Proportional Integral (PI) Control
The PI Controller

“Ideal” form of the PI Controller

\[
CO = CO_{\text{bias}} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt
\]

where:
- \( CO \) = controller output signal
- \( CO_{\text{bias}} \) = controller bias or null value
- \( PV \) = measured process variable
- \( SP \) = set point
- \( e(t) \) = controller error = \( SP - PV \)
- \( K_c \) = controller gain (a tuning parameter)
- \( \tau_I \) = controller **reset time** (a tuning parameter)

- \( \tau_I \) is in denominator so smaller values provide a larger weighting to the integral term
- \( \tau_I \) has units of time, and therefore is always positive
Function of the Proportional Term

- The proportional term, $K_c \cdot e(t)$, immediately impacts CO based on the size of $e(t)$ at a particular time $t$

- The past history and current trajectory of the controller error have no influence on the proportional term computation
Class Exercise – Calculate Error and Integral

![Graph of Setpoint (SP) and Process Variable (PV)]
Control Calculation is Based on Error, $e(t)$

- Here is identical data plotted two ways
- To the right is a plot of error, where: $e(t) = \text{SP} - \text{PV}$
- Error $e(t)$ continually changes size and sign with time

![Graph showing error $e(t)$ with points $e(25) = 4$, $e(40) = -2$.]
Function of the Integral Term

• The integral term continually sums up error, e(t)

• Through constant summing, integral action accumulates influence based on how long and how far the measured PV has been from SP over time.

• Even a small error, if it persists, will have a sum total that grows over time and the amount added to CO$_{bias}$ will similarly grow.

• The continual summing of integration starts from the moment the controller is put in automatic
Integral Term Continually Sums the Value: SP – PV

- The integral is the sum of the area between SP and PV
- At t = 32 min, when the PV first reaches the SP, the integral is:

\[
\int_{0}^{32 \text{ min}} e(t) \, dt = 135
\]

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Integral of Error is the Same as Integral of: \( SP - PV \)

- At \( t = 60 \) min, the total integral is: \( 135 - 34 = 101 \)
- When the dynamics have ended, \( e(t) \) is constant at zero and the total integral has a final residual value: \( 135 - 34 + 7 = 108 \)
Advantage of PI Control – No Offset

• The PI controller stops computing changes in CO when $e(t)$ equals zero for a sustained period

$$CO = CO_{bias} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt$$

• At that point, the proportional term equals zero, and the integral term may have a residual value

$$CO = CO_{bias} + 0 + \frac{K_c}{\tau_I} (108)$$

*Integral acts as “moving bias” term*

• This residual value, when added to $CO_{bias}$, essentially creates an overall “moving bias” that tracks changes in operating level

• This moving bias eliminates offset, making PI control the most widely used industry algorithm
Disadvantages of PI Control - Interaction

- Integral action tends to increase the oscillatory or rolling behavior of the PV

- There are two tuning parameters ($K_c$ and $\tau_I$) and they interact with each other

\[
C_O = C_{O_{bias}} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt
\]

- This interaction can make it challenging to arrive at “best” tuning values
PI Controller Tuning Guide (Figure 8.9)
Integral Action and Reset Windup

• The math makes it possible for the error sum (the integral) to grow very large.

\[ CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I} \int e(t) dt \]

• The integral term can grow so large that the total CO signal stops making sense (it can be signaling for a valve to be open 120% or negative 15%)

• “Windup” is when the CO grows to exceed the valve limits because the integral has reached a huge positive/negative value

• It is associated with the integral term, so it is called reset windup

• The controller can’t regulate the process until the error changes sign and the integral term shrinks sufficiently so that the CO value again makes sense (moves between 0 – 100%).
Reset Windup and Jacketing Logic

• Industrial controllers employ jacketing logic to halt integration when the CO reaches a maximum or minimum value

• Beware if you program your own controller because reset windup is a trap that novices fall into time and again

• If two controllers trade off regulation of a single PV (e.g. select control; override control), jacketing logic must instruct the inactive controller to stop integrating. Otherwise, that controller’s integral term can wind up.
Evaluating Controller Performance

- Bioreactors can’t tolerate sudden operating changes because the fragile living cell cultures could die.
  » “good” control means PV moves \textit{slowly}

- Packaging/filling stations can be unreliable. Upstream process must ramp back quickly if a container filling station goes down.
  » “good” control means PV moves \textit{quickly}

- The operator or engineer defines what is good or best control performance based on their knowledge of:
  - goals of production
  - capabilities of the process
  - impact on down stream units
  - desires of management
Performance Analysis

- Rise Time = When PV first reaches SP
- Peak Time = Time of first peak
- Overshoot Ratio = B/A
- Decay Ratio = C/B
- Settling Time = Time when PV remains < 5% of SP
Class Exercise

- Calculate:
  - Rise Time
  - Peak Time
  - Overshoot Ratio
  - Decay Ratio
  - Settling Time

![Graph showing a process variable vs. time with points A, B, and C highlighted.](image)
Performance Analysis - Time Related Criteria

• The clock for time related events begins when the SP is stepped

- Rise time
- Peak time
- Settling time

±5% of ΔPV
Performance Analysis - Time Related Criteria

- \( t_{\text{rise}} = 43 - 30 = 13 \text{ min} \)
- \( t_{\text{peak}} = 51 - 30 = 13 \text{ min} \)
- \( t_{\text{settle}} = 100 - 30 = 70 \text{ min} \)
Performance Analysis - Peak Related Criteria

- A = (30 - 20) = 10%
- B = (34.5 - 30) = 4.5%
- C = (31 - 30) = 1%

Here:
- Overshoot = 4.5/10 = 0.45 or 45%
- Decay ratio = 1/4.5 = 0.22 or 22%
Performance Analysis Note

• The classical criteria are not independent:
  • if decay ratio is large, then likely will have a long settling time
  • if rise time is long, then likely will have a long peak time
Performance Analysis – What If No Peaks?

- Old rule-of-thumb is to design for a 10% Overshoot Ratio and/or a 25% decay ratio (called a quarter decay)
- Yet many modern operations want no PV overshoot at all, making $B = C = 0$
- With no peaks, the performance criteria are of limited value