Maintain Levels in a Network of Water Reservoirs

A network of tanks can be used as a simulator for management of water reservoir levels that provide water resources to distributed agricultural and residential communities. This network of tanks is a highly coupled system that leads to interacting controllers. In this case, the split valve fractions γ_1 and γ_2 are occasionally modified to rebalance the levels across all of the reservoirs. Your objective is to design a control system that will maintain levels in the two lower reservoirs (*height*$_1$ and *height*$_2$) when the split values change or when the lower level set points change.

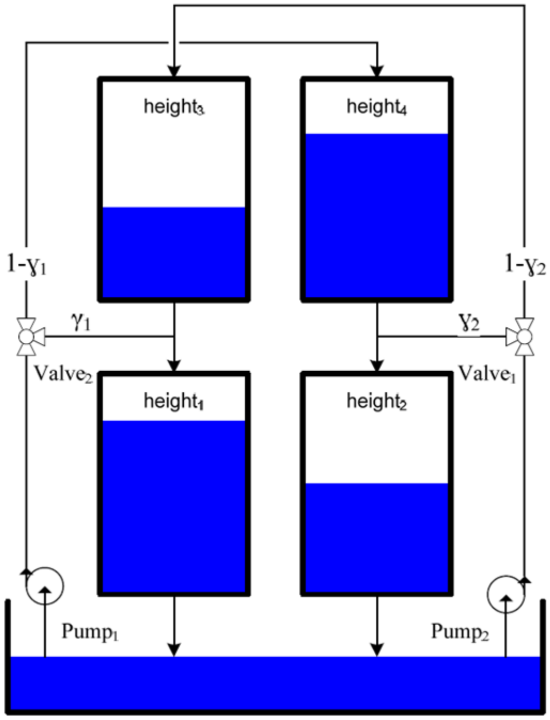


Figure 1. Network of 4 Gravity Drained Reservoirs

Some useful techniques for designing the control system are:

- Empirical model fitting of the Multiple Input Multiple Output (MIMO) System
- Relative Gain Array (RGA) analysis to determine best pairing of Manipulated and Controlled Variables
- Derivation of a first principles model

The appendices provide some assistance in designing and one of three control systems:

1. Non-interacting PID controllers
2. Interacting PID controllers (feedforward information from other controller)
3. Optional: Nonlinear Control (NLC)

Your task is to evaluate the three control systems to determine which will provide the best performance for disturbance rejection (e.g. changing the split fraction) and set point changes.

Appendix A – Below is additional information on the derivation of a first principles model for the 4 tank system.

- Four gravity drained tanks
- Disturbances
  - Split Fraction 1 (γ_1)
  - Split Fraction 2 (γ_2)
- Manipulated Variables
  - Flow rate from Pump 1 (q_a)
  - Flow rate from Pump 2 (q_b)
- Controlled Variables
  - Level in Tank 1 (h_1)
  - Level in Tank 2 (h_2)

First Principles Model Equations

$$\begin{aligned} q_{1,in} &= \gamma_1 q_a + q_{3,out} \\ q_{2,in} &= \gamma_2 q_b + q_{4,out} \\ q_{3,in} &= \left(1-\gamma_2\right) q_b \\ q_{4,in} &= \left(1-\gamma_1\right) q_a \end{aligned}$$

$$\begin{aligned} q_{1,out} &= c_1\sqrt{2gh_1} \\ q_{2,out} &= c_2\sqrt{2gh_2} \\ q_{3,out} &= c_3\sqrt{2gh_3} \\ q_{4,out} &= c_4\sqrt{2gh_4} \end{aligned}$$

$$\begin{aligned} A_1\frac{\partial h_1}{\partial t} &= q_{1,in} - q_{1_{out}} \\ A_2\frac{\partial h_2}{\partial t} &= q_{2,in} - q_{2_{out}} \\ A_3\frac{\partial h_3}{\partial t} &= q_{3,in} - q_{3_{out}} \\ A_4\frac{\partial h_4}{\partial t} &= q_{4,in} - q_{4_{out}} \end{aligned}$$

Appendix B: Calculate transfer functions for the distillation column model response and put model into the form

$$\begin{bmatrix} H_1(s) \\ H_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{K_{p,11}}{\tau_{p,11}s+1} & \dfrac{K_{p,12}}{\tau_{p,12}s+1} \\ \dfrac{K_{p,21}}{\tau_{p,21}s+1} & \dfrac{K_{p,22}}{\tau_{p,22}s+1} \end{bmatrix} \begin{bmatrix} Q_a \\ Q_b \end{bmatrix} =$$

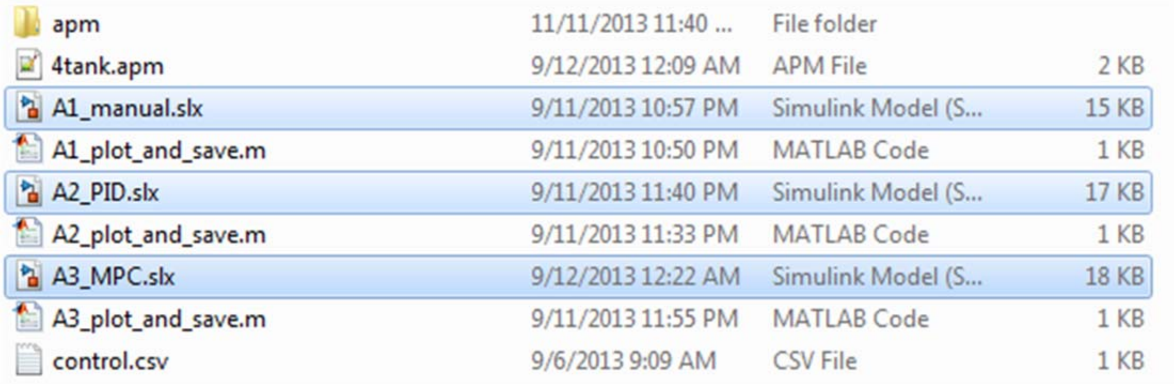
Appendix C - Using Relative Gain Array (RGA) analysis, suggest best pairing options for the MVs (Pump 1 / Pump 2) and CVs (Height 1 / Height 2). To complete this analysis, you will need to calculate the steady state gains at the nominal operating conditions. The RGA analysis can be completed by using the gains.

$$\lambda_{11} = \lambda_{22} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}$$

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} =$$

Appendix D - Simulate a PID controller for the 4 tank system using files provided with this assignment (open **A1_manual.slx** for step testing and **A2_PID.slx** as a starting template for PID control). Adjust the PID controller tuning parameters to achieve acceptable set point and disturbance tracking.



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A3_plot_and_save.m	9/11/2013 11:55 PM	MATLAB Code	1 KB
control.csv	9/6/2013 9:09 AM	CSV File	1 KB

(Optional) - Compare the performance of the PID controller with Nonlinear Control.

Run the optimizing Nonlinear Controller with **A3_MPC.slx** and compare to the performance with the PID controller with changes in γ_1 and γ_2 or other disturbances or set point changes.