Waterflood Optimization with Reduced Order Modeling

Jackson Udy

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Abstract

In this paper, the input output relationships of petroleum reservoirs under mature production are simulated using first order ordinary differential equations. Reservoir simulation seeks to understand the dynamics of petroleum reservoirs to determine optimal production strategies. Often these systems are simulated using finite element analysis using thousands of equations with millions of state variables. These simulations are computationally expensive and make optimization schemes impractical for large reservoirs with multiple wells. To reduce computational expense we model reservoir dynamics using the Capacitance Resistance Model (CRM) [1]. The CRM model is coupled with the Fractional Flow Model (FFM) to predict the fraction of production fluid that is oil. These simple first order approximations allow for optimization algorithms to be performed on the time scale of minutes and hours instead of days and weeks, giving engineers the ability to rapidly evaluate many scenarios throughout the life of the reservoir.

Keywords: Reduced Order Modeling, Non-linear estimation, Optimization, Enhanced Oil Recovery

1. Introduction

The CRM is a reduced-order model that allows for the evaluation and optimization of waterflood injection schemes over the time scale of months [2]. Only the injection and production data are required, although bottom hole data can be used to obtain a more accurate model [1] [2]. Mamghaderi et al. [3] developed a CRM model that accounts for the cross flow of reservoir fluids between reservoir layers. This increases the computation time and number of parameters of the model, but allows for more accurate production predictions to be made in layered reservoirs. The CRM model is best suited to legacy assets, allowing engineers to quickly and easily estimate the connectivity and time constants between wells. However the CRM parameters are time invariant, therefore the model may not predict well over the whole life of the well without refitting the parameters.

Other low order approximations of reservoir systems exist in the literature. Lee et al. used a finite impulse response model (FIR) to determine flow units between injection and production wells [4]. The FIR model requires a large number of parameters to achieve comparable accuracy with other empirical models, making it computationally inefficient. Lee et al. use a multivariate autoregressive model to quantify the relationship between injection and production wells. The model was found to handle noise better than the FIR Model [5]. An autoregressive model with two parameters per injector employs an extended Kalman filter to continually update the model parameters [6]. The filter is used to quickly infer relationships between wells and even determine faults and other geological heterogeneities. In the paper by Daoyuan, the study was furthered to validate this model and more easily determine relationships between injection and production wells [7]. A constrained Kalman filter is used to ensure that the injector-producer relationships are constrained to physically possible values. These data driven models allow engineers without prior knowledge of reservoir geology to understand the dynamics and infer geologic structures between different wells within the reservoir. The lack of fundamental insight provided by data driven models, and the inability to extrapolate beyond the training data, are weaknesses of data driven models when compared to physics based models. However, with constraints or other information to improve the models, considerable insight can be achieved.

In this paper, CRM parameters are estimated using a constrained solver to improve model accuracy. Constrained estimated allows for better model fit with less data when compared to unconstrained estimation. After model indentification, injection rates are optimized according to Net Present Value. Results and future work are discussed, highlighting the need for comparative studies and improved solvers.

2. Model Description

The CRM model is similar to the First order plus dead time (FOPDT) model common in process control. The model looks at the relationship between a single input (An injection well) and a single output (A production well). The model attempts to predict the flow rate of fluid out of the production well based on the variation in flow rate of water entering the reservoir from the injection well. Two parameters are used to fit the model, a connectivity and a time constant which are analogous to the gain and time constant of an FOPDT model. Figure 1 shows an example reservoir with four injection wells and two production wells. The relationship between each well is modeled by the equation below:

$$q_{ij} = f_{ij}I(t) - \tau_{ij}\frac{dq_{ij}(t)}{dt} - J_j\tau_{ij}\frac{dP_{wf}^{(i)}}{t} \tag{1}$$

Where: $q_{ij}(t)$ is the production of producer j attributed to injector i, f_{ij} is the connectivity or gain between injector i and producer j, $I(t)$ is the injection flow rate, τ_{ij} is the time constant between injector i and producer j, $p_{wf}^{(j)}$ is the bottom hole pressure at producer j, and J_{ij} is the productivity index, which can be defined as $q_{ij} = J_{ij}(p_{ij} - p_{wf}^{(j)})$ where p_{ij} is the average pressure for the control volume between producer j and injector i.

