HFC* can be calculated as a function of fish size, water temperature and water quality:

$$HFC = FBh_h f_h f_{sm} f_2(T) f(WQ). \tag{6.34}$$

Fish biomass (FB) can be calculated with Eq. 6.31, h_h is described in Tables 6.5-6.7, $f_s m$, $f_2(T)$ and $f(WQ)$ can be calculated with Eqs. 6.8, 6.22 and 6.9 respectively. The remaining term, f_h in the equation above is subject to heterotrophic food availability and fish size.

$$f_h = 1 - e^{-s(HF_e/FB)^{2.2}} \tag{6.35}$$

Similar to Eq. 6.13, s represents a proportionality coefficient of food nutrient quantity to fish biomass, and HF_e represents the availability of heterotrophic food source in terms of energy. The dynamics of heterotrophic food availability can be described with these equations:

$$\frac{dHF_e}{dt} = AFM + FW - HFC - HFS - HFD \tag{6.36}$$

$$HFS = HF_e k_s \tag{6.37}$$

$$HFD = HF_e k_d f_1(DO) \tag{6.38}$$

$$\frac{dHF_p}{dt} = \frac{dHF_e}{dtk_{p2}} \tag{6.39}$$

Where dHF_p is the heterotrophic food amount in terms of protein, FW is fish fecal wastes (Eq. 6.5), HFC, HFD and HFS are heterotrophic food loss rate due to tilapia grazing, decomposition and sedimentation respectively. The term $f_1(DO)$ is the effect of DO on an aerobic process and is given by Eq. 6.11. AFM is calculated as shown in Eq. 6.17.

6.1.6 Fish Nutrient Submodel

Inorganic nitrogen and phosphorus are considered as necessary ingredients of nearly all fishpond fertilization [2]. The fish nutrient submodel describes the elementary nutrient pool available for the fish, and the amount of inorganic nitrogen is another state variable that we are interested in. Elementary nitrogen, phosphorus and dissolved oxygen are limiting factors for fish appetite, they show up in equations that describe the water quality's effect on fish appetite (e.g. Eqs. 6.34, 6.12). Furthermore they are also an influence on the amount of heterotrophic and autotrophic food supplies. Mass balance equations for nitrogen (TIN) and phosphorus (TIP) are expressed as:

$$\frac{dTIN}{dt} = FCk_{fn} + AFRk_{an} + HFDk_{hn} + TNSk_{sn} + FT_N - (AFGk_{an} - FIXN) - TINk_{nl}, \tag{6.40}$$

$$\frac{dTNS}{dt} = HFSk_{hn} - TNSk_{sn}, \tag{6.41}$$

$$\frac{dTIP}{dt} = FCk_{fp} + AFRk_{ap} + HFDk_{hp} + TPSk_{pr}/d_w + FT_p - AFGk_{ap} - TIPk_{ps}/d_w, \tag{6.42}$$

$$\frac{dTPS}{dt} = TIPk_{ps}/d_w + HFSk_{hp} - TPSk_{pr}/d_w. \quad (6.43)$$

Here *TNS*, *TPS* represent total nitrogen and phosphorus in sediment, FT_N and FT_P are inorganic nitrogen and phosphorus from ferilization, *FIXN* is N-fixation rate by phytoplankton, and d_w is water depth. Values of the parameters and their descriptions can be found in Tables 6.5-6.7, except for k_{an} and k_{hn} which are calculated by k_{p1} and k_{p2} divided by a N to protein coefficient (6.25 g protein/gN). The value of *FIXN* is given by Eq. 6.44.

$$FIXN = k_n AF_e e^{-k_{nf} INC^2}, \quad (6.44)$$

where AF_e is given by Eq. 6.14 and *INC* is the inorganic nitrogen content. The parameters k_n and k_{nf} are described in Tables 6.5-6.7. The content of dissolved oxygen is also calculated in the Fish nutrient submodel. The oxygen system can be modelled as:

$$\frac{dQ_{DO}}{dt} = AFGk_{ao} + / - DO_{df} - FCk_{fo} - AFRk_{ao} - HFDk_{ho} - TNSk_{sn}/k_{hn}k_{sno}, \quad (6.45)$$

$$DO_{df} = [DICO(DOC - DOS)/(AWFTd_w)V_{pond}]/1000 \quad (6.46)$$

where Q_{DO} is DO quantity in pond water column, AFG is calculated by Eq. 6.15, FC is calculated by Eq. 6.27, AFR is calculated by Eq. 6.16, HFD is calculated by Eq. 6.38. The value of DO_{df} represents oxgen exchange rate between air and water body, and DOS is the saturation content of oxygen in water, V_{pond} is pond water volume and AWFT is airwater interface film thickness. The value of 1000 is to convert from mg to g. All values of the parameters can be found in Table 6.5-6.7.

6.2 Model Validation

Each of the fish submodels were built and tested individually for the correct response to dissolved oxygen concentration, pH, feeding rate, water temperature and solar radiation. The submodels were tested separately, and compared against the results from [2]. Shown below are the results of the heterotrophic and autotrophic food models. The heterotrophic and autotrophic food pools increase rapidly in the first days and then level off. Figure 6.2 shows the result when the 5 submodels are simulated with an initial population of 10 fish, with no control over the feed rate or other MV's. The result correctly displays a decline in fish population and the nitrogen concentration reaching saturation within the 14-day period.

The ability of the fish component model to interact with other models was also tested. The connecting variables of NH_3, NO_2, and NO_3 were communicated between the bacteria and the fish component models and demonstrated a correct response, as shown in Figure 6.3.

6.3 Conclusion

The fish component can be interfaced with a simple model such as the bacteria component of aquaponics system. It displays correct behavior and matches the experimental behavior

Variable	Initial	Units	Description
FA	0.0	kcal / day / pond	Fish anabolism
AFC	0.0	kcal / day / pond	Autotrophic food consumption
HFC	0.0	kcal / day / pond	Heterotrophic food consumption
FW	0.0	kcal / day / pond	Fish fecal waste
TFC	0.0	kcal / day / pond	Total food consumption
AFe	0.0	kcal / pond	The autotrophic food quantity in terms of energy
AFp	0.0	g protein / pond	The autotrophic food quantity in terms of protein
AFM	0.0	kcal / day / pond	Autotrophic food entering heterotrophic food pool
AFR	0.0	kcal / day / pond	Autotrophic food loss due to phytoplankton respiration
AFG	0.0	kcal / day / pond	Autotrophic food loss due to phytoplankton growth
FB	2250	kcal / pond	Total fish biomass
FP	10	fish / pond	Total fish population
HFe	0.0	kcal / pond	Quantity of heterotrophic food nutrients in terms of energy
HFp	0.0	kcal / pond	Quantity of heterotrophic food nutrients in terms of protein
f1DO	0.0	-	Decomposition of heterotrophic particles
HFS	0.0	-	Heterotrophic food loss rate due to sedimentation
HFD	0.0	-	Heterotrophic food loss rate due to decomposition

Table 6.1 State Variables for the fish submodel with their default initial values, units, and descriptions.

