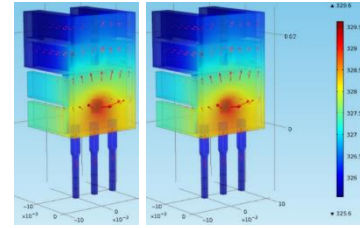
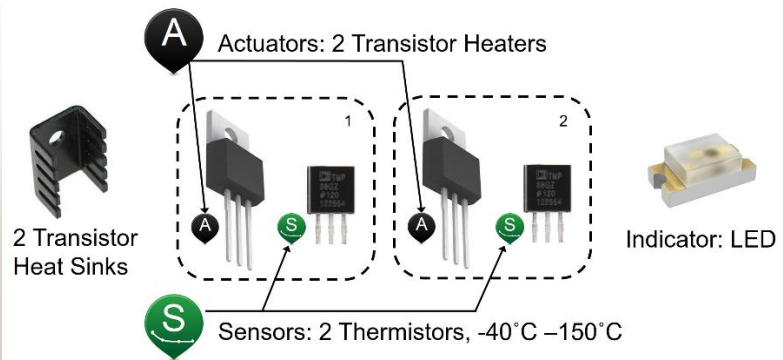


Temperature Control Lab B: MIMO (Multiple Input, Multiple Output) Model

Develop a transient model between the two heater power outputs and the two temperature sensors. An energy balance describes the transient temperature response of a single heater with one temperature sensor. The heater and temperature sensor are assumed to be at the same temperature. You can assume that conduction is negligible and that the only heat transferred is through radiation to the surroundings or convection or radiation to the surrounding air or from the heater nearby. The heaters are initially off and the heaters and sensors are initially at ambient temperature. Use the following values:



Quantity	Value	Value (SI Units)
Initial temperature (T_0)	23°C	296.15 K
Ambient temperature (T_∞)	23°C	296.15 K
Heater Output (Q)	0 to 1 W	0 to 1 W
Heat Capacity (C_p)	500 J/kg-K	500 J/kg-K
Surface Area Not Between Heat Sinks (A)	10 cm ²	1x10 ⁻³ m ²
Surface Area Between Heat Sinks (A_s)	2 cm ²	2x10 ⁻⁴ m ²
Mass (m)	4 gm	0.004 kg
Overall Heat Transfer Coefficient (U)	10 W/m ² -K	10 W/m ² -K
Emissivity (ϵ)	0.9	0.9
Stefan Boltzmann Constant (σ)	5.67x10 ⁻⁸ W/m ² -K ⁴	5.67x10 ⁻⁸ W/m ² -K ⁴



Use the following energy balance equation with convective heat transfer, radiative heat transfer, and the heater energy inputs. The additional blue terms are heat transfer

$$mC_p \frac{dT_1}{dt} = UA(T_\infty - T_1) + \epsilon \sigma A(T_\infty^4 - T_1^4) + UA_s(T_2 - T_1) + \epsilon \sigma A_s(T_2^4 - T_1^4) + Q_1$$

$$mC_p \frac{dT_2}{dt} = UA(T_\infty - T_2) + \epsilon \sigma A(T_\infty^4 - T_2^4) + UA_s(T_1 - T_2) + \epsilon \sigma A_s(T_1^4 - T_2^4) + Q_2$$

Simulate a temperature change when changing the heater output from 0.0 to 1.0 W for Q_1 at $t=0.1$ min and from 0.0 to 0.75 W for Q_2 at $t=5.0$ min. Continue the simulation period for 10 minutes. Show the temperature responses.

If you have a temperature control lab kit, compare the simulated response to the actual response by setting each heater output to 100% at the appropriate times ($Q_1 = 100\%$ at $t=0.1$ min and $Q_2 = 100\%$ at $t=5.0$ min). The heater 1 is more powerful than heater 2 by design. The heater output limits are set in the Arduino firmware (tclab.ino) for Python labs and tclab.m for the MATLAB labs.

Questions for consideration:

How much effect does Q_1 have on T_2 starting at $t=0.1$ min?

How much effect does Q_2 have on T_1 starting at $t=5$ min?

How would the response change if you were to slide a coin between the two heat sinks so that there is a continuous metallic interface between the two heaters? How would you change the model equations?

Is model error due to incorrect parameters or due to incorrect equations?

See <http://apmonitor.com/pdc/index.php/Main/ArduinoModeling2> for source code solutions (if needed).