

Chemical Engineering Department  
Brigham Young University  
350 Clyde Building  
PO Box 24100  
Provo, UT 84602-4100  
(801) 422-2586 / FAX (801) 422-01



April 14<sup>th</sup>, 2016

Optimization Algorithm for Hybrid Vehicle Using APMonitor Optimization Suite

Dr. John Hedengren,

We have completed our final project: Optimization Algorithm for Hybrid Vehicle Using APMonitor Optimization Suite. The objective of this project is to minimize fuel usage in hybrid cars using synchronization with a GPS instrument. To develop this algorithm we began with developing a model to simulate the system. This involved gathering parameters from literature sources and developing a series of equations to describe the system. Using online software we gathered elevation profiles for multiple routes. Then using APMonitor and Python we developed an algorithm that was capable of decreasing the amount of gasoline used by up to 23%. This project primarily contributes the following:

- A method for gathering discretized elevation profiles for driving routes
- A simulator which uses actual elevation profiles to calculate work done by a vehicle
- An optimization algorithm which minimizes fuel consumption for a given driving route

We look forward to your feedback.

Regards,

Shane Gallagher  
Garret Laugenour  
Stephen Miller

# Optimization Algorithm for Hybrid Vehicle Using APMonitor Optimization Suite

Authors: Shane Gallagher, Garret Laugenour, Stephen Miller

## Abstract

The interest in hybrid vehicles has been increasing over the last few decades due to their excellent fuel efficiency and the general push to become less dependent on fossil fuels. This project aims to further increase the fuel efficiency of hybrid vehicles by minimizing the amount of fuel for a trip with a specified distance, velocity and elevation profile. The model uses an energy balance at many steps during the trip to accurately predict when the battery in the hybrid vehicle will be charged and discharged. Assumptions in the model parameters such as battery capacity and charge/discharge efficiencies are presented. The estimation of remaining variables such as vehicle mass is also discussed since these parameters will vary between vehicle make, model, and number of passengers. Using the parameters and the model, an optimization algorithm will be designed for the system. The implementation of an accurate model, an estimator and an optimization algorithm will allow the maximization of fuel efficiency for a given trip of a hybrid vehicle.

## Introduction

Conventional cars have always had a major drawback: as soon as the fuel is consumed to provide energy to accelerate the car, that energy can't be recovered. Hybrid cars provide a solution to this problem; when the car applies its braking mechanism to reduce its kinetic energy, that energy is collected by the breaks and used to charge the car's Li-ion battery. In this way, a percentage of the energy from the fuel is recycled (depending on efficiencies). In this way, the fuel consumption required can be greatly reduced through effective use of the rechargeable battery. Today, hybrid car systems are becoming more prevalent due to advancing technology and increased concern about reducing fuel usage. Optimization applications such as minimizing the fuel required to travel to a certain route by optimizing the engine and Li-ion battery integration would further increase the fuel economy appeal of hybrid vehicles.

The objective of this paper is to minimize fuel usage in hybrid cars using synchronization with a GPS instrument. To do this, data from the GPS and vehicle including elevation, position, and velocity are used to schedule the charging routine of the battery. Other parameters come from literature sources or estimation using artificial data (due to current lack of experimental data). The optimization objective was to minimize the final value of the integral of the work performed by the motor.

## Literature Review

Not surprisingly, a number of research articles specifically analyze and address the opportunity for optimizing fuel efficiency in hybrid cars. The ones discussed in this section utilize energy management strategies such as Dynamic Programming (DP), Equivalent Consumption Minimization Strategy (ECMS), and a Model Predictive Controller (MPC).

Han et al. present that the elevation profile of a road significantly influences the fuel economy of a hybrid car, due to the large changes in power demand. In their research, they utilize future altitude profile information to optimize the hybrid cars performance and battery scheduling along the

elevation profile. To do this, they apply two of the energy management strategies: DP as well as ECMS.<sup>1</sup>

A similar study by Yu et al. incorporates a MPC approach with slope information. The slope information contains both an elevation profile and a speed model. A full-order model was developed for the power-split hybrid electric vehicle system and results showed that the MPC approach effectively manages energy consumption with slope information.<sup>2</sup>

Sinoquet et al. focus on driving conditions without future information available. To do this, they also use the ECMS because it is a real-time control strategy. As part of their model, they include the engine as a second state, allowing for better representation of the engine's contribution and requirements. The battery is then considered as an auxiliary reversible fuel reservoir.<sup>3</sup>