Alternatively, if we assume the control volume around the production well is geologically homogeneous, we can construct our control volume around each producer instead of between each injector producer pair:

$$q_{ij} = f_{ij}I(t) - \tau_j\frac{dq_{ij}(t)}{dt} - J_j\tau_j\frac{dP_{wf}^{(i)}}{t} \tag{2}$$

This reduces the number of parameters in our model to one gain value for each injector producer pair and one time constant for each injector. Further simplification can be made if we assume the bottom hole pressure to be constant over the time horizon. This reduces our model to the following form:

$$q_{ij} = f_{ij}I(t) - \tau_j\frac{dq_{ij}(t)}{dt} \tag{3}$$

where

$$\sum_{i=1}^{n} f_{ij} \leq 1 \tag{4}$$

and

$$\tau_j > 0 \tag{5}$$

are constraints to 4.

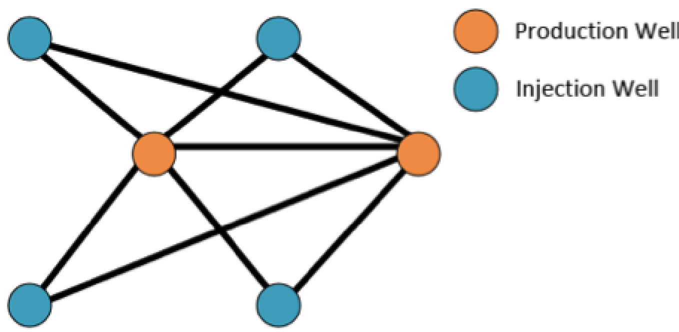

Figure 1: Example reservoir with 4 injectors and 2 producers. Each line represents a single equation with a single connectivity and time constant.

The CRM model relates injection flow rate to total production rate, however another model must be used to determine the oil/water cut in each producer. The Fractional Flow Model by Gentil (2005) is used in this study [8]:

$$qo_j(t) = \frac{1}{1 + a_j CWI_j^{b_j}} q_j \tag{6}$$

Where a_j and b_j are model parameters for each producer j.

3. Model Identification

Moving horizon estimation (MHE) is applied to both the CRM and FFM. Analysis of the fitting procedures is performed and explained. Noise is added to the synthetic data to simulate real data. A sensitivity analysis is performed to determine important parameters. Estimation is also performed on larger fields to better understand the scalability of constrained estimation. All estimation is executed using the APMonitor modeling language [9].

3.1. Estimation Methods

Different estimation methods are performed in this study. The CRM and FFM are fit to past dynamic data using MHE. MHE seeks to minimize the error between past data and model prediction, by adjusting unknown model parameters. In the CRM model these parameters are the gain and time constants. MHE in an optimization method, and can be implemented with various objective functions. Two of the most common objective functions are the squared error and l_1-norm objective functions (eq. 7 and 8 respectively).

$$min\Phi = (y_x - y)^T W_m (y_x - y) + \Delta p^T C_{\Delta P} + (y - \hat{y}) \tag{7}$$

$$min\Phi = W_m^T (e_u - e_l) + \Delta p^T C_{\Delta P} + W_p^T (c_u - c_l) \tag{8}$$

where 7 and 8 are subject too:

$$0 = f(\frac{dx}{dt}, x, y, p, d, u) \tag{9}$$

$$0 = g(x, y, p, d, u) \tag{10}$$

$$0 \leq h(x, y, p, d, u) \tag{11}$$

In the application described in this paper, cost of movement $c_{\Delta p}$ and W_p are set to zero because the estimation is offline and the optimal parameter values are desired regardless of parameter and solution movement. This reduces the two equations above to:

$$min\Phi = (y_x - y)^T W_m (y_x - y) \tag{12}$$

$$min\Phi = W_m^T (e_u - e_l) \tag{13}$$

Equations 12 and 13 are the objective functions used in this paper for estimation of CRM and FFM parameters and are subject to equations 9, 10, and 11.

3.2. Estimation Results and Sensitivity Analysis

Figures 2 and 3 shows the estimation results from the moving horizon estimation using the summed squared error objective function. The reservoir in this analysis is based on the SPE 10 benchmark field and consists of 2 injection and two production wells. The wells are placed close together, so system dynamics are fast. The fast dynamics and simplicity of the system allow us to fit our model with excellent precision. The l_1-norm objective provides similar results for both wells. Table 3.2 displays the solve times for this 2x2 system.