Variable	Initial	Units	Description
T	20	deg C	Temperature
FB	2250	kcal / pond	Fish biomass, obtained from fish growth submodel
FP	10	fish / pond	Fish population, obtained from fish growth submodel
AF_e	0.0	kcal / pond	Autotrophic food quantity in terms of energy, obtained from fish autotrophic submodel
HF_e	0.0	kcal / pond	Heterotrophic food quantity in terms of energy, obtained from fish heterotrophic submodel
f1DO	0.0	-	Decomposition of heterotrophic particles, obtained from fish heterotrophic submodel
INC	1.63	mg N / l	Incorporated nitrogen, obtained from connecting nitrogen
I0	0.0	10^6 cal / m^2 / day	The light reaching the surface of the water
IPC	16.86	mg P / l	Total inorganic phosphorus concentration
AFC	0.0	kcal / day / pond	Autotrophic food loss rate due to tilapia grazing, obtained from fish anabolism submodel

Table 6.2 Manipulated Variables for the fish submodel with their default initial values, units, and descriptions.

Variable	Initial	Units	Description
DO	5	-	Dissolved oxygen
DOC	5	mg O / l	Dissolved oxygen concentration
TFC	0	kcal / day / pond	Total food consumption, obtained from fish anabolism submodel
FPs	0.0	fish / day / pond	Fish stocking number
FA	0.0	kcal / day / pond	Fish anabolism, recieved from fish anabolism submodel
FC	0.0	kcal / day / pond	Fish catabolism, received from fish catabolism submodel
AFM	0.0	kcal / day / pond	Autotrophic food entering heterotrophic food pool
FW	0.0	kcal / day / pond	Fish fecal waste, recieves from fish anabolism submodel
HFC	0.0	kcal / day / pond	Heterotrophic food loss from fish grazing, obtained from fish anabolism submodel
FTN	0.0	g N / day / pond	Inorganic nitrogen added from fertilization
FTP	0.0	g P / day / pond	Inorganic phosphorus added from fertilization

Table 6.3 Manipulated Variables for the fish submodel with their default initial values, units, and descriptions.

Variable	Units	Description
FB_m	kcal / fish	Biomass per fish
f_a	-	Autotrophic food availability, Eq. 6.13
f_h	-	Heterotrophic food availability, Eq. 6.35
f_{sm}	-	Effect of fish size on food consumption, Eq. 6.8
S_1	-	Effect of temperature on fish food consumption, Eq. 6.25
S_2	-	Effect of temperature on fish food consumption, Eq. 6.26
x	-	Effect of temperature on fish food consumption, Eq. 6.24
V	-	Effect of temperature on fish food consumption, Eq. 6.23
$f_2(T)$	-	Effect of temperature on fish food consumption, Eq. 6.22
TI	-	Toxicity index, Eq. 6.10
f(WQ)	-	Effect of water quality on food consumption, Eq. 6.9
FAPP	kcal / day / pond	Fish appetite satiation, Eq. 6.7
f(WQ)	-	Elementary nutrient limitation, Eq. 6.19
f(I)	- day	Light limitation, Eq. 6.20
$f_1(T)$	-	Temperature effect on phytoplankton growth, Eq. 6.21
f_{sn}	-	Effect of fish size on fasting catabolism, Eq. 6.28
$f_2(DO)$	-	Effect of dissolved oxygen on fasting catabolism, Eq. 6.29
$f_3(T)$	-	Effect of temperature on fasting catabolism, Eq. 6.30
k_{m2}	1 / day	Fish mortality coefficient, Eq. 6.33
$f_1(DO)$	-	Decomposition of heterotrophic particles, Eq. 6.11
k_{an}	gN / kcal	Nitrogen content of phytoplankton components, see section 6.1.6
k_{hn}	gN / kcal	Nitrogen content of heterotrophic components, see section 6.1.6
FIXN	gN / day / pond	N-fixation rate, Eq. 6.44

Table 6.4 Intermediate equations for the fish submodel with their units, and descriptions. They are listed here in rough order of their introduction.

Parameter	Value	Units	Description
s	21.08	-	Proportionality coefficient of food nutrient quantity to fish biomass
m	-0.3	-	The exponent on effect of fish biomass on food consumption
Q10	2.37	-	Expresses the relative increase in the rate of biological activity with a 10 deg C increase in temperature
h_a	0.51	1 / day	Autotrophic food consumption coefficient
h_h	0.05	1 / day	Heterotrophic food consumption coefficient
FB_s	20	kcal / fish	Average fish size at which f_{sm} is normalized at 1
k_{maxa}	0.75	-	Maximum assimilation coefficient
k_{DOT}	4.0	-	Weighting factor for DO depletion toxicity to food consumption
k_{NHT}	4.0	1 / $\frac{mgN}{l}$	Weighting factor for un-incorporated ammonia toxiticy to food consumption
k_{T1}	0.004	1 / $(degC)^2$	Effect of temperatures below T_{opta} on growth
k_{T2}	0.008	1 / $(degC)^2$	Effect of temperatures above T_{opta} on growth
I_r	6.547	$10^6 cal$ / m^2 day	reference sunlight intensity for phytoplankton growth
T_{opta}	30	deg C	Optimal temperature
k_{feed}	0.31	1 / day	Feeding catabolism coefficient
k_{fast}	0.005	1 / day	Fasting catabolism coefficient
n	-0.12	-	Exponent for calculating fish size effect on fish fasting
c	0.59	-	Regressed parameters describing effect of water temperature on fasting catabolism
d	0.027	1 / deg C	Regressed parameters describing effect of water temperature on fasting catabolism

Table 6.5 Parameters for the fish submodel with their values, units, and descriptions. They are listed here in rough order of their introduction.