A fourth study, by Yun et al., incorporates DP to design a control algorithm to optimize fuel consumption in a heavy class hybrid vehicle (such as a city transit bus). This control algorithm imitates the behavior of the DP control signal to actuate a CNG engine, generator, and battery. The objective of this algorithm is to minimize fuel consumption while maintaining the battery state of charge within a proper region. Their results show that the fuel economy can be enhanced by up to 30%. They conclude by proposing that future research should address a multi-variable DP to develop an advanced rule-based control algorithm.<sup>4</sup>

As mentioned before, optimizing fuel efficiency in hybrid electric vehicles is being thoroughly researched due to pressures to reduce dependency on fossil fuels. It is difficult to find an area that isn't already being studied. However, it seems like an integration of optimization methods still needs to be pursued. Most research (including ours) assume foreknowledge of the route conditions such as elevation and speed limit. These utilize the DP control method and are ideal for finding the overall optimal solution. Less research has focused on real-time control, utilizing the ECMS or similar methods. Real-time control is the most pertinent to real-life applications. Integrating both types of control to utilize the unique benefits of each would ensure applicable optimization to real driving situations.

The optimization program presented in this paper utilizes strategies from both Yu et al. and Yun et al. Elevation and speed information are initially collected using GPS. This information is then used in conjunction with a model that imitates the behavior of an engine and a battery to minimize the fuel consumption while maintaining an appropriate battery charge state. The program takes the entire route into account when finding the optimal battery schedule before even starting. In this way, it acts more as a feedforward optimizer than a model predictive controller.

---

<sup>1</sup> Han, J., Kum, D., & Park, Y. (2014). Impact of hilly road information on fuel economy of FCHEV based on parameterization of hilly roads.

<sup>2</sup> Yu, K., Yang, H., Kawabe, T., & Tan, X. (2015). Model predictive control of a power-split hybrid electric vehicle system with slope preview.

<sup>3</sup> Sinoquet, D., Rousseau, G., & Milhau, Y. (2009). Design optimization and optimal control for hybrid vehicles.

<sup>4</sup> Yun, S., Lee, K., & Yi, K. (2015). Development of a power management strategy to minimize the fuel consumption of a heavy-duty series hybrid electric vehicle.

## Theory / Methods

The theoretical analysis began with an overall energy balance for a vehicle traveling between two discretized time points. (Definition of Symbols provided below this section)

$$E_{motor} = KE + PE + FE + BE \quad (1)$$

This equation can be further expanded by applying the definitions of kinetic energy, potential energy and frictional losses.

$$E_{motor} = \left\{ \frac{1}{2} * m_{car} * (v_2^2 - v_1^2) \right\}_{KE} + \{g * m_{car} * (h_2 - h_1)\}_{PE} + \{C_{friction} * m_{car} * g * \cos(\theta) * (x_2 - x_1)\}_{FE} + BE \quad (2)$$

Equation 2 adjusts the energy balance to include  $v_1, v_2, h_1, h_2, x_1$  and  $x_2$  which are all parameters given to the system by the user. These parameters can be obtained using applications such as Google Earth, GPS Visualizer, and Geocontext. In addition,  $m_{car}$  and  $C_{friction}$  are parameters that were held constant and verified by research. The verification of parameters are discussed in the next section of this report. The variable  $\theta$  is an intermediate which is calculated from  $h_1, h_2, x_1$  and  $x_2$  using trigonometry. The frictional losses component of the model is highly simplified and further work can be done to increase frictional loss accuracy. Equation 2 would be sufficient for a system in which the engine and batteries operate at perfect efficiency; however, when efficiencies are incorporated into the system, the model is expanded even further to the following:

$$E_{motor} = \left[ \left\{ \frac{1}{2} * m_{car} * (v_2^2 - v_1^2) \right\}_{KE} + \{g * m_{car} * (h_2 - h_1)\}_{PE} + \{C_{friction} * m_{car} * g * \cos(\theta) * (x_2 - x_1)\}_{FE} - Int_C * \eta_C * BE \right] \frac{1}{\eta_M} + (1 - Int_C) * \eta_D * BE \quad (3)$$

Equation 3 is the final form of the energy balance used in the model. Next, equation 4 was applied to the model to ensure that the charge of the battery did not leave the permissible charge state range.

