<u>Temperature Control Lab D: Empirical Estimation</u>

Up to this point, the temperature control lab analysis has been with a first principles model, derived from an energy balance equation. Another popular approach to model identification is to use an empirical model form to derive dynamic input to output relationships. This exercise is to fit a Single Input, Single Output (SISO) empirical model described by the three parameters through Moving Horizon Estimation or parameter estimation from batch data:

- Gain (K)
- Time Constant (τ)
- Damping Factor (ζ)

These three parameters are the unknown parameters in the second order system:

$$\tau^2 \frac{dT_c^2}{d^2t} + 2\zeta\tau \frac{dT_c}{dt} + (T_c - T_a) = K\,Q \tag{1}$$

with input heater (Q), temperature (T_c), and ambient temperature (T_a). An overdamped second order system can also be represented as a set of two first order differential equations with two time constants (τ_1 and τ_2):

$$\tau_1 \frac{dT_h}{dt} + (T_h - T_a) = K\,Q \tag{2}$$

$$\tau_2 \frac{dT_c}{dt} + (T_c - T_a) = T_h - T_a \tag{3}$$

where temperature of the sensor (T_c) has a different value than the temperature of the heater (T_h). The two differential equations relate to the second order equation with $\tau^2 = \tau_1\tau_2$ and $2\zeta\tau = (\tau_1 + \tau_2)$. Use either the second order equation (Equation 1) or the two separate equations (Equation 2 and 3) to estimate K, τ_1, and τ_2 or else the one equation to estimate K, τ, and ζ. Although we are limiting this exercise to just 3 parameters, identification methods can estimate higher order systems. These higher order empirical systems are then analyzed to determine the appropriate order for each input to output relationship.

<u>Questions for Consideration</u>

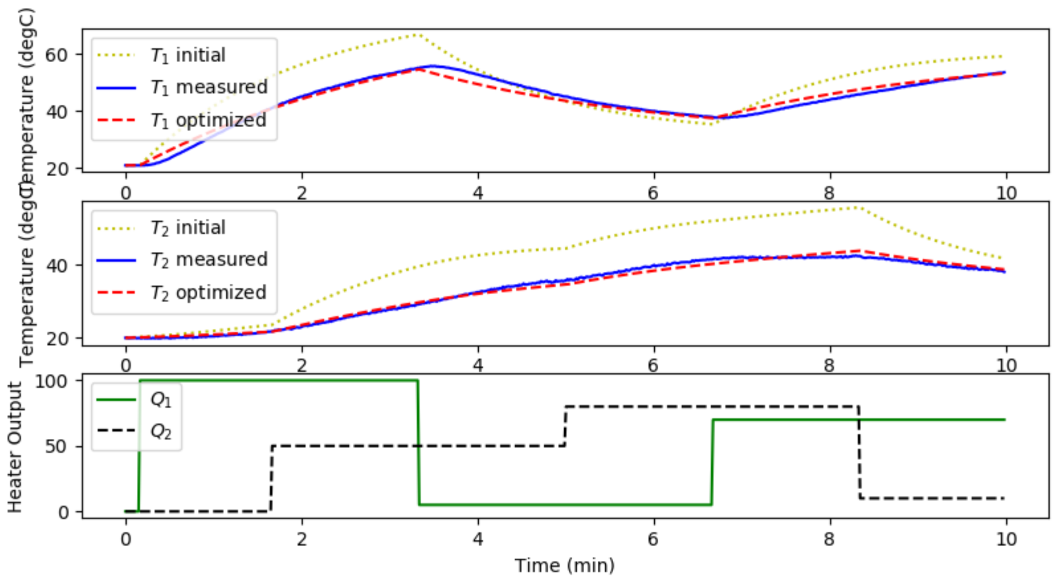
1. A first order model such as the beginning energy balance approach, responds immediately to changes in the heater output while the measured values appear to have a delay in the response. What is a physical justification for selecting a 2nd order (or higher order) equation versus using a 1st order equation?

2. What other empirical model forms (ARX, FIR, State Space) could be used to perform model identification? See the Github repository for a variety of regression methods (empirical and first principles) associated with this lab, including source code for this exercise (2nd Order Folders).

   Parameter Regression: https://github.com/APMonitor/arduino/tree/master/2_Regression
   MHE: https://github.com/APMonitor/arduino/tree/master/5_Moving_Horizon_Estimation

3. The second order system for this exercise is linear set of equations. Are there any nonlinear aspects to the data that don't fit the linear predicted response such as at high temperature versus low temperature?

4. How well does the empirical model fit the TCLab data versus the MIMO energy balance model from [Lab C](https://github.com/APMonitor/arduino/tree/master/2_Regression/Energy_balance_MIMO)?



Lab C Source: https://github.com/APMonitor/arduino/tree/master/2_Regression/Energy_balance_MIMO