Parameter	Value	Units	Description
k_{NH}	0.365	-	Fraction of un-incorporated ammonia to inorganic nitrogen content
k_{TI}	0.012	-	Coefficient of toxicity index on food consumption
$FAPP_{max}$	0.17	1 / day	Maximum fish appetite (FAPP)
PE_{min}	0.025	g protein / kcal	Minimum P:E ratio for tilapia growth
PE_{opt}	0.09	g protein / kcal	Optimum P:E ratio for tilapia growth
k_{PE}	0.45	$kcal^2$ / $(gprotein)^2$	Coefficient of PE on food assimilation
k_{p1}	0.14	g protein / kcal	Protein content of phytoplankton
k_{p2}	0.12	g protein / kcal	Protein content of heterotrophic food
T_{optf}	30	deg C	Optimum temperature for tilapia
T_{maxf}	41	deg C	Maximum temperature for tilapia
μ_{max}	1.6	1 / day	Maximum growth coefficient for phytoplankton growth
k_{m1}	0.6	1 / day	Autotrophic food entering heterotrophic food pool
k_r	0.1	1 / day	Coefficient of phytoplankton respiration
h_N	0.2	mg N / l	Half-saturation inorganic nitrogen concentration
h_P	0.02	mg P / l	Half-saturation inorganic phosphorus concentration
a	0.000017	pond / kcal	Autotrophic light extinction coefficient
b	0.000015	pond / kcal	Heterotrophic light extinction coefficient

Table 6.6 Parameters for the fish submodel with their values, units, and descriptions. They are listed here in rough order of their introduction.

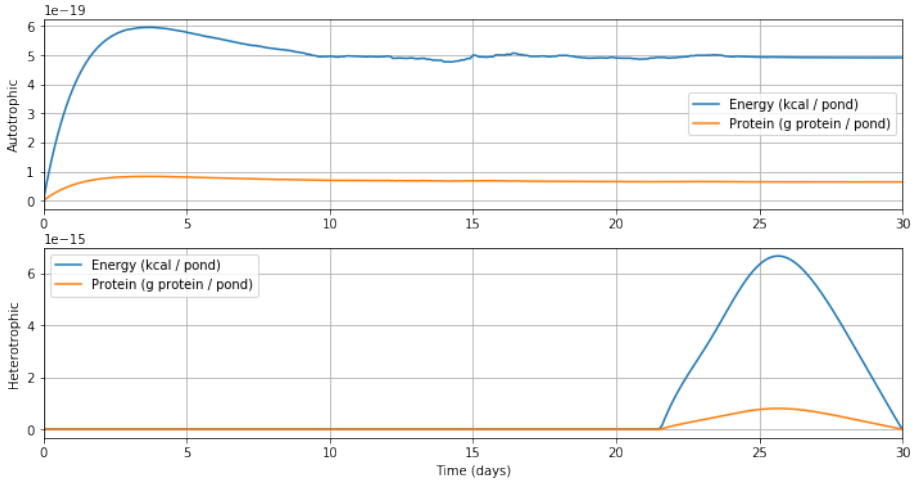


Figure 6.1 Autotrophic and heterotrophic food content per pool

Parameter	Value	Units	Description
DO_{crit}	1.0	mg DO / l	Critical DO limit above which fasting catabolism is not affected by DO
k_{DOf}	2.5	l^2 / $(mgDO)^2$	Coefficient of DO on fasting catabolism
FB_i	225	kcal / fish	Individual fish biomass during stocking
k_s	0.14	1 / day	Coefficient of heterotrophic food sedimentation
k_d	0.12	1 / day	Coefficient of heterotrophic food decomposition
k_{DO}	0.14	l^2 / $(mgDO)^2$	Coefficient of DO on aerobic biological activity
k_n	0.01	g N / kcal day	N-fixation coefficient of phytoplankton
k_{nf}	0.47	l^2 / mg^2	Coefficient of INC on N-fixation
k_{fn}	0.017	g N / kcal	Nitrogen content of fish tissue
k_{fp}	0.002	g P / kcal	Phosphorous content of fish tissue
k_{sn}	0.003	1 / day	Release coefficient of nitrogen in sediment
k_{nl}	0.17	1 / day	Coefficient of inorganic nitrogen loss to air
k_{hp}	0.001	g P / kcal	Phosphorus content of heterotrophic components
k_{pr}	0.06	cm / day	Release coefficient of phospohrus to sediment
k_{ps}	28	cm / day	Coefficient of inorganic phosphorus sedimentation to sediment
k_{ap}	0.001	g P / kcal	Phosphorus content of phytoplankton

Table 6.7 Parameters for the fish submodel with their values, units, and descriptions. They are listed here in rough order of their introduction.

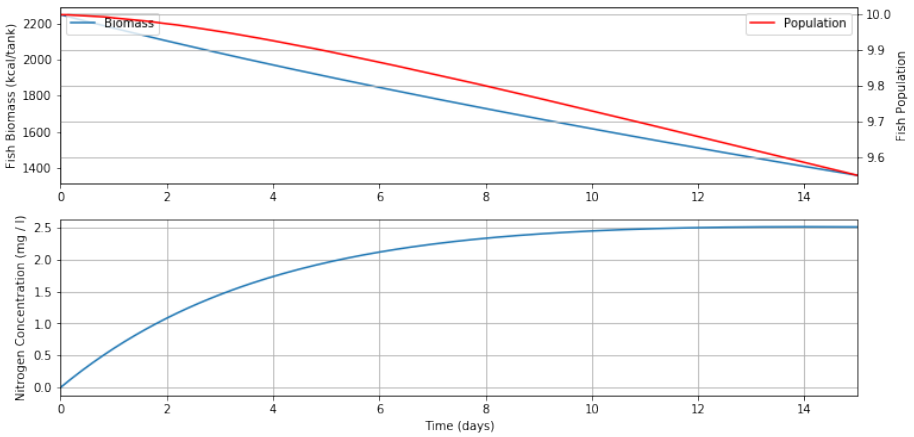


Figure 6.2 Fish component test with correct population, biomass and nitrogen concentration dynamics