$$\frac{\Delta C_C}{\Delta t} = BE * Int_C - BE * (1 - Int_C) \quad \text{where } .25 * Capacity \leq C_C \leq Capacity \quad (4)$$

$$\text{minimize } \int E_{motor\_final} \quad (5)$$

Finally an objective was added to the model to minimize the overall energy used by the motor energy (Equation 5). This enables us to use the APOPT dynamic optimization solver in APMonitor Optimization Suite to find the optimal conditions for the charging profile of the battery. Unfortunately the model only works while  $Int_C$  is non-discretized and due to project time constraints a solution to this problem was not able to be found. Because an integer value of  $Int_C$  was no longer being required, the IPOPT dynamic optimization solver was used instead of APOPT. The next section will discuss how the accuracy of the results were improved by parameter verification.

### Definition of Symbols

- $x_1$  = the distance the vehicle has traveled at time  $t_1$
- $x_2$  = the distance the vehicle has traveled at time  $t_2$
- $h_1$  = the elevation of the vehicle at time  $t_1$
- $h_2$  = the elevation of the vehicle at time  $t_2$
- $v_1$  = the velocity of the vehicle at time  $t_1$

$v_2$  = the velocity of the vehicle at time  $t_2$   
 $m_{car}$  = the combine mass of the vehicle and passengers  
 $g$  = gravitational constant  
 $C_{friction}$  = coefficent of friction  
 $\theta$  = approximate slope of the road between time  $t_1$  and time  $t_2$   
 $KE$  = kinetic energy change of the vehicle due to velocity change  
 $PE$  = potential energy change of the vehicle due to elevation change  
 $FE$  = loss of kinetic energy due to friction over a change in distance  
 $BE$  = energy usage or expenditure by the battery over a change in time  
 $E_{motor}$  = the energy required by the motor  
 $\eta_M$  = efficiency of motor  
 $\eta_C$  = efficiency of charging the battery  
 $\eta_D$  = efficiency of discharging the battery  
 $Int_C$  = integer of charge, 1 for charging and 0 for discharging the battery  
 $C_C$  = current charge of battery

## Discussion

### Parameter Validation

The various parameters needed in this model were either found in literature or approximated from vehicle specifications. However, the coefficient of friction ( $C_{friction}$ ) was approximated due to the simplified energy balance. The Battery Max Capacity was found by using the specifications for an electric car, the Tesla Roadster. The Tesla Roadster is around 1,400 kg<sup>5</sup> and has a range of approximately 150 miles when traveling at 80 mph.<sup>6</sup> Using the effective coefficient of friction, this equates to approximately 3 MJ of energy stored in the battery as shown in Equation 6.

$$(0.001)(150 \text{ mi}) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) (3000 \text{ lb}) = 3.221 \text{ MJ} \quad (6)$$

The Tesla Roadster is a comparatively smaller car and 100% electric, therefore a 2000 kg mass of car and passenger assumption is realistic. Battery charging and discharging efficiencies were found in multiple sources and ranged between 80-90%. Therefore this model uses an 86% efficiency which was the most typical value reported.<sup>7,8</sup> Charging rates can vary depending upon the difference in voltage across the system and can also affect the efficiency. Max charge or discharge rates were not found to be documented in literature sources and have therefore not been included in this analysis.

---

<sup>5</sup> Tesla Roadster. Wikipedia. (2016)

<sup>6</sup> Roadster, C. (2010) How Far Can You Really Go in an Electric Vehicle.

<sup>7</sup> Charging Efficiency. Telsa Motors Forums. (2012)

<sup>8</sup> Sun, J. (2010) Car Battery Efficiencies.

Table 1: Model Parameters

Parameter	New
Mass of Car and Passengers	2000 kg
Coefficient of Friction (estimated to simplify model)	0.001
Battery Max Capacity	3,000,000 J
Battery Charge Efficiency	0.86
Battery Discharge Efficiency	0.86
Motor Efficiency	0.65

## Simulation and Estimation

In order to develop an accurate simulation of the model, real world elevation profiles were necessary. These are easily accessible through GPS applications such as Google Earth or GPS Visualizer. However, the difficult part is transferring this information to a CSV format to be utilized by the optimization algorithm. A website called Geocontext provides the solution to this problem. Using either Google Earth or GPS Visualizer, a route can be specified and saved as a KMZ or KML file. This file can then be imported into Geocontext which will then display the information for the elevation profile in a text format, which can easily be copied and pasted into a CSV file for use.