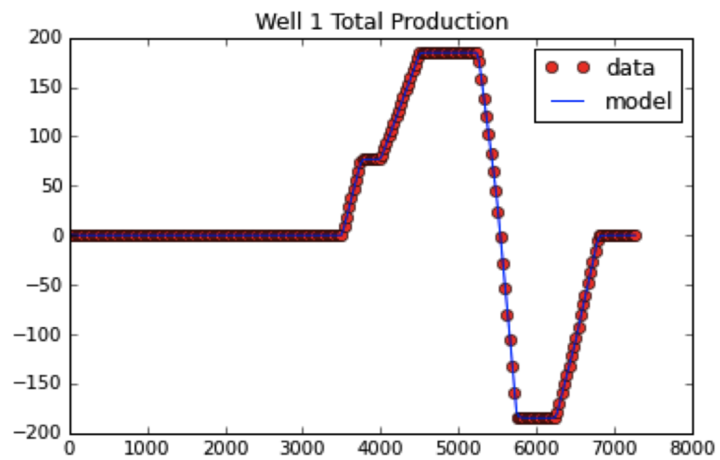

Figure 2: CRM fit with summed squared error objective.

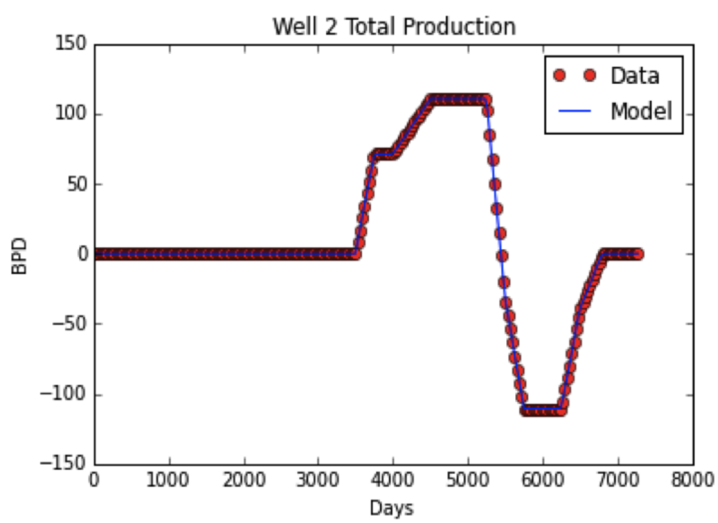

Figure 3: CRM fit with summed squared error objective.

	Constrained	Unconstrained
APOPT	8.3	0.7
IPOPT	2.5	1.3
BPOPT	2.7	1.9

Table 1: Solve times for the parameter estimation of the 2x2 reservoir system in seconds

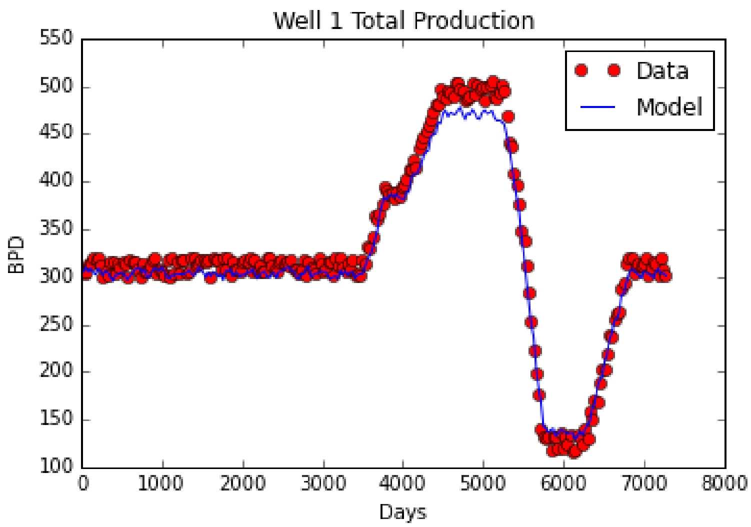

Figure 4: CRM fit with noisy data: Production Well 1.