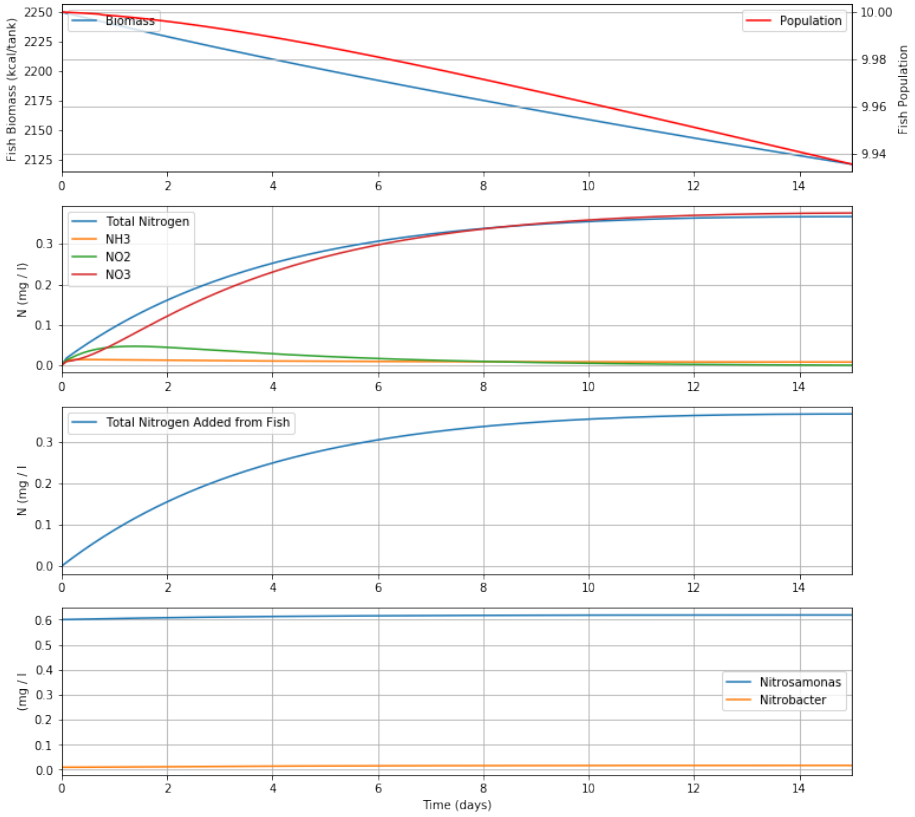


Figure 6.3 Simulation test to verify the fish model's ability to receive inputs from other models.

and model behavior found in [2]. The initial decision of how much fish to stock at a time is an important control element in the aquaponics system, and along with how many plants and plant beds to grow at once is one of the key control outcomes of this project.

7

Connecting the Full Aquaponics System

As shown in Figure 7.1, all three submodels (fish, plant, and bacteria) can be connected in feedback to simulate a complete aquaponics system. The nitrogen pools managed by the bacteria component are the key connections between the three models, though an intermediate nitrogen conversion module is required to convert the units used by each respective model. We discuss the nitrogen conversion component and implementation issues here.

7.1 Nitrogen Conversion

In order to convert the organic nitrogen from the fish model into a nitrogen concentration used by the bacteria model, we need to know the size of the fish tank. As shown in Table 7.1, we use two parameters describing the radius and the depth of the fish tank (120cm and 40cm respectively). We make the simplifying assumption that all water in the system is contained in the tank (which is reasonable since we can fill the tank up to a 40cm depth, and this depth will drop somewhat as the pumps are turned on and the water is cycled to the plants). We also assume that the nitrogen concentration in the water is uniform throughout the system.

We then compute the volume of the tank (in meters cubed) as

$$v_w = \frac{\pi r_w^2 d_w}{1000}, \tag{7.1}$$

and the conversion from the change in TIN from the fish model (specifically the fish elementary nutrient component) and the change in x_a (ammonia added by fish) from the bacteria model is given by

$$\frac{dx_a}{dt} = \left(\frac{1000}{v_w}\right)\left(\frac{dTIN}{dt}\right). \tag{7.2}$$

The conversion between x (ammonia), y (nitrite), and z (nitrate) from the bacteria model to INC used by the fish model is given by

$$INC = x + y + z. \tag{7.3}$$

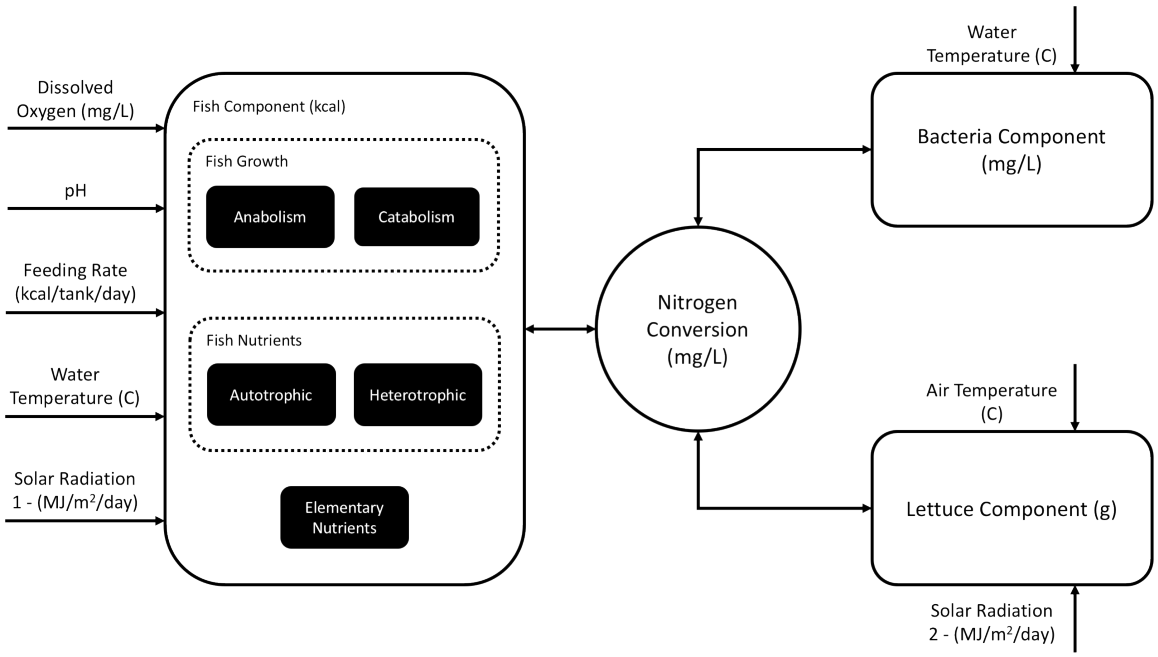


Figure 7.1 The aquaponics model, consisting of the plant, bacteria, and fish submodels connected in feedback.

Parameter	Value	Units	Type	Description
mm	62.0049	Param	Molar mass of nitrate	
d_w	40	cm	Param	Water depth in the tank
r_w	120	cm	Param	Radius of the tank
v_w	-	m^3	Volume of the tank	

Table 7.1 Parameters and intermediates for the nitrogen connection component.

With mm as the molar mass of nitrate, the conversion between N used by the plant model and z used by the bacteria model is given by

$$N = \frac{z}{mm}. \tag{7.4}$$

And the conversion between z_{up} (nitrate uptake) used by the plant model and N_{up} (nitrogen uptake) used by the plant model is given by

$$\frac{dz_{up}}{dt} = \left(\frac{mm}{v_w}\right)\left(\frac{dN_{up}}{dt}\right). \tag{7.5}$$

7.2 Implementation Issues

Unfortunately, we were unable to simulate the complete system in Gekko. The plant component connected in feedback with the bacteria and nitrogen conversion components would

simulate. Likewise, the fish component connected in feedback with the bacteria and nitrogen conversion components would also simulate. However, when all were stitched together, the simulation would wander into an infeasible region from which it was never able to recover. This was true under every one of the many simulation modes, time discretizations, and initial conditions that we tried.