Using data collected from Google Earth about the elevation profile for a drive from Santa Cruz to Monterey California, a successful simulation was produced. The velocity, elevation, and energy profiles are shown in Figure 1. For this simulation a constant velocity was assumed with a varying elevation profile. This would simulate driving with cruise control on roads with many hills.

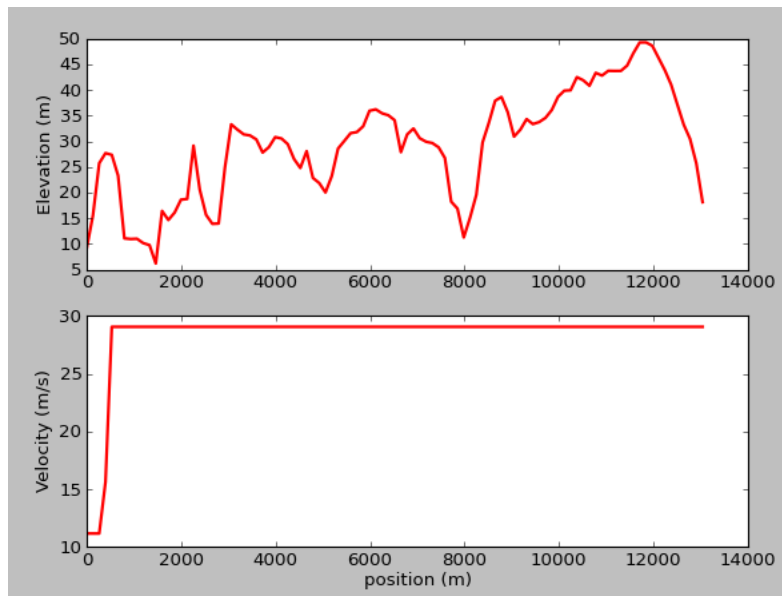
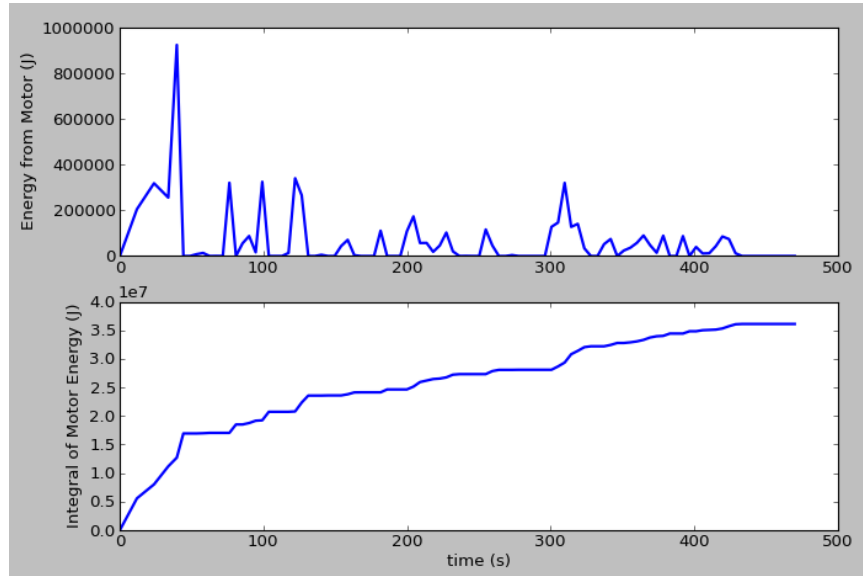


Figure 1: Elevation and velocity profiles of route.<sup>9</sup>

<sup>9</sup> Pietruszka, K. (2010, April). Geocontext.

Feeding this data into the model gives a profile for the work required by the motor of the vehicle. Taking the integral of the work shows the overall energy required by the motor for the designated route (see Figure 2).



*Figure 2: Results of trip without using any battery. (Top) Instantaneous energy required from the motor throughout the trip. (Bottom) Total amount of energy required from the motor throughout the trip.*

Since experimental data is currently unavailable to the authors of this paper to validate the parameters in Table 1, artificial data was produced in order to mimic how a potential controller could use data to update the parameters and improve the accuracy of the model. The simulation was used to produce a battery percent charged profile. This profile was then compared to the artificial data to demonstrate that the estimator could estimate parameters such as mass of the vehicle and passengers, which can vary from trip to trip. The artificial battery charge profile data and battery charge profile using the estimated vehicle mass are shown in Figure 3. Excel Solver was used to minimize the Sum Square Error to calculate the estimated parameters.