The data used in 2 and 3 is from the CMG simulator and is noise free. Real field data has significant noise due to measurement inaccuracy, sensor noise, broken sensors, and other factors. Figures 4 and 5 show the fit to noisy data using the l_1-norm objective function.

MHE is also used to predict FFM parameters to a high degree of accuracy. Figure 6 and 7 compare the model prediction with simulator data. There is good agreement between the model and data.

A sensitivity analysis demonstrates how variations in parameters affect the quality of model fit with the data. Figure 8 and **??** show the system sensitivity to changes in the gain values. The reservoir model is heavily influenced by gain values therefore accurate estimation of these parameters is important for accuracy. Proper perturbation of the system that excites all of the dynamics of the system is important to properly fit the gain values. The model time constants are much less sensitive to change. In figure 10, a

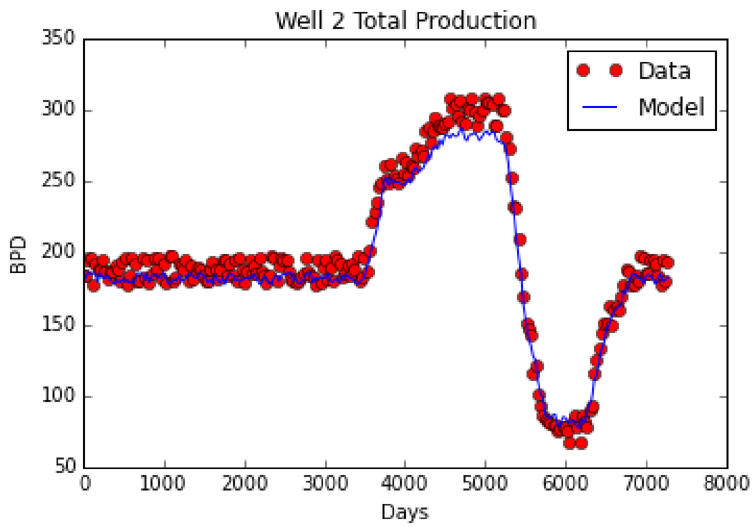

Figure 5: CRM fit with noisy data: Production Well 2.

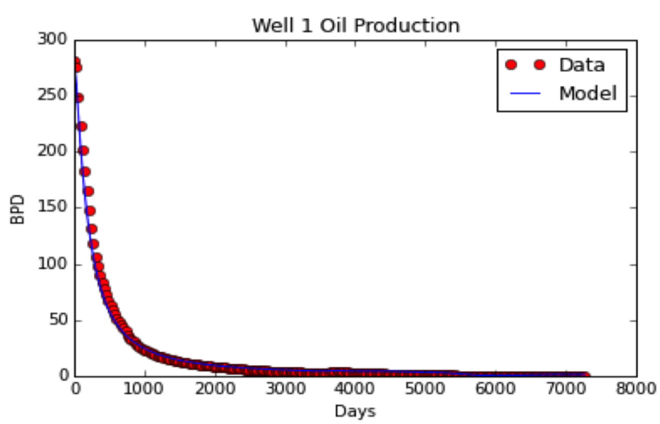

Figure 6: FFM fit for production well 1 with l_1-norm objective.

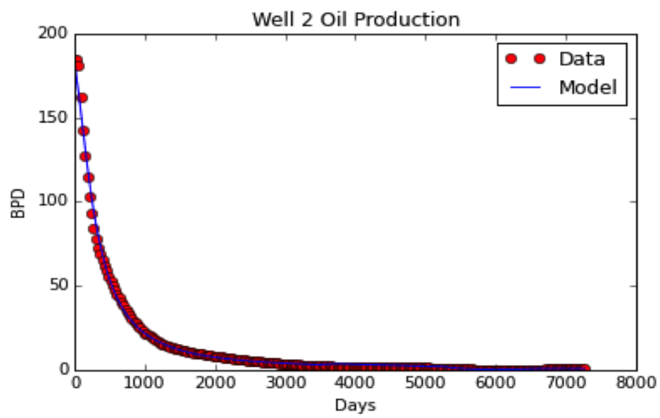

Figure 7: FFM fit for production well 2 with l_1-norm objective.