As a work around, we implemented these models in MATLAB's Simulink, which was capable of running these models. Furthermore, we did not have to approximate the discontinuous equations in Simulink as we did with Gekko. Unfortunately, full MPC experiments in Simulink are unwieldy at best; therefore, we simply chose some fixed variables of interest and gridded different choices of those fixed variables in order to find an optimal solution (described in Chapter 8).

8

Optimization and Control of the Full Aquaponics System

We now attempt to optimize the full aquaponics system. Two of the most important decisions available to any aquaponics farmer are (1) how many fish to stock in a single tank and (2) how many plants to grow on each bed.

8.1 Experimental Setup

We set up the aquaponics system using the default values described in the previous section, using the Simulink model of the system instead of the Gekko model. We choose a tank with a radius of 1.2 meters and a water depth of 0.4 meters, which has a capacity of approximately 181 liters. To this tank we attach six growing beds with the planting and harvest dates given in Table 8.1. Note that, at any given point of time, only three beds are active, meaning that instead of attaching six growing beds, we can actually attach just three and replant each once they are harvested.

We also select our two main decision variables, plants per bed and fish stocking number, and set those as fixed variables.

We then simulate over an 80-day horizon and collect plant weight, fish biomass, and fish population at the end of the simulation. We have four main metrics of interest:

- **Total plant weight:** This is the sum of the weight of all heads of lettuce across all

Bed No.	Planting Day	Harvest Day
1	0	30
2	10	40
3	20	50
4	30	60
5	40	70
6	50	80

Table 8.1 Growing bed planting and harvest dates.

beds. A simple economic model would choose to maximize this measure as lettuce is often sold by weight.

- **Average plant weight per head:** This is a surrogate for the quality of the head of lettuce. We also seek to maximize this variable.
- **Total fish biomass:** This is the sum of the biomass of all fish in the tank alive at the end of the simulation. A simple economic model would choose to maximize this measure as fish is often sold by weight.
- **Average fish biomass:** This is a surrogate for the health of the fish. We also seek to maximize this variable.
- **Fish mortality:** The total number of fish that have died throughout the simulation. For a moral model, we consider variable choices where mortality is non-zero to be infeasible. For an economic model, we simply wish to minimize this number.

Since we have five different objectives, which at times may conflict with one another, the true optimal solution will depend on an economic model that specifies a linear (or some other) combination of these objectives. We do not specify such a model here, as ranges of feasible solutions where the optimal will lie will be apparent even without specifying an economic model.

8.2 Experiment 1 - Fixed Stocking Number at Beginning of Simulation

For the first experiment, we fix the fish stocking number to 10 and simulate. The results of this experiment are contained in Figure 8.1. As can be seen, no matter the plants per bed selected, six of the fish will always die. As such, all planting densities could be considered infeasible from a morality perspective.

From an economics perspective, however, if we were to maximize total plant weight, fish biomass, and average fish biomass, our optimal solution would be to plant 200 or more heads of lettuce per bed. However, the quality of lettuce sharply declines after about 10 heads; as such, in order to maintain quality as well, we would recommend planting 10 to 40 heads of lettuce under these circumstances.

8.3 Experiment 2 - Staggered But Fixed Stocking Number

Upon closer investigation into Experiment 1, we find that the fish that die die early in the simulation. This happens because of the following problems:

- Bacteria populations are small at the beginning of the horizon, and must be given time to grow in order to efficiently convert ammonia into nitrates.
- Plant sizes are small in the first 15 days, where only one bed is growing and the plants are small. For the remainder of the simulation, two beds are growing plants and/or the plants on one bed are nearing the end of their growth.

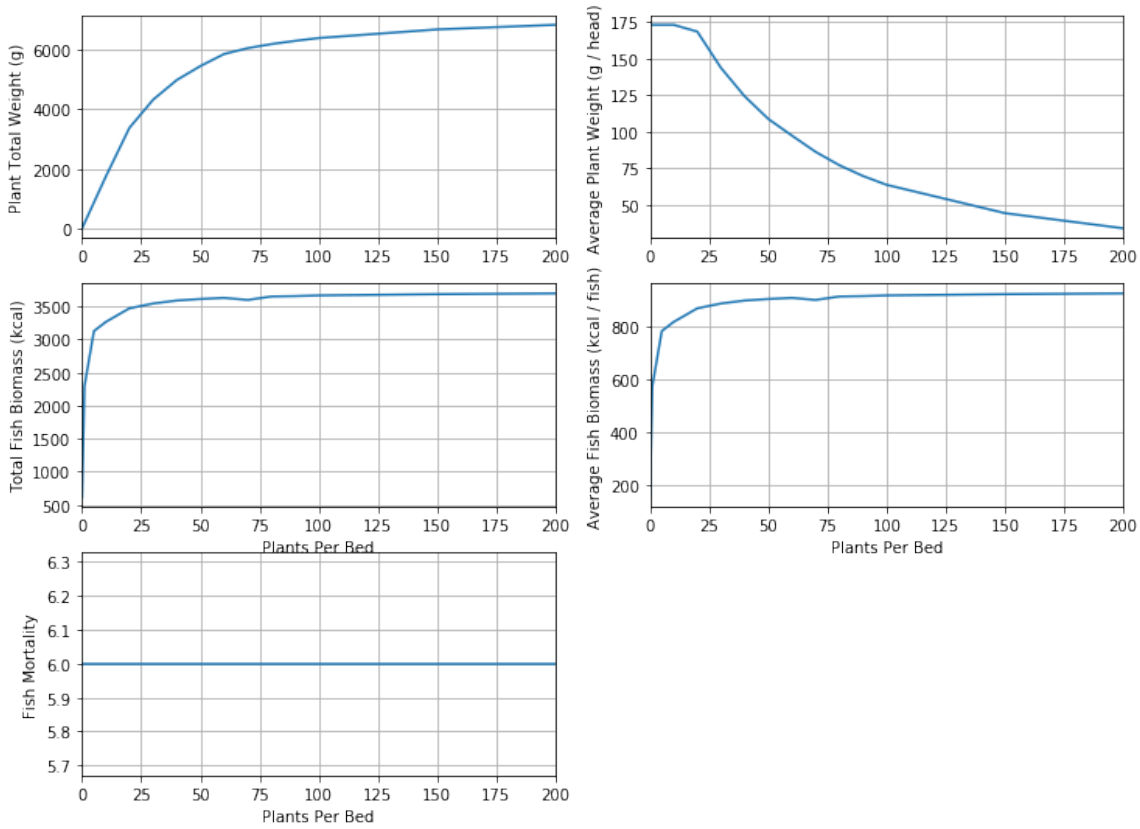


Figure 8.1 The results of Experiment 1. The optimal decision would be to plant roughly 10 to 40 heads of lettuce per bed.

