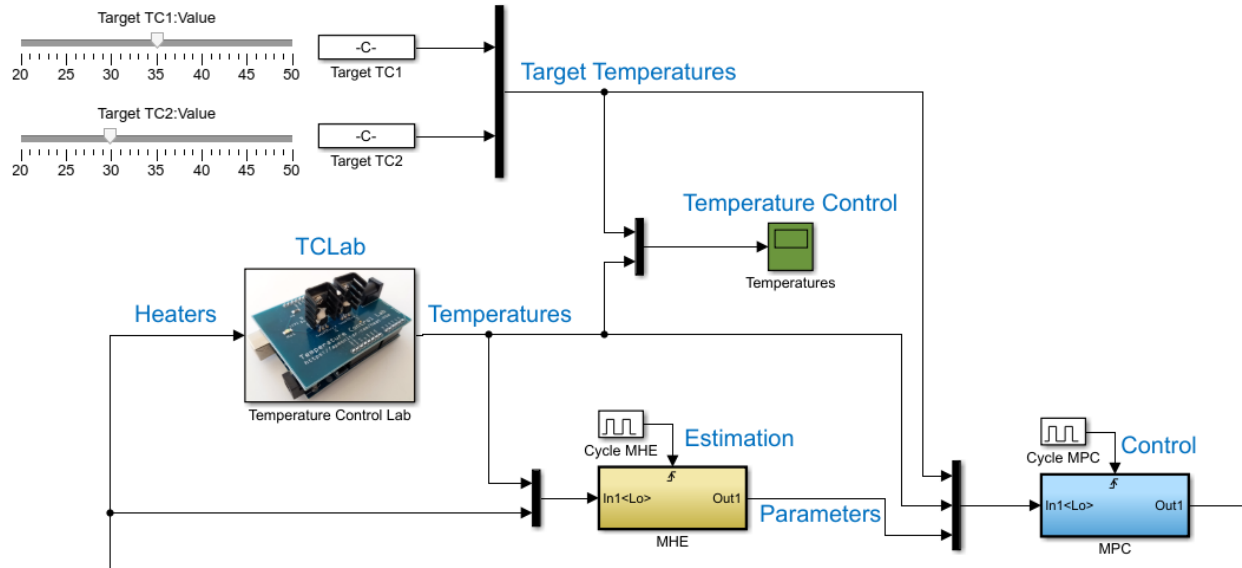


## Temperature Control Lab H: MHE and MPC

**Objective:** Combine Moving Horizon Estimation (MHE) and Model Predictive Control (MPC) into one unified application for the Temperature Control lab. This document describes a possible hybrid first-principles and empirical model that was also detailed in Labs E and G but another model form can be used. The combined MHE and MPC is shown in the flowchart Figure 1. A simple combined MHE and MPC application is available at <http://apmonitor.com/do/index.php/Main/EstimatorTypes> in Python and MATLAB.

### Adjust Temperature Targets



**Figure 1.** Flowchart for Combined MHE and MPC

The adjustable inputs to the system are the desired temperatures for sensors 1 and 2. The TCLab block has heater inputs ( $Q_1$  and  $Q_2$ ) and reports the new temperature values ( $T_{c1}$  and  $T_{c2}$ ). The measured temperatures are given to the MHE and MPC applications. MHE reads the temperatures and current heater values to predict updated parameters ( $U$ ,  $\tau$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $T_\infty$ ) every cycle time (4 sec recommended). MPC reads the target temperatures (2), measured temperatures (2), and updated parameters (5) for a total of 9 inputs. MPC produces new heater values every cycle time (4 sec recommended). MHE and MPC do not need to operate at the same cycle time but typically operate successively where the MHE updates the new parameters and MPC updates the manipulated variables at regular intervals. The new heater values from MPC are injected into the system (TCLab) and the cycle repeats every cycle interval.

**Table 1.** Parameters

Adjustable from MHE	Default Values
Overall Heat Transfer Coefficient ( $U$ )	4.05 W/m <sup>2</sup> -K
Empirical time constant ( $\tau$ )	15.4 sec
Heater Factor ( $\alpha_1$ )	0.0061 W/%
Heater Factor ( $\alpha_2$ )	0.0031 W/%
Ambient temperature ( $T_\infty$ )	23 °C
Manipulated Variables from MPC	Range
Heater Output ( $Q_1$ )	0 to 100%
Heater Output ( $Q_2$ )	0 to 100%
Common Constants	Values

Initial temperature ( $T_0$ )	23 °C
Heat Capacity ( $C_p$ )	500 J/kg-K
Surface Area Not Between Heat Sinks ( $A$ )	$1 \times 10^{-3} \text{ m}^2$
Surface Area Between Heat Sinks ( $A_s$ )	$2 \times 10^{-4} \text{ m}^2$
Mass ( $m$ )	0.004 kg
Emissivity ( $\epsilon$ )	0.9
Stefan Boltzmann Constant ( $\sigma$ )	$5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4$

Four differential equations are one possible description of the temperature dynamics. The first two equations come from an energy balance for each heater.

$$mC_p \frac{dT_{h1}}{dt} = UA (T_\infty - T_{h1}) + \epsilon \sigma A (T_\infty^4 - T_{h1}^4) + UA_s (T_{h2} - T_{h1}) + \epsilon \sigma A_s (T_{h2}^4 - T_{h1}^4) + \alpha_1 Q_1$$

$$mC_p \frac{dT_{h2}}{dt} = UA (T_\infty - T_{h2}) + \epsilon \sigma A (T_\infty^4 - T_{h2}^4) + UA_s (T_{h1} - T_{h2}) + \epsilon \sigma A_s (T_{h1}^4 - T_{h2}^4) + \alpha_2 Q_2$$

Additional 2 empirical equations capture the temperature delay between the heater and the temperature sensor as:

$$\tau \frac{dT_{c1}}{dt} = -T_{c1} + T_{h1} \quad \text{and} \quad \tau \frac{dT_{c2}}{dt} = -T_{c2} + T_{h2}$$

where  $T_c$  is the temperature of the sensor and  $T_h$  is the temperature of the heater. The additional number 1 or 2 refers to which heater and sensor.  $T_{c1}$  is the temperature of sensor 1 and  $T_{c2}$  is the temperature of sensor 2. The heater temperatures,  $T_{h1}$  and  $T_{h2}$ , are predicted from the equations but are not directly measured.

With this model or a custom model, implement the combined MHE and MPC application that follows set point changes for both temperatures. It is recommended to use a 15 minute test period where each set point changes 2-3 times during the testing. Include a plot of the set point values ( $T_{1,sp}$  and  $T_{2,sp}$ ), measured temperatures ( $T_1$  and  $T_2$ ), and heater values ( $Q_1$  and  $Q_2$ ). Discuss the estimator and controller performance.

### **Questions for Consideration**

1. This lab combines the parameter estimation and predictive control in a unified approach. How does the temperature control perform relative to Lab G (Nonlinear MPC) where the parameter estimation is performed before the controller is implemented?
2. How does MHE affect MPC when there is overly aggressive tracking of the measured temperatures with large parameter changes?
3. How does MPC affect MHE when there are no set point changes for a sustained period of time?

See <https://github.com/APMonitor/arduino> (7\_MHE\_and\_MPC) for source code examples related to this lab.