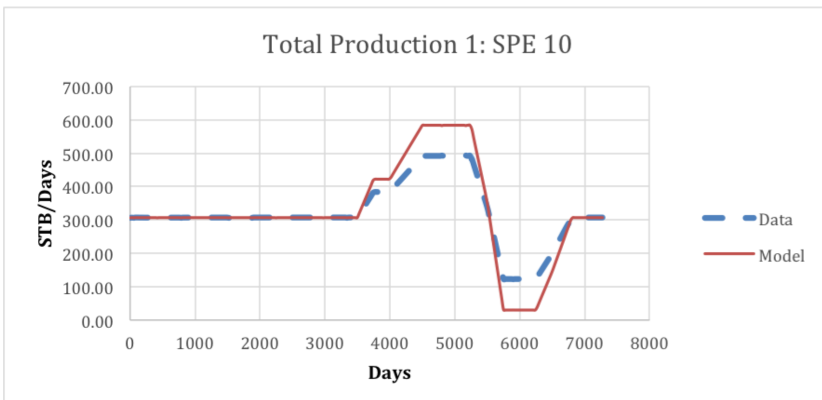

Figure 8: Model gains increased by 150%.

1000% increase in values has a small impact on the model.

MHE accurately predicts model parameters in the CRM and FFM. The l_1-norm objective function outperforms the SSE objective function when noise is introduced to the data. The CRM model is very sensitive to changes in gain but is quite insensitive to changes in time constant on this synthetic reservoir. MHE accurately predicts the parameters for the FFM.

3.3. Larger Systems

The methods described in section 3.1 are scaled to larger systems to see the effects of constrained estimation on solve time and model accuracy.

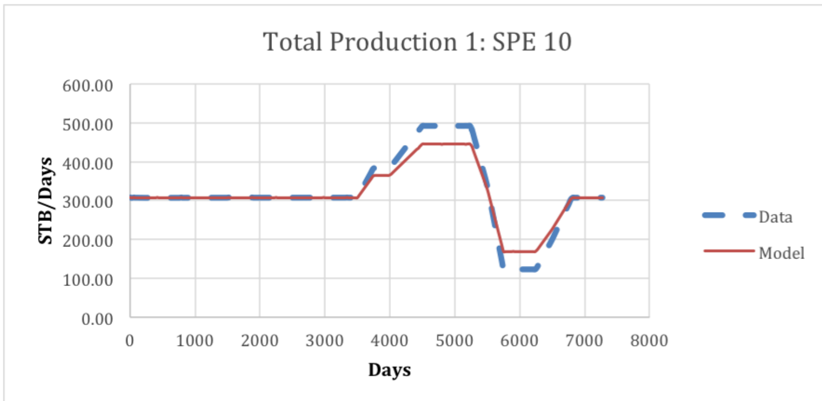

Figure 9: Model gains decreased by 25%.

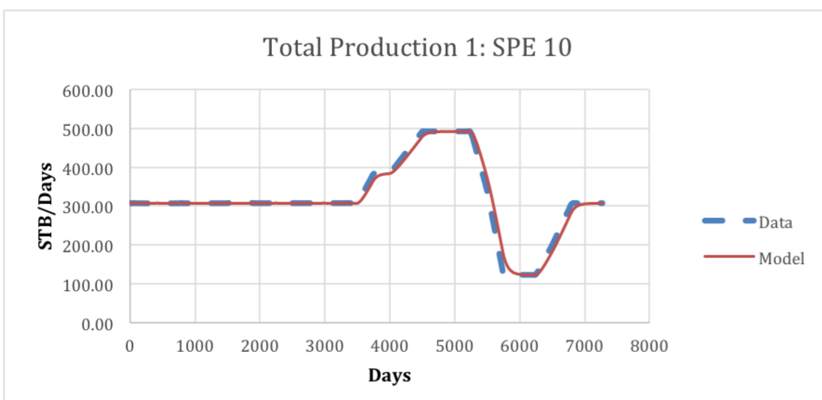

Figure 10: Model time constants increased by 1000%.