These problems are seen in actual aquaponics systems, and strategies used to mitigate these problems are known as *seasoning*. There are several seasoning strategies available, including

- Taking water from an already seasoned system and using it in the new system. This essentially sets the initial bacteria populations to a higher value.
- Stocking large populations of fish at the beginning of a growing horizon, allow them to die, and then restocking the fish at a later date.
- Stocking smaller populations of fish at the beginning of a growing horizon (which allows the bacteria to grow without overwhelming the system with nitrogen) and then adding more fish at a future date

The first of these strategies does not solve the problem where plant sizes are small at the beginning of the horizon and the second is amoral. And so we choose to adopt the third.

In particular, we once again choose to stock 10 fish in total. We start the simulation stocking only 4 fish, and then 15 days into the simulation, we add the remaining 6. The results of this simulation are given in Figure 8.2.

Notice how this time if more than 5 plants per bed are planted, no fish die. Again, biomass and total plant weight are optimized by maximizing plants per bed. Quality of lettuce is also maximized for any planting density of 60 heads of lettuce per bed or less. Thus, we would recommend planting at exactly 60 heads of lettuce per bed in this situation.

8.4 Experiment 3 - Optimal Fish Stocking and Planting Density

We now continue Experiment 2, this time allowing the fish stocking number to change. Let n be the number of fish we choose to stock. As before, we add 4 fish at the beginning of the simulation and $n - 4$ fish on day 15.

The results of this experiment are contained in Figure 8.3. If more than 10 fish are added to the tank, then the mortality is at least 1 fish, no matter the number of plants per bed. Thus, if we consider fish mortality to be in-feasible, we would choose 10 fish maximum. Furthermore, as fish stocking increases, plant total weight, plant average weight, and fish biomass all increase, and so we would choose exactly 10 fish.

From Experiment 2 above, we already know that the optimal decision with 10 fish in the tank is to plant 60 heads of lettuce per bed.

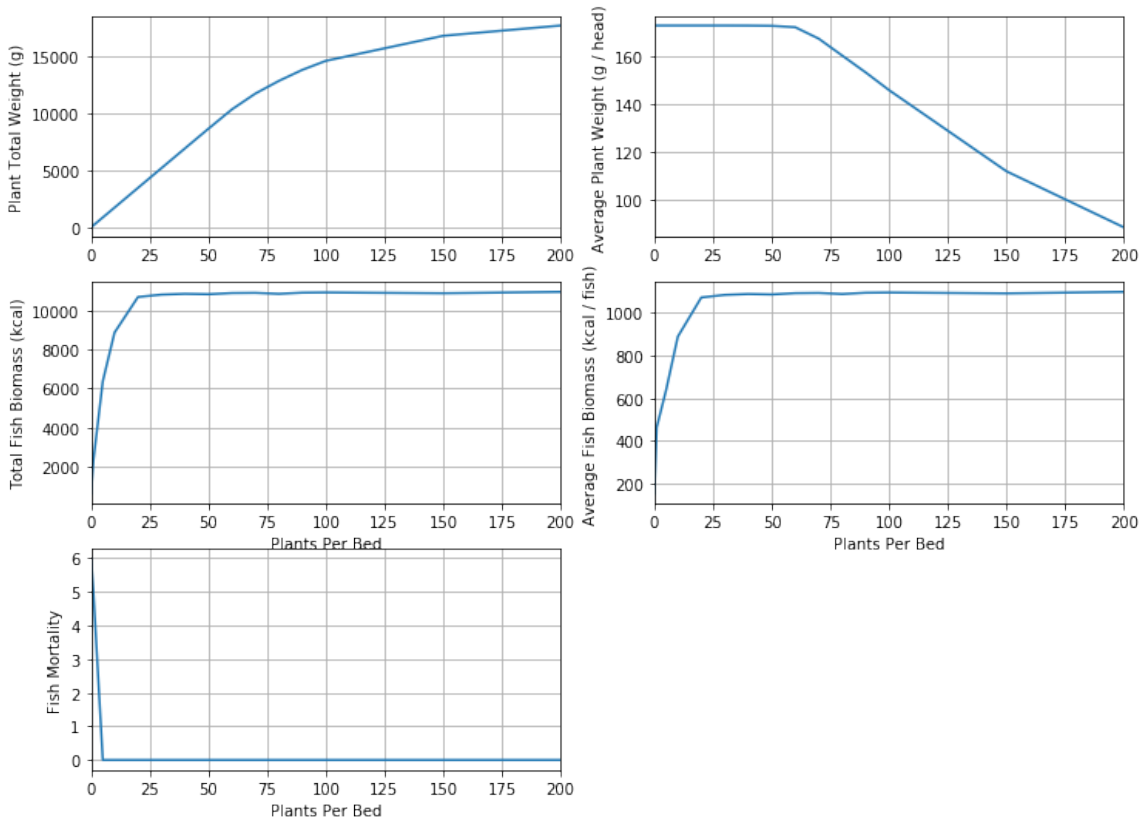


Figure 8.2 The results of Experiment 2. The optimal decision would be to plant 60 heads of lettuce per bed.