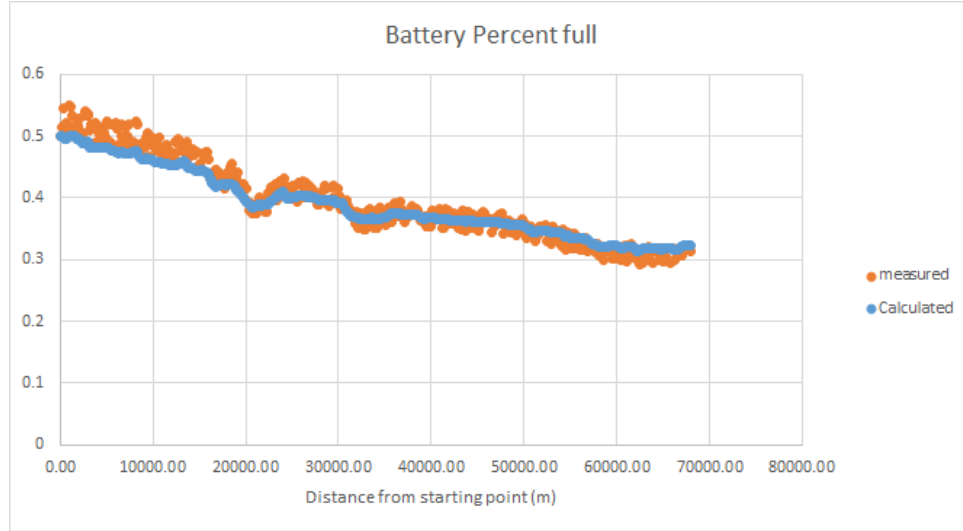


Figure 3: Artificial battery charge profile data used to estimate model parameters

### Optimization and Sensitivity Analysis

Applying these parameters and this model, an optimization algorithm to schedule the battery usage has been achieved. The optimization algorithm determines whether the battery is storing or discharging energy with the objective of minimizing the total amount of energy required from the motor throughout the trip. A successful solution of the optimized model is represented in the results displayed in Figure 4 and Figure 5.

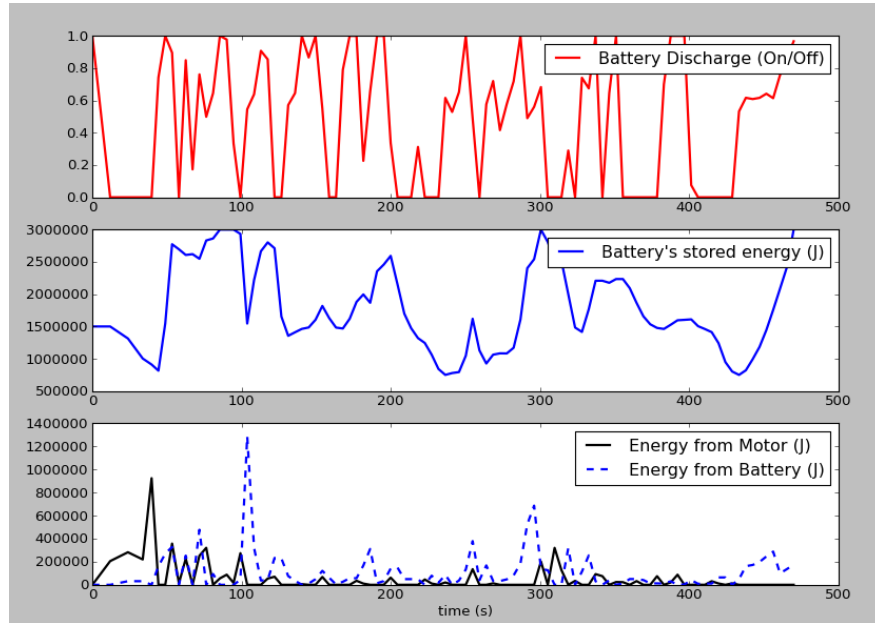
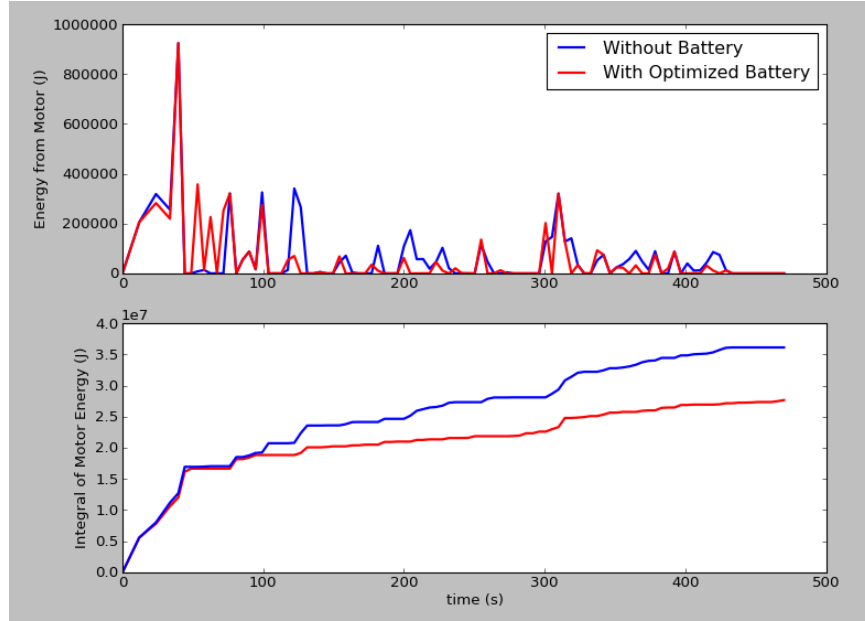