Estimation was performed on a four injector, four producer field and a eight injector, eight producer field. Field data for both simulations was gathered using the SPE 10 benchmark with the CMG black oil simulator. Table 3.3 shows the solve times for both constrained and unconstrained estimation of the four injector four producer field. The four injector four producer field contains 3672 variables and 3640 equations, with 48 estimated parameters. Table 3.3 shows the the solve times for the eight injector, eight producer field. The eight injector eight producer field has 11744 variables and 11616 equations, with 128 estimated parameters. The 8x8 system was solved using the squared error objective function to improve simulation time. Simulation for most scenarios on this field were improved by an order of magnitude when compared to the l_1-norm objective function. Figure 11 shows the solve time for the 2,4,8 and producer system using the l_1-norm and SSE objective functions. Significant improvements in solve time are achieved using the SSE objective function.

	Constrained	**Unconstrained**
APOPT	34.9	19.3
IPOPT	4.5	4.2
BPOPT	7.0	6.8

Table 2: Solve times for the 4x4 reservoir system in seconds (l_1-norm objective)

	Constrained	**Unconstrained**
APOPT	32	56
IPOPT	34	4.8
BPOPT	Did not converge	Did not converge

Table 3: Solve times for the 8x8 reservoir system in seconds (Squared Error Objective)

4. Optimization

4.1.

Optimization Equations and Theory

Reservoir optimization seeks to maximize the value of a particular reservoir by producing as much oil as quickly as possible at the lowest cost. Net Present Value (NPV) is the most common method in finance to quantify

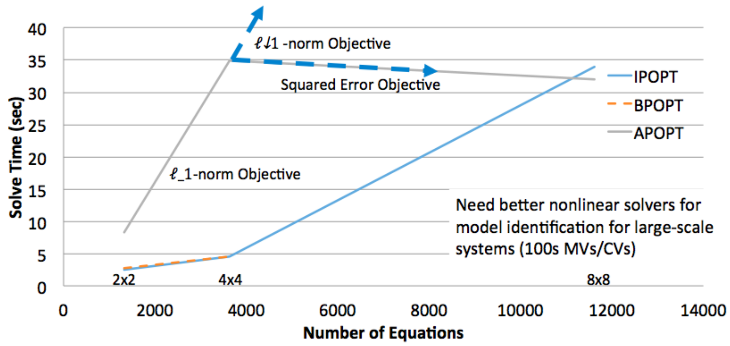

Figure 11: Comparison of solve times for various solvers using both the l_1-norm and SSE objective functions. Note that BPOPT did not converge for the 8x8 system.

value. NPV is the present value of all future cash flows from the reservoir, and is defined as:

$$NPV = \sum_{n=1}^{m} (Revenue_n - Expenses_n) \frac{(1+r)^n - 1}{r} \tag{14}$$

Where r is the required rate of return and n is the number of years in the future the cash flow will occur. In this study we use the continuous time version of this equation as shown below.

$$NPV = \int_0^{T_final} (Revenue(t) - Expenses(t)) e^{-rt} dt \tag{15}$$

Revenue and Expenses are defined as:

$$Revenue = P_{oil}(t) q_{oi}(t) \tag{16}$$

$$Expenses = P_{water}(t) I_i(t) \tag{17}$$

where j is the number of production wells, k is the number of injection wells, P_{oil} is the price of oil, P_{water} is the price of water, q_{oi} is the amount of oil produced from production well i, and I_i is the amount of water injected at injection well k. q_{oi} is calculated from the Fractional Flow Model and I_i is the manipulated variable for the optimization problem. We also restrict the values of I_i to be greater than 0 and less than 1000 STB/Day.

4.2. Results and Discussion

The CRM and FFM are fitted to a small synthetic oil field and NPV is optimized by adjusting injection flow rate. The APOPT solver successfully finds a solution to the objective function by varying the injection flow rates. Figure 12 depicts the optimized NPV for the two injector two producer reservoir. Figures 13 and 14 show the production and injection profiles for the same reservoir. Both injection wells have a total gain close to one, meaning that all of the water injected into the well is not lost to the reservoir but instead returns to the surface at the production wells (See Table 4.2). However Figure 14 shows that it is optimal to inject more water into well two than well one.