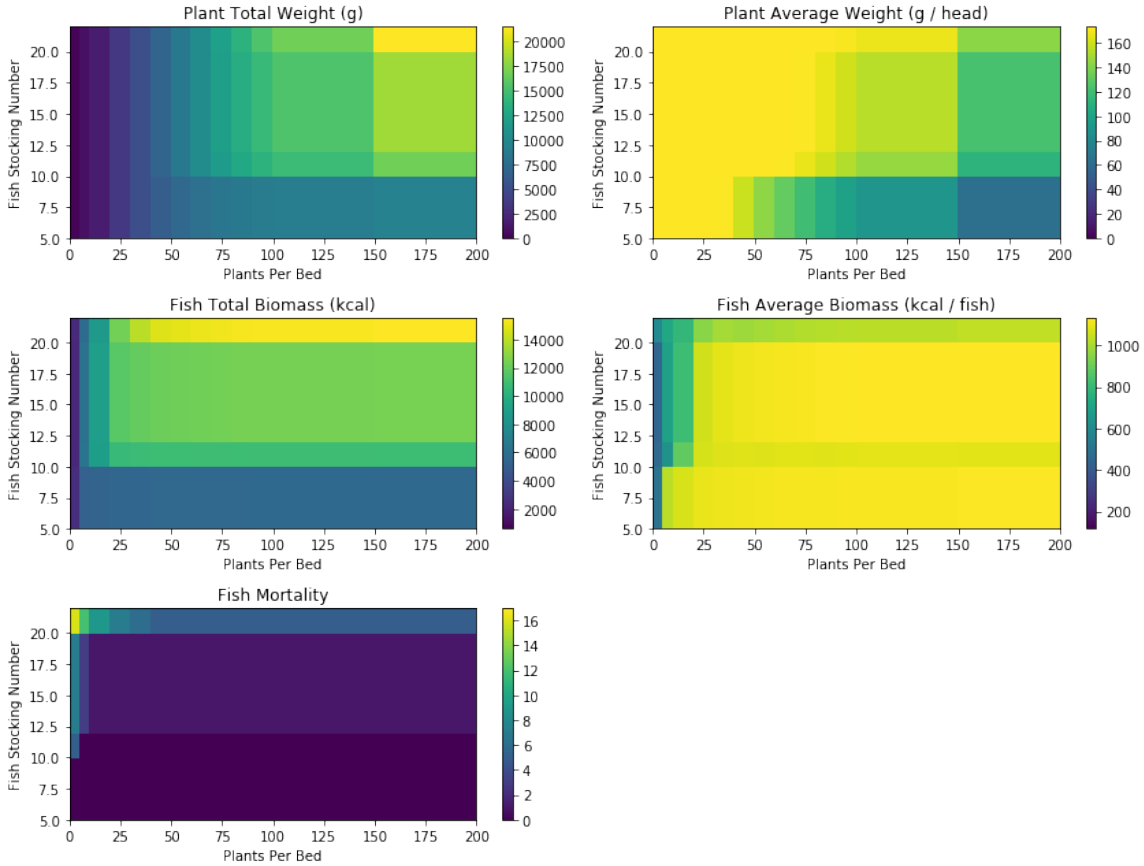


Figure 8.3 The results of Experiment 3. The optimal decision would be to plant 60 heads of lettuce per bed with 10 fish in the tank, same as experiemnt 2.

9

Conclusions

In conclusion, we have demonstrated the use of an aquaponics model to determine best-practices for growing lettuce in an aquaponics system. In particular, we found that the growth of lettuce is insensitive to temperature and solar radiation, as long as these values are within reasonable ranges; therefore, strong effort to control these is unnecessary. However, by increasing both to higher levels, nitrogen uptake requirements is reduced, which would allow for higher planting densities to be utilized.

We also showed that so long as the lettuce is not nitrogen starved, it will reach a saturation at full growth, thus strict nitrogen control is not terribly necessary either.

We then integrated this model with a model of fish and bacteria growth to form a model of an aquaponics system. Using this model, we demonstrated the ability to determine optimal decisions for number of heads of lettuce to grow and number of fish to stock given a fixed planting strategy and fish tank size. In particular, if we have three beds, each planted 10 days before the previous, each harvested after 30 days, and each replanted once immediately after harvest, and if we have a fish tank that contains roughly 180 liters of water, then the optimal strategy would be to add 10 fish to the tank and plant 60 heads of lettuce per growing bed.

Appendices

Appendix A

Source Code

The source code to generate all figures and results in this project is publicly found at `https://gitlab.com/nwoodbury/aquaponics`.

The `/aquaponics/Aquaponics.py` module is a wrapper around Gekko which registers user-specified aquaponics subcomponents, including their variables and equations, with Gekko. The aquaponics subcomponents are also found in the `/aquaponics/` directory. The Gekko extensions, described in Appendix B, are found in `/aquaponics/gekko_extensions.py` and are automatically registered with the `Aquaponics` wrapper. All experiments, including figures and plots, are contained in Jupyter notebooks found at `/notebooks/`.

Additional documentation is found within the source code itself.

Appendix B

Gekko Extensions

The dynamics of various components in the Aquaponics model contained piece-wise continuous functions which Gekko is unable to natively simulate. We have therefore created several extensions to Gekko using continuous approximations of these functions using logistics curves. We describe each of these functions as well as how to use them in Gekko here.

B.1 Implemented Gekko Extensions

The primary Gekko extension used is the *switch* function, all others are derivatives of this function. We define them here.

B.1.1 The Switch Function

The *switch* function essentially models a conditional. Formally, it is defined as

$$switch(left, right, on, loc) = \begin{cases} left & on \leq loc \\ right & on > loc \end{cases}. \tag{B.1}$$

We approximate the *switch* function using a logistics curve. Specifically, let

$$\sigma(on, loc) = \frac{1}{1 + e^{-k(on-loc)}}. \tag{B.2}$$

We have that, as $k \to \infty$,

$$\sigma(on, loc) = \begin{cases} 0 & on \leq loc \\ 1 & on > loc \end{cases}. \tag{B.3}$$

For smaller k, this holds true far from loc, and will have some value between 0 and 1 near loc. Thus smaller k is only an approximation of this switch. We choose $k = 100$ by default in the code, though the user can specify other values.

With $\sigma(on, loc)$ defined, we can implement *switch* as

$$switch(left, right, on, loc) = (1 - \sigma(on, loc)) \cdot left + \sigma(on, loc) \cdot right. \tag{B.4}$$

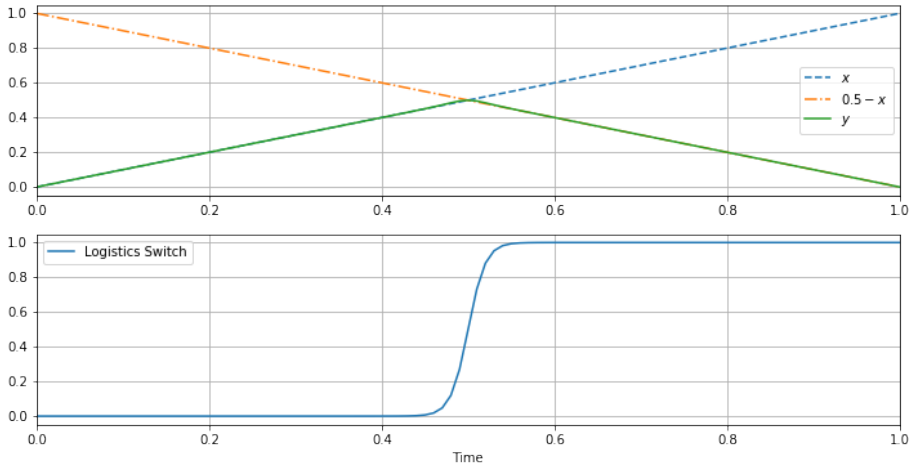


Figure B.1 The *switch* implementation in Equation (B.5). For further illustration, $\sigma(x, 0.5)$ is shown in the bottom figure.