Figure 4: Results of trip with optimized battery scheduling. (Top) Battery state. 0 = Charging, 1 = Discharging. (Middle) Instantaneous amount of energy stored in battery. (Bottom) Comparison of energy used from motor and battery.





*Figure 5: Comparison of trip without battery and trip with optimized battery scheduling. (Top) Instantaneous energy required from motor throughout the trip. (Bottom) Total energy required from motor throughout the trip.*

As shown in the bottom plot of Figure 5, the optimized battery schedule successfully reduced the amount of energy required from the motor. The total energy required from the motor without using the battery was 36,110,940.0 J. The total energy required from the motor with the assistance of a scheduled battery was 27,657,270 J. This means that the optimized battery schedule saved 8,453,670 J, which amounts to 23.4% of the original amount required.

This simulation specifically tested the effects of a varying elevation profile. The velocity profile was held constant at 29 m/s (65 mph). This is shown in Figure 1. Additional trials performed to test the effects of varying velocity will be discussed next.

The middle plot in Figure 4 shows that the charge in the battery climbs to its peak at the end of the trip. While it makes more sense that the battery would discharge the last of its energy at the end of the trip, the final rise is explained by the decrease in the elevation profile during this time (as shown in Figure 1); energy from the motor was not required so the battery charged itself rather than discharging energy.

Finally, a sensitivity analysis was performed on this optimization algorithm to identify how changes in certain parameters would affect the solution. The results of this sensitivity analysis is summarized in Table 2. As shown, the mass of the vehicle had the smallest effect on the results of the optimization. This significant because an additional passenger for instance would not significantly change the results. The largest changes resulted from changing the effective Coefficient of Friction and efficiencies. These parameters are also likely not constant and therefore a more sophisticated model would be necessary to accurately describe how they change on a given route and how they affect the optimization.

Table 2: Sensitivity Analysis

	Using Original Parameters	Mass = 2100 kg	Coefficient of Friction = 0.01	Motor Efficiency = 0.6	Charge and Discharge Efficiency = 0.70
<b>Simulation</b>	36,110,940 J	37,916,488 J	44,582,608 J	39,120,180 J	36,110,950 J
<b>Optimization</b>	27,657,270 J	29,265,250 J	37,796,100 J	31,468,260 J	16,136,760 J
<b>Difference</b>	8,453,670 J	8,651,238 J	6,786,508 J	7,651,920 J	19,974,190 J
<b>% Saved</b>	23.41%	22.82%	15.22%	19.56%	55.31%

## Future work

A few adjustments still need to be made to the model. The most prominent of these is to make the battery discharge/charge variable ( $Int_c$ ) a Boolean type, specifically an integer type constrained to 0 and 1. This method has proven difficult for the solver to handle, so the current simplified version was required. Another important feature is to determine appropriate limits to the battery charge and discharge rates. This will greatly enhance the accuracy of the optimizer for real-life applications. A third adjustment that would be beneficial is to replace the current friction model with a more accurate one. This will specifically include a rolling friction factor. Finally, more tests should be performed to analyze the effect of changes in velocity on the optimized result. This report focuses on changes in elevation with a constant velocity profile, so another test to consider would be a varying velocity profile with a constant elevation. After this, a combination of the two varying profiles would be appropriate.