	Gain11	**Gain21**	**Tau11**	**Tau21**
Production Well 1	-9.22	-1.9	0.109	-0.022
	Gain12	**Gain22**	**Tau12**	**Tau22**
Production Well 2	-7.37	0.279	0.023	-0.063

Table 4: Sensitivities for the 2x2 reservoir system

	Producer 1	**Producer 2**	**Total**
Injector 1	0.732	0.268	1.00
Injector 2	0.520	0.480	1.00

Table 5: Model gains for injector producer pairs

A sensitivity analysis performed on the reservoir reveals how NPV is affected by changes in injection rate. Table 4.2 shows the sensitivity of NPV to injection rates in each well. Well two has a much larger effect on the NPV of the reservoir when compared to well one. For this reason the optimizer chooses to inject more from well two than from well one.

Sensitivities	**NPV**
Well 1	-0.020735
Well 2	-48.391

Table 6: Objective function sensitivity to changes in injection rate at both injection wells.

Assumptions in the objective function can lead to non-optimal performance. One of the difficulties of finding the optimal solution is predicting

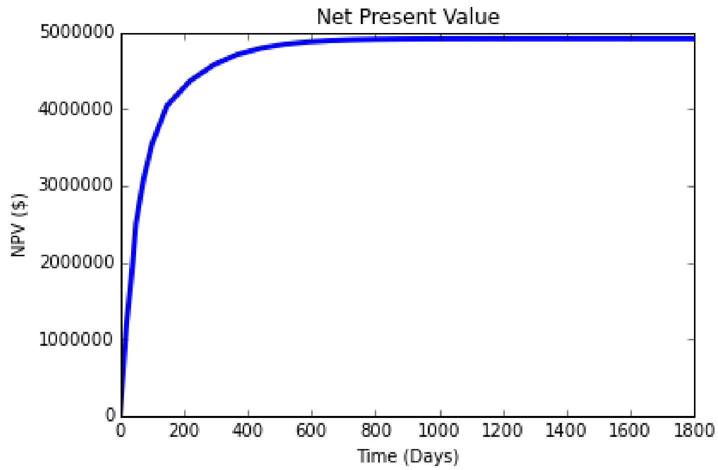

Figure 12: Optimal NPV for a synthetic two injector two producer field

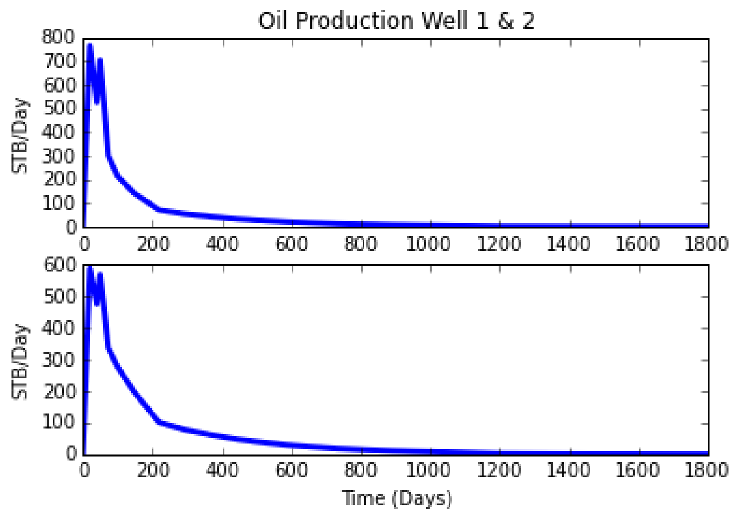

Figure 13: Optimal Oil Production for a synthetic two injector two producer field

the future price of oil and water. For example, an oil over supply may make it more economical for the reservoir to produce at lower levels until the price of oil rebounds, even though the NPV function discounts future cash flows. Conversely, in a high oil price environment, it may be favorable to produce more oil quickly at the expense of water breakthrough in the reservoir, and leaving oil trapped in the reservoir. These types of price events are difficult to

