Derivative Action and PID Control

Learn in This Section

• Essential Elements of the PID Controller
• Derivative on Measurement is Used in Practice
• PV Noise Degrades Derivative Action
• Case Study to Design and Apply a PID Controller
The PID Controller

“Ideal” form of the PID Controller

\[
CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_i} \int e(t)dt + Kc \cdot \tau_D \frac{de(t)}{dt}
\]

where:
- \(CO\) = controller output signal
- \(CO_{bias}\) = controller bias or null value
- \(PV\) = measured process variable
- \(SP\) = set point
- \(e(t)\) = controller error = \(SP - PV\)
- \(Kc\) = controller gain (a tuning parameter)
- \(\tau_i\) = controller reset time (a tuning parameter)
- \(\tau_D\) = controller derivative time (a tuning parameter)
The PID Controller

- Ideal PID Controller

\[
CO = CO_{bias} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt + K_c \cdot \tau_D \frac{de(t)}{dt}
\]

- A derivative is a slope or rate of change

- \( \tau_D \) provides a separate weight to the derivative (or rate of change) of error, \( e(t) = SP - PV \), as it changes over time

- \( \tau_D \) has units of time so it is always positive

- Larger values of \( \tau_D \) increase influence of the derivative term
Function of the Derivative Term

• Proportional term considers *how far* PV is from SP at any instant in time and adds or subtracts from $C_{O_{bias}}$ accordingly (recall $e(t) = SP - PV$)

• Integral term addresses *how long* and how far PV has been from SP by continually summing $e(t)$ over time

• Derivative term considers *how fast* $e(t)$ is changing at any instant using the rate of change or slope of the error curve

  rapidly changing $e(t) = large derivative = large impact on CO$

• Derivative doesn’t consider if $e(t)$ is positive, negative or how much time has passed, just how fast $e(t)$ is changing
Derivative on Measurement

Consider that if the set point (SP) is constant, then:

\[
\frac{de(t)}{dt} = \frac{d(SP - PV)}{dt} = -\frac{dPV}{dt}
\]

That is, as long as SP is constant, then:

deriv on error = − deriv on measurement
Heat Exchanger under PID control shows CO kick with derivative on e(t)

Impact of CO kick on PV performance depends on sample time (T) relative to $\tau_p$ (fast/small sample time gives little chance for impact)

But potential for wear on mechanical FCE (e.g., valve) is always a concern
Understanding Derivative Action

- Assuming $K_c$ and $\tau_D$ are positive and appropriate size:
  - when $\frac{dPV}{dt}$ (the slope) is positive, the derivative contribution works to decrease CO from its current value
  - when $\frac{dPV}{dt}$ is negative, derivative contribution increases CO

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PID Controllers Work in Harmony

• **Proportional term** provides a rapid response to controller error

• **Integral term** eliminates offset but increases the oscillatory or rolling behavior of the PV

• **Derivative term** works to *decrease oscillations* in the PV because its largest influence is when PV is rapidly changing
PID Set Point Tracking

- PID shows decreased oscillations compared to PI performance

PID has somewhat:
- shorter rise time
- faster settling time
- smaller overshoot

Heat Exchanger – Aggressive Tuning
Disadvantages of Derivative

• Measurement Noise is a Problem:
  • Derivative action loses its benefits when there is random error (noise) in the measured PV – a common occurrence
  • The derivative action causes PV measurement noise to be amplified and reflected in the CO signal
  • This is because a noisy PV signal has changing derivatives as the slope switches direction at every sample
Noise Degrades Derivative Action

- Slope (derivative) switches direction every sample
- This produces alternating CO actions (called “chatter”) from the PID algorithm
- The CO chatter is amplified based on the size of $\tau_D$
Noise Degrades Derivative Action

- As noise level increases, its impact on CO chatter is apparent.
- If CO hits a constraint, lack of “symmetry in randomness” can impact PV.
Comparing Controller Performance

Heat Exchanger – Aggressive Tuning

- IMC tuned ideal algorithm: PI vs PID vs PID w/ CO Filter