While the current program is an optimization algorithm, real-life applications would benefit more from a model predictive controller. Even a feedforward controller would have its drawbacks because variations during the traversal of the route are inevitable, thus decreasing the effectiveness of the initial optimization. While an initial optimization of the route is beneficial, a future version would work best using an integrated model that would update the optimized schedule throughout the time horizon of the route.

## Conclusions

In conclusion, this optimization algorithm is able to reduce the total energy used over a trip using the APMonitor Optimization Suite. Future work is necessary to stream line the code to decrease the solve time of the optimizer as well as continued research into developing a more sophisticated model to more accurately represent the physics of a hybrid vehicle. Also further testing is required to validate the optimizer under different conditions such as variable velocity. This optimization algorithm however lays the groundwork for a potential controller to use GPS information in addition to our model to minimize the total cost (fuel + battery replacement) of using a hybrid vehicle.

## Acknowledgments

We would like to thank Dr. John Hedengren, an assistant Professor of Chemical Engineering at Brigham Young University and creator of APMonitor Optimization Suite, as well as Dr. Aditya Tulsyan, a Postdoctoral Associate at the Massachusetts Institute of Technology Department of Chemical Engineering. Their mentoring through this project was essential to our success.

## References

- Awerbuch, Jonathan J. (2008) Control of Ultracapacitor-Battery Hybrid Power Source for Vehicular Applications. Conference on Global Sustainable Energy Infrastructure: Energy2030.
- Bo Long et al. (2014) Energy Management and Control of Electric Vehicles, Using Hybrid Power Source in Regenerative Braking Operation. *Energies*.
- Charging Efficiency. (2012) Tesla Motors Forums.  
<https://forums.teslamotors.com/forum/forums/charging-efficiency-0>
- Dao, T., Seaman, A., & McPhee, J. (2010) Mathematics-Based Modeling of a Series-Hybrid Electric Vehicle. 5th Asian Conference on Multibody Dynamics.
- Han, J., Kum, D., & Park, Y. (2014). Impact of hilly road information on fuel economy of FCHEV based on parameterization of hilly roads. *International Journal of Automotive Technology Int.J Automot. Technol.*, 15(2), 283-290.
- Hedengren, J.D., R. Asgharzadeh Shishavan, K.M. Powell, T.F. Edgar, (2014) Nonlinear Modeling, Estimation and Predictive Control in APMonitor, *Computers & Chemical Engineering*.
- Pietruszka, K. (2010, April). Geocontext. Retrieved April 13, 2016, <http://www.geocontext.org/>
- Roadster, C. (2010) How Far Can You Really Go in an Electric Vehicle.  
<http://www.pluginamerica.org/drivers-seat/how-far-can-you-really-go-electric-vehicle>.
- Schneider, A. (2003). GPS Visualizer: Do-It-Yourself Mapping. Retrieved April 13, 2016, from <http://www.gpsvisualizer.com/>
- Sinoquet, D., Rousseau, G., & Milhau, Y. (2009). Design optimization and optimal control for hybrid vehicles. *Optimization and Engineering Optim Eng*, 12(1-2), 199-213.
- Sun, J. (2010) Car Battery Efficiencies. Stanford University.  
<http://large.stanford.edu/courses/2010/ph240/sun1/>
- Tesla Roadster. Wikipedia. (2016) [https://en.wikipedia.org/wiki/Tesla\\_Roadster#Specifications](https://en.wikipedia.org/wiki/Tesla_Roadster#Specifications)
- Yu, K., Yang, H., Kawabe, T., & Tan, X. (2015). Model predictive control of a power-split hybrid electric vehicle system with slope preview. *Artif Life Robotics Artificial Life and Robotics*, 20(4), 305-314.
- Yun, S., Lee, K., & Yi, K. (2015). Development of a power management strategy to minimize the fuel consumption of a heavy-duty series hybrid electric vehicle. *Journal of Mechanical Science and Technology J Mech Sci Technol*, 29(10), 4399-4406.