We test *switch* with two sample systems. The first is

$$\begin{aligned} \dot{x} &= 1, \\ y &= switch(x, 1-x, x, 0.5) = \begin{cases} x & x \leq 0.5 \\ 1-x & x > 0.5 \end{cases}. \end{aligned} \tag{B.5}$$

In other words, $x(t)$ is a line of slope 1 starting at 0 on $t \in [0, 1]$, and $y(t)$ is a line of slope 1 starting at 0 on $t \in [0, 0.5]$ and $y(t)$ is a line of slope -1 on $t \in [0.5, 1]$ with $y(0.5) = 0.5$. Figure B.1 shows that GEKKO simulates this behavior perfectly.

The second example is a clamp function which involves the embedding of two switches. Mathematically, it is given by

$$\begin{aligned} \dot{x} &= x, \\ y &= \begin{cases} 0.3 & x \leq 0.3 \\ x & 0.3 < x \leq 0.7 \\ 0.7 & x > 0.7 \end{cases}. \end{aligned} \tag{B.6}$$

In essence, $x(t)$ grows exponentially, and $y(t)$ is $x(t)$ but clamped to be no less than 0.3 and no greater than 0.7. Figure (B.2) shows that, once again, GEKKO is capable of capturing this behavior perfectly.

Note that this second example can also be implemented using $y = clamp(x, 0.3, 0.7)$, where *clamp* is an alias for the nested switch described above.

B.1.2 The Max Function

The $max(left, right)$ function returns the maximum of its two arguments. Mathematically, it is written as

$$max(left, right) = \begin{cases} right & left \leq right \\ left & left < right \end{cases} = switch(right, left, left, right). \tag{B.7}$$

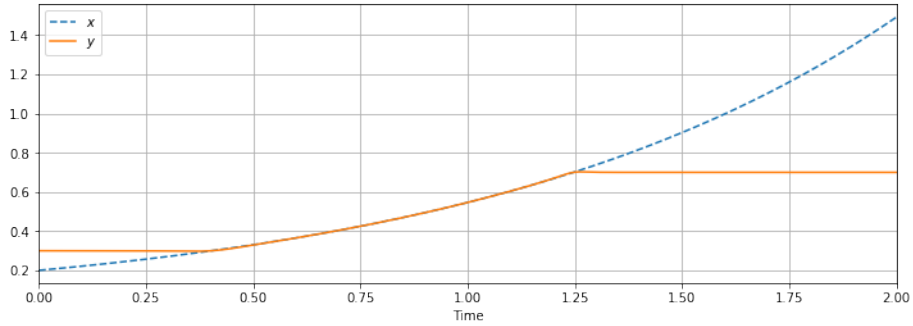


Figure B.2 The *switch* implementation in Equation (B.6).

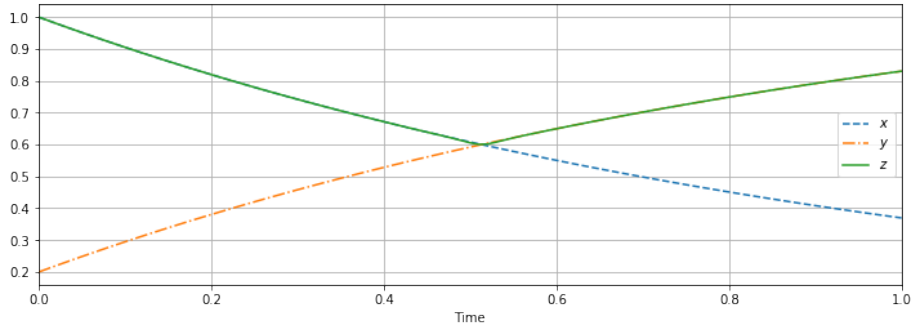


Figure B.3 The *max* implementation in Equation (B.8).

Thus, the *max* function can be implemented as a wrapper around the *switch* function.

To test this function, we implement the following system:

$$\begin{aligned} \dot{x} &= -x, \\ \dot{y} &= x, \\ z &= max(x, y). \end{aligned} \tag{B.8}$$

As shown in Figure B.3, GEKKO also approximates this function nearly perfectly.

B.1.3 The Min Function

The $max(left, right)$ function returns the minimum of its two arguments, and is very similar to the *max* function. Mathematically, it is written as

$$max(left, right) = \begin{cases} left & left \leq right \\ right & left < right \end{cases} = switch(left, right, left, right). \tag{B.9}$$

Thus, the *min* function can be implemented as a wrapper around the *switch* function.

To test this function, we implement the following system:

$$\begin{aligned} \dot{x} &= -x, \\ \dot{y} &= x, \\ z &= min(x, y). \end{aligned} \tag{B.10}$$

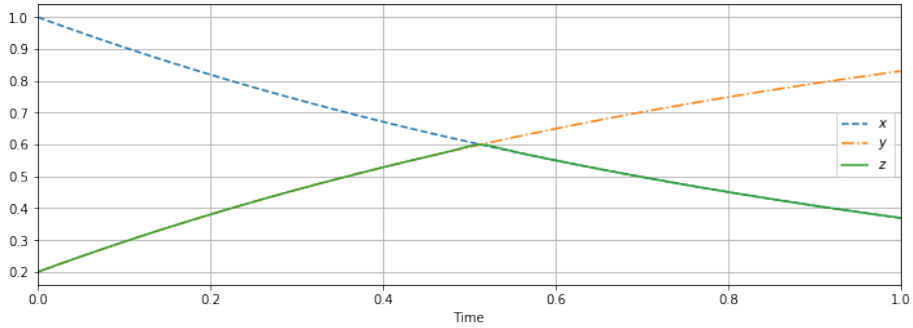


Figure B.4 The *min* implementation in Equation (B.10).

As shown in Figure B.4, GEKKO also approximates this function nearly perfectly.

B.2 Registering the Extensions with Gekko

The Gekko extensions (along with further documentation) are found in the Aquaponics source code (see Appendix A) at `/aquaponics/gekko_extensions.py`. With the Gekko model loaded as `m = Gekko()`, the extensions can then be loaded with `m = register_extensions(m)`. Then, the above functions can be accessed just like a normal function. For example, if `x` and `y` are defined as Gekko parameters, variables, or intermediaries, the max of these can be created with `z = m.max(x, y, k=100)`.

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