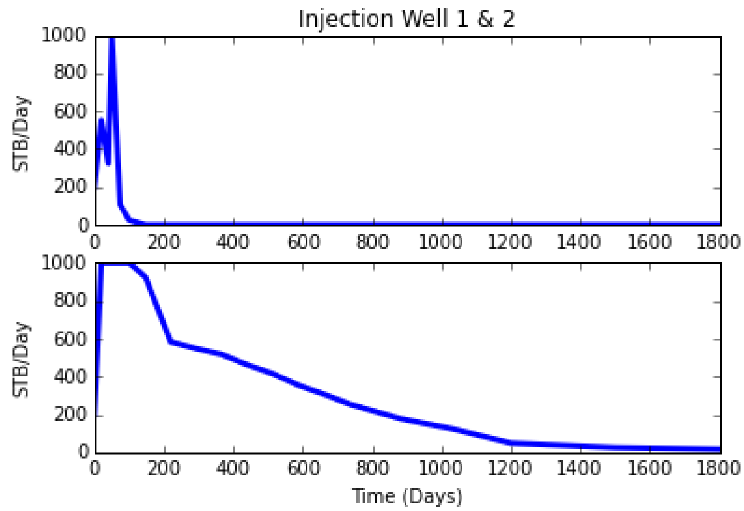

Figure 14: Optimal injection schedule for a synthetic two injector two producer field

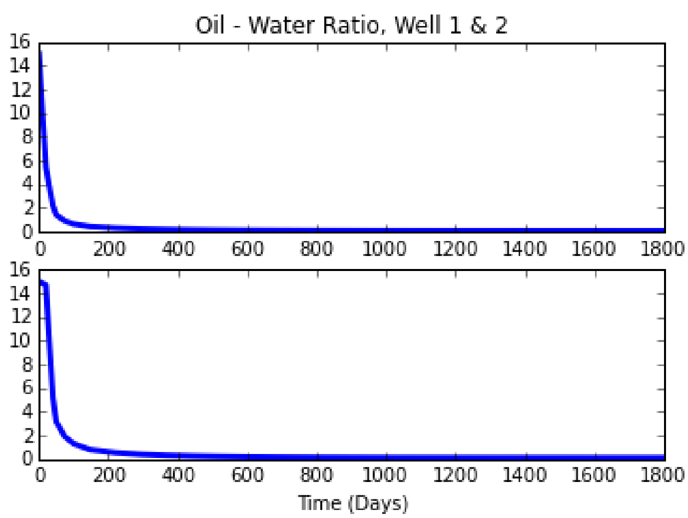

Figure 15: Oil - Water ratio for a synthetic two injector two producer field

187 model and introduce a significant amount of error into the optimal solution.

5. Conclusion and Future Work

Optimization coupled with estimation and modeling provides a methodology for engineers and managers to reduce water usage, shut in ineffective wells and increase oil production. In this study, constrained estimation and optimization were implemented to optimize the economic value of a small synthetic reservoir. The optimization was performed at one time step, however in real reservoirs, optimization can occur many times. As new data is received from oil fields, estimation and optimization procedures may be repeated to achieve better model fits, and new injection profiles scheduled into the future. Closed loop control using model predictive control may be implemented, however given the timescale of these systems, may not be necessary in most cases. Nevertheless closed loop control may provide benefits by calculating the optimal responses to disturbances such as well shut ins, and unplanned well maintenance.

The linearity of the models used in this optimization scheme limit their validity to certain production and injection rates. In real reservoirs, wells are periodically shut in for maintenance, providing step data for dynamic estimation. However if injection rates are small, extrapolating the model to higher flow rates creates inaccuracies due to the non-linearity of the reservoir system. It is also important to fit these models to data during the 'mature' reservoir phase as the reservoir behaves more linearly during this phase [1]. Low order non-linear models may provide improved fit, such as the auto-regressive exogenous inputs (ARX) model. However ARX models are strictly empirical and do not factor in information such as bottom-hole pressure, unlike the CRM. A direct comparison of low order models should be made in future work to determine accuracy in various reservoir types.

The systems modeled in this study were small and highlight the need for improved solvers. The largest reservoir in this paper is an eight injector eight producer field. In this case only 128 parameters were estimated. In a 100 injector 100 producer system, 20,000 parameters would need to be estimated. Current nonlinear solvers may struggle to solve problems of this scale quickly. Improvements in solver design are a potential solution to solve large non-linear systems.

6. Nomenclature

Φ - objective function

y_x - system measurements
y - model measurements
$\hat{y}$ - prior model values
W_m - measurement deviation
W_p - penalty for movement from prior solution
$C_{\Delta P}$ - parameter movement penalty
ΔP - change in parameters
e_u, e_l - slack variables above and below measurement deadband
c_u, c_l - slack variables above and below a previous model value
x, u, p, d - states, inputs, parameters, and disturbances
f, g, h - equation residuals, output function, and inequality